Analog Models of Gravity using Bose Einstein Condensates

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Quantum Matter in Low Dimensions:
Opportunities and Challenges

Stockholm, Sept 6-10 , 2010

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- I. Cirac , B. Horstamnn , Max Planck Institut fur Quantenoptik , Munich
- B. Reznik , Tel Aviv University, Israel
- E. Altman , Weizmann Institute , Israel
- E. Demler , Harvard University , US
- P. Krüger , Cold Atoms Group , Nottingham, UK
- E. Copeland's Group , Nottingham , UK

Outline

Quantum Field Theory in Curved Spacetime:

Semiclassical Gravity studies the quantum effects due to the propagation of quantum field in the presence of strong gravitational fields

- Gravity treated classically (Einstein Theory)
- Matter fields are quantized

Important and amazing results:

- Hawking Radiation
- Cosmological particle production
- Super-radiance
- Moving Mirror particle production
- ....

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Yet no experimental observation!

Analog Models of Gravity in Hydrodynamics

. W.G. Unruh, PRL 46 (1981)

- Field propagation in curved background is geometrical (metric)
- Sound-phonon propagation $\rightarrow$ massless scalar field in curved space-time
- Hydrodynamical fluids $\rightarrow$ curved space-time
- HOMOGENEOUS SYSTEM $\longrightarrow$ FLAT SPACETIME
- INHOMOGENEITIES $\longrightarrow$ CURVED SPACETIME
- Tool to investigate effects otherwise NOT accessible

ANALOGY WITH HYDRODYNAMICS

- Continuity and Bernoulli Eqs.
  for irrotational, inviscid fluid
  n= density, $\vec{v} = \vec{\nabla}\theta$= flow velocity, $\mu(n)$= specific enthalpy

$$\dot{n} + \vec{\nabla} \cdot (n\vec{v}) = 0 \qquad \dot{\theta} + \frac{1}{2}v^2 + \mu(n) = 0$$

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- Fluctuations on top of mean field solution: $n + n_1\,, \theta + \theta_1$
  Linearized Eqs:

$$\dot{n}_1 + \vec{\nabla} \cdot (n\vec{v}_1 + n_1\vec{v}) = 0 \qquad \dot{\theta}_1 + \vec{v} \cdot \vec{v}_1 + \frac{c^2}{n} n_1 = 0$$

with the sound velocity: $c^2 = nd\mu/dn$.

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  with the sound velocity: $c^2 = nd\mu/dn$.
- Putting n_1 into the first eq....

Analog Models

....one gets:

$$\left\{-\partial_t\left[\frac{n}{2c^2}(\partial_t+\vec{v}\cdot\vec{\nabla})\right]+\vec{\nabla}\cdot\left[\frac{\vec{v}n}{c^2}(\partial_t+\vec{v}\cdot\vec{\nabla})+n\vec{v}\cdot\vec{\nabla}\right]\right\}\theta_1=0$$

$$\longrightarrow \quad \Box\theta_1=\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\right)\theta_1=0$$

$\Box$: D'Alembertian in curved space
described by the "acoustic metric" $g_{\mu\nu}$:

$$g_{\mu\nu}\equiv\frac{n}{mc}\begin{pmatrix}-(c^2-v^2) & -\vec{v}^T \\ -\vec{v} & \mathbf{1}\end{pmatrix}$$

Analog Models

Core of the Analogy: $\Box\theta_1 = 0$

Acoustic metric:

$$g_{\mu\nu} \equiv \frac{n}{mc} \begin{pmatrix} -(c^2 - v^2) & -\vec{v}^T \\ -\vec{v} & \mathbf{1} \end{pmatrix}$$

- Sound propagates along null geodesics of $g_{\mu\nu}$.
- Geometrical analogy
- θ_1 massless scalar field propagating on curved spacetime with c, v, n functions of $(t, \vec{x})$.
- Choosing different space-time profiles for $c, v, n \rightarrow$ different metrics
- For $v = c \rightarrow g_{\mu\nu}$ black hole metric.

ANALOG MODELS OF GRAVITY IN CONDENSED MATTER

POWERFUL TOOL TO THEORETICALLY AND EXPERIMENTALLY INVESTIGATE QFT IN CURVED SPACES PROBLEMS (and not only)

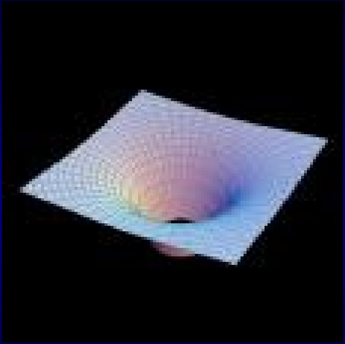
EXPERIMENTAL:

- Black Holes evaporation: Hawking Effect
- Cosmological expansion
- Dynamical Casimir effect
- ...

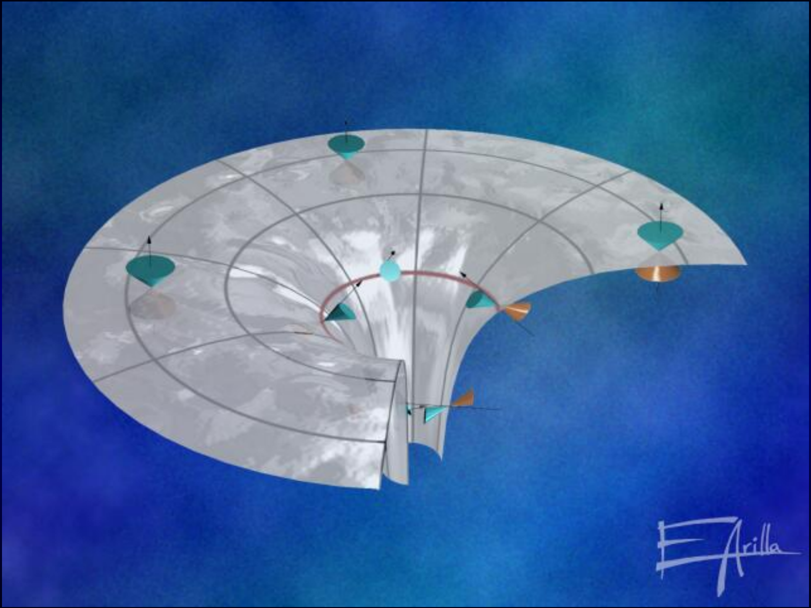
THEORETICAL:

- Trans-Planckian problem: Effects of non-linear dispersion relations
- Emergent Gravity
- Black Holes thermodynamics

GRAVITATIONAL BLACK HOLES



- Geometrical Objects
- Curvature so high that even light cannot escape
  Trapped regions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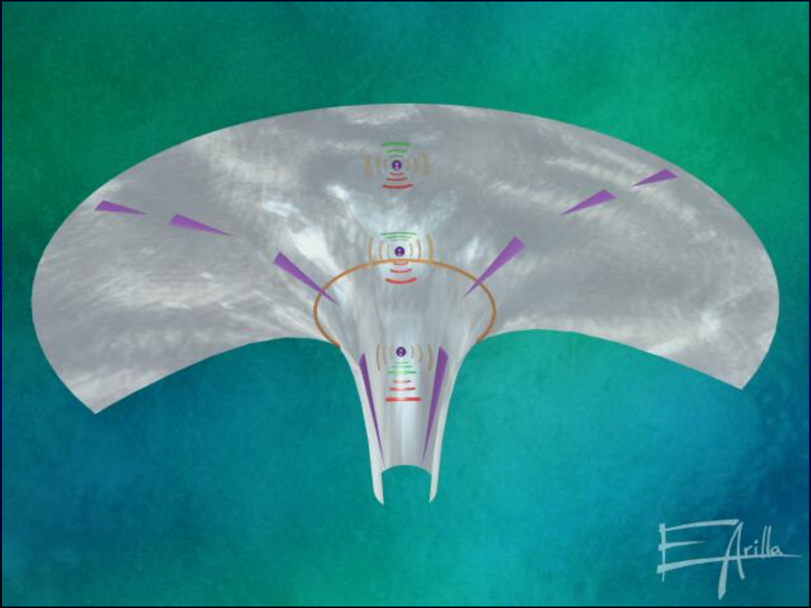
What is an acoustic Black Hole ?



Hawking Radiation

. S.W. Hawking, Nature 248 (1974)

Semiclassically, black holes are not "black" objects, but radiate particles after the horizon fomation. Thermal flux of particles detected asymptotically far from a black hole.

- Quantum effect
- Stationary emission
- Thermal spectrum
- Pure geometrical effect $\rightarrow$ independent on dynamics
- Still unobserved: 10^{-8} K (CMB: ~ 3 K)

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- Quantum effect
- Stationary emission
- Thermal spectrum
- Pure geometrical effect $\rightarrow$ independent on dynamics
- Still unobserved: 10^{-8} K (CMB: ~ 3 K)
- Since the analogy with fluids is both classical and quantum, in acoustic black holes Hawking radiation is expected as a thermal phonons emission

Acoustic Black Holes

Possible candidates:

- Atomic Bose-Einstein condensates
  . Garay, Anglin, Cirac, Zoller, PRL 85(2000)
- Quasi-particle excitations in superfluid Helium
  . Jacobson, Volovik, PRD 58(1998)
- Fermi gases . Giovanazzi, PRL 94 (2005)
- Slow-light . Leonhardt, Piwnicki, PRL 85(2000)
- Nolinear electromagnetic waveguides
  . Schutzhold, Unruh, PRL 95 (2005)
  . Philbin et al., Science 319 (2008)
- Ion rings . Horstmann, Reznik, SF, Cirac, PRL 104 (2010)
- ...

Ultra-Cold Atoms

Bose Einstein Condensates – Ion Rings

probably the most promising candidates to observe gravitational quantum effects:

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Bose Einstein condensates:

- Pure quantum systems
- Ultra-cold temperature (< 100 nK)
  $\rightarrow$ Hawking temperature ~ 10 nK
- Huge technological improvement
- Hydrodynamical description

Hydrodynamical Description for BEC

Gross-Pitaevskii Equation

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{ext} + g(a)|\psi|^2\right)\psi \qquad (1)$$

Madelung representation: $\psi = \sqrt{n}e^{-i\theta/\hbar}$
$|\psi|^2 = n$, $a =$scattering length

- Continuity and Bernoulli Eqs.
  $n=$ density, $\vec{v} = \vec{\nabla}\theta/m=$ flow velocity,

$$\dot{n} + \vec{\nabla}\cdot(n\vec{v}) = 0 \qquad \dot{\theta} + \frac{1}{2}v^2 + V_{ext} + gn - \frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{n}}{\sqrt{n}} = 0$$

Hawking Radiation's direct detection
is still very difficult

In BEC: $T_{Hawking} \sim$ few nK
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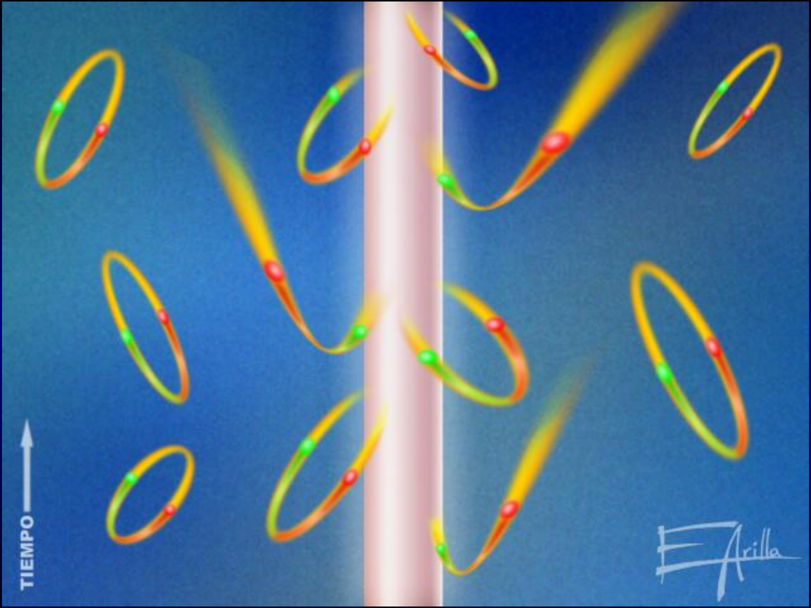
- New Idea: Non-local Correlations



PARTICLES PRODUCED IN PAIRS !!

Non-local Density Correlations

Acoustic Black Hole using supersonic BEC

Diluted BEC: n (density) and θ (phase) solution of the GP eqs.
Fluctuations: $n + \hat{n}_1$; $\theta + \hat{\theta}_1$

- $G_2(x, x') = \langle \hat{n}(x)\hat{n}(x')\rangle - \langle \hat{n}(x)\rangle\langle \hat{n}(x')\rangle = \langle \hat{n}_1(x)\hat{n}_1(x')\rangle$
  $\rightarrow$ x and x' on opposite sides of the horizon

- Technical details: Density Correlations

$$G_2(x,x') = \frac{1}{g(x)g(x')} \lim_{t'\to t} \mathcal{D}_{xx'} \langle \hat{\theta}_1(x,t)\hat{\theta}_1(x',t')\rangle$$

$$\mathcal{D}_{xx'} = \left[\partial_t\partial_{t'} + v(\vec{x})\vec{\nabla}_{\vec{x}}\partial_{t'} + v(\vec{x}')\partial_t\vec{\nabla}_{\vec{x}'} + v(\vec{x})v(\vec{x}')\vec{\nabla}_{\vec{x}}\cdot\vec{\nabla}_{\vec{x}'}\right].$$

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FIND $G(x,x') = \langle\hat{\theta}_1(x)\hat{\theta}_1(x')\rangle$!

Correlations : Technical details

- QFT in curved space - Black Holes

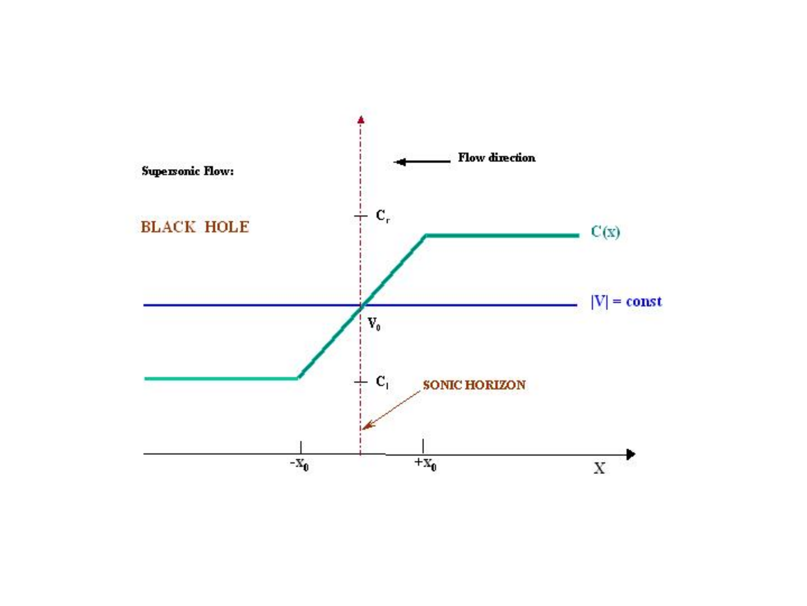
In 1D: $\langle \hat{\theta}_1(x)\hat{\theta}_1(x')\rangle \propto \ln\left[\Delta X^-(x,x')\Delta X^+(x,x')\right]$

- $\Delta X^-, \Delta X^+$: light-cone distance
- Motion of downstream modes unaffected: $X^+ = ct + x$
- Distortion of upstream propagating modes is UNIVERSAL: $X^- = \pm 1/k\; e^{-kx^-}$
- Surface gravity on the sonic horizon: $k = \frac{1}{2}\frac{d}{dx}(c^2 - v^2)|_H$

. Balbinot, SF, Fabbri, Procopio, PRL 94 (2005)

. Balbinot, SF, Fabbri, PRD 71 (2005)

BEC Setup Proposal



Non-Local IN/OUT Density Correlations

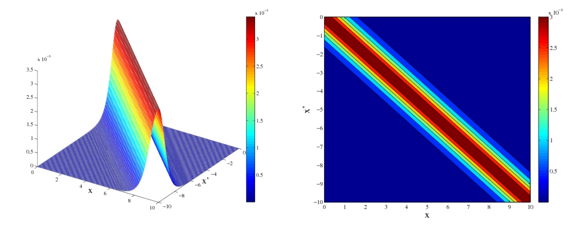
$$\langle \hat{n}_1(x)\hat{n}_1(x')\rangle = \frac{\hbar^2}{16\pi g_1 g_2}\frac{1}{n\sqrt{\xi_1\xi_2}}\frac{c_1 c_2}{(c_1-v_0)(c_2-v_0)}$$
$$\times \frac{k^2}{\cosh^2\left[\frac{k}{2}\left(\frac{x}{c_1-v_0}-\frac{x'}{c_2-v_0}\right)\right]} + O(x-x')^{-2}.$$

- Balbinot, Fabbri, SF, Recati, Carusotto, PRA 78 (2008)

Hawking correlations in BEC

Non-Local IN/OUT Density Correlations

$$\langle \hat{n}_1(x)\hat{n}_1(x')\rangle \propto \frac{T^2_{Hawking}}{\cosh^2(k(x+x')/2v_0)}$$



Peak for $x' = -x$!

The Hawking and the partner particles are exactly opposite with respect to the horizon

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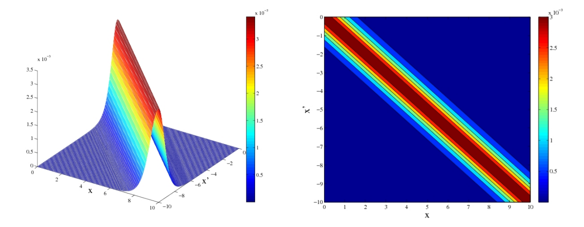

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The Hawking and the partner particles are exactly opposite with respect to the horizon

No Black Hole: $\langle \hat{n}_1(x)\hat{n}_1(x')\rangle \propto (x-x')^{-2}$

Open issues:

- Role of high-frequency modes (beyond the analogy)
  - Brout *et al*, PRD 52 (1995)
  - Corley, Jacobson, PRD 54 (1996),
  - Unruh, Schutzhold, PRD 71 (2005) and 0804.1686 (2008)
- Role of the back-scattering of the modes across the horizon
- Role of finite temperature fluctuations
  (Here $T = 0$.)
- Actual experimental difficulties to measure so tiny effects.

TIME DEPENDENT FORMATION OF AN ACOUSTIC HORIZON

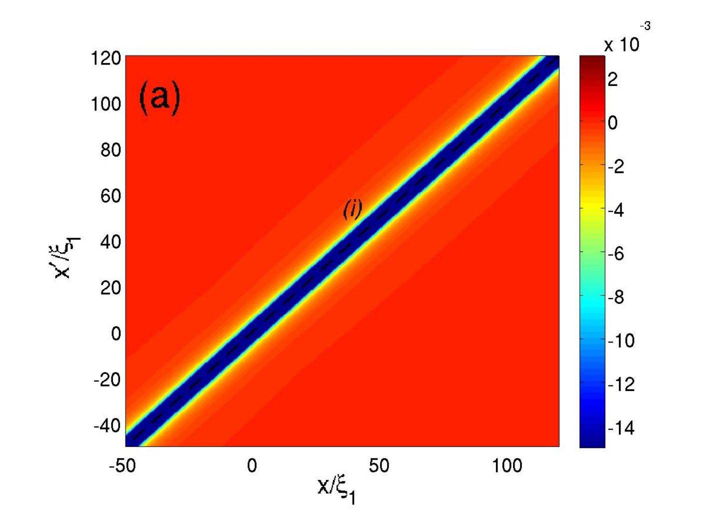
FULL MICROSCOPIC MANY-BODY PHYSICS TAKEN INTO ACCOUNT IN A BOGOLIUBOV-LIKE NUMERICAL SIMULATION

- Carusotto, SF, Recati, Balbinot, Fabbri, New J. Phys. 10 (2008)

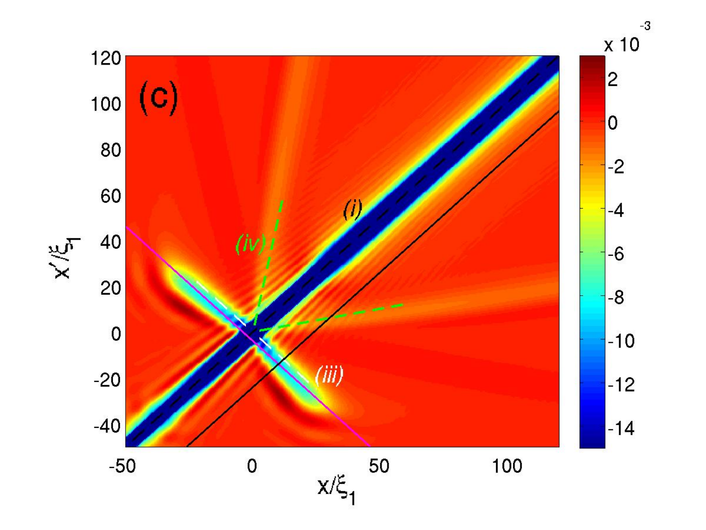
Semi-analytical analysis:

- Recati, Pavloff, Carusotto, 0907.4305

Numerical simulation - Initial: $G_2(x, x')/n^2$



Numerical simulation - Results: $G_2(x, x')/n^2$



System parameters: $v_0/c_1 = 0.75$, $v_0/c_2 = 1.5$,
Horizon width= $\sigma/\xi_1 = 0.5$, $T_0 = 0$

Numerical simulation Results

The result does NOT depend on:

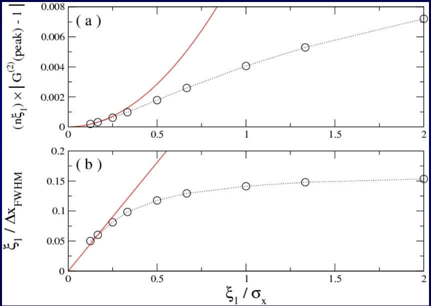
- UV cutoff k_{max}
- Box size L
- time-dep switching

The Signal DOES depend on:

- Horizon shape
- How steep is the spatial variation on the horizon: the steeper $\rightarrow$ the higher $T_{Hawking}$
- Back-scattering

Numerical simulation Results

Comparison with the analytical predictions:



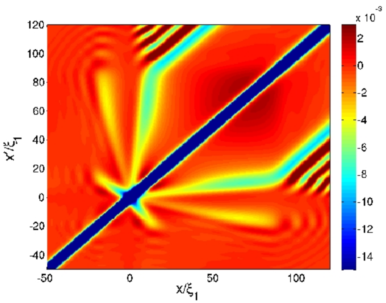
Excellent agreement in the hydrodynamical limit.

Still Hawking effect robust even beyond

Numerical simulation Results

Role of thermal fluctuations:

- Numerical simulation with $T = 0.1\mu \gg T_{Hawking} \sim 10^{-2}\mu$

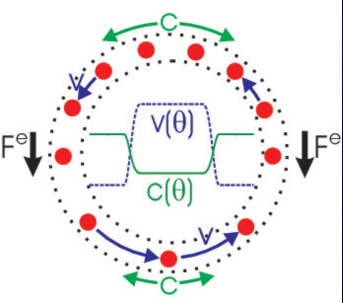


ION RINGS

First proposal to observe Hawking rad. in discrete systems.

. Horstmann, Reznik, SF, Cirac, PRL 104 (2010)

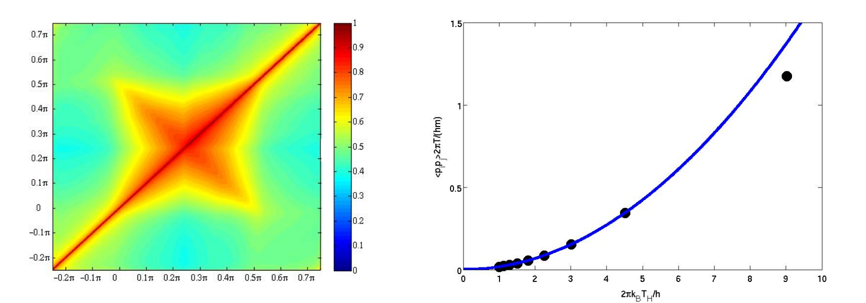
- Non homogenous spacing - quadrupole ring trap.
- Rotation imposed.



High control in ions manipulation: trapping and measurements

. Schatz *et al.*, Nature 412, 717 (2001)

ION RINGS



- Good agreement with analytic prediction
- HR robust even in discrete system
  $\rightarrow$ non linear dispersion even at very low energy
- High signal, visible with current technology

Realizing any concrete experiment to observe Hawking radiation still is a non trivial task.
These works represent the first necessary steps towards real experiments.
Major issues solved before any concrete experimental realization.
Experimental realization is getting closer and closer.

First Acoustic Black Hole created:
. Lahav, Itah, Blumkin, Gordon, Steinhauer, arXiv:0906.1337 (2009)

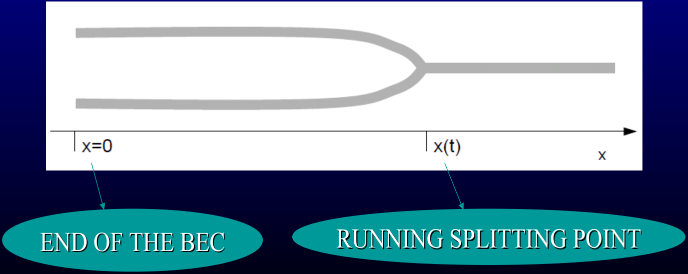
Dynamical Casimir effect

- **Casimir Effect**:
  Two conducting plates in vacuum attract each other due to em vacuum. V
- **Dynamical Casimir**:
  Plates in non uniformly accelerated motion $\rightarrow$ real photons are produced. Not V
- Originally DCE is a Moving Mirrors problem
  . Moore, JMP 11 (1970)
  . Carlitz, Willey, PRD 36 (1987)

The Moving mirror problem : THE BEC ZIPPER

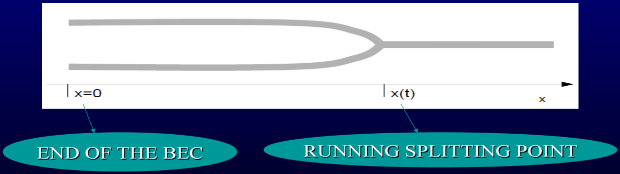
. SF, Altman, Demler, to appear

UNZIPPING A 1-D BEC through a Y-junction.
Dynamics of the phase difference φ between the two arms $\rightarrow$ well known QFT description. (. A. Polkovnikov et al, PNAS (2006))



THE BEC ZIPPER– QFT mapping :

- $1D$-problem for the phase difference $\varphi = \varphi_1 - \varphi_2$:
  $S = -\frac{K}{2} \int dt\, dx \left[(\partial_t \hat{\varphi})^2 - \frac{1}{c^2} (\partial_x \varphi)^2 \right]$
- Eq. of Motion for φ:
  $(c^2 \partial_t^2 - \partial_x^2)\varphi = 0$
- with two boundary conditions:
  $\varphi|_{x_m(t)} = 0 \rightarrow$ at the spitting point
  $\partial_x \varphi|_{x=0} = 0 \rightarrow$ at the edge (to avoid current)



THE BEC ZIPPER

- Natural realization of Dynamical Casimir effect in an ultra-cold atom interferometer
- Emitted radiation can be directly measured by the interference fringes
- Outcomes will depend just on the trajectory of the splitting point
  i.e. exponential traj.$\rightarrow$thermal emission
  (link to Hawking radiation)

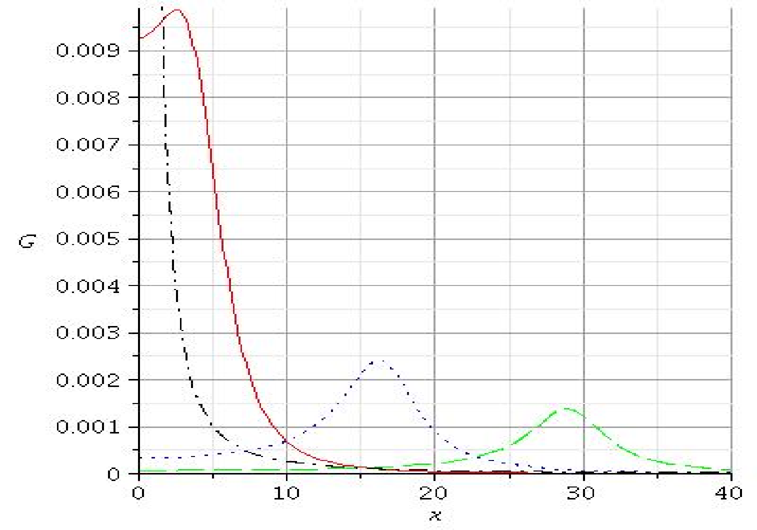
Cosmological Quantum Emission

Cosmology can be simulated by:

- Imposing special time dependent profile in either c or v (or both)
- Expanding BEC after releasing the trapping potential

. Schutzhold, et al, PRD (2005)
. Liberati et al, PRD (2005)
...

Density correlation in a $3D$ Expanding BEC after trap release
. Prain, SF, Liberati, arXiv:1009.0647)



As times goes (dashed-dotted $\rightarrow$ red $\rightarrow$ dashed $\rightarrow$ green)

Conclusion

- Importance of Analog Models to test undetectable effects and to suggest new physics
- Importance of Correlation measurement for testing non-equilibrium dynamics
- Major issues solved before real experiments to detect Hawking radiation
- The BEC UNZIPPING is a very interesting non equilibrium process mapping in a well known QFT problem: The Dynamical Casimir Effect
- Cosmological Particle emission

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Thank you!

Numerical Simulations

Space-time modulation of V_{ext} and g: $V_{ext} + gn = const$

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Space-time modulation of V_{ext} and g: $V_{ext} + gn = const$

- Mean-field condensate wavefunction is a plane wave solution of GPE at any time
- Fluctuations dynamics separated from the mean-field dynamics

Propagation according to deterministic GPE of classical wavefunction $\Phi(x,t)$ with random initial wavefunction Φ_0:

. Sinatra, Lobo, Castin, J. Phyis. B (2002)

$$\Phi_0 = e^{i(k_0 x - \omega_0 t)} \sum_{k \neq 0} \left(\alpha_k u_k e^{ikx} + \alpha^* v_k e^{-ikx} \right)$$

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- $u_k \pm v_k = (\epsilon_k / E_k)^{\pm 1/4}$
  $E_k = \hbar^2 k^2 / 2m$
  ϵ_k Bogoliubov spectrum

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- Sum running over $k = 2\pi j/L$, $L=$ box size
- UV cutoff k_{max} needed
- $u_k \pm v_k = (\epsilon_k / E_k)^{\pm 1/4}$
  $E_k = \hbar^2 k^2 / 2m$
  ϵ_k Bogoliubov spectrum
- α_k random variables: $\langle \alpha_k \rangle = \langle \alpha_k^2 \rangle = 0$