On Ions and Neutrals in Disks and Pencils

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Pencil-Code User Meeting

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NORDITA

1. Introduction

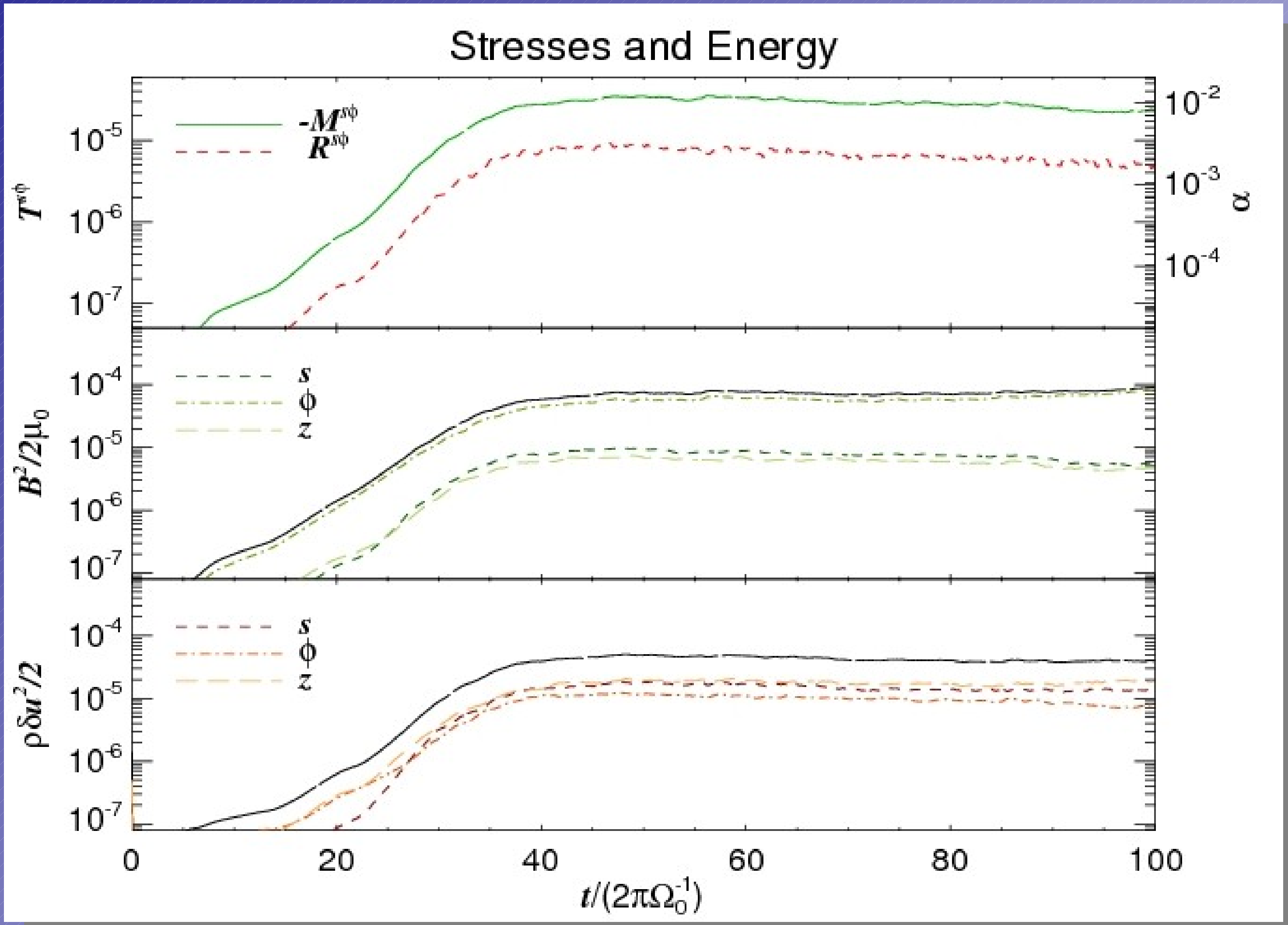
Turbulence is important for accretion

Turbulence is important for planet formation

Turbulence in disks is probably hydromagnetic (MRI)

1.1 Turbulence stresses transport angular momentum

$$\partial_t \overline{L_\phi} + \nabla\cdot(\overline{L_\phi}\,\overline{\boldsymbol{u}}) = -\nabla\cdot(r\,(R^{r\phi} - M^{r\phi}))$$



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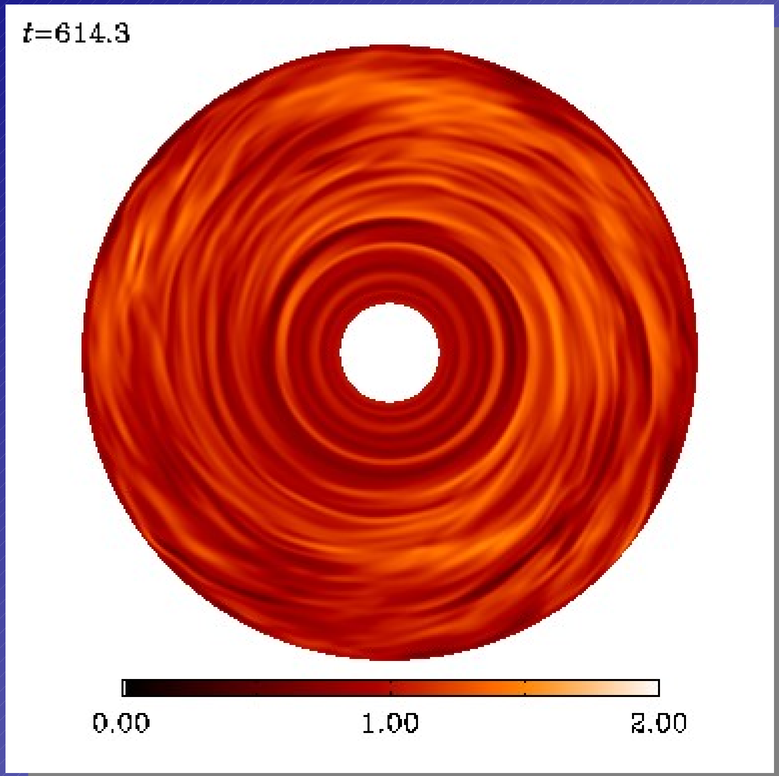
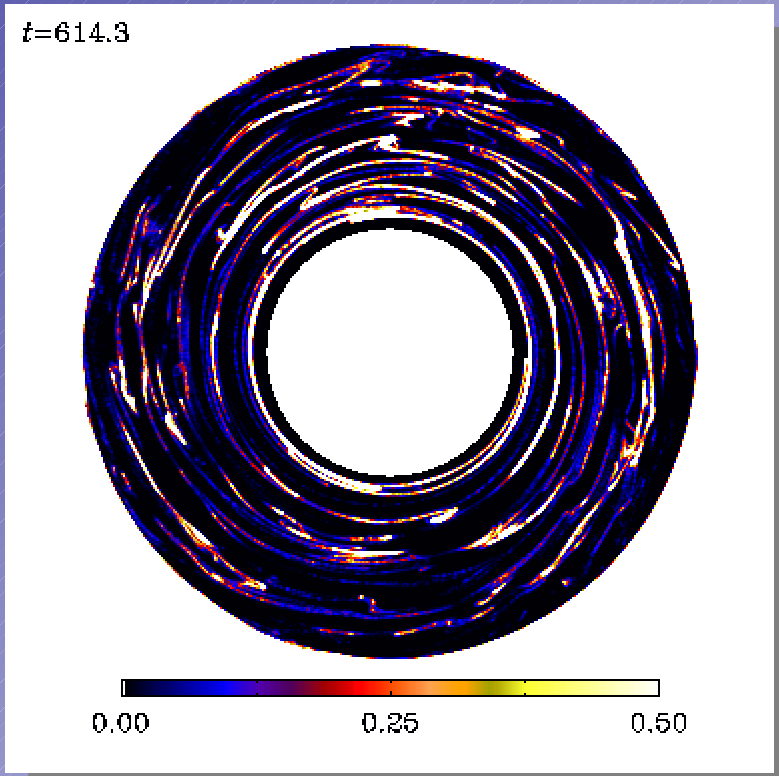
1.2 Turbulent eddies are efficient particle traps



Gas Density



Bulk Density of Solids

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But the midplanes of protoplanetary disks are cold and therefore only WEAKLY IONIZED

Solve for neutrals as well as ions!

2. Ideal MHD

$$\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{u} \times \boldsymbol{B}$$

$$\frac{\partial (\rho \boldsymbol{u})}{\partial t} = -\nabla \cdot (\rho \boldsymbol{u} : \boldsymbol{u}) + \nabla \left(p + \frac{B^2}{2\mu_0} \right)$$

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \left(\rho \boldsymbol{u} \right)$$

2. Two-fluid MHD with partial ionization

$$\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{u}_i \times \boldsymbol{B}$$

$$\frac{\partial \rho_i}{\partial t} = -\nabla \cdot \left(\rho_i \boldsymbol{u}_i\right) + \boldsymbol{G}$$

$$\frac{\partial \rho_n}{\partial t} = -\nabla \cdot \left(\rho_n \boldsymbol{u}_n\right) - \boldsymbol{G}$$

$$\frac{\partial \left(\rho_i \boldsymbol{u}_i\right)}{\partial t} = -\nabla \cdot \left(\rho_i \boldsymbol{u}_i : \boldsymbol{u}_i\right) - \nabla \left(p_i + p_e + \frac{B^2}{2\mu_0}\right) + \boldsymbol{F}$$

$$\frac{\partial \left(\rho_n \boldsymbol{u}_n\right)}{\partial t} = -\nabla \cdot \left(\rho_n \boldsymbol{u}_n : \boldsymbol{u}_n\right) - \nabla p_n - \boldsymbol{F}$$

$$\boldsymbol{G} = \zeta \rho_n - \alpha \rho_i^2$$

ionization

recombination

$$\boldsymbol{F} = \zeta \rho_n \boldsymbol{u}_n - \alpha \rho_i^2 \boldsymbol{u}_i + \gamma \rho_i \rho_n \left(\boldsymbol{u}_n - \boldsymbol{u}_i\right)$$

collisional drag

2. Two-fluid MHD with partial ionization in the Pencil Code

$$\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{u}_i \times \boldsymbol{B}$$

$$\frac{\partial \rho_i}{\partial t} = -\boldsymbol{u}_{\boldsymbol{i}} \cdot \nabla \rho_i - \rho_i \nabla \cdot \boldsymbol{u}_{\boldsymbol{i}} + \zeta \rho_n - \alpha \rho_i^2$$

$$\frac{\partial \boldsymbol{u}_{\boldsymbol{i}}}{\partial t} = -(\boldsymbol{u}_{\boldsymbol{i}} \cdot \nabla) \boldsymbol{u}_{\boldsymbol{i}} - \rho_i^{-1} \nabla (p_i + p_e) - \frac{\boldsymbol{J} \times \boldsymbol{B}}{\mu_0} - \rho_n \left(\gamma + \frac{\zeta}{\rho_i} \right) (\boldsymbol{u}_{\boldsymbol{i}} - \boldsymbol{u}_{\boldsymbol{n}})$$

$$\frac{\partial \rho_n}{\partial t} = -\boldsymbol{u}_{\boldsymbol{n}} \cdot \nabla \rho_n - \rho_n \nabla \cdot \boldsymbol{u}_{\boldsymbol{n}} - \zeta \rho_n + \alpha \rho_i^2$$

$$\frac{\partial \boldsymbol{u}_{\boldsymbol{n}}}{\partial t} = -(\boldsymbol{u}_{\boldsymbol{n}} \cdot \nabla) \boldsymbol{u}_{\boldsymbol{n}} - \rho_n^{-1} \nabla p_n + \rho_i \left(\gamma + \frac{\alpha \rho_i}{\rho_n} \right) (\boldsymbol{u}_{\boldsymbol{i}} - \boldsymbol{u}_{\boldsymbol{n}})$$

2. Two-fluid MHD with partial ionization in the Pencil Code

$$\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{u}_i \times \boldsymbol{B}$$

$$\frac{\partial \ln \rho_i}{\partial t} = -\boldsymbol{u}_i \cdot \nabla \ln \rho_i - \nabla \cdot \boldsymbol{u}_i + \zeta \frac{\rho_n}{\rho_i} - \alpha \rho_i$$

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$$\frac{\partial \ln \rho_n}{\partial t} = -\boldsymbol{u}_n \cdot \nabla \ln \rho_n - \nabla \cdot \boldsymbol{u}_n - \zeta + \alpha \frac{\rho_i^2}{\rho_n}$$

$$\frac{\partial \boldsymbol{u}_n}{\partial t} = -(\boldsymbol{u}_n \cdot \nabla) \boldsymbol{u}_n - \rho_n^{-1} \nabla p_n + \rho_i \left(\gamma + \frac{\alpha \rho_i}{\rho_n} \right) (\boldsymbol{u}_i - \boldsymbol{u}_n)$$

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(ion) density.f90

$$\frac{\partial \boldsymbol{u}_i}{\partial t} = -(\boldsymbol{u}_i \cdot \nabla) \boldsymbol{u}_i - \rho_i^{-1} \nabla (p_i + p_e) - \frac{\boldsymbol{J} \times \boldsymbol{B}}{\mu_0} - \rho_n \left(\gamma + \frac{\zeta}{\rho_i} \right) (\boldsymbol{u}_i - \boldsymbol{u}_n)$$

(ion) hydro.f90

$$\frac{\partial \ln \rho_n}{\partial t} = -\boldsymbol{u}_n \cdot \nabla \ln \rho_n - \nabla \cdot \boldsymbol{u}_n - \zeta + \alpha \frac{\rho_i^2}{\rho_n}$$

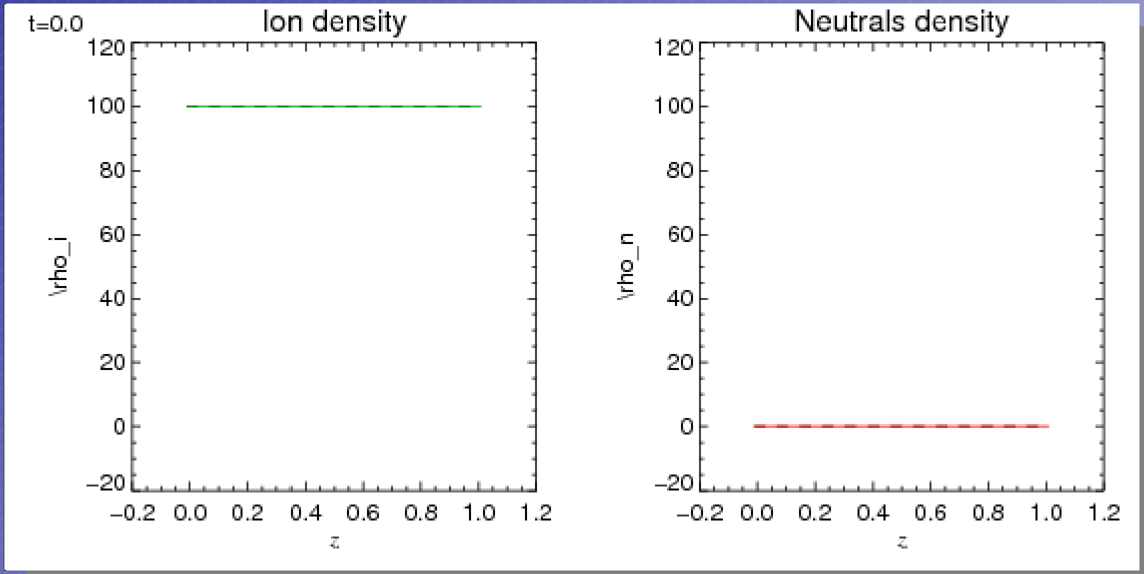
neutraldensity.f90

$$\frac{\partial \boldsymbol{u}_n}{\partial t} = -(\boldsymbol{u}_n \cdot \nabla) \boldsymbol{u}_n - \rho_n^{-1} \nabla p_n + \rho_i \left(\gamma + \frac{\alpha \rho_i}{\rho_n} \right) (\boldsymbol{u}_i - \boldsymbol{u}_n)$$

neutralvelocity.f90

3. Basic tests

Recombination

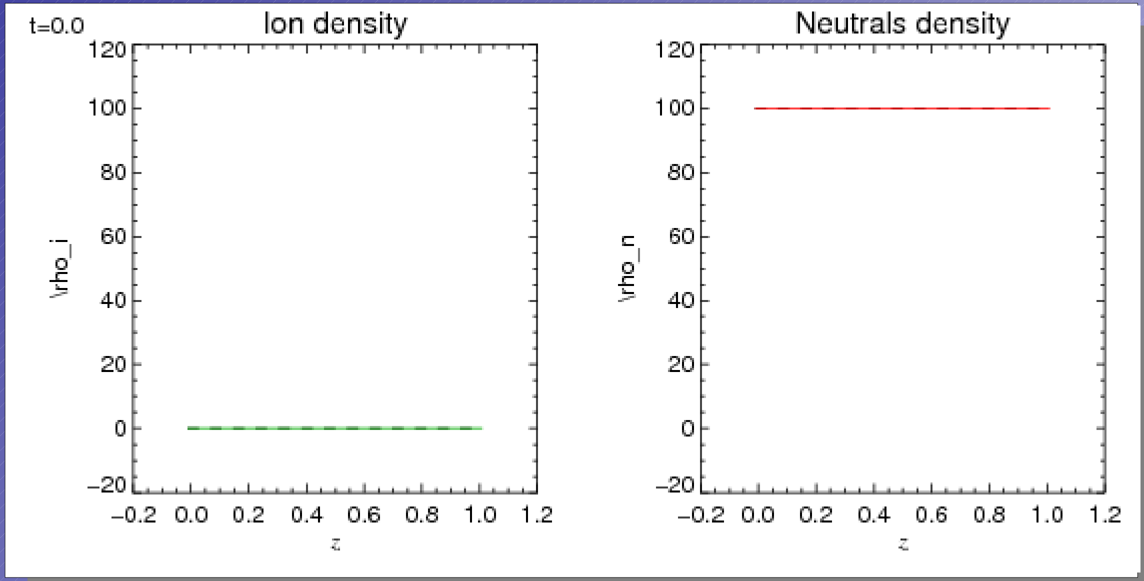


$$\alpha = 1$$

$$\zeta = 0$$

3. Basic tests

Ionization



$\alpha = 0$

$\zeta = 1$

3. Comparison Sample

Effects of pressure and resistivity on the Ambipolar diffusion singularity – Brandenburg & Zweibel 1995

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EFFECTS OF PRESSURE AND RESISTIVITY ON THE AMBIPOLAR DIFFUSION SINGULARITY: TOO LITTLE, TOO LATE

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ABSTRACT

Ambipolar diffusion, or ion-neutral drift, can lead to steepening of the magnetic field profile and even to the formation of a singularity in the current density. These results are based on an approximate treatment of ambipolar drift in which the ion pressure is assumed vanishingly small and the frictional coupling is assumed to be very strong, so that the medium can be treated as a single fluid. This steepening, if it really occurs, must act to facilitate magnetic reconnection in the interstellar medium, and so could have important consequences for the structure and evolution of the galactic magnetic field on both global and local scales.

In actuality, the formation of a singularity must be prevented by physical effects omitted by the strong coupling approximation. In this paper we solve the coupled equations for charged and neutral fluids in a simple slab geometry, which was previously shown to evolve to a singularity in the strong coupling approximation. We show that both ion pressure and resistivity play a role in removing the singularity, but that, for parameters characteristic of the interstellar medium, the peak current density is nearly independent of ion pressure and scales inversely with resistivity. The current gradient length scale, however, does depend on ion pressure. In the end, effects outside the fluid approximation, such as the finite ion gyroradius, impose the strictest limit on the evolution of the magnetic profile.

Subject headings: diffusion — ISM: magnetic fields — MHD

3. Comparison Sample

Effects of pressure and resistivity on the Ambipolar diffusion singularity – Brandenburg & Zweibel 1995

Ionization equilibrium $\alpha = \zeta \rho_n / \rho_i^2$

Strong coupling $\gamma = 10$

almost fully neutral ($\rho_i = 0.1$, $\rho_n = 10$)

Isothermal

Linear toroidal field $\mathbf{B} = B_0 z$, vanishing at the origin

3. Comparison Sample

Diffusion singularity – Densities

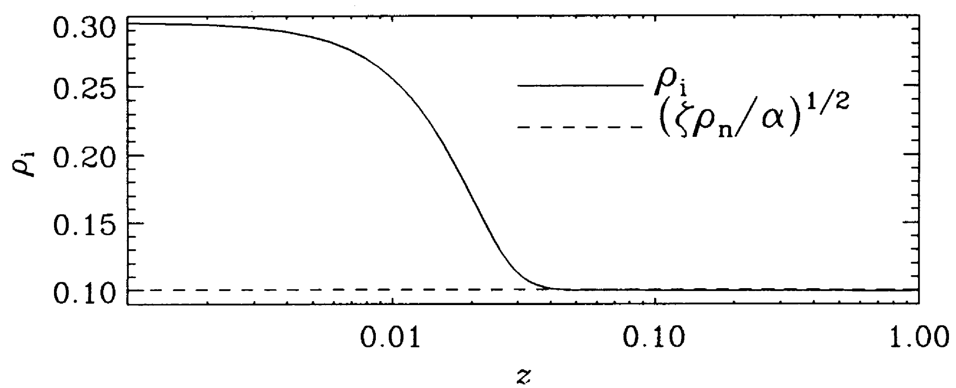
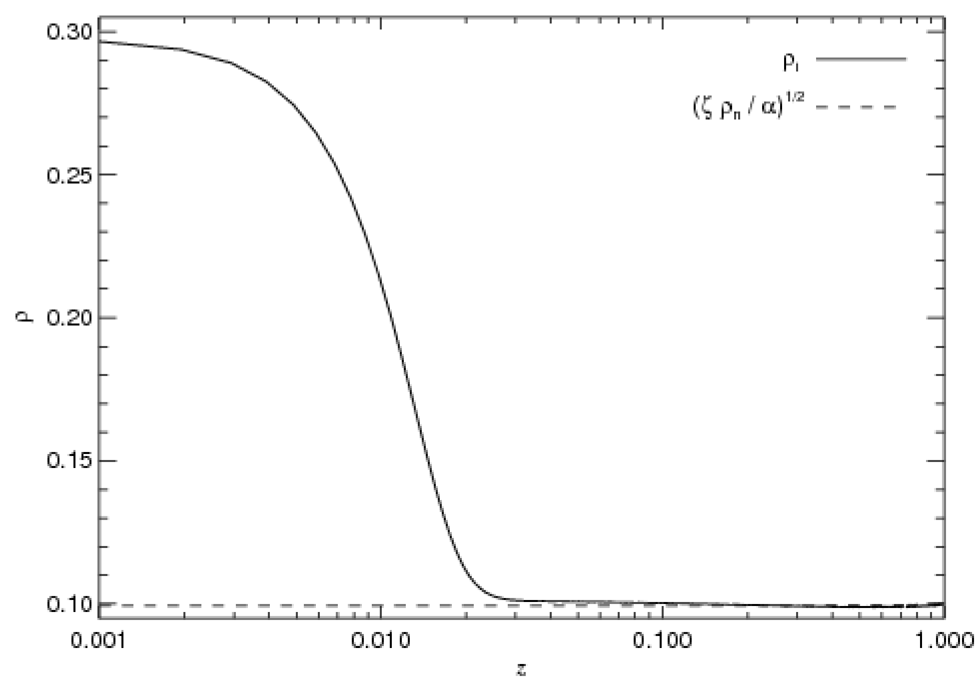


FIG. 3.—Profile of ion density and comparison with $(\zeta\rho_n/\alpha)^{1/2}$ for run B0

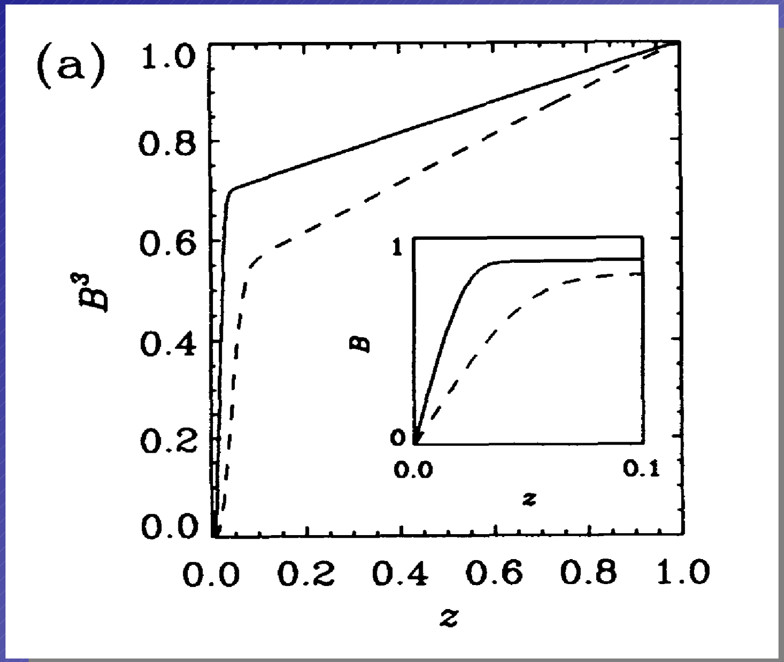
Brandenburg & Zweibel 1995

Solved with Pencil's neutrals module



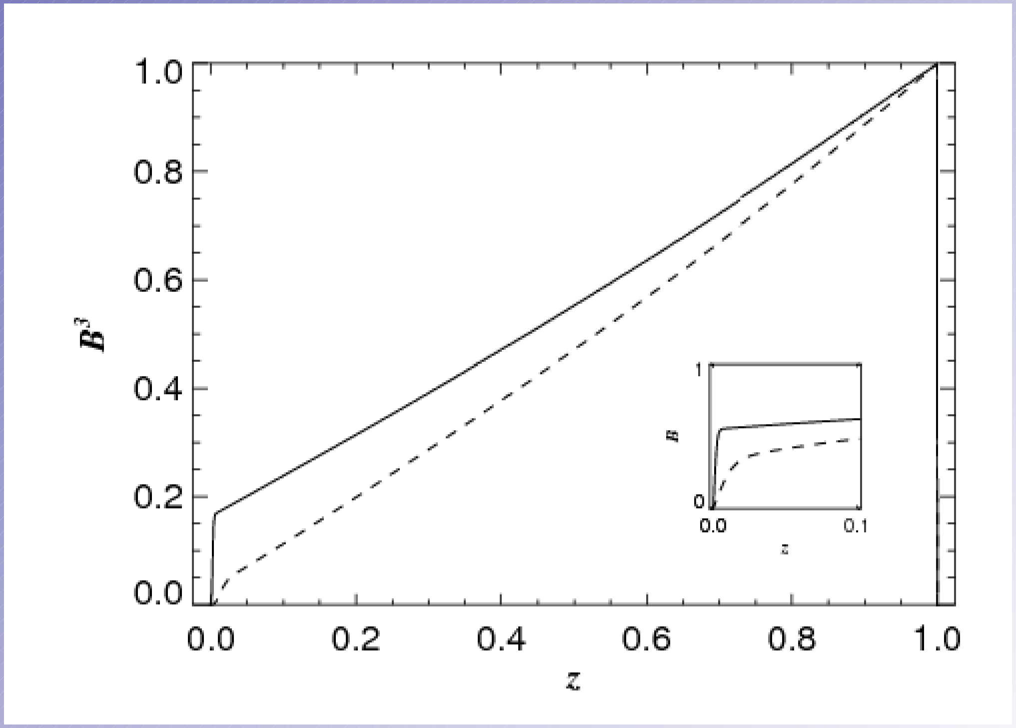
3. Comparison Sample

Diffusion singularity – The field approaches $B=z^{1/3}$, depending on the resistivity



Brandenburg & Zweibel 1995

Solved with Pencil's neutrals module



Summary and Conclusions: To do list

- Not solving for electron pressure (assume $p_e = p_i$)

- Works only for isothermal neutrals.

Have other entropy+eos files?

neutral_entropy.f90, neutral_eos_idealgas.f90?

And for electrons?

electron_velocity.f90 electron_density.f90, electron_entropy.f90?

Doesn't seem practical, buy maybe unavoidable

Summary and Conclusions:

It works!

But can be made more general.