



UNIVERSITY OF LEEDS

1. Planetary magnetic fields and interior structure

Chris Jones

Department of Applied Mathematics

University of Leeds, UK

Nordita Winter School January 11th Stockholm

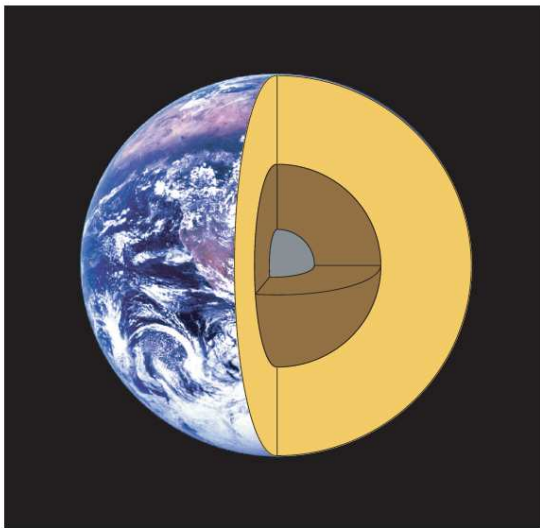
Outline

1. Internal structure of the planets
2. Planetary magnetic fields and dynamo theory
3. Reconstructing the core flow
4. Energy sources for planetary dynamos
5. Energy and entropy balance in the core
6. Features of individual planetary dynamos and proposed models

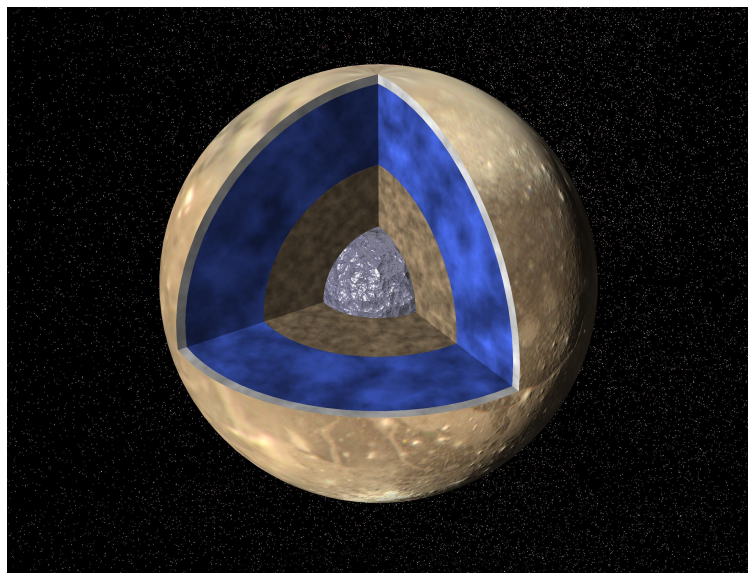
Internal Structure of the Planets

Planets can be divided into three classes:

(i) Terrestrial planets have rock mantles, and many have metallic cores. We include the larger satellites of Jupiter and Saturn. Some terrestrial planets have magnetic fields: the metallic core is believed to be where the field is generated.



Interior structure of Earth

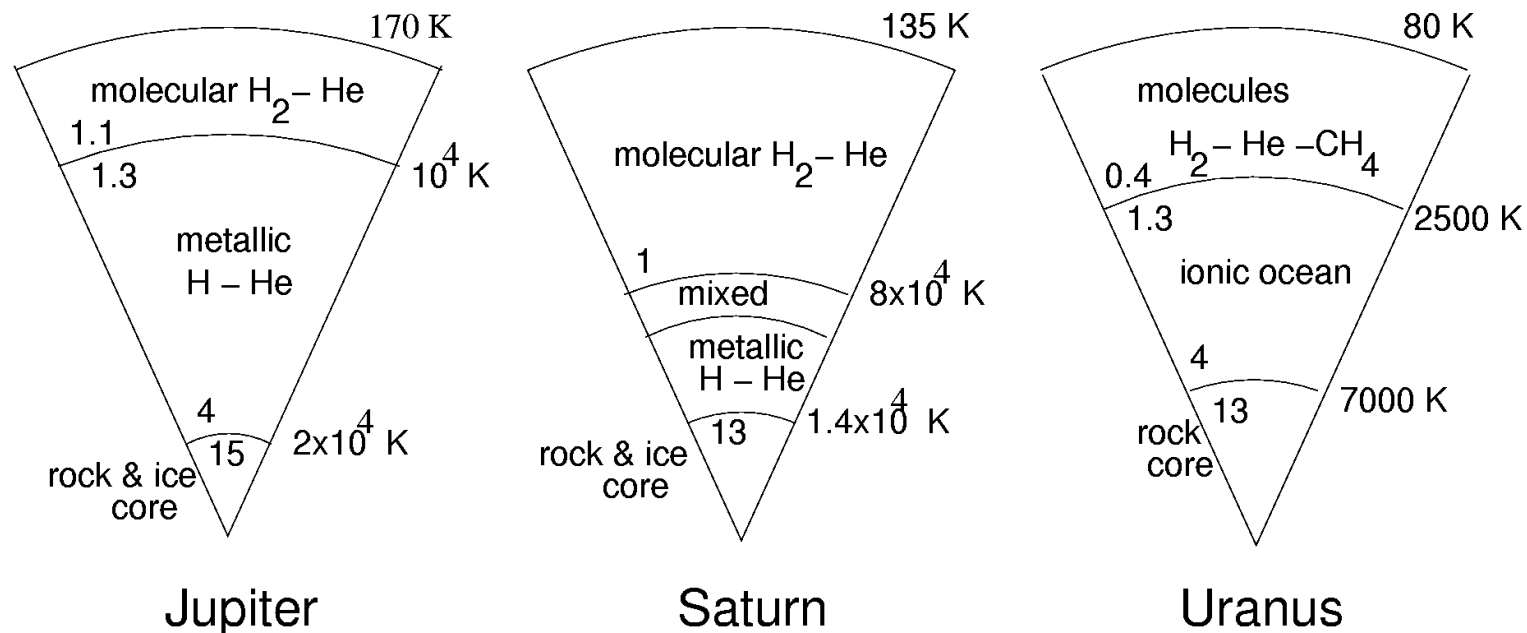


Interior structure of Ganymede

Earth's fluid outer core radius 3480 km, solid inner core radius 1220 km.

Internal Structure of Giant Planets

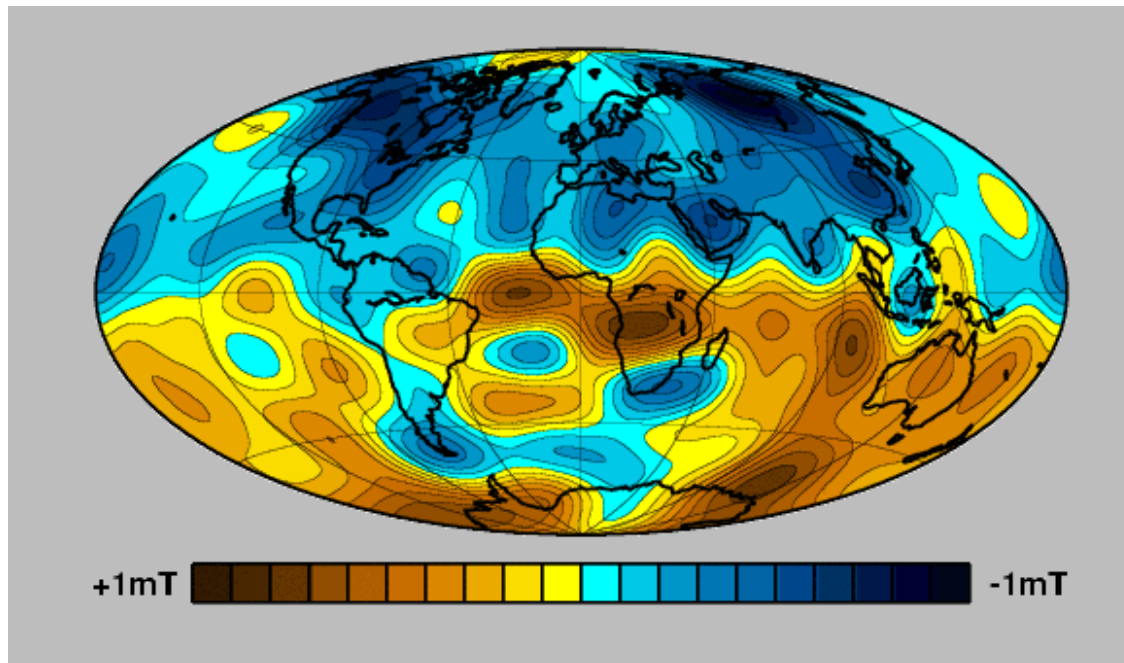
(ii) Gas giants, Jupiter and Saturn in our solar system, are mostly hydrogen and helium, with (probably) comparatively small rocky cores. Very high pressure forms **metallic hydrogen** which is electrically conducting. Dynamo is believed to occur in this metallic hydrogen region.



(iii) Ice Giants, Uranus and Neptune. These planets have a mantle believed to be a water-ammonia ocean, which has an ionic electrical conductivity. This is where the dynamo is believed to be.

Planetary magnetic fields: Terrestrial planets

Earth: mainly dipolar magnetic field. Inclination of dipole axis to rotation axis currently 11.5° . Field at Core-Mantle boundary, CMB, can be reconstructed from satellite and observatory data. Strength $\sim 8 \times 10^{-4} \text{ Tesla} = \sim 8 \text{ Gauss}$.



Field at the CMB in year 2000, units 10^{-3} T . Note the intense flux patches near Canada and Siberia. Patches under Africa are moving westward. Field at poles surprisingly low.

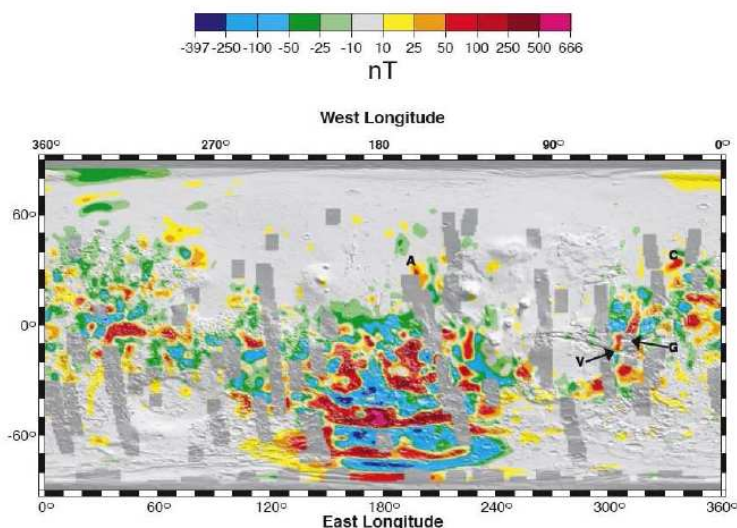
Magnetic fields of other terrestrial planets

Mercury: internal field of dipolar structure. Strength at CMB $\sim 1.4 \times 10^{-6} \text{T}$. Rotates only once every 57 days. Liquid iron core, radius 1900 km.

Venus: no magnetic field.

Ganymede: approx dipolar, inclination $\sim 10^\circ$. Strength at CMB $\sim 2.5 \times 10^{-4} \text{T}$. Rotates once every 7 days. Core radius 480 km.

Extinct Martian field is deduced from strong crustal magnetism, which suggests Mars had a dipole field in the past. Lunar rocks also have remanent magnetism.



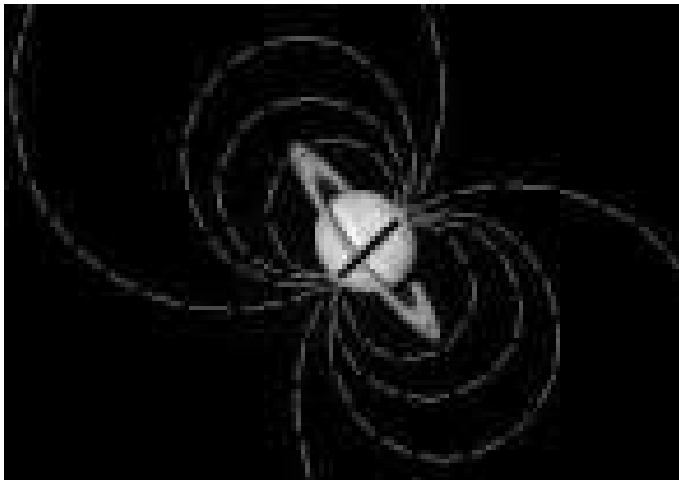
Remanent magnetic field of Mars, from Mars Global Surveyor (M. Purucker). Surprisingly strong field suggests Mars once had a dynamo.

Planetary magnetic fields: Giant planets

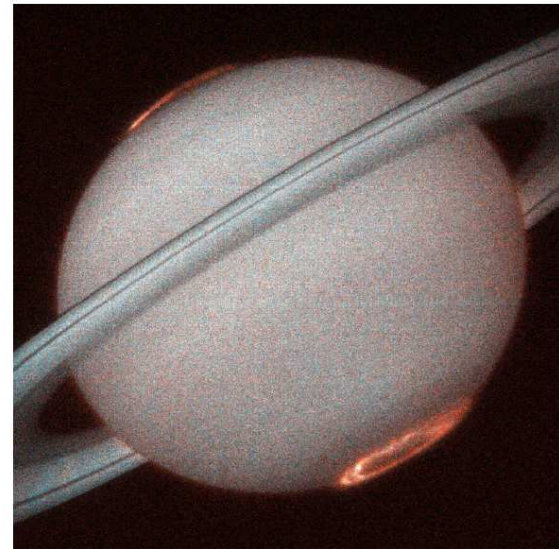
Giant planets are all rotating rapidly.

Jupiter: strong magnetic field, basically dipolar (more higher harmonics than Earth), inclined 10° to rotation axis, about 17 gauss at surface.

Saturn: very axisymmetric field, about 2.5 gauss at surface.

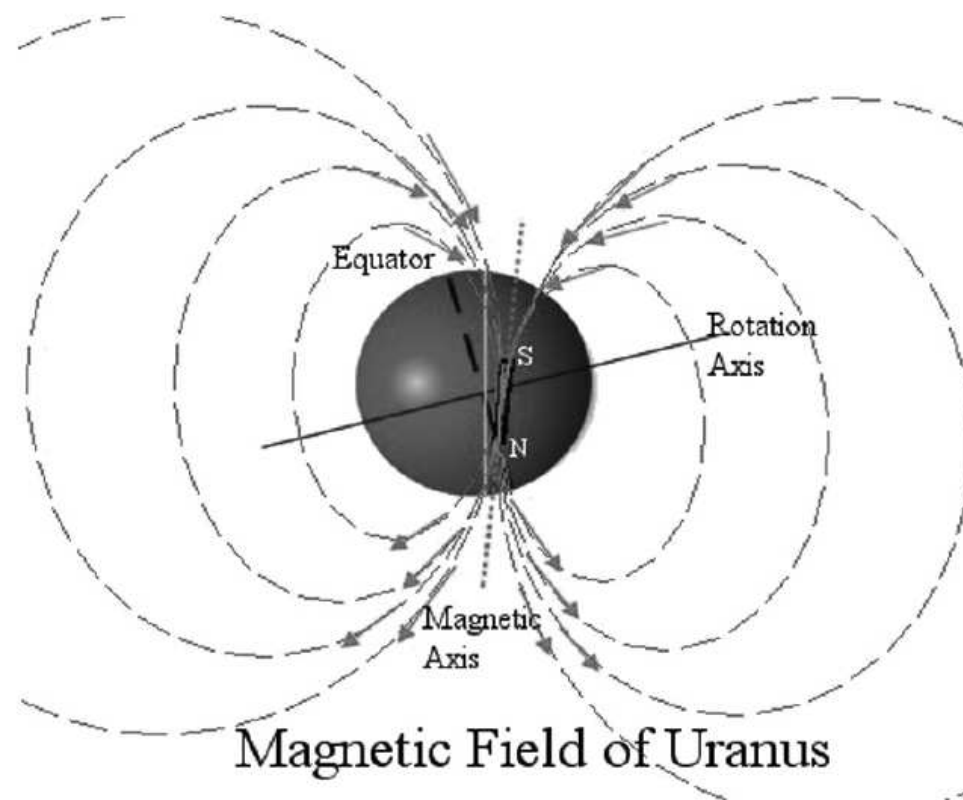


Dipole field of Saturn



Aurora of Saturn, produced by particles moving along field lines

Dipole model of Uranus magnetic field



An off-centre dipole model of Uranus magnetic field. Note the strong inclination to the rotation axis.

External magnetic fields

Outside the core, no currents so field is potential $\mathbf{B} = -\nabla\Psi$.

$$\Psi = r_s \sum_{n=1}^{\infty} \sum_{m=0}^n \left(\frac{r_s}{r}\right)^{n+1} P_n^m(\cos\theta) (g_n^m \cos m\phi + h_n^m \sin m\phi)$$

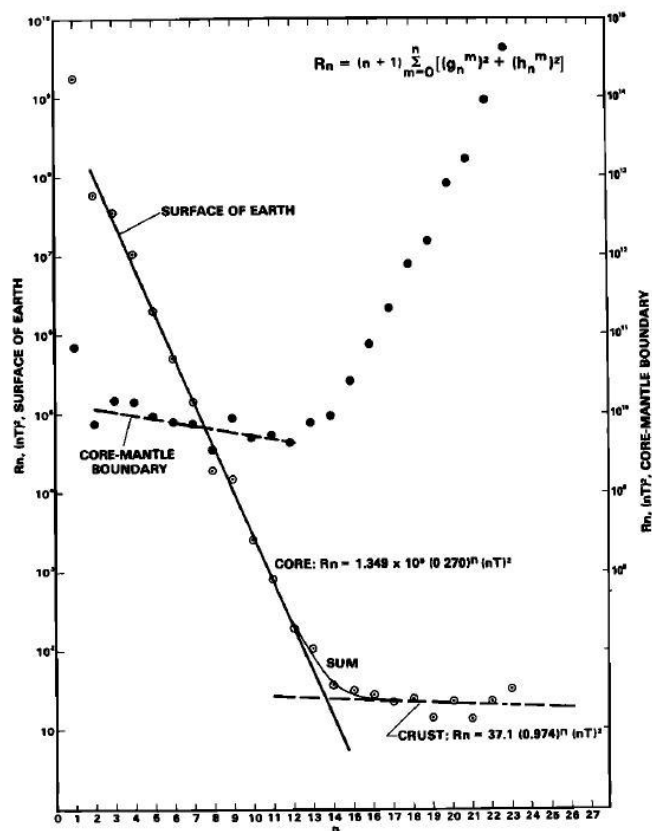
where r , θ , ϕ are planet centred spherical polar coordinates, P_n^m are the Schmidt normalized associated Legendre polynomials, r_s is surface radius and g_n^m and h_n^m are the Gauss coefficients of degree n and order m .

This is the field due to an internal source: the fields around planets also have external sources from the solar wind. Short timescales, allow them to be filtered out.

Note that for large n , the field at the CMB is much larger than the field at the surface. The small surface crustal field dominates the harmonics above $n \sim 14$, so no higher harmonics from the core can be reliably determined.

Observed geomagnetic field spectrum

Langel and Estes GRL Vol 9 1982 pp250-253: Lowes-Mauersberger spectrum.

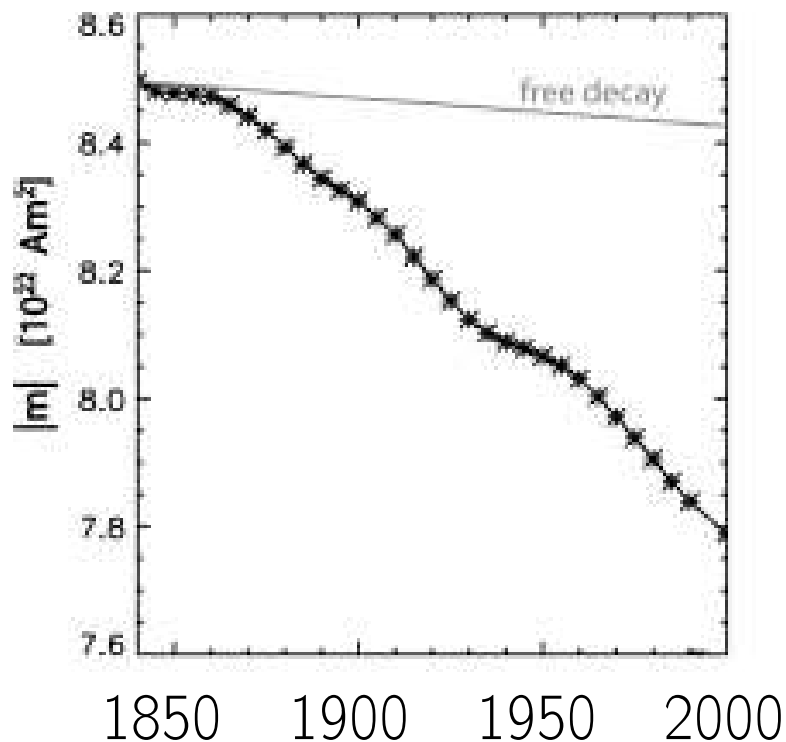


Note that the spectrum at the CMB is much flatter than at the surface. Even at the CMB, the dipole dominates.

Dipole moment

The dipole moment is $M = \frac{4\pi r_s^3}{\mu_0} g_1$, where

$$g_1 = \left((g_1^0)^2 + (g_1^1)^2 + (h_1^1)^2 \right)^{1/2}.$$



The Earth's dipole moment is currently decreasing faster than free decay rate, i.e. if there were no dynamo! About half its value in Roman times.
Growing reversed flux patches under S. Africa?

Gauss coefficients for Jupiter and Saturn

Gauss coefficients for Jupiter and Saturn in mT.

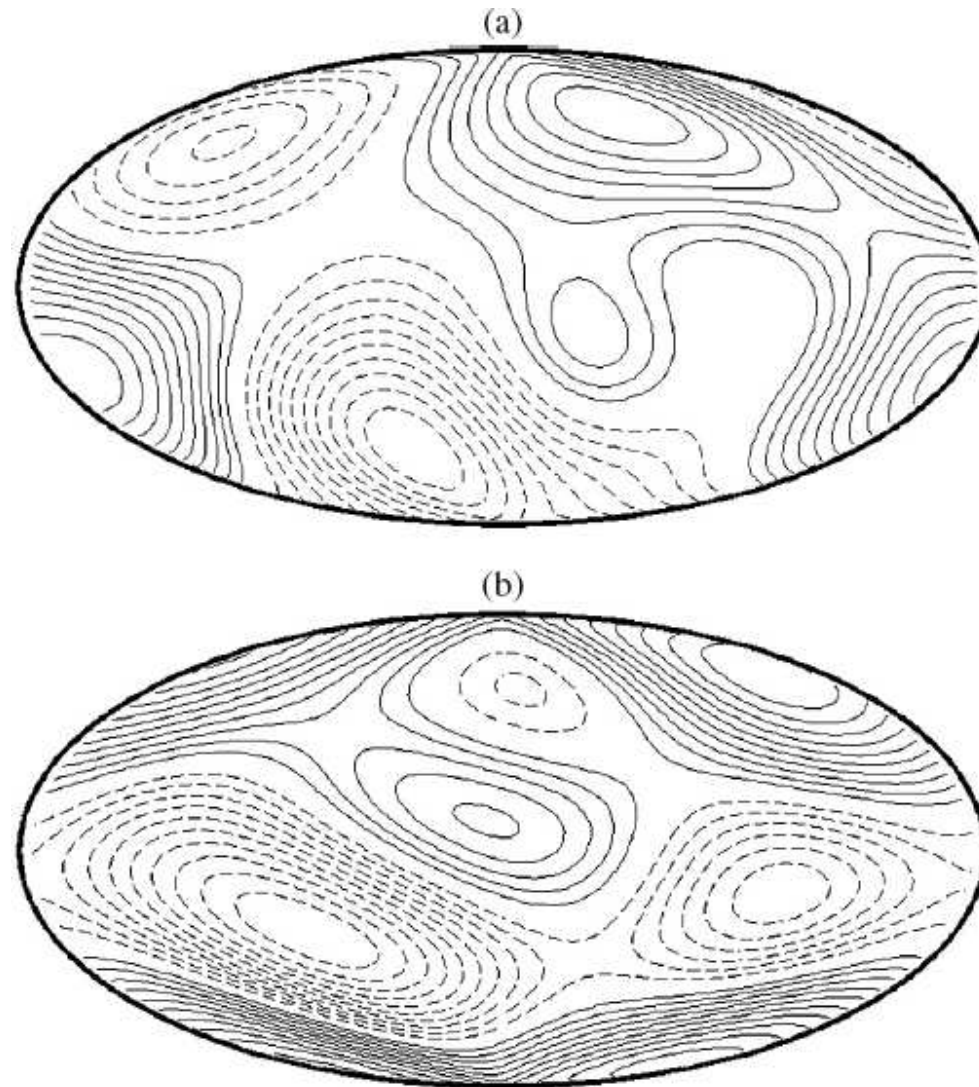
n	m	Jupiter ^a		Saturn ^b	
		g_n^m	h_n^m	g_n^m	g_n^m
1	0	4.208	0.	0.2144	0.
1	1	−0.660	0.261	−0.0014	0.0014
2	0	−0.034	0.	0.0188	0.
2	1	−0.759	−0.294	−0.0052	−0.0043
2	2	0.483	0.107	0.0050	−0.0004
3	0	...	0.	...	0.
3	1	...	...	...	...
3	2	0.263	0.695	...	...
3	3	−0.069	−0.247	...	...

^a from Connerney et al. (1982).

^b from Acuña et al. (1983).

Note that the dipole dominates on both planets, particularly Saturn. The octupole h_3 is larger in Jupiter than Earth.

Uranus magnetic field



Radial field at the surface of (a) Uranus, (b) Neptune reconstructed from Voyager data.

Dynamo theory for planetary magnetism

No permanent magnetism above the Curie point, 800°C for iron. Earth's outer core between 4000K and 5500K.

Thermoelectric effects, field induced by solar wind, suggested for Mercury, too small for Earth.

Fluid motion in the core produces dynamo action (Larmor, 1919).

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} \quad \text{Pre-Maxwell equation}$$

$$\mathbf{j} = \sigma(\mathbf{E} + \mathbf{u} \times \mathbf{B}) \quad \text{Ohm's law in a fluid moving at velocity } \mathbf{u}$$

Currents are generated by motion across magnetic field lines.

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} = \nabla \times (\mathbf{u} \times \mathbf{B}) - \nabla \times (\mathbf{j}/\sigma), \quad \text{Faraday's law}$$

Induction equation

So the magnetic field obeys the induction equation which is

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B},$$

if the electrical conductivity is assumed constant.

The induction term must overcome the diffusion term. The magnetic diffusivity $\eta = 1/\mu_0\sigma$, where σ is the electrical conductivity and $\mu_0 = 4\pi \times 10^{-7}$ is the permeability.

Earth's core magnetic diffusivity $\eta \sim 2 \text{ m}^2\text{s}^{-1}$

The ratio of induction to diffusion is approximately the Magnetic Reynolds number $R_m = U_*\ell/\eta$. Here U_* is the typical fluid velocity, ℓ the typical length scale, either outer core radius or gap between inner/outer core.

It can be proved that for induction to overcome diffusion in spherical geometry, R_m must be greater than π^2 , and in practice in the numerical dynamo simulations R_m of around 50 is needed for dynamo action.

Earth's core velocity $U_* \sim 4 \times 10^{-4} \text{ ms}^{-1}$, and Earth's core size $\ell \sim 3.5 \times 10^6 \text{ m}$, giving $R_m \sim 700$

U_* from secular variation studies 'Westward Drift'

Ohmic decay time $\ell^2 / \pi^2 \eta \sim 20,000$ years

How is the velocity at the CMB found from observing the geomagnetic field?

Core-flow inversion

Ignore diffusion, (large R_m) and take radial component of induction equation,

$$\frac{\partial \mathbf{B}}{\partial t} = -(\mathbf{u} \cdot \nabla) \mathbf{B} + (\mathbf{B} \cdot \nabla) \mathbf{u},$$

and since $u_r = 0$ and $du_r/dr = 0$ at CMB, get

$$\frac{\partial B_r}{\partial t} = -(\mathbf{u} \cdot \nabla) B_r. \quad (1)$$

Since B_r and $\partial B_r / \partial t$ can be observed, we might hope to use this equation to determine $\mathbf{u}$. Unfortunately, there are two unknown components of $\mathbf{u}$ and only one equation. Velocities along contours of constant B_r give no signal.

Assume tangential geostrophy, that is near CMB

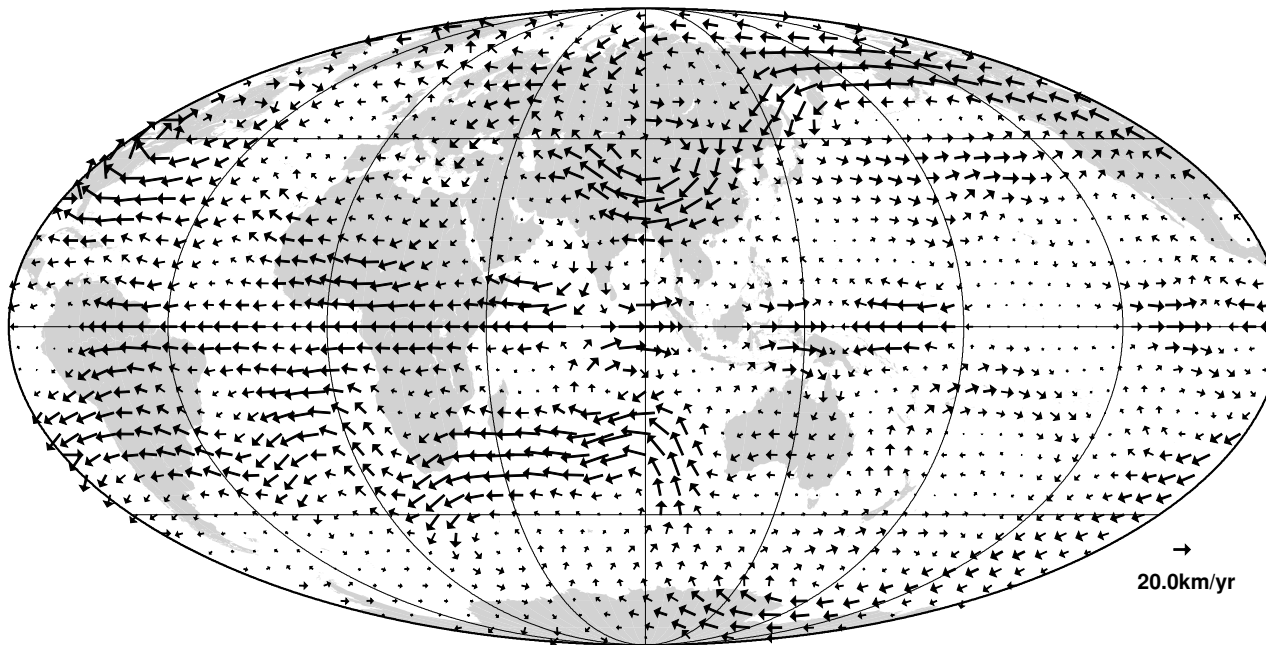
$$2\rho\Omega\hat{\mathbf{r}} \cos \theta \times \mathbf{u} = -\nabla p,$$

Reasonable, as buoyancy and Lorentz forces small close to CMB.

Taking the radial component of the curl of this force balance gives

$$\nabla \cdot \cos \theta \mathbf{u} = 0, \quad (2)$$

and (1) and (2) are enough to reconstruct $\mathbf{u}$.



20 km per year is 6×10^{-4} metres/sec. Slower than a snail! Note westward drift in S. Atlantic, Indian Ocean. Pacific has low secular variation. Waves or flow?

Magnetic field strength?

Equation of motion is

$$\rho \frac{D\mathbf{u}}{Dt} + 2\rho\boldsymbol{\Omega} \times \mathbf{u} = -\nabla p + \mathbf{j} \times \mathbf{B} + \rho\nu\nabla^2\mathbf{u} + \rho g\alpha\theta\hat{\mathbf{r}}$$

Induction equation is linear in $\mathbf{B}$ so expect exponential growth until Lorentz force modifies the velocity. How much modification necessary?

Simplest assumption is $|2\rho\boldsymbol{\Omega} \times \mathbf{u}| \sim |\mathbf{j} \times \mathbf{B}|$.

$$|\mathbf{j}| \sim |\mathbf{B}|/\mu\ell, \text{ so } |\mathbf{B}|^2 \sim 2\mu\rho\Omega\ell U_*.$$

Now $U_*\ell = \eta Rm$, so $|\mathbf{B}|^2 \sim 2\mu\rho\Omega\eta Rm$. This suggests a natural unit for $|\mathbf{B}|$ is $(\mu\rho\Omega\eta)^{1/2}$. For Earth values, this is about $13 \times 10^{-4}\text{T}$, about the observed value.

Elsasser number and Rossby number

The Elsasser number Λ is defined as $B_*^2/\rho\mu\Omega\eta$. Naive argument suggests $\Lambda \sim Rm$. Actually simulations give $\Lambda \sim 1$. Primary balance is geostrophic, pressure against Coriolis, and only small ageostrophic components are required to limit the dynamo.

Some more sophisticated magnetic field estimates later.

Ratio of kinetic energy to magnetic energy? If Inertia balances Lorentz force,

$\mathbf{u} \cdot \nabla \mathbf{u} \sim \mathbf{j} \times \mathbf{B}$, $u_*^2 \sim B_*^2/\mu$, so kinetic energy density and magnetic energy density are in balance. With rapid rotation,

$\mathbf{u} \cdot \nabla \mathbf{u} \ll 2\boldsymbol{\Omega} \times \mathbf{u}$, or the Rossby number $Ro = U_*/\ell\Omega \ll 1$. Since Lorentz force approximately balances Coriolis force, kinetic energy is much smaller than magnetic energy in the core, by a factor 10^3 .

Energy Sources for Planetary Dynamos

Dynamo energy source: Precession, Tidal interactions, Thermal Convection, Compositional Convection

Tides and precession derive their energy from the Earth's core rotation. Tides distort the CMB, precession is caused by the torques on the Earth's equatorial bulge. Earth's axis of rotation precesses once every 26,000 years.

$$\rho \frac{D\mathbf{u}}{Dt} + 2\rho(\boldsymbol{\Omega} + \boldsymbol{\Omega}_p) \times \mathbf{u} = -\nabla p + \mathbf{j} \times \mathbf{B} + \rho\nu\nabla^2\mathbf{u} - \rho(\boldsymbol{\Omega}_p \times \boldsymbol{\Omega}) \times \mathbf{r}$$

in the mantle frame. $\boldsymbol{\Omega}_p$ is the precessional rotation, 10^{-8} of daily rotation. Last term is the Poincaré force, replacing the buoyancy force. Poincaré force drives a large scale flow, which can become unstable, and the secondary flows, which are high frequency inertial waves, can drive a dynamo. Successful numerical simulations have been done, but only with rotation ratios of 10^{-3} . Stabilised even by very small viscosity, so not clear it works in Earth's core.

Compositional convection

Compositional convection (Braginsky, 1963): as the inner core grows, dense iron freezes at the ICB

This releases lighter buoyant material which rises, stirring the fluid outer core.

This process liberates about 0.5 TW of gravitational energy.

Latent heat is also released (about 4TW), driving thermal convection.

Cooling may also induce phase changes at the outer core boundary. This could release heavier material which falls through the fluid core.

‘Helium rain-out’ in Saturn.

Thermal convection

Thermal convection only occurs if the heat flux produced is greater than the amount that can be carried by conduction. A plausible reason for some planets like Venus not having a dynamo is that all the heat flux in the core is carried by conduction.

When convection occurs, it is very efficient, so although the temperature gradient must be steeper than adiabatic (superadiabatic), it is only a tiny amount steeper than adiabatic.

For deriving the basic structure of the core, assuming adiabatic is a good approximation.

If $\alpha = \rho^{-1}(\partial\rho/\partial T)_{ad}$ is the coefficient of thermal expansion, and $c_p = \alpha T \rho^{-1}(\partial P/\partial T)_{ad}$ is the specific heat at constant pressure, then hydrostatic balance gives

$$\frac{dp}{dr} = -\rho g, \quad \frac{dT}{dr} \left(\frac{\partial P}{\partial T} \right)_{ad} = -\rho g.$$

Adiabatic temperature equation

The adiabatic temperature gradient is then

$$T_{ad}^{-1} \left(\frac{dT}{dr} \right)_{ad} = -\frac{g\alpha}{c_p},$$

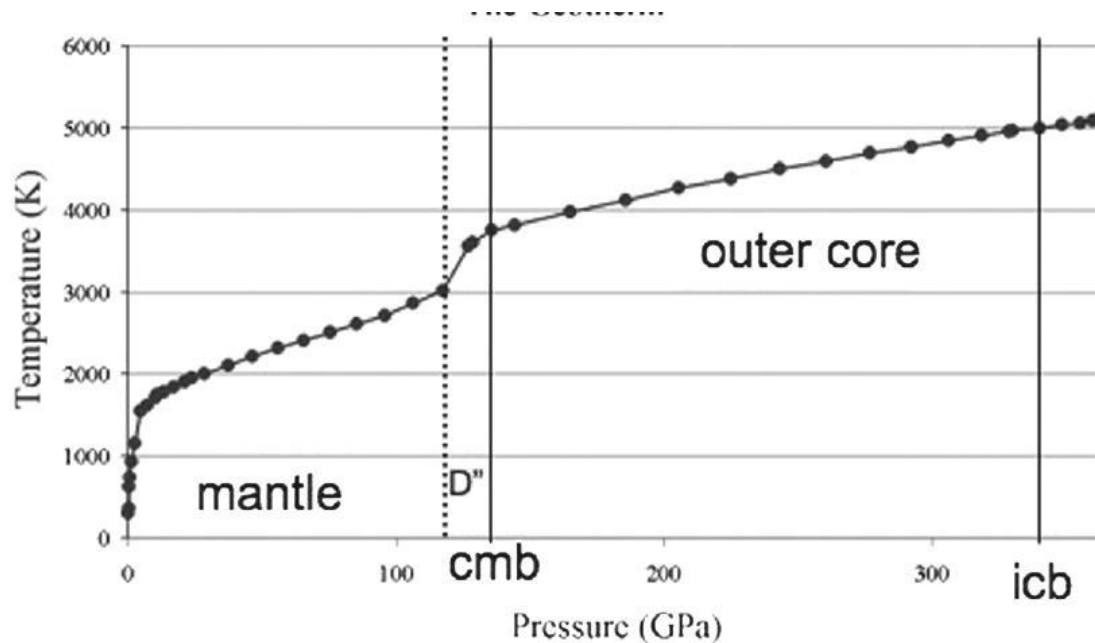
The heat flux carried down this gradient by conduction is

$$F_{ad} = -\kappa \rho c_p \left(\frac{dT}{dr} \right)_{ad}$$

For convection need the actual heat flux $F > F_{ad}$

Even if no convection occurs, heat is still being conducted down the adiabat, so the core is cooling and the inner core is growing.

Temperature inside the Earth



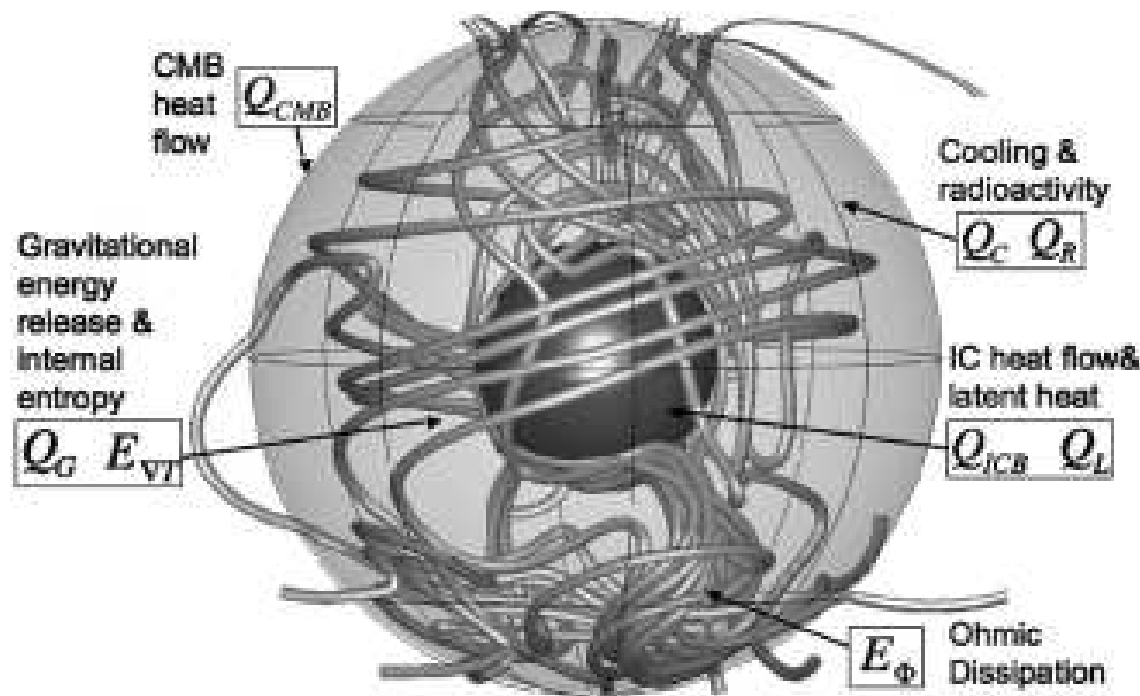
At bottom of mantle, thermal boundary layer D'' .

In the outer core, geotherm is close to adiabatic.

Heat flux carried by conduction only ~ 0.3 TW at the ICB, rising to ~ 6 TW at the CMB. Mostly because of larger surface area.

If latent heat is dominant heat source, possible that core is superadiabatic (convecting) near ICB and subadiabatic (stable) near CMB.

Energy sources in Earth's core



Amount of heat taken out by the mantle determines rate of core cooling, hence growth rate of inner core.

All sources are uncertain to a factor 2. $Q_L \sim 4\text{TW}$, $Q_C \sim 2\text{TW}$, $Q_{CMB} \sim 6 - 12\text{TW}$. Major uncertainty is whether there is significant radioactivity in the core. Very controversial.

Efficiency of thermal convection

Entropy balance Q/T needed to estimate what fraction of thermal heat flux can be extracted to drive dynamo.

Inner Core Boundary $\approx 5,300 \pm 200^\circ\text{K}$

Core-Mantle Boundary $\approx 4,000 \pm 100^\circ\text{K}$

Energy balance in the fluid outer core gives

$$Q_{\text{CMB}} = Q_{\text{ICB}} + Q^L + Q^S + Q^G + Q^R, \quad (*)$$

where Q^L is latent heat, Q^S is secular cooling, Q^G is gravitational energy, Q^R is radioactivity.

Entropy balance

$$\frac{Q_{\text{CMB}}}{T_{\text{CMB}}} = \frac{Q_{\text{ICB}} + Q^L}{T_{\text{ICB}}} + \Sigma + \frac{Q^D}{T_D} + \frac{Q^S + Q^R}{\bar{T}}, \quad (**)$$

Here Σ is entropy source due to thermal conduction, Q^D is the dissipation, which we assume is mainly ohmic. The T are the temperatures at which these processes occur. From (*) and (**)

$$Q^D = \frac{T_D}{T_{\text{CMB}}} \left[(Q_{\text{ICB}} + Q^L) \left(1 - \frac{T_{\text{CMB}}}{T_{\text{ICB}}} \right) + (Q^S + Q^R) \left(1 - \frac{T_{\text{CMB}}}{\bar{T}} \right) + Q^G - \Sigma T_{\text{CMB}} \right].$$

This suggests only about 0.5 TW of the heat flux can go into dissipation, which means that goes into magnetic field creation.

Total available is therefore order of 1 TW. Assumes viscous dissipation probably small compared to ohmic dissipation.

Wiedemann-Franz law

Thermal conductivity is linked to electrical conductivity in the cores of terrestrial planets.

Wiedemann-Franz law $\kappa\rho c_p = 0.02T/\eta$.

High electrical conductivity (low η) implies high thermal conductivity, making F_{ad} large.

Stevenson's paradox: high electrical conductivity is **bad** for dynamos, because it makes F_{ad} larger than F thus stopping convection!

Inner Core problem

Inner core solid because of high pressure, but temperature must be less than 6000K.

If we use all these estimates to work out Earth's rate of cooling, we discover that the inner core has only been in existence for about 1 billion years.

Rocks aged 3.5 billion years have remanent magnetism, and paleomagnetic record shows no sign of interruption.

No inner core means no compositional convection, and no latent heat, so much less heat production. Estimates suggest all core heat production could then be carried by conduction, so no dynamo!

Reason for believing in radioactive potassium in the core?

Other terrestrial planets

Three ideas for Mercury's weak field:

- (i) No dynamo, but solar wind field amplified.
- (ii) As Mercury cooled, inner core grew. Remaining liquid outer core has progressively higher impurity content, so thin shell of liquid iron. Thin shell dynamos have high harmonic content, low dipole content.
- (iii) Most of Mercury's core is liquid, but mostly stably stratified as heat produced mostly carried by conduction. Dynamo only near ICB, only small fraction of field penetrates stably stratified upper core.

Messenger should be able to decide between these theories.

Venus rotates only once every 243 days. But Rossby number $U_*/\ell\Omega$ with Earth-like U_* would still be small.

Venus appears to have no plate tectonics. This could reduce F and hence make Venus's core subadiabatic.

Venus seems to have been resurfaced 300 million years ago. Possibly there was mantle convection then, and possibly a dynamo. High surface temperature unfavourable for remanent magnetism.

Ganymede has a weak surface field, but a small core, so at CMB field is ~ 2 Gauss, giving an Elsasser number of order unity. Surprising that there is sufficient heating in the core to make it convect.

Collapse of Martian dynamo possibly due to core becoming stably stratified.

Dynamos in the Giant Planets

Electrical conductivity is due to very high pressure ionising electrons (quantum effect). Earth-like conductivity in the deep interior, gradually falling to low values near the surface.

Giant planets are convecting, so thermal convection is most natural energy source.

Some secular variation occurs on Jupiter, suggesting core flow $\sim 10^{-3}$ metres/sec. Rm large.

Where is the dynamo most efficient? Possibly nearer outside, where local Rm not so big?

Why is Saturn's field so axisymmetric? Stably stratified region with strong differential rotation outside the dynamo region? This could axisymmetrise the observed field.

Dynamos in the Ice Giants

Uranus and Neptune have very unusual fields.

Two suggestions for this:

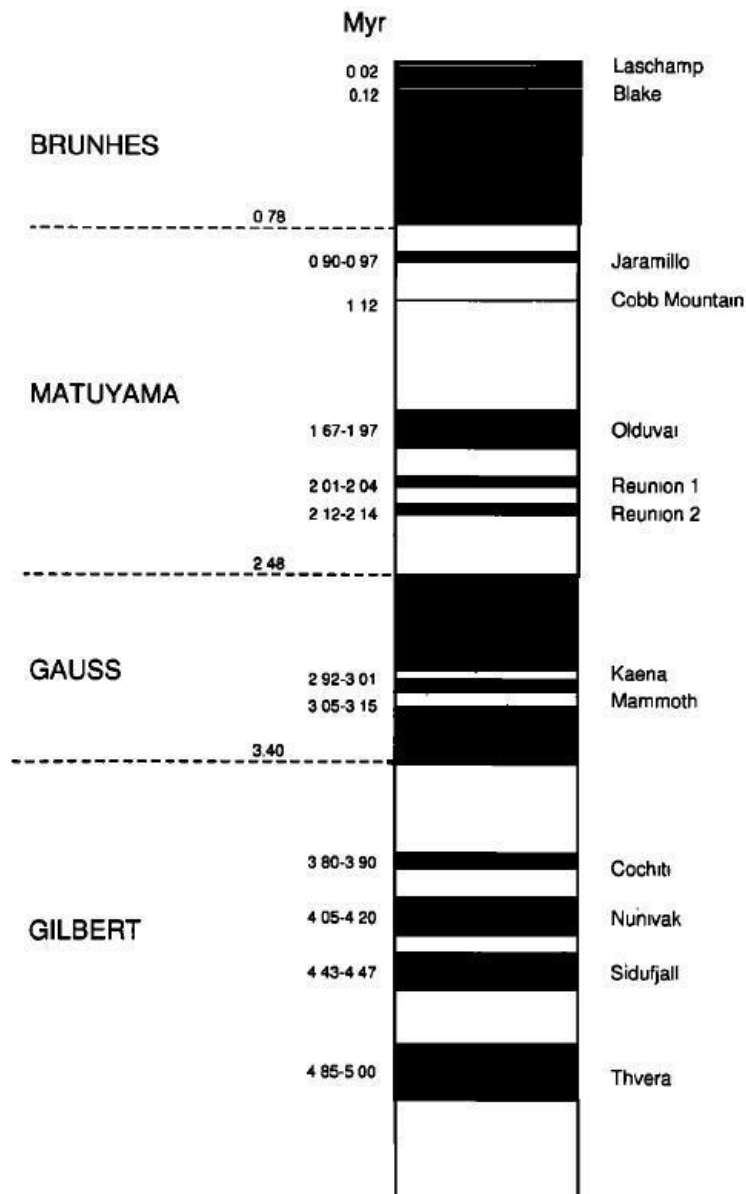
(i) Thin shell dynamo. In a thin shell there is not much connection between the different hemispheres, so more complex field is possible. Problem is that the location of the thin shell is far below the surface, so non-dipolar components would be strongly attenuated.

(ii) Sabine Stanley suggested that a dynamo shell above a stably stratified fluid layer can produce nondipolar fields. The fluid motion in the core reduces the stabilising effect of a solid inner core, leading to more complex fields.

Not much known about internal structure and heat flux. Ionic conductivity is low, so Rm cannot be very large.

Reversals

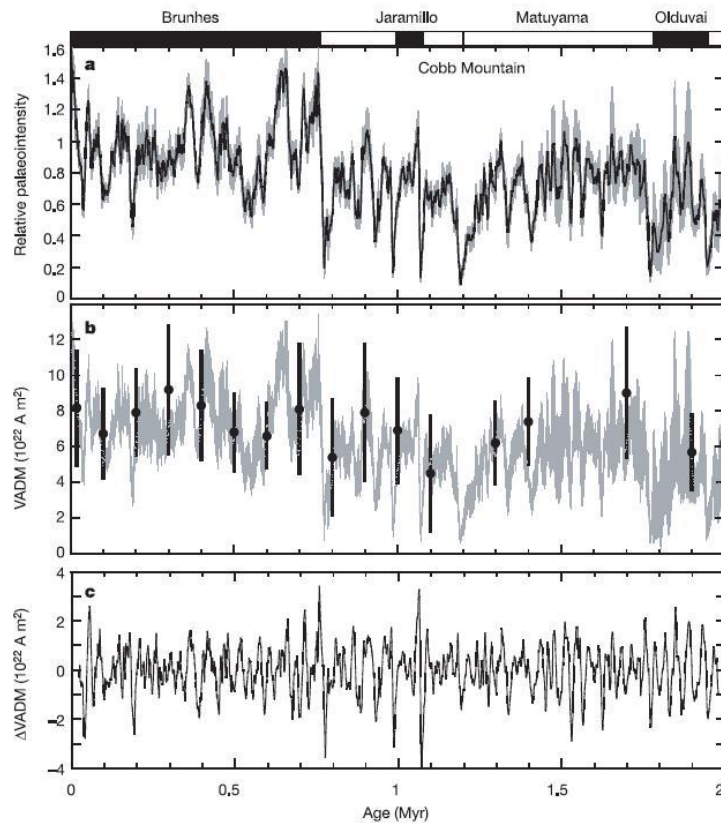
Geomagnetic reversal record: last 5 million years



The Earth's field reverses randomly about once every 0.3 million years. There have been periods as long as 60 million years with no reversals (superchrons). Reversals occur relatively quickly $\sim 10,000$ years. During a reversal, the dipole axis traces out a path at the Earth's surface: are there preferred longitudes?

Excursions

Dipole component varies significantly between reversals, the so-called excursions.



Current decline may be an excursion rather than a reversal.

Major Problems in Planetary Dynamo theory

- (i) Why is Mercury's field so weak?
- (ii) Why does Venus not have a magnetic field?
- (iii) Why are most planets dipole dominated? Why do geomagnetic reversals occur?
- (iv) What powered the geodynamo before inner core formation?
- (v) What killed off the Martian dynamo?
- (vi) How does Ganymede maintain a dynamo when its core is so small?
- (vii) Why is Saturn's field so axisymmetric?
- (viii) Why are the fields of Uranus and Neptune non-dipolar?