A Markov Chain Monte Carlo technique to sample transport and source parameters of Galactic cosmic rays

Antje Putze

Oskar Klein Centre for Cosmo Particle Physics

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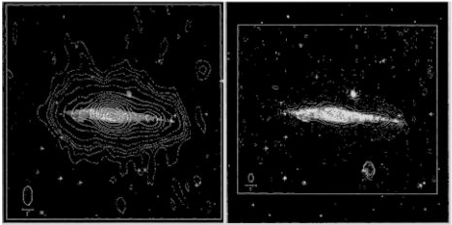
Questions on cosmic-ray propagation



NGC 4631 (610 MHz)
[Ekers & Sancisi, A&A **54** (1977), 973]

Galactic halo model

observation of radio halo which is due to cosmic rays around the galactic disc

$\Longrightarrow$ galactic halo

Mechanisms

- diffusion: $K(E)$
  $\Longrightarrow$ magnetic field;
  Kolmogorov: $K \propto E^{1/3}$?
- convection: V_c
  $\Longrightarrow$ galactic wind;
- reacceleration: V_a
  $\Longrightarrow$ magnetohydrodynamic waves.

Bayesian approach

Identification and quantification of m parametres $\boldsymbol{\theta} = \left\{\theta^{(1)}, \theta^{(2)}, \ldots, \theta^{(m)}\right\}$ of an theoretical model

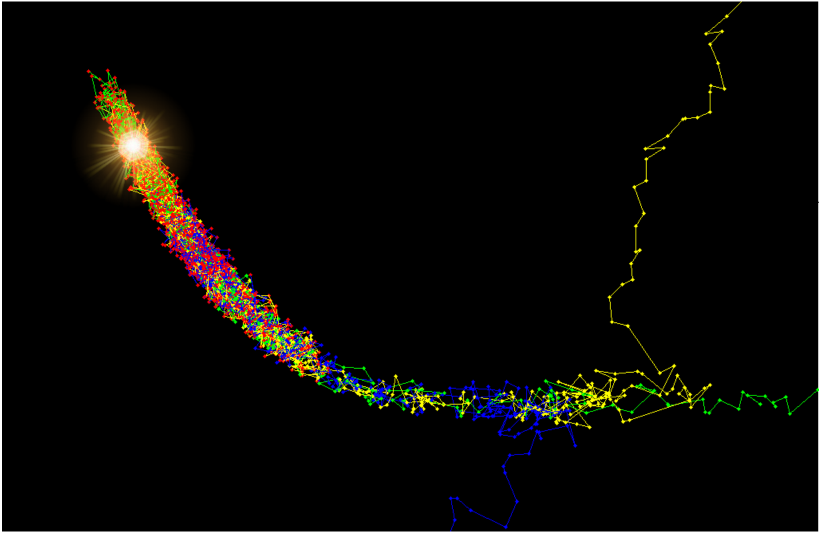
$$\underbrace{P(\boldsymbol{\theta}|\mathrm{data})}_{\text{posterior probability}} \propto \underbrace{P(\mathrm{data}|\boldsymbol{\theta})}_{\text{likelihood}} \cdot \underbrace{P(\boldsymbol{\theta})}_{\text{prior probability}}$$

Extraction of the marginalised posterior PDFs by multi-dimensional integration

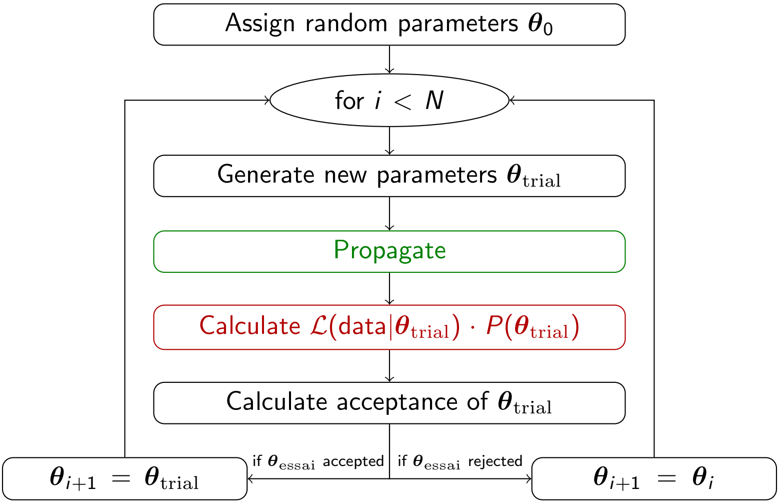
$\Longrightarrow$ Sampling of $P(\boldsymbol{\theta}|\mathrm{data})$ with an Markov Chain Monte Carlo

An example

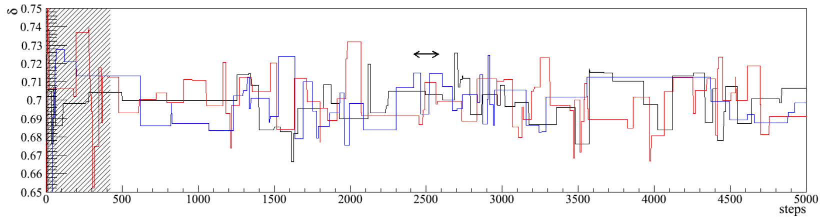
Markov chains sampling a 3D function



Implementation the propagation code USINE

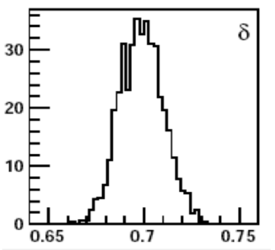


Chain analysis



Evaluation of the burn-in and correlation lengths for independent sample extraction

⇒



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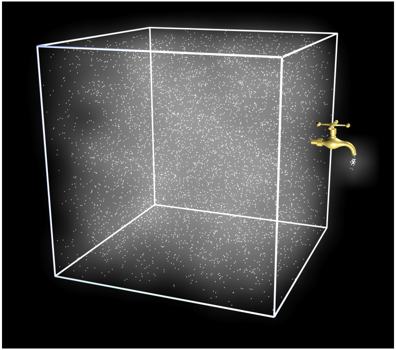
Estimation of the posterior PDF

Leaky-Box Model



$$\frac{N_i}{\tau_{esc}} + \bar{n} v \sigma_i N_i = \bar{q}_i + \sum_{j>i} \bar{n} v \sigma_{ij} N_j$$

$$n \Leftrightarrow \bar{n}, \quad q_i \Leftrightarrow \bar{q}_i$$

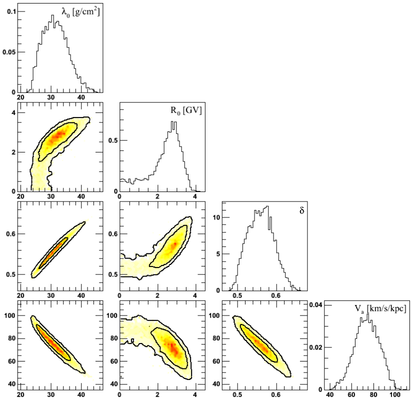
$$\frac{S}{P} = \frac{\sigma_{PS}}{\bar{m}/\lambda_{esc} + \sigma_S}$$

$$\lambda_{esc} = \bar{m}\bar{n} v \tau_{esc}$$

$$\lambda_{esc}(R) = \lambda_0 \beta \begin{cases} R_0^{-\delta} & \text{for } R < R_0, \\ R^{-\delta} & \text{sinon} \end{cases} \quad \text{with} \quad R = \frac{pc}{Ze}$$

4 free parameters: λ_0 in $\mathrm{g\,cm^{-2}}$, R_0 in GV, δ, $\mathcal{V}_a$ in $\mathrm{km\,s^{-1}}$

Leaky-Box Model: B/C ratio



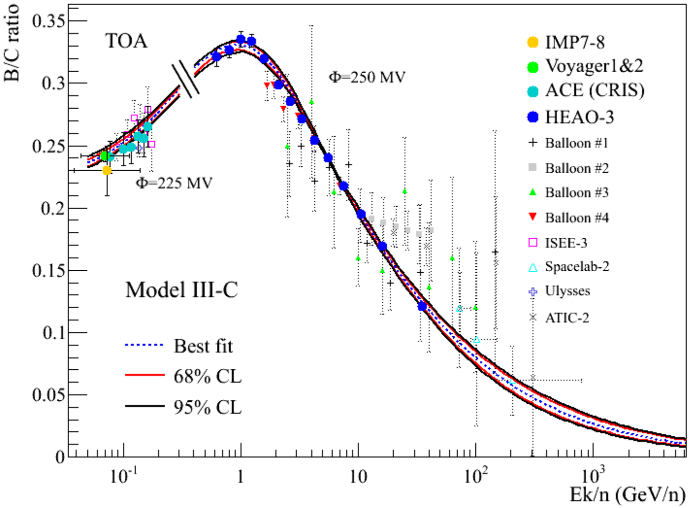
[Putze *et al.*, A&A **497** (2009), 991]

- Configuration with critical rigidity and reacceleration preferred:

$$\lambda_0 = 27^{+2}_{-2}\,\mathrm{g\,cm^{-2}}$$
$$R_0 = 2.6^{+0.4}_{-0.7}\,\mathrm{GV}$$
$$\delta = 0.53^{+0.02}_{-0.03}$$
$$\mathcal{V}_a = 86^{+9}_{-5}\,\mathrm{km\,s^{-1}\,kpc^{-1}}$$

- Kolmogorov spectral index ($\delta = 1/3$) disfavoured for all configurations by used data;
- Break in the $\lambda_{\rm esc}$ spectrum favoured by used data.

Leaky-Box Model: B/C ratio (envelope)



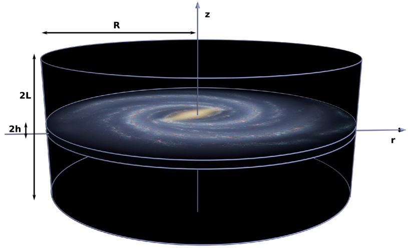
[Putze *et al.*, A&A **497** (2009), 991]

Leaky-Box Model: Conclusion

- MCMC successfully implemented and tested in the framework of the simple Leaky-Box model;
- comparison of the relative merit of different configurations:
  - configuration with critical rigidity and reacceleration preferred by used data;
  - Kolmogorov spectral index disfavoured by used data;
  - break in the $\lambda_{\rm esc}$ spectrum favoured by used data;
- correlation study between the propagation and source parameters.

MCMC is a powerful statistical tool which allows us to constrain efficiently the propagation model parameters!

Diffusion Model



Galaxy is divided into two zones:

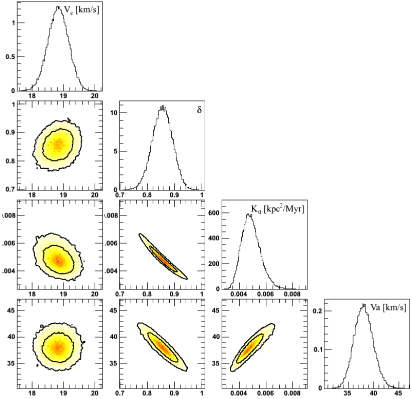
1. a thin disk of size h;
2. a diffusive halo of size $L \gg h$.

$$K(R) = K_0 \beta R^{\delta}$$

$$n_d = n, \quad n_h = 0$$

5 free parameters: K_0 in kpc^2/Myr, δ, V_c in km/s, L in kpc, V_a in km/s.

1D Diffusion Model: stable nuclei



[Putze *et al.*. ICRC 2009]

[Putze *et al.*, A&A, submitted, arxiv:1001.0551]

- Configuration with convection and reacceleration preferred:

$$L = 4\,\text{kpc fixed}$$
$$V_c = 18.8^{+0.3}_{-0.3}\,\text{km/s}$$
$$\delta = 0.86^{+0.04}_{-0.04}$$
$$K_0 = 0.0046^{+0.0008}_{-0.0006}\,\text{kpc}^2/\text{Myr}$$
$$V_a = 38^{+2}_{-2}\,\text{km/s}$$

- Kolmogorov spectral index ($\delta = 1/3$) disfavoured by used data.

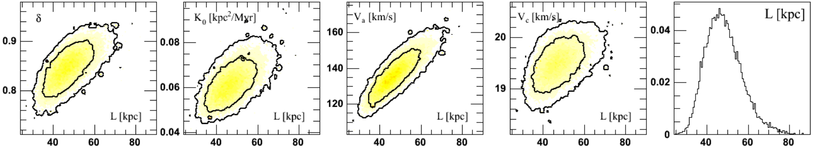
1D Diffusion Model: radioactive nuclei

Stable secondary-to-primary ratios: degeneracy between K_0 and L

$$\lambda_{\rm esc} = nmvh \frac{L}{K(E)}$$

$\Longrightarrow$ Radioactive secondaries needed to lift degeneracy

Results with ^{10}Be/^{9}Be data

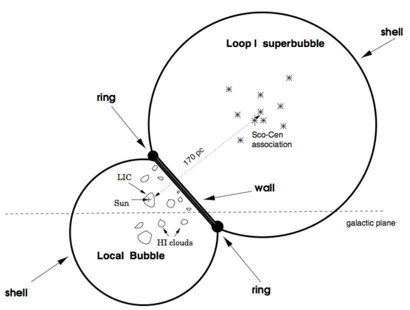


[Putze *et al.*, A&A, submitted, arxiv:1001.0551]

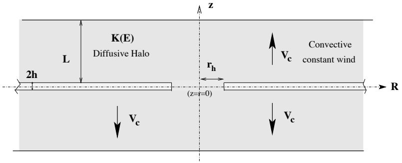
$L = 46^{+9}_{-8}$ kpc

Modified 1D Diffusion Model

The local bubble



Modelisation: hole in the disk



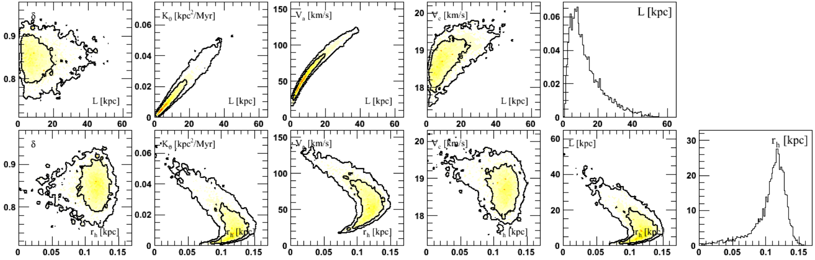
$$\frac{N_{r_h}}{N_{r_h=0}} = \exp\left(\frac{-r_h}{l_{\rm rad}}\right),$$

where $l_{\rm rad}$ is the typical distance on which a radioactive nucleus diffuses before decaying [Donato *et al.*, A&A **381** (2002), 539]

Supplementary parameter: the radius r_h (in pc) of the local bubble

Modified 1D Diffusion Model: radioactive nuclei

Results with ^{10}Be/^{9}Be data

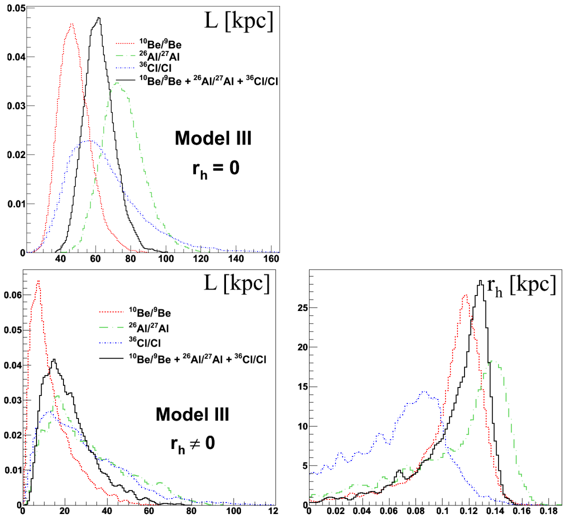


[Putze *et al.*, A&A, submitted, arxiv:1001.0551]

$$L = 8^{+8}_{-7}\,\text{kpc} \quad r_h = 120^{+20}_{-20}\,\text{pc}$$

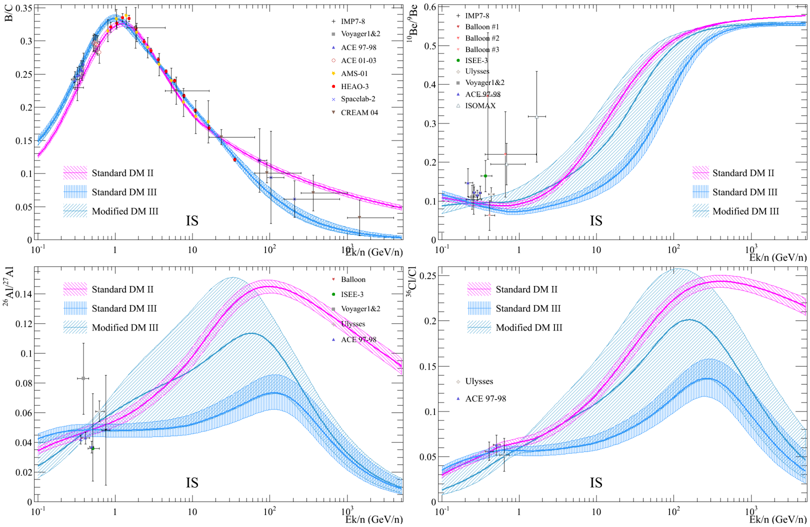
Better radioactive nuclei data needed for more precise constraints!

Modified 1D Diffusion Model: radioactive nuclei



[Putze *et al.*, A&A, submitted, arxiv:1001.0551]

1D Diffusion Model: radioactive nuclei (envelopes)



[Putze *et al.*, A&A, submitted, arxiv:1001.0551]

1D Diffusion Model: Conclusion

- Successful posterior PDF extraction of the propagation parameters of the one dimensional diffusion model:
  - configuration with reacceleration and convection favoured by used data;
  - Kolmogorov spectral index disfavoured by used data;
- Study of parameters describing the geometry of the Galaxy:
  - estimation of the halo size L and the radius r_h of the local bubble;
  - taking into account the local bubble decreases the halo size L;
  - values found compatible with observations.

MCMC is a robust tool allowing an excellent parameter estimation.

Determination of L and r_h values strongly depends on the δ value!
$\Longrightarrow$ high-energy data of secondary-to-primary ratios and/or radioactive elemental ratios are necessary!