Mean field dynamos and magnetic helicity

Kandaswamy Subramanian [a]

Axel Brandenburg (Nordita), Anvar Shukurov (Newcastle Univ.), Sharanya Sur (Heidelberg), Dimitry Sokoloff (Moscow), Sanved Kolekar (IUCAA), S. Sridhar (RRI)

[a] Inter-University Centre for Astronomy and Astrophysics, Pune 411 007, India.

Plan

- MHD Basics
- The fluctuation dynamo
- Magnetic Helicity
- The mean-field dynamo at large R_m
- Magnetic Helicity and mean field dynamos
- MTA and The dynamical quenching model
- Magnetic helicity density, flux and dynamical quenching

A. Brandenburg & K. Subramanian, Physics Reports, 417, 1-205 (2005)

The Magnetic Universe

Why Magnetic Universe?

- Electrically Neutral Universe
  - Both $+$ve and $-$ve Charges Present
- Free Electic charges + No magnetic charges $\Rightarrow$
  - Can short out strong $\mathbf{E}$ in plasma rest frame
- Non Relativistic Velocities $\Rightarrow$
  - Simpler to think interms of $\mathbf{B}$ than $\mathbf{E}$

Why think about B fields?

- **Gives "fluid" like property to hot/dilute plasma**
  - $\lambda_{spitzer} \approx 5\mathrm{kpc}\ (c_s/10^3 \mathrm{kms}^{-1})^4\ (n/10^{-3}\mathrm{cm}^{-3})^{-1}$
- **Affects transport: Thermal conduction, viscosity, resistivity**
- **Controls star formation? Typical masses and IMF?**
  - **Is B feedback as important as Z feedback for first stars?**
- **Helps Accretion - Jets: Relevance to first AGN?**
- **Cosmic ray confinement/transport/heating in ISM, IGM?**
- **Makes objects "visible" : Synchrotron/Inverse compton**
- **Primordial fields modify/induce structure formation**
- **Need to explain ordered fields in Sun (11 yr cycle), Galaxies** ($\sim 10\mu$G, upto 10 kpc scale) **and clusters** (few μ G, kpc to 10 kpc scale)

Fun to think about!

MHD basics: The Induction equation

- **Maxwell equations**

$$\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \quad \nabla \cdot \mathbf{B} = 0,$$

$$\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{B} - \frac{4\pi}{c}\mathbf{J}, \quad \nabla \cdot \mathbf{E} = 4\pi\rho_{\mathrm{e}},$$

- **Ohm's law**

$$\mathbf{J} = \rho_{\mathrm{e}}\mathbf{U} + \sigma\left(\mathbf{E} + \frac{\mathbf{U}\times\mathbf{B}}{c}\right)$$

- **Eliminate $\mathbf{J}$ using Ohm+Maxwell, define resitivity** $\eta = c^2/(4\pi\sigma)$

$$\frac{\eta}{c^2}\frac{\partial \mathbf{E}}{\partial t} + \frac{\eta}{c^2}\mathbf{U}(\nabla\cdot\mathbf{E}) + \mathbf{E} = \frac{\eta}{c}(\nabla\times\mathbf{B}) - \frac{\mathbf{U}\times\mathbf{B}}{c}.$$

- **Faraday timescale** $\eta/c^2 \sim 10^{-14} T_4^{-3/2} s$ **for ionized plasma!**

MHD basics: The Induction equation

- Variation timescale (or advective timescale) of E usually much smaller than Faraday time

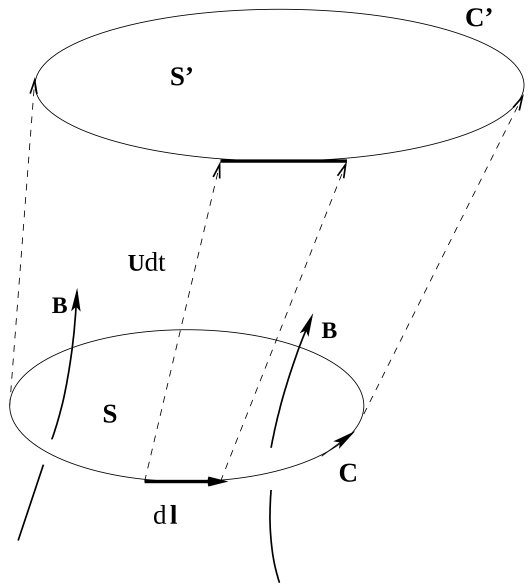
$$\frac{\eta}{c^2}\frac{\partial \mathbf{E}}{\partial t} + \frac{\eta}{c^2}\mathbf{U}(\nabla\cdot\mathbf{E}) + \mathbf{E} = \frac{\eta}{c}(\nabla\times\mathbf{B}) - \frac{\mathbf{U}\times\mathbf{B}}{c}.$$

- So neglect displacement and advective currents terms, take curl, use Faraday $\Rightarrow$ Induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{U}\times\mathbf{B} - \eta\nabla\times\mathbf{B}).$$

  - $\mathbf{U}=0 \Rightarrow$ pure diffusion and decay
  - If $\eta\to 0$, the flux $\Phi=\int_S \mathbf{B}\cdot d\mathbf{S}$ is 'frozen' $\to d\Phi/dt \to 0$.
  - Mag. Reynolds number $R_{\mathrm{M}} = (UB)/(\eta B/L) = UL/\eta \gg 1$

Flux Freezing



- **Magnetic flux:** $\Phi = \int_S \mathbf{B} \cdot \mathrm{d}\mathbf{S}$.

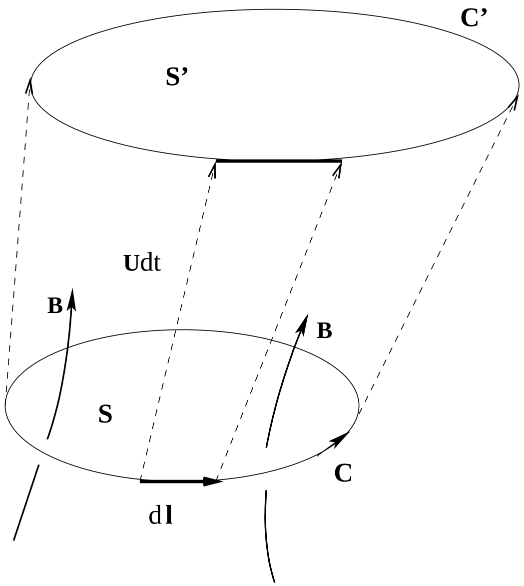
$$\Delta\Phi = \int_{S'} \boldsymbol{B}(t+dt)\cdot\mathrm{d}\mathbf{S} - \int_S \boldsymbol{B}(t)\cdot\mathrm{d}\mathbf{S}.$$

- $\nabla \cdot \boldsymbol{B} = 0$ **at time** $t + dt \Rightarrow$

$$\int_{S'} \boldsymbol{B}(t+dt) \cdot \mathrm{d}\mathbf{S} = \int_S \boldsymbol{B}(t+dt) \cdot \mathrm{d}\mathbf{S} - \oint_C \boldsymbol{B}(t+dt) \cdot (d\mathbf{l} \times \mathbf{U}dt),$$

$$\Delta\Phi = \int_S [\boldsymbol{B}(t+dt) - \boldsymbol{B}(t)] \cdot \mathrm{d}\mathbf{S} - \oint_C \boldsymbol{B}(t+dt) \cdot (d\mathbf{l} \times \mathbf{U})dt.$$

Flux Freezing



$$\Delta\Phi = \int_S [\boldsymbol{B}(t+dt) - \boldsymbol{B}(t)]\cdot \mathrm{d}\mathbf{S} - \oint_C \boldsymbol{B}(t+dt)\cdot(d\mathbf{l}\times\mathbf{U})dt.$$

$$\frac{d\Phi}{dt} = \int_S \frac{\partial \boldsymbol{B}}{\partial t}\cdot \mathrm{d}\mathbf{S} - \oint_C (\mathbf{U}\times\boldsymbol{B})\cdot d\mathbf{l}$$

- Using $\oint_C (\mathbf{U}\times\boldsymbol{B})\cdot d\mathbf{l} = \int_S \nabla\times(\mathbf{U}\times\boldsymbol{B})\cdot\mathrm{d}\mathbf{S}$

$$\frac{d\Phi}{dt} = \int_S \left[\frac{\partial \boldsymbol{B}}{\partial t} - \nabla\times(\mathbf{U}\times\boldsymbol{B})\right]\cdot \mathrm{d}\mathbf{S} = \eta\int_S (\nabla^2\boldsymbol{B})\cdot\mathrm{d}\mathbf{S}.$$

- **So as $\eta\to 0$, $d\Phi/dt \to 0$ and so Φ is constant.**

Flux freezing applications

- **Consider a flux tube with field B, length l, density ρ**

$$BA = \text{Constant}, \qquad \rho A l = \text{Constant} \rightarrow \frac{B}{\rho l} = \text{Constant}$$

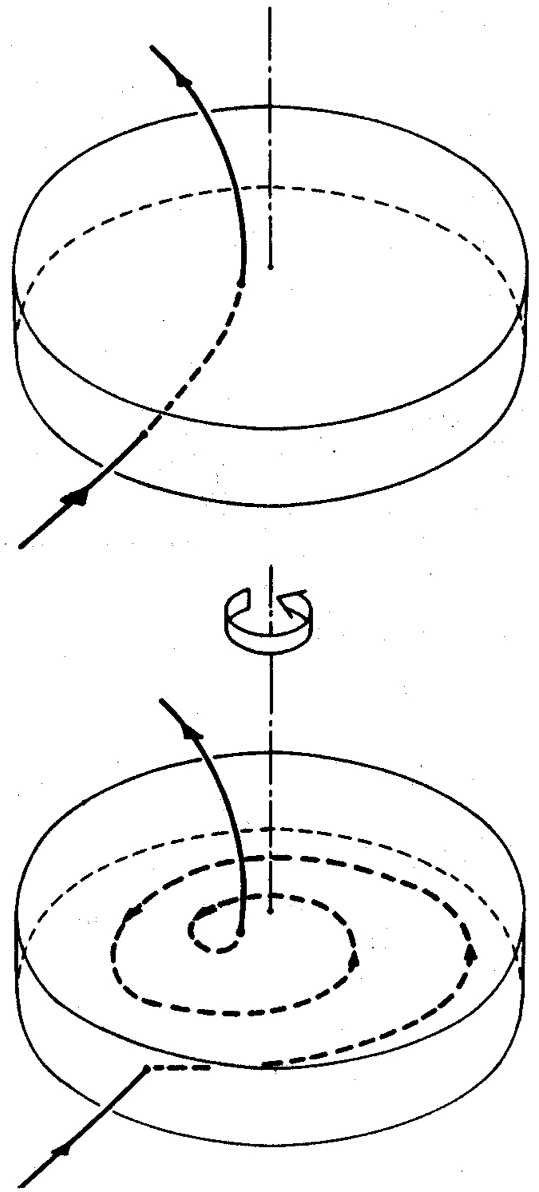
- **Compressing a flux tube of constant lenth l increases B**

- **Incompressible fluid motion with $\rho = \text{Constant}$ gives $B \propto l$ and $A \propto 1/l$: Application to small-scale dynamo**

- **Ingoring diffusion, one can expand induction equation as**

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{U} \times \boldsymbol{B}) = -\underbrace{(\boldsymbol{U} \cdot \boldsymbol{\nabla})\boldsymbol{B}}_{\text{advection}} + \underbrace{(\boldsymbol{B} \cdot \boldsymbol{\nabla})\boldsymbol{U}}_{\text{stretching}} - \underbrace{\boldsymbol{B}(\boldsymbol{\nabla} \cdot \boldsymbol{U})}_{\text{compression}},$$

Galactic Shear



- Stretching due to differential rotation in galaxy
- Let $U = (0, r\Omega(r), 0)$, initially axisymmetric B
- Leads to winding up the field: $B_r = B_r(r,z)$
  $B_\phi =$
  $B_\phi(r,z,t_0) + r(d\Omega/dr)B_r t$
  $B_z = B_z(r,z))$

The Lorentz Force

- **The Lorentz force on a charge:** $\mathbf{F} = q[\boldsymbol{E} + (\boldsymbol{u} \times \boldsymbol{B})/c]$

- **The Lorentz force on fluid $\mathbf{F}$ is** :

$$
\begin{aligned}
\mathbf{F} &= +en_p\left[\mathbf{E} + \frac{\mathbf{u_p} \times \boldsymbol{B}}{c}\right] - en_e\left[\mathbf{E} + \frac{\mathbf{u_e} \times \boldsymbol{B}}{c}\right] \\
&= [en_p - en_e]\mathbf{E} + [en_p\mathbf{u_p} - en_e\mathbf{u_e}] \times \boldsymbol{B}/c \\
&= \rho_e\mathbf{E} + \frac{\mathbf{J} \times \boldsymbol{B}}{c}
\end{aligned}
$$

- **Charge density** $\rho_e = (+en_p - en_e)$

- **Current density:** $\mathbf{J} = (+en_p\mathbf{u_p} - en_e\mathbf{u_e})$

The neglect of the Electric force

- Including the charge density the Lorentz force is:

$$\mathbf{F} = \rho_e \mathbf{E} + \frac{\mathbf{J} \times \boldsymbol{B}}{c}$$

- The ratio of the electric to magnetic parts is:

$$\frac{|\rho_e \mathbf{E}|}{|(\mathbf{J} \times \boldsymbol{B}/c)|} = \frac{|(\nabla \cdot \mathbf{E})\mathbf{E}|}{|(\nabla \times \boldsymbol{B}) \times \boldsymbol{B})|} \sim \frac{U^2}{c^2} \ll 1,$$

- The last part follows from Ohms law applied to a highly conducting medium: $\mathbf{E} + (\mathbf{U}/c) \times \boldsymbol{B} = \mathbf{J}/\sigma \sim 0$.

- So electric part of force can be neglected compared to magnetic part for $U/c \ll 1$.

The Lorentz force

- Using $\nabla \times \boldsymbol{B} = (4\pi/c)\mathbf{J}$

$$\mathbf{F} = \frac{(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}}{4\pi} = \underbrace{-\nabla\left(\frac{\boldsymbol{B}^2}{8\pi}\right)}_{\text{Pressure}} \underbrace{+\frac{(\boldsymbol{B}\cdot\nabla)\boldsymbol{B}}{4\pi}}_{\text{Tension}}$$

- **Magnetic force broken into a "Pressure gradient" and Tension force**

Magnetic energy evolution

- **Start from Faraday's Law and dot with B**

$$\frac{1}{2}\frac{\partial B^2}{\partial t} = \boldsymbol{B}\cdot\frac{\partial \boldsymbol{B}}{\partial t} = -c\boldsymbol{B}\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}) = -c\boldsymbol{\nabla}\cdot(\boldsymbol{E}\times\boldsymbol{B}) - c\boldsymbol{E}\cdot(\boldsymbol{\nabla}\times\boldsymbol{B})$$

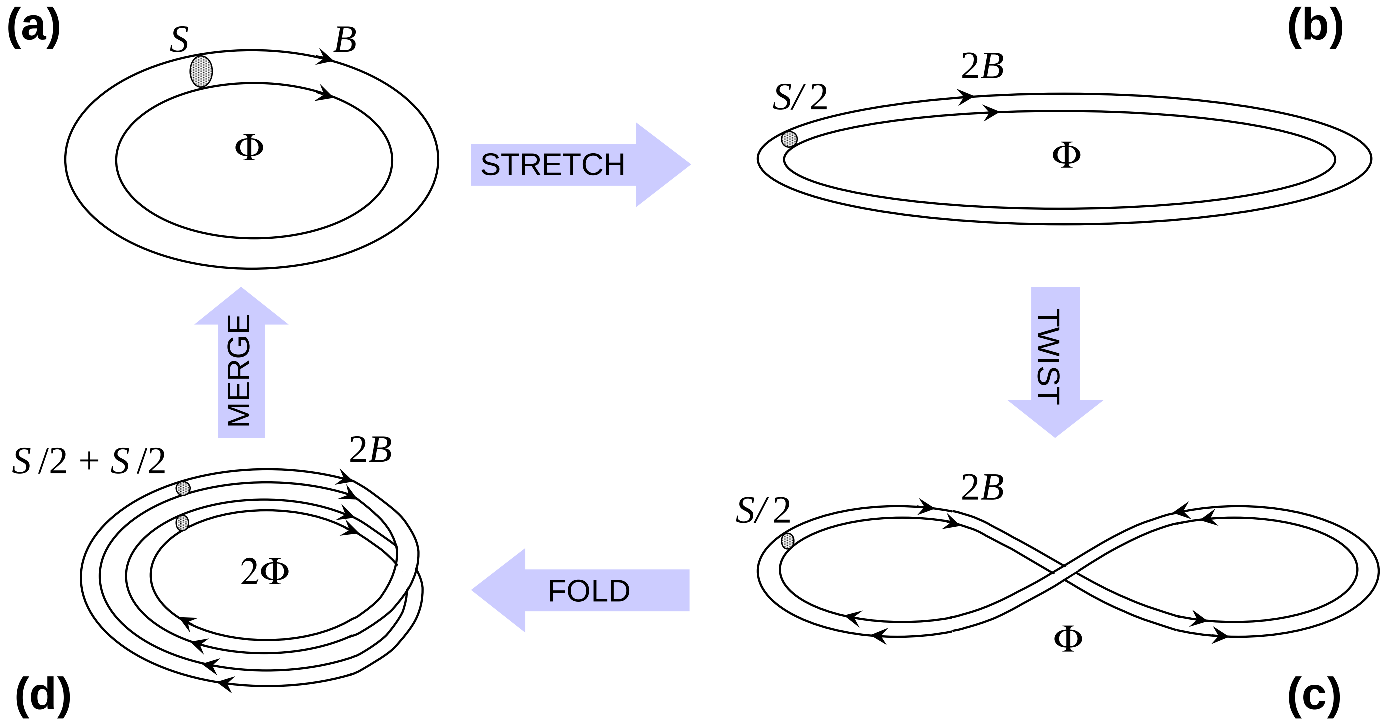
- **Use Ohm's Law**: $\boldsymbol{E} = -(\boldsymbol{U}\times\boldsymbol{B})/c + (4\pi/c^2)\eta\boldsymbol{J}$

$$\frac{1}{2}\frac{\partial B^2}{\partial t} = -c\boldsymbol{\nabla}\cdot(\boldsymbol{E}\times\boldsymbol{B}) - (4\pi/c^2)\eta\boldsymbol{J}\cdot(\boldsymbol{\nabla}\times\boldsymbol{B}) + (\boldsymbol{U}\times\boldsymbol{B})\cdot(\boldsymbol{\nabla}\times\boldsymbol{B})$$

- **Shuffle the triple product, devide by 4π**

$$\frac{1}{8\pi}\frac{\partial B^2}{\partial t} + \underbrace{\boldsymbol{\nabla}\cdot\left[\frac{c}{4\pi}\boldsymbol{E}\times\boldsymbol{B}\right]}_{\text{Poynting flux}} = \underbrace{\frac{4\pi}{c^2}\eta\mathbf{J}^2}_{\text{Dissipation}} \underbrace{-\mathbf{U}\cdot\frac{(\mathbf{J}\times\mathbf{B})}{c}}_{\text{LF Work}}$$

Zeldovich STF dynamo

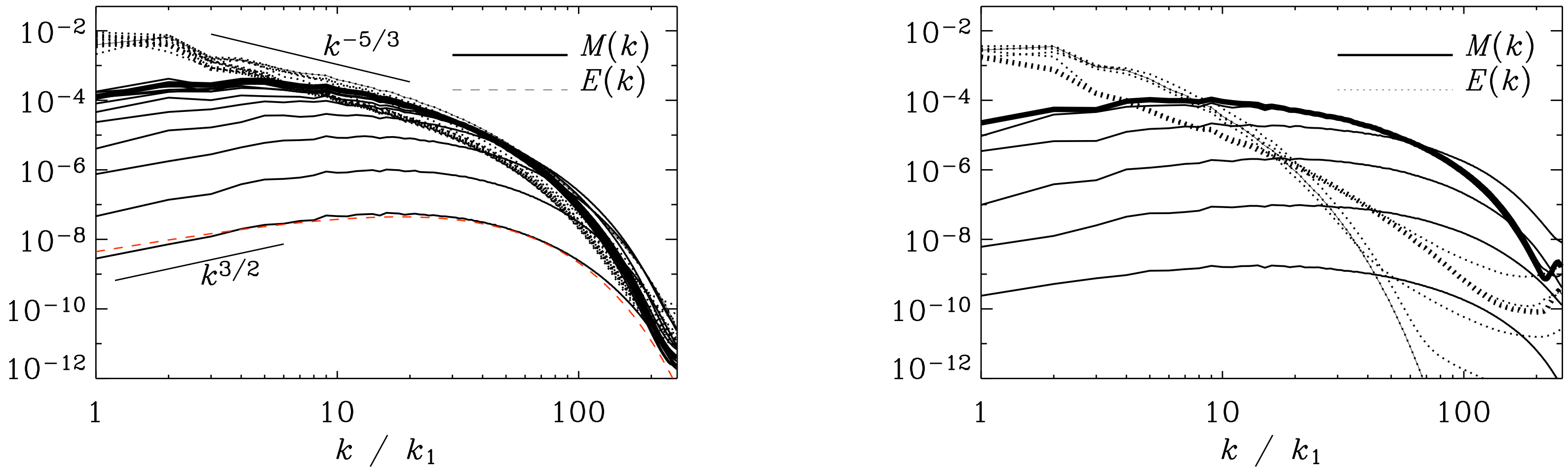


- Field grows by factor 2^N; Energy from motions
- Flux through Eulerian surface grows though nearly frozen!
- Twist and 3d motion required for coherent growth
- Eventual saturation through Lorentz forces.

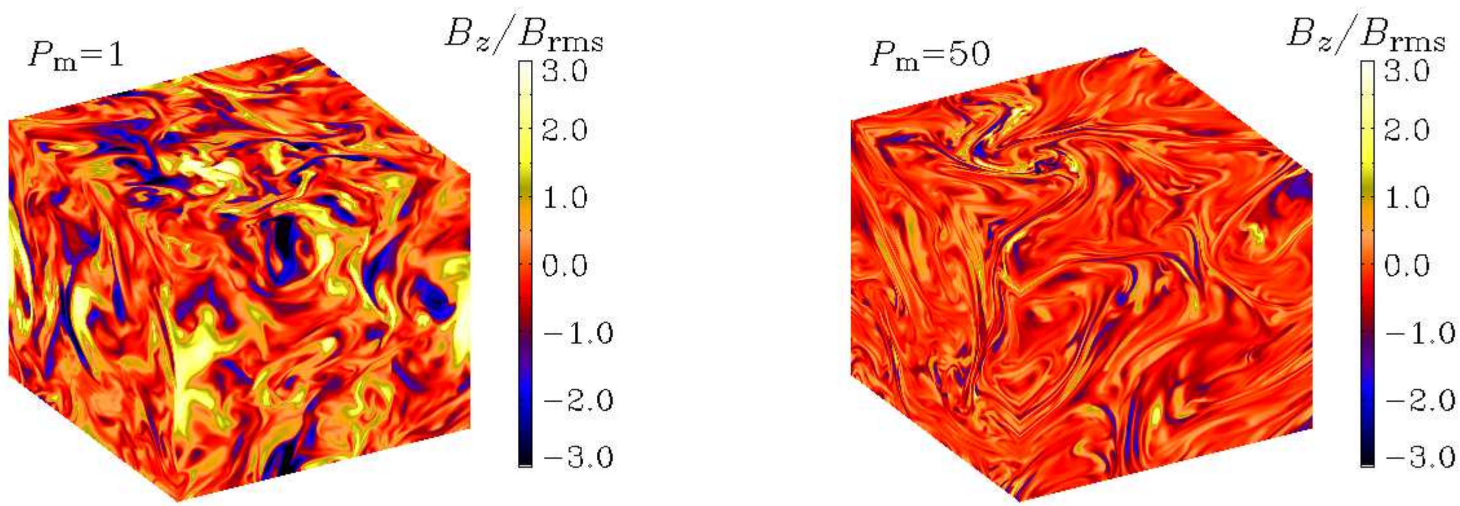
The fluctuation dynamo at large R_m

- **Turbulence $\rightarrow$ Random Stretching;**
- **Then Flux freezing $\Rightarrow$ Growth of B**
- $BA =$ **constant and** $\rho AL =$ **constant** $\rightarrow B/\rho \propto L$**, and** $A \propto 1/(\rho L)$
- **Diffusion $\sim$ growth when** $v/L \sim \eta/l_B^2$ **or** $l_B \sim L/R_{\mathrm{M}}^{1/2}$
- **Random B grows if** $R_{\mathrm{M}} > R_{crit} \sim 30 - 100$ (Kazantsev 1967)
- **Growth rate $\Gamma \sim v/L$, field intermittent, concentrated in scales** $\sim L/R_m^{1/2}$
- **For $l \sim 10$ km s^{-1} and $L \sim 100$ pc, growth time $\Gamma^{-1} \sim 10^7$ yr**
- **How does it saturate? Impt. for mean field dynamo?**

Fluctuation dynamo simulations

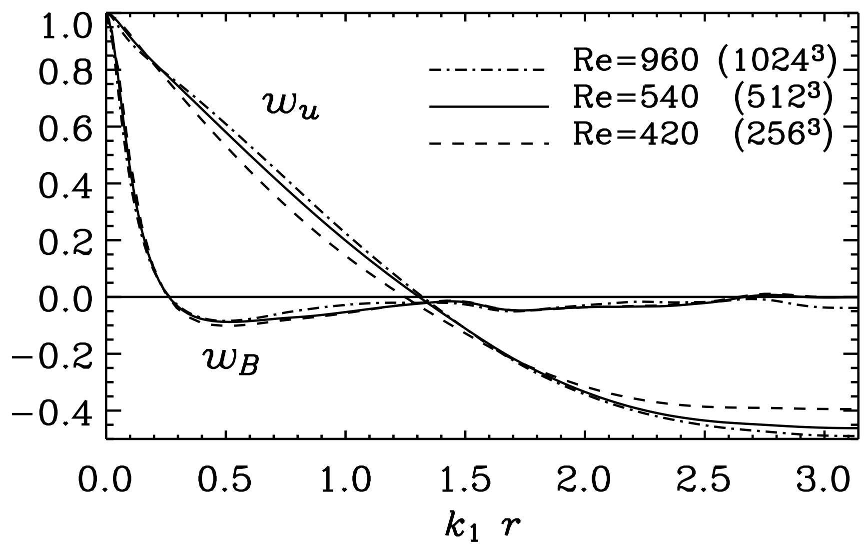


Generated B intermittent :Simulations by Axel Brandenburg, 2005



Fluctuation dynamo saturation?

- **Renormalized η drives efective $R_{\rm M} \to R_{crit}$, $l_B \sim L/R_{crit}^{1/2}$,** **Saturated state universal** (Subramanian, PRL, 1999; 2003).
- **Magnetic autocorrelation function independent of $R_{\rm M}$, for $P_m = \nu/\eta = 1$** (Haugen, Brandenburg, Dobler, 2003, 2004)

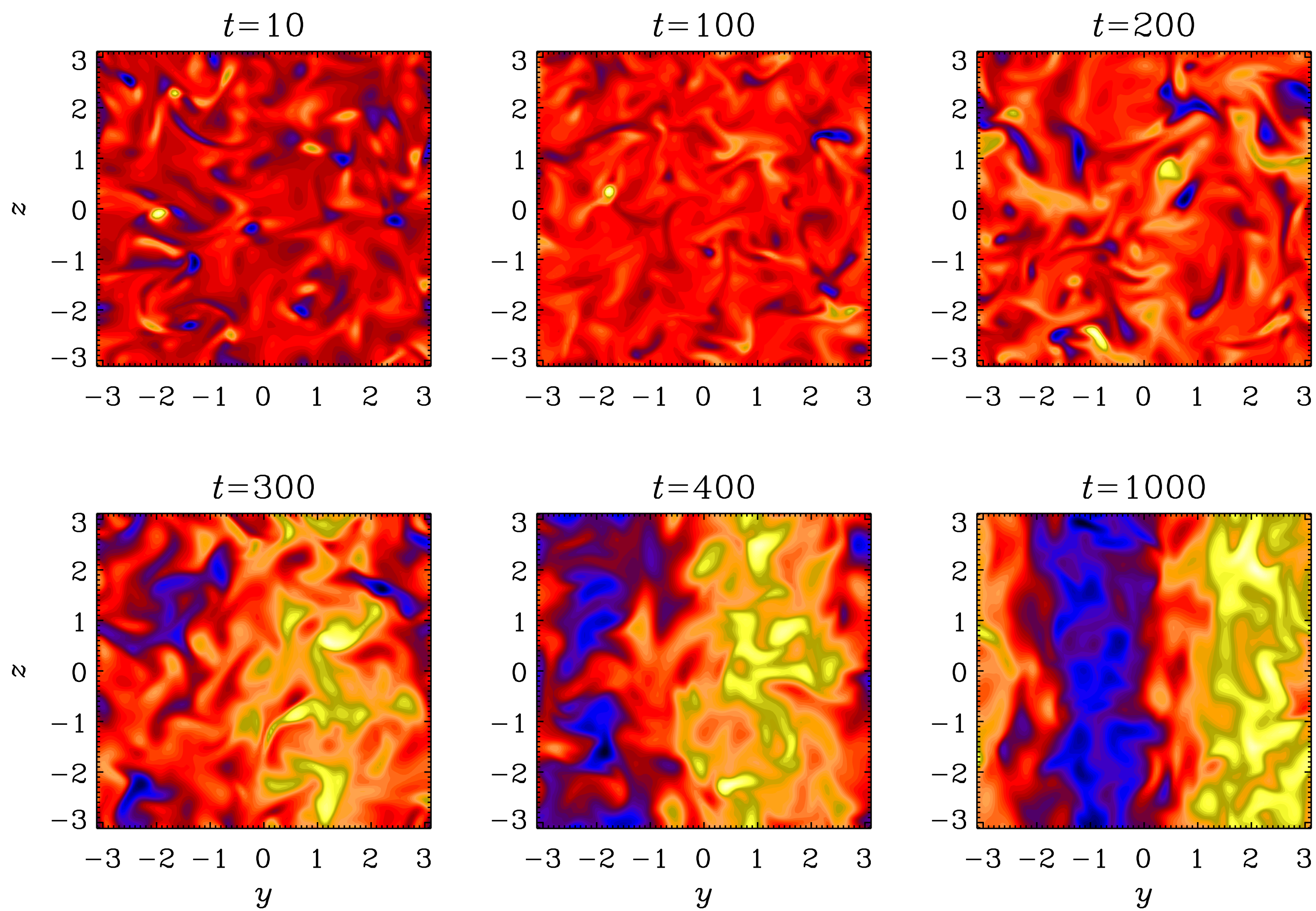


- **Saturation due to Reduced stretching BUT $l_B \sim L/R_{\rm M}^{1/2}$!** **Plasma effects crucial** (Schekochihin, Cowley et al., ApJ, 2004, 2006)

Important for Cluster/young galaxy Faraday RM/CR confinement

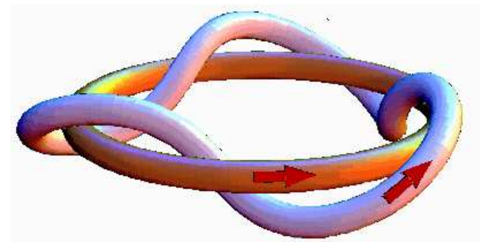
Helically forced turbulent dynamos

Axel Brandenburg, Ap.J. 550, 824 (2001)



Magnetic Helicity

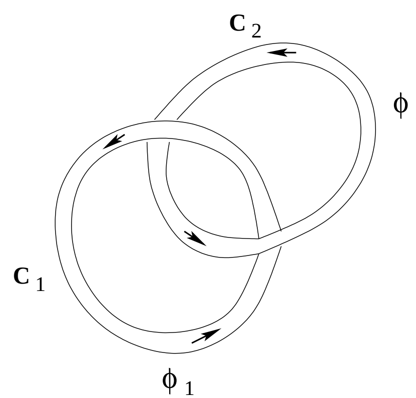
- **Magnetic Helicity** $H = \int_V \mathbf{A} \cdot \mathbf{B} \, dV$, $\quad \nabla \times \boldsymbol{A} = \boldsymbol{B}$
  $\mathbf{A}$ is vector potential, V is **closed** volume
- **Measures links and twists in** B



- H **invariant under gauge transformation for closed fields**

$$H' = \int_V \boldsymbol{A}' \cdot \boldsymbol{B}' \,\mathrm{d}V = H + \int_V \nabla\Lambda \cdot \boldsymbol{B} \,\mathrm{d}V = H + \oint_{\partial V} \Lambda \boldsymbol{B} \cdot \hat{\boldsymbol{n}} \mathrm{d}S = H,$$

- $H = \Phi_1 \oint_{C_1} \boldsymbol{A} \cdot \mathrm{d}\mathbf{l} + \Phi_2 \oint_{C_2} \boldsymbol{A} \cdot \mathrm{d}\mathbf{l}, = 2\Phi_1\Phi_2$



Magnetic helicity evolution

- **Using Faraday's law and** $(\partial \boldsymbol{A}/\partial t) = -c(\boldsymbol{E} + \nabla\phi)$

$$
\begin{aligned}
\frac{1}{c}\frac{\partial}{\partial t}(\boldsymbol{A}\cdot\boldsymbol{B}) &= (-\boldsymbol{E} - \nabla\phi)\cdot\boldsymbol{B} + \boldsymbol{A}\cdot(-\nabla\times\boldsymbol{E}) \\
&= -2\boldsymbol{E}\cdot\boldsymbol{B} + \nabla\cdot(\boldsymbol{A}\times\boldsymbol{E} - \phi\boldsymbol{B}).
\end{aligned}
$$

- **Use Ohm's Law**: $\boldsymbol{E} = -(\boldsymbol{U}\times\boldsymbol{B})/c + (4\pi/c^2)\eta\boldsymbol{J}$

$$
\begin{aligned}
\frac{\mathrm{d}H}{\mathrm{d}t} &= -2\int_V \boldsymbol{E}\cdot\boldsymbol{B}\,\mathrm{d}V + \oint_{\partial V}(\boldsymbol{A}\times\boldsymbol{E} - \phi\boldsymbol{B})\cdot\hat{\boldsymbol{n}}\,\mathrm{d}S \\
&= -2\eta\int_V \left(\frac{4\pi}{c}\right)\boldsymbol{J}\cdot\boldsymbol{B}\;\mathrm{d}V
\end{aligned}
$$

Helicity Conservation

- **Helicity evolution**

$$\frac{dH}{dt} = -2\eta \int_V dV \; \frac{4\pi}{c}\mathbf{J}\cdot\mathbf{B}.$$

- **For energy**

$$\frac{dE_B}{dt} = -\eta \int_V dV \; \frac{4\pi}{c^2}\mathbf{J}^2 - \int_V dV \; \mathbf{U}\cdot\frac{(\mathbf{J}\times\mathbf{B})}{c}$$

- **As $\eta \to 0$, $dE_B/dt \to$ constant with $|\mathbf{J}| \propto \eta^{-1/2}$, $B \propto \eta^0$.**
- **BUT $dH/dt \to 0$!**
- **Helicity is nearly conserved even when energy dissipated**

How does the mean field helicity arise?

IUCAA

IUCAA Linking number is

$$(\mathbf{4} \times \mathbf{1}) + (\mathbf{4} \times -\mathbf{1}) = \mathbf{0}!!$$

LECTURE 2

MHD Basics

- **Maxwell equations + Ohms law $\Rightarrow$ Induction equation**

$$(\partial \mathbf{B}/\partial t) = \nabla \times (\mathbf{U} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) \, .$$

- **The field back reacts on $\mathbf{U}$ via the Lorentz force $\mathbf{J} \times \mathbf{B}$**

$$\rho(\mathrm{D}\mathbf{U}/\mathrm{D}t) = -\nabla p + \mathbf{J} \times \mathbf{B}/c + \mathbf{f} + \mathbf{F}_{\mathrm{visc}},$$

  - $\mathbf{U} = 0 \Rightarrow$ **pure diffusion and decay**
  - **If $\eta \to 0$, the flux $\Phi = \int_S \mathbf{B} \cdot d\mathbf{S}$ is 'frozen' $\to d\Phi/dt \to 0$.**
  - **Mag. Reynolds number $R_{\mathrm{M}} = (UB)/(\eta B/L) = UL/\eta \gg 1$**
  - **$\mathbf{B} = 0$ is solution! $\Rightarrow$ Need Batteries to generate B_{seed}**
  - $B_{seed} \ll B_{observed} \Rightarrow$ **Need $\mathbf{U}$ to act as Dynamo**

Crucial to understand how dynamos work/saturate ?

The first "seed" fields in the universe

- **Primordial fields from Early Universe? Uncertain Physics**
  Constrained by observations of CMB, FRM

- **Astrophysical Batteries using postive/negative charge asymmetry**

- **Biermann Batteries**: $\mathbf{E}_{Bier} = -\nabla p_e / e n_e + \dots$

$$(\partial \mathbf{B} / \partial t) = -c \nabla \times \mathbf{E}_{Bier} = -(ck/e n_e) \nabla n_e \times \nabla T_e$$

  - **Re-Ionization fronts**: $B < 10^{-19}$ G (Subramanian, Narasimha, Chitre, MN, 1994; Gnedin, Ferrara and Zweibel, ApJ, 2000)
  - **Structure formation Shocks** (Kulsrud et al, 1997)

- **During recombination:** $\gamma - e/p$ **scattering asymmetry**
  $B_0 \sim 10^{-30}$ G at MPc (Gopal & Sethi, 2005; Mattarrese et al, 2005);
  $B_0 \sim 10^{-21}$ G at pc (Ichiki et al 2007)

- **Seed fields from first supernovae and AGN outflows**

Need Dynamos to explain observed fields and maintain against decay

Biermann Battery

- **Write 2-fluid equations for electrons/protons and subtract**

$$\frac{D_e \boldsymbol{v}_e}{Dt} = -\frac{\boldsymbol{\nabla} p_e}{n_e m_e} - \frac{e}{m_e}(\boldsymbol{E} + \boldsymbol{v}_e \times \boldsymbol{B}) - \boldsymbol{\nabla}\phi_g - \frac{(\boldsymbol{v}_e - \boldsymbol{v}_p)}{\tau_{ei}},$$

$$\frac{D_i \boldsymbol{v}_i}{Dt} = -\frac{\boldsymbol{\nabla} p_i}{n_i m_i} + \frac{e}{m_i}(\boldsymbol{E} + \boldsymbol{v}_i \times \boldsymbol{B}) - \boldsymbol{\nabla}\phi_g + \frac{m_e n_e}{m_i n_i}\frac{(\boldsymbol{v}_e - \boldsymbol{v}_i)}{\tau_{ei}}.$$

- **Assume $m_e \ll m_i$, neglect nonlinear terms, to get Generalized Ohm's law**

$$\boldsymbol{E} + \boldsymbol{v}_i \times \boldsymbol{B} = -\frac{\boldsymbol{\nabla} p_e}{e n_e} + \frac{\boldsymbol{J}}{\sigma} + \frac{1}{e n_e}\boldsymbol{J} \times \boldsymbol{B} + \frac{m_e}{e^2}\frac{\partial}{\partial t}\left(\frac{\boldsymbol{J}}{n_e}\right)$$

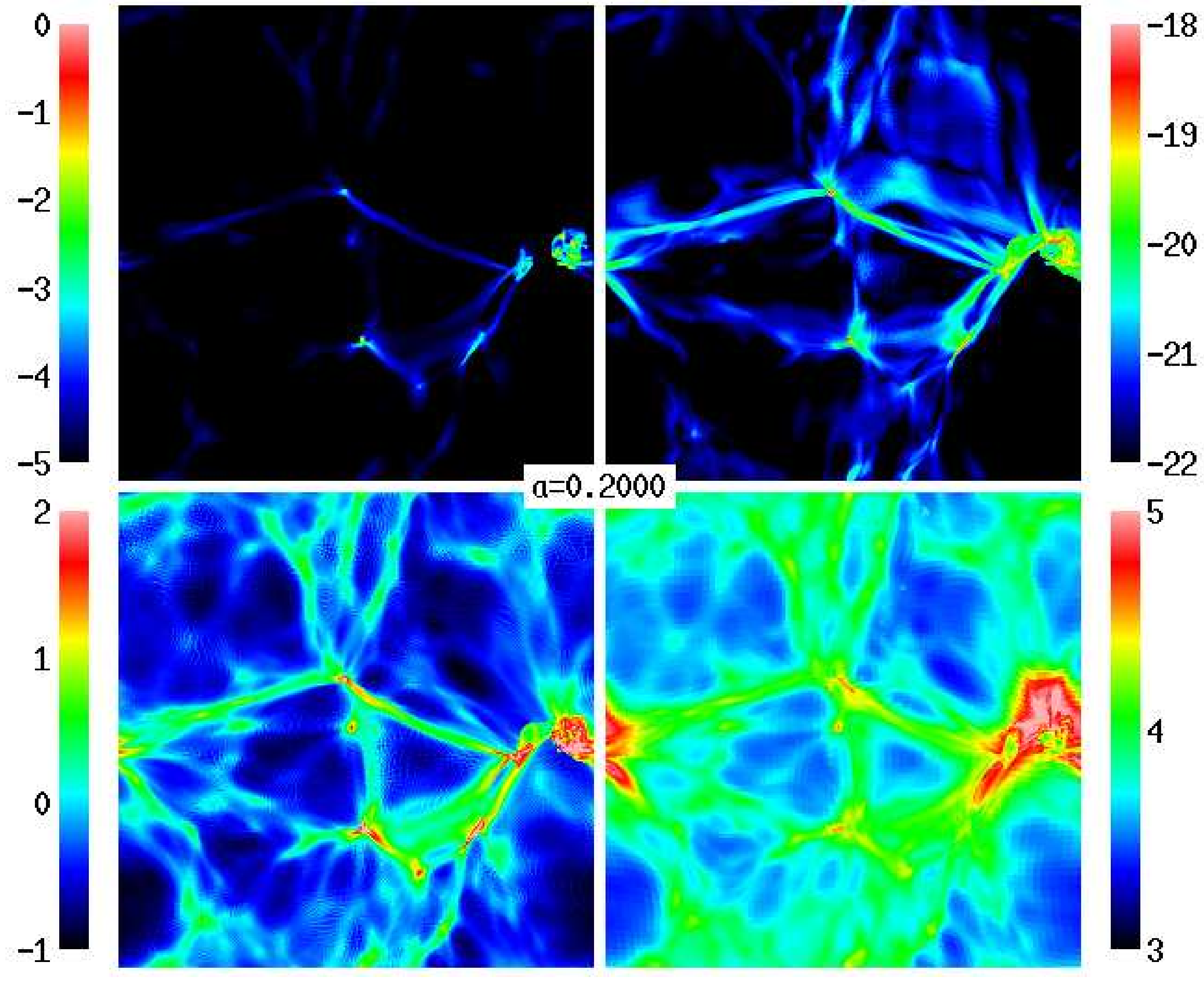
- **Take curl, use Ampere, neglect Hall, inertial terms**

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times [\boldsymbol{U} \times \boldsymbol{B} - \eta(\boldsymbol{\nabla} \times \boldsymbol{B})] - \frac{c k_{\rm B}}{e}\frac{\boldsymbol{\nabla} n_e}{n_e} \times \boldsymbol{\nabla} T.$$

- **Fields generated from zero if $\boldsymbol{\nabla} T$ not parallel to $\boldsymbol{\nabla} n_e$**

Magnetic fields from Reionization

HI, gas density, temperature and B field; Gnedin, Ferrara, Zweibel, 2000, ApJ



Turbulent Mean-Field Dynamo

- Consider flow with large-scale velocity $\overline{\mathrm{U}}$ and a 'turbulent' stochastic velocity $\mathbf{u}$
- $\mathbf{U} = \overline{\mathbf{U}} + \mathbf{u}, \mathbf{B} = \overline{\mathbf{B}} + \mathbf{b}$: Mean + Stochastic fields
- Average can be volume average over 'intermediate' scales or ensemble average
- Averages satisfy Reynolds rules

$$\overline{\boldsymbol{U}_1 + \boldsymbol{U}_2} = \overline{\boldsymbol{U}}_1 + \overline{\boldsymbol{U}}_2, \quad \overline{\overline{\boldsymbol{U}}} = \overline{\boldsymbol{U}}, \quad \overline{\overline{\boldsymbol{U}}\boldsymbol{u}} = 0, \quad \overline{\overline{\boldsymbol{U}}_1\,\overline{\boldsymbol{U}}_2} = \overline{\boldsymbol{U}}_1\,\overline{\boldsymbol{U}}_2,$$

$$\overline{\partial \boldsymbol{U}/\partial t} = \partial \overline{\boldsymbol{U}}/\partial t, \quad \overline{\partial \boldsymbol{U}/\partial x_i} = \partial \overline{\boldsymbol{U}}/\partial x_i.$$

- Average now the induction equation

Turbulent Mean-Field Dynamo

- $\mathbf{U} = \overline{\mathbf{U}} + \mathbf{u}$, $\mathbf{B} = \overline{\mathbf{B}} + \mathbf{b}$: **Mean + Stochastic fields**
- **Mean satisfies DYNAMO equation, with $\overline{\boldsymbol{\mathcal{E}}} = \overline{\mathbf{u} \times \mathbf{b}}$:**

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times \left(\overline{\mathbf{U}} \times \overline{\mathbf{B}} + \overline{\boldsymbol{\mathcal{E}}} - \eta (\nabla \times \overline{\mathbf{B}}) \right) ;$$

- **The stochastic small-scale field satisfies:**

$$\frac{\partial \boldsymbol{b}}{\partial t} = \nabla \times (\overline{\boldsymbol{U}} \times \boldsymbol{b} + \boldsymbol{u} \times \overline{\boldsymbol{B}} - \eta \nabla \times \boldsymbol{b}) + \boldsymbol{G}$$

- **Here G is the ”pain in neck” nonlinear term in u and b.**

$$\boldsymbol{G} = \nabla \times (\boldsymbol{u} \times \boldsymbol{b})' = \nabla \times [\boldsymbol{u} \times \boldsymbol{b} - \overline{\boldsymbol{u} \times \boldsymbol{b}}]$$

- **Finding $\overline{\boldsymbol{\mathcal{E}}} = \overline{\mathbf{u} \times \mathbf{b}}$ is a closure problem:** $\overline{\boldsymbol{\mathcal{E}}} = \alpha \overline{\mathbf{B}} - \eta_{turb} (\nabla \times \overline{\mathbf{B}})$

The kinematic limit of $\overline{\boldsymbol{\mathcal{E}}}$

- **For short correlation times ($\tau_{\rm cor}$), neglect G, also assume statistical isotropy of the random $\boldsymbol{u}$:**

$$\overline{\boldsymbol{\mathcal{E}}} = \overline{\boldsymbol{u} \times \int^t dt' (\partial \boldsymbol{b}/\partial \tau)} = \overline{\boldsymbol{u}(t) \times \int^t dt' [-\boldsymbol{u}(t') \cdot \nabla \overline{\boldsymbol{B}} + \overline{\boldsymbol{B}} \cdot \nabla \boldsymbol{u}(t')]}$$

$$\overline{\mathcal{E}}_i = \int_0^t \left[\epsilon_{ijk} \overline{u_j(t) u_{k,p}(t')}\, \overline{B}_p(t') + \epsilon_{ijp} \overline{u_j(t) u_l(t')}\, \overline{B}_{p,l}(t') \right] \mathrm{d}t',$$

- **So**: $\overline{\boldsymbol{\mathcal{E}}} = \alpha \overline{\boldsymbol{B}} - \eta_t \nabla \times \overline{\boldsymbol{B}}$, where

$$\alpha = -\tfrac{1}{3} \int_0^t \overline{\boldsymbol{u}(t) \cdot \boldsymbol{\omega}(t')}\, dt' \approx -\tfrac{1}{3} \tau_{\rm cor} \overline{\boldsymbol{u} \cdot \boldsymbol{\omega}},$$

$$\eta_{\rm t} = \tfrac{1}{3} \int_0^t \overline{\boldsymbol{u}(t) \cdot \boldsymbol{u}(t')}\, dt' \approx \tfrac{1}{3} \tau_{\rm cor} \overline{\boldsymbol{u}^2},$$

MFD in δ-correlated velocity field

- **Iterate the formal solution for small δt**

$$\boldsymbol{B}(\delta t)=\boldsymbol{B}(0)+\int_0^{\delta t} dt'\,\{\boldsymbol{\nabla}\times[\boldsymbol{U}(t')\times\boldsymbol{B}(t')]-\eta\boldsymbol{\nabla}\times\boldsymbol{B}(t')\}\,.$$

- **Zeroth order: ignore integral and put $\boldsymbol{B}^{(0)}(\boldsymbol{x},\delta t)=\boldsymbol{B}(\boldsymbol{x},0)$.**
- **Substitute B^0 for B to get $B^{(1)}$, then $B^{(1)}$ for B, to get $B^{(2)}$ and then average.**

$$\begin{aligned}\overline{\boldsymbol{B}}(\delta t) \quad &= \quad \overline{\boldsymbol{B}}(0)-\delta t[\boldsymbol{\nabla}\times(\overline{\boldsymbol{U}}\times\overline{\boldsymbol{B}}(0))-\eta\boldsymbol{\nabla}\times\overline{\boldsymbol{B}}(0)] \\ &+ \quad \int_0^{\delta t} dt'\int_0^{t'} dt''\boldsymbol{\nabla}\times\left\{\overline{\boldsymbol{v}(t')\times[\boldsymbol{\nabla}\times(\boldsymbol{v}(t'')\times\boldsymbol{B}(0))]}\right\} \quad (1)\end{aligned}$$

- **Use v at time t not correlated with $B(\boldsymbol{x},0)$, and take limit $\delta t\to 0$**

$$\frac{\partial\overline{\boldsymbol{B}}}{\partial t}=\boldsymbol{\nabla}\times\left[\overline{\boldsymbol{U}}\times\overline{\boldsymbol{B}}+\alpha\overline{\boldsymbol{B}}-(\eta+\eta_{\rm t})\boldsymbol{\nabla}\times\overline{\boldsymbol{B}}\right]\,.$$

Kazantsev Model

- **Isotropic, homogeneous, gaussian random with zero mean, δ-correlated in time velocity field**

$$\langle v_i(\boldsymbol{x},t)v_j(\boldsymbol{y},s)\rangle = \delta(t-s)\left[\left(\delta_{ij} - \frac{r_i r_j}{r^2}\right) T_{\mathrm{N}}(r) + \frac{r_i r_j}{r^2} T_{\mathrm{L}}(r)\right]$$

- **Here** $r = |\boldsymbol{x} - \boldsymbol{y}|$ **and** $T_{\mathrm{N}} = (1/2r)[\partial(r^2 T_{\mathrm{L}})/\partial r]$
- **Turbulent diffusion:** $T_{\mathrm{L}}(0) = (1/3)\int_0^t \langle \boldsymbol{v}(t)\cdot\boldsymbol{v}(t')\rangle\, dt'$
- **Induction equation is now stochastic. Look for solutions for magnetic correlations:**

$$\Big\langle B_i(\boldsymbol{x},t)B_j(\boldsymbol{y},t)\Big\rangle = \left(\delta_{ij} - \frac{r_i r_j}{r^2}\right) M_{\mathrm{N}}(r,t) + \frac{r_i r_j}{r^2} M_{\mathrm{L}}(r,t)$$

Kazantsev Model: Bound states

- **Iterate the induction equation, average and get**

$$\frac{\partial M_\mathrm{L}}{\partial t} = \frac{2}{r^4}\frac{\partial}{\partial r}\left[r^4 \eta_\mathrm{T}(r)\frac{\partial M_\mathrm{L}}{\partial r}\right] + G M_\mathrm{L},$$

- η_T **is Scale dependent turbulent diffusion,** G **velocity shear**

$$\eta_\mathrm{T}(r) = \eta + T_\mathrm{L}(0) - T_\mathrm{L}(r), \quad G = -2(T_\mathrm{L}'' + 4T_\mathrm{L}'/r)$$

- **Eigenmode solutions of form:** $\Psi(r)\exp(2\Gamma t) = r^2\sqrt{\eta_\mathrm{T}}M_\mathrm{L}$

$$-\Gamma\Psi = -\eta_\mathrm{T}\frac{d^2\Psi}{dr^2} + U_0(r)\Psi$$

- **Boundary condition is** $\Psi \to 0$ **for** $r \to 0, \infty$
- **Growing modes if there are bound states in potential** $U_0(r)$**.**
- **Growth rate** $\sim v/L$**, field concentrated in scales** $\sim L/R_m^{1/2}$

Including helicity

- **Now let us include kinetic helicity** (VK; 96, RK, 97; S 99, KH, 99)

$$T_{ij} = \left(\delta_{ij} - \frac{r_i r_j}{r^2}\right) T_N(r) + \frac{r_i r_j}{r^2} T_L(r) + \epsilon_{ijk} r_k F(r),$$

- **Here** $-2F(0) = -\frac{1}{3}\int_0^t \langle \boldsymbol{v}(t) \cdot \boldsymbol{\nabla} \times \boldsymbol{v}(t') \rangle \, \mathrm{d}t'$**.**

$$M_{ij} = \left(\delta_{ij} - \frac{r_i r_j}{r^2}\right) M_{\mathrm{N}} + \frac{r_i r_j}{r^2} M_{\mathrm{L}} + \epsilon_{ijk} r_k C,$$

$$\frac{\partial M_{\mathrm{L}}}{\partial t} = \frac{2}{r^4}\frac{\partial}{\partial r}\left[r^4 \eta_{\mathrm{T}} \frac{\partial M_{\mathrm{L}}}{\partial r}\right] + G M_{\mathrm{L}} + 4\alpha C,$$

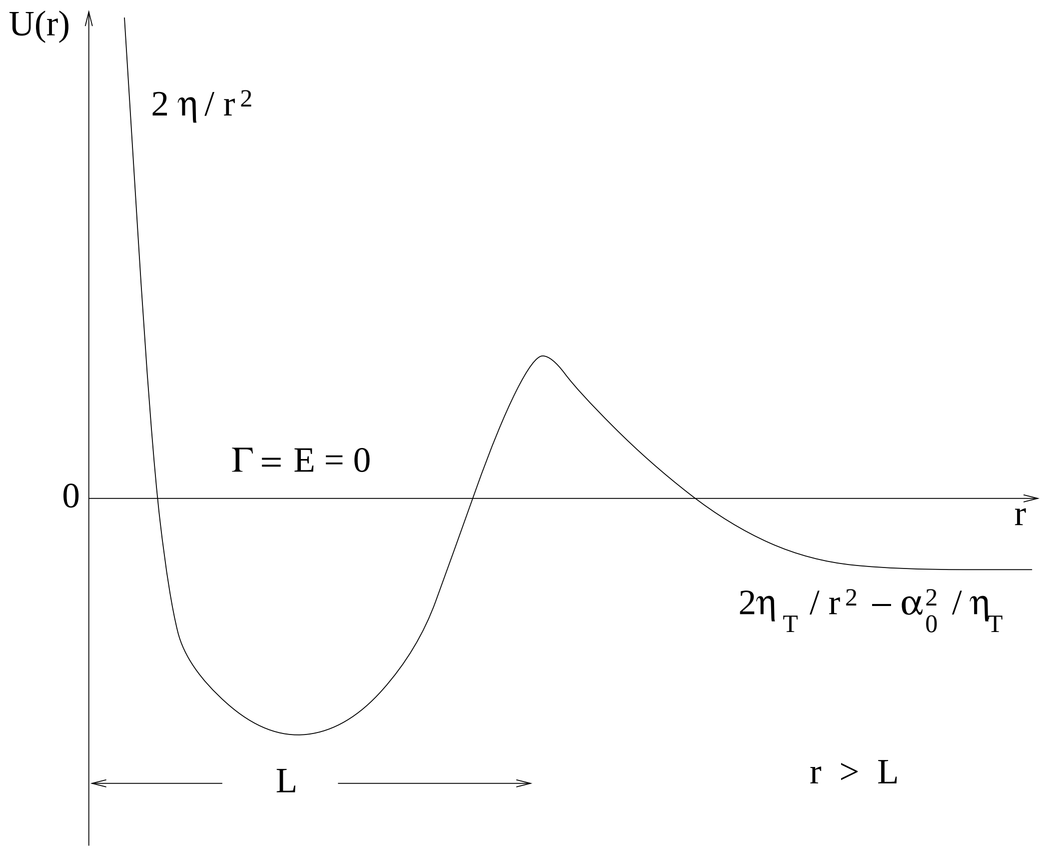
$$\frac{\partial H}{\partial t} = -2\eta_{\mathrm{T}} C + \alpha M_{\mathrm{L}}, \quad C = -\left(H'' + \frac{4H'}{r}\right),$$

- $H(0,t) = \frac{1}{6}\overline{\boldsymbol{A} \cdot \boldsymbol{B}}$, $C(0,t) = \frac{1}{6}\overline{\boldsymbol{J} \cdot \boldsymbol{B}}$
- **Suppose** $\alpha_0 = -2F(0) \neq 0$ $\dot{M}_{\mathrm{L}} = ... + 4\alpha_0 C$ **and** $\dot{H} = ... + \alpha_0 M$**.**

Kazantsev with Helicity: Tunneling?

- **For** $\dot{M}_{\rm L} \approx 0$, $\dot{H} \approx 0 \rightarrow -\eta_{\rm T}(d^2\Psi/dr^2) + \Psi\left[U_0 - (\alpha^2(r)/\eta_{\rm T}(r))\right] = 0$,
- $r \gg L$, $M_{\rm L}(r) = \bar{M}_{\rm L}(r) \propto r^{-3/2} J_{\pm 3/2}(\mu r)$,

Helicity opens up possibility of 'tunneling'



MFD in Renovating Flows

- **Consider a flow which renovates every τ.**
  (Gilbert and Bayly, 1992, JFM).

  $$\mathbf{u}(\mathbf{x}) = \mathbf{a}\sin(\mathbf{q}\cdot\mathbf{x}+\psi) + \mathbf{b}h\cos(\mathbf{q}\cdot\mathbf{x}+\psi);$$

  $$\mathbf{a}\cdot\mathbf{q} = 0, \quad \mathbf{b} = \mathbf{q}/q \times \mathbf{a}.$$

- **h controls helicity; $h = \pm 1$ for maximal helicity.**

- **In any time interval τ, randomly chosen ψ, $\mathbf{q}/q$ and a.**

  $$B_i(\mathbf{x}, n\tau) = \int \mathcal{G}_{ij}(\mathbf{x},\mathbf{y}) B_j(\mathbf{y},(n-1)\tau) d^3\mathbf{y},$$

  $$\overline{\tilde{B}}_i(\mathbf{k}, n\tau) = \overline{G_{ij}}(\mathbf{k}) \overline{\tilde{B}}_j(\mathbf{k},(n-1)\tau)$$

- **Split τ into 2 subintervals, (1) flux frozen (2) pure diffusion.**

- **Exonential growth if eigenvector of G_{ij} has eigenvalue σ, s.t $|\sigma| > 1$. Growth rate: $\lambda = \tau^{-1}\ln(\sigma)$.**

Flux freezing revisited

$$\frac{\partial \boldsymbol{B}}{\partial t} + (\boldsymbol{U} \cdot \boldsymbol{\nabla})\boldsymbol{B} = \frac{\mathrm{D}\boldsymbol{B}}{\mathrm{D}t} = \boldsymbol{B} \cdot \boldsymbol{\nabla}\boldsymbol{U} - \boldsymbol{B}(\boldsymbol{\nabla} \cdot \boldsymbol{U}),$$

$$\frac{\partial \rho}{\partial t} = -\boldsymbol{\nabla} \cdot (\rho \boldsymbol{U}),$$

$$\frac{\mathrm{D}}{\mathrm{D}t}\left(\frac{\boldsymbol{B}}{\rho}\right) = \frac{\boldsymbol{B}}{\rho} \cdot \boldsymbol{\nabla}\boldsymbol{U}$$

$$\frac{\mathrm{D}\delta\boldsymbol{x}}{\mathrm{D}t} = \delta\boldsymbol{x} \cdot \boldsymbol{\nabla}\boldsymbol{U},$$

- **If at $t = t_0$, fluid particles were on a given magnetic field line with $(\boldsymbol{B}/\rho)(\boldsymbol{x}_0, t_0) = c_0 \delta\boldsymbol{x}(t_0) = c_0 \delta\boldsymbol{x}_0$, then for all times we will have $\boldsymbol{B}/\rho = c_0 \delta\boldsymbol{x}$.**

$$B_i(\boldsymbol{x}, t) = \rho c_0 \delta x_i = \frac{\mathsf{G}_{ij}(\boldsymbol{x}_0, t)}{\det(\mathsf{G})} B_{0j}(\boldsymbol{x}_0), \qquad \mathsf{G}_{ij} = \partial x_i / \partial x_{0j}$$

MFD in renovating flows

- For $h = \pm 1$

$$G_{ij}(\mathbf{k}) = \delta_{ij}\ j_0(ak\tau) + j_1(\tau ak\chi)(iaqh\tau/2k)\epsilon_{ijm}k_m$$

- Eigenvectors are $\mathbf{k}, (\mp i, 1, 0)$ and the eigenvalues $1, \sigma_\pm$.

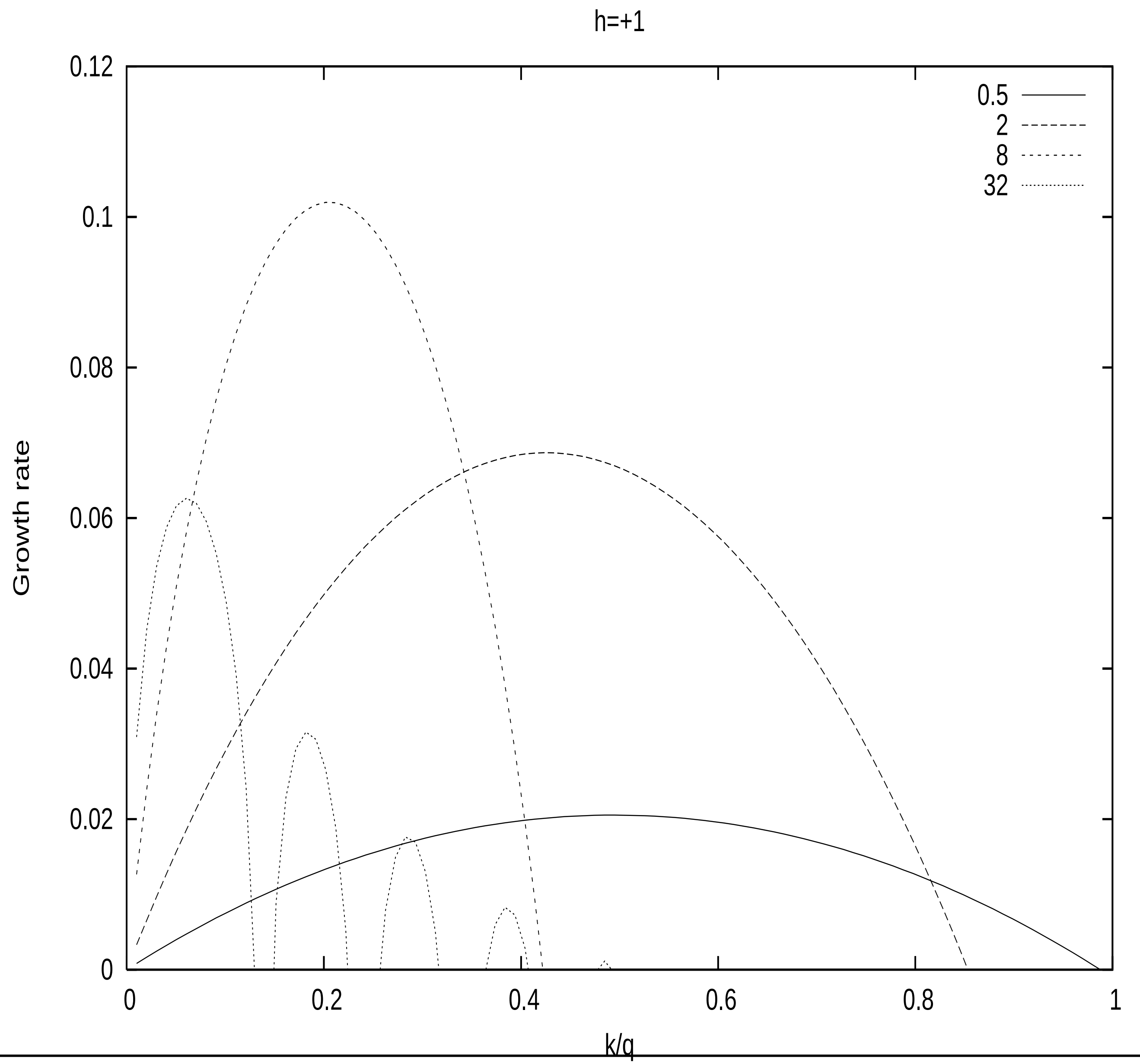
$$\sigma_\pm = j_0(\tau ak) \mp \frac{\tau aqh}{2} j_1(\tau ak)$$

- The growth rate in terms of eddy turnover rate T^{-1} is

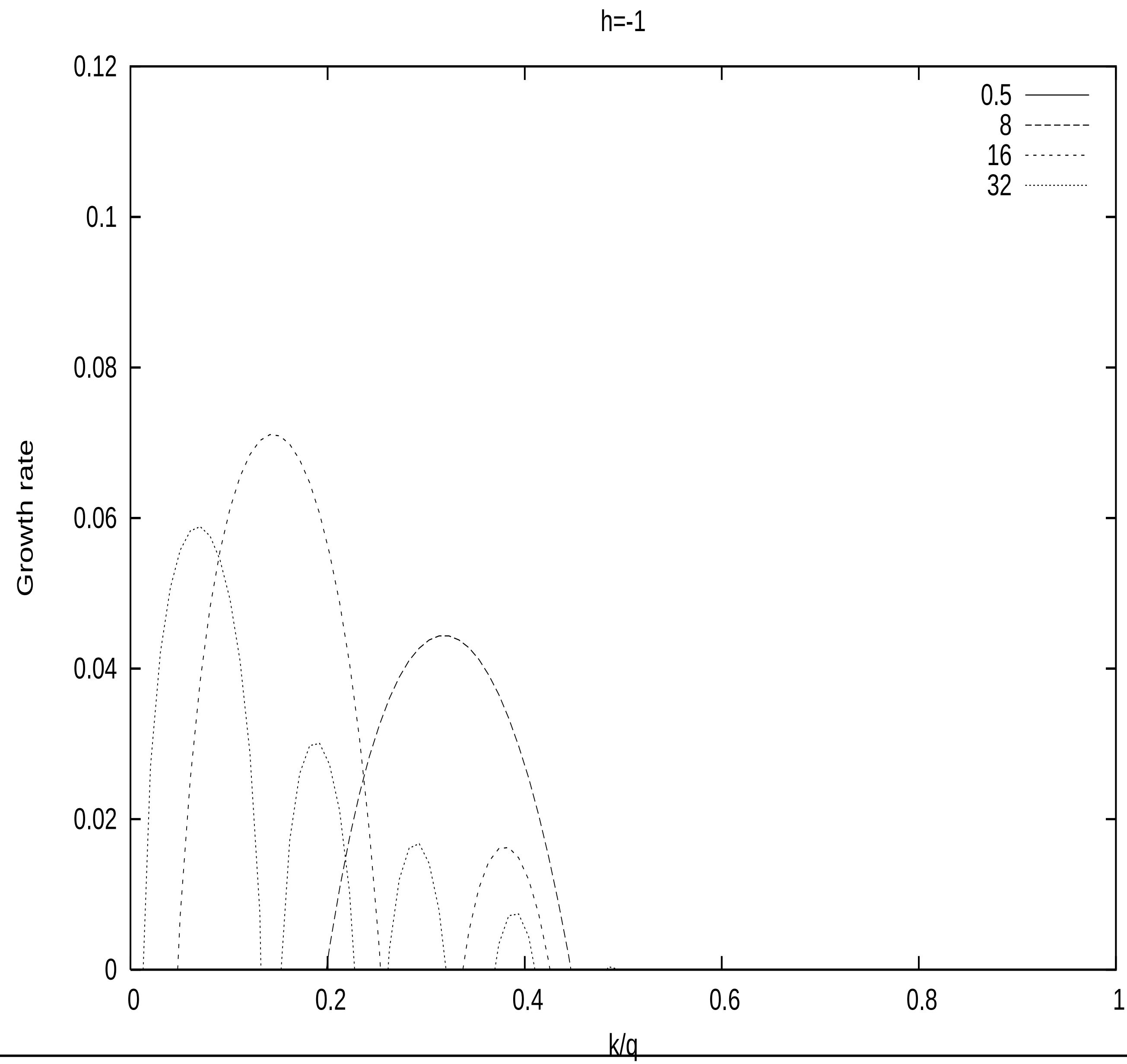
$$T\lambda_\pm = (\frac{\tau}{T})^{-1} \ln\left[j_0\left(\frac{\tau}{T}\frac{k}{q}\right) \mp \left((h/2)\frac{\tau}{T}\frac{k}{q}\right) j_1\left(\frac{\tau}{T}\frac{k}{q}\right)\right]$$

- For $\tau/T \ll 1 \rightarrow \lambda_- = \alpha k - \eta_t k^2$

Growth rate for renovating dynamo



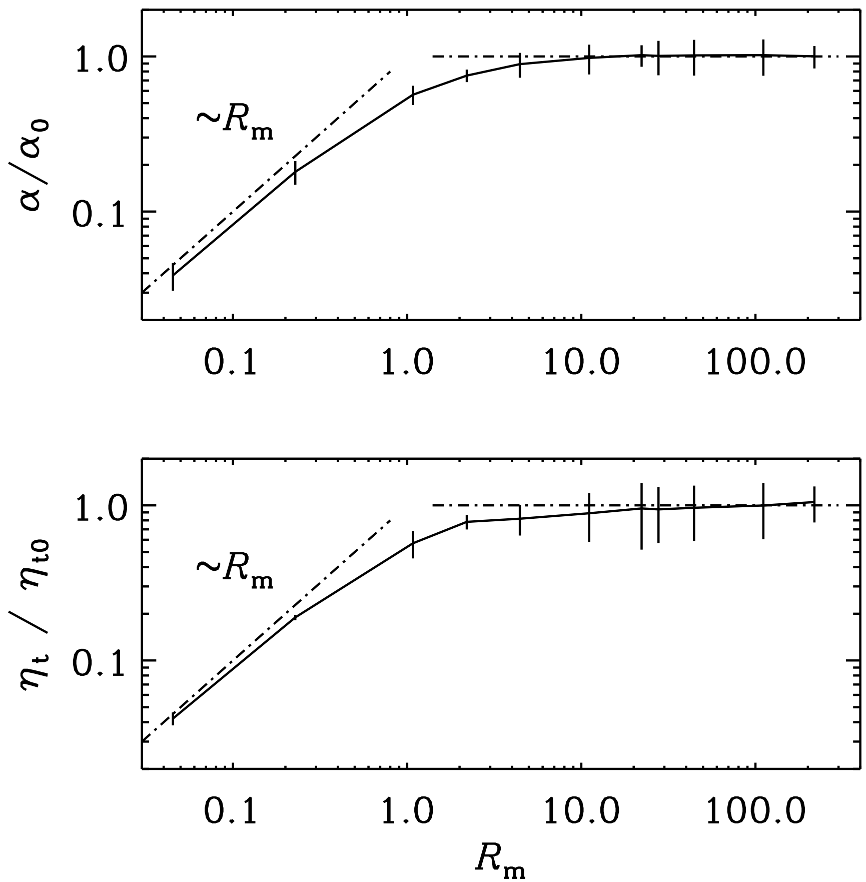
Growth rate for renovating dynamo



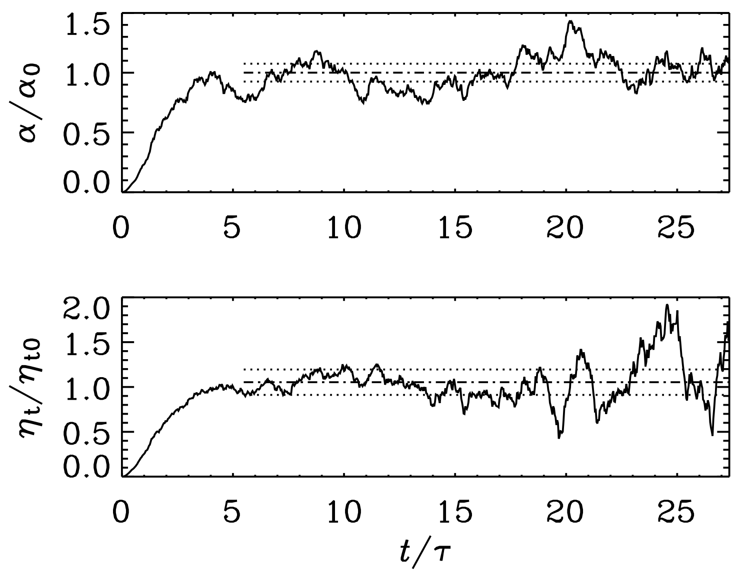
Kinematic α-effect from simulations

Sur, Brandenburg, Subramanian, 2008

Normalized α, η_t for $R_e = 2.2$



Time series for $R_e = 2.2$, $R_m = 220$



- **Measure directly $\overline{\mathcal{E}} = \overline{\mathbf{u}\times\mathbf{b}}$ in isotropic turbulence simulations.**
- α, η_t **as expected. Independent of R_m,**
  Even in presence of Fluctuation dynamo $\Rightarrow$ Kinematic regime OK?

Sensitivity to fluctuations

- Formal Greens function solution of b

$$\begin{aligned} \boldsymbol{b}_i(\boldsymbol{x},t) &= \int_{t_i}^{t} dt' \int d^3\boldsymbol{x}'\; G^u_{ij}(\boldsymbol{x},\boldsymbol{x}',t,t') \times \left[\boldsymbol{\nabla} \times (\boldsymbol{u}(\boldsymbol{x}',t') \times \overline{\boldsymbol{B}}(\boldsymbol{x}',t')\right]_j \\ &+ \int d^3\boldsymbol{x}' G^u_{ij}(\boldsymbol{x},\boldsymbol{x}',t,t_i) b_j(\boldsymbol{x}',t_i) \end{aligned} \quad (2)$$

- If $\overline{B} = 0$ only the second term survives and this equation then has to describe a growing small scale dynamo.
- So Exponential growing fluctuations in G^u_{ij}
- But same Green function also term $\propto \overline{B}$
- So $\overline{u \times b}$ will have exponentially growing fluctuations.
- Still both mean field and fluctuations can grow

LECTURE 3

Magnetic Helicity and dynamos

- Helicity and the Galactic dynamo
- What is magnetic helicity?
- Helicity evolution/conservation
- Picture and Revised picture for α-effect
- Catastrophic α-quenching?
- Helical dynamo in a periodic box.
- Helicity constraint in a closure model.
- Helicity density and flux for random fields
- The nonlinear alpha effect?

Turbulent Mean-Field Dynamo: Galactic

- $\mathbf{U} = \overline{\mathbf{U}} + \mathbf{u}$, $\mathbf{B} = \overline{\mathbf{B}} + \mathrm{b}$: **Mean + Stochastic fields**
- **Mean field satisfies DYNAMO equation**

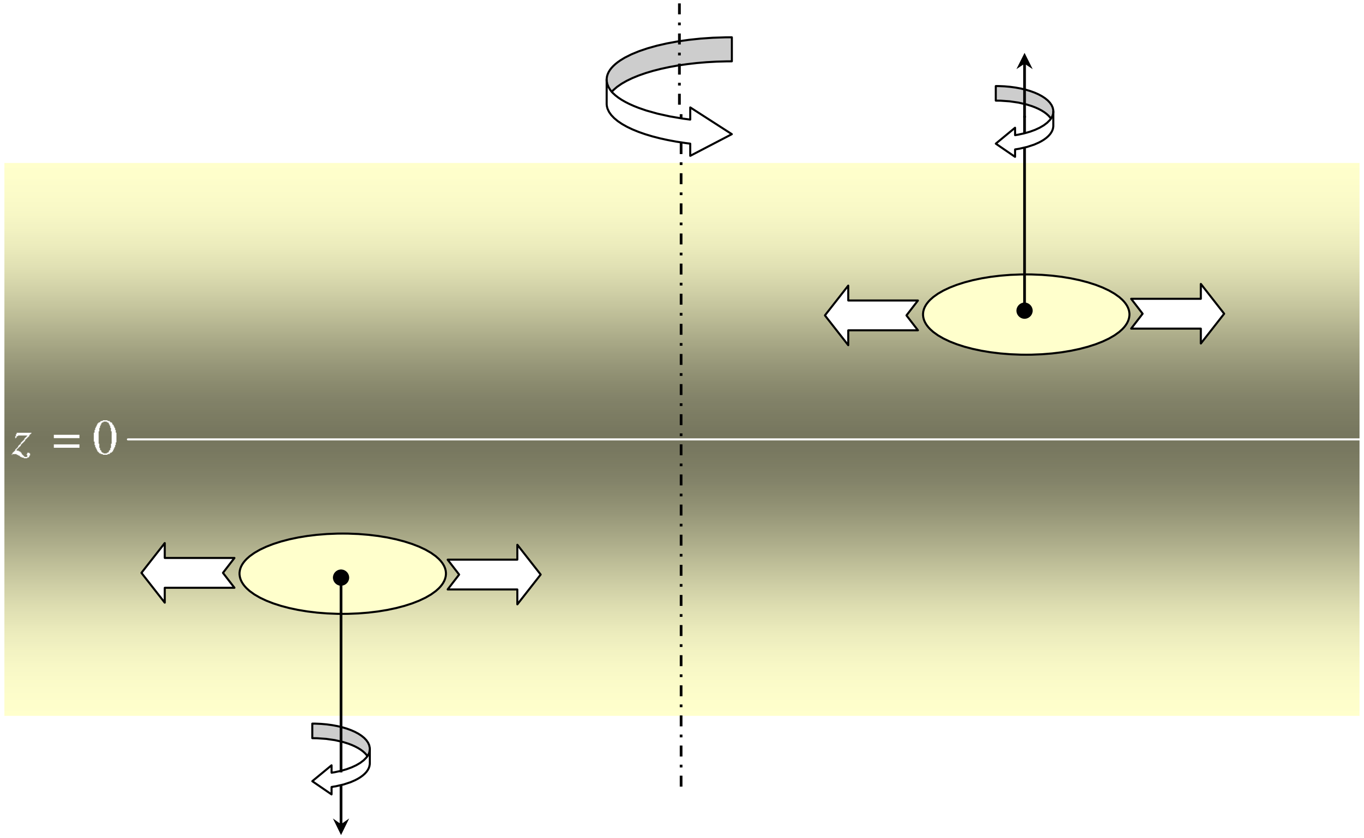
$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times \left(\overline{\mathbf{U}} \times \overline{\mathbf{B}} + \overline{\mathcal{E}} - \eta(\nabla \times \overline{\mathbf{B}}) \right) ;$$

- **Finding $\overline{\mathcal{E}} = \overline{\mathbf{u} \times \mathrm{b}}$ is a closure problem**: $\overline{\mathcal{E}} = \alpha \overline{\mathbf{B}} - \eta_{turb}(\nabla \times \overline{\mathbf{B}})$

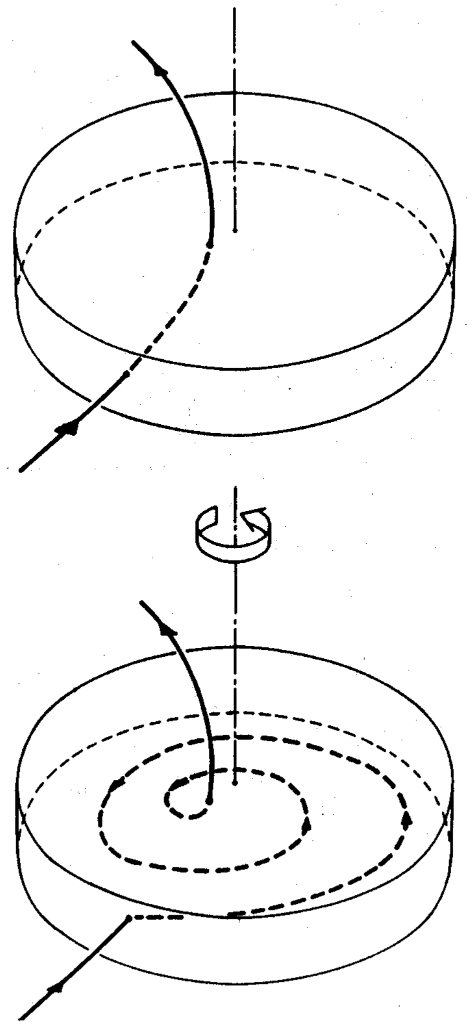
$$\underbrace{\alpha = -(\tau/3)\langle \mathbf{u} \cdot \omega \rangle ;}_{\text{alpha-effect}} \qquad \underbrace{\eta_{turb} = (\tau/3)\langle \mathbf{u}^2 \rangle}_{\text{Turbulent diffusion}}$$

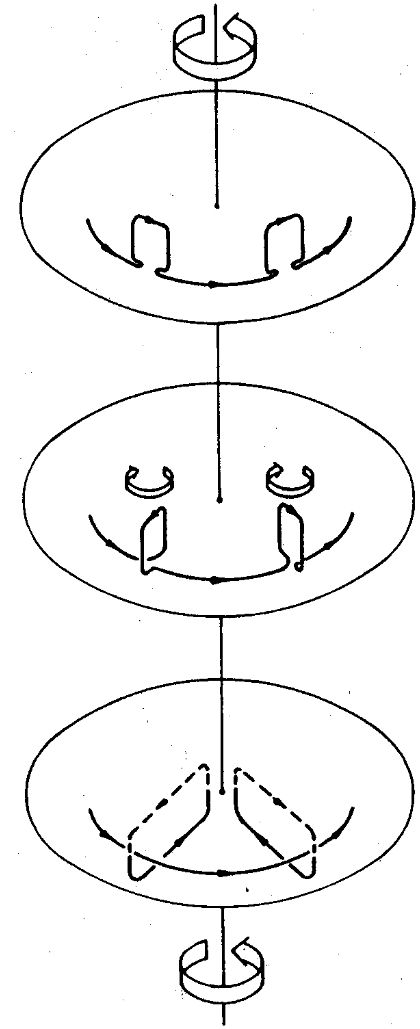
- **Galactic Shear generates B_ϕ from B_r**
- **Supernovae drive HELICAL turbulence** (**Due to Rotation + Stratification**)
- **Helical motions generate B_r from B_ϕ**
- **Exponential growth of $\overline{\mathrm{B}}$**, $t_{growth} \sim 10^8 - 10^9$ yr

Supernovae Drive Helical turbulence



Galactic Shear and α effect: RSS





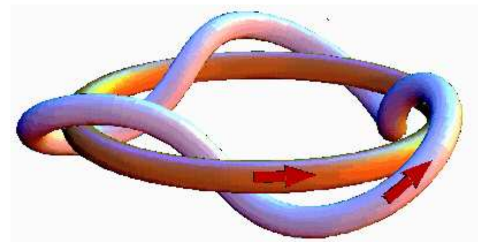
Mean vs fluctuations?

Helicity (links) conservation? Ejection of flux?

Magnetic Helicity

- **Magnetic Helicity** $H = \int_V \mathbf{A} \cdot \mathbf{B}\, dV$, $\nabla \times \boldsymbol{A} = \boldsymbol{B}$
  $\mathbf{A}$ is vector potential, V is **closed** volume

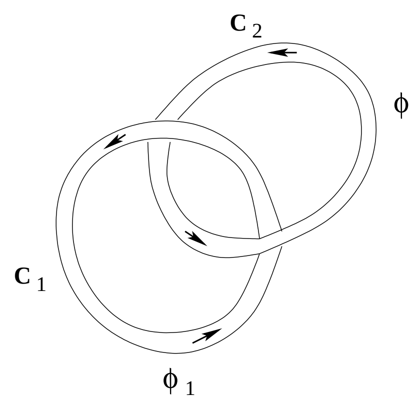
- **Measures links and twists in** B



- H **invariant under gauge transformation for closed fields**

$$H' = \int_V \boldsymbol{A}' \cdot \boldsymbol{B}'\, \mathrm{d}V = H + \int_V \nabla\Lambda \cdot \boldsymbol{B}\, \mathrm{d}V = H + \oint_{\partial V} \Lambda \boldsymbol{B} \cdot \hat{\boldsymbol{n}} \mathrm{d}S = H,$$

- $H = \Phi_1 \oint_{C_1} \boldsymbol{A} \cdot \mathrm{d}\mathbf{l} + \Phi_2 \oint_{C_2} \boldsymbol{A} \cdot \mathrm{d}\mathbf{l}, = 2\Phi_1\Phi_2$



Vetenskapens Hus

Magnetic helicity evolution

- **Using Faraday's law and** $(\partial \boldsymbol{A}/\partial t) = -c(\boldsymbol{E} + \nabla\phi)$

$$\begin{aligned}
\frac{1}{c}\frac{\partial}{\partial t}(\boldsymbol{A}\cdot\boldsymbol{B}) &= (-\boldsymbol{E} - \nabla\phi)\cdot\boldsymbol{B} + \boldsymbol{A}\cdot(-\nabla\times\boldsymbol{E}) \\
&= -2\boldsymbol{E}\cdot\boldsymbol{B} + \nabla\cdot(\boldsymbol{A}\times\boldsymbol{E} - \phi\boldsymbol{B}).
\end{aligned}$$

- **Use Ohm's Law**: $\boldsymbol{E} = -(\boldsymbol{U}\times\boldsymbol{B})/c + (4\pi/c^2)\eta\boldsymbol{J}$

$$\begin{aligned}
\frac{\mathrm{d}H}{\mathrm{d}t} &= -2\int_V \boldsymbol{E}\cdot\boldsymbol{B}\mathrm{d}V + \oint_{\partial V}(\boldsymbol{A}\times\boldsymbol{E} - \phi\boldsymbol{B})\cdot\hat{\boldsymbol{n}}\mathrm{d}S \\
&= -2\eta\int_V (\frac{4\pi}{c})\boldsymbol{J}\cdot\boldsymbol{B}\ \mathrm{d}V
\end{aligned}$$

Magnetic energy evolution

- **Start from Faraday's Law and dot with B**

$$\frac{1}{2}\frac{\partial B^2}{\partial t} = \boldsymbol{B}\cdot\frac{\partial \boldsymbol{B}}{\partial t} = -c\boldsymbol{B}\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}) = -c\boldsymbol{\nabla}\cdot(\boldsymbol{E}\times\boldsymbol{B}) - c\boldsymbol{E}\cdot(\boldsymbol{\nabla}\times\boldsymbol{B})$$

- **Use Ohm's Law**: $\boldsymbol{E} = -(\boldsymbol{U}\times\boldsymbol{B})/c + (4\pi/c^2)\eta\boldsymbol{J}$

$$\frac{1}{2}\frac{\partial B^2}{\partial t} = -c\boldsymbol{\nabla}\cdot(\boldsymbol{E}\times\boldsymbol{B}) - (4\pi/c^2)\eta\boldsymbol{J}\cdot(\boldsymbol{\nabla}\times\boldsymbol{B}) + (\boldsymbol{U}\times\boldsymbol{B})\cdot(\boldsymbol{\nabla}\times\boldsymbol{B})$$

- **Shuffle the triple product, devide by 4π**

$$\frac{1}{8\pi}\frac{\partial B^2}{\partial t} + \underbrace{\boldsymbol{\nabla}\cdot\left[\frac{c}{4\pi}\boldsymbol{E}\times\boldsymbol{B}\right]}_{\text{Poynting flux}} = \underbrace{\frac{4\pi}{c^2}\eta\mathbf{J}^2}_{\text{Dissipation}} \underbrace{-\mathbf{U}\cdot\frac{(\mathbf{J}\times\mathbf{B})}{c}}_{\text{LF Work}}$$

Helicity Conservation

- **Helicity evolution**

$$\frac{dH}{dt} = -2\eta \int_V dV\ \frac{4\pi}{c}\mathbf{J}\cdot\mathbf{B}.$$

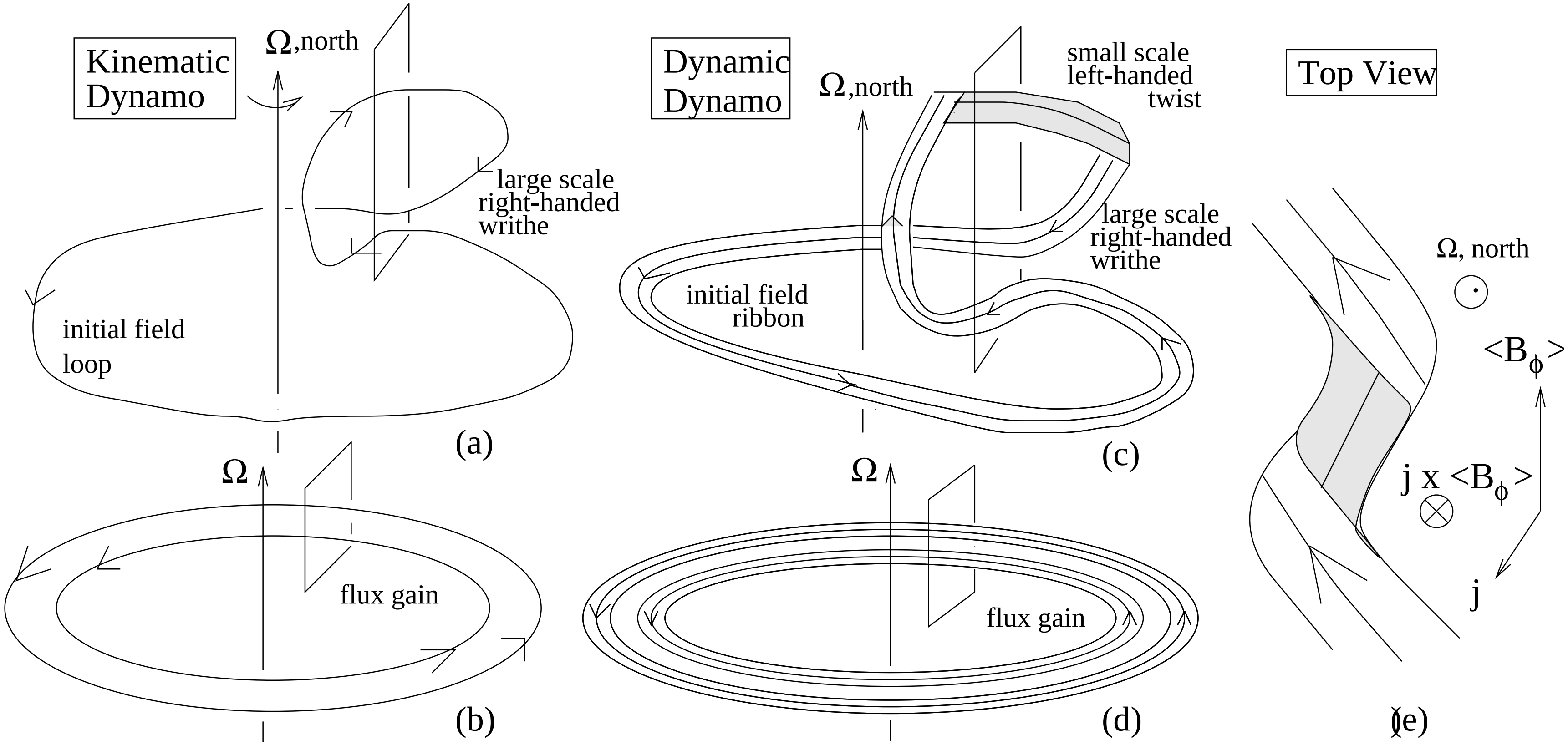
- **For energy**

$$\frac{dE_B}{dt} = -\eta \int_V dV\ \frac{4\pi}{c^2}\mathbf{J}^2 - \int_V dV\ \mathbf{U}\cdot\frac{(\mathbf{J}\times\mathbf{B})}{c}$$

- **As $\eta \to 0$, $dE_B/dt \to$ constant with $|\mathbf{J}| \propto \eta^{-1/2}$, $B \propto \eta^0$.**
- **BUT $dH/dt \to 0$!**
- **Helicity is nearly conserved even when energy dissipated**

How does the mean field helicity arise?

A revised picture for α-effect



Blackman,Brandenburg 2003

WRITHE AND TWIST Helicities

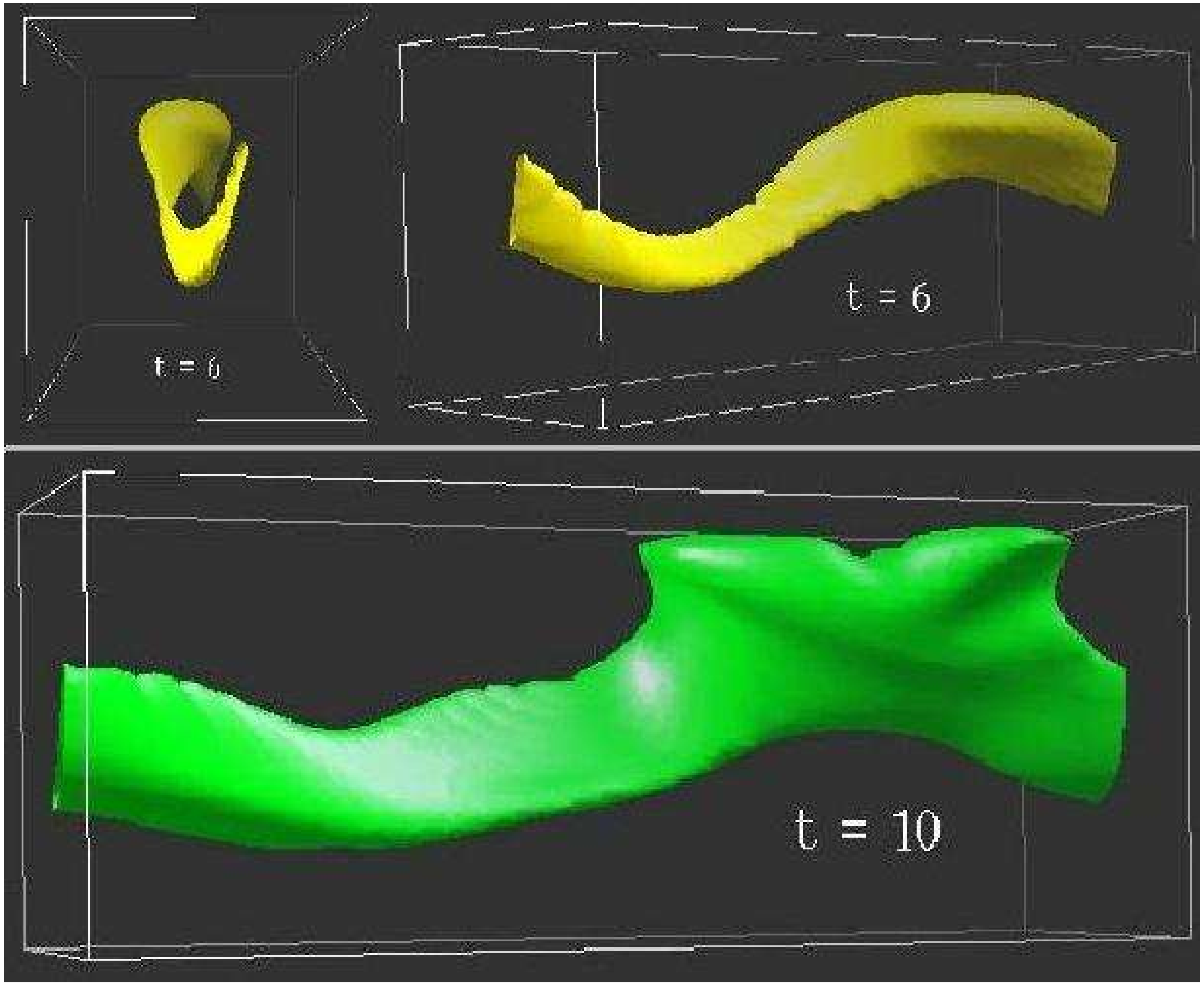
A revised picture for α-effect

Anvar and Natasha Shukurov 2009

WRITHE AND TWIST Helicities

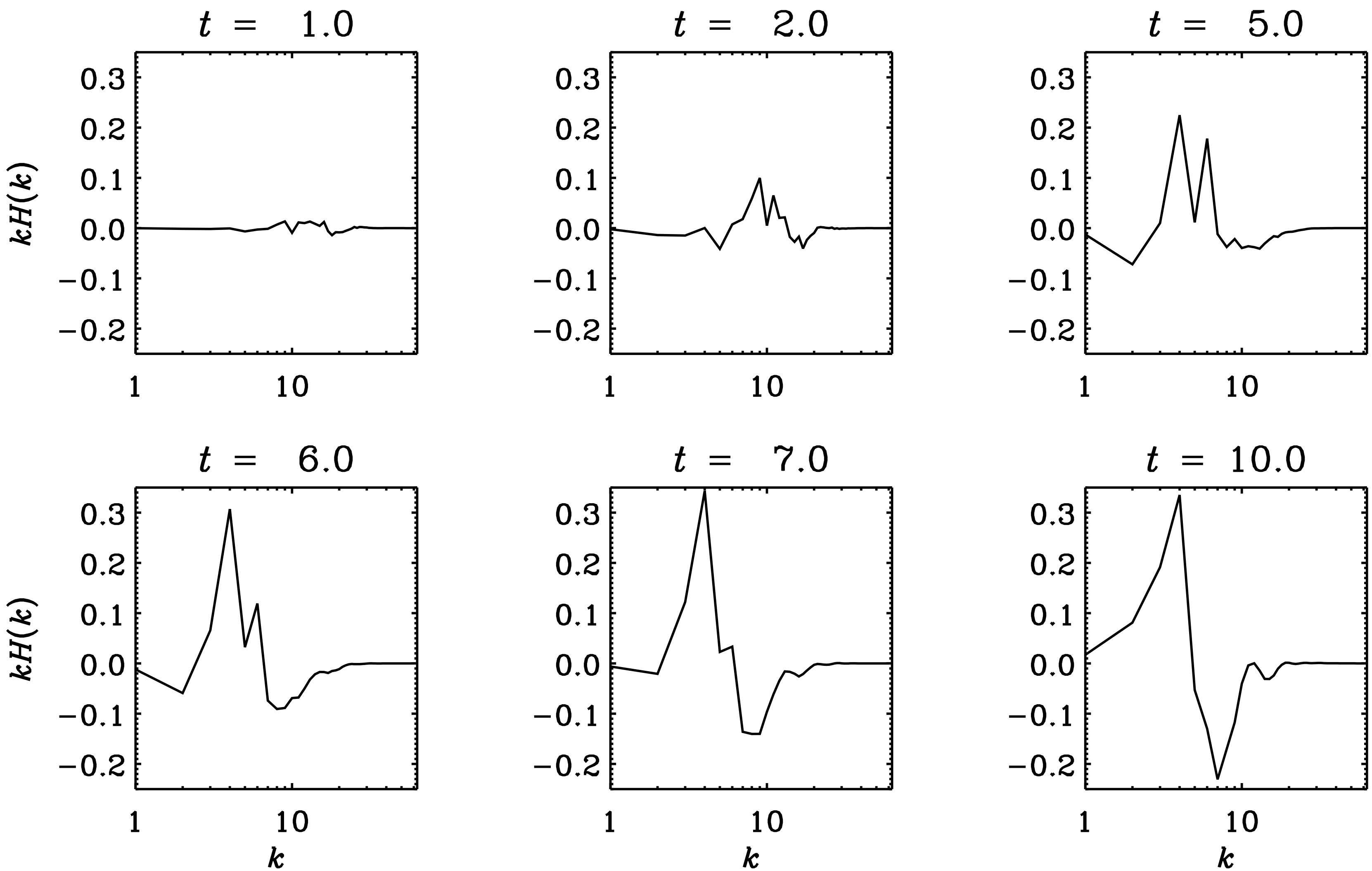
Twist/Writhe in Buoyant flux tube

Brandenburg, Blackman, Sarson, 2003



Twist/Writhe in Buoyant flux tube

Brandenburg, Blackman, Sarson, 2003



Helicity Power spectrum

Helicity and catastrophic α quenching?

- $\overline{\mathcal{E}} = \overline{\mathbf{u} \times \mathbf{b}}$ **transfers helicity between $\overline{\mathrm{B}}$ and b fields**
- **Now ohms law for mean field is:** $\overline{\boldsymbol{E}} = \overline{\boldsymbol{J}}/\sigma - (\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}})/c - \overline{\boldsymbol{\mathcal{E}}}/c$

$$\frac{d}{dt}\langle\overline{\mathbf{A}} \cdot \overline{\mathbf{B}}\rangle = -2\langle\overline{\boldsymbol{E}} \cdot \overline{\boldsymbol{B}}\rangle = 2\langle\overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\mathbf{B}}\rangle - 2\eta\frac{4\pi}{c}\langle\overline{\mathbf{J}} \cdot \overline{\mathbf{B}}\rangle$$

- **Subtracting from total helicity equation**

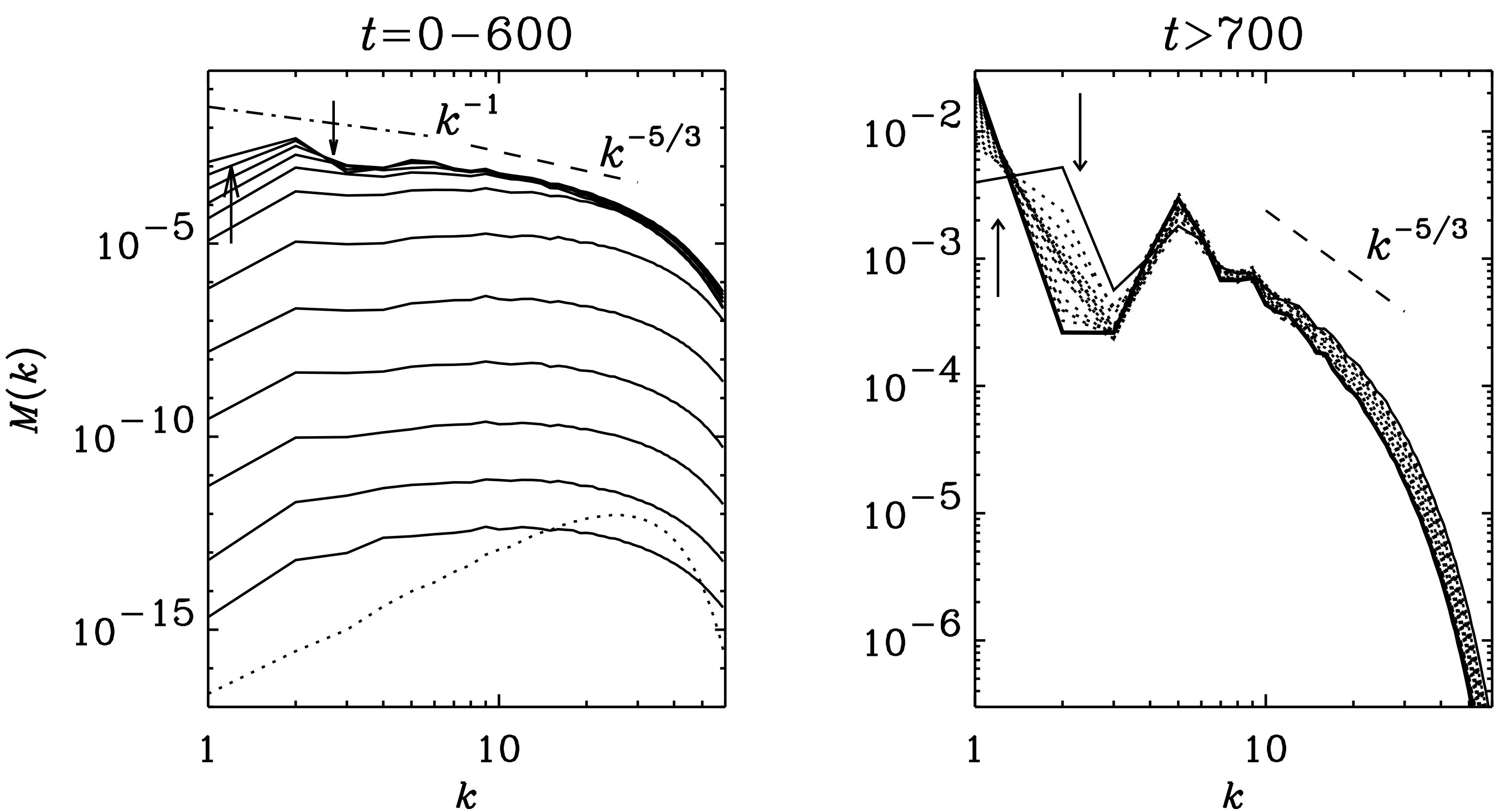
$$\frac{d}{dt}\langle\mathbf{a} \cdot \mathbf{b}\rangle = -2\langle\overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\mathbf{B}}\rangle - 2\eta\frac{4\pi}{c}\langle\mathbf{j} \cdot \mathbf{b}\rangle$$

- **Stationary limit**: $\langle\overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\mathbf{B}}\rangle = -2\eta\frac{4\pi}{c}\langle\mathbf{j} \cdot \mathbf{b}\rangle \rightarrow 0$ as $\eta \rightarrow 0$
- **Catastrophic R_{M} dependent quenching** of $\overline{\mathcal{E}}$??

Large scale dynamos need helicity fluxes

Turbulent helical dynamos in a box

Brandenburg, 2001, $k_f = 5$



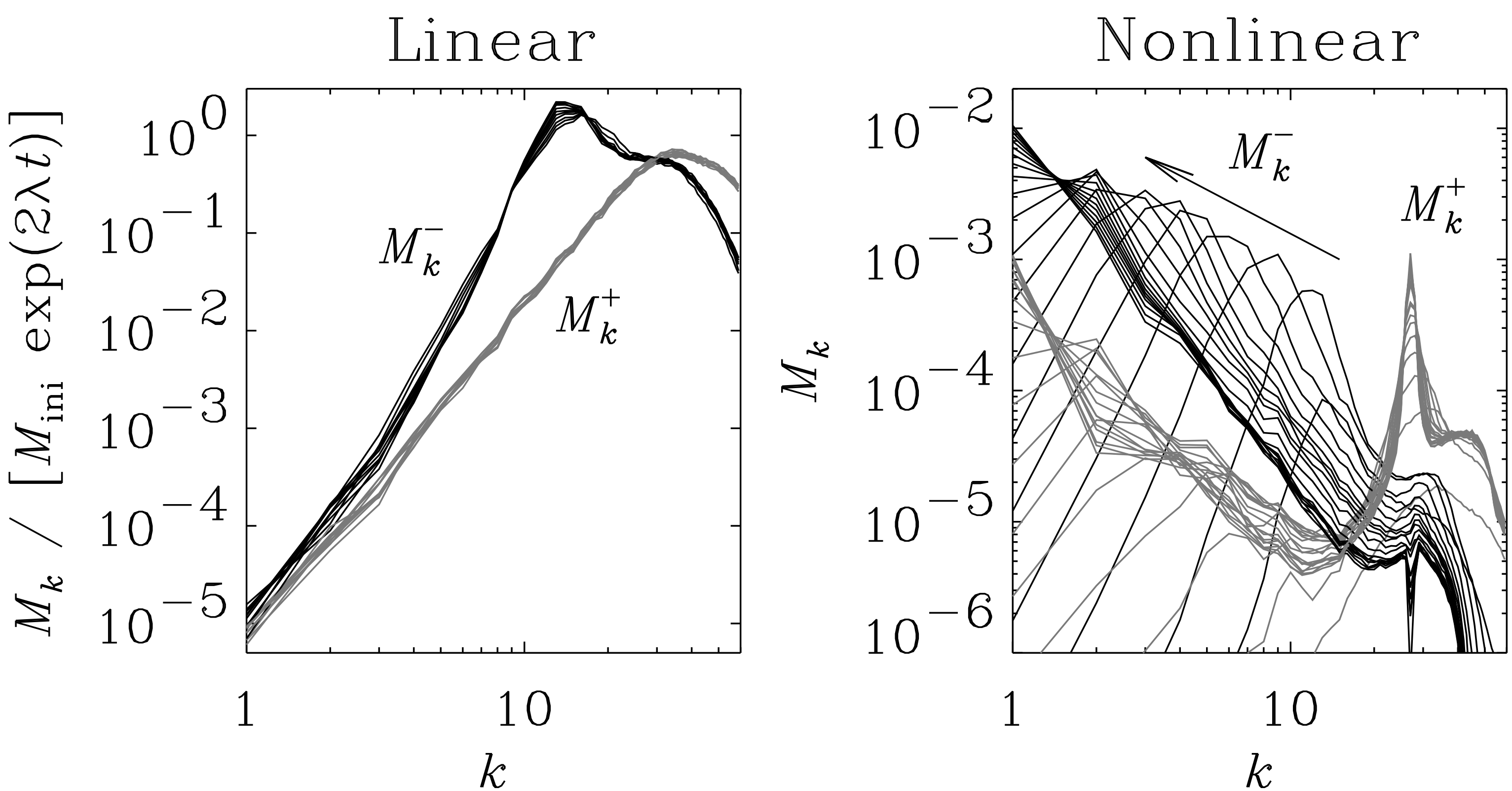
Rapid growth in kinematic stage conserving helicity.

Growth only on long resistive timescale at nonlinear stage.

(dissipating small-scale helicity)

Turbulent helical dynamos in a box

Brandenburg, $k_f = 27$



Helicity transferred between small-large scales

Rapid growth in kinematic stage conserving helicity.

Growth only on long resistive timescale at nonlinear stage.

(dissipating small-scale helicity)

Helicity constraint in a closure model

KS, 99; Brandenburg, KS, 2000

Ambipolar drift nonlinearity, $\mathbf{u} = \mathbf{u}_0 + a(\boldsymbol{J} \times \boldsymbol{B})$

$$\frac{\partial M_{\rm L}}{\partial t} = \frac{2}{r^4}\frac{\partial}{\partial r}\left(r^4 \eta_{\rm N}(r)\frac{\partial M_{\rm L}}{\partial r}\right) + GM_{\rm L} + 4\alpha_{\rm N}(r)C,$$

$$\frac{\partial H}{\partial t} = -2\eta_{\rm N}C + \alpha_{\rm N}M_{\rm L},$$

$$\alpha_{\rm N} = \alpha(r) + 4aC(0,t), \quad \eta_{\rm N} = \eta_T(r) + 2aM(0,t).$$

$$\alpha_\infty \equiv \alpha_{\rm N}(r\to\infty) = -\tfrac{1}{3}\tau\langle\boldsymbol{\omega}\cdot\boldsymbol{u}\rangle + \tfrac{1}{3}\tau_{\rm AD}\langle\boldsymbol{J}\cdot\boldsymbol{B}\rangle/\rho_0,$$

$$\eta_\infty \equiv \eta_{\rm N}(r\to\infty) = \tfrac{1}{3}\tau\langle\boldsymbol{u}^2\rangle + \tfrac{1}{3}\tau_{\rm AD}\langle\boldsymbol{B}^2\rangle/\mu_0\rho_0,$$

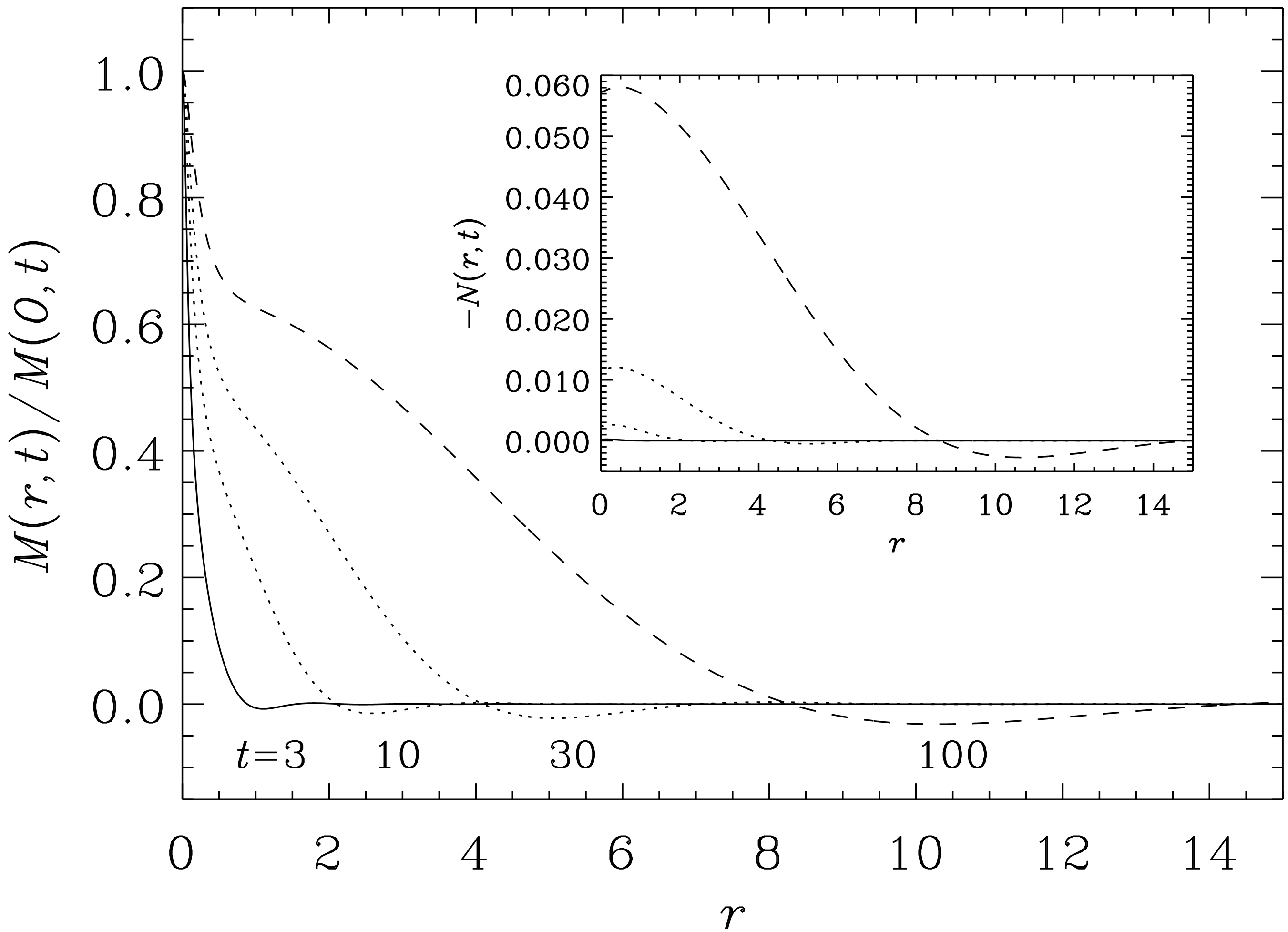
The system conserved magnetic helicity

$$\dot{H}(0,t) = -2\eta C(0,t), \quad {\rm d}\langle\boldsymbol{A}\cdot\boldsymbol{B}\rangle/{\rm d}t = -2\eta\langle\boldsymbol{J}\cdot\boldsymbol{B}\rangle,$$

Helicity constraint in a closure model

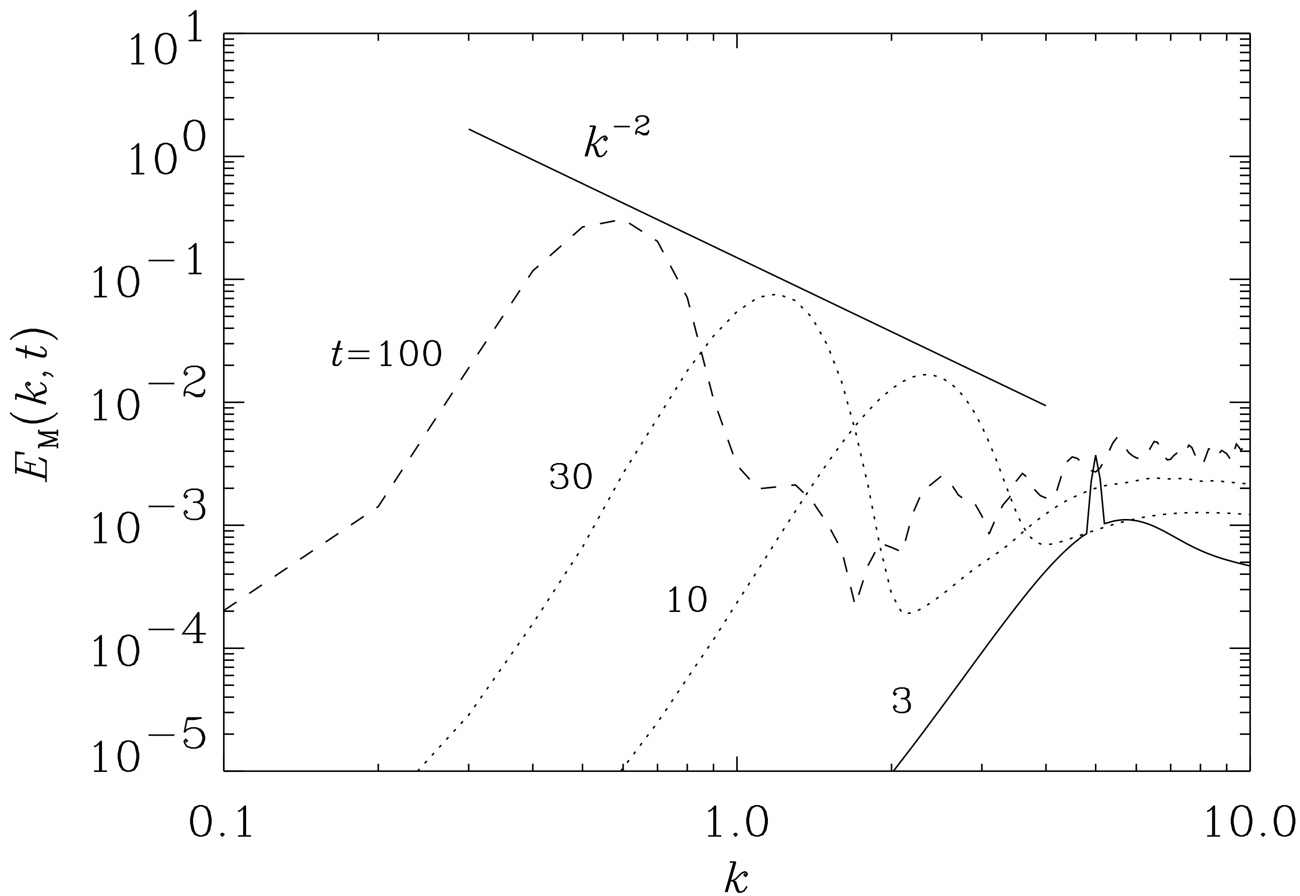
KS, 99; Brandenburg, KS, 2000

Ambipolar drift nonlinearity, $\mathbf{u} = \mathbf{u}_0 + a(\boldsymbol{J} \times \boldsymbol{B})$, $M(r,t) = \langle \boldsymbol{B}(\mathbf{x},t)\boldsymbol{B}(\mathbf{x}-\mathbf{r})\rangle$



Helicity constraint in a closure model

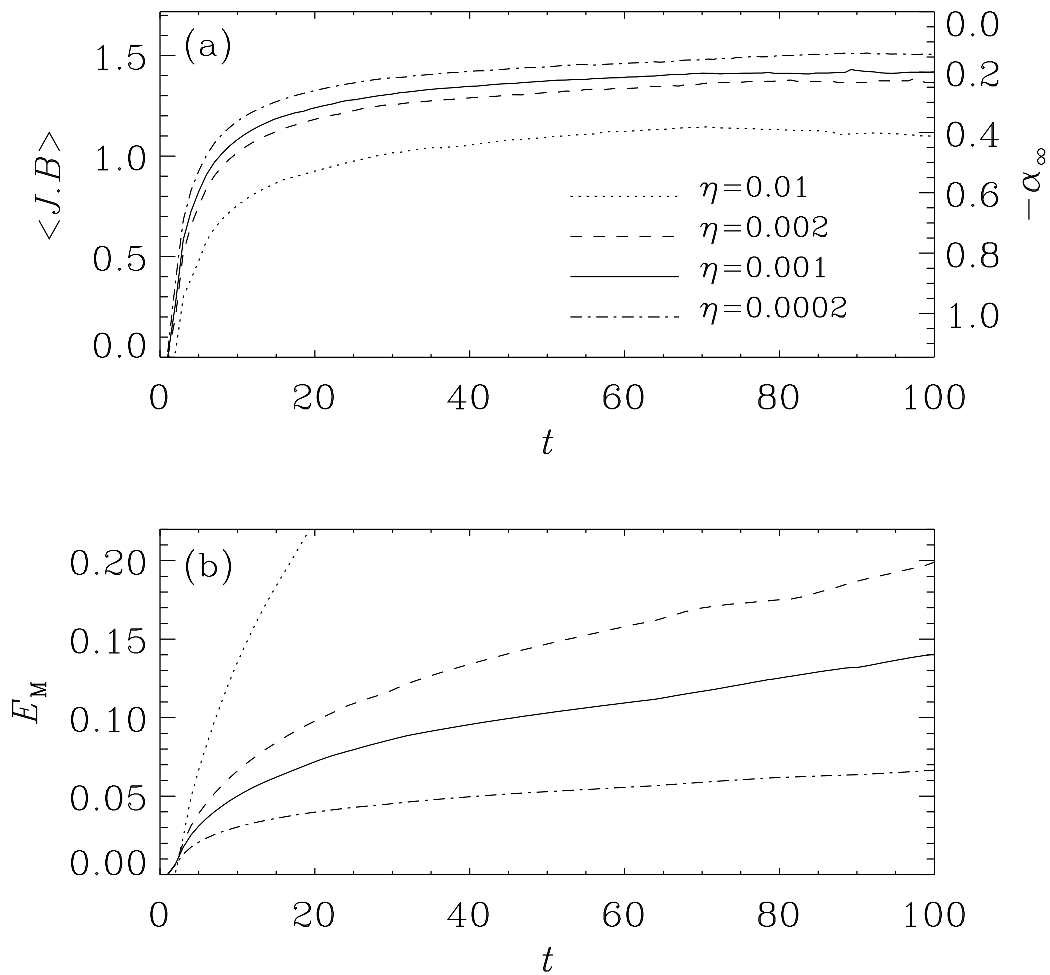
KS, 99; Brandenburg, KS, 2000



Large-scale field Grows only on long resistive timescale.

Helicity constraint in a closure model

KS, 99; Brandenburg, KS, 2000



Large-scale field Grows only on long resistive timescale.

Helicity and catastrophic α quenching?

- **The small-scale helicity equation**

$$\frac{d}{dt}\langle \mathbf{a}\cdot\mathbf{b}\rangle = -2\langle \overline{\boldsymbol{\mathcal{E}}}\cdot\overline{\mathbf{B}}\rangle - 2\eta\frac{4\pi}{c}\langle \mathbf{j}\cdot\mathbf{b}\rangle$$

- **Stationary limit**: $\langle \overline{\boldsymbol{\mathcal{E}}}\cdot\overline{\mathbf{B}}\rangle = -2\eta\frac{4\pi}{c}\langle \mathbf{j}\cdot\mathbf{b}\rangle \to 0$ as $\eta \to 0$

- **Catastrophic R_{M} dependent quenching** of $\overline{\boldsymbol{\mathcal{E}}}$??

- **Change the helicity density conservation equation?** :

$$\partial h/\partial t + \nabla\cdot\mathbf{F} = -2\overline{\boldsymbol{\mathcal{E}}}\cdot\overline{\mathbf{B}} - 2\eta(4\pi/c)\overline{\mathbf{j}\cdot\mathbf{b}}$$

- In Stationary limit: $2\overline{\boldsymbol{\mathcal{E}}}\cdot\overline{\mathbf{B}} \to -\nabla\cdot\mathbf{F}$?

Large scale dynamos need helicity fluxes?

What is helicity density or its flux?

A gauge-invariant helicity density

- **Gauss's linking formula for magnetic helicity**

$$h_{\mathrm{G}} = \frac{1}{4\pi} \int \int \boldsymbol{b}(\boldsymbol{x}) \cdot \left[\boldsymbol{b}(\boldsymbol{y}) \times \frac{\boldsymbol{x} - \boldsymbol{y}}{|\boldsymbol{x} - \boldsymbol{y}|^3} \right] \mathrm{d}^3x \, \mathrm{d}^3y == \int \boldsymbol{a} \cdot \boldsymbol{b} \, \mathrm{d}^3x$$

- **Here:** $\boldsymbol{a} = (1/4\pi) \int \boldsymbol{b}(\boldsymbol{y}) \times (\boldsymbol{x} - \boldsymbol{y})/(|\boldsymbol{x} - \boldsymbol{y}|^3) \, \mathrm{d}^3y$
- $\boldsymbol{a}$ **vector potential in "Coulmb gauge"**: $\boldsymbol{\nabla} \times \boldsymbol{a} = \boldsymbol{b}$, and $\boldsymbol{\nabla} \cdot \boldsymbol{a} = 0$.
- Let $\overline{b_i(\boldsymbol{x}, t) b_j(\boldsymbol{y}, t)} = M_{ij}(\boldsymbol{r}, \boldsymbol{R})$, with $\boldsymbol{r} = \boldsymbol{x} - \boldsymbol{y}$, $\boldsymbol{R} = (\boldsymbol{x} + \boldsymbol{y})/2$
- Correlation scale $l \ll L \ll R_s$ the system scale (Formally let $L \to \infty$)
- **The ensemble average helicity is:** $\overline{h}_{\mathrm{G}} = \int \mathrm{d}^3R \, h(\boldsymbol{R})$

$$h(\boldsymbol{R}) = \frac{1}{4\pi} \int_{L^3} \mathrm{d}^3r \, \epsilon_{ijk} M_{ij}(\boldsymbol{r}, \boldsymbol{R}) \, \frac{r_k}{r^3},$$

$h(\boldsymbol{R})$: **Gauge invariant helicity density of the random small scale field b.**

LECTURE 4

The dynamical quenching model

- Magnetic helicity density and its flux
- Calculating the turbulent $\overline{\mathcal{E}}$
- The minimal τ-approximation (MTA) closure
- Testing MTA in a box
- The dynamical quenching models for MFD
- Conclusions/questions

A gauge-invariant helicity density

- **Gauss's linking formula for magnetic helicity (Moffatt, 1969)**

$$h_{\rm G} = \frac{1}{4\pi}\int\int \boldsymbol{b}(\boldsymbol{x})\cdot\left[\boldsymbol{b}(\boldsymbol{y})\times\frac{\boldsymbol{x}-\boldsymbol{y}}{|\boldsymbol{x}-\boldsymbol{y}|^3}\right]\mathrm{d}^3x\,\mathrm{d}^3y = \int \boldsymbol{a}\cdot\boldsymbol{b}\,\mathrm{d}^3x$$

- **Let** $\overline{b_i(\boldsymbol{x},t)b_j(\boldsymbol{y},t)} = M_{ij}(\boldsymbol{r},\boldsymbol{R})$, **with** $\boldsymbol{r}=\boldsymbol{x}-\boldsymbol{y}$, $\boldsymbol{R}=(\boldsymbol{x}+\boldsymbol{y})/2$
- **Correlation scale** $l \ll L \ll R_s$ **the system scale (Formally let** $L\to\infty$**)**
- **The ensemble average helicity is:** $\overline{h}_{\rm G} = \int \mathrm{d}^3R\, h(\boldsymbol{R})$

$$\overline{h}_{\rm G} = \int \mathrm{d}^3R\left[\frac{1}{4\pi}\int_{L^3}\mathrm{d}^3r\,\epsilon_{ijk}M_{ij}(\boldsymbol{r},\boldsymbol{R})\,\frac{r_k}{r^3}\right] = \int \mathrm{d}^3R\,h(\boldsymbol{R})$$

$h(\boldsymbol{R})$: **Gauge invariant helicity density of the random small scale field** b.

density of correlated b **field links**

(Subramanian & Brandenburg, ApJ Lett., 2006)

Local Helicity conservation

- **For weakly inhomogeneous system, useful to employ double Fourier transforms:** Define $\boldsymbol{k}=(\boldsymbol{k}_1-\boldsymbol{k}_2)/2, \boldsymbol{K}=\boldsymbol{k}_1+\boldsymbol{k}_2$

$$\overline{f(\boldsymbol{x}_1)g(\boldsymbol{x}_2)}=\int \Phi(\hat{f},\hat{g},\boldsymbol{k},\boldsymbol{R})\ \mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{r}}\ \mathrm{d}^3k$$

$$\Phi(\hat{f},\hat{g},\boldsymbol{k},\boldsymbol{R})=\int \overline{\hat{f}(\boldsymbol{k}+\tfrac{1}{2}\boldsymbol{K})\hat{g}(-\boldsymbol{k}+\tfrac{1}{2}\boldsymbol{K})}\ \mathrm{e}^{i\boldsymbol{K}\cdot\boldsymbol{R}}\,\mathrm{d}^3K.$$

- **For example the EMF is** $\overline{\mathcal{E}}_i(\boldsymbol{R})=\epsilon_{ijk}\int\chi_{jk}(\boldsymbol{k},\boldsymbol{R})\,\mathrm{d}^3k$
$\chi_{jk}(\boldsymbol{k},\boldsymbol{R})=\Phi(\hat{u}_j,\hat{b}_k,\boldsymbol{k},\boldsymbol{R})$

$$h(\boldsymbol{R})=\int\int\epsilon_{ijk}\ \overline{\hat{b}_i(\boldsymbol{k}+\tfrac{1}{2}\boldsymbol{K})\hat{b}_j(-\boldsymbol{k}+\tfrac{1}{2}\boldsymbol{K})}\times(\mathrm{i}k_k/k^2)\mathrm{e}^{i\boldsymbol{K}\cdot\boldsymbol{R}}\ \mathrm{d}^3k\ \mathrm{d}^3K$$

- **Use the evolution equation:** $\partial\hat{b}_i(\boldsymbol{k})/\partial t=-\epsilon_{ipq}\mathrm{i}k_p\hat{e}_q$
$\boldsymbol{e}=-\boldsymbol{u}\times\overline{\boldsymbol{B}}-\overline{\boldsymbol{U}}\times\boldsymbol{b}-\boldsymbol{u}\times\boldsymbol{b}+\overline{\boldsymbol{u}\times\boldsymbol{b}}+\eta\boldsymbol{j}.$

Local Helicity conservation

- The evolution of h is given by

$$\begin{aligned}\frac{\partial h(\boldsymbol{R})}{\partial t} &= \int\int\Big\{-2\int \overline{\hat{e}_q(\boldsymbol{k}+\tfrac{1}{2}\boldsymbol{K})\hat{b}_q(-\boldsymbol{k}+\tfrac{1}{2}\boldsymbol{K})} \\ &+ 2(K_j k_q/k^2)\ \overline{\hat{e}_q(\boldsymbol{k}+\tfrac{1}{2}\boldsymbol{K})\hat{b}_j(-\boldsymbol{k}+\tfrac{1}{2}\boldsymbol{K})} \\ &- (K_s k_s/k^2)\ \overline{\hat{e}_q(\boldsymbol{k}+\tfrac{1}{2}\boldsymbol{K})\hat{b}_q(-\boldsymbol{k}+\tfrac{1}{2}\boldsymbol{K})}\Big\} \\ &\times\ \mathrm{e}^{i\boldsymbol{K}\cdot\boldsymbol{R}}\ \mathrm{d}^3K\ \mathrm{d}^3k.\end{aligned}$$

$$A_1 = -2\overline{\boldsymbol{e}\cdot\boldsymbol{b}} = 2\overline{\boldsymbol{b}\cdot(\boldsymbol{u}\times\overline{\boldsymbol{B}})} - 2\eta\overline{\boldsymbol{j}\cdot\boldsymbol{b}} = -2\overline{\boldsymbol{\mathcal{E}}}\cdot\overline{\boldsymbol{B}} - 2\eta\overline{\boldsymbol{j}\cdot\boldsymbol{b}}$$

- From the $\boldsymbol{e} = -\boldsymbol{u}\times\overline{\boldsymbol{B}} + \ldots$ term, get fluxes proportional to $\overline{\boldsymbol{B}}$
- From the $\boldsymbol{e} = -\overline{\boldsymbol{U}}\times\boldsymbol{b} + \ldots$ term, get advective flux
- The $\boldsymbol{e} = -\boldsymbol{u}\times\boldsymbol{b}$ gives triple correlations in the flux, needing closure.

A local helicity conservation equation

- **Local helicity conservation equation** :

$$\partial h/\partial t + \nabla \cdot \mathbf{F} = -2\overline{\mathcal{E}} \cdot \overline{\mathbf{B}} - 2\eta(4\pi/c)\overline{\mathbf{j} \cdot \mathbf{b}}$$

- **Helicity flux:** $F_i = F_i^{\mathrm{VC}} + F_i^{\mathrm{A}} + F_i^{\mathrm{bulk}} + F_i^{\mathrm{triple}}$.

- **Simplest flux due to advection**: $\mathbf{F}^{\mathrm{bulk}} = h\overline{\boldsymbol{U}}$

- **Other fluxes driven by (u, b) correlations and anisotropy**:

$F_i^{\mathrm{VC}} = 2\epsilon_{qlm}\overline{B}_l(\boldsymbol{R}) \int \mathrm{i}k_q\chi_{mi}k^{-2}\mathrm{d}^3k$: (Vishniac and Cho, 2001)

$F_i^{\mathrm{A}} = -\epsilon_{qlm}\overline{B}_l(\boldsymbol{R}) \int \mathrm{i}k_i\chi_{mq}k^{-2}\mathrm{d}^3k$: (Kleeorin et al 2000) .

- **Here** $\chi_{jk}(\boldsymbol{k}, \boldsymbol{R}) = \int \overline{\hat{u_j}(\boldsymbol{k} + \frac{1}{2}\boldsymbol{K})\hat{b_k}(-\boldsymbol{k} + \frac{1}{2}\boldsymbol{K})}\ \mathrm{e}^{i\boldsymbol{K}\cdot\boldsymbol{R}}\,\mathrm{d}^3K$ (cf. Roberts & Soward 1975)

- **Now in stationary limit;** $\overline{\mathcal{E}} \cdot \overline{B} = -\frac{1}{2}\boldsymbol{\nabla} \cdot \mathbf{F} - \eta\overline{\boldsymbol{j} \cdot \boldsymbol{b}}$

- **Large χ_{ij} not guaranteed if $\mathbf{F}^{bulk} = 0$.**

The turbulent $\overline{\mathcal{E}}$

- Need to find $\overline{\mathcal{E}} = \overline{\mathrm{u} \times \mathrm{b}}$ under influence of Lorentz forces

$$\frac{\partial \boldsymbol{b}}{\partial t} = \nabla \times (\overline{\boldsymbol{U}} \times \boldsymbol{b} + \boldsymbol{u} \times \overline{\boldsymbol{B}} - \eta \nabla \times \boldsymbol{b}) + \boldsymbol{G}$$

$$\begin{aligned} \frac{\partial \boldsymbol{u}}{\partial t} &= -\frac{1}{\rho} \boldsymbol{\nabla} \left(p + \frac{1}{\mu_0} \overline{\boldsymbol{B}} \cdot \boldsymbol{b} \right) + \nu \boldsymbol{\nabla}^2 \boldsymbol{u} \\ &+ \frac{1}{\rho} \left[(\overline{\boldsymbol{B}} \cdot \boldsymbol{\nabla}) \boldsymbol{b} + (\boldsymbol{b} \cdot \boldsymbol{\nabla}) \overline{\boldsymbol{B}} \right] + \mathbf{f} + \boldsymbol{T}. \end{aligned} \tag{3}$$

- Here G and T are the "pain in neck" nonlinear terms in u and b.

$$\boldsymbol{G} = \nabla \times (\boldsymbol{u} \times \boldsymbol{b})' = \nabla \times [\boldsymbol{u} \times \boldsymbol{b} - \overline{\boldsymbol{u} \times \boldsymbol{b}}]$$

$$\boldsymbol{T} = -(\boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u})' - \frac{1}{\mu_0 \rho} \left[(\boldsymbol{b} \cdot \boldsymbol{\nabla} \boldsymbol{b})' - \tfrac{1}{2} \boldsymbol{\nabla} (\boldsymbol{b}^2)' \right]$$

- Calculating $\overline{\mathcal{E}}$ requires a closure even in the kinematic limit

The kinematic limit of $\overline{\mathcal{E}}$

- For short correlation times ($\tau_{\rm cor}$), neglect G, also assume statistical isotropy of the random u :

$$\overline{\boldsymbol{\mathcal{E}}} = \overline{\boldsymbol{u} \times \int^t dt' (\partial \boldsymbol{b}/\partial\tau)} = \overline{\boldsymbol{u}(t) \times \int^t dt' [-\boldsymbol{u}(t') \cdot \nabla \overline{\boldsymbol{B}} + \overline{\boldsymbol{B}} \cdot \nabla \boldsymbol{u}(t')]}$$

$$\overline{\mathcal{E}}_i = \int_0^t \left[\epsilon_{ijk} \overline{u_j(t) u_{k,p}(t')} \overline{B}_p(t') + \epsilon_{ijp} \overline{u_j(t) u_l(t')} \overline{B}_{p,l}(t') \right] \mathrm{d}t',$$

- So: $\overline{\boldsymbol{\mathcal{E}}} = \alpha \overline{\boldsymbol{B}} - \eta_t \nabla \times \overline{\boldsymbol{B}}$, where

$$\alpha = -\tfrac{1}{3} \int_0^t \overline{\boldsymbol{u}(t) \cdot \boldsymbol{\omega}(t')}\, dt' \approx -\tfrac{1}{3} \tau_{\rm cor} \overline{\boldsymbol{u} \cdot \boldsymbol{\omega}},$$

$$\eta_{\rm t} = \tfrac{1}{3} \int_0^t \overline{\boldsymbol{u}(t) \cdot \boldsymbol{u}(t')}\, dt' \approx \tfrac{1}{3} \tau_{\rm cor} \overline{\boldsymbol{u}^2},$$

The "Minimal τ approximation" closure

- **Closure for the triple correlations arising in $\overline{\mathcal{E}}$**
  (**Blackman, Field, 2002; Radler, Kleeorin, Rogachevski, 2003; KS/AB 2005**)

$$
\begin{aligned}
\frac{\partial \overline{\mathcal{E}}}{\partial t} &= \overline{\boldsymbol{u} \times \dot{\boldsymbol{b}}} + \overline{\dot{\boldsymbol{u}} \times \boldsymbol{b}}, = \overline{\boldsymbol{u} \times [-\boldsymbol{u} \cdot \nabla \overline{\boldsymbol{B}} + \overline{\boldsymbol{B}} \cdot \nabla \boldsymbol{u} + \boldsymbol{G}]} \\
&+ \overline{[\frac{\boldsymbol{b} \cdot \nabla \overline{\boldsymbol{B}} - \boldsymbol{\nabla} p + \overline{\boldsymbol{B}} \cdot \nabla \boldsymbol{b}}{\rho} + \mathrm{f} + \boldsymbol{T}] \times \boldsymbol{b}} \\
&= \tilde{\alpha}\, \overline{\boldsymbol{B}} - \tilde{\eta}_{\mathrm{t}}\, (\nabla \times \overline{\boldsymbol{B}}) + 0 + \mathrm{N}
\end{aligned}
$$

- $\tilde{\alpha} = -\frac{1}{3}\left(\overline{\boldsymbol{\omega} \cdot \boldsymbol{u}} - (4\pi / c\rho)\overline{\boldsymbol{j} \cdot \boldsymbol{b}}\right) \qquad \tilde{\eta}_{\mathrm{t}} = \frac{1}{3}\overline{\boldsymbol{u}^2}.$

- **Closure hypothesis; triple correlations provide damping proportional to $\overline{\mathcal{E}}$ over a relaxation time τ:** $\quad \mathrm{N} = -\overline{\mathcal{E}}/\tau$

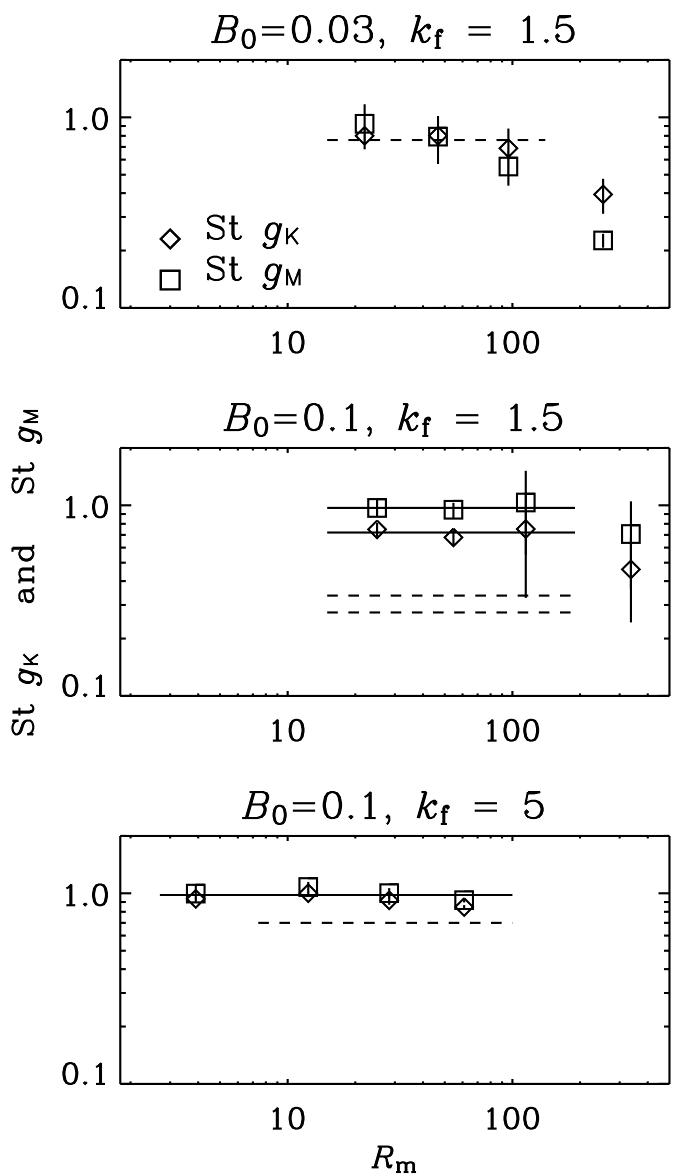
- **In steady state** $\overline{\mathcal{E}} = \tau\tilde{\alpha}\, \overline{\boldsymbol{B}} - \tau\tilde{\eta}_{\mathrm{t}}\, (\nabla \times \overline{\boldsymbol{B}})$

α-effect gets renormalized by a term proportional to $j \cdot b$

(Also in Pouquet et al., 75 EDQNM closure)

Testing MTA in a box

Brandenburg, KS, 2005, 2007



- $\alpha = \tau(g_K\tilde{\alpha}_K + g_M\tilde{\alpha}_M)$
- $St = \tau u_{rms} k_f$
- τ is positive and ~ 1 in natural units!

Dynamical quenching model for MFD

- **Solve dynamo equation with local helicity conservation**

- $\alpha = \alpha_{\rm K} + \alpha_{\rm M}$, **with** $\alpha_{\rm M} = (\tau/3\rho)(4\pi/c)\langle \mathbf{j}\cdot\mathbf{b}\rangle$ (Pouquet et a. 1976, Gruzinov & Diamond 1994)

- **Is there a magnetic** $\alpha_{\rm M}$**? What is** τ**?**

- **For small** $R_m \ll 1$, $R_e \ll 1$**, can use FOSA in Induction and Momentum equation. Then** α **either** $(\boldsymbol{u}, \boldsymbol{u})$ **helical correlation or sum** $(\mathbf{f}, \boldsymbol{b}) + (\boldsymbol{b}, \boldsymbol{b})$ **helical correlations** (Sur et al 2007).

- **In 2-d, for large** R_m, $(\boldsymbol{u}^2 - \boldsymbol{b}^2)$ **structure appears in the effective** $\tau_{\rm eff}$ **or cross-phase leading to suppression** (Diamond, INI Talk, 2004).

- **Good to transport away magnetic helicity by flux: to prevent catastrophic** $\overline{\mathcal{E}}_{||}$ (α) **suppression?**

Dynamical quenching model for MFD

- **Solve dynamo equation with local helicity conservation**

$$\frac{\partial \overline{\boldsymbol{B}}}{\partial t} = \boldsymbol{\nabla} \times \left(\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}} + \alpha \overline{\boldsymbol{B}} - (\eta + \eta_t)(\nabla \times \overline{\boldsymbol{B}}) \right), \quad \alpha = (\alpha_{\rm K} + \alpha_{\rm m})$$

- $\alpha_{\rm m} = (\tau/3\rho)(4\pi/c)\langle \mathbf{j} \cdot \mathbf{b} \rangle \simeq (\tau/3\rho) k_0^2 \, h$
- **Helicity conservation with flux becomes**, $R_m = \eta_t/\eta$, $B_{\rm eq}^2 = \rho\overline{\boldsymbol{u}^2}$

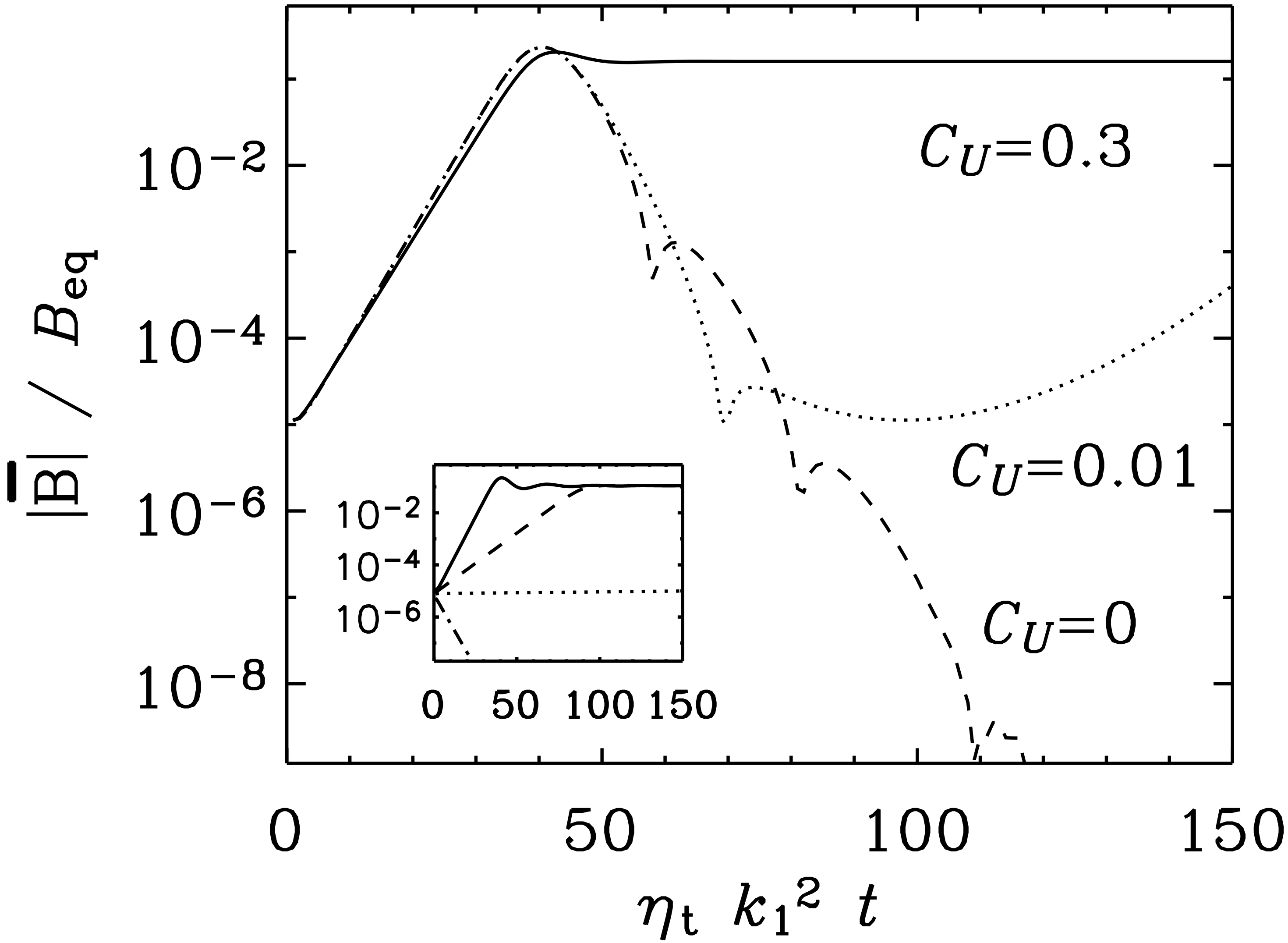
$$\frac{\partial \alpha_{\rm m}}{\partial t} = -2\eta_{\rm t} k_0^2 \left(\frac{(\alpha_{\rm K} + \alpha_{\rm m})\overline{\boldsymbol{B}}^2 - \eta_t (\nabla \times \overline{\boldsymbol{B}}) \cdot \overline{\boldsymbol{B}} + \frac{1}{2} \boldsymbol{\nabla} \cdot \mathbf{F}}{B_{\rm eq}^2} + \frac{\alpha_{\rm m}}{R_m} \right)$$

- **For $\dot{\alpha}_{\rm m} = 0$, $\mathbf{F} = 0$, "old algebraic quenching"**

$$\alpha = \frac{\alpha_{\rm K} + \eta_t R_m \nabla \times \overline{\boldsymbol{B}} \cdot \overline{\boldsymbol{B}}}{1 + R_m \overline{\boldsymbol{B}}^2 / B_{\rm eq}^2}$$

Effect of advective helicity flux

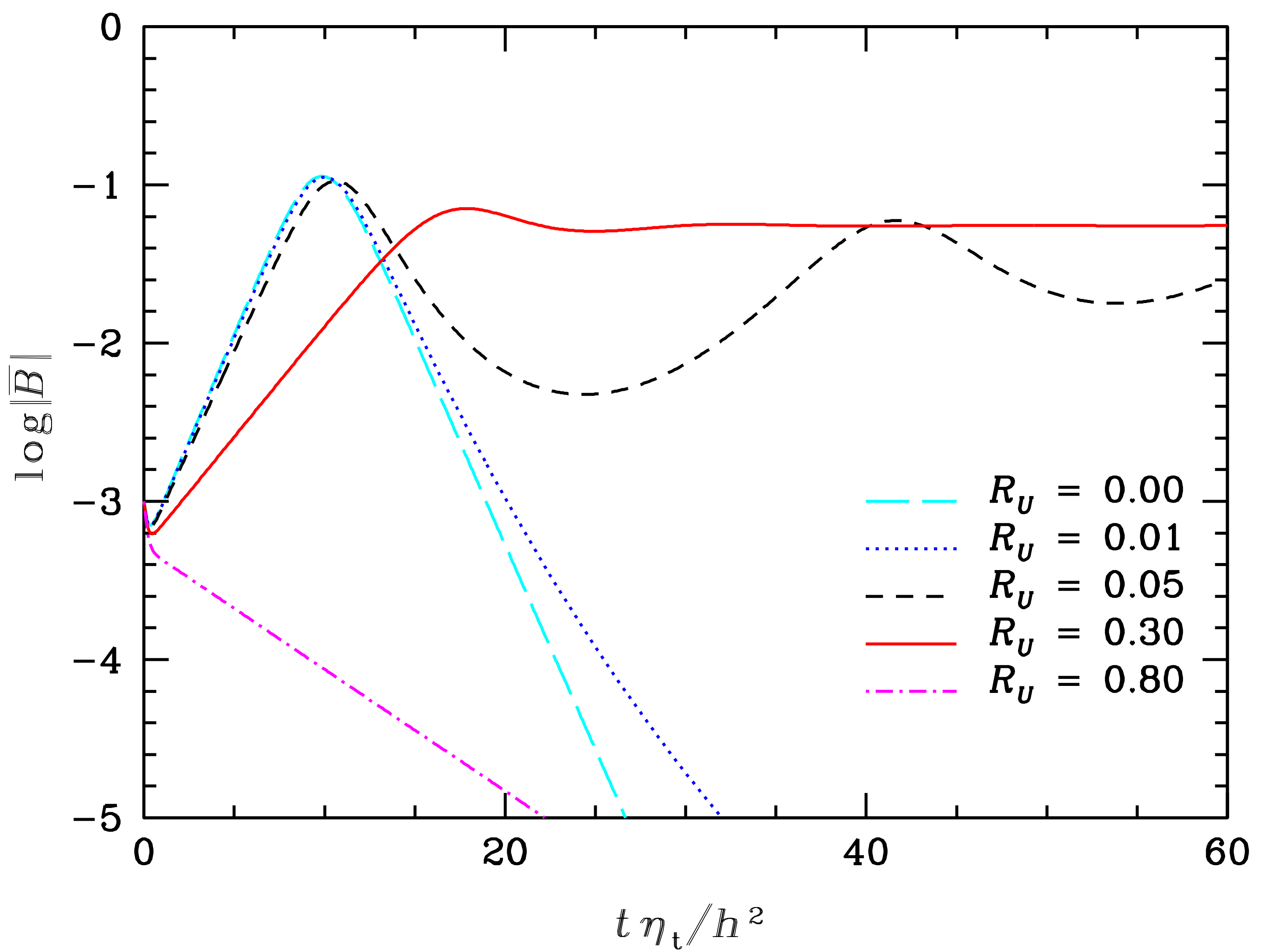
Shukurov, Sokoloff, Subramanian, Brandenburg, AA Lett., 2006



$C_\Omega = -2, C_\alpha = 1, R_{\rm m} = 10^5$

Effect of advective flux:0-d model

Sur, Shukurov, Subramanian, MNRAS, 2007



Helicity fluxes involving χ

- **These fluxes zero for isotropic turbulence, or if $v_{mi} = m_{mi}$**

- **Shear can generate anisotropy $+$ break v-b symmetry**

$(\partial \boldsymbol{u}/\partial t \sim -\boldsymbol{u} \cdot \boldsymbol{\nabla} \overline{\boldsymbol{U}} + ...$ and $\partial \boldsymbol{b}/\partial t = +\boldsymbol{b} \cdot \boldsymbol{\nabla} \overline{\boldsymbol{U}} + ...)$

- **Estimate of kinetic V-C Flux**: (Brandenburg, Subramanian, AN, 2005)

$$F_i^{\mathrm{VC}} \sim (u\tau)(u\tau^*)\epsilon_{ijl}\frac{1}{2}(\overline{U}_{l,k} + \overline{U}_{k,l})\overline{B}_j\overline{B}_k \sim (u\tau)^2[\overline{\boldsymbol{B}} \times (\mathbf{S} \cdot \overline{\boldsymbol{B}})]_i$$

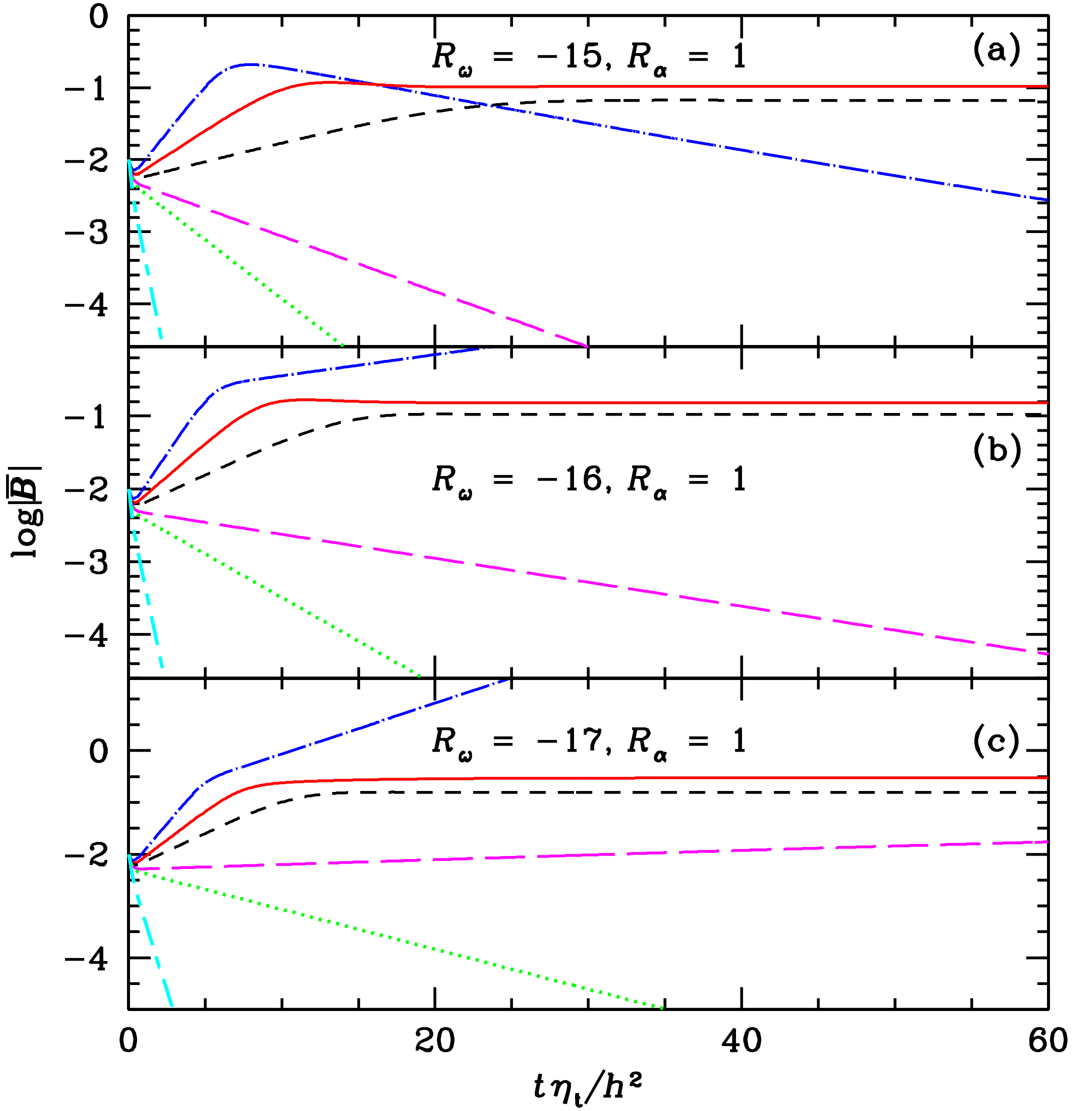
- **Other closures could lead to suppression of "effective τ"**

- **V-C flux works to enhance finite amplitude fields**

Effect of Vishniac-Cho flux

Sur, Shukurov, Subramanian, MNRAS, 2007

$R_U = 0, 0.3, 0.5, 0.8, 1.0$ and 3.0



More thoughts

- Mean field dynamos at large R_m have to work in presence of strong fluctuations
- Mean field dynamos work with helicity fluxes
- Need a gauge invariant description of helicity density
- How does dynamo generation work for high redshift galaxies?
- Fluctuation dynamo saturation?
- For mean field dynamos: How to compute dynamo co-efficients for systems with large $R_{\rm M}$ and P_m?
- When is the IGM significantly polluted with magnetic fields?
- Primordial fields?

SKA will probe the RM sky and could yield surprises...