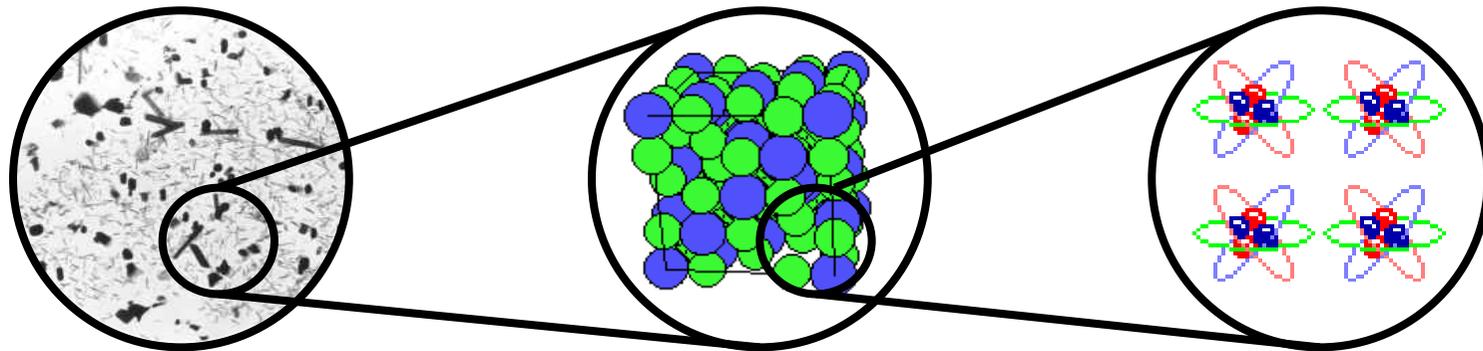


Challenges and Progress in Computational Materials Science



P. A. Korzhavyi

Applied Materials Physics

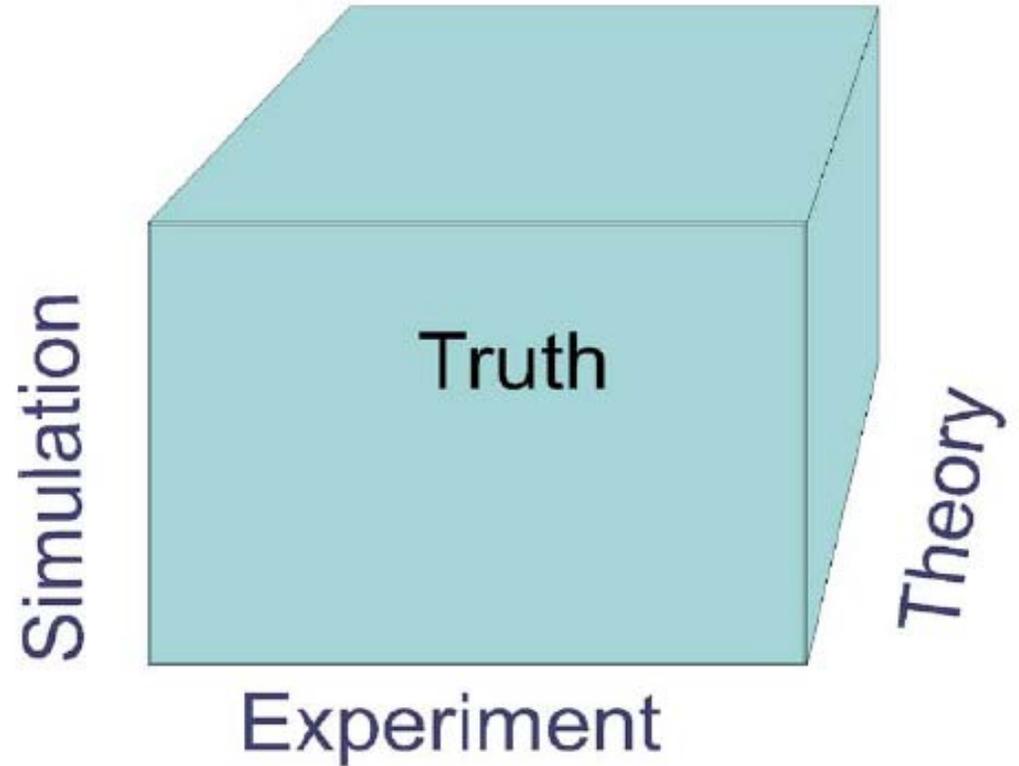
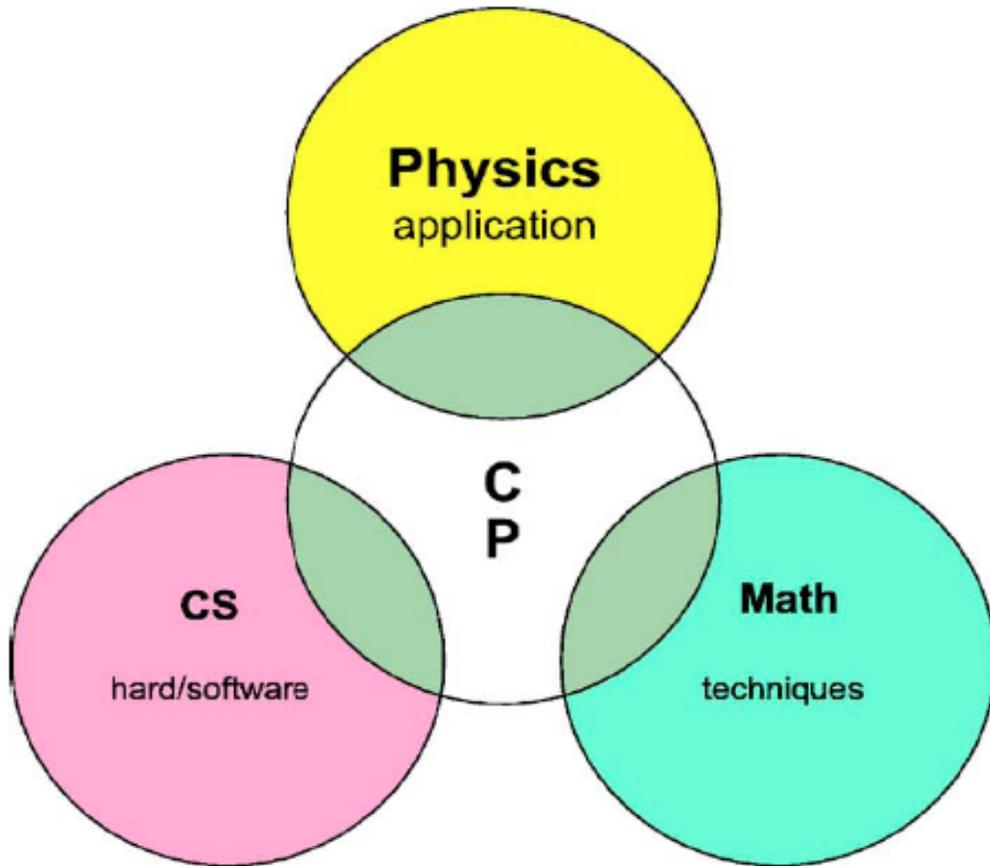
Department of Materials Science and Engineering

School of Industrial Engineering and Management

Royal Institute of Technology (KTH), Stockholm



Computational Physics (CP)



Source: R. H. Landau, Resource Letter CP-2: Computational Physics, *American Journal of Physics* **76**, 296 (2008).

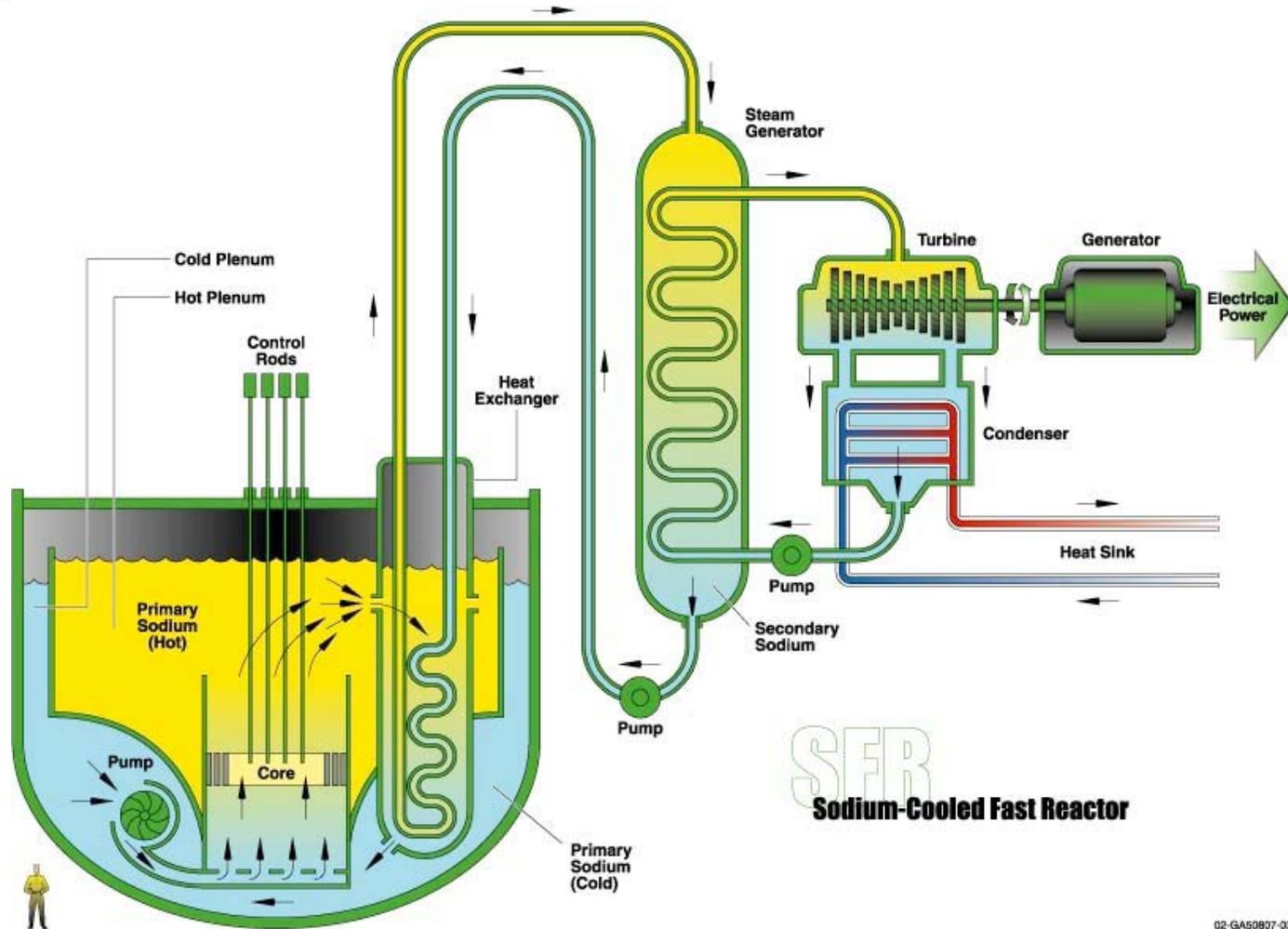


Challenges for Materials Science

- Construction materials for the new-generation **energy** production plants and engines:
 - New (higher temperature) coal-burning plants;
 - Generation-IV (fast neutron) nuclear reactors;
 - Thermo-nuclear reactors;
 - Wave, wind, solar, *etc.* power stations.

(lifetime ~ 10-50 years)
- Materials for **safe** immobilization and disposal of nuclear waste (industrial & military): (lifetime ~ 100 000 years)

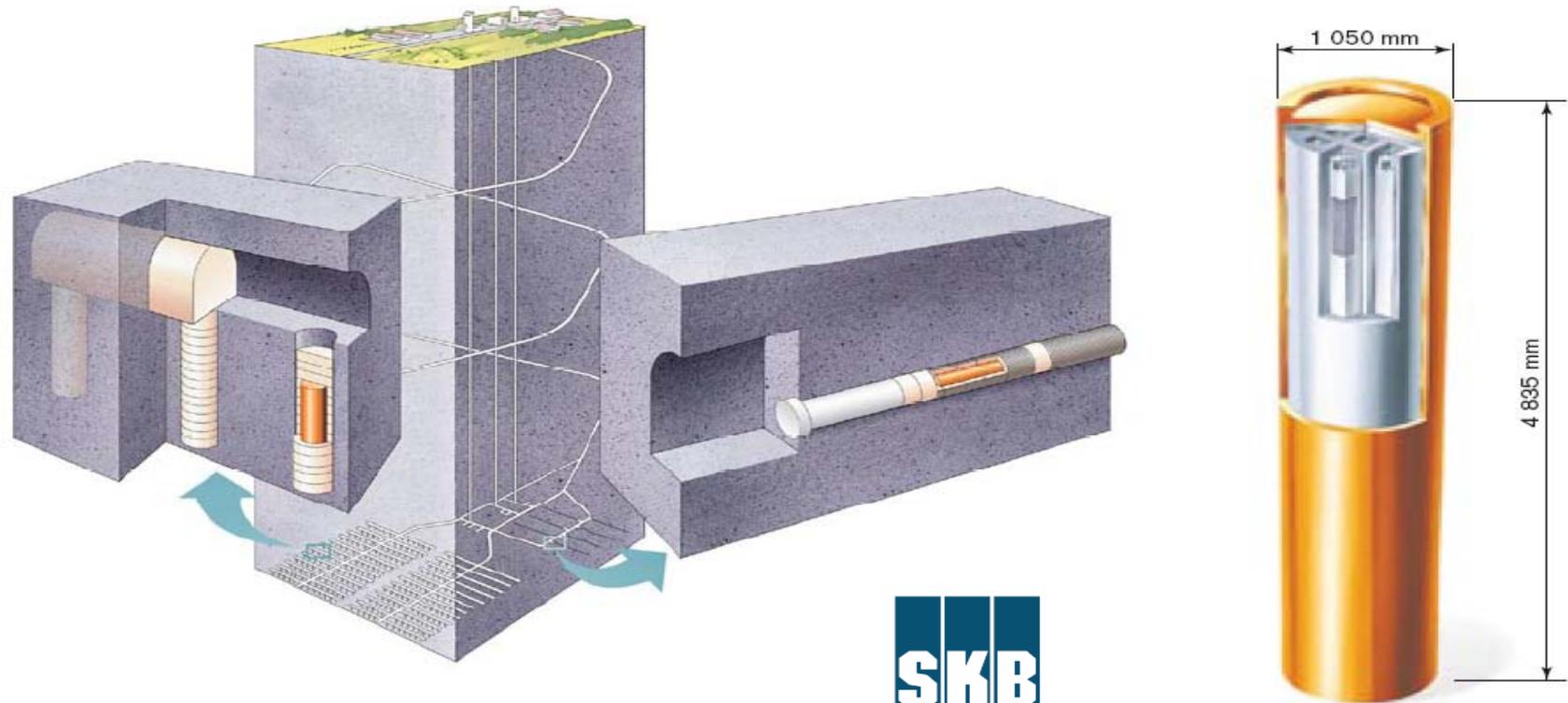
Challenge: Generation-IV nuclear power reactors (breeders)



The economical feasibility of Gen-IV reactors is directly related to the lifetime of structural materials.

Desired materials lifetime ~ 30 - 50 y.

Challenge: Safe disposal of nuclear waste



Svensk Kärnbränslehantering AB
Swedish Nuclear Fuel and Waste Management Co
Box 5864, SE-102 40 Stockholm, Sweden
Telephone +46 8 459 84 00 Fax +46 8 661 57 19
www.skb.se info@skb.se

Reliable predictions of materials' behaviour (spent UO₂ fuel, iron insert, and copper canister) should be made for the whole period of storage.
Storage period about 300 000 y.

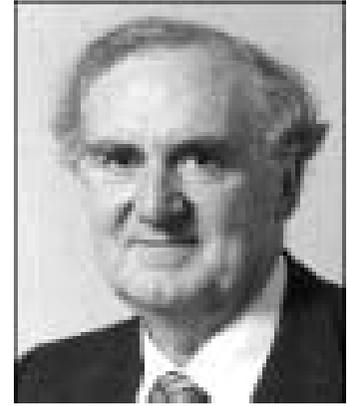


Challenges for Materials Science

- Materials engineers need **novel design approaches**: the new materials should be developed during a relatively short period (as compared to the materials' lifetime).
- Normal experimental testing of materials (during 1/3 of the expected lifetime) is not affordable.
- We have to rely on our understanding of materials as well as on computer models.
- Fundamental understanding and accurate models of materials are needed.



Impact of XXth century physics on Materials Science



Sir Alan Cottrell

*It was the century when we understood materials, scientifically, for the first time-what they are and how they perform. Two advances have been outstanding in making this possible: **X-ray analysis**, which showed us where the atoms are inside materials; and **quantum mechanics**, which explained how they interact and produce the bulk properties of matter.*

Alan Cottrell

Centennial Report [MRS Bulletin **25**, 43 (2000)]



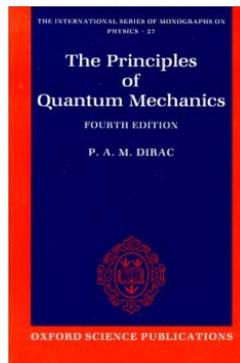
Computational experiments on "virtual" matter

INPUT

Periodic Table

Legend:
 ■ Alkali Metal ■ Metal ■ Rare Earth
 ■ Alkali Earth ■ Non-Metal ■ Noble Gas
 ■ Trans. Met. ■ Halogen

SOFTWARE



Physical laws
 Universal constants
 Methods
 Approximations

OUTPUT

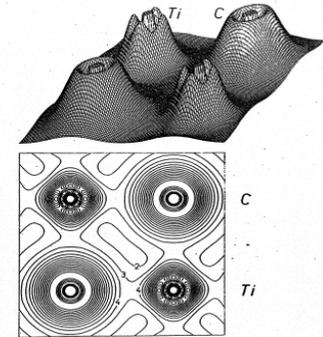
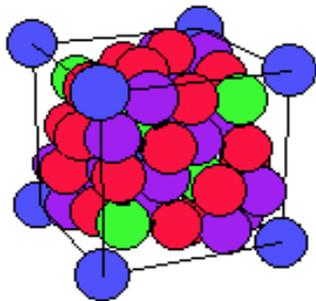


FIG. 1. Theoretical (static) valence-electron density of TiC in the (100) plane obtained by LAPW calculations (Ref. 18). Contour intervals, $0.1 e \text{ \AA}^{-3}$, cutoff at $1.7 e \text{ \AA}^{-3}$.

Atomic numbers & coordinates.

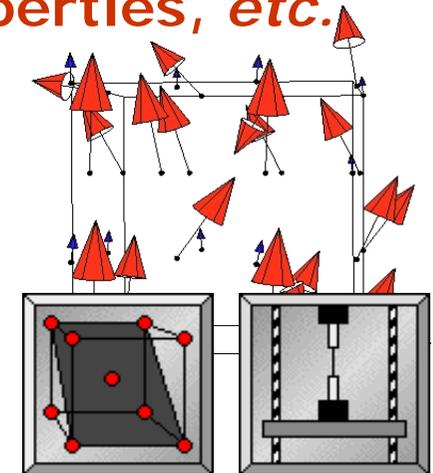


HARDWARE



1. Electron density
2. Total energy

Derivatives: forces, properties, etc.





HERO-M: Hierarchic Engineering of Industrial Materials



J. Ågren

Thermo-Calc
DICTRA
Phase Field
Life time assessment
Many years

FEM
CAD-CAM

Engineering design

Continuum models

Phenomenological models

- Macro simulation
- Micro structure



Atomic level simulation

- Statistical mechanics
- Classical dynamics
- Force field

Kinetic Monte Carlo
>1 million atoms,
20 nm, long times

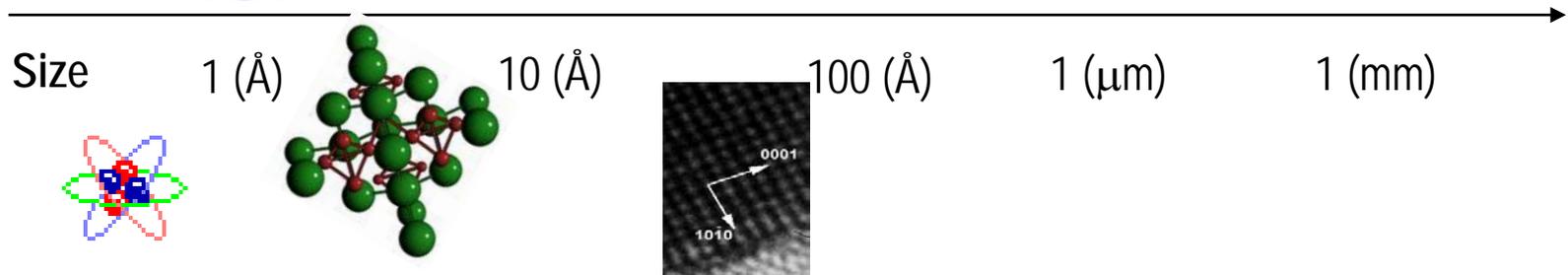
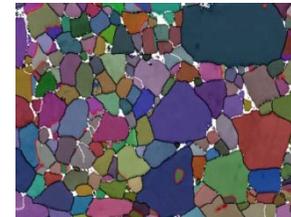
Molecular dynamics
100000 ca 10 nm 10⁻⁶ s

Memika

Fundamental models

- Quantum mechanics
- Quantum theory

Density functional theory:
ca 100 atoms
1 nm, 0 K





Schrödinger equation

$$\hat{H}\Psi(\mathbf{r}, \mathbf{R}) = \left[\hat{H}_{\mathbf{r}} + \hat{H}_{\mathbf{R}} + \hat{H}_{\mathbf{r},\mathbf{R}} \right] \Psi(\mathbf{r}, \mathbf{R}) = E\Psi(\mathbf{r}, \mathbf{R})$$

Depends on the coordinates of all the electrons (\mathbf{r}_i) and nuclei (\mathbf{R}_I) in the system:

$$\Psi(\mathbf{r}, \mathbf{R}) \equiv \Psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \dots, \mathbf{R}_1, \mathbf{R}_2, \mathbf{R}_3, \dots) = \underbrace{\psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \dots)}_{\text{electron wavefunction}} \cdot \underbrace{\varphi(\mathbf{R}_1, \mathbf{R}_2, \mathbf{R}_3, \dots)}_{\text{wavefunction of nuclei}} \equiv \psi(\mathbf{r}) \cdot \varphi(\mathbf{R})$$

$$\hat{H}_{\mathbf{r}} = \left[-\sum_i \nabla_i^2 + \frac{1}{2} \sum_{i \neq j} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|} \right]; \quad \hat{H}_{\mathbf{R}} = \left[-\sum_I \nabla_I^2 + \frac{1}{2} \sum_{I \neq J} \frac{Z_I Z_J}{|\mathbf{R}_I - \mathbf{R}_J|} \right]; \quad \hat{H}_{\mathbf{r},\mathbf{R}} = \left[-\sum_{i,J} \frac{Z_J}{|\mathbf{r}_i - \mathbf{R}_J|} \right]$$

el. kin. energy
el.-el. interaction
nuc. kin. energy
nuc.-nuc. interaction
el.-nuc. interaction

For a solid with $\sim 10^{24}$ charged particles the equation cannot be solved exactly!

Adiabatic (Born-Oppenheimer) approximation makes use of the fact that nuclei are much heavier (and, therefore, slower) than electrons. Therefore, electronic wave function can be determined first, while the nuclei are considered as static point-charges.

$$m_p \approx 1840 \cdot m$$

Self-consistent field approach: reduce the many body problem to a one-particle problem. Each electron moves in the effective potential created by the nuclei and *all the other* electrons. The problem is to be solved iteratively (*self-consistently*).



Density functional theory (DFT)

Total energy of the ground state is a unique functional

$$E = T_s[\rho] + \int d^3r v_{ext}(\mathbf{r})\rho(\mathbf{r}) + \frac{1}{2} \int d^3r' \int d^3r \frac{\rho(\mathbf{r})\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} + E_{xc}[\rho]$$

of electron density, $\rho(\mathbf{r}) = \sum_i^{occ} |\varphi_i(\mathbf{r})|^2$, where

$$T_s[\rho] = \sum_i^{occ} \int d^3r \varphi_i^*(\mathbf{r}) \left(-\frac{\nabla^2}{2} \right) \varphi_i(\mathbf{r})$$

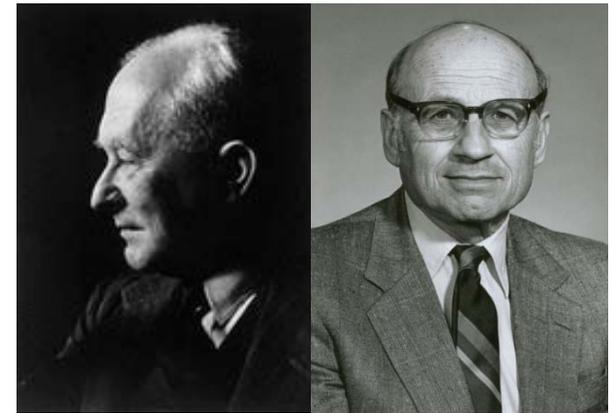
is the kin. energy,

$\varphi_i(\mathbf{r})$ is a formal solution of Kohn-Sham equation:

$$\left(-\frac{\nabla^2}{2} + v_{ext}(\mathbf{r}) + \int d^3r' \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} + v_{xc}(\mathbf{r}) \right) \varphi_i(\mathbf{r}) = \varepsilon_i \varphi_i(\mathbf{r})$$

where $v_{xc}(\mathbf{r}) = \frac{\delta E_{xc}[\rho]}{\delta \rho(\mathbf{r})}$.

Note that $\varphi_i(\mathbf{r})$ is **just an auxiliary function!**



M. Born W. Kohn
Interpretation and use
of the electron density

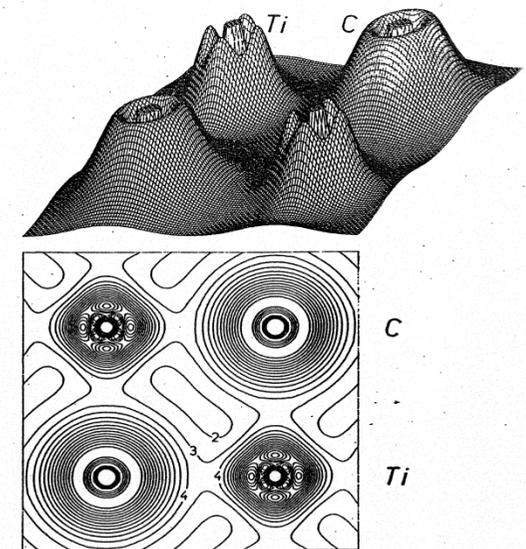


FIG. 1. Theoretical (static) valence-electron density of TiC in the (100) plane obtained by LAPW calculations (Ref. 18). Contour intervals, $0.1e \text{ \AA}^{-3}$, cutoff at $1.7e \text{ \AA}^{-3}$.

P. Blaha and K. Schwarz, (1983).



Exchange-correlation functionals E_{xc}

LDA*: $E_{xc}^{LDA} = \int d^3r \rho(\mathbf{r}) \varepsilon_{xc}^{UEG}(\rho(\mathbf{r}))$, where $\varepsilon_{xc}^{UEG}(\rho)$ is the exchange-correlation energy per particle of a uniform electron gas (UEG) with density ρ . This quantity is known from quantum Monte Carlo calculations [D.M. Ceperley & B.J. Alder, *Phys. Rev. Lett.* **45**, 566 (1980)].

GGA*: $E_{xc}^{GGA} = \int d^3r \rho(\mathbf{r}) \varepsilon_{xc}^{GGA}(\rho, \nabla\rho)$. Here $\varepsilon_{xc}^{GGA}(\rho, \nabla\rho)$ is not uniquely defined, many different forms have been proposed, e.g. **PBE** [J.P. Perdew, K. Burke, & M. Ernzerhof, *Phys. Rev. Lett.* **77**, 3865 (1996); *Ibid.* **78**, 1396 (1997)].

Meta-GGA: $E_{xc}^{MGGA} = \int d^3r \rho(\mathbf{r}) \varepsilon_{xc}^{MGGA}(\rho, \nabla\rho, \nabla^2\rho, \tau)$, where $\tau = \frac{1}{2} \sum_i^{occ} |\nabla\varphi_i(r)|^2$ is the kinetic energy density. E.g., **PKZB** [J.P. Perdew, S. Kurth, A. Zupan, & P. Blaha, *Phys. Rev. Lett.* **82**, 5179 (1999)].

Hybride functionals, “exact exchange”: $E_{xc}^{hyb}[\rho] = a \cdot (E_x - E_x^{GGA}) + E_{xc}^{GGA}$, where some fraction, a , of the exact exchange energy, E_x , is admixed to the GGA XC energy.

$$E_x = -\frac{1}{2} \sum_{i,j}^{occ} \int d^3r \int d^3r' \frac{\varphi_i^*(\mathbf{r}) \varphi_j^*(\mathbf{r}') \varphi_i(\mathbf{r}') \varphi_j(\mathbf{r})}{|\mathbf{r} - \mathbf{r}'|}$$

* **LDA**: local density approximation; **GGA**: generalized gradient approximation



Popular software packages for electronic structure calculations

1. Vienna Ab-initio Simulation Package (VASP)
<http://cms.mpi.univie.ac.at/vasp/>
2. Quantum ESPRESSO
<http://www.pwscf.org/>
3. ABINIT <http://www.abinit.org/>
<http://www.abinit.org/about/presentation.pdf>
4. SIESTA
<http://www.icmab.es/siesta/>



Ab initio = from the beginning, from first principles

Input to *ab-initio* calculations

➤ Atomic numbers of the constituents

Periodic Table

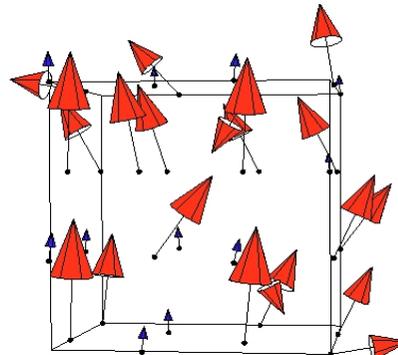
1a																	0	
1																	2	
H	2a												3a	4a	5a	6a	7a	He
3	4											5	6	7	8	9	10	
Li	Be											B	C	N	O	F	Ne	
11	12	3b	4b	5b	6b	7b	8				1b	2b	13	14	15	16	17	18
Na	Mg											Al	Si	P	S	Cl	Ar	
19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr	
37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe	
55	56	57	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	
Cs	Ba	La	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn	
87	88	89	104	105	106													
Fr	Ra	Ac	Rf	Ha														

■ Alkali Metal ■ Metal ■ Rare Earth
■ Alkali Earth ■ Non-Metal Noble Gas
■ Trans. Met. ■ Halogen

Lanthanides	58	59	60	61	62	63	64	65	66	67	68	69	70	71
	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu
Actinides	90	91	92	93	94	95	96	97	98	99	100	101	102	103
	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr

➤ Crystal and magnetic structure

No experimental data



No adjustable parameters

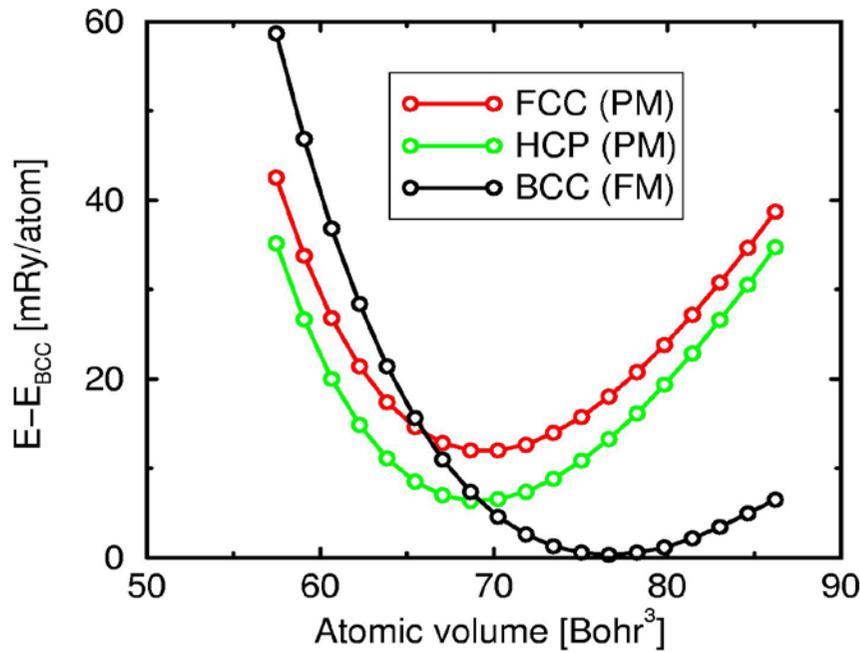


Output from *ab-initio* calculations

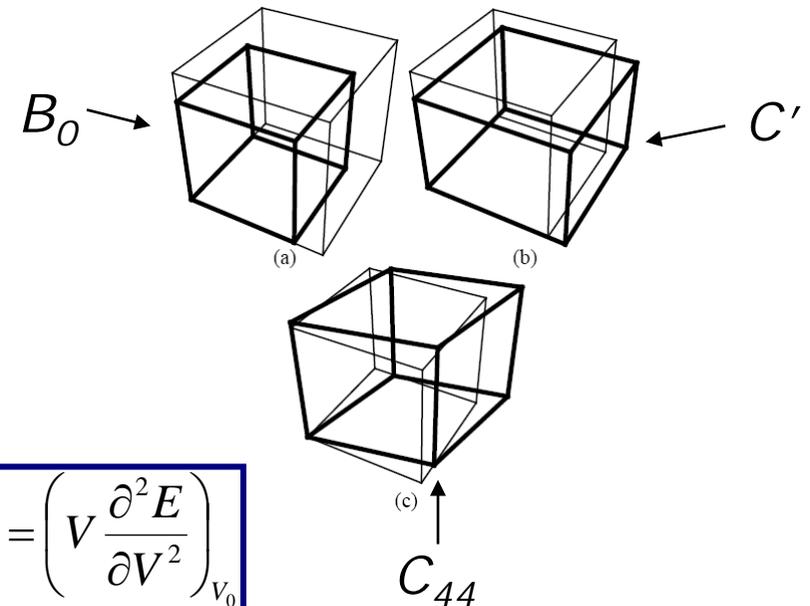
➤ Total energy as a function of ...

- Volume
- Crystal structure
- Distortion
- Magnetic structure
- Composition
- Chemical/magnetic order
- Defect concentration
- *etc.*

Pure Fe, FP-LMTO, GGA



Elastic constants: $C \sim d^2E/d\varepsilon^2$

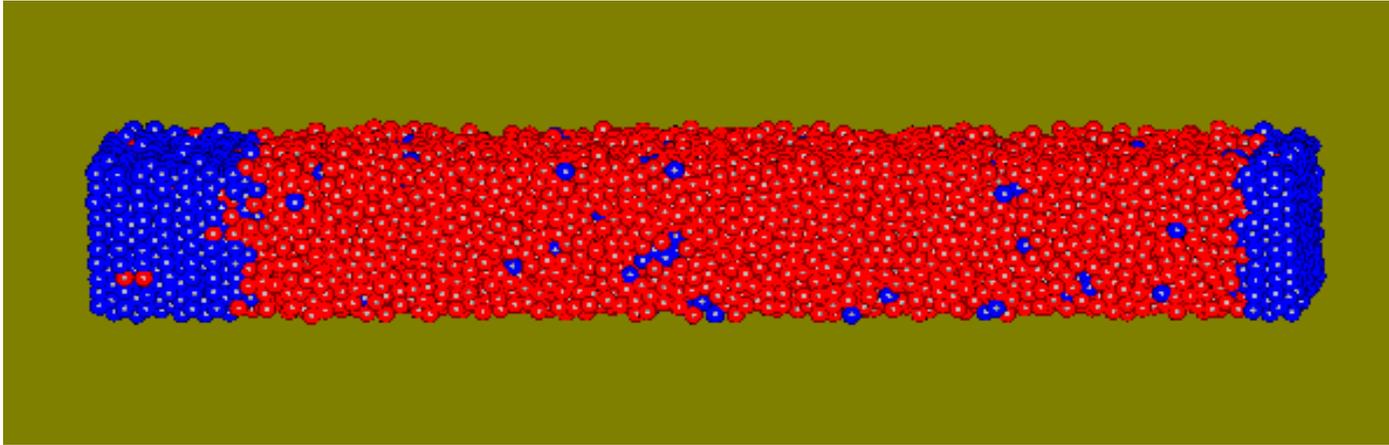


Equilibrium volume, V_0 : $\left(\frac{\partial E}{\partial V}\right)_{V_0} = 0$ or $P(V_0) = 0$

Pressure: $P(V) = -\frac{\partial E}{\partial V}$

Bulk modulus: $B_0 = -\left(V \frac{\partial P}{\partial V}\right)_{V_0} = \left(V \frac{\partial^2 E}{\partial V^2}\right)_{V_0}$

Including atomic motion (molecular dynamics).



Article: J. J. Hoyt, M. Asta and A. Karma, *Method for Computing the Anisotropy of the Solid-Liquid Interfacial Free Energy*, Phys. Rev. Lett. **86**, 5530-5533 (2001). **See also:** <http://cms.northwestern.edu/Intro.html>

Potential energy for a system of N particles

$$\begin{aligned}
 U = U(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) &= U_0 + \sum_i U(\mathbf{r}_i) \\
 &+ \frac{1}{2} \sum_{i,j} V^{(2)}(\mathbf{r}_i, \mathbf{r}_j) + \frac{1}{6} \sum_{i,j,k} V^{(3)}(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_k) + \dots \\
 &+ \frac{1}{N!} \sum_{i,j,\dots,n} V^{(N)}(\mathbf{r}_i, \mathbf{r}_j, \dots, \mathbf{r}_n)
 \end{aligned}$$

(pair potential)
(3-body potential)

(N-body potential)

Atomic motion: Phonons.

Hamiltonian for atomic motion (harmonic approx.)

$$H = \frac{1}{2} \sum_i M_i [\dot{u}(i)]^2 + \frac{1}{2} \sum_{i,j} u^T(i) \Phi(i,j) u(j),$$

Theory: X.Dai *et al.*, *Science* **300**, 953 (2003).

Expt: J. Wong *et al.*, *Science* **301** 1078 (2003).

Force constants

$$\Phi_{\alpha\beta}(i,j) = \left. \frac{\partial^2 E}{\partial u_\alpha(i) \partial u_\beta(j)} \right|_{u(l)=0 \forall l}$$

Dynamical matrix to diagonalize

$$D = \begin{pmatrix} \frac{\Phi(1,1)}{\sqrt{M_1 M_1}} & \dots & \frac{\Phi(1,N)}{\sqrt{M_1 M_N}} \\ \vdots & \ddots & \vdots \\ \frac{\Phi(N,1)}{\sqrt{M_N M_1}} & \dots & \frac{\Phi(N,N)}{\sqrt{M_N M_N}} \end{pmatrix}$$

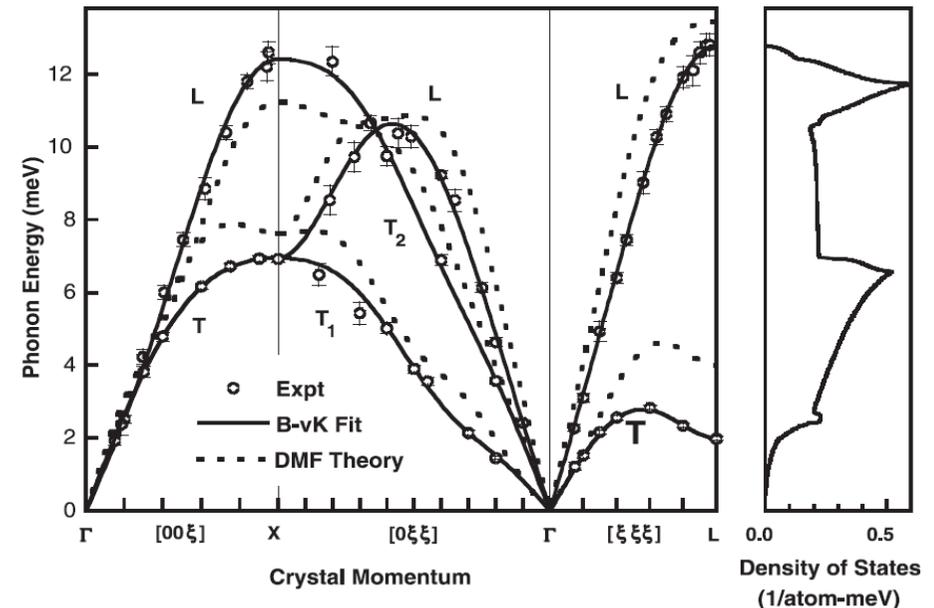
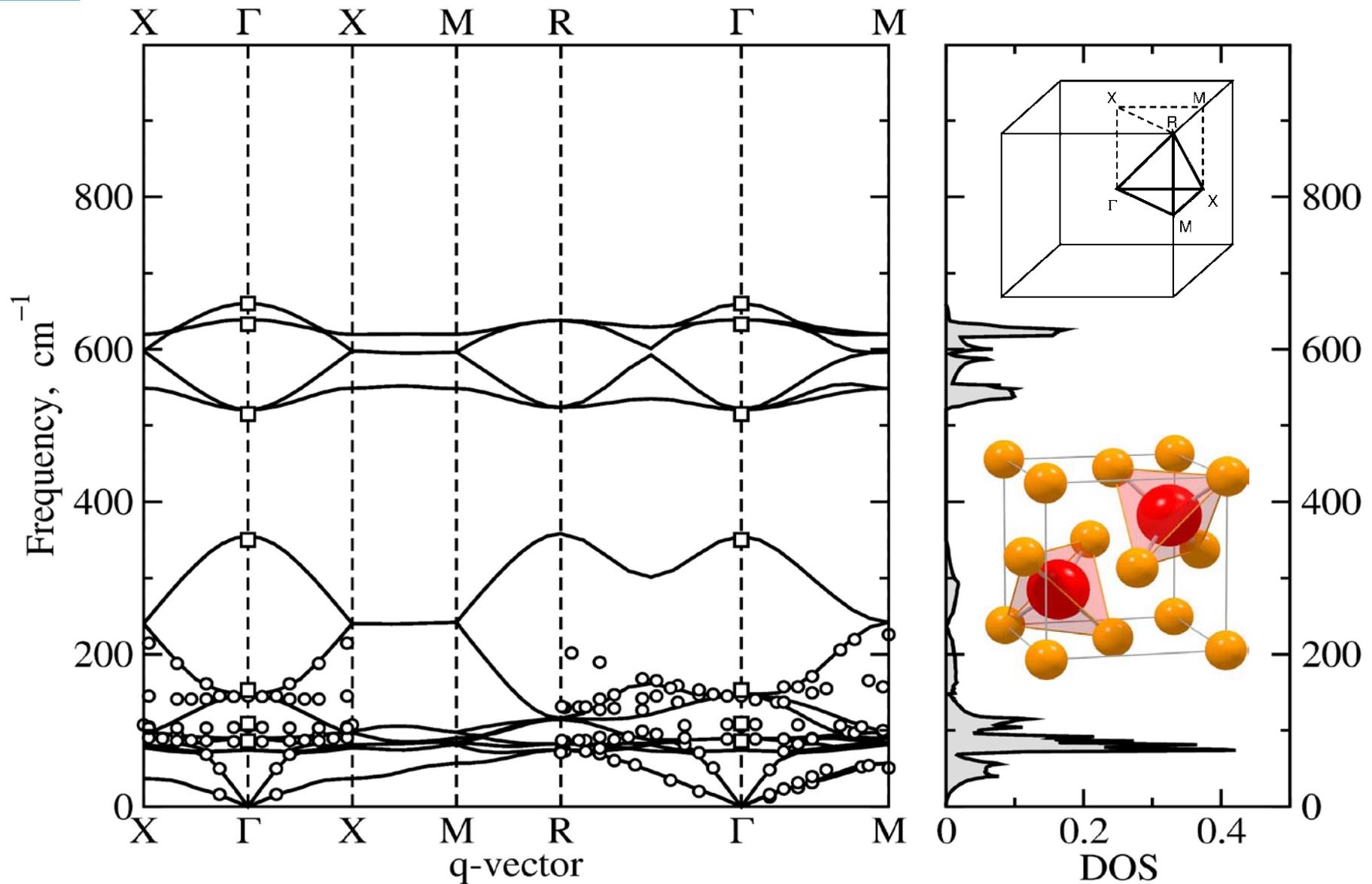


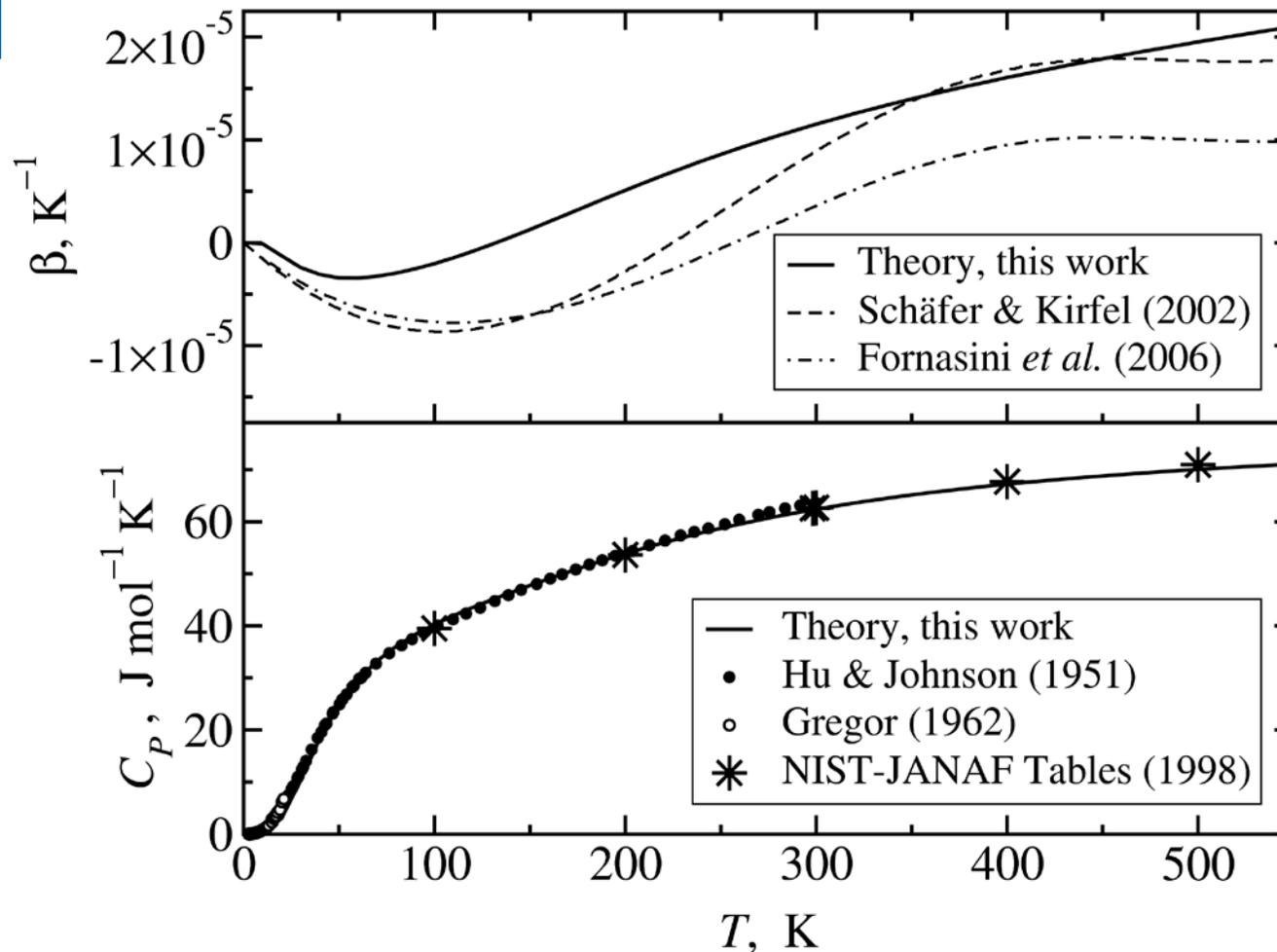
Fig. 2. Phonon dispersions along high-symmetry directions in δ -Pu-0.6 weight % Ga alloy. The longitudinal and transverse modes are denoted L and T, respectively. The experimental data are shown as circles. Along the $[0\xi\xi]$ direction, there are two transverse branches, $[011]\langle 011 \rangle$ (T_1) and $[011]\langle 100 \rangle$ (T_2). The softening of the TA $[111]$ branch toward the L point is apparent. The lattice parameter of our samples is $a = 0.4621$ nm. The solid curves are the fourth-nearest neighbor Born-von Kármán (B-vK) model fit. The derived phonon density of states, normalized to three states per atom, is plotted in the right panel. The dashed curves are calculated dispersions for pure δ -Pu based on DMFT (6).

See A. van de Walle and G. Ceder, *RMP* **74**, 11 (2002) for a review.

Phonon spectrum of Cu_2O



Thermodynamic properties of Cu₂O



Volume coefficient
of thermal
expansion

Heat capacity

$$E_0 = U_0 + \frac{1}{2} \int_0^{\infty} \hbar \omega g(\omega) d\omega$$

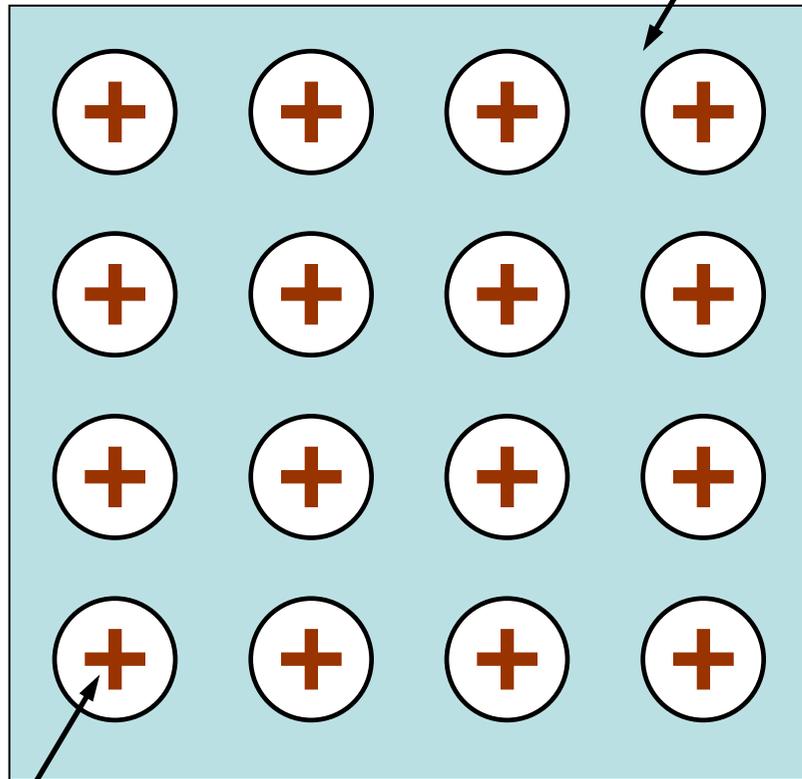
$$C_V = k_B \int_0^{\infty} \left(\frac{\hbar \omega}{2k_B T} \right)^2 \frac{g(\omega) d\omega}{\sinh^2(\hbar \omega / 2k_B T)}$$

$$E(T) = E_0 + \int_0^{\infty} \frac{\hbar \omega g(\omega) d\omega}{\exp(\hbar \omega / k_B T) - 1}$$

$$F(T) = U_0 + k_B T \int_0^{\infty} \log [2 \sinh(\hbar \omega / 2k_B T)] d\omega$$

Bonding in simple metals

"Sea" of valence electrons
(the outermost shell).



Ionic "core" (localized electrons
belonging to inner shells).

Pair potential between two ions in a simple metal is the result of a direct Coulomb interaction between ions and of an *indirect* interaction due to scattering of valence electrons on ions.

The *indirect* interaction is a purely quantum-mechanical effect. Many ions can participate in the interaction process. This leads, in general, to multi-atom (many-body) interactions.



Thermodynamics and magnetism of iron

PHYSICAL REVIEW

VOLUME 102, NUMBER 6

JUNE 15, 1956

Components of the Thermodynamic Functions of Iron

R. J. WEISS AND K. J. TAUER*

Ordnance Materials Research Office, Watertown Arsenal, Watertown, Massachusetts

(Received January 23, 1956)

Hillert & Jarl (1978)

Magnetic energy, α -Fe:

$$E_{\text{magn}} = \int_0^{\infty} C_{\text{magn}} dT \cong \mathbf{1103 \text{ K}}$$

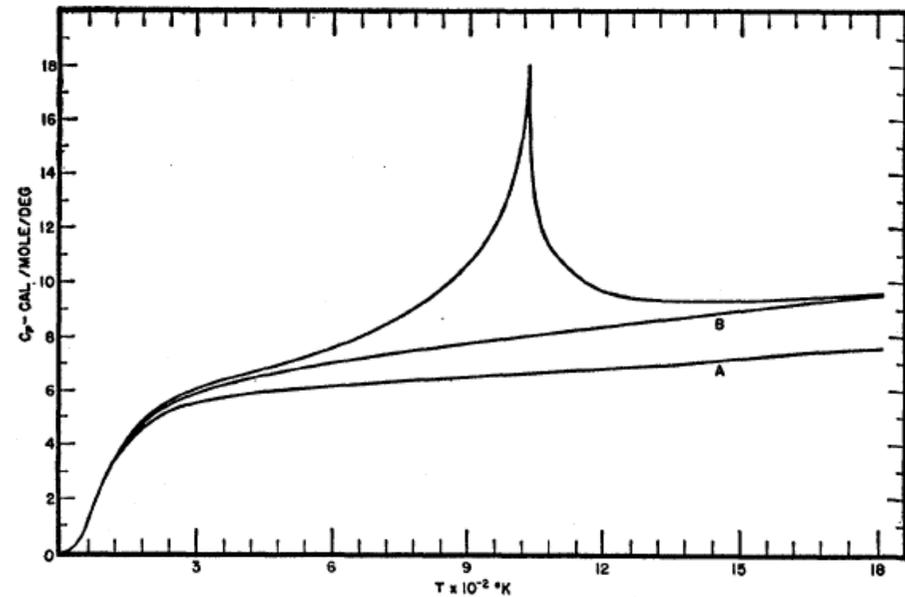


FIG. 1. Specific heat of α (b.c.c.) Fe. Curve A is the calculated lattice specific heat for a Debye $\theta=420^\circ\text{K}$. Curve B includes the electronic specific heat.

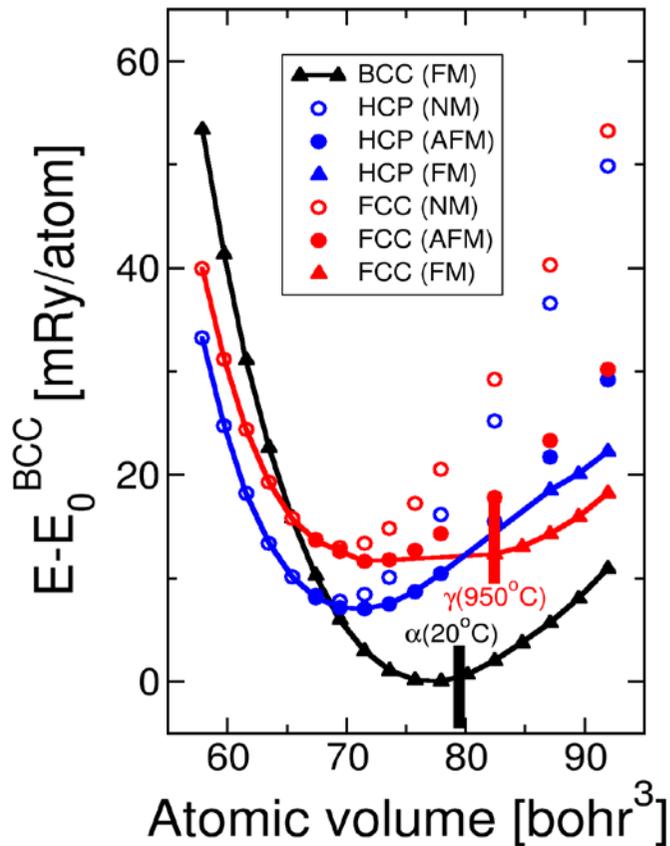
Magnetic entropy, α -Fe:

$$S_{\text{magn}} = \int_0^{\infty} \frac{C_{\text{magn}}}{T} dT \cong R \ln(2s + 1) \cong \mathbf{1.169 \cdot R}$$



Levente Vitos Hans Skriver Andrei Ruban Igor Abrikosov

Pure Fe: EMTO, GGA

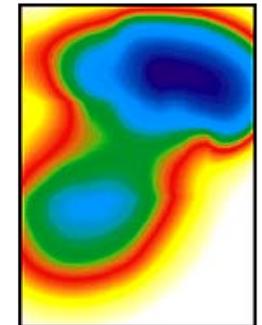


Exact Muffin-Tin Orbitals theory

(O.K. Andersen 1994, L. Vitos, H.L. Skriver 2000)

Coherent Potential Approximation

-chemical and magnetic disorder



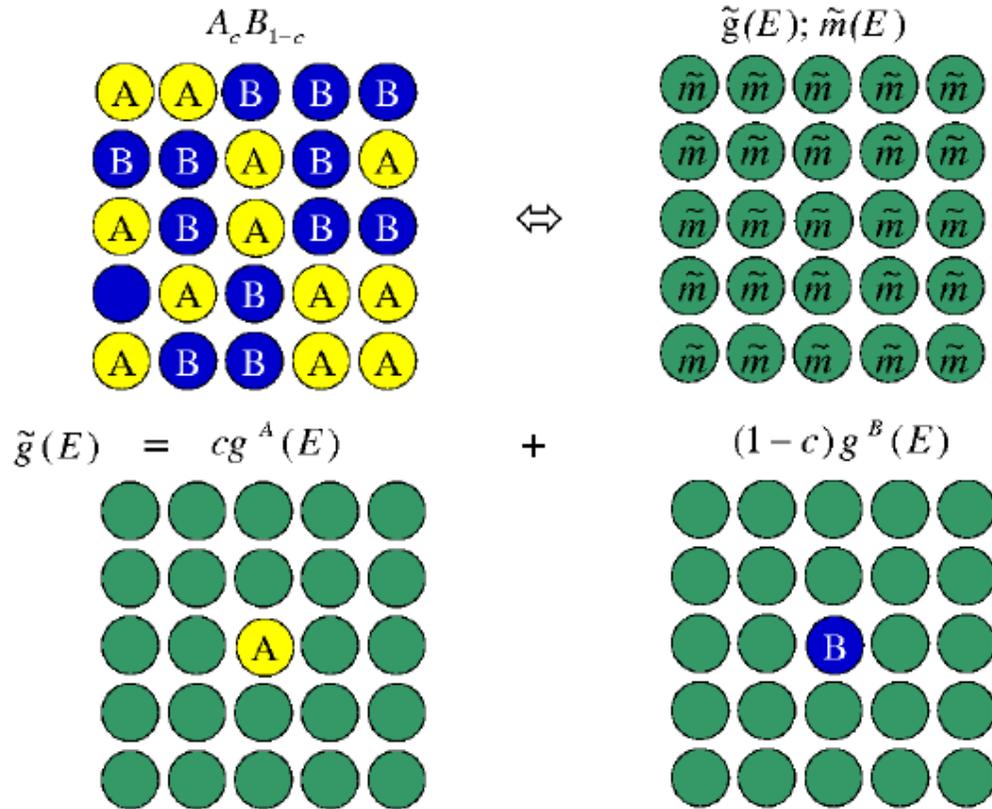
EMTO-CPA method

L. Vitos, I.A. Abrikosov, B. Johansson, Phys. Rev. Lett. **87**, 156401 (2001).

Coherent potential approximation (CPA)

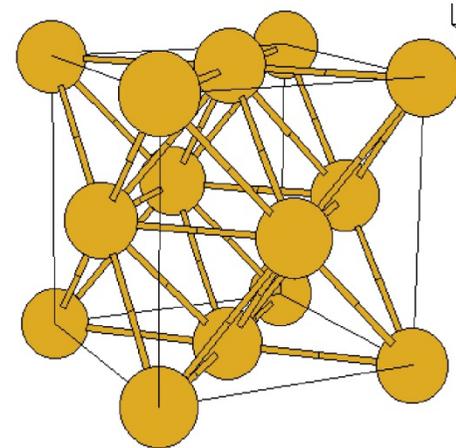
P.Soven, Phys. Rev. **156**, 809 (1967).

D.W.Taylor, Phys. Rev. **156**, 1017 (1967).



Disordered Local Moment (DLM) model

J. Staunton, B.L. Györfy A.J.Pindor, G.M. Stocks, and H. Winter, JMMM **45**, 15 (1984).



50% Fe \uparrow + 50% Fe \downarrow

Magnetic entropy:

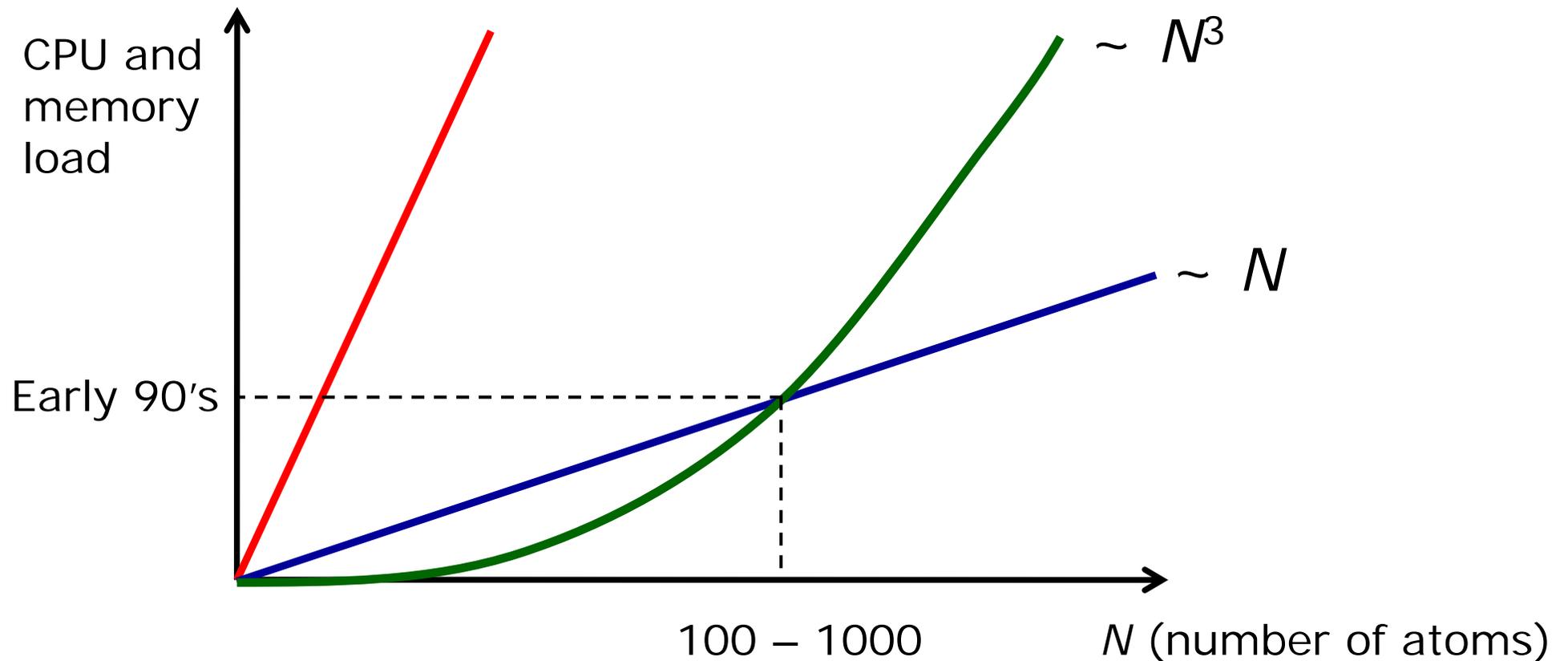
$$S_{\text{magn}} = k_B \ln(M_s + 1)$$

J.S. Faulkner: "The modern theory of alloys", Progr. Mater. Science **27**, 1 – 187 (1982).

A.V. Ruban and I.A. Abrikosov: "Configurational thermodynamics of alloys from first principles: effective cluster interactions", Rep. Prog. Phys. **71**, 046501 (2008)



Order(N) – Linear Scaling Methods



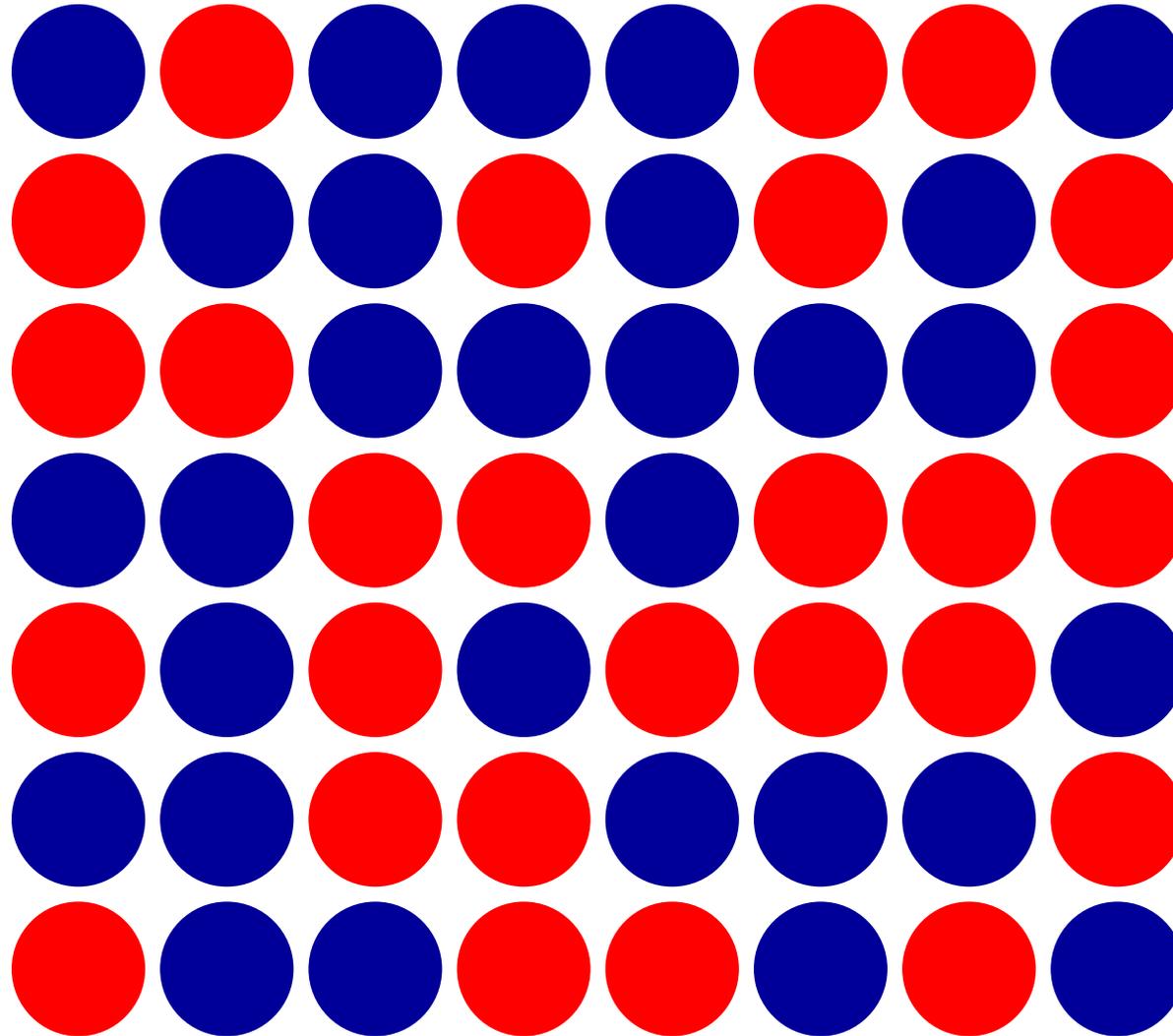
Reviews on $O(N)$ Methods:

S. Goedecker, Rev. Mod. Phys. 71, (1999); <http://arXiv.org/abs/cond-m3at/980607>.

P. Ordejón, Comp. Mat. Sci. 12, 157 (1998).



Key to linear scaling: locality



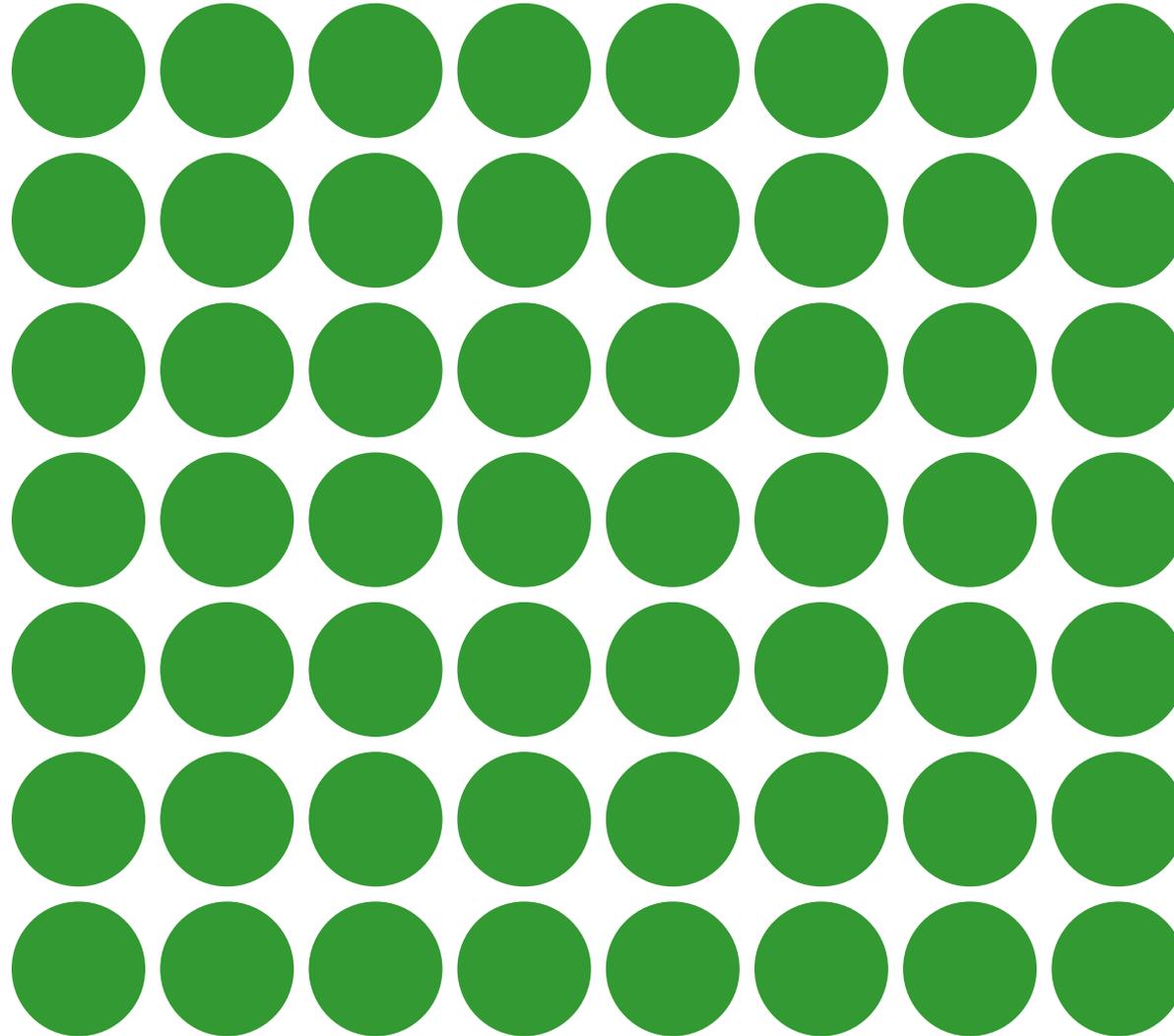
Locally Self-consistent Green's Function (LSGF) method:

I.A. Abrikosov *et. al.*, , Phys. Rev. Lett. **76**, 4203 (1996);

I.A. Abrikosov *et. al.*, , Phys. Rev. B **56**, 9319 (1997).

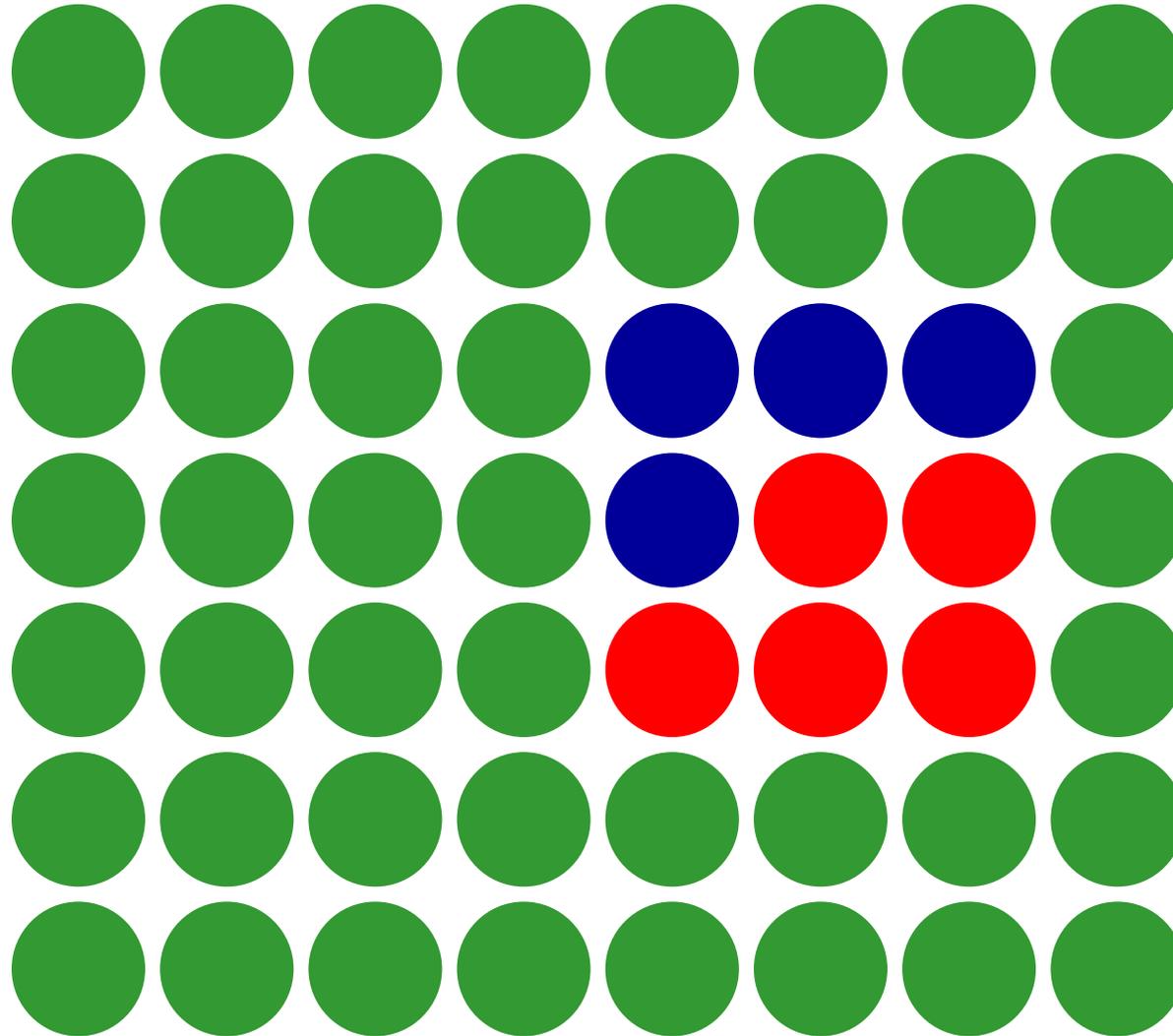


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Locally Self-consistent Green's Function (LSGF) method:
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I.A. Abrikosov *et. al.*, , Phys. Rev. B **56**, 9319 (1997).

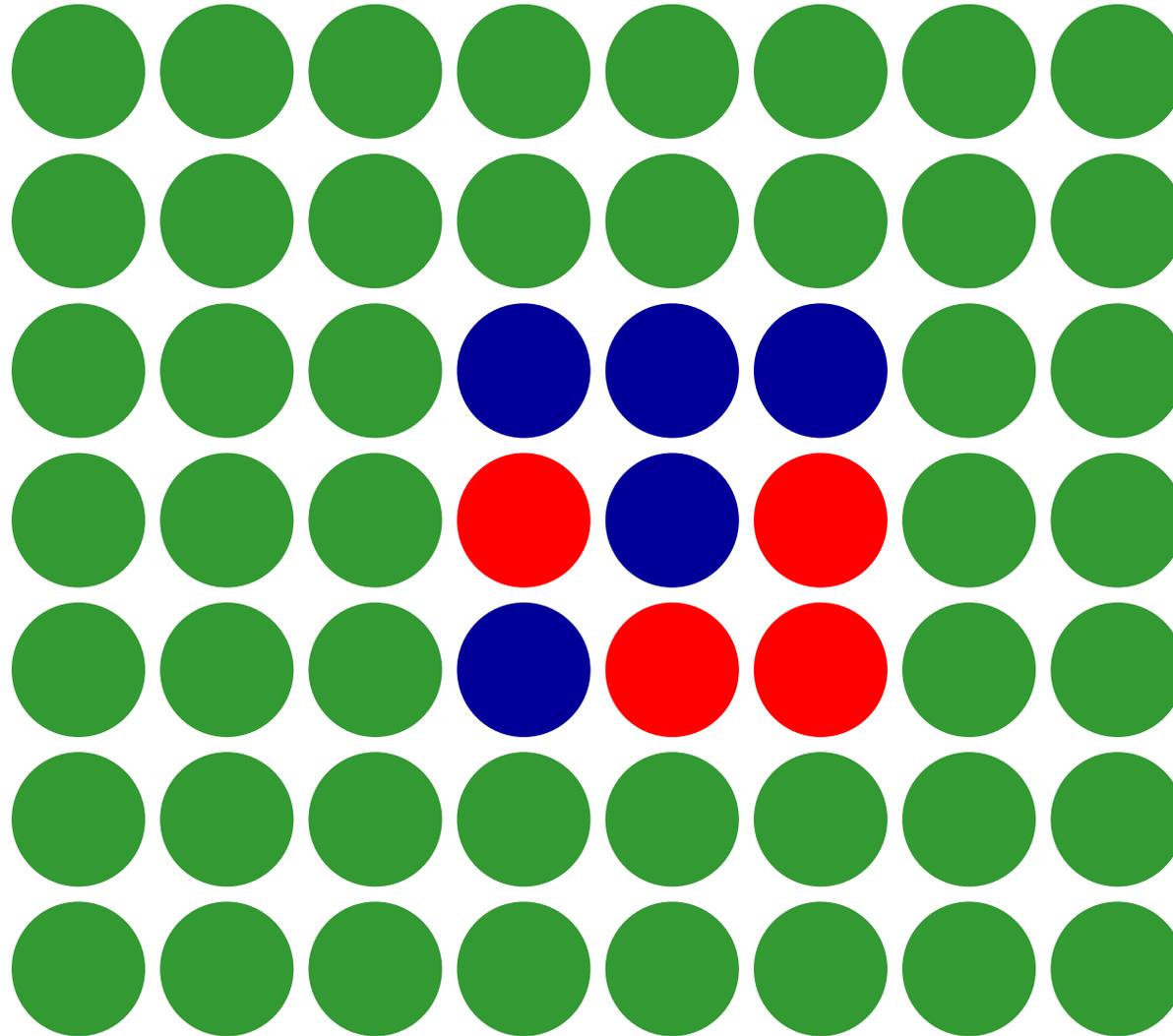
Key to linear scaling: locality



Locally Self-consistent Green's Function (LSGF) method:
I.A. Abrikosov *et. al.*, , Phys. Rev. Lett. **76**, 4203 (1996);
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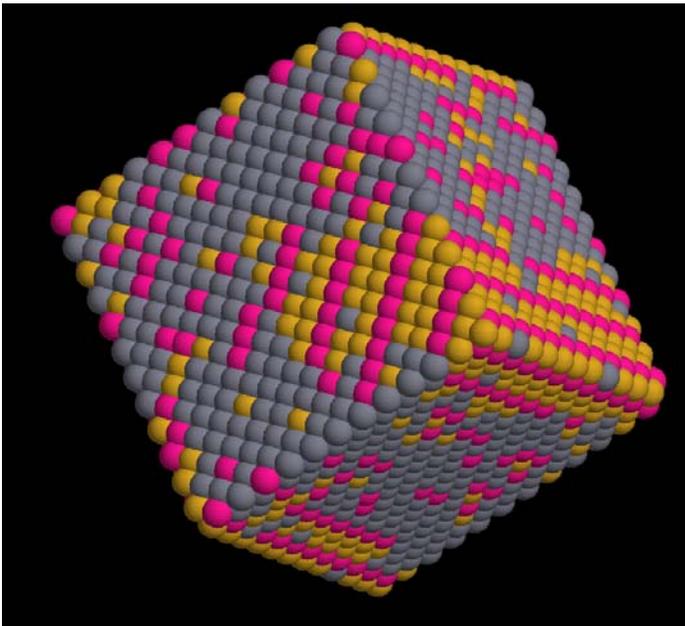
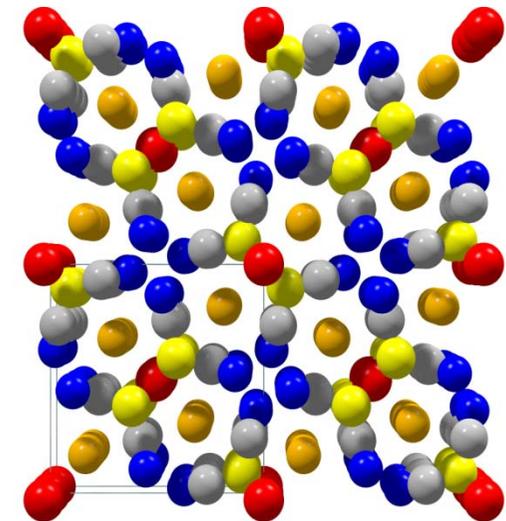
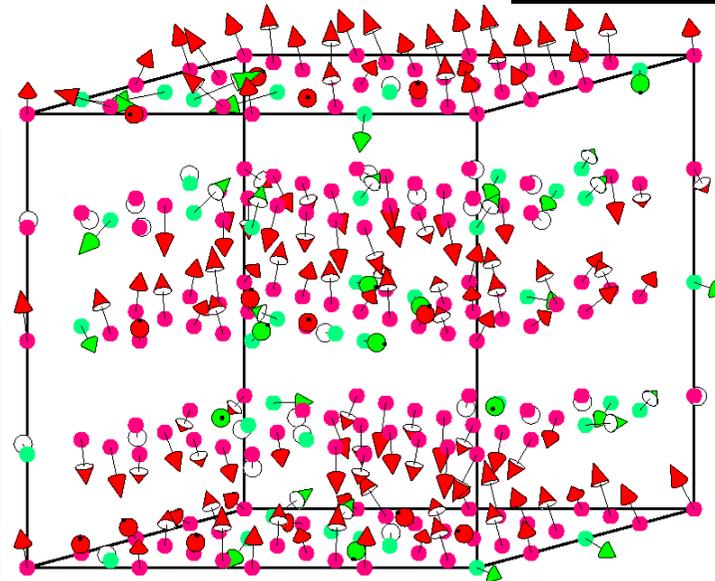
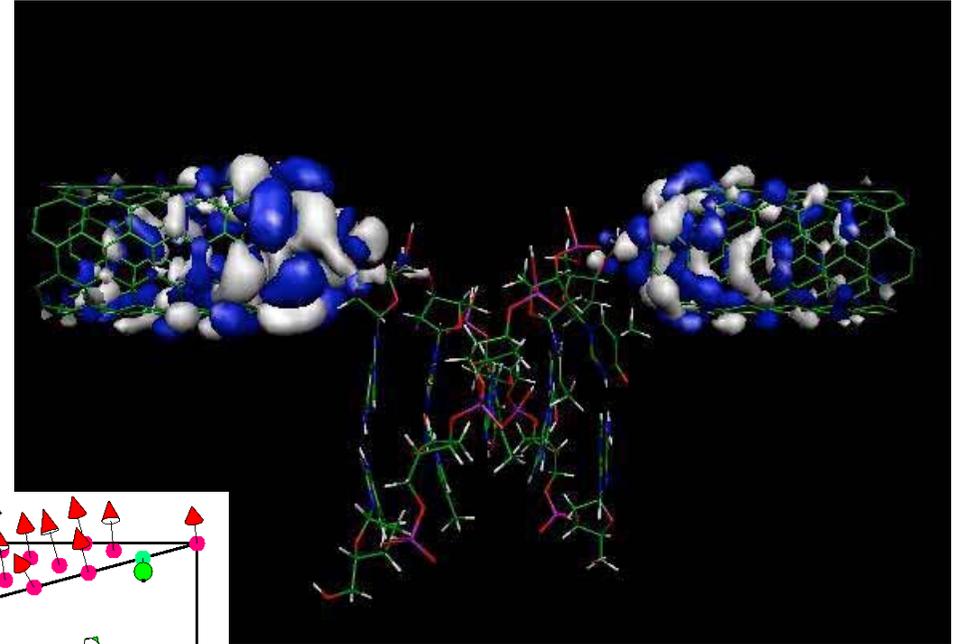
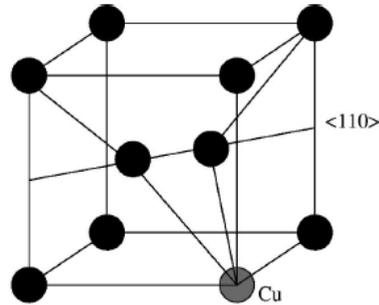
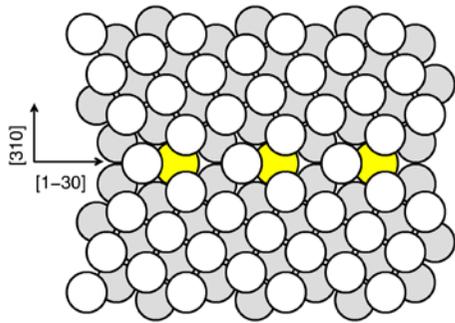


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Linear scaling methods (LSGF, SIESTA): access to large systems (disorder, defects).

Sulfur-segregated GB

$$\Sigma = 5(310)[001]$$



Cu precipitation in Fe: a serious issue for nuclear reactor safety

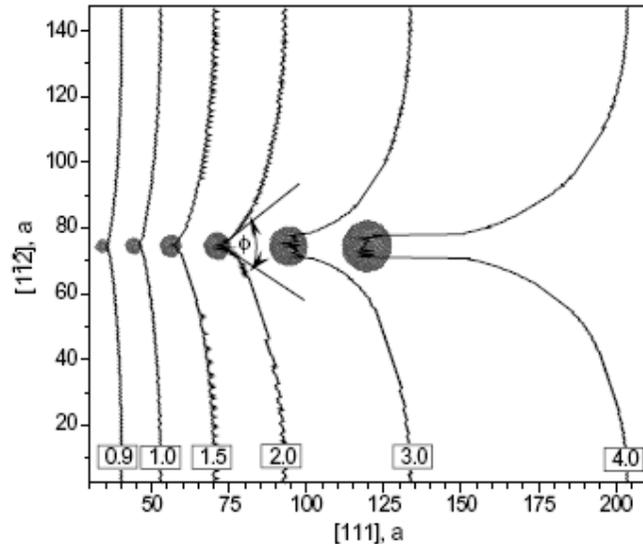


Fig. 2. Dislocation line in the $(1\bar{1}0)$ slip plane at the critical stress τ_c for different precipitate sizes at 0 K.

- Nano-sized precipitates of bcc Cu form under irradiation of reactor pressure vessel (RPV) steels
- Cu particles impede dislocation motion by pinning the dislocation lines – strengthening effect
- Increase in yield strength correlates with the increase of brittle-to-ductile transition temperature – embrittlement

K.C. Russell and L.M. Brown: *A dispersion strengthening model based on differing elastic moduli applied to the iron-copper system*, Acta Metall. 20, 969 – 974 (1972).

D.J. Bacon and Yu.N. Osetsky: *Hardening due to copper precipitates in α -iron studied by atomic-scale modelling*, Journal of Nuclear Materials 329–333, 1233–1237 (2004).

N. Sandberg and P. Korzhavyi: *Theoretical study of irradiation induced hardening and embrittlement in spent nuclear fuel holders, relevant for the Swedish long-term storage*, SKB R-09-15 (2009).

Energies of defect clusters from first-principles calculations

In order to make reliable long-term predictions, a firm knowledge of the driving forces and of the atomic mechanisms for precipitation at low temperatures. The challenging task is to deduce these mechanisms and to compute the relevant energies from first principles.

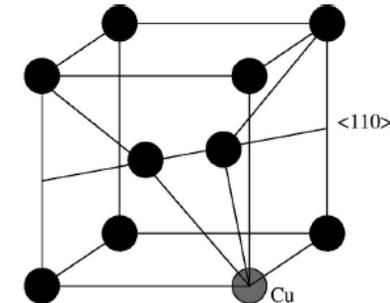
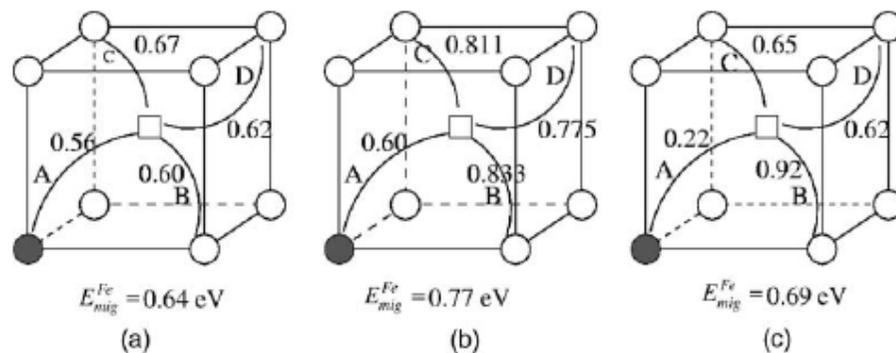


Fig. 2. Vacancy migration energy in a presence of one Cu atom: (a) ab initio, (b) potential FS-1, (c) potential EAM-1. The empty squares are vacancies, the empty circles are Fe atoms and the grey circles are Cu atoms. The bottom caption is the vacancy migration energy in pure Fe.

C.S. Becquart and C. Domain: "Ab initio contribution to the study of complexes formed during dilute FeCu alloys radiation", Nuclear Instruments and Methods in Physics Research B, 202, 2003, pp. 44 – 50

Solubility and diffusivity of Cu in Fe from first principles!

F. Soisson and C.-C. Fu: PHYSICAL REVIEW B 76, 214102 (2007)

Cu-PRECIPIATION KINETICS IN α -Fe FROM...

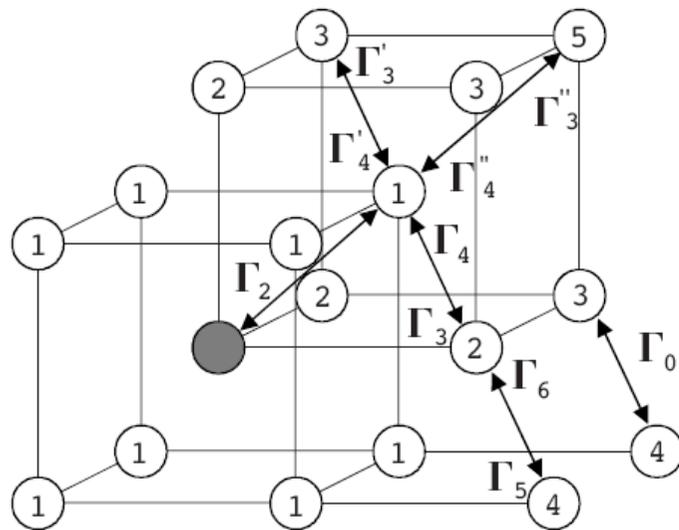


FIG. 2. Vacancy jump frequencies (Γ_i) around a copper atom (gray) and its neighboring iron atoms (white). The arrows indicate the direction of the vacancy jumps; the circled numbers are the order of neighbors to the copper atom.

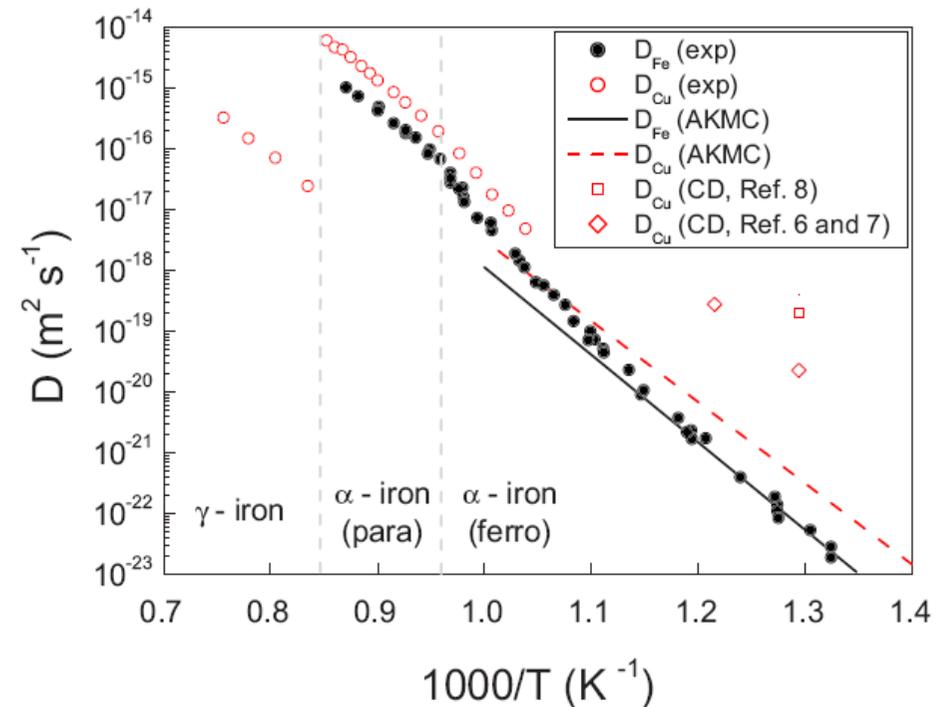
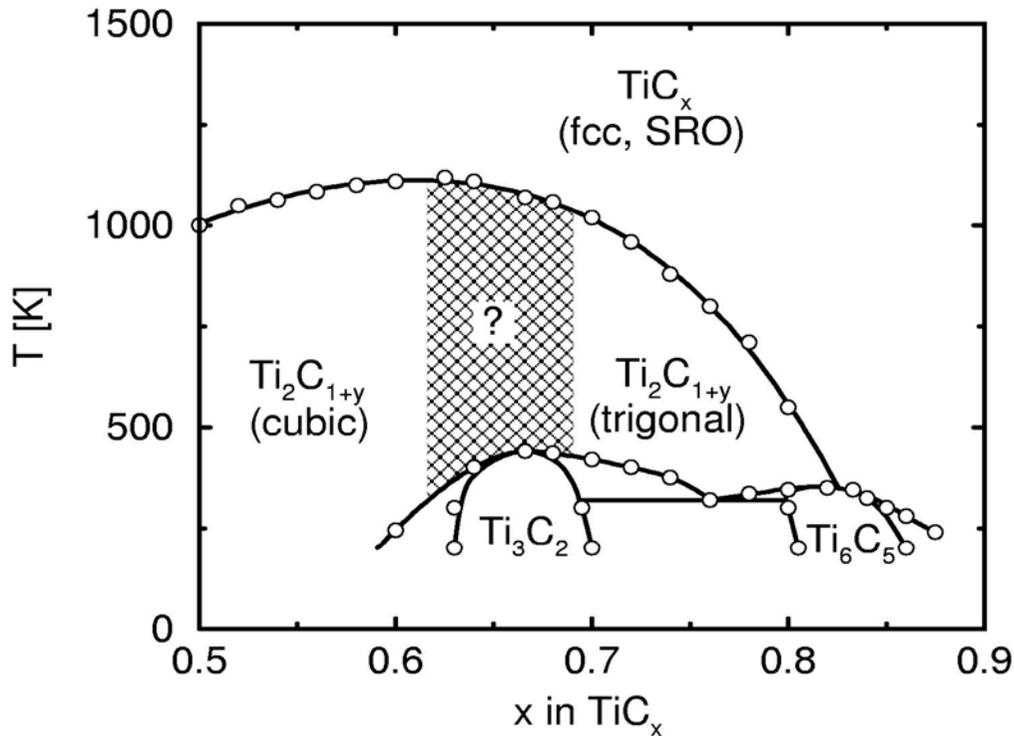


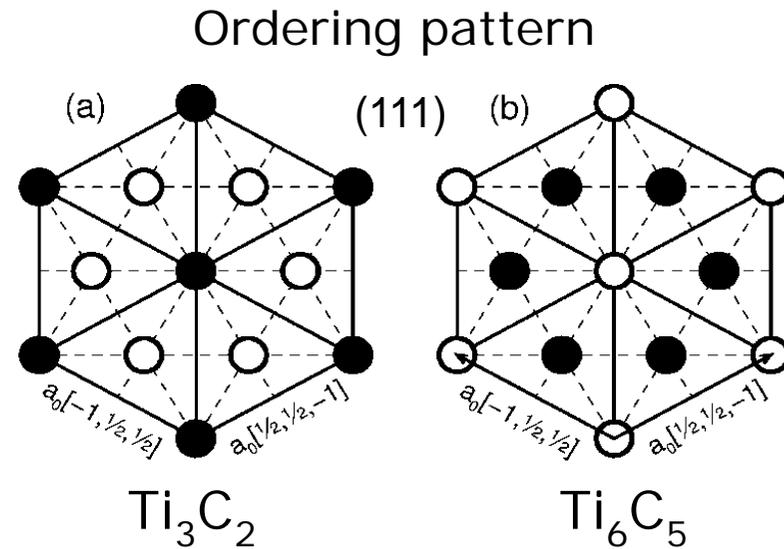
FIG. 5. (Color online) Iron self-diffusion and copper impurity diffusion coefficients. Comparison between the experimental data (Refs. 1 and 36); the coefficients used in our AKMC simulations and the coefficients used in the cluster dynamics modeling of Christian and Barbu (Ref. 8) and Golubov *et al.* (Refs. 6 and 7).

Monte Carlo study of vacancy ordering

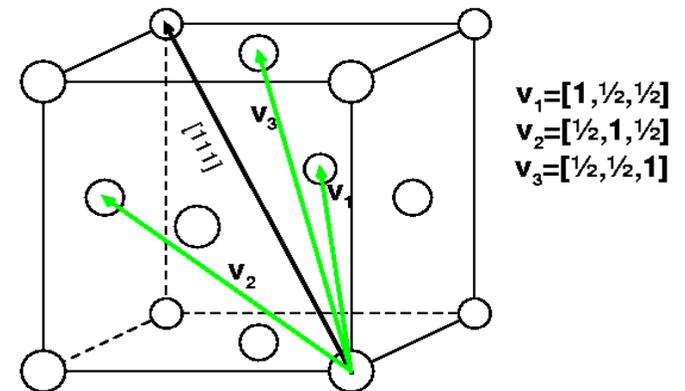
Calculated phase diagram



Order of vacancies in the (111) layers of Ti_3C_2 and Ti_6C_5 :



Stacking between layers



P. Korzhavyi *et al.*, Phys. Rev. Lett. 88, 15505 (2002).

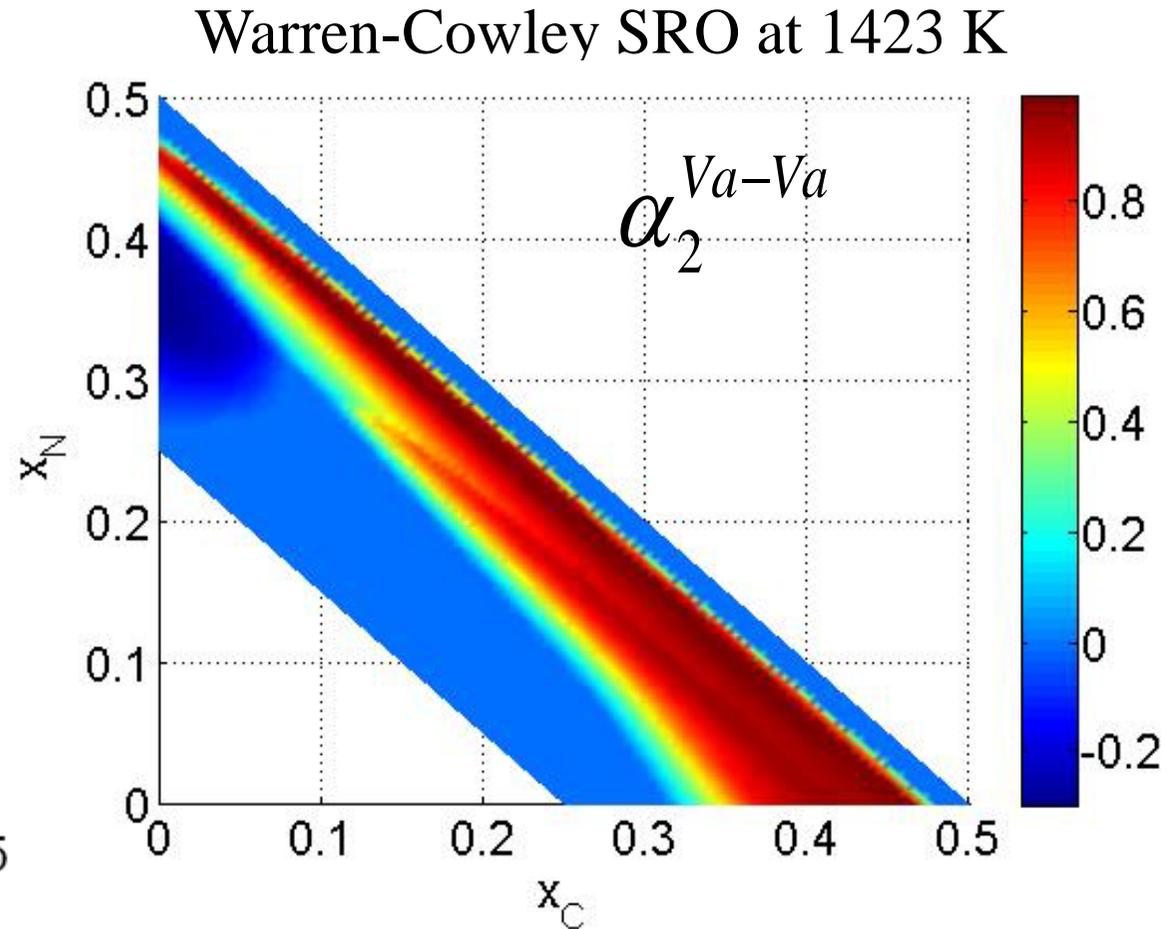
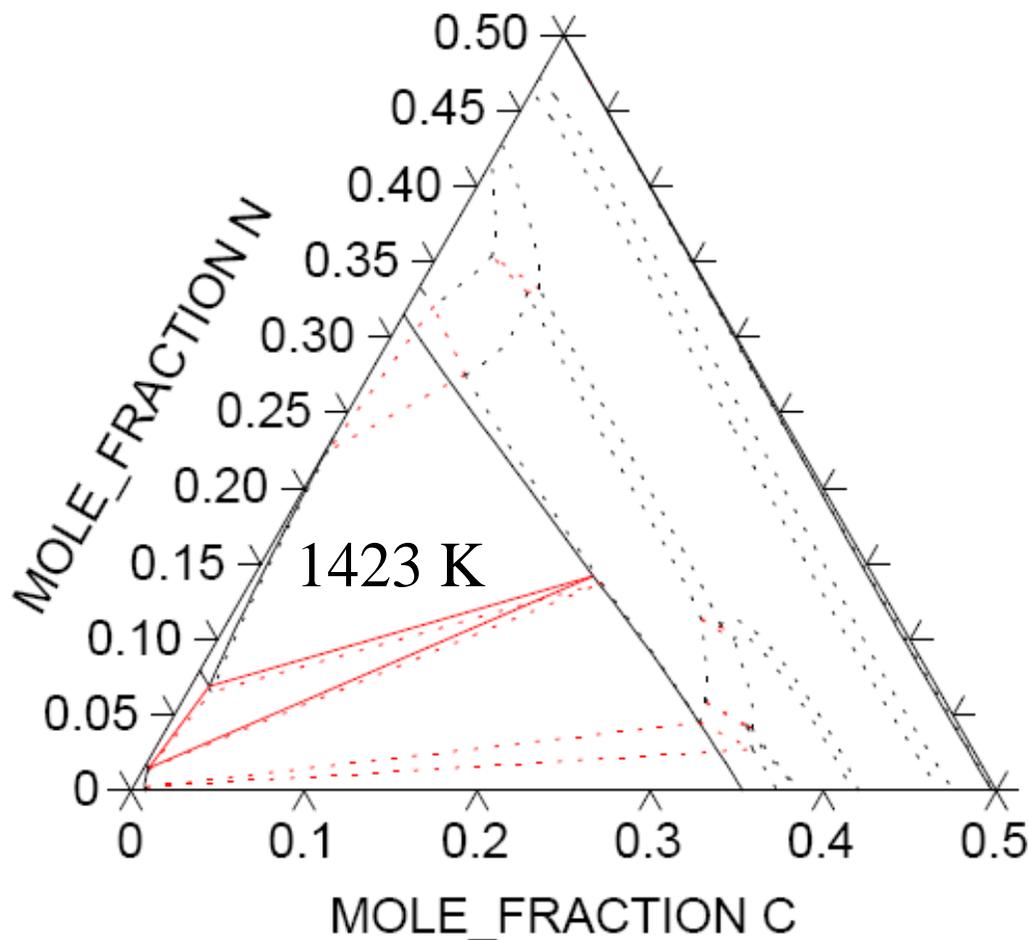
Monte Carlo

- Simulation box: 32x32x32 (32768 carbon sites)
- 5000 time steps per site



Combined ab initio and Calphad study of vacancy order in $\text{Ti}(\text{C},\text{N})_x$

D.A. Andersson, P.A. Korzhavyi and B. Johansson, "First-principles based calculation of binary and multicomponent phase diagrams for titanium carbonitride", CALPHAD, **32**, 543 (2008) **BEST PAPER AWARD (CALPHAD 2009)**





Distinguishing features of *ab-initio* calculations as a modern research tool

- Atomic-scale resolution
- Access to the properties of metastable or unstable systems.
- Access to phase equilibria at low temperatures.
- Excellent control of "experimental" conditions.
- *Ab initio* calculations are restricted to simple structures, typically < 1000 atoms. With $O(N)$ – up to 10^5 atoms.
- In order to access the properties of more realistic (complex) systems, *ab initio* calculations **must be combined with statistical-mechanical or thermodynamic modeling** (e.g., Molecular Dynamics, Monte Carlo, etc.).