Dynamos on galactic scales,

or

Dynamos around us.

Part IV. Cluster dynamos

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Outline

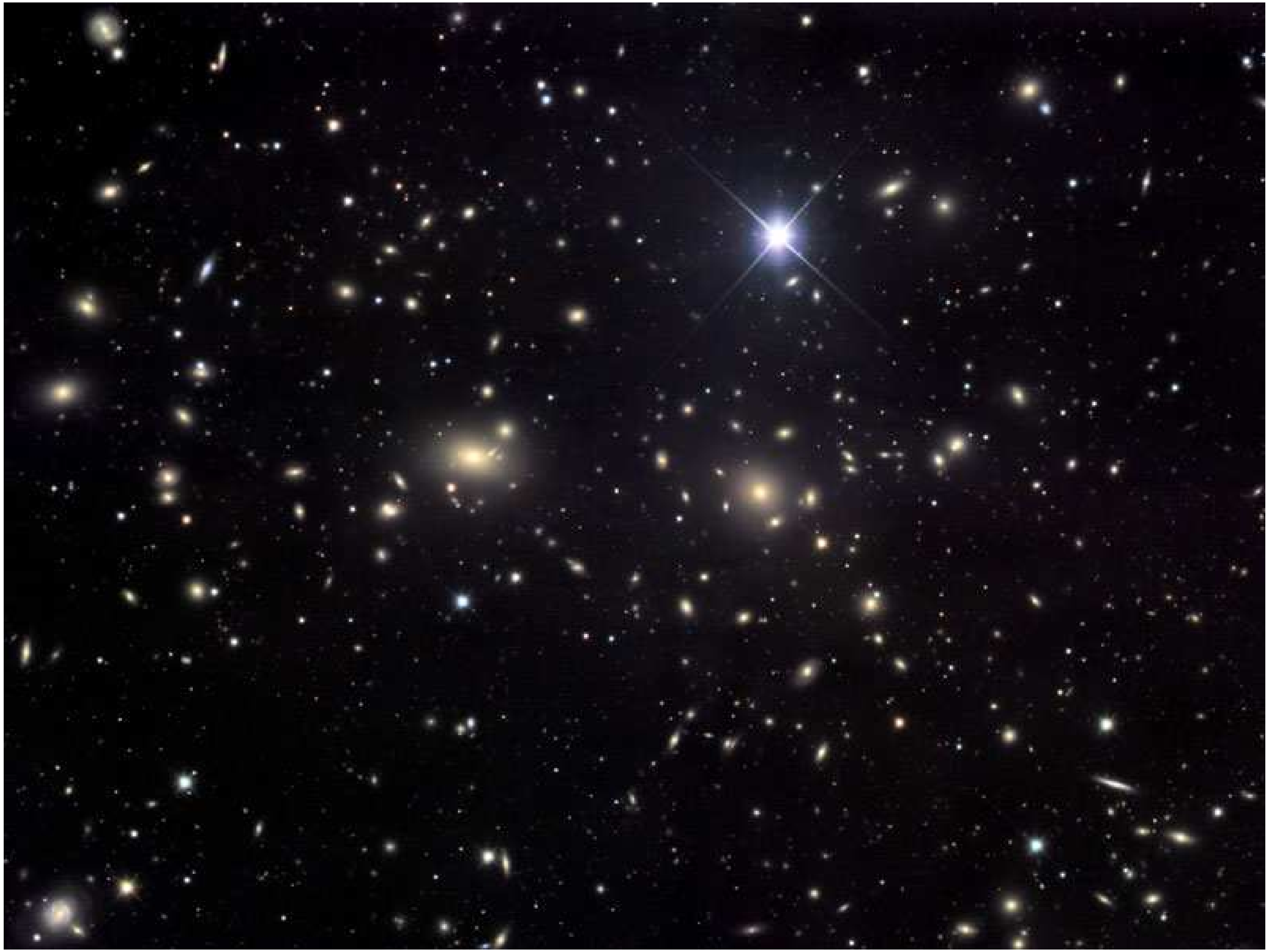
1. Evidence for intracluster magnetic fields

2. Young clusters: decaying turbulence

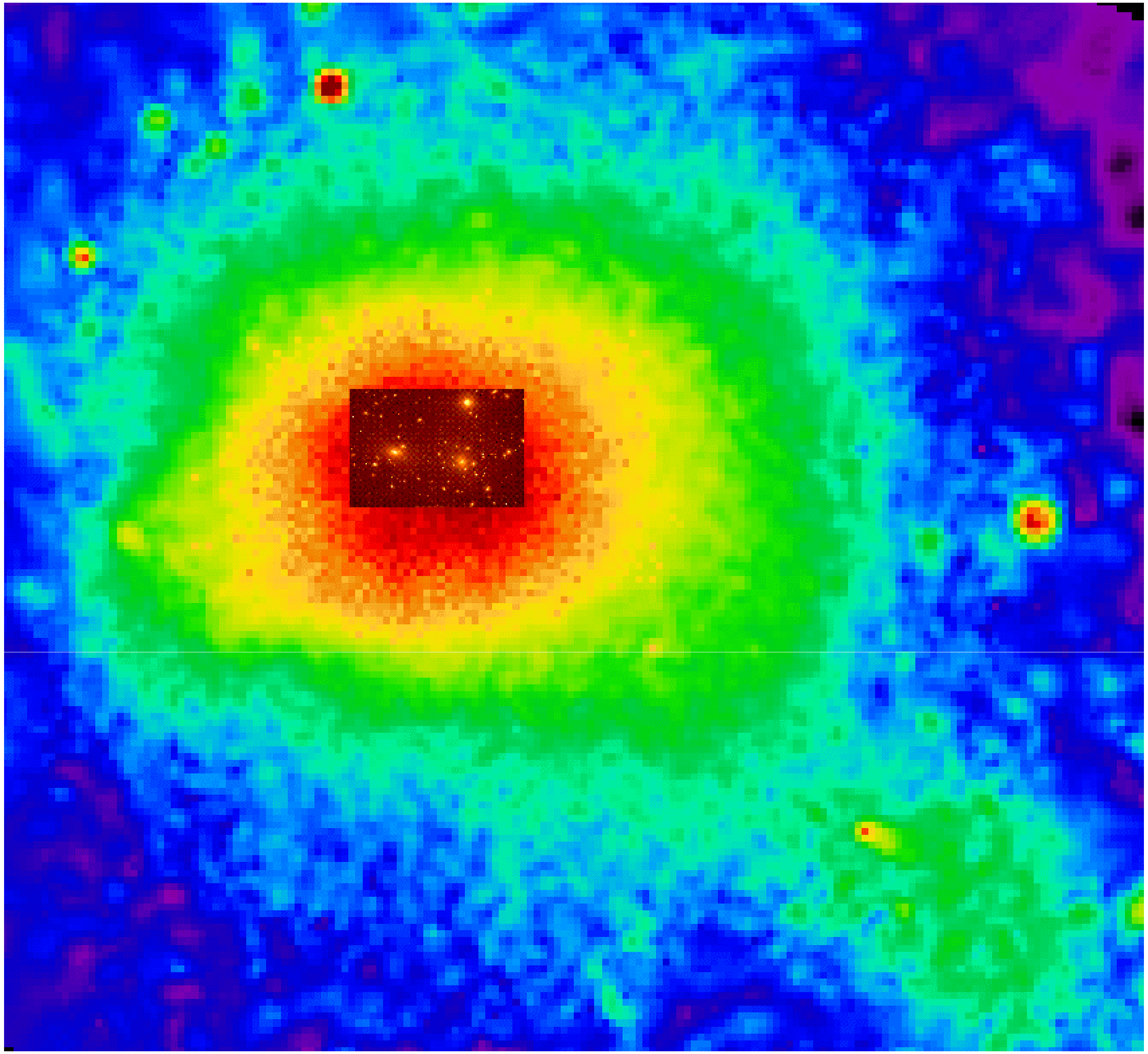
3. Three evolutionary stages

4. Morphology of magnetic structures produced by the fluctuation (small-scale) dynamo

1. Evidence for intracluster magnetic fields



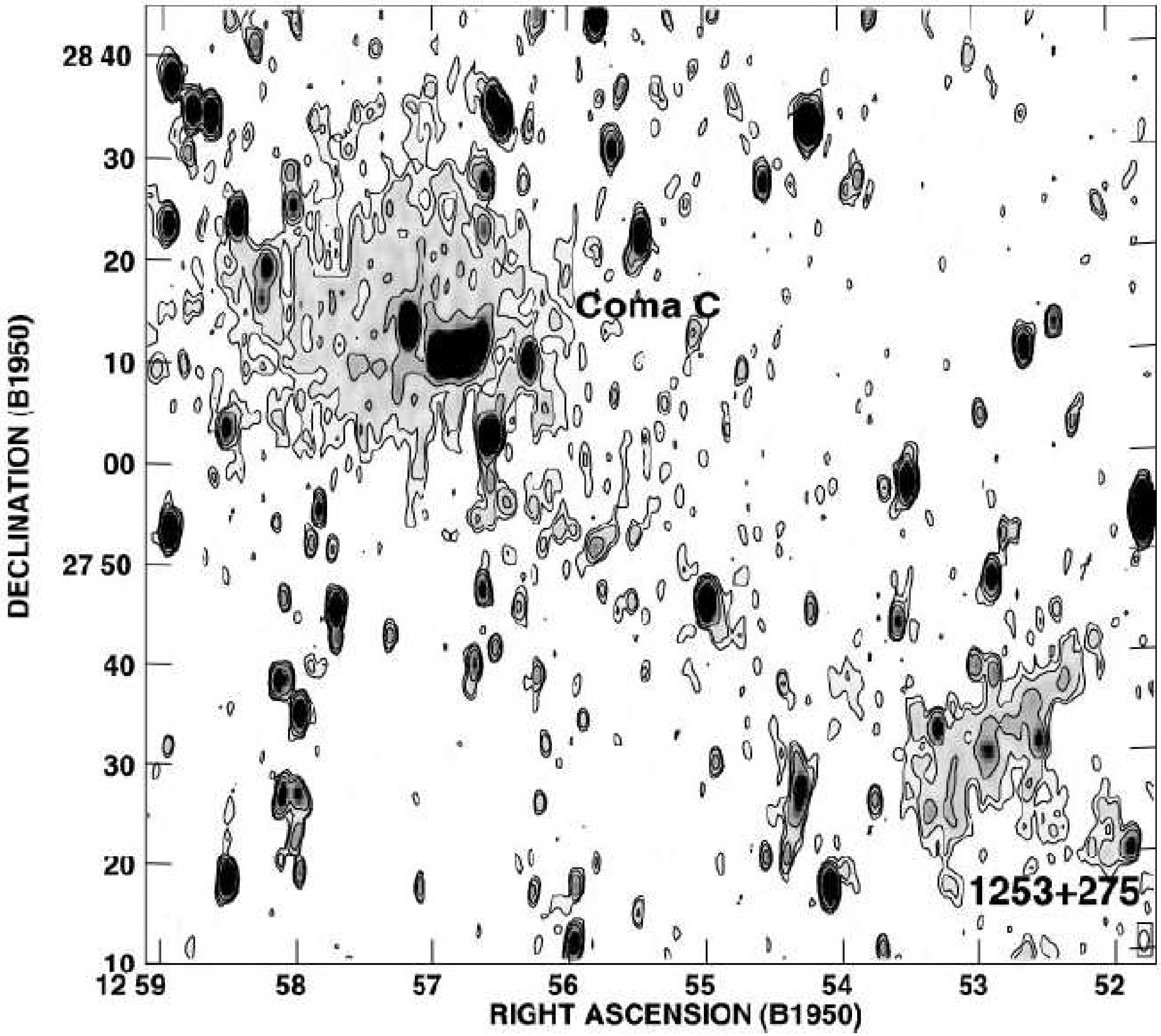
Coma cluster, central part, > 1000 galaxies (the Coma Berenices Constellation)



Coma cluster in X-rays (ROSAT)**: evidence for intergalactic gas**

$T = 10^6$ K, $n = 10^{-3}$ cm^{-3}, $R = 500$ kpc, deviations from symmetry indicate recent merger

Radio halo: synchrotron emission of Coma at λ90 cm, tracer of magnetic fields and relativistic electrons

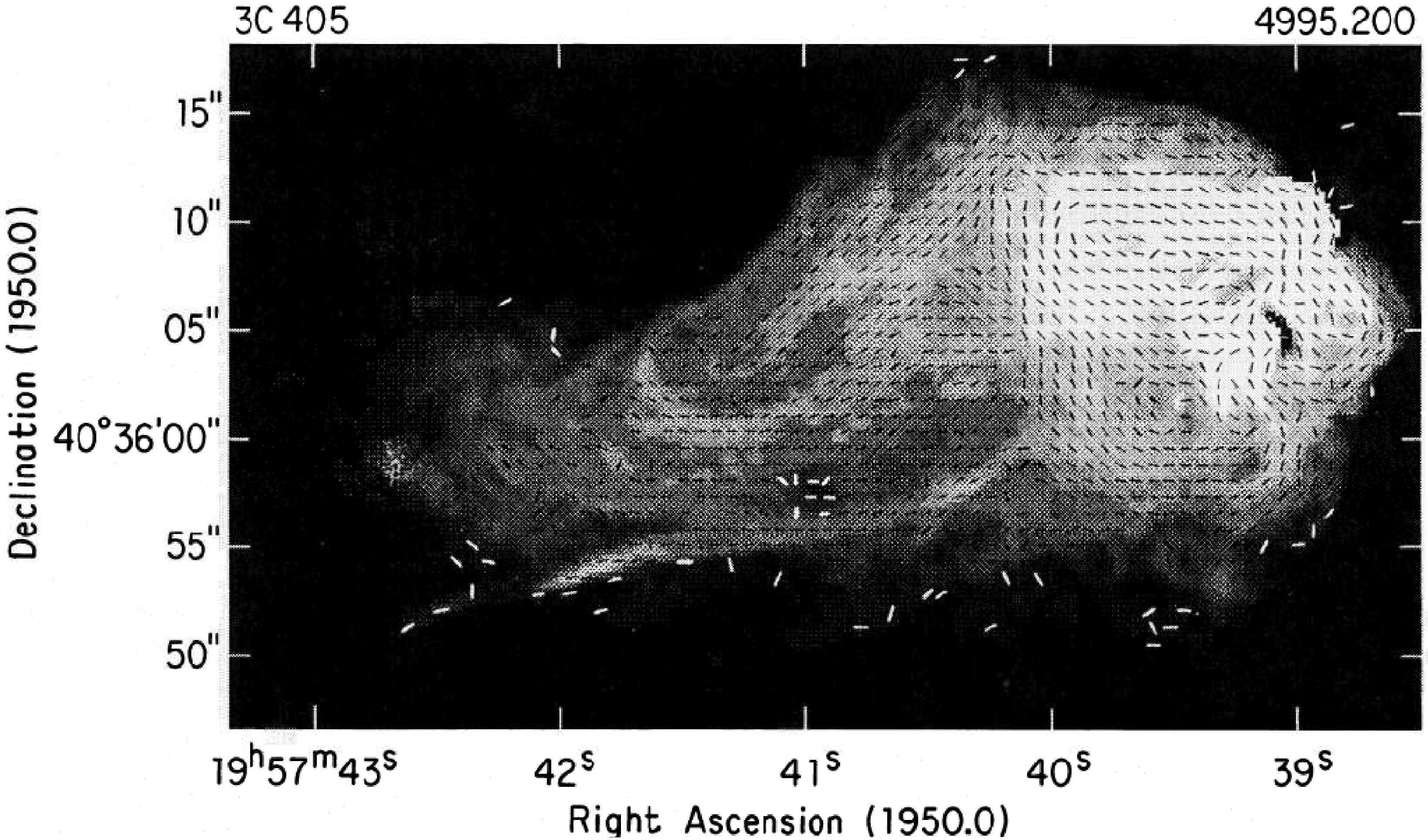


Radio halos are observed in some (5–10%) clusters (lack of relativistic electrons?).

The occurrence of radio halos seems to correlate with existence of substructure in X-rays indicative of a recent merger.

Feretti & Giovannini (1998), resolution 25×50 kpc (RA×DEC) .

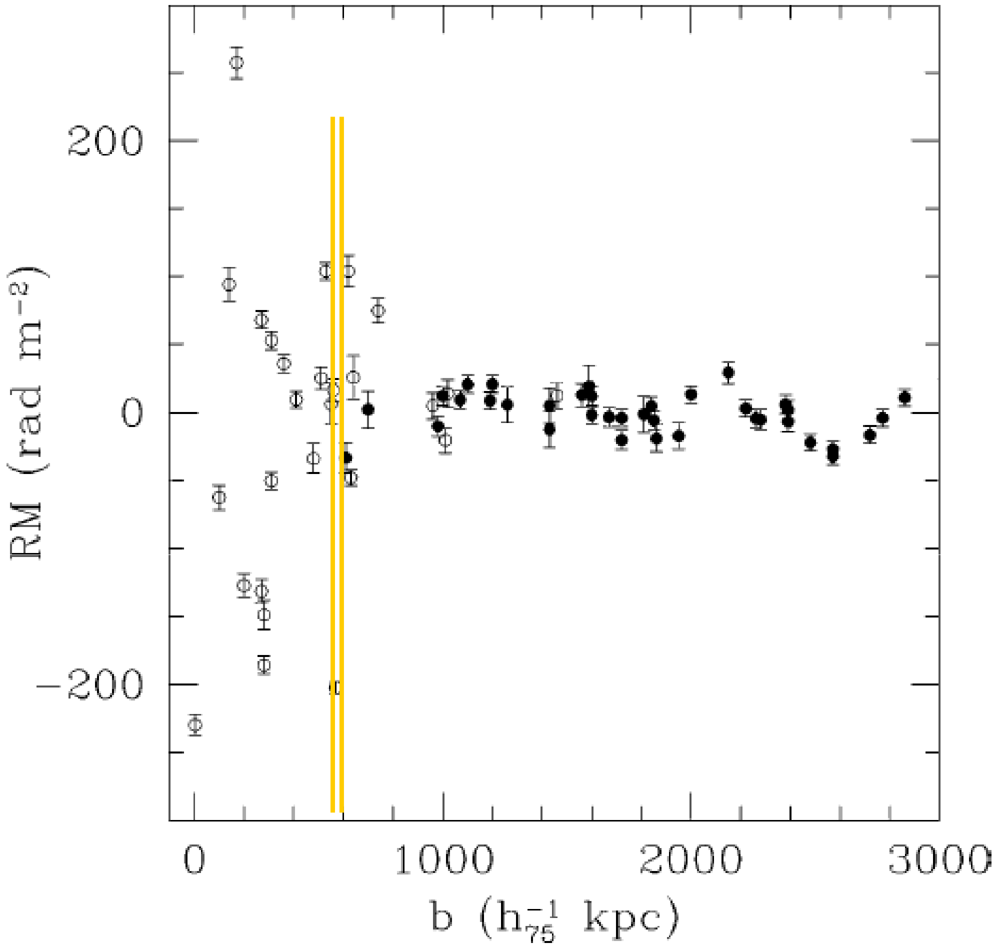
Faraday rotation in a cluster gas: radio lobes of Cyg A



Dreher et al., ApJ, 316, 611, 1987: magnetic field in the intracluster gas,

B = 2–10 μG, l = 20–30 kpc

Faraday rotation: evidence of magnetic fields in many clusters



RM of background radio sources versus distance from the cluster centre for 16 galaxy clusters;

filled symbols: field sources

(Clarke et al., ApJ, 547, L111, 2001)

$B = 2\ \mu\text{G}, \quad L = 500\ \text{kpc},$
$n_e = 10^{-3}\ \text{cm}^{-3}$

$\Rightarrow \text{RM} = 1000\ \text{rad/m}^2$

$= 10\ \text{RM}_{\text{observed}}$

Random magnetic field b, scale $l_0 = 10$ kpc:

$$\sigma_{\text{RM}} = 0.81\ b n_e (L l_0)^{1/2} = 100\ \text{rad/m}^2 \Rightarrow b = 2\ \mu\text{G}$$

Observational estimates of magnetic fields in galaxy clusters

Method	Strength μG	Model parameters
Synchrotron halos	0.4–1	Minimum energy, $k = \eta = 1$, $\nu_{low} = 10$ MHz, $\nu_{high} = 10$ GHz
Faraday rotation (embedded)	3–40	Cell size = 10 kpc
Faraday rotation (background)	1–10	Cell size = 10 kpc
Inverse Compton	0.2–1	$\alpha = -1$, $\gamma_{radio} \sim 18000$, $\gamma_{xray} \sim 5000$
Cold fronts	1–10	Amplification factor ~3
GZK	>0.3	AGN = site of origin for EeV CRs

From Carilli & Taylor, Ann. Rev. Astron. Astrophys., 40, 319, 2002

2. Young clusters: decaying turbulence

- Galaxy clusters: merger of smaller structures
- Turbulence driven in the merger events
- Decaying turbulence after the merger
- No direct evidence of turbulence in the intracluster gas (no line emission/absorption)
- Indirect evidence from pressure fluctuations

$l_0 \simeq 100\,\mathrm{kpc}, \;\; v_0 \simeq 250\,\mathrm{km/s}$ (Schuecker et al. (2004)

- Possibility of Fe XXV line observations in X-rays (Inogamov & Sunyaev 2003)
- Upper limit: heating rate < X-ray luminosity, $v_0 < 200\,\mathrm{km/s}$

(Subramanian et al., 2006)

Coulomb mean free path in the intracluster gas:

$$\lambda \simeq 5\,\mathrm{kpc}\left(\frac{c_\mathrm{s}}{10^3\,\mathrm{km\,s^{-1}}}\right)^4\left(\frac{n_\mathrm{e}}{10^{-3}\,\mathrm{cm^{-3}}}\right)^{-1}$$

$\Rightarrow$Collisionless gas at scales O(10 kpc).

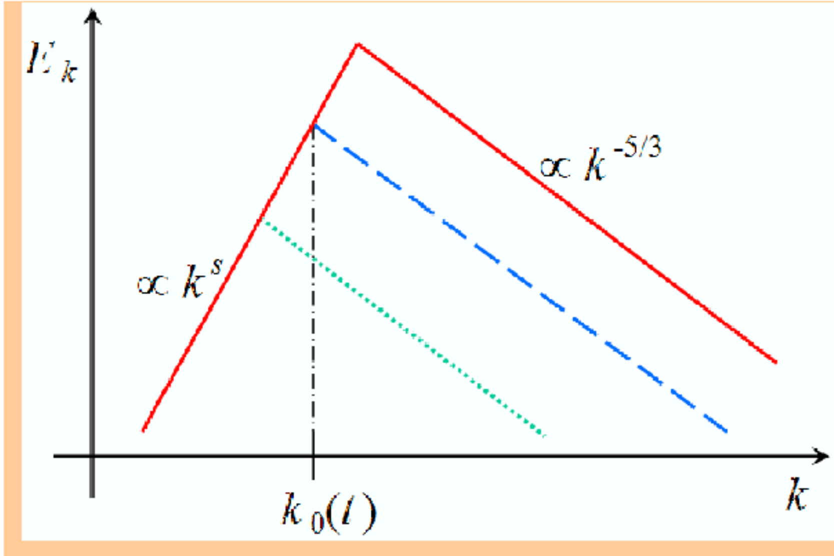
However, Larmor radius << λ

$\Rightarrow$ an effectively collisional plasma?

$$\mathrm{Re} = \frac{v_0 l_0}{\nu} \simeq 3\frac{v_0 l_0}{c_\mathrm{s}\lambda\delta} = 3\mathcal{M}\frac{l_0}{\lambda\delta}, \qquad \nu = \tfrac{1}{3}c_\mathrm{s}\lambda\delta\,, \quad \delta \cong 0.1\ (?)$$

Turbulence past a solid sphere: $\mathrm{Re} > 400$

Decaying turbulence



$$E_k = Ck^s,\ k \le k_0,\ s = 2, 4$$

Total specific energy: $E(t) \simeq \frac{1}{2}v_0^2$

On the other hand, $E(t) = \int_0^\infty E_k\, dk \propto k_0^{s+1}.$

$$\frac{dE}{dt} = -\frac{v_0^2}{t_0} \simeq -v_0^3 k_0, \quad t_0 = 1/(k_0 v_0)$$

$$v_0 \propto E^{1/2},\ k_0 \propto E^{1/(s+1)} \quad \Rightarrow \quad \frac{dE}{dt} = -AE^{(3s+5)/[2(s+1)]}$$

$$E \propto t^{-\alpha},\ l_0 = 2\pi/k_0 \propto t^\beta, \quad \alpha = 2\frac{s+1}{s+3},\ \beta = \frac{2}{s+3}$$

3. Three evolutionary stages

Stage 1. Cluster formation, $0 \lesssim t \lesssim 4$ Gyr

- Volume-filling random flow, $v_0 \simeq 300$ km/s, $\ell_0 \simeq 150$kpc,
- produced in the major merger event
  (e.g., wakes of merging subclusters).
- Re $\gtrsim 100 \Rightarrow$ turbulence.

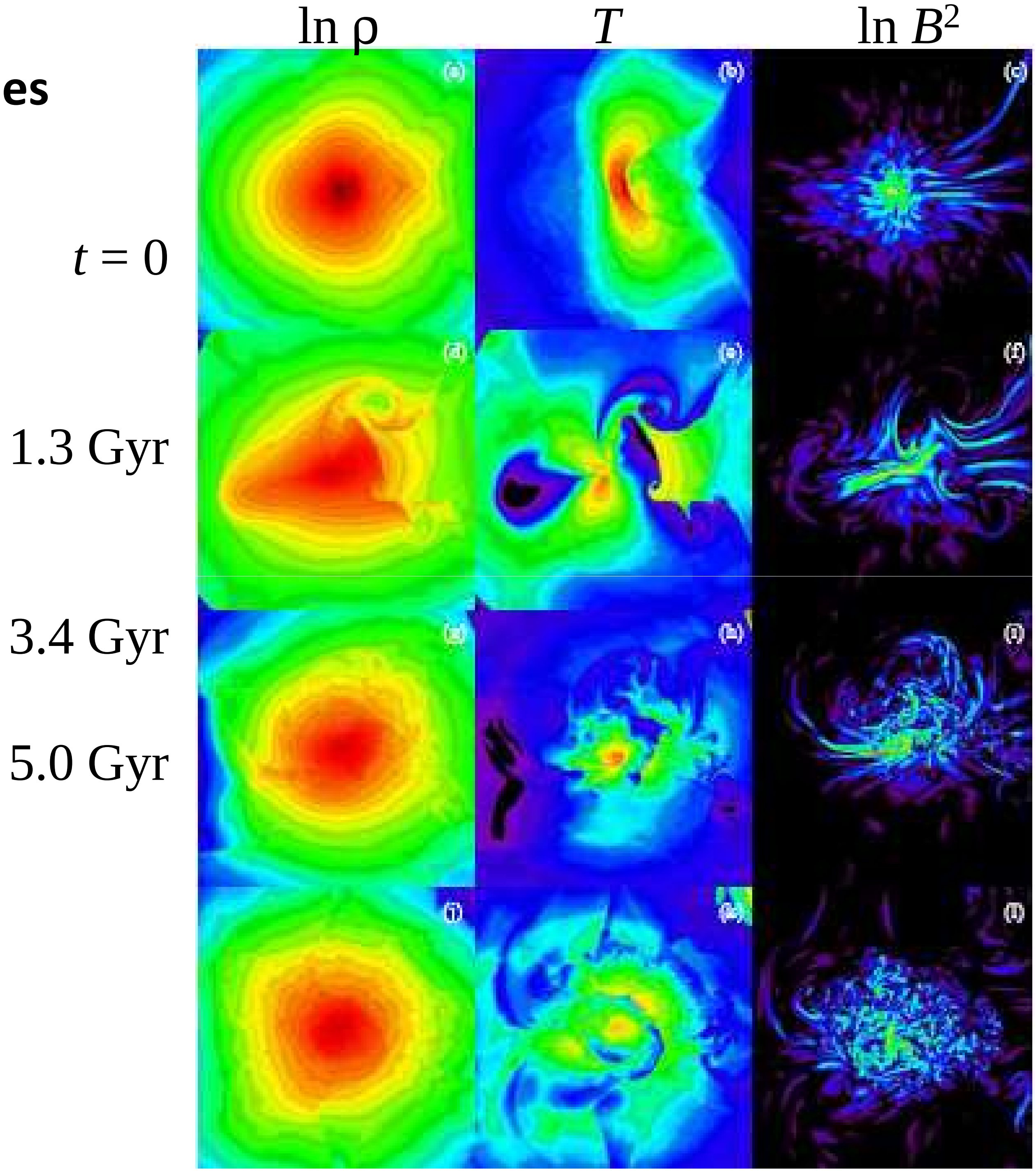
- Fluctuation dynamo: B amplified by a factor $A > 3000$,

- $B \simeq 2\ \mu$G, $\ell_B \simeq$ 20–30 kpc (if $B_0 > 10^{-9}$ G),

- $\sigma_{RM} \cong 200$ rad/m^2

Magnetic field in a merging cluster of galaxies

Roettiger et al, ApJ, 518, 594, 1999



Stage 2. Decay after major mergers, $4 \lesssim t \lesssim 9$ Gyr

$v_0 \propto t^{-3/5}, \ \ell_0 \propto t^{2/5}$

$\Rightarrow v_0 \simeq 150$ km/s, $\ell_0 \simeq 300$ kpc at $t = 9$ Gyr

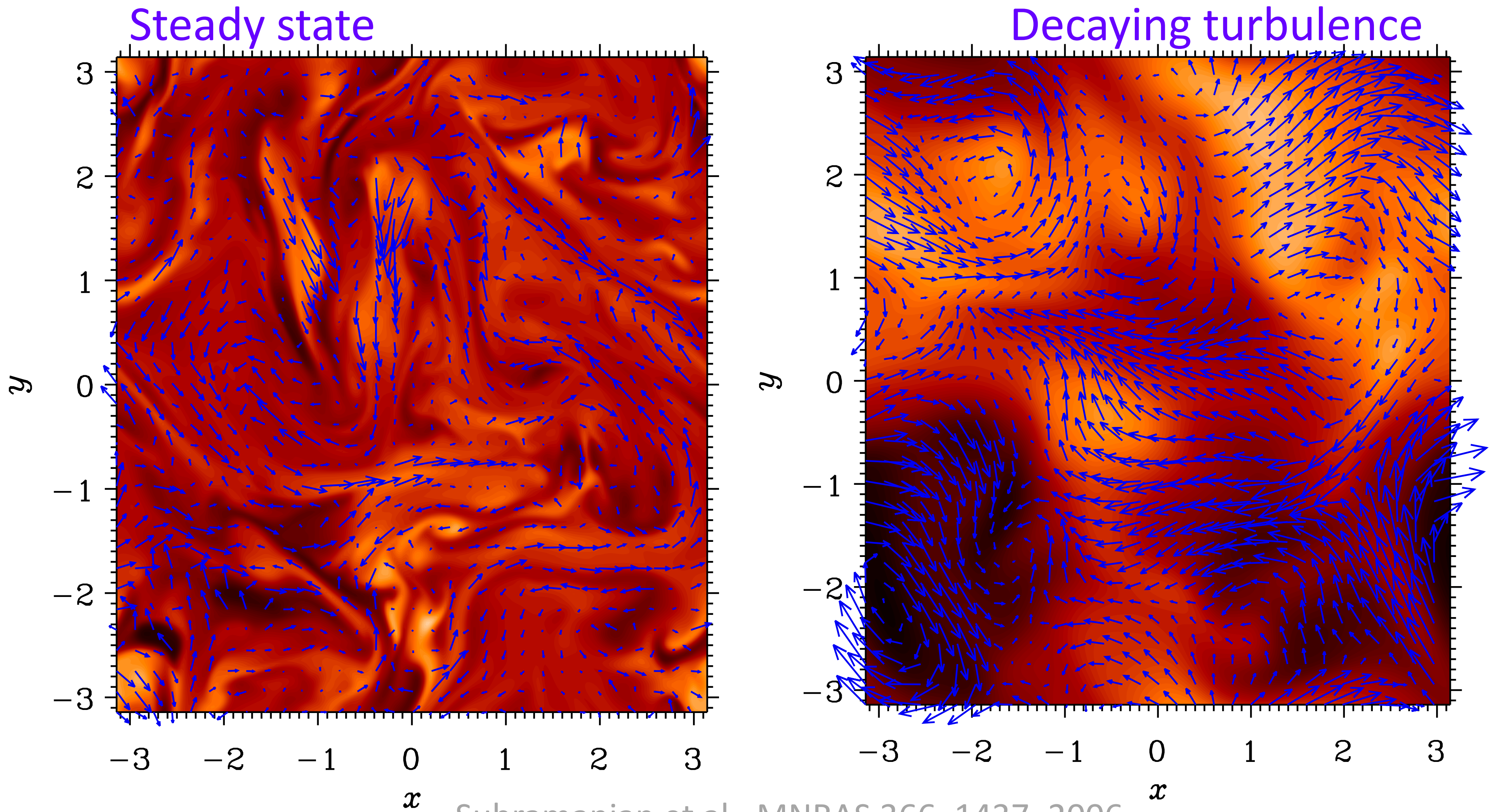
Dynamo action, $A > 2\times10^4$, $B \simeq 1\ \mu$G, $\ell_B \simeq 40$ kpc

R_m, Re $\propto t^{-1/5}$, $\sigma_{RM} \propto t^{-2/5}$

Magnetic field in a decaying turbulent flow

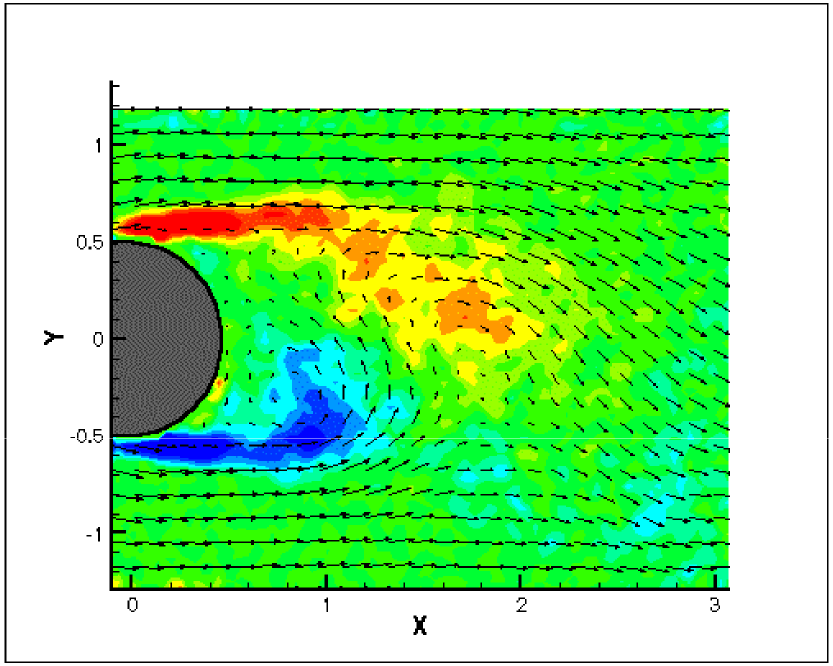
256^3 resolution, $\ell_0 = L_{box}/1.5$, $\mathcal{M} \simeq 0.1$, $\mathrm{Re} = R_\mathrm{m} = 420$.

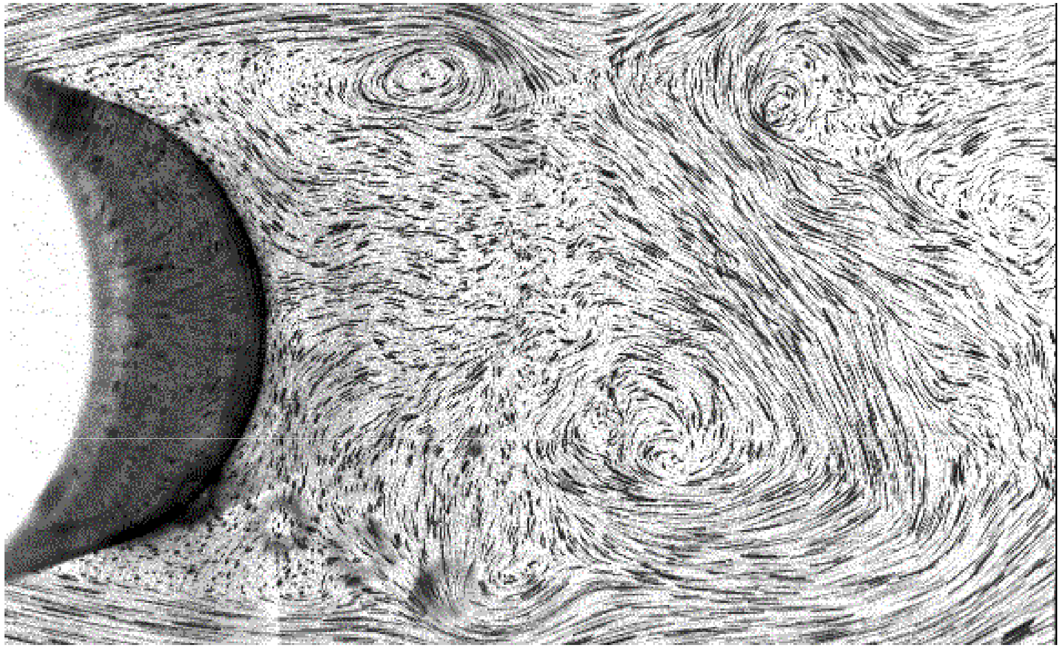
Colour: $B_\parallel$; vectors: $\mathbf{B}_\perp$



Subramanian et al., MNRAS 366, 1437, 2006

Stage 3. Mature cluster: turbulence in the wakes of galaxies and galaxy groups





Turbulent wake (Prandtl: see Landau & Lifshitz, *Hydrodynamics*):

$$l_{0x} \simeq L_{\rm i}(x/L_{\rm i})^{1/3}\ , \qquad v_{0x} \simeq V_{\rm i}(x/L_{\rm i})^{-2/3},$$

x = distance along the wake, $L_{\rm i}$ & $V_{\rm i}$ = size & speed of the body.

http://www.eng.fsu.edu/~shih/succeed/cylinder/cylinder.htm

- Clumps $m = 3\times10^{13}M_\odot$ falling into cluster $M = 10^{15}M_\odot$ every $\Delta t \propto m^{-1/2} \simeq 0.3$ Gyr (Lacey & Cole 1993),

- gas stripping radius $R_0 \simeq 100$ kpc,

- wake length $$\frac{X}{R_0} = 27\left(\frac{\mathrm{Re}}{10^3}\right)^3$$

- $v_0 \simeq 250$ km/s, $\ell_0 \simeq 200$ kpc, $B \simeq 2\ \mu$G, $\ell_B \simeq 30$ kpc

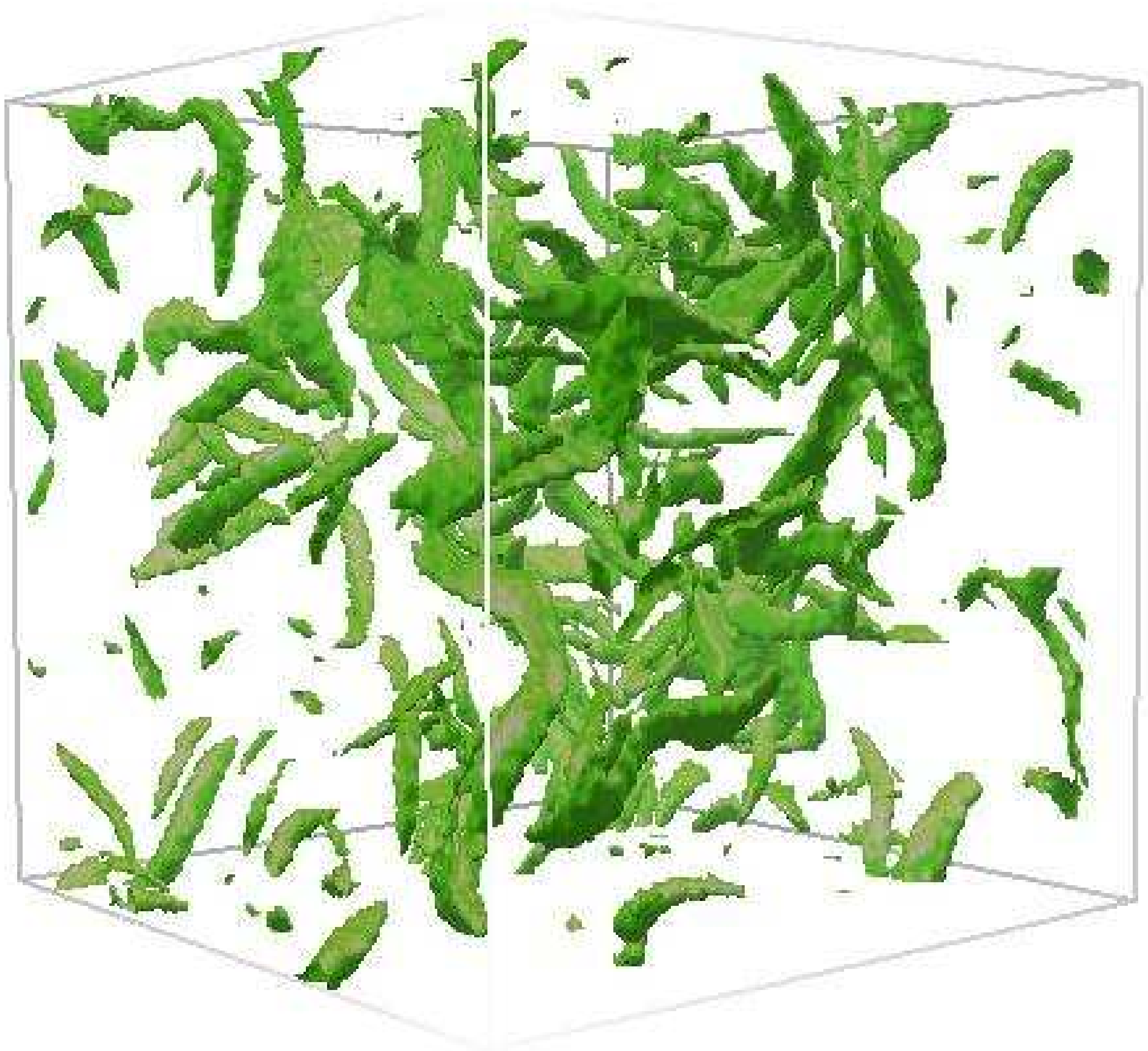
- Volume filling factor: $f_V \simeq 0.02\left(\frac{\mathrm{Re}}{10^3}\right)^5$

- Area covering factor: $f_S \simeq 0.2\left(\frac{\mathrm{Re}}{10^3}\right)^4$

Turbulence and magnetic fields at various stages of a galaxy cluster evolution (the Coma cluster)

Evolution stage	Duration [Gyr]	v_0 [km/s]	l_0 [kpc]	t_0 [Gyr]	B_{eq} [μG]	l_B [kpc]	$\langle B^2 \rangle^{1/2}$ [μG]	σ_{RM} [rad/m^2]
Major mergers	4	300	150	0.5	4	25	1.8	200
Decaying turbulence	5	130	260	2.0	2	44	0.8	120
Subcluster wakes		260	200	0.8	4	34	1.6	110
Galactic wakes		300	8	0.03	4	1.4	1.6	5

4. Morphology of magnetic structures produced by the fluctuation (small-scale) dynamo



Wilkin et al., PRL 2007

Filaments?

Sheets?

Ribbons?

Anything else?

Would different people see the same?

Are the conclusions robust?

Minkowski functionals

Morphology of structures in 3D is completely characterised by FOUR *Minkowski functionals*:

(Hadwiger's theorem, 1957)

$$V_0 = \iiint dV$$ ➔ Volume

$$V_1 = \frac{1}{6}\iint dS$$ ➔ Surface area

$$V_2 = \frac{1}{6\pi}\iint (\kappa_1 + \kappa_2) dS$$ ➔ Integral mean curvature

$$V_3 = \iint \kappa_1 \kappa_2 dS$$ ➔ Euler characteristic

κ_1, κ_2 = principal curvatures

V. Sahni et al., ApJ 1998

Computing Minkowski functionals

n_0 = number of grid points within the structure,

n_1 = number of complete edges,

n_2 = number of faces within the structure,

n_3 = total number of grid cubes,

N = total number of grid points in the domain.

$$V_0 = n_3 \qquad V_2 = \frac{2(n_1 - 2n_2 + 3n_3)}{9N^2}$$

$$V_1 = \frac{2(n_2 - 3n_3)}{9N} \qquad V_3 = \frac{n_0 - n_1 + n_2 - n_3}{N^3}$$

(J. Schmalzing et al., ApJ 1997 & 1999)

Shapefinders

$$V_0 = \iiint dV$$

$$V_1 = \frac{1}{6}\iint dS$$

$$V_2 = \frac{1}{6\pi}\iint (\kappa_1 + \kappa_2) dS$$

$$V_3 = \iint \kappa_1 \kappa_2 dS$$

Thickness, **W**idth, **L**ength

$$T = \frac{V_0}{2V_1}, \quad W = \frac{2V_1}{\pi V_2}, \quad L = \frac{3V_2}{4V_3}$$

Planarity and **F**ilamentarity

$$P = \frac{W-T}{W+T} \quad \text{and} \quad F = \frac{L-W}{L+W}$$

- Filament: $P = 0$, $F = 1$;
- Pancake: $P = 1$, $F = 0$;
- Sphere: $P = F = 0$

$(P, \quad F)$ =

(a) (0.096, 0.81);

(b) (0.66, 0.23);

(c) (0.66, 0.12);

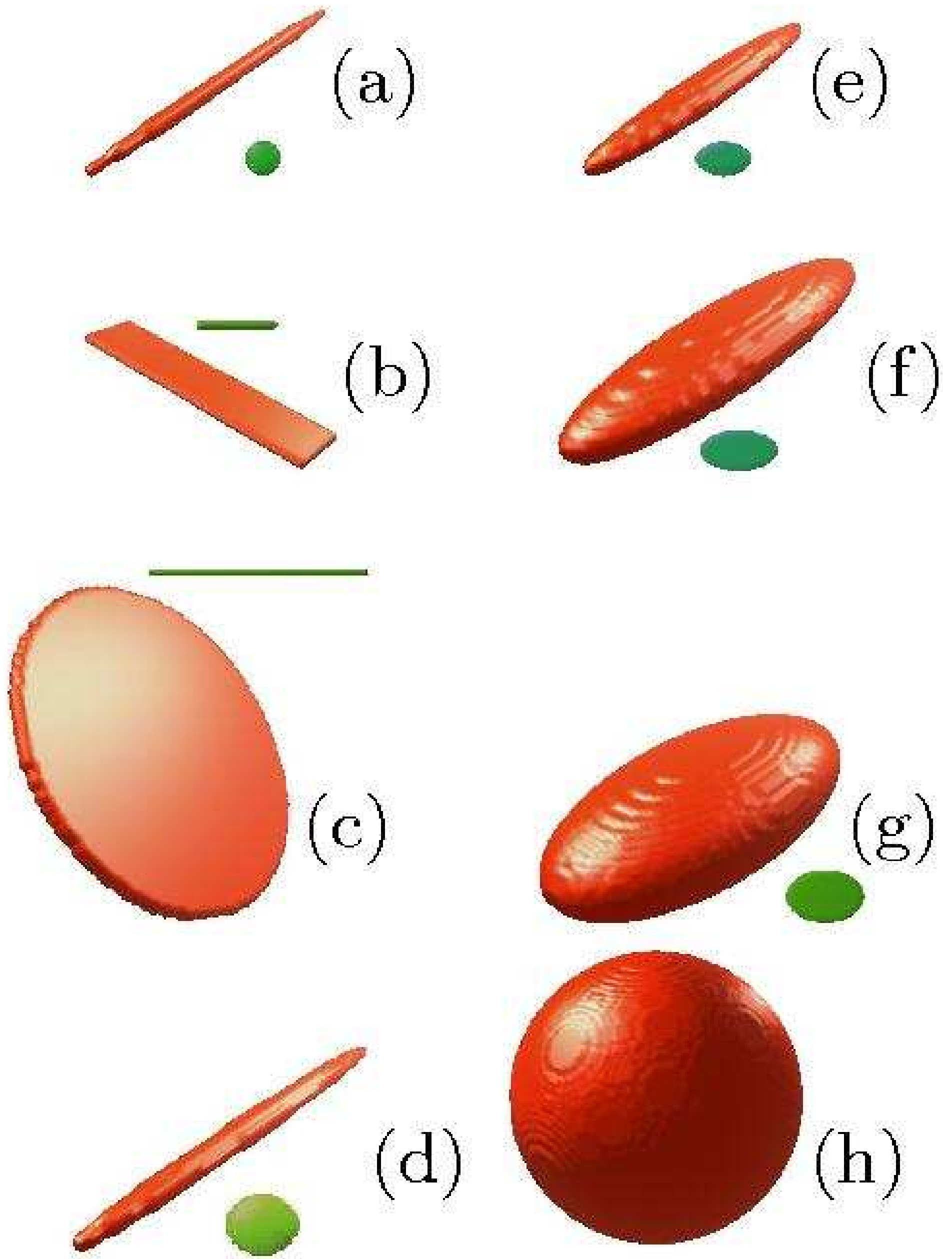
(d) (0.25, 0.66);

(e) (0.18, 0.43);

(f) (0.14, 0.23);

(g) (0.087, 0.073);

(h) (0.0036, –0.0047).



- Application to a kinematic simulation of the fluctuation dynamo in a periodic box (Wilkin et al., PRL, 99, 134501, 2007).

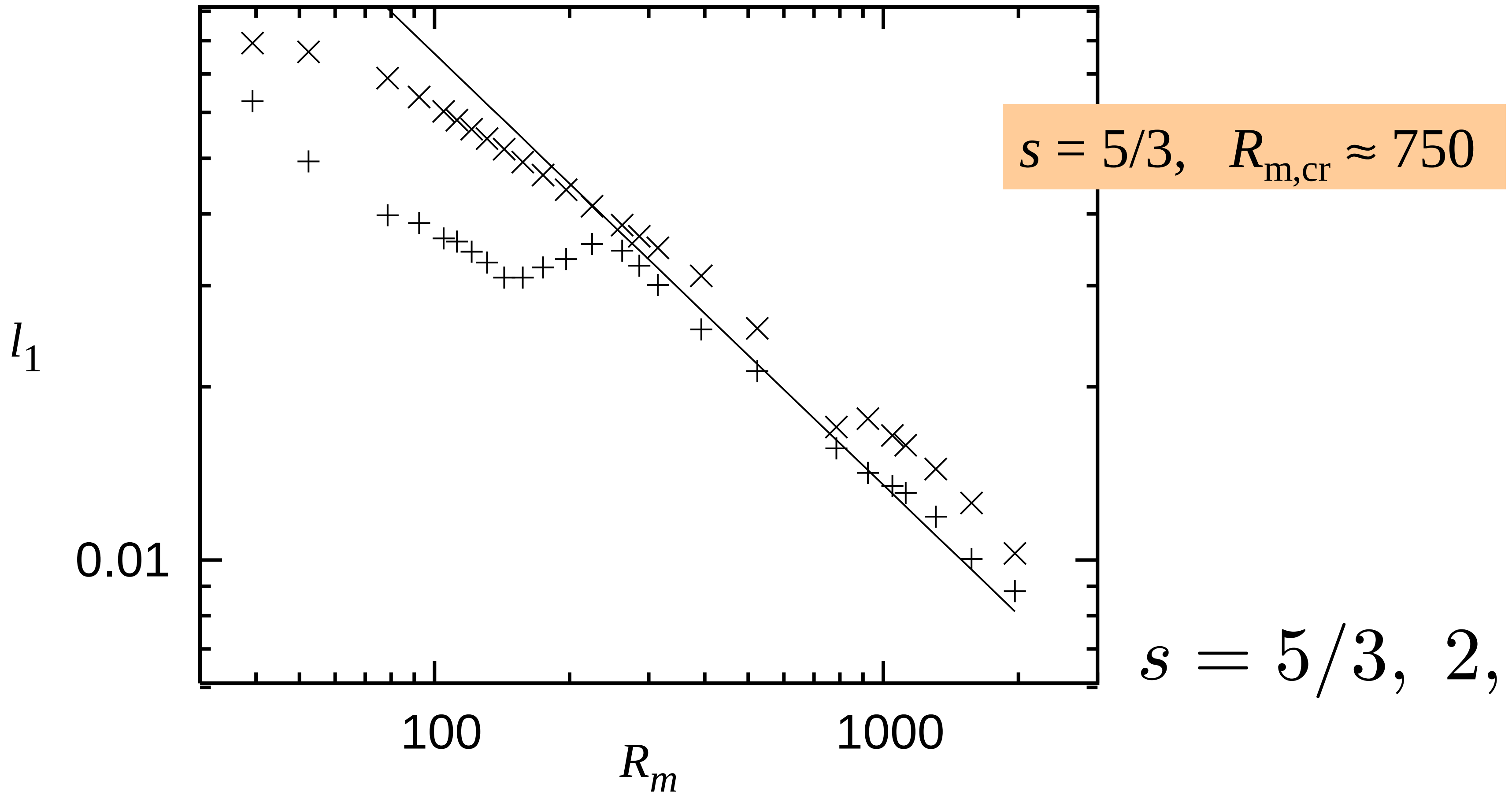
- Velocity field:

$$\mathbf{u}(\mathbf{x}, t) = \sum_{n=1}^{N} [\mathbf{C}_n \times \hat{\mathbf{k}}_n \cos\phi_n + \mathbf{D}_n \times \hat{\mathbf{k}}_n \sin\phi_n],$$

$$\phi_n = \mathbf{k}_n \cdot \mathbf{x} + \omega_n t,$$

$\mathbf{C}_n$, $\mathbf{D}_n$: $$E(k_n) = a k_n^4 \left[1 + \left(\frac{k_n}{k_0}\right)^2\right]^{(s-4)/2} \exp\left[-\frac{1}{2}\left(\frac{k_n}{k_d}\right)^2\right]$$

$$l_1 = \text{min}(T, W, L), \quad l_2 = \text{med}(T, W, L), \quad l_3 = \text{max}(T, W, L)$$

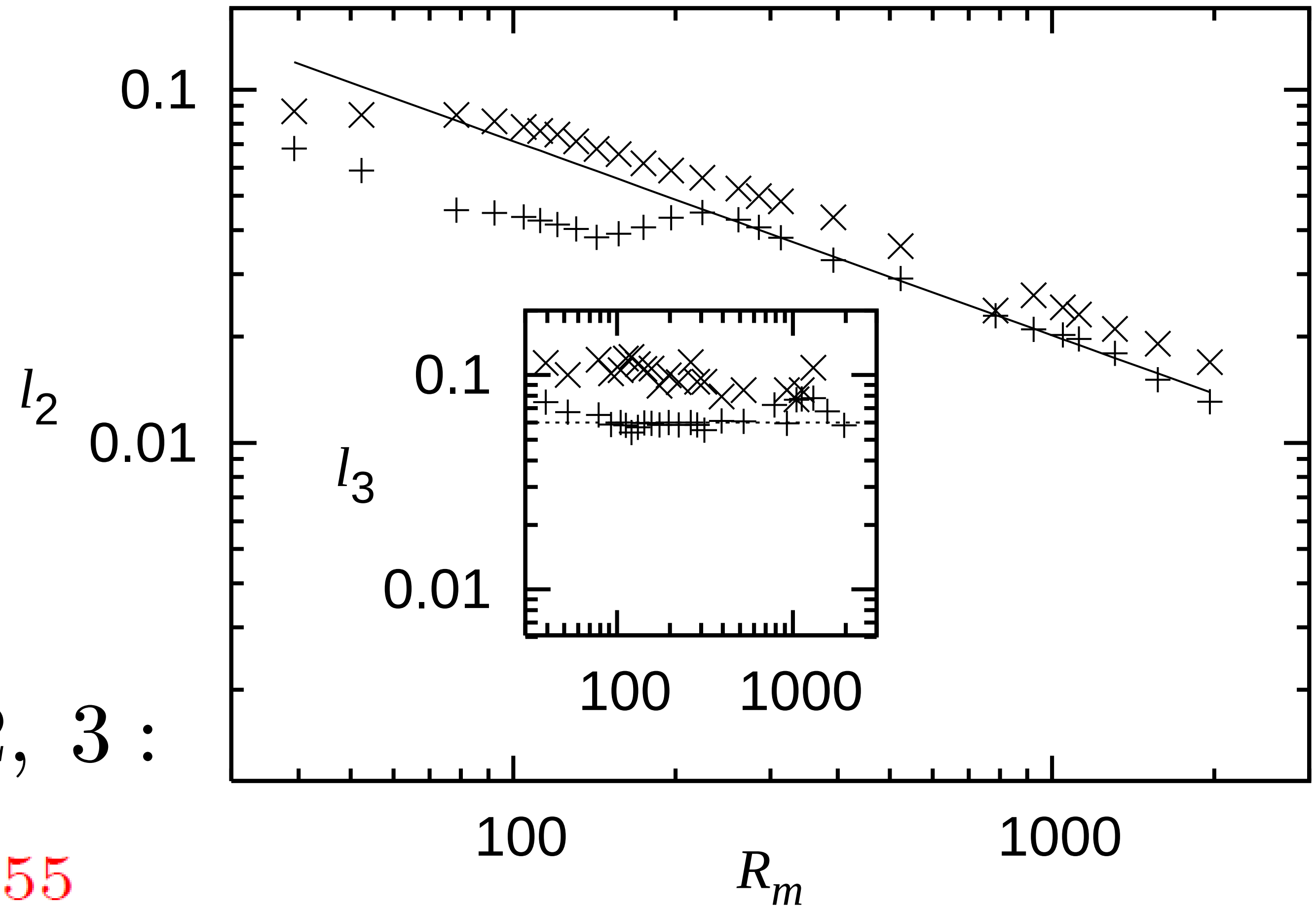


$$s = 5/3,\ 2,\ 3:$$

$$l_1 \simeq l_0 R_{\mathrm{m}}^{-2/(s+1)} \equiv l_B$$

Different scaling for s =1
(slope -2/3 instead of -1)

$$l_1 = \min(T, W, L), \quad l_2 = \text{med}(T, W, L), \quad l_3 = \text{med}(T, W, L)$$



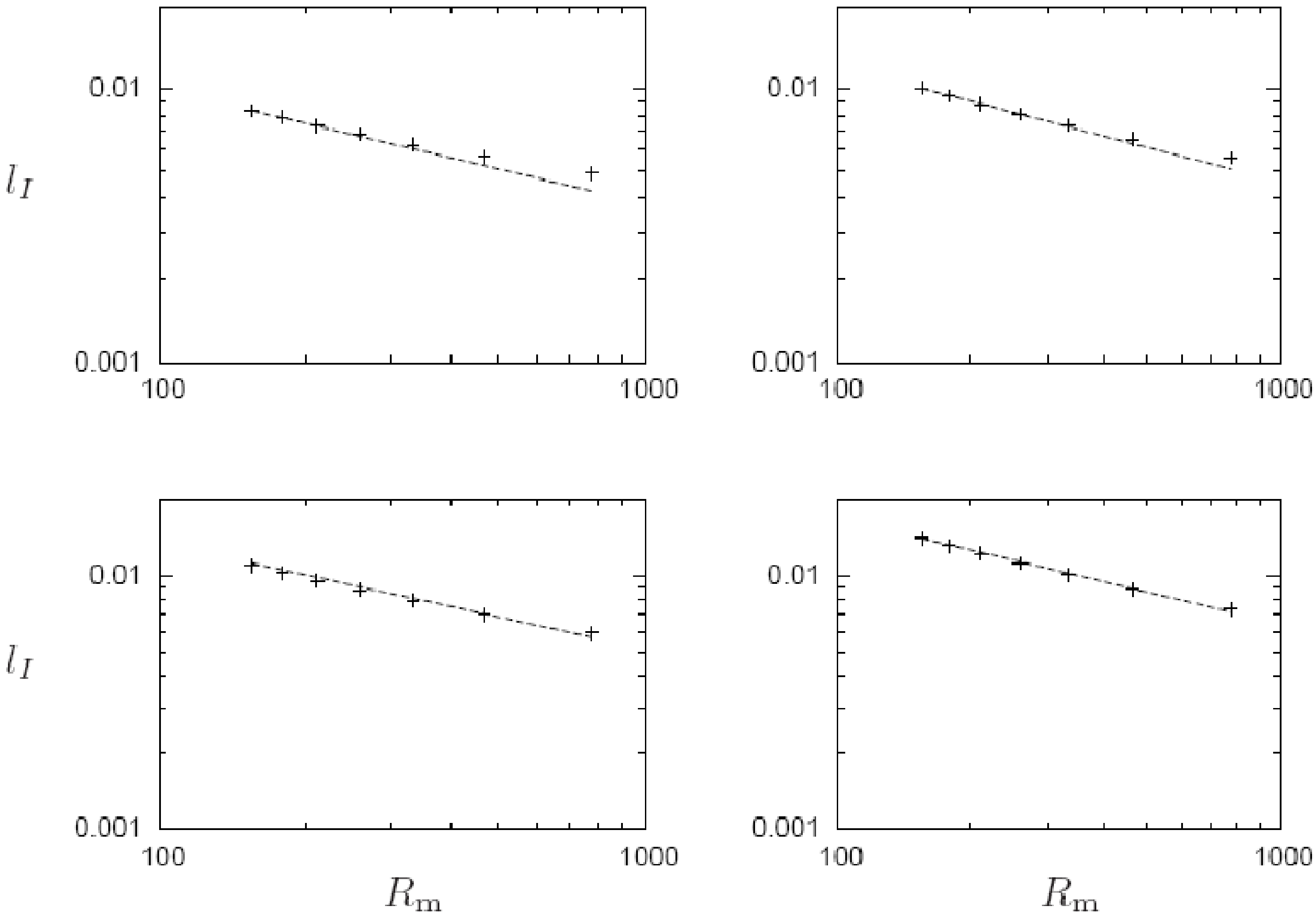
$$s = 1,\ 5/3,\ 2,\ 3:$$

$$l_2 \simeq l_0 R_{\rm m}^{-0.55}\ ,$$

$$l_3 \simeq \text{const}\ .$$

$s = 5/3$, $R_{\rm m,cr} \approx 750$

An integral scale, $l_I = 2\pi \dfrac{\int M(k)\, dk}{\int k M(k)\, dk} \propto R_{\mathrm{m}}^{-0.42}$ for all s



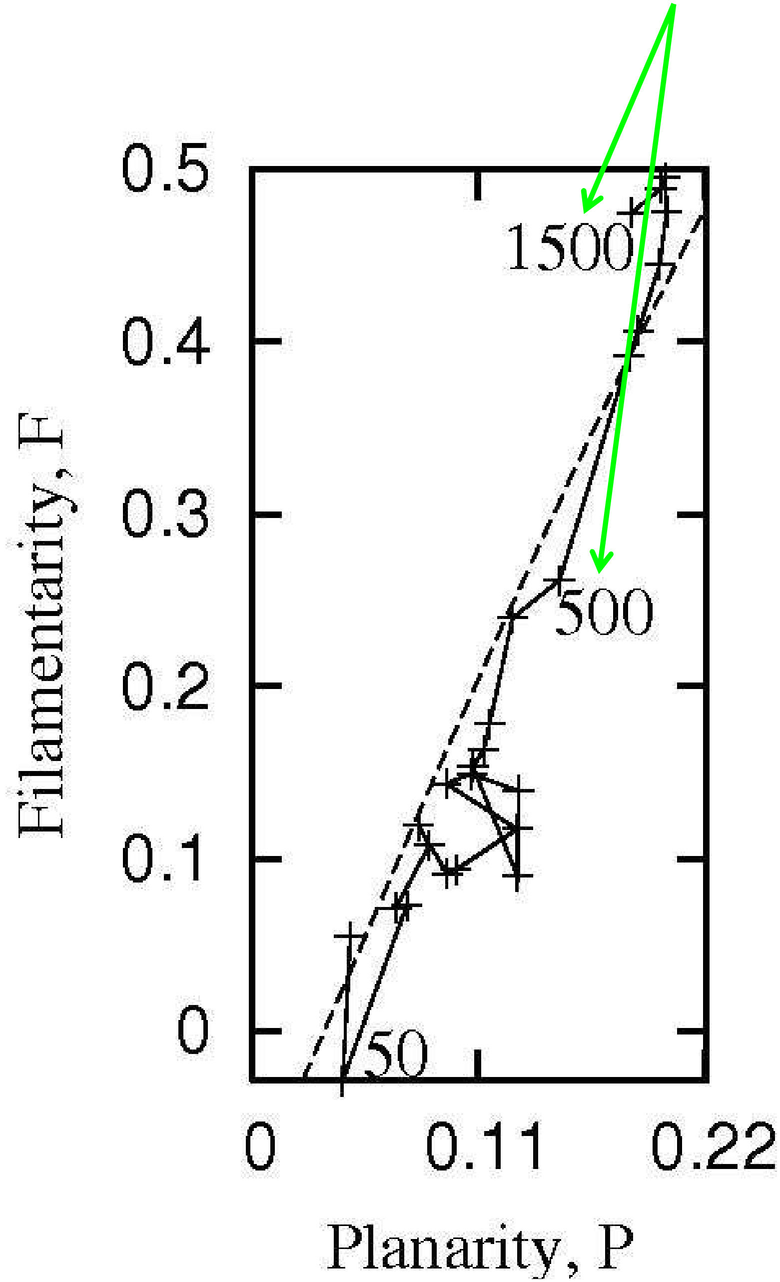
$s = 1$

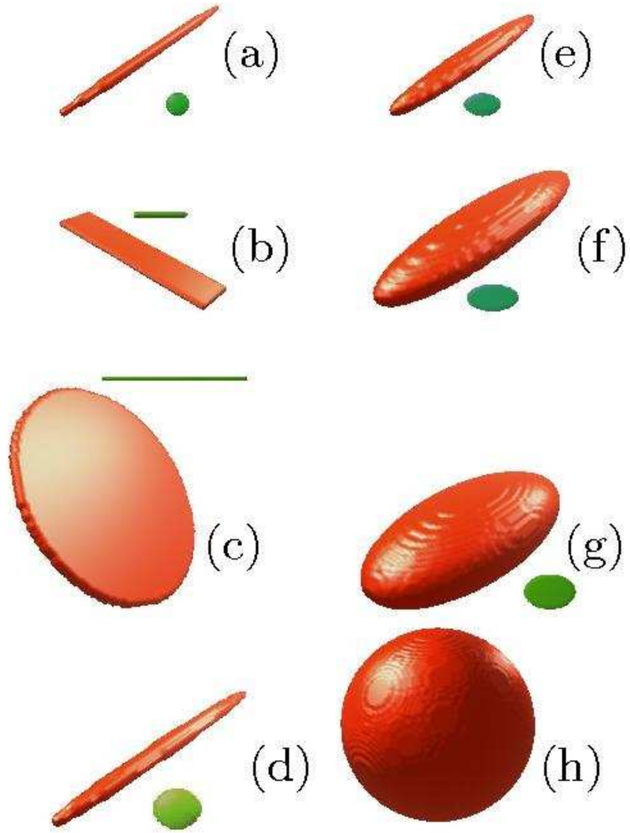
$s = 5/3$

$s = 2$

$s = 3$

Morphology at varying R_m (the kinematic stage)





(P, F) = (a) (0.096, 0.81); (b) (0.66, 0.23);
(c) (0.66, 0.12); (d) (0.25, 0.66);
(e) (0.18, 0.43); (f) (0.14, 0.23);
(g) (0.087, 0.073); (h) (0.0036, -0.0047).

$$P = \frac{W - T}{W + T} = \frac{l_2 - l_1}{l_2 + l_1} \qquad F = \frac{L - W}{L + W} = \frac{l_3 - l_2}{l_3 + l_2}$$

$$P \sim 1 - 2[1 + \tfrac{3}{8} R_{\rm m}^{0.2}]^{-1}, \quad F \sim 1 - 2[1 + \tfrac{1}{18} R_{\rm m}^{0.55}]^{-1}$$

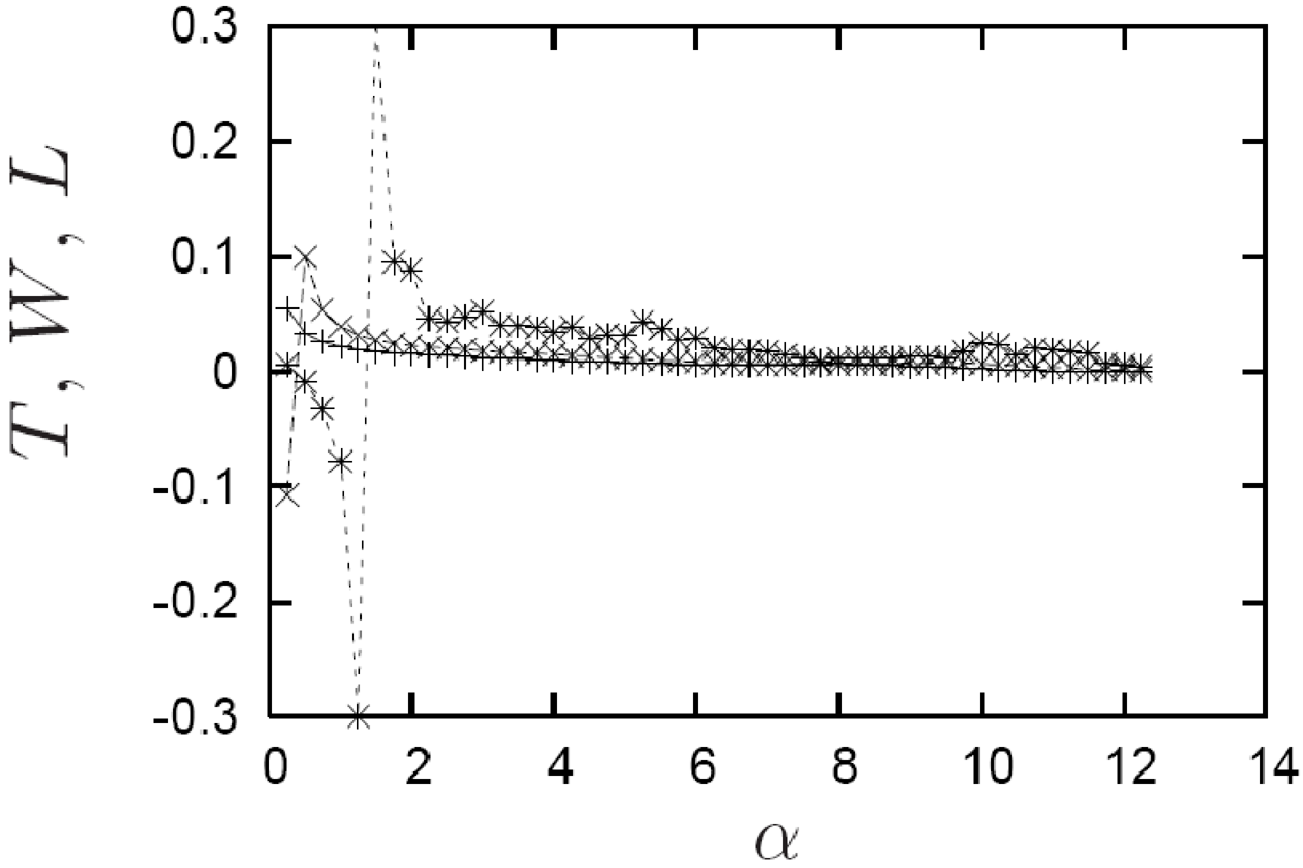
$R_{\rm m} > 200$ ➔ $F > P$, **filamentary magnetic structures**

Current $\vec{J} = \nabla \times \vec{B}$: ribbons,

$(P, F) = (0.57, 0.82)$ for $J = 2J_{\rm rms}$, $R_{\rm m} = 1500$

Dependence on the isosurface level

$$B = \alpha \, B_{\mathrm{rms}}$$



The working range: 2 < α < 5