Interaction-Assisted Transport and Mass Generation in Graphene

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- **1D defects in graphene: interaction-assisted transport**
- **mass generation in multilayer graphene**

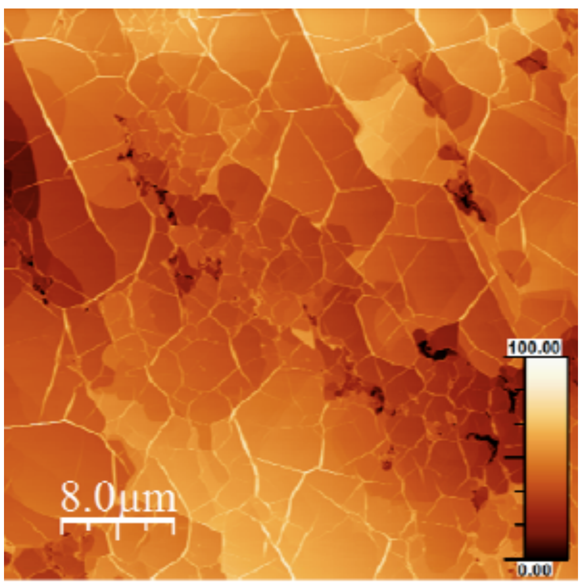
With: Lee Miller, Walt de Heer,
Phil First (Georgia Tech),
Joe Stroscio (NIST)

1D defects in graphene: many-body interactions

MK, arXiv:10032414

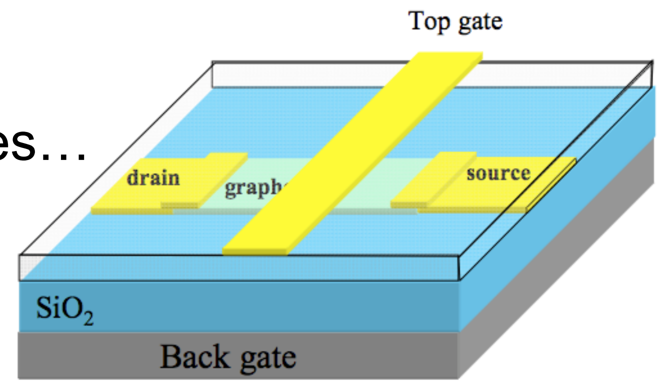
1D defects abound in graphene devices, e.g. …

… step edges

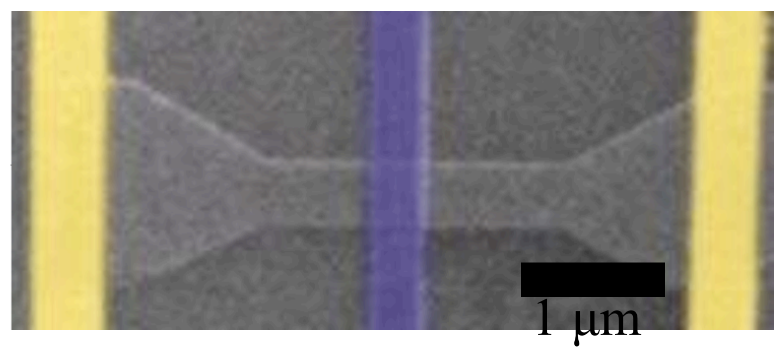


de Heer (2008)

… gates, sample edges…



Oezyilmaz, *et al.*, PRL (2007)



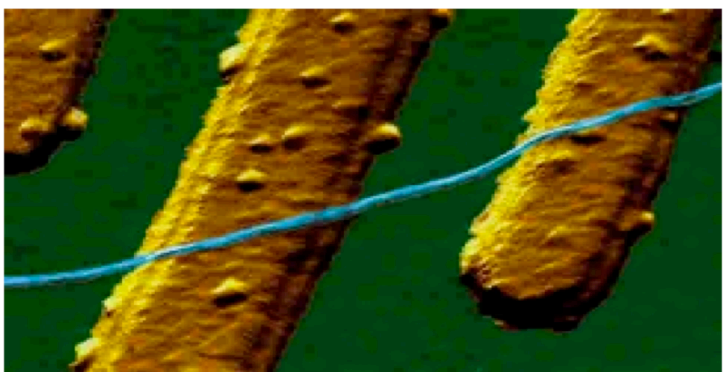
Many-body interactions strong in unscreened graphene; dramatic effects in 1D …

Interacting electrons in 1D

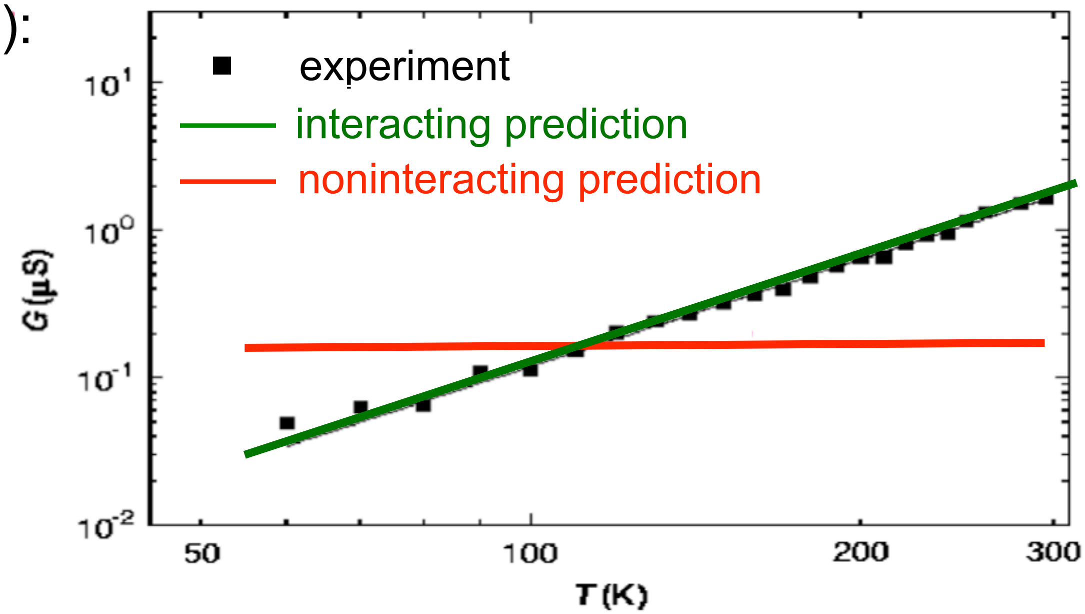
• *drastic effects of electron-electron interactions on scattering*:

$$G \sim T^{\alpha},\ G \sim V^{\alpha}$$

experiment (carbon nanotubes):



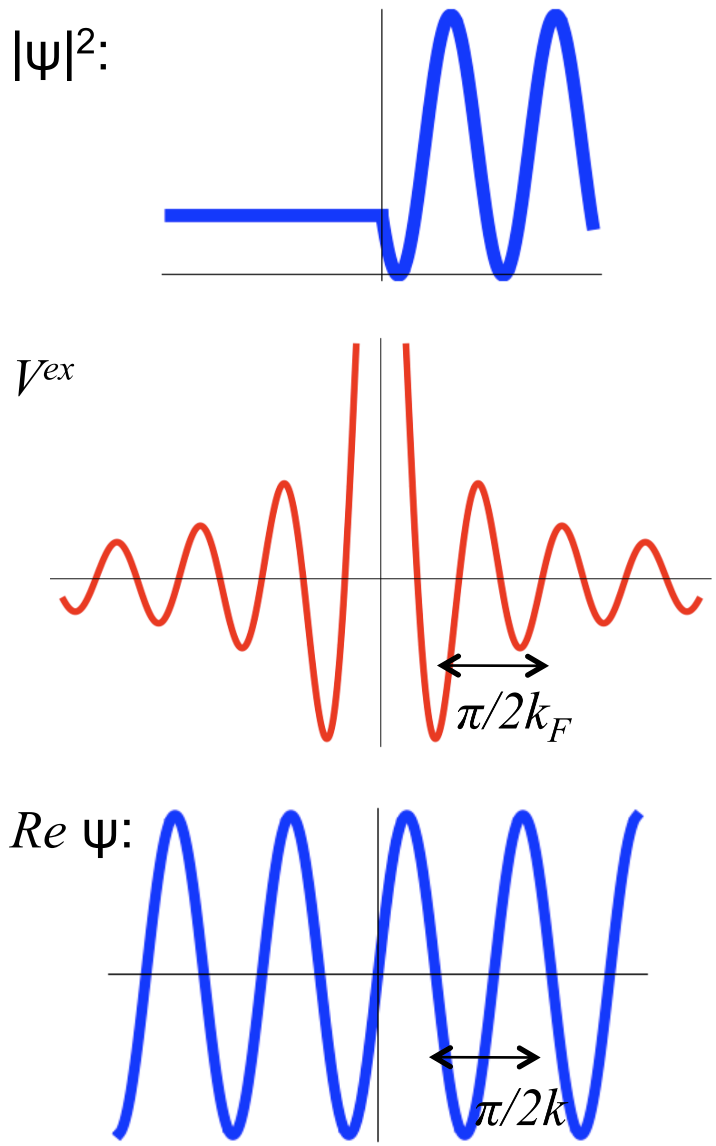
Yao *et al*., Nature ('99);
Bockrath *et al*., Nature ('99)



• *universal low-energy description* by the *Luttinger liquid* Haldane, JPC ('81)

Friedel oscillations

Friedel oscillations: interference of incoming & backscattered waves



➔ Hartree and exchange potentials

$$V^{\rm ex}(x,x') = -V(x-x') \sum_{\mu\in\{\rm L,R\}} \int_0^{k_{\rm F}} dk'\, \psi_{\mu k'}(x)\psi^*_{\mu k'}(x')$$

$$V(x-x') = u\delta(x-x')$$

$$V^{\rm ex}_{\rm osc}(x,x') = u\delta(x-x')\frac{|r|}{2\pi|x|}\sin(2k_{\rm F}|x| - \arg r)$$

➔ extra scattering

Matveev, Yue, Glazman, PRL (1993)

Lowest order Born approximation at $k \approx k_{\rm F}$:

$$\delta r^{\rm ex} \propto \int dx dx'\, \psi^*_{{\rm L}k}(x) V_{\rm ex}(x,x')\psi_{{\rm R}k}(x')$$

➔ Log. divergent scattering at $k \rightarrow k_{\rm F}$

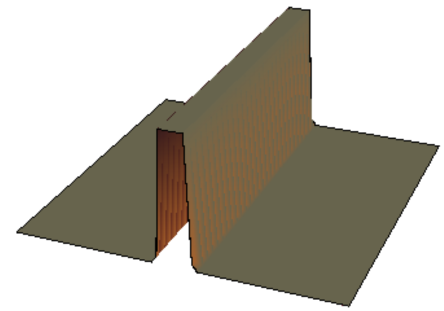
➔ Current blocked at T=0 (Luttinger liquid)

Scattering from 1D defects in 2D conductors

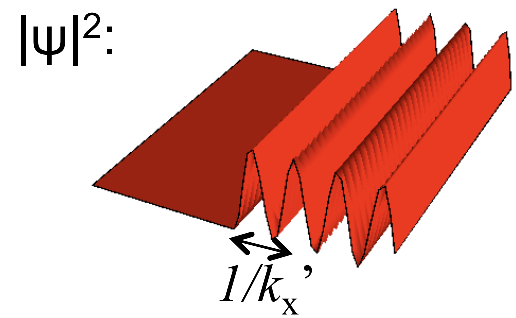
1D scatterer in a 2D electron gas:

$$V^{\text{ex}}(\mathbf{r},\mathbf{r}') = -V_{\text{C}}(\mathbf{r}-\mathbf{r}') \sum_{\mu\in\{\text{L,R}\}} \int \frac{d^2\mathbf{k}'}{(2\pi)^2} \psi_{\mu\mathbf{k}'}(\mathbf{r})\psi^*_{\mu\mathbf{k}'}(\mathbf{r}')$$

$$\delta r^{\text{ex}} \propto \int d\mathbf{r}d\mathbf{r}'\, \psi^*_{\text{L}k_{\text{F}}}(\mathbf{r})V_{\text{ex}}(\mathbf{r},\mathbf{r}')\psi_{\text{R}k_{\text{F}}}(\mathbf{r}')$$



Scattering state producing a Friedel oscillation:

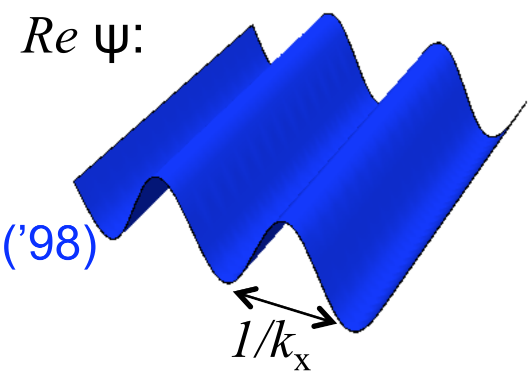


Wave at at $k=k_F$:

➔ In 2D: $k_x \neq k_x$' even at $k=k'=k_F$ in generic directions

➔ oscillations suppress δr^{ex}

Shekhtman, Glazman, PRB ('95); Alekseev, Cheianov, PRB ('98)

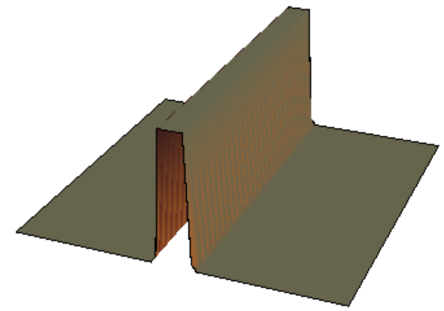


Similarly: point defects

Stauber, Guinea, Vozmediano, PRB ('05); Foster, Aleiner, PRB ('08)

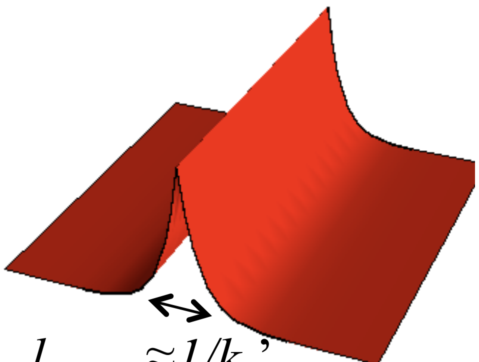
Scattering from 1D defects in intrinsic graphene

Not at $k_F=0$ ➔ the Dirac point of graphene:



$$V^{\rm ex}(\mathbf{r},\mathbf{r}') = -V_{\rm C}(\mathbf{r}-\mathbf{r}') \sum_{\mu\in\{\rm L,R\}} \int \frac{d^2\mathbf{k}'}{(2\pi)^2}\, \psi_{\mu\mathbf{k}'}(\mathbf{r})\psi^*_{\mu\mathbf{k}'}(\mathbf{r}')$$

$$\delta r^{\rm ex} \propto \int d^2\mathbf{r} d^2\mathbf{r}'\, V^{\rm ex}(\mathbf{r}-\mathbf{r}')$$



$$\delta r^{\rm ex} \propto \int d^2k' \quad l_{\rm Friedel} \times V_{\rm C}(k') \propto r_{\rm s} \int d^2k' \frac{1}{k_x' k'}$$

$$\delta r_0^{\rm ex} \propto \int_0^\infty d\left(\frac{x+x'}{2}\right) \int d(x-x')d(y-y') \frac{dk_x'}{2\pi}\frac{dk_y'}{2\pi} V_{\rm C}(x-x',y-y') r_{\mathbf{k}'}\, e^{ik_y'(y-y')+ik_x'(x+x')}$$

$$\delta r_0^{\rm ex} \propto \int_0^\infty d\left(\frac{x+x'}{2}\right) \frac{dk_x'}{2\pi}\frac{dk_y'}{2\pi} r_{\mathbf{k}'} \frac{2\pi r_{\rm s} v}{|k_y'|} e^{ik_x'(x+x')} \sim r_{\rm s} \int \frac{d^2k'}{k_x' k_y'} r_{\mathbf{k}'}$$

If $r_{\mathbf{k}'}$ is k'-independent: $\delta r_0^{\rm ex} \propto r_{\rm s} r \int \frac{dk'}{k'}$

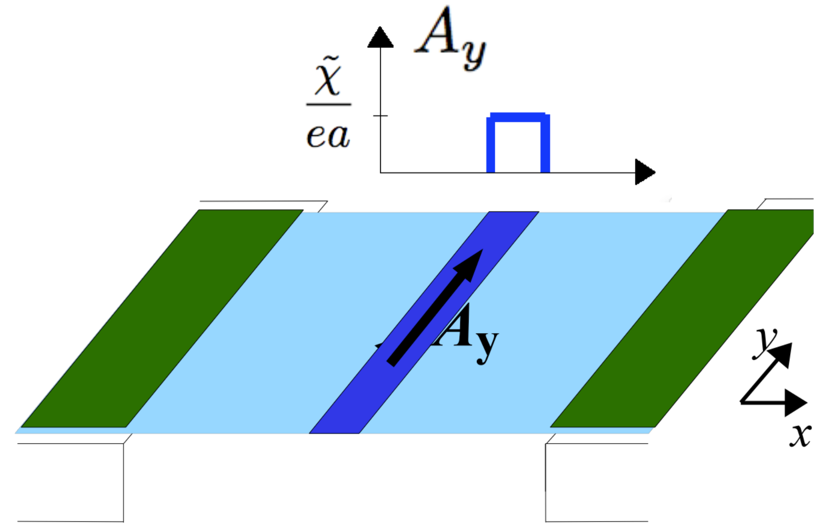
➔ divergent interaction effects at low T if Hartree potentials are absent

Model

Dirac Hamiltonian with **1D vector potential** $\mathbf{A}(x,y) = (0, A_y)$

$$A_y = \frac{\tilde{\chi}}{a}\left[\Theta\left(x+\frac{a}{2}\right) - \Theta\left(x-\frac{a}{2}\right)\right]$$

$$H = v\,\vec{\sigma}\cdot\left(\vec{p} - e\vec{A}\right)$$



H purely pseudospin-off-diagonal

➔ particle-hole symmetry

$$\sigma_z H \sigma_z = -H$$

➔ **no Hatree** potential $\propto \sigma_0$

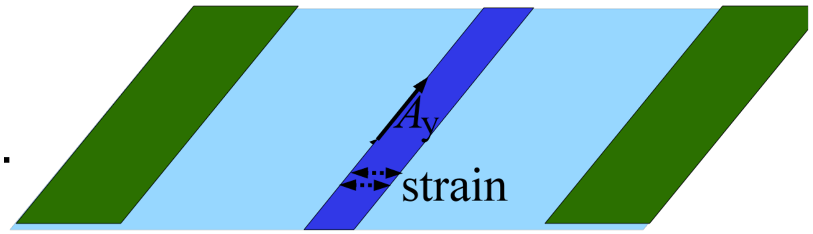
➔ expect logarithmically divergent interaction corrections

Implementation (1): Strain

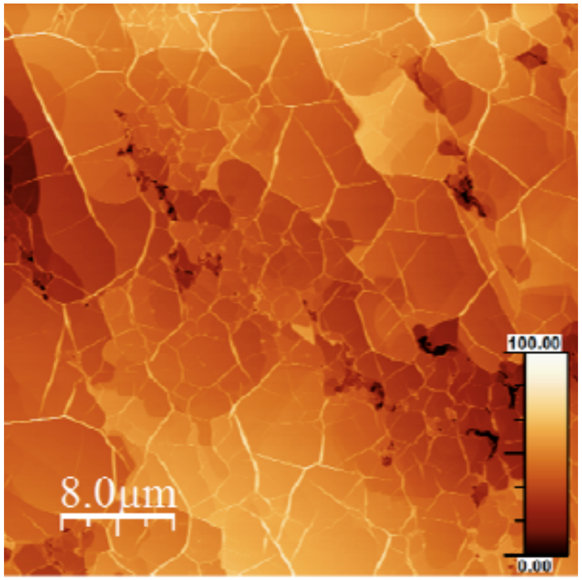
Strain u ➔ vector potential $\vec{A} = -\frac{\beta}{a}\begin{pmatrix} 2u_{xy} \\ u_{xx} - u_{yy} \end{pmatrix}$

Fogler, Guinea, Katsnelson, PRL (2008); Guinea, Katsnelson, Geim, Nat. Phys (2010)

➔ 1D vector potentials in strips under strain:

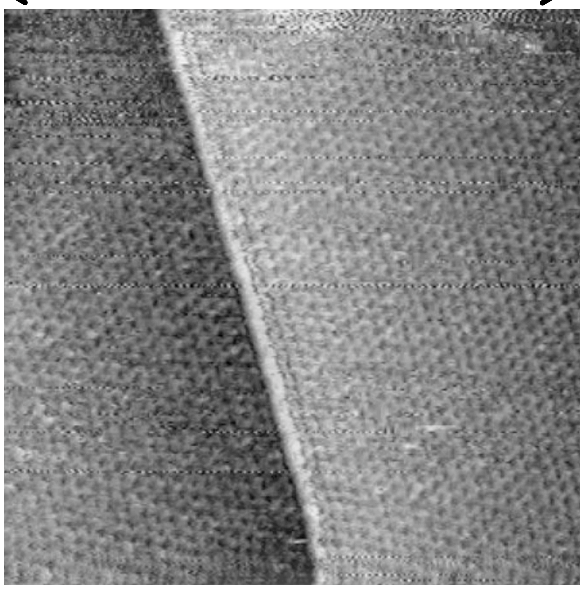


Strain appears at steps in the substrate, …

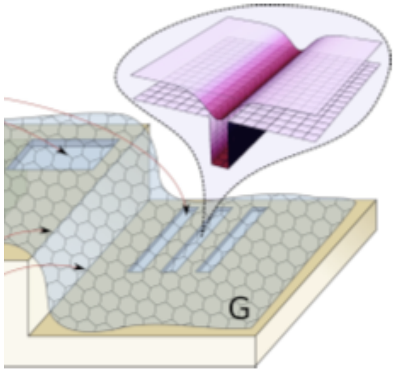


de Heer (2008)

60 nm



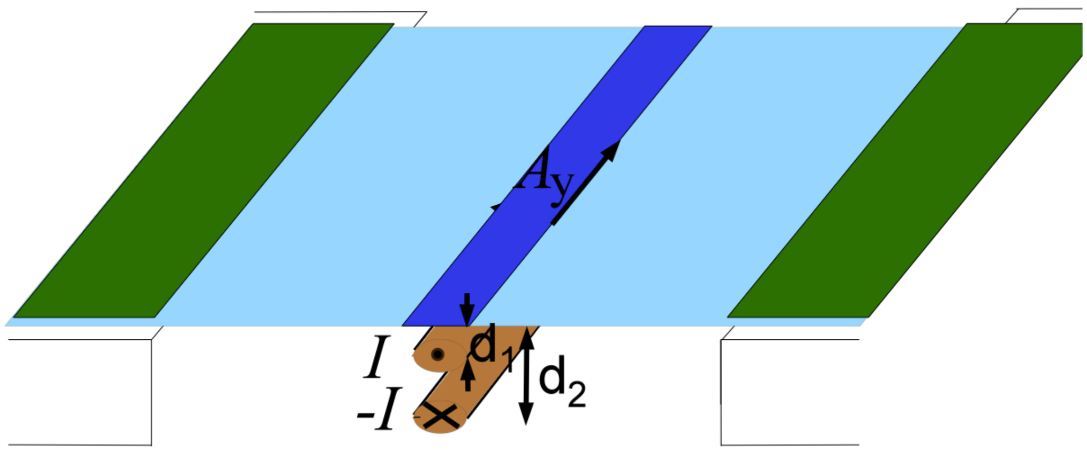
… and can be engineered:



Pereira, Castro Neto, PRL (2009)

Implementation (2): electrical currents

Two wires, carrying anti-parallel currents produce 1D vector potentials:

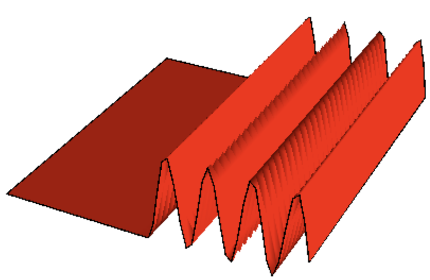


$$A_y = \frac{\mu_0 I}{2\pi} \ln \frac{\sqrt{x^2 + d_1^2}}{\sqrt{x^2 + d_2^2}} \underset{x \to \infty}{\sim} \boxed{\frac{\mu_0 I (d_1^2 - d_2^2)}{2\pi x^2}}$$

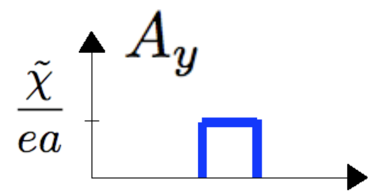
Single-Particle Physics

Fogler, Guinea, Katsnelson, PRL (2008)

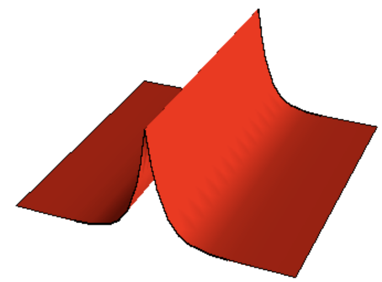
A_y induces scattering states, … … and bound states:



$$\varepsilon^{\mathrm{s}}_{s\mathbf{k}} = sv|\mathbf{k}|$$



$$\tilde{\chi} = \int dx\, A_y(x)$$



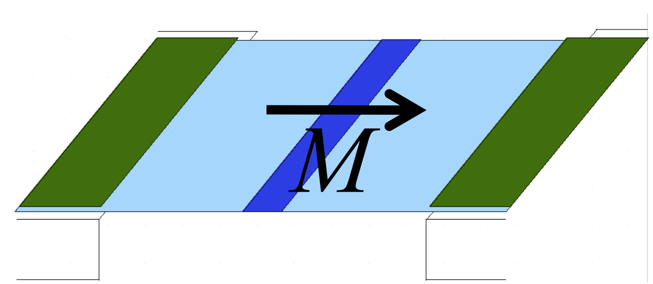
$$\varepsilon^{\mathrm{b}}_{sk_y} = -svk_y \operatorname{sech} \tilde{\chi}$$

Characterize low energy scattering by the transfer matrix:

$$M = T(\infty, -\infty)|_{\varepsilon=0}$$

Find:

$$M = e^{\tilde{\chi}\sigma_z}$$



Conductance: $G = G(\tilde{\chi})$

Electron-electron interactions

Unscreened Coulomb interaction (insulating substrate or suspended sample):

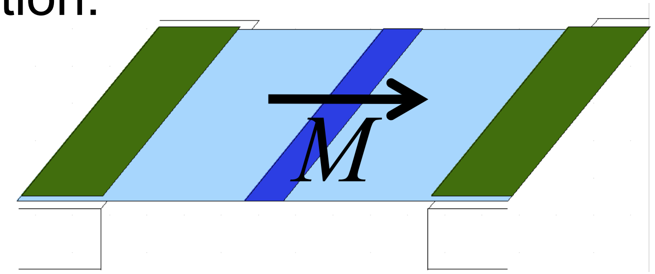
$$H_{\rm int} = \int d^2x\, d^2y\, \psi^\dagger(\vec{x})\psi(\vec{x}) \frac{e^2}{\kappa|\vec{x}-\vec{y}|}\psi^\dagger(\vec{y})\psi(\vec{y})$$

Interaction parameter $r_{\rm s} = \dfrac{e^2}{\kappa v}$

At $r_s \ll 1$ many-body scattering (inelastic processes) is suppressed at low T (inelastic: $\mathcal{O}(r_{\rm s})$; elastic: $\mathcal{O}[r_{\rm s}\ln(v/akT)]$)

Single-particle, low-energy scattering still described by *M;*
By parity, particle-hole symmetry, current conservation:

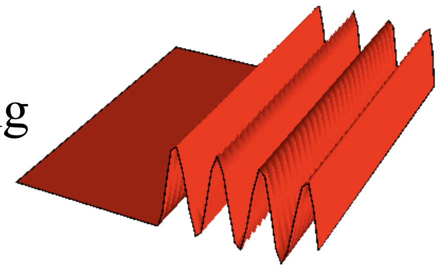
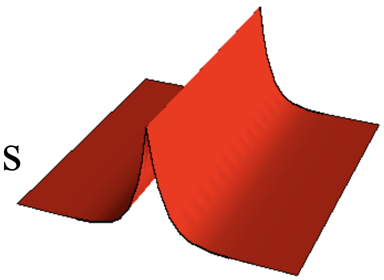
$$M = e^{\chi\sigma_z}$$



➔ Characterize interaction effects at $r_s \ll 1$ by renormalizing χ

First order in r_s

Compute $V^{\text{ex}}(\mathbf{r},\mathbf{r}') = -V_{\text{C}}(\mathbf{r}-\mathbf{r}') \sum_{\mu\in\{\text{L,R}\}} \int \frac{d^2\mathbf{k}'}{(2\pi)^2} \psi_{\mu\mathbf{k}'}(\mathbf{r})\psi^*_{\mu\mathbf{k}'}(\mathbf{r}')$

from the scattering  and bound states 

and obtain $\delta r^{\text{ex}} \propto \int d^2\mathbf{r} d^2\mathbf{r}'\, V^{\text{ex}}(\mathbf{r}-\mathbf{r}')$ in Born approximation.

Find: i) x=x': $V^{\text{ex}}_{\text{bound}} \propto \frac{\ln(x\tanh\chi/a)-1}{x^2}$

➔ no low-energy divergence due to the local part of V^{ex}

Luttinger liquid (LL): $V^{\text{ex}}_{\text{osc}}(x,x') = u\delta(x-x')\frac{|r|}{2\pi|x|}\sin(2k_{\text{F}}|x| - \arg r)$

But: the non-locality of V^{ex} produces the same divergence as in LL:

ii) e.g. $x>0$, $x'<0$: $V^{\text{ex}}_{\text{bound}}(x,x') \propto \frac{1}{(x-x')^2}$

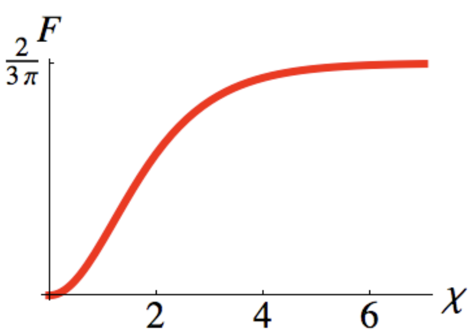
Interaction correction

Diagrammatically: extract $\chi - \tilde{\chi}$ from

,

(symmetry)

Find: $$\chi = \tilde{\chi} - \tilde{r}_s \sinh \tilde{\chi} F(\tilde{\chi}) \, [l - \tilde{\chi}] + \mathcal{O}(\tilde{r}_s^2)$$ $$l = \ln \frac{v}{akT}$$



$$F(\chi) = \frac{\coth \chi (\cosh 3\chi - 3 \cosh \chi) \arcsin(\tanh \chi) + 2 \cosh 2\chi}{2\pi} - \cosh \chi \sinh^2 \chi.$$

Discussion

$$\chi = \tilde{\chi} - \tilde{r}_s \sinh \tilde{\chi} F(\tilde{\chi}) \, [l - \tilde{\chi}] + \mathcal{O}(\tilde{r}_s^2)$$

Note:

- **logarithmic divergence at low *T*,** $l = \ln \frac{v}{akT}$
- **minus sign**
  ➔ interactions suppress scattering (similar to the Kondo effect in 1D)

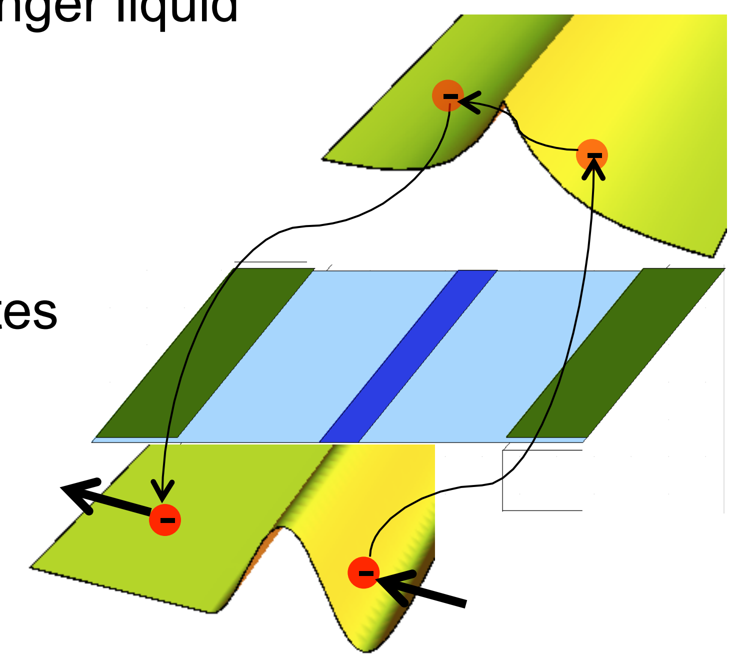
Luttinger liquid

- **exponential enhancement at** $\chi \gg 1$

Origin: exchange with electrons in bound states

➔ increase transmission amplitude by

$$\delta t \propto r_s l \gg t \propto e^{-\tilde{\chi}}$$



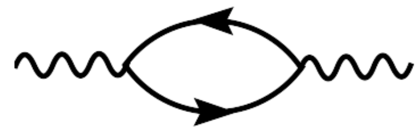
1-loop RG

Find: - IR-divergent interaction correction to χ

- IR-divergent correction to velocity ➔ correction to $r_s = \dfrac{e^2}{\kappa v}$

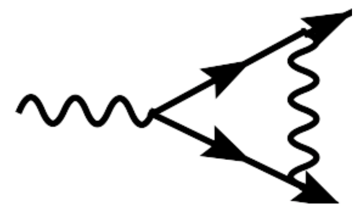
  Gonzalez, Guinea, Vozmediano, PRB ('99)

- no IR-divergence of the polarization

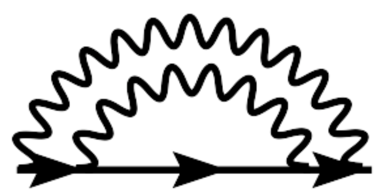


- no IR-divergence of the first vertex correction

  Kotov, Uchoa, Castro Neto, PRB ('08)



➔ at $\tilde{r}_s \sinh\tilde{\chi} F(\tilde{\chi})/\tilde{\chi} \ll 1$, but $\tilde{r}_s \sinh\tilde{\chi} F(\tilde{\chi}) l/\tilde{\chi} \gtrsim 1$ the corrections

 +  + …

are dominant.

Sum them up by the RG eqs.

$$\frac{d\chi}{dl} = -r_s \sinh\chi F(\chi), \qquad \frac{dr_s}{dl} = -\frac{r_s^2}{4}$$

Results (1)

i) $\chi \ll 1$: $\frac{d\chi}{dy} = -4\left(\frac{10}{3\pi} - 1\right)\chi^3$, $y = \ln(1 + \tilde{r}_s l/4)$

➔ $$\chi = \frac{\tilde{\chi}}{\sqrt{1 + 8\tilde{\chi}^2(10/3\pi - 1)y}}$$

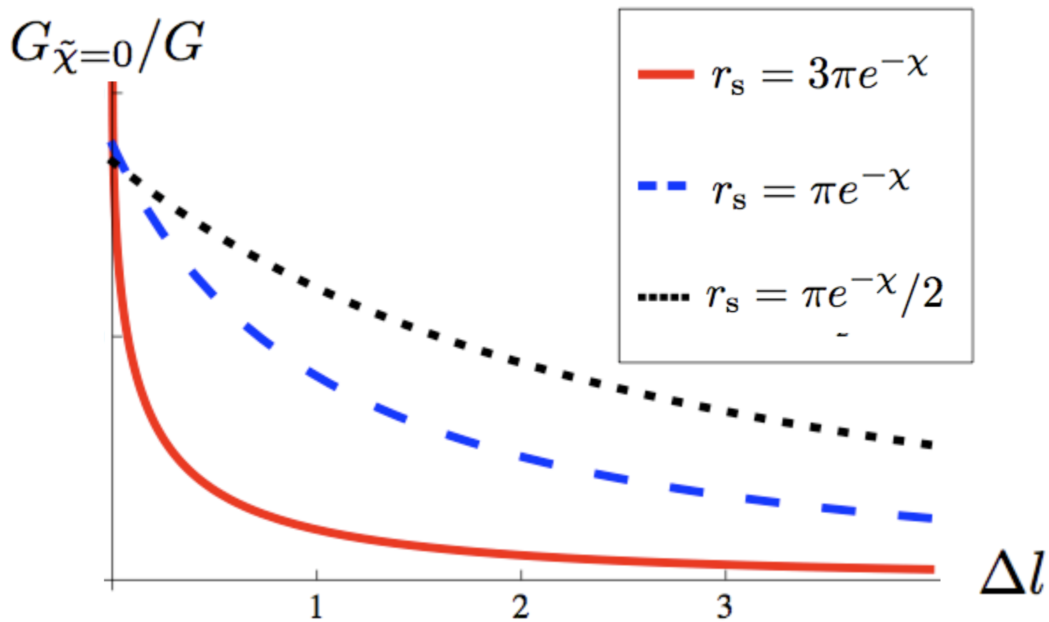
- **no scattering at *T*=0** (w/o bulk instabilities) ⚡ Luttinger liquid (LL)
- χ *marginally* irrelevant $(d\chi/dl \propto \chi^3)$ ⚡ scattering v in LL $(dv/dl \propto v)$

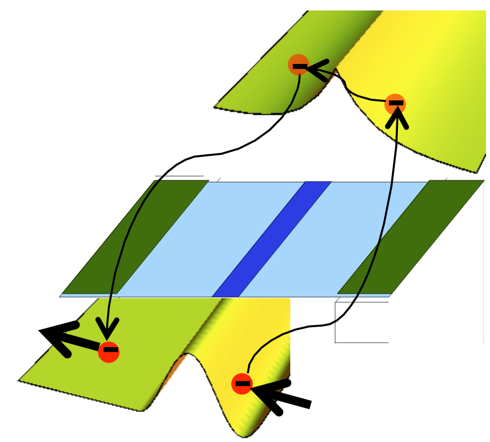
➔ Much slower scaling than in the LL

Results (2)

ii) $\chi \gg 1$: exponential renormalization by bound states $\frac{d\chi}{dy} = -\frac{4}{3\pi} e^{\chi}$,
cut-off at $\kappa^{\rm b} = e^{\chi|_{k'=\kappa^{\rm b}}} kT/v$

➔ **Strong signatures:**



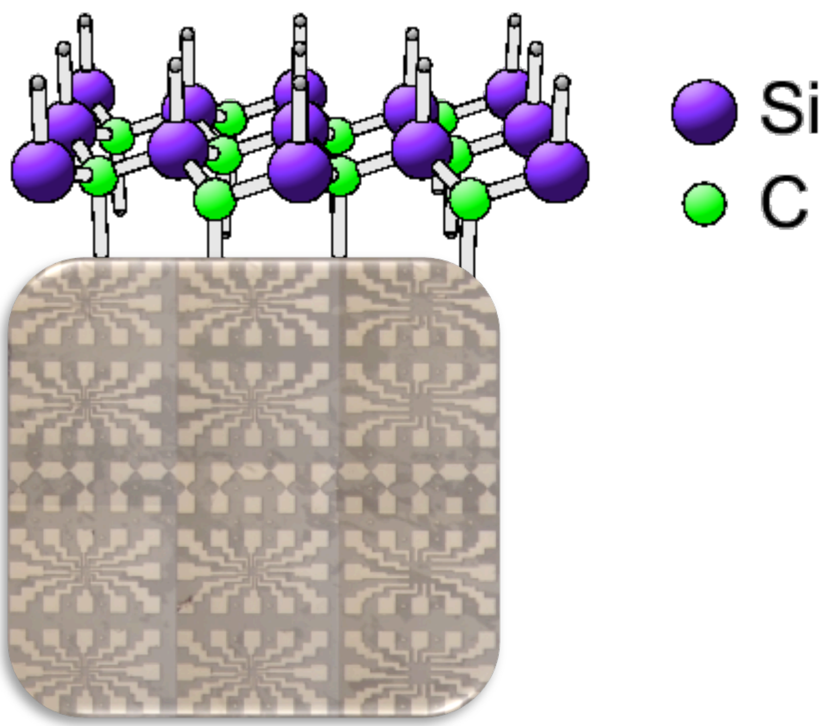


➔ **"unitary transport"** below temperature $kT^* = \frac{v}{a} e^{-3\pi/\tilde{r}_{\rm s}}$

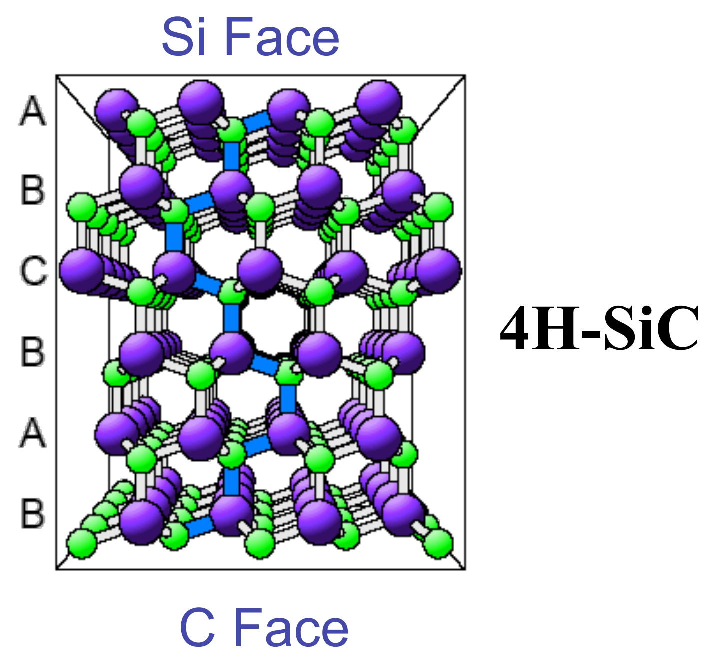
Epitaxial Graphene on SiC: Mass Generation

Miller, Kubista, Rutter, Ruan, de Heer, MK,
First, Stroscio, Nature Physics (2010)

Thermal desorption of Si at high temperatures to form graphene:

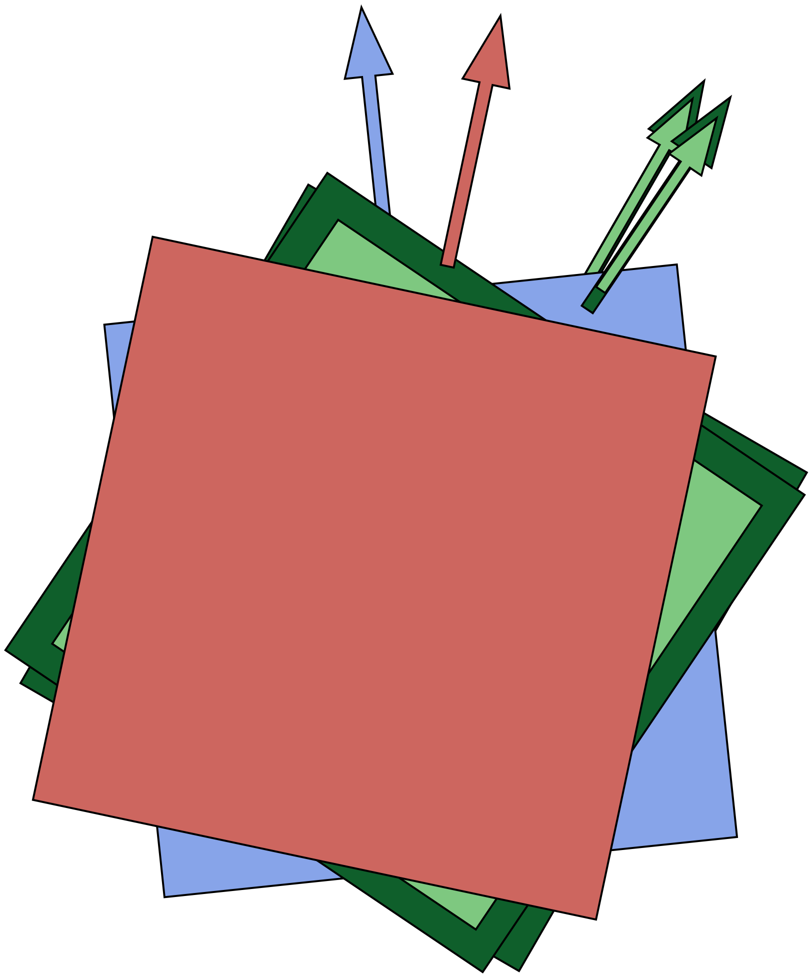


Courtesy of Walt de Heer, GT



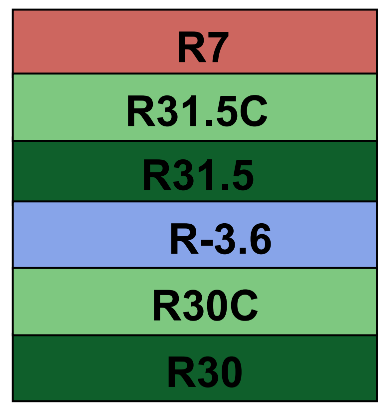
Berger *et al*., *J. Phys Chem B* (2004), *Science* (2006), First *et al*., MRS Bulletin (2010)

Multilayer Graphene on C-face SiC



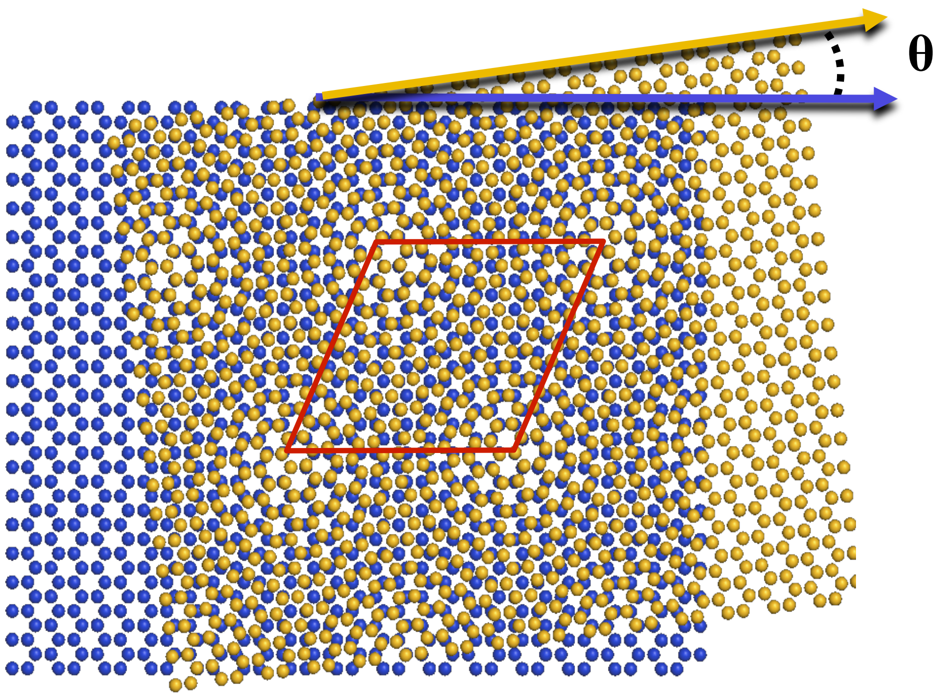
- Layer stacking

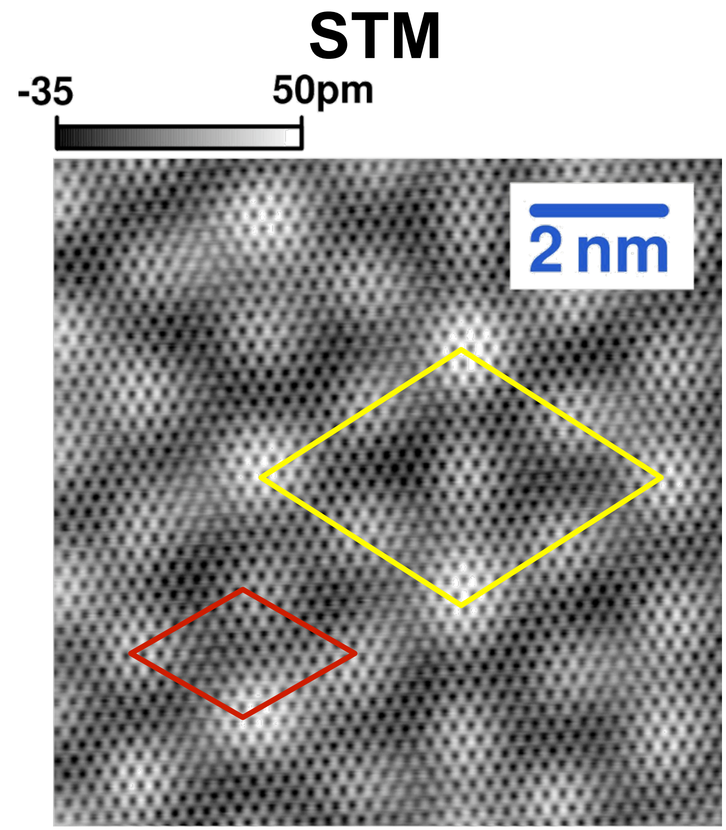
Alternating between:
NEAR 30˚ & NEAR 0˚



Hass et al., PRL ('08)

Multilayer Graphene on C-face SiC

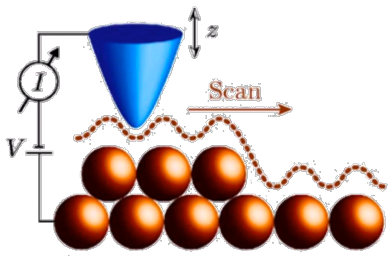




Electronic "decoupling"

Sadowski et al., PRL, **97,** 266405 (2006)

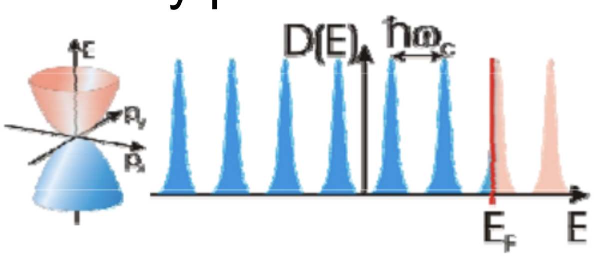
STS in a B-field



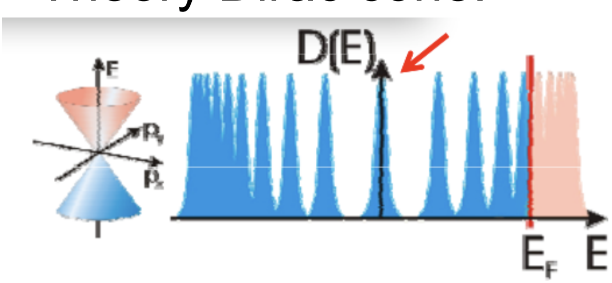
Experiment epitaxial graphene:

Miller *et al.*, Science (2009)

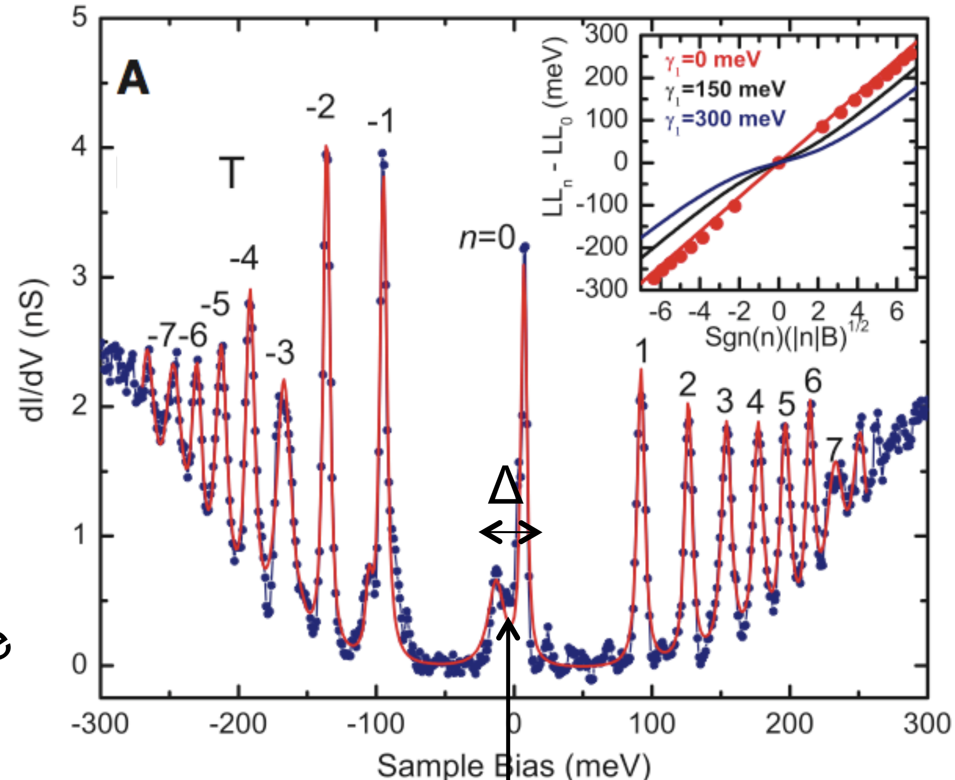
Theory parabolic band:



Theory Dirac cone:



evidence

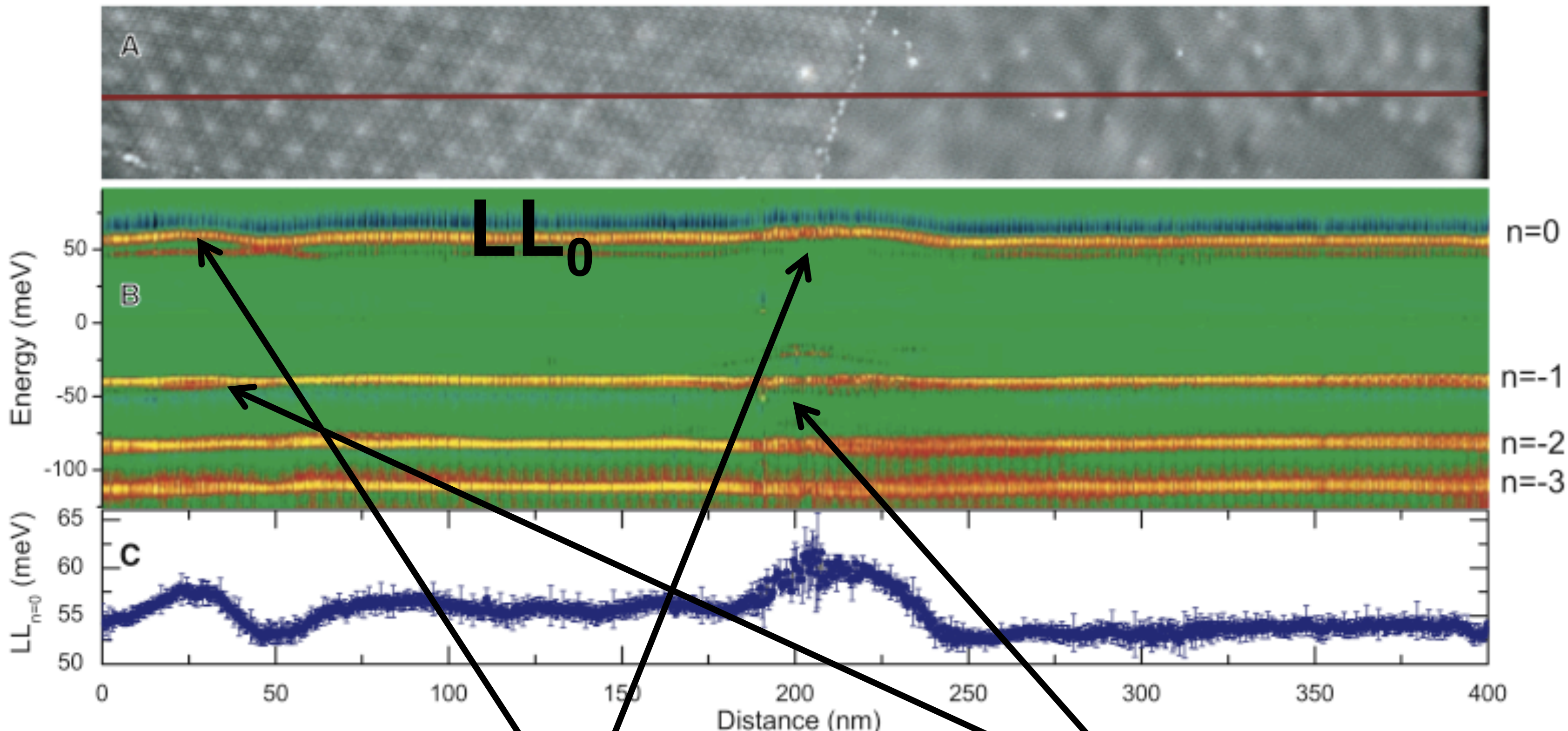


Observe: splitting $\Delta \approx 10$ meV of LL_0

➔ Electron-electron interactions??

Spatially resolved STS

Line scan of STS spectra in B=5T; Miller *et al.*, Science (2009):



Find: spatially inhomogeneous splitting Δ of LL_0

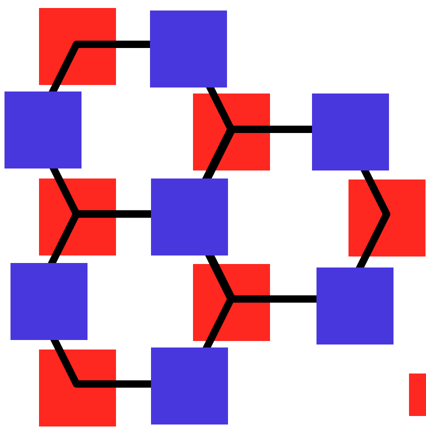
weak space-dependence of higher LL.

Conjecture: spatially inhomogeneous mass term?

Mass in the Dirac Equation

Dirac equation with mass m:

$$H_\gamma = v\boldsymbol{\sigma}_\gamma \cdot (\boldsymbol{p} - e\boldsymbol{A}) + m\sigma_z$$



recall: $\psi = \begin{pmatrix} \Phi_\mathrm{A} \\ \Phi_\mathrm{B} \end{pmatrix}$ $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

➔ m: potential with opposite sign on the sublattices ("staggered potential")

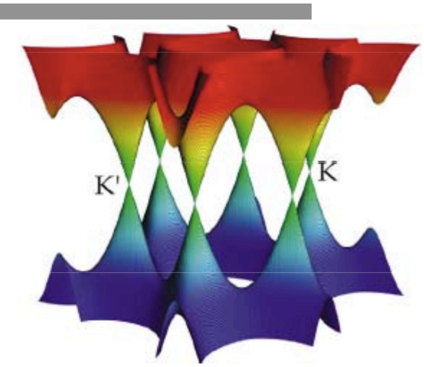
■ (A-sublattice): V=m

■ (B-sublattice): V=-m

LL spectrum: $$\varepsilon_\mathrm{n} = \pm\sqrt{n\,(\hbar\omega_c)^2 + m^2}$$

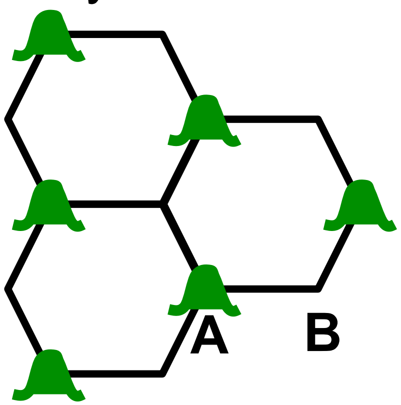
➔ consistent with experiment for a space-dependent $m \ll \hbar\omega_c$

LL_0 in single layer graphene



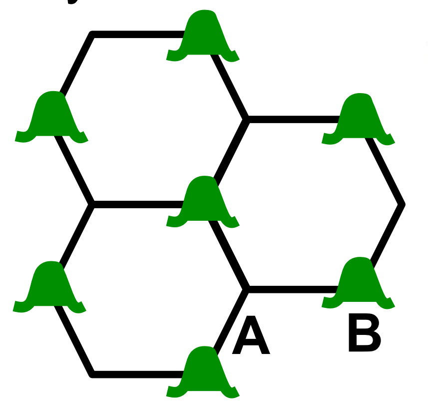
LL_0 wavefunction: sublattice-polarized

valley K':



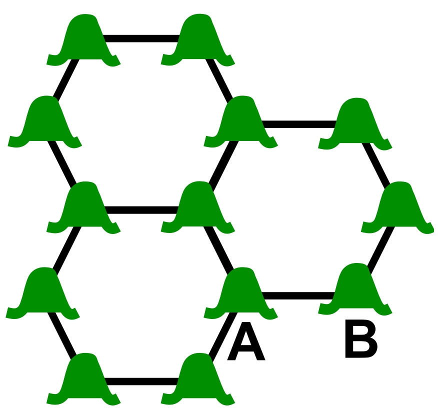
$$\psi' = \begin{pmatrix} \Phi_0 \\ 0 \end{pmatrix}$$

valley K:

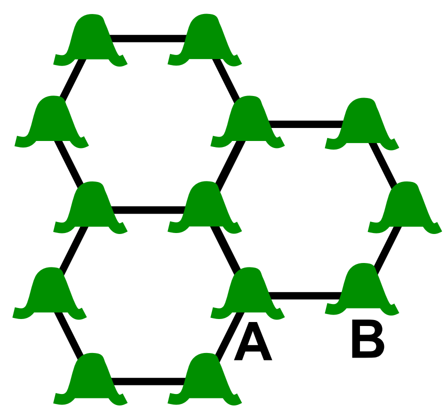


$$\psi = \begin{pmatrix} 0 \\ \Phi_0 \end{pmatrix}$$

LL_n ($n>0$): unpolarized



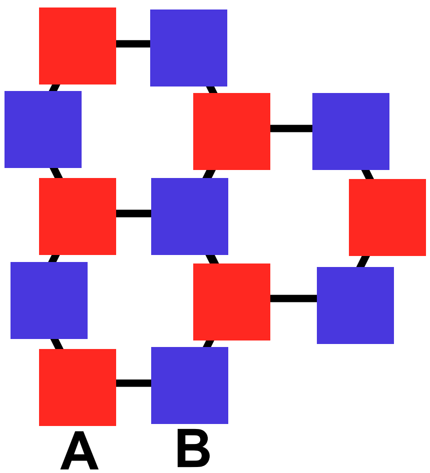
$$\psi' = \frac{1}{\sqrt{2}} \begin{pmatrix} \Phi_n \\ \Phi_{n-1} \end{pmatrix}$$



$$\psi = \frac{1}{\sqrt{2}} \begin{pmatrix} \Phi_{n-1} \\ \Phi_n \end{pmatrix}$$

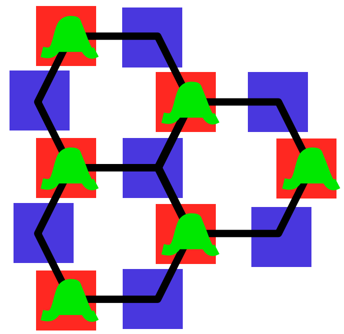
LL_0–splitting

"Staggered potential" m (with sublattice-dependent sign):

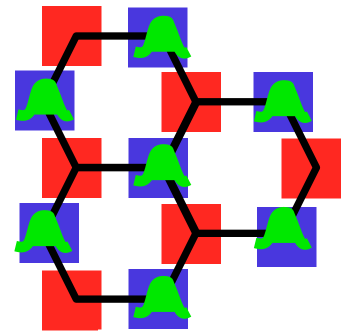


: $V=m$

: $V=-m$

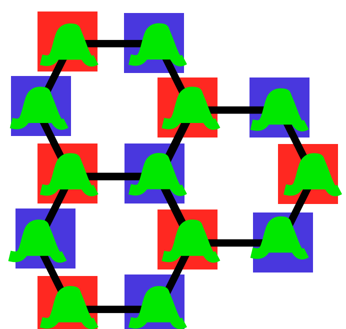


$$\Delta\varepsilon'^{(1)}_0 = m$$

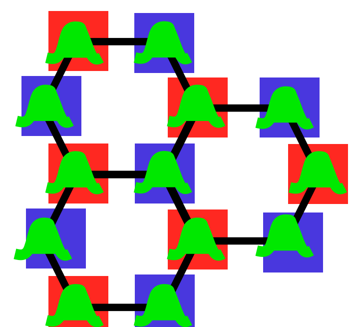


$$\Delta\varepsilon^{(1)}_0 = -m$$

➔ splitting of LL_0 by $\Delta=2m$



$$\Delta\varepsilon'^{(1)}_n = 0$$



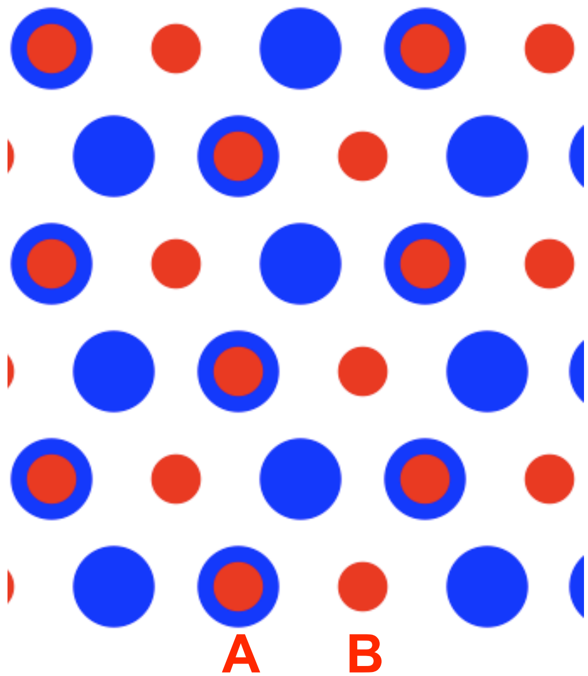
$$\Delta\varepsilon^{(1)}_n = 0$$

➔ weak perturbation on LL_n (n>0)

Interlayer interaction

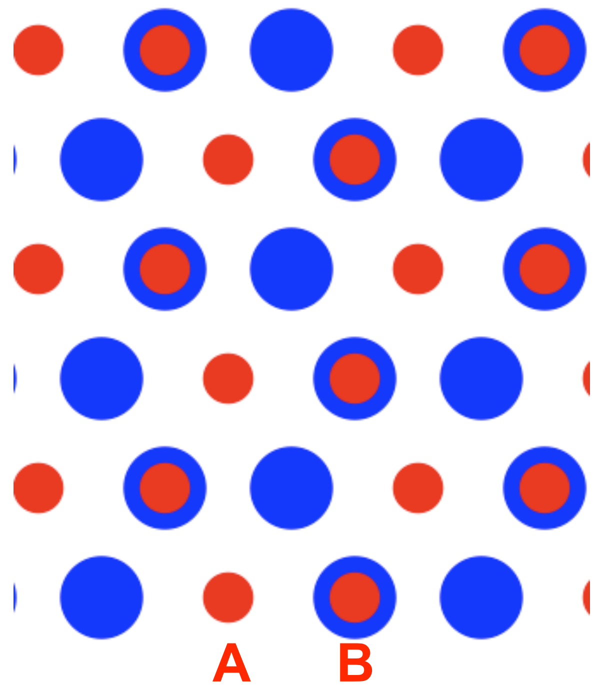
For short range interaction between top (red) and bottom (blue) layer:

AB-stacking



$V_A > V_B$

BA-stacking



$V_A < V_B$

➔ staggered potential

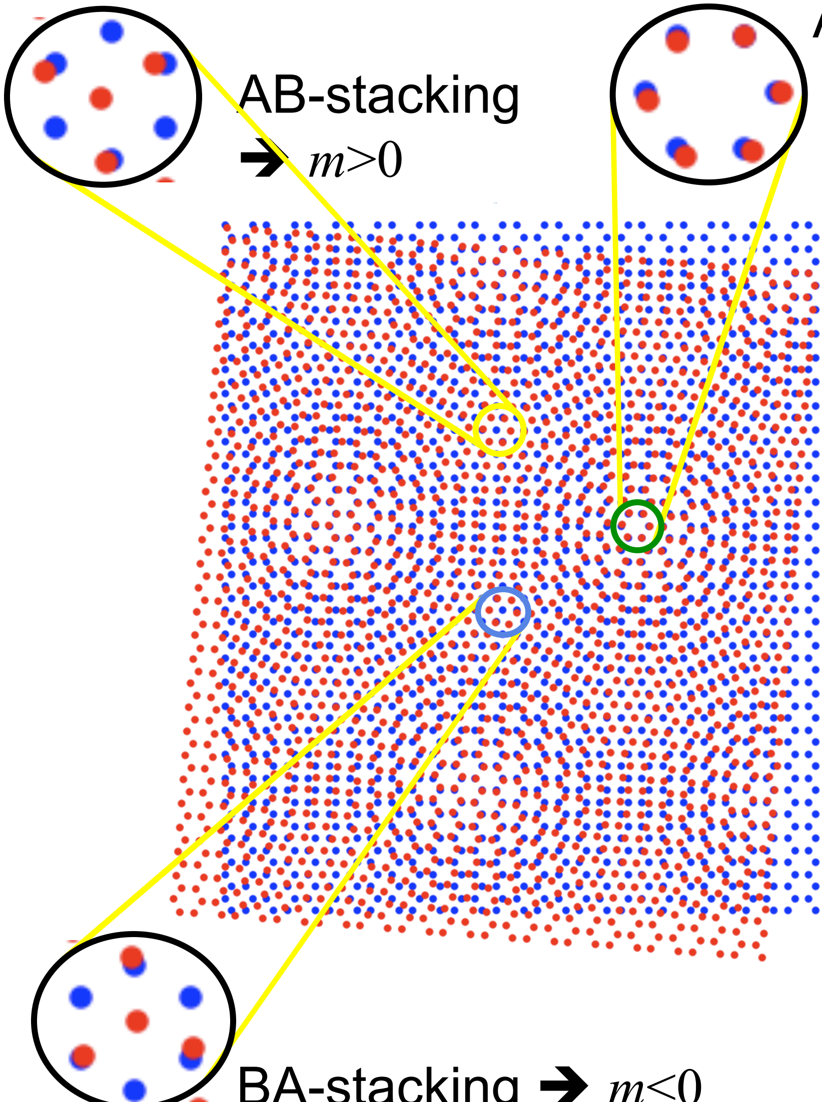
➔ mass in the top layer: $m > 0$

$m < 0$

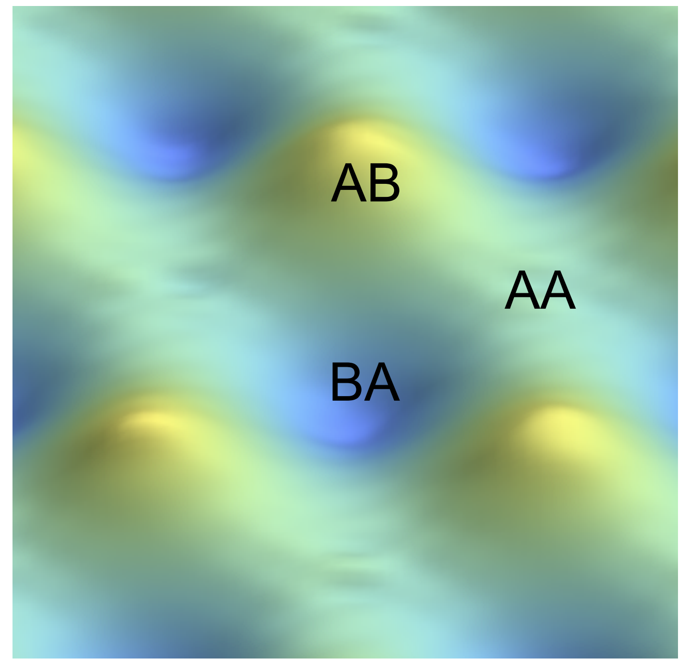
➔ stacking order-dependent mass

Local sublattice symmetry breaking

Spatially varying stacking order:



➔ **space-dependent mass *m***



: $m<0$: $m>0$

commensurate rotations:
m has trigonal superlattice

Experiment & Phenomenological Theory (I)

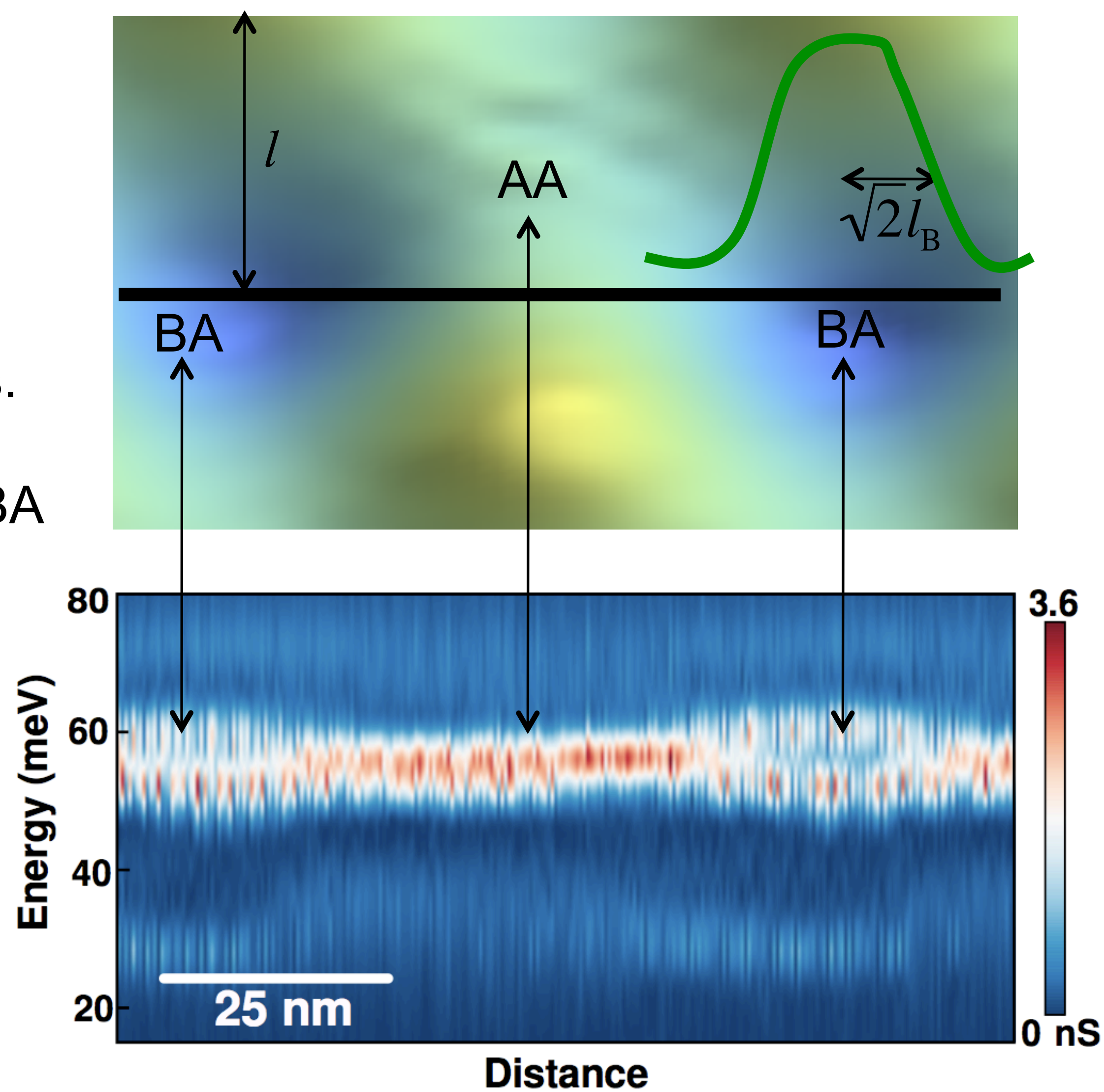
Miller, Kubista, Rutter, Ruan, de Heer, MK, First, Stroscio, Nature Physics (2010)

Postulate an m oscillating on the scale $l \approx 30\,\mathrm{nm}$

LL wavefunctions have spatial extent $\approx l_\mathrm{B} = 26\,\mathrm{nm}/\sqrt{B/\mathrm{T}}$

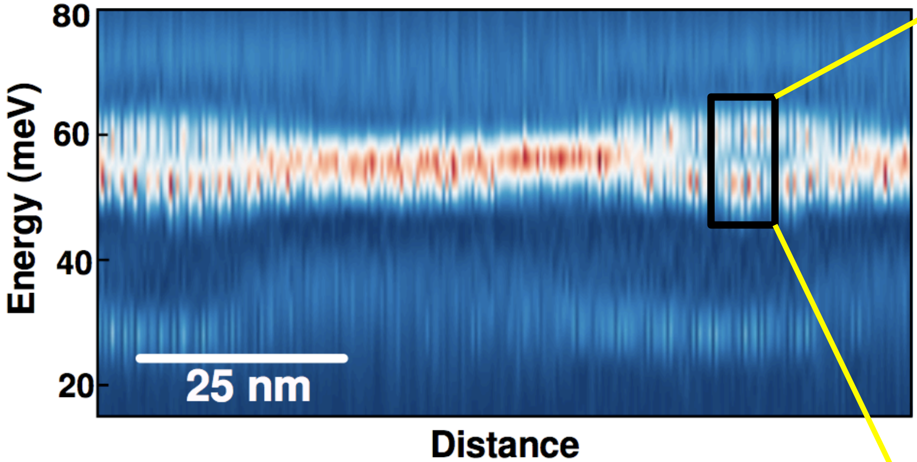
Have $l_\mathrm{B} \lesssim l$ ➔ the wavefcts. are confined to AB/BA regions
➔ expect splitting of LL_0 at AB/BA

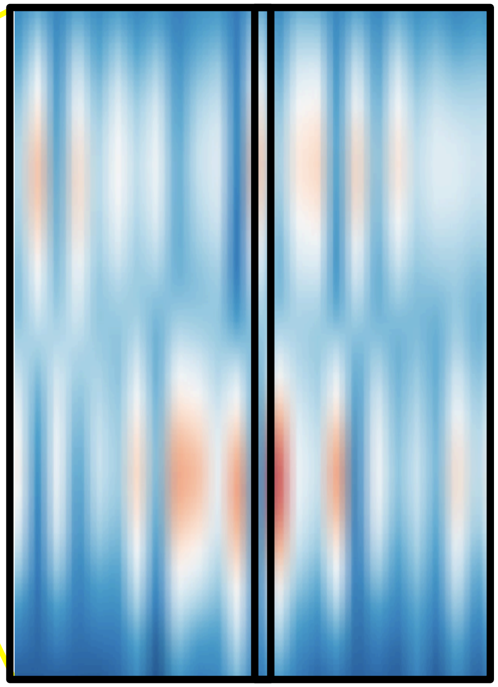
Compare to STS line scan ($8\mathrm{T}$):

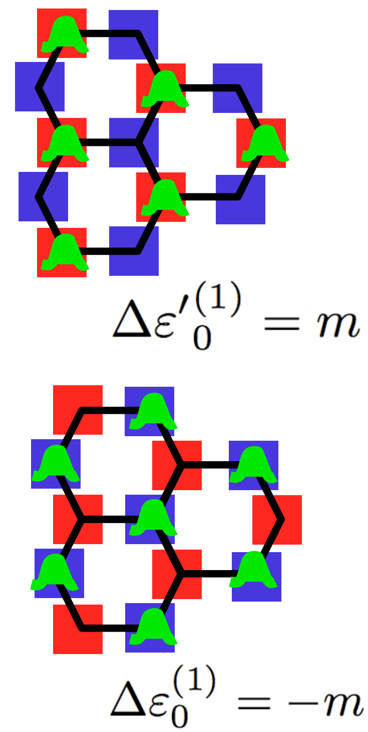


➔ qualitative agreement

Experiment & Phenomenological Theory (II)





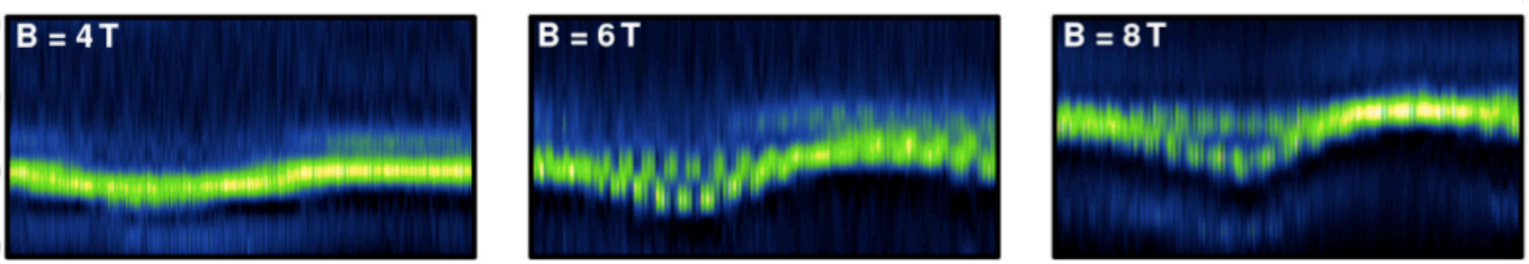


Observe: anticorrelation on lattice scale

Theory: sublattice polarization of LL_0 ✔

Theory: suppression of LL_0 splitting for $l_B \gtrsim l$ (weak B): $\Delta \sim m\, e^{-2(2\pi l_B/3l)^2}$

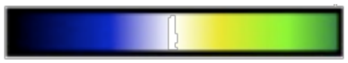
Observe:



✔

2D STS

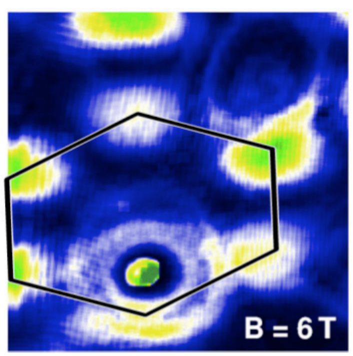
2D map of LL_0 splitting:

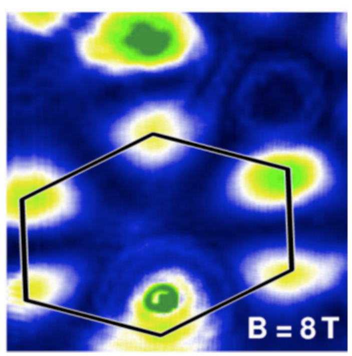
0  **10 meV**

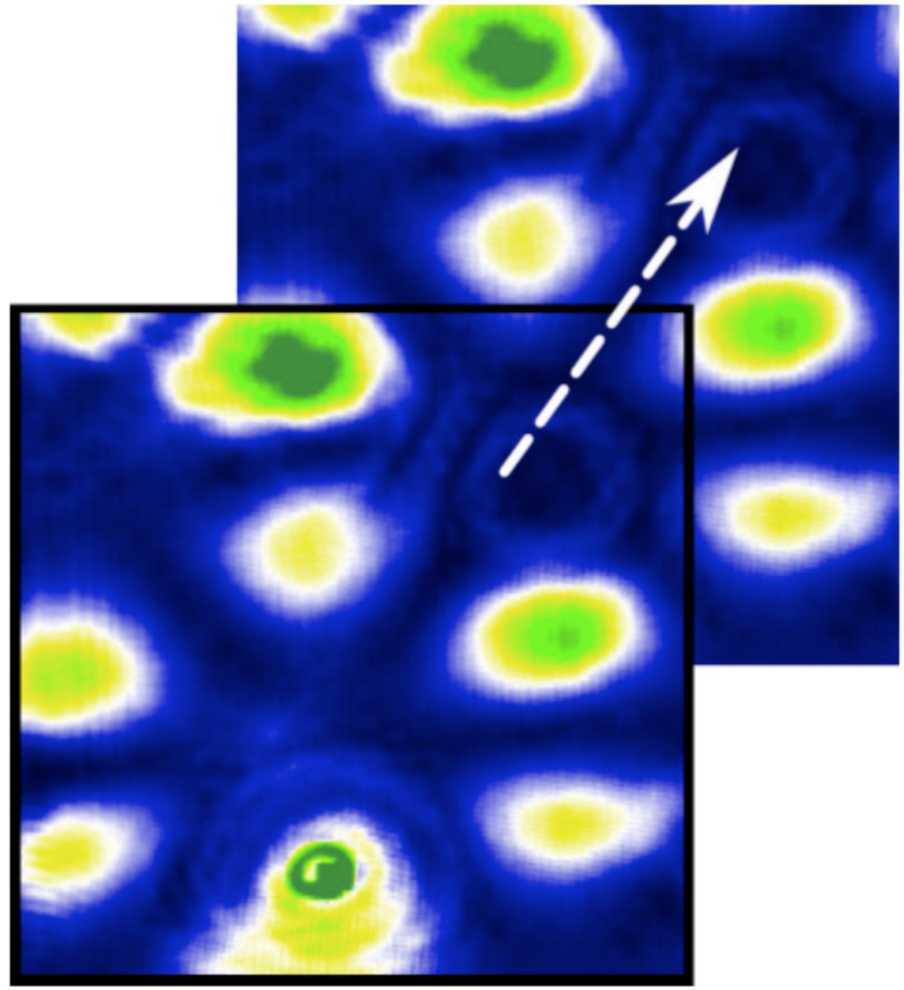
***B*=8T**

➔ hexagonal superlattice ✔

➔ hints at continuation of superlattice ✔





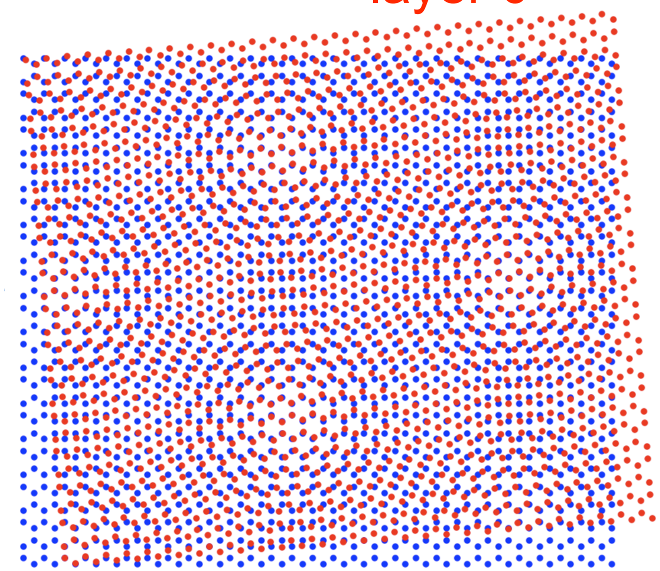


➔ superlattice B-independent ✔

≠ Wigner crystal, other correlation effects

Microscopic Theory

i) starting point: tight-binding model of bilayer graphene fitted to experiment
Dresselhaus, Dresselhaus, Adv. Phys. (2002)



$$H = \overbrace{H_0 + H_1}^{\text{Single-layer graphene}} + \underbrace{H_{01} + H_{10}}_{\text{interlayer hopping}}$$

ii) integrate out layer 1 ➔ effective theory for layer 0:

$$\boxed{H_0^{\text{eff}}(\omega) = H_0 + H_{01}(\omega - H_1)^{-1} H_{10}}$$

➔ Quantitative agreement with experiment.

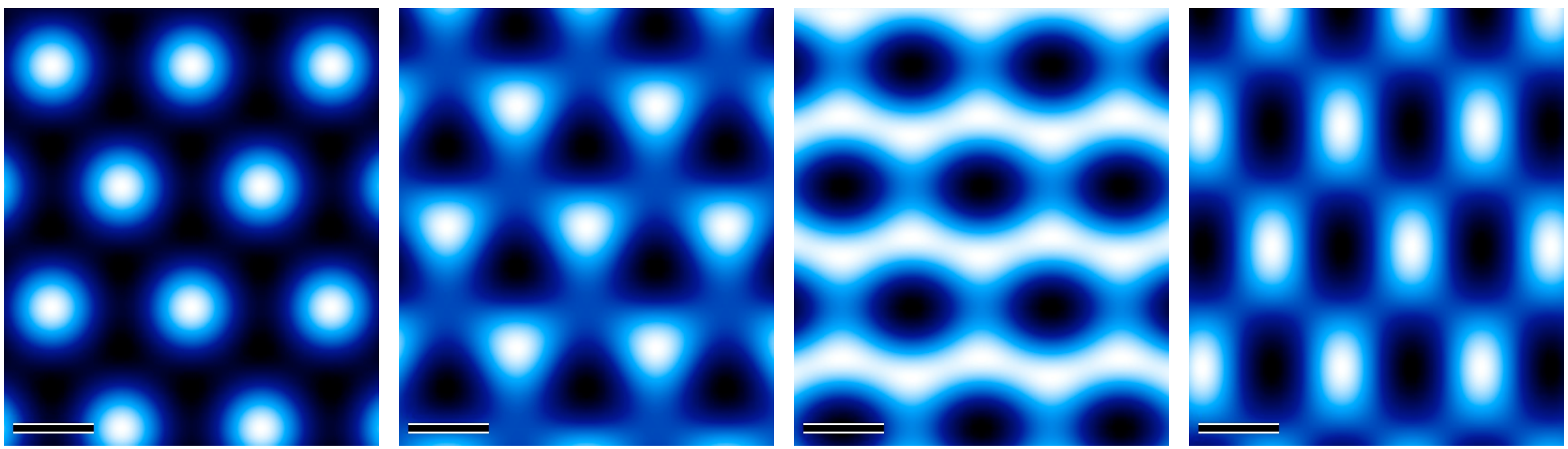
➔ Phenomenological theory if $H_{01}(\omega - H_1)^{-1} H_{10}$ is a (vector) potential, i.e.

- ω-dependence of H_0^{eff} may be neglected
- spatial non-locality of H_0^{eff} may be neglected

Large Interlayer Bias

$|V| \gg \omega, \gamma, \theta v/a$ ➔ local Hamiltonian for layer 0:

$$H_0^{\text{eff}} = H_0 + V^{\text{eff}}(\mathbf{r}) + \boldsymbol{\sigma}_\nu \cdot \boldsymbol{A}^{\text{eff}}(\mathbf{r}) + m^{\text{eff}}(\mathbf{r})v^2\sigma_z$$



V^{eff} m^{eff} A_x^{eff} A_y^{eff}

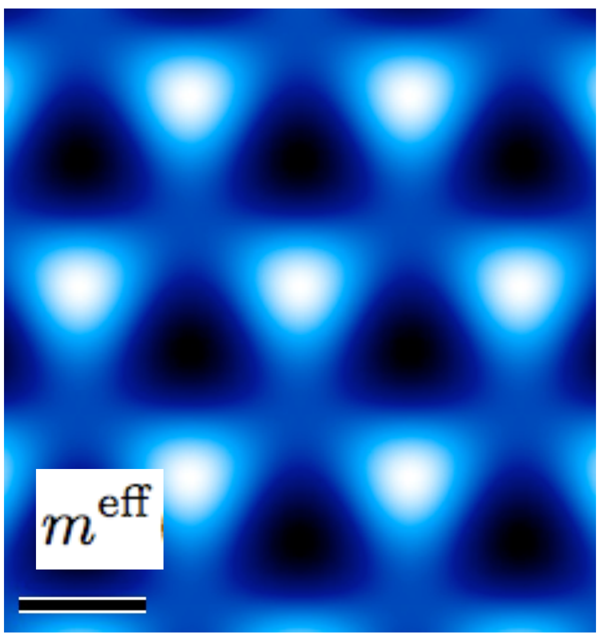
Dirac Mass

Doping of the layer closest to the substrate: $\mu \approx 400\,\mathrm{meV}$

Expect: $V \lesssim \mu$

➔ $|V| \gg \omega, \gamma, \theta v/a$ for some pairs of layers ➔ local H_0^{eff}
(next-nearest layer coupling $\gamma \lesssim 40\,\mathrm{meV}$, $\theta v/a \approx 15\,\mathrm{meV}$ in exp.)

➔ Dirac electrons with space-dependent mass.

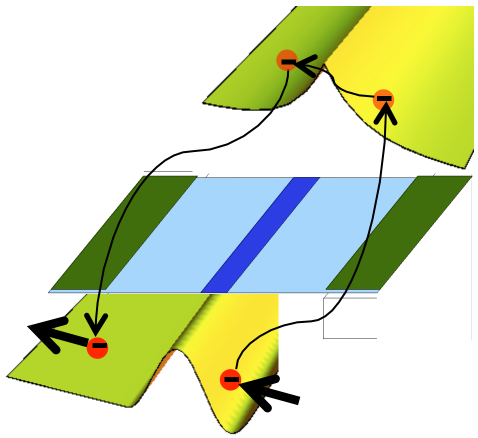


➔ topologically confined states
Yao *et al*., PRL (2008);
Semenoff, *et al*., PRL (2008);
Martin, *et al*., PRL (2008).

➔ qualitative agreement with numerics on twisted bilayers (velocity suppression, …)
Trambly de Laissardière et al., Nano Lett. (2009)

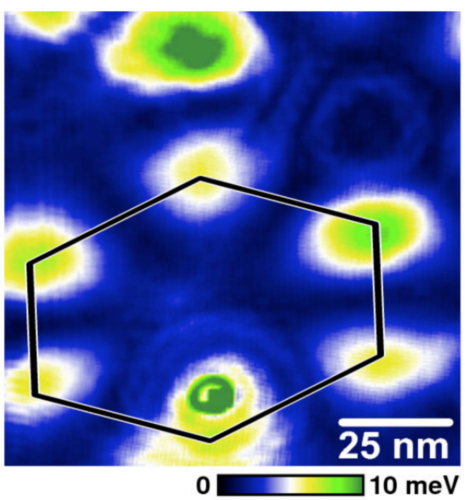
Summary

Graphene with 1D vector potentials:



- exponential renormalization
- many-body scattering resonance

Epitaxial Multilayer Graphene:



- Space-dependent splitting of LL_0
- Local sublattice symmetry breaking – spatially inhomogeneous mass generation