Long life
to quantum correlations!

Sabrina Maniscalco
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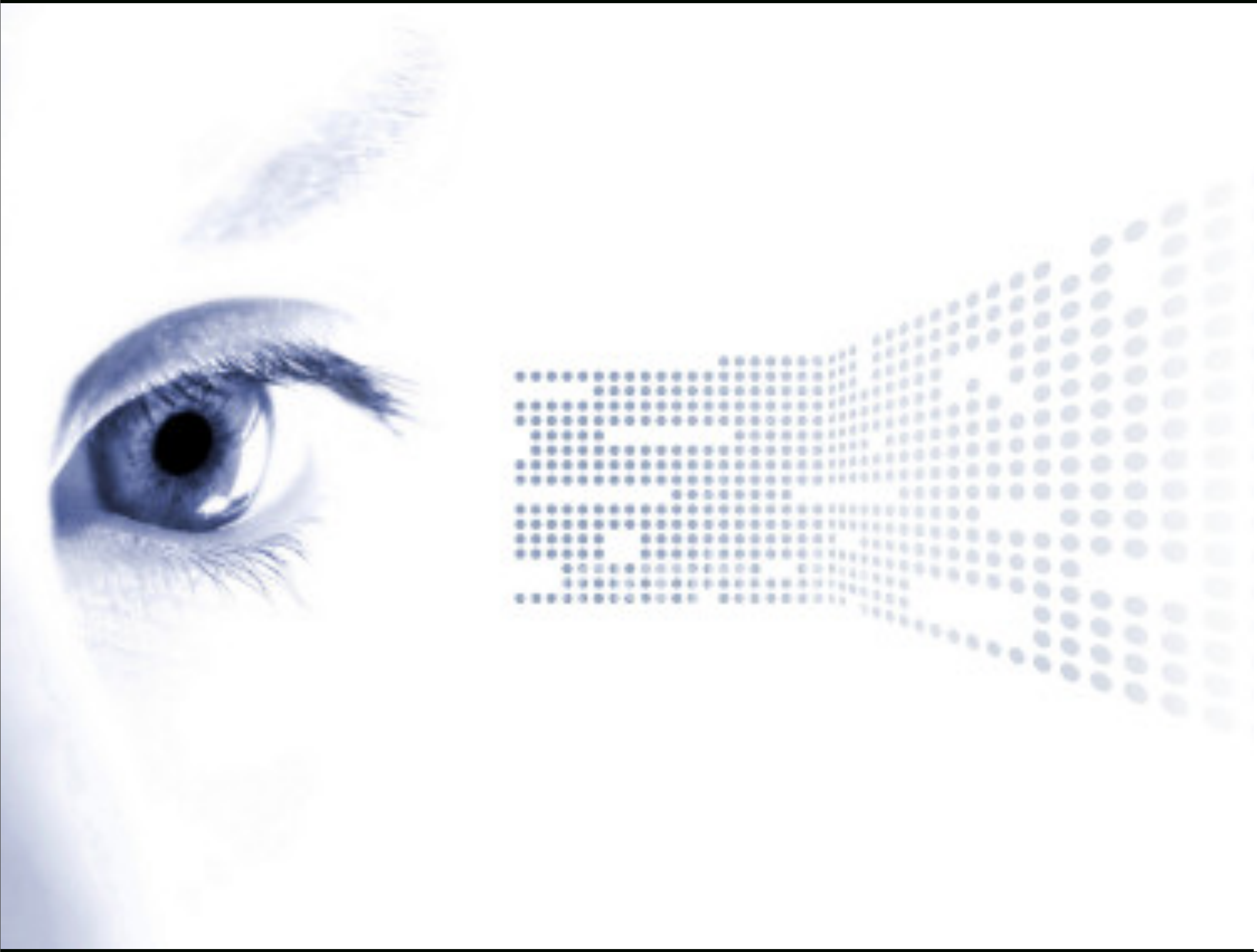
Finland

TCQP

OQS&E

SW ON ENTANGLEMENT
SCIENCE WORKSHOP ON ENTANGLEMENT



change in paradigm

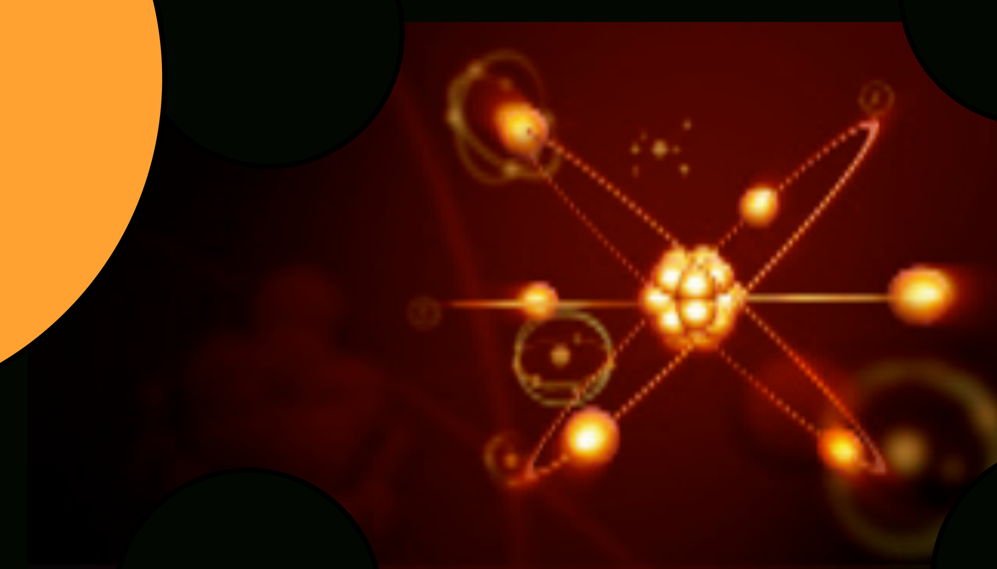
"Quantum mechanics is very much more than just a theory, it is a ***completely new way of looking at the world***, involving a change in paradigm perhaps more radical than any other in the history of human thought"

A. J. Leggett
Nobel Prize in Physics 2003

cosmology

laser

human vision

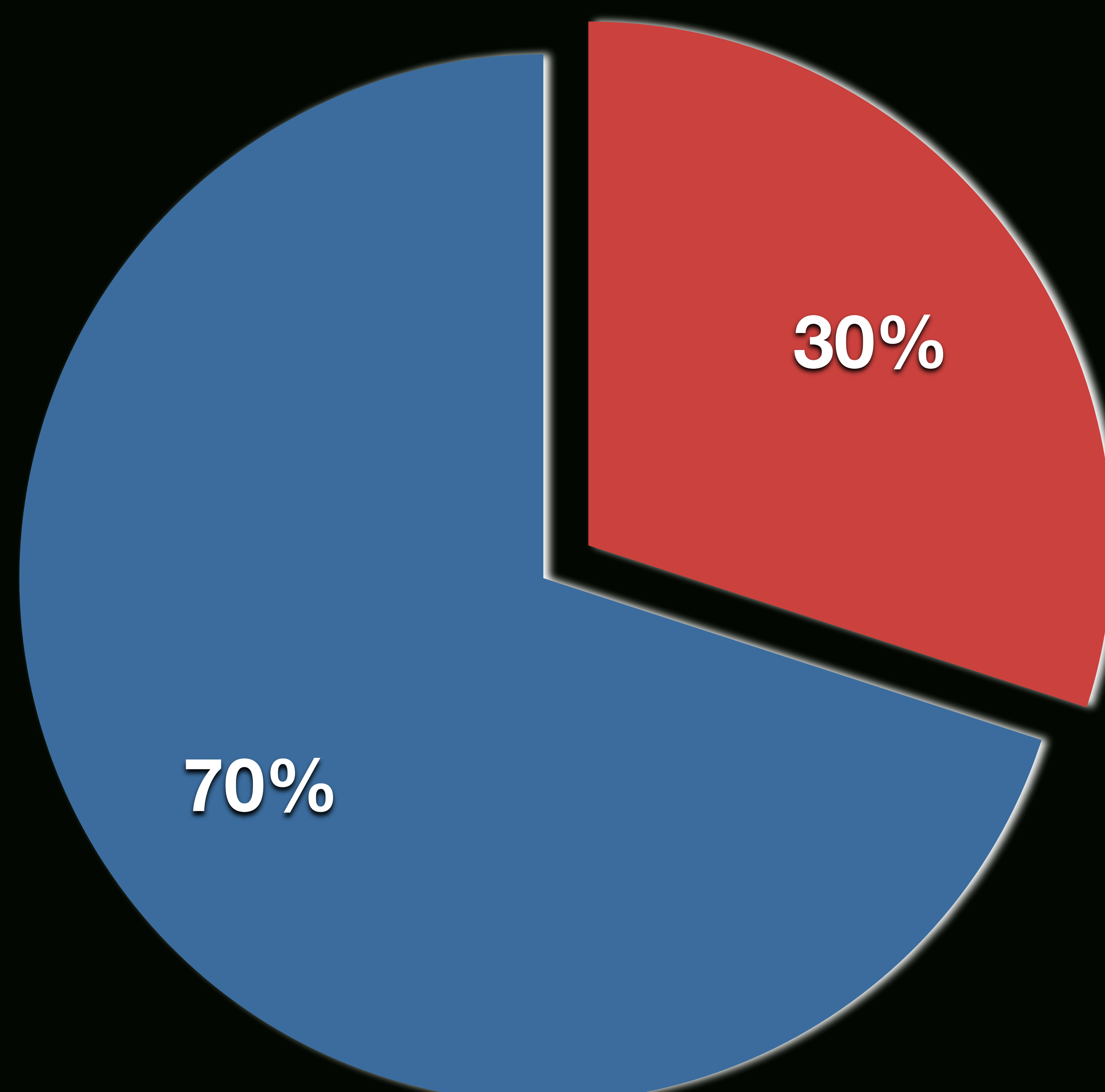


nucleus

supercon ductors

30%

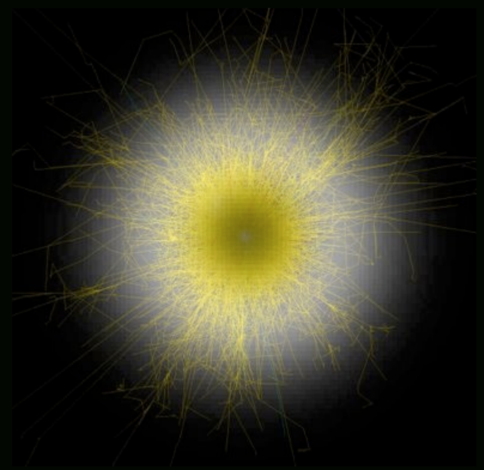
US GROSS NATIONAL PRODUCT



Tegmark and Wheeler **2001**


100

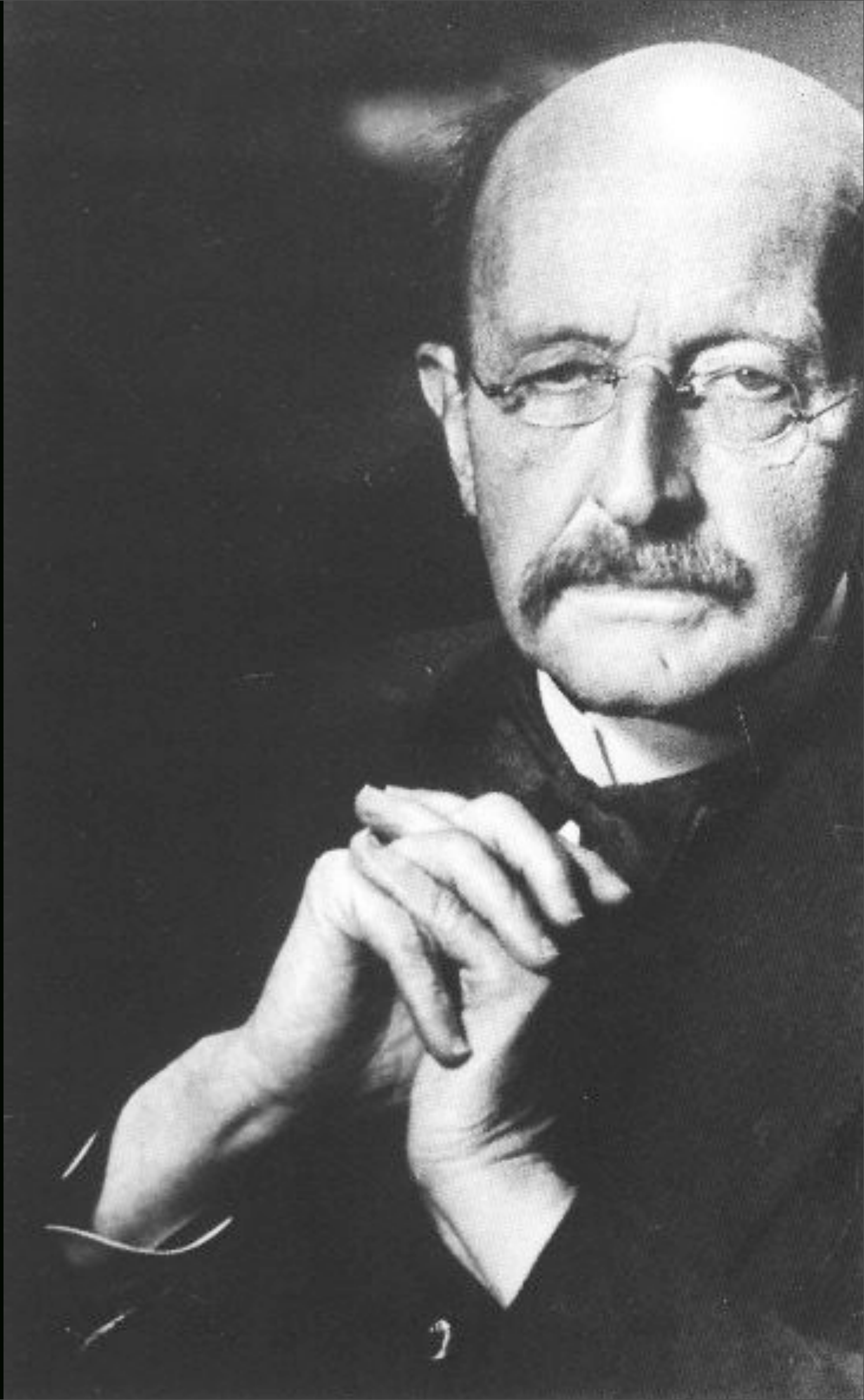
1905

Einstein's **quantum** of light 

THE BIRTH OF QUANTUM THEORY

"It was an act of desperation."

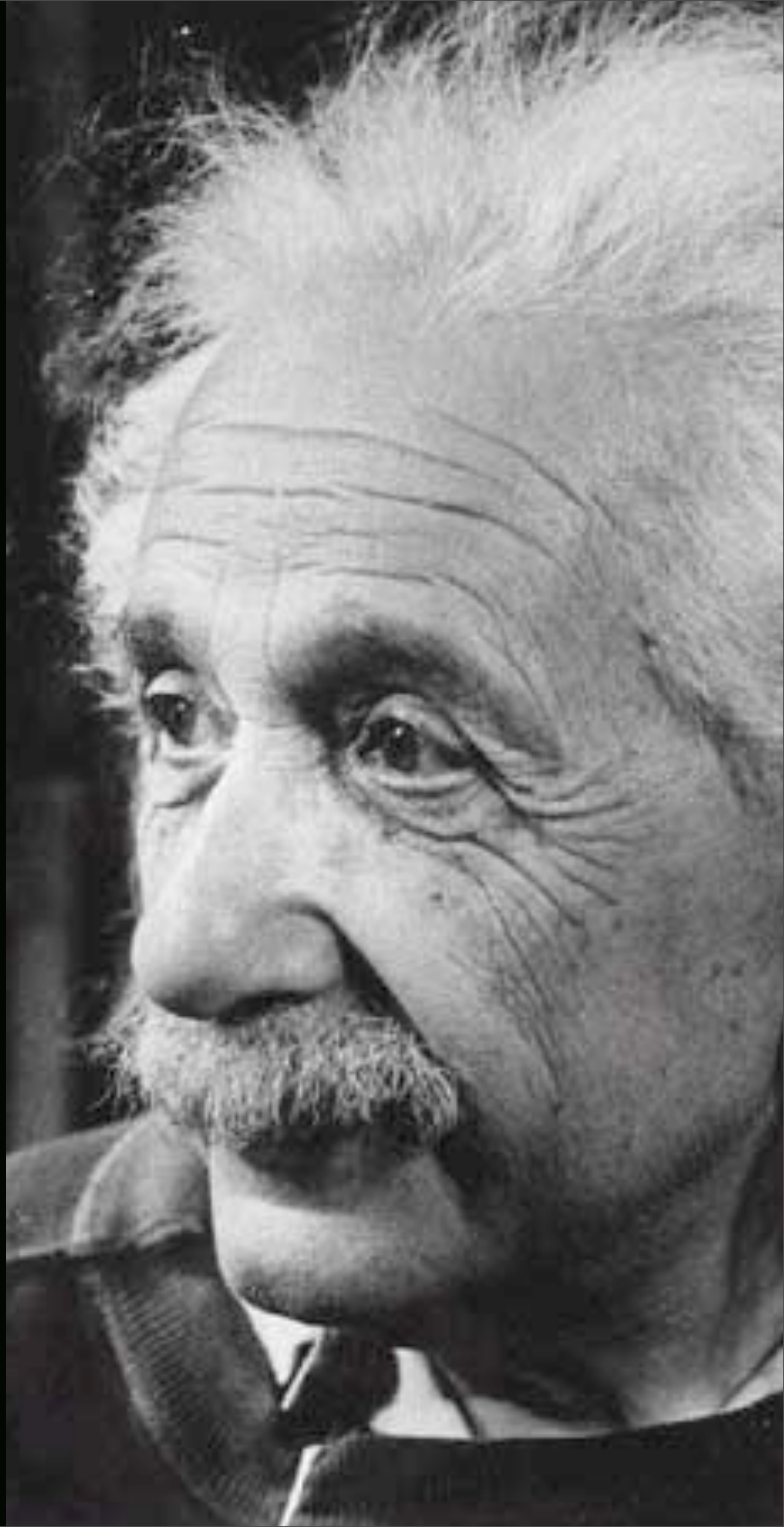
Max Planck



PROBABILISTIC NATURE

"God does not play dice with the Universe."

Albert Einstein



SINGLE ATOMS

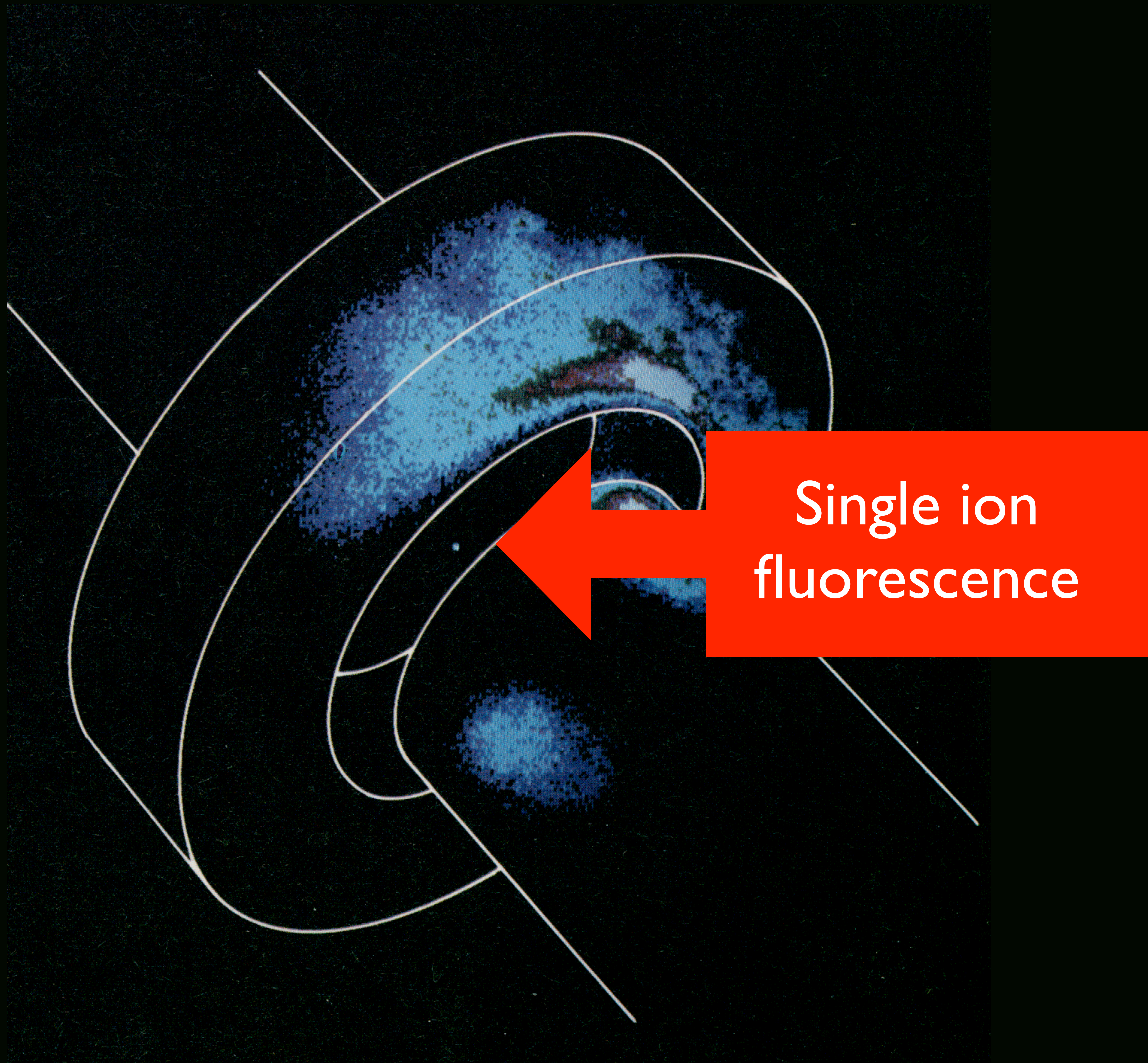
“We never experiment with just one atom, in thought experiments we sometimes assume that we do, this invariably entails ridiculous consequences”

Erwin Schrödinger

QUANTUM JUMPS

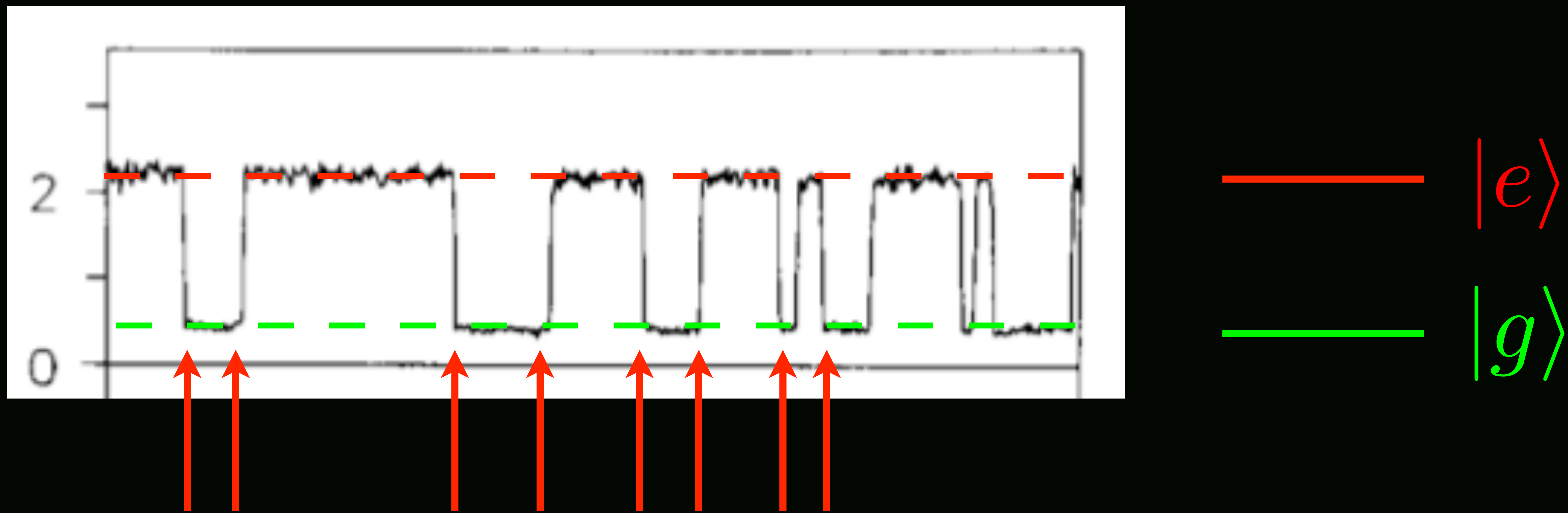
"If we are still going to put up with these damn quantum jumps, I am sorry that I ever had anything to do with quantum theory"

Erwin Schrödinger



W Neuhauser, M Hohenstatt, PE Toschek, H Dehmelt **1980**

SINGLE ION FLUORESCENCE



QUANTUM JUMPS

Th. Sauter, W. Neuhauser, R. Blatt, and P. E. Toschek

1986

Entanglement

Erwin Schrödinger

"***The*** characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought"

Bank robbery

Bank robbery

The gun has not fired

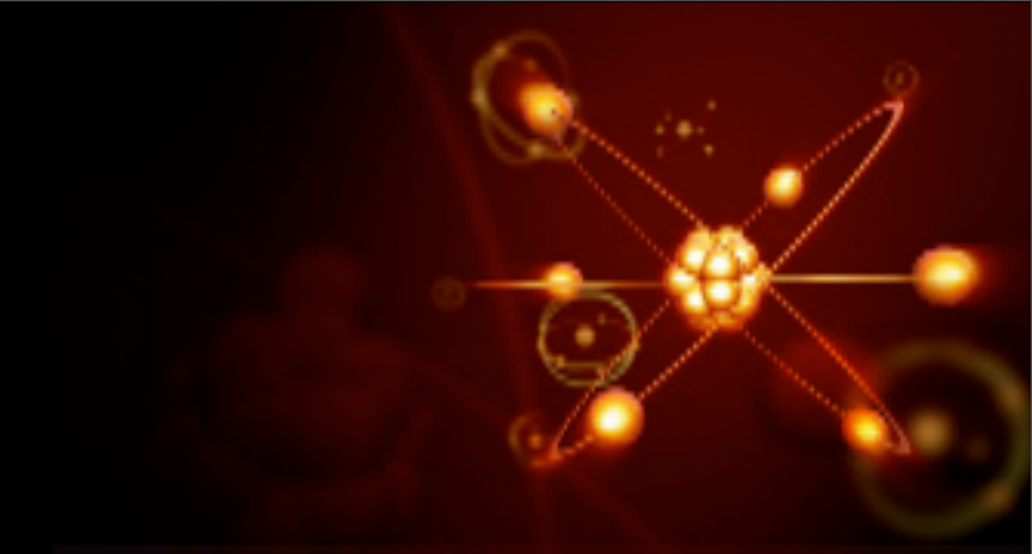
The teller is alive

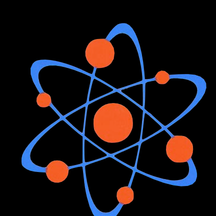
The gun has fired

The teller is dead

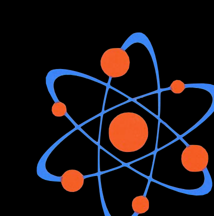
Quantum Bank robbery

Quantum Bank robbery

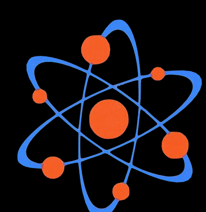


|  decayed ⟩ |  dead ⟩

+

|  not decayed ⟩ |  alive ⟩

Quantum Bank robbery

|  decayed ⟩ |  dead ⟩

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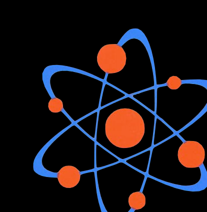
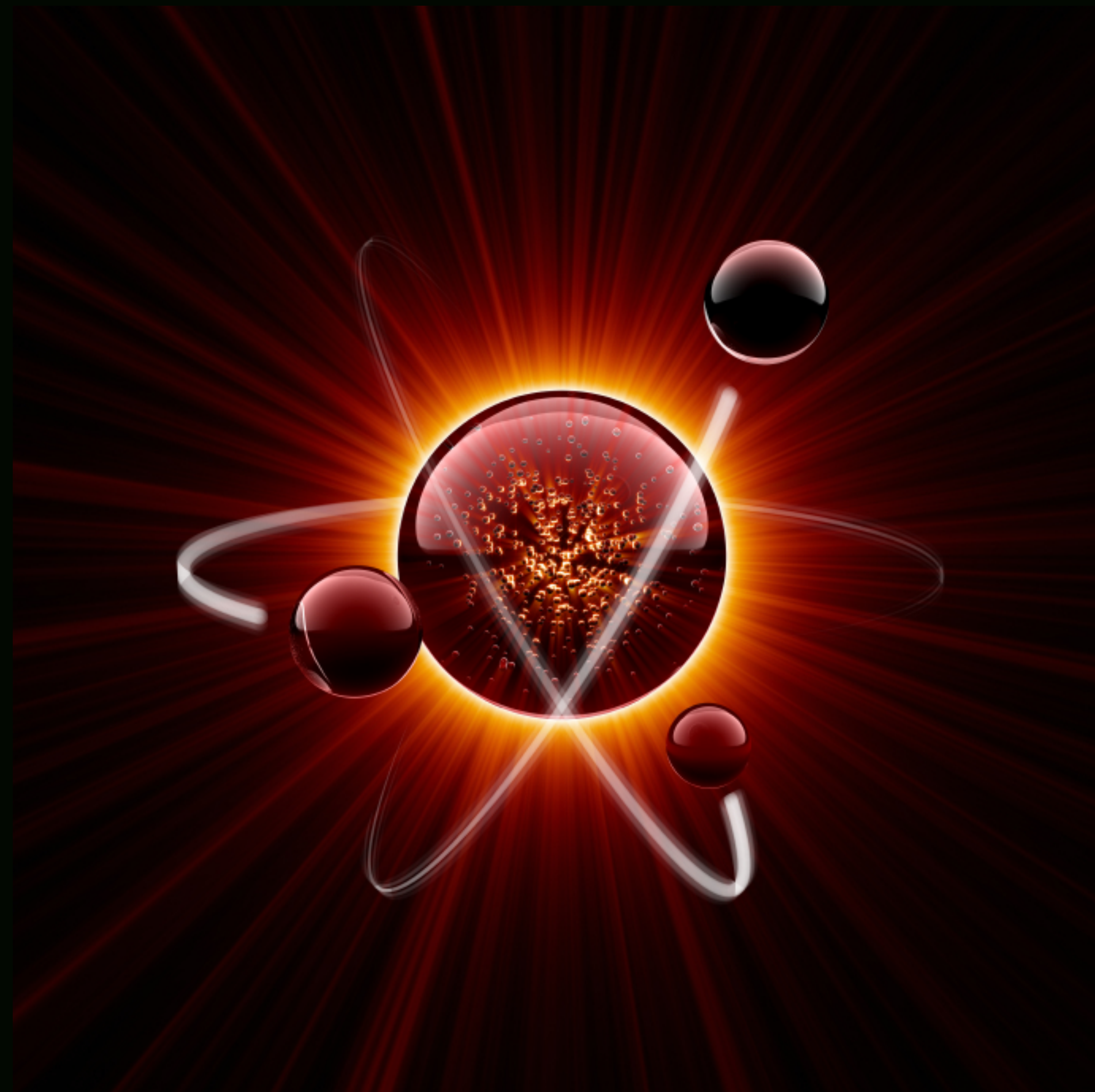
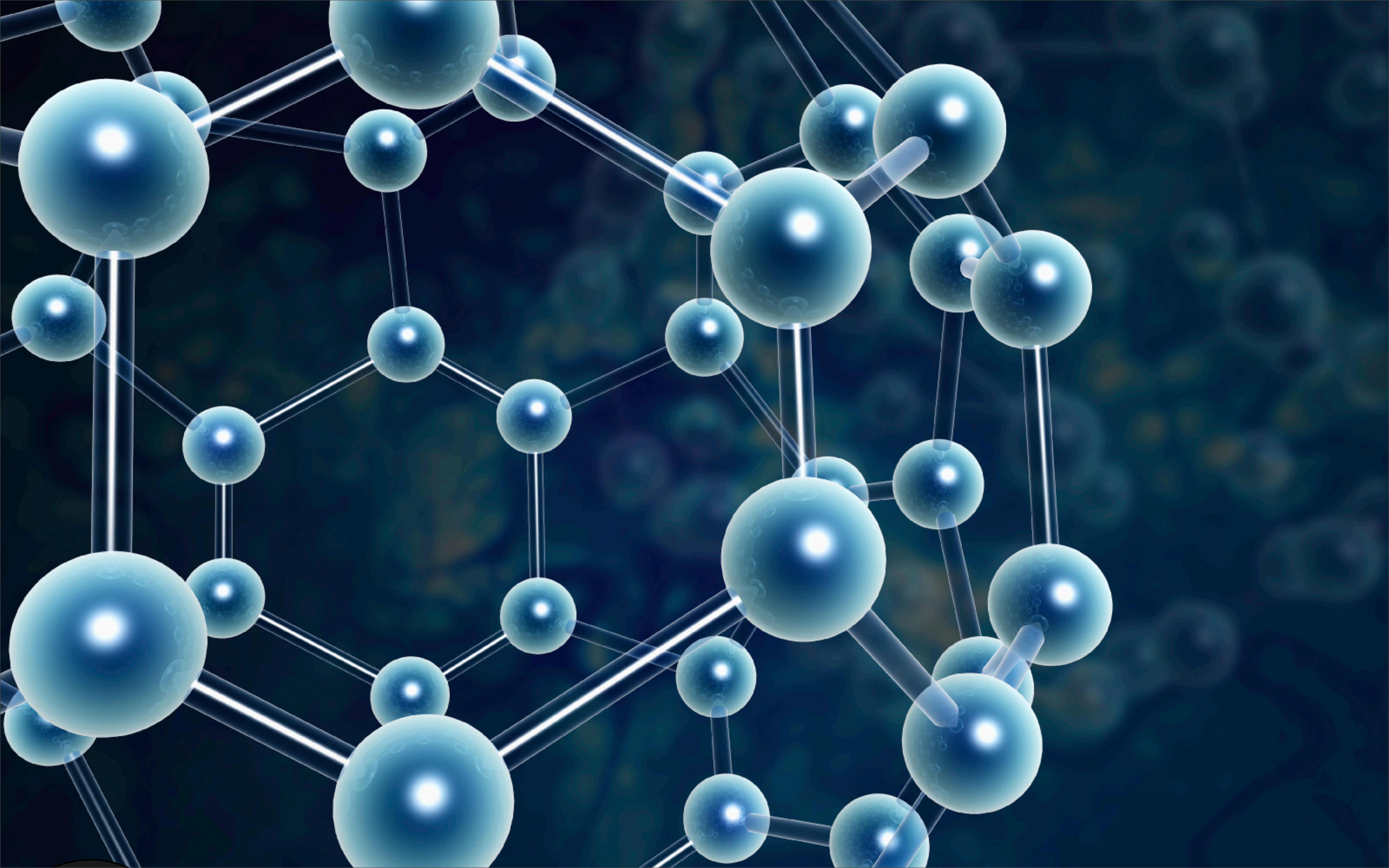
|  not decayed ⟩ |  alive ⟩

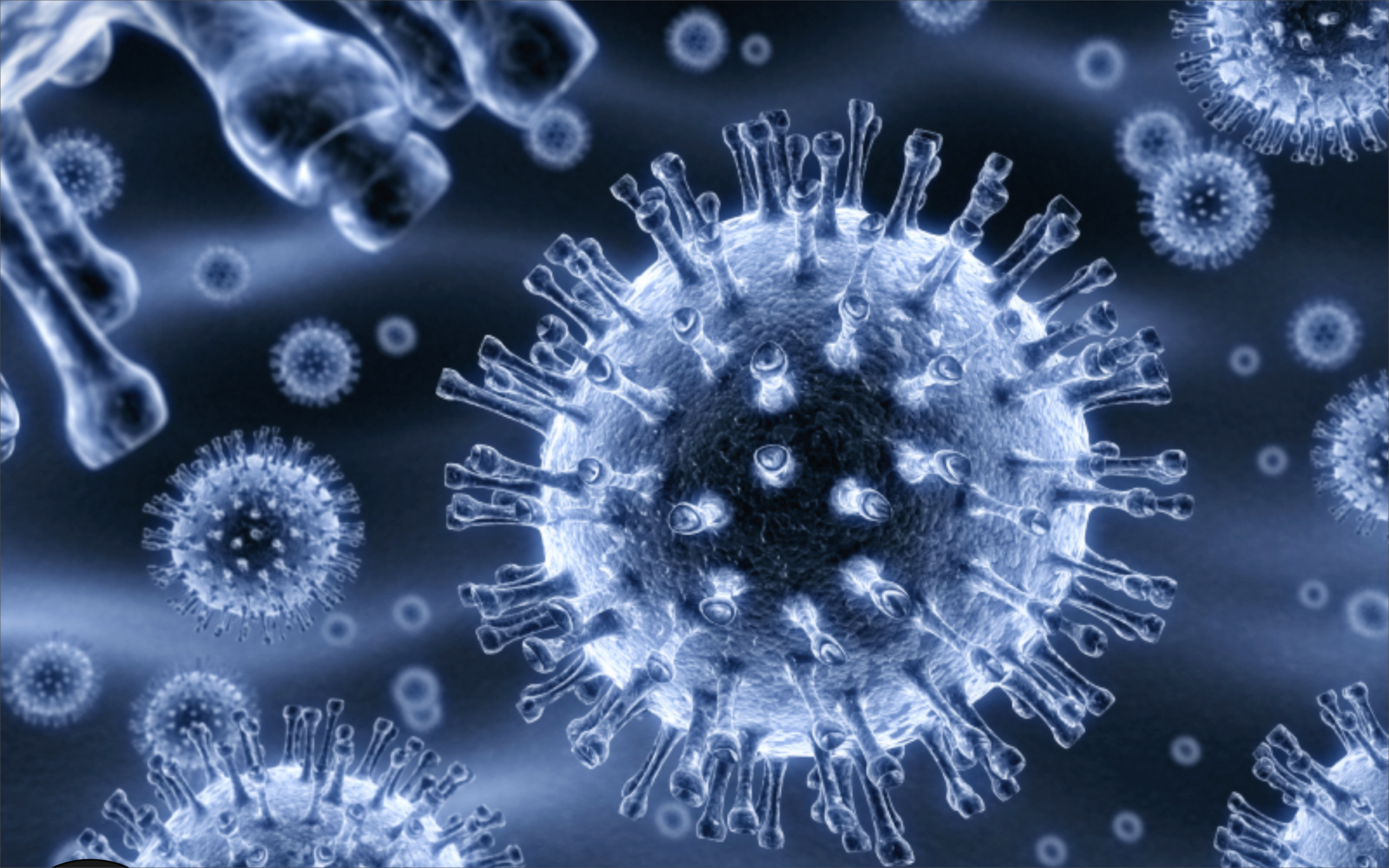
ENTANGLEMENT

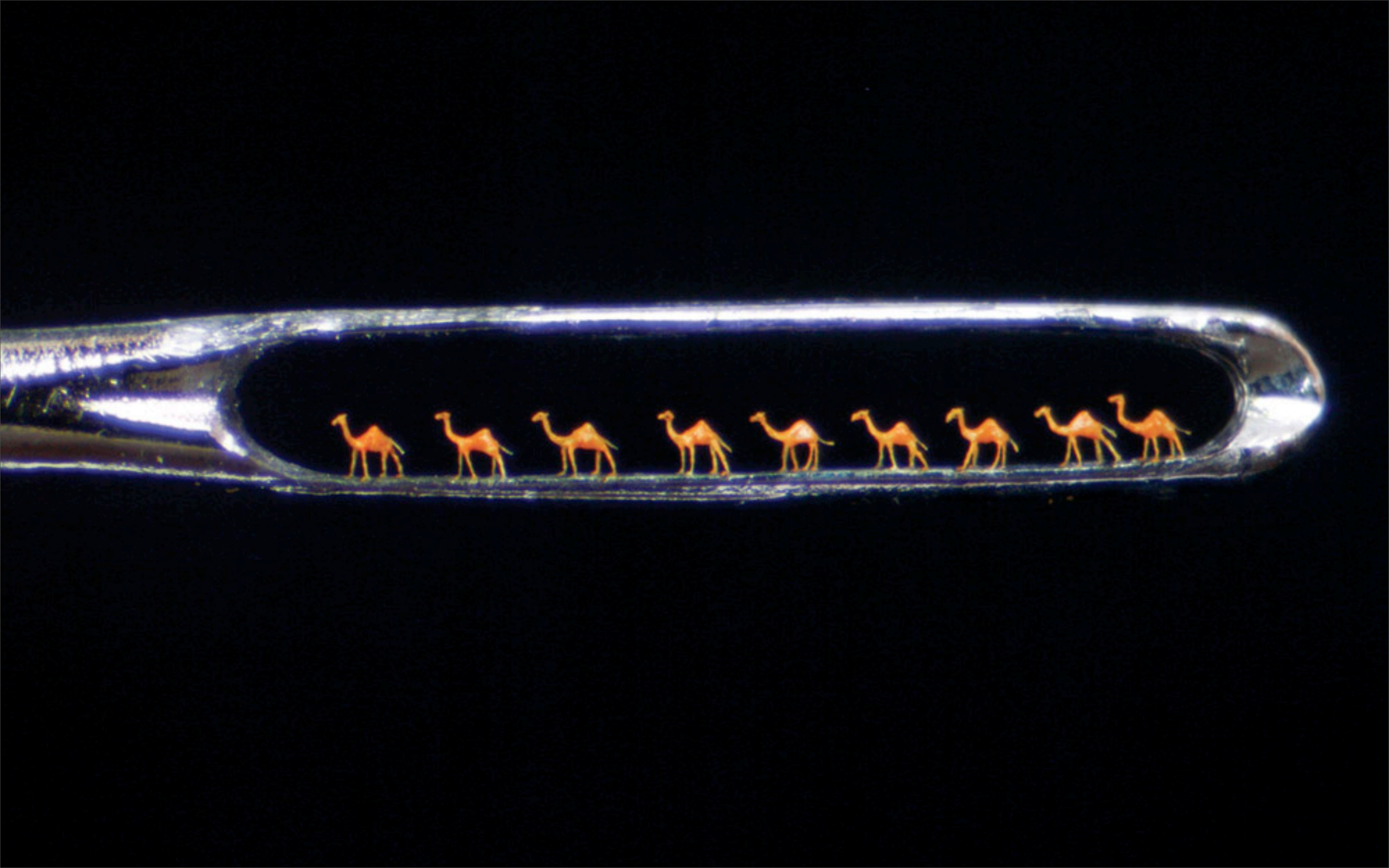
QUANTUM COMPUTERS

ENVIRONMENT

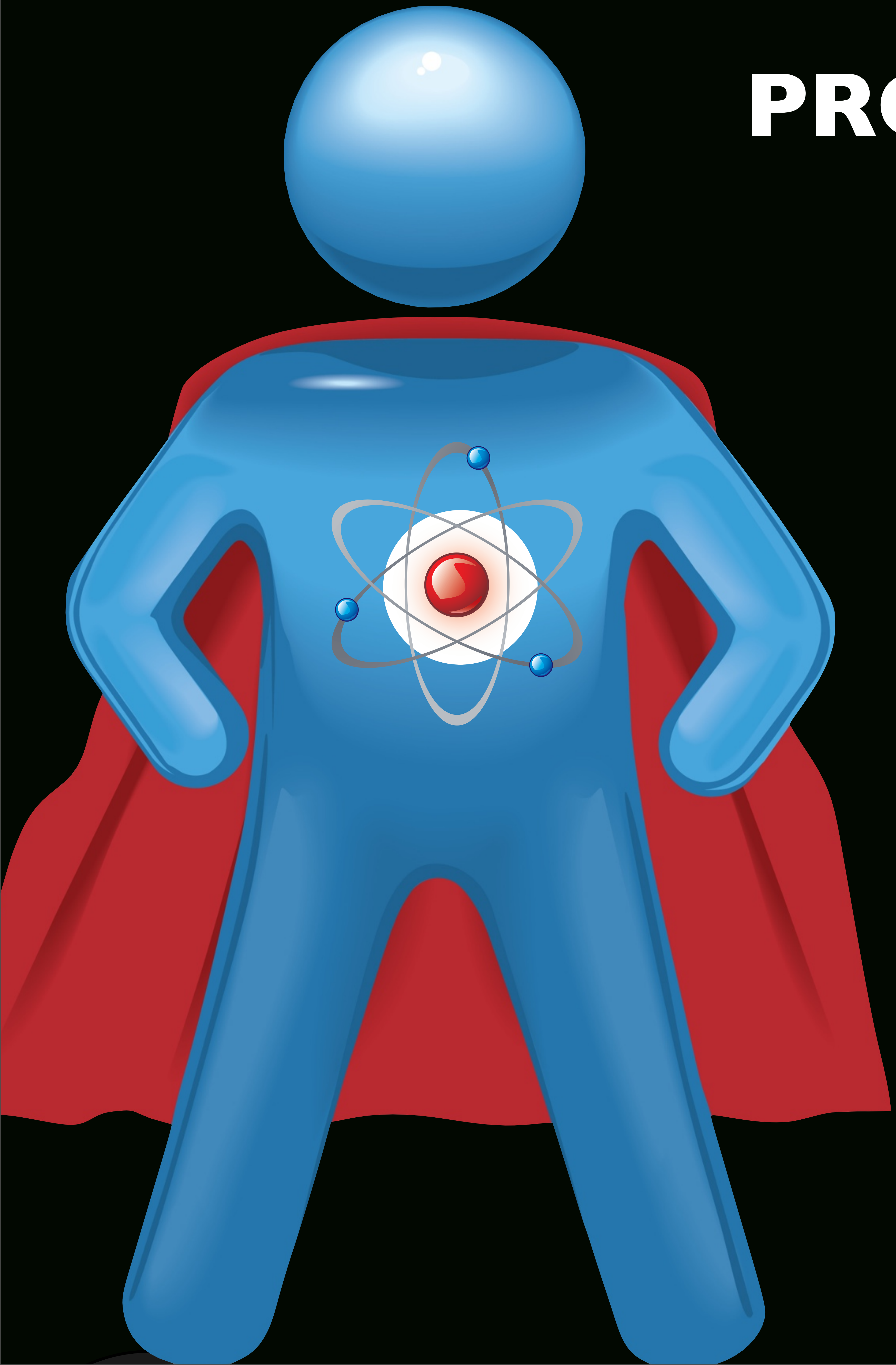




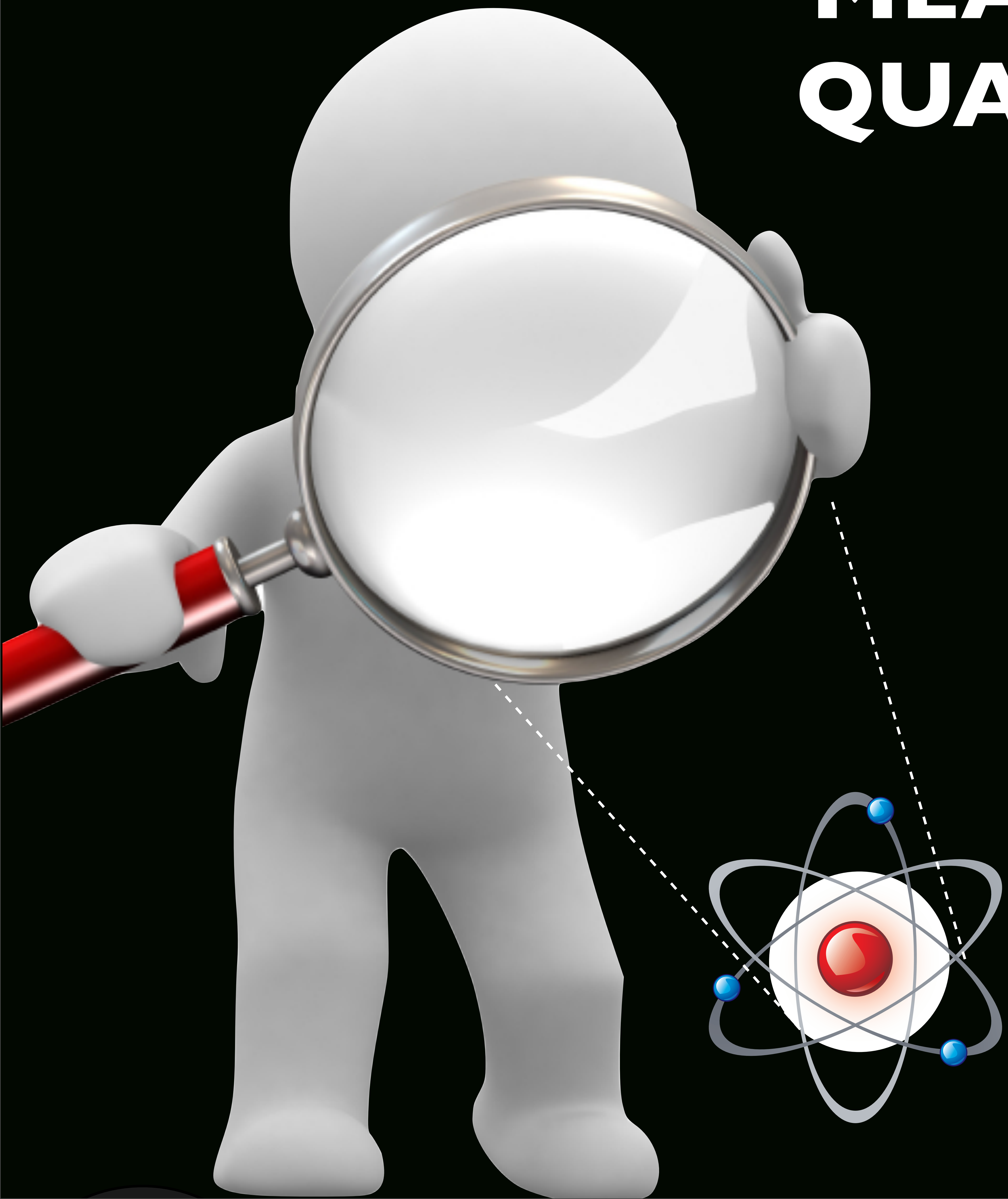




PROTECTING THE QUANTUM



MEASUREMENTS AFFECT QUANTUM SYSTEMS



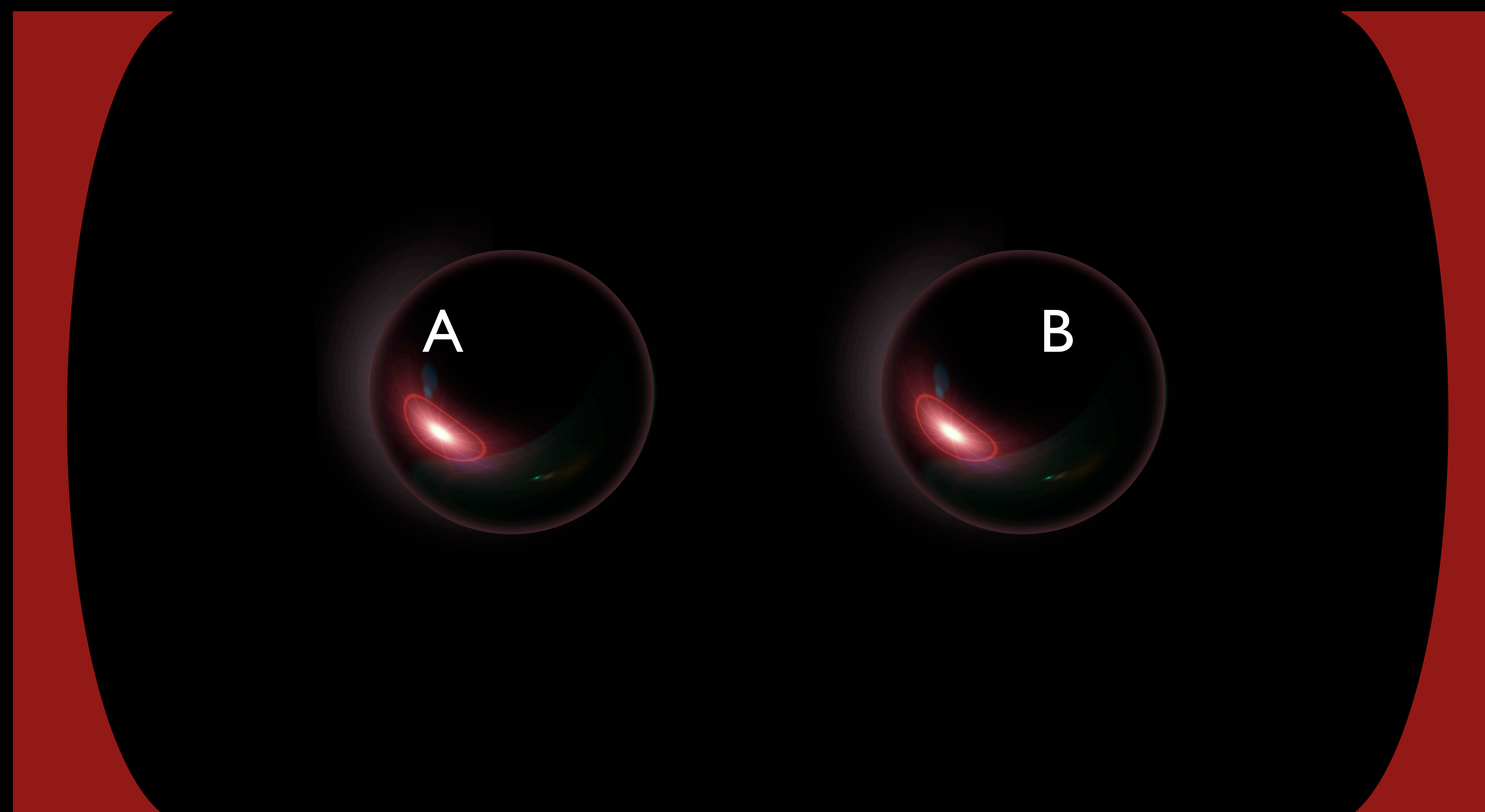
Quantum Zeno effect

allows to protect quantum properties

S. Maniscalco, J. Piilo, K.-A. Suominen
Phys. Rev. Lett. 97, 130402 (2006)

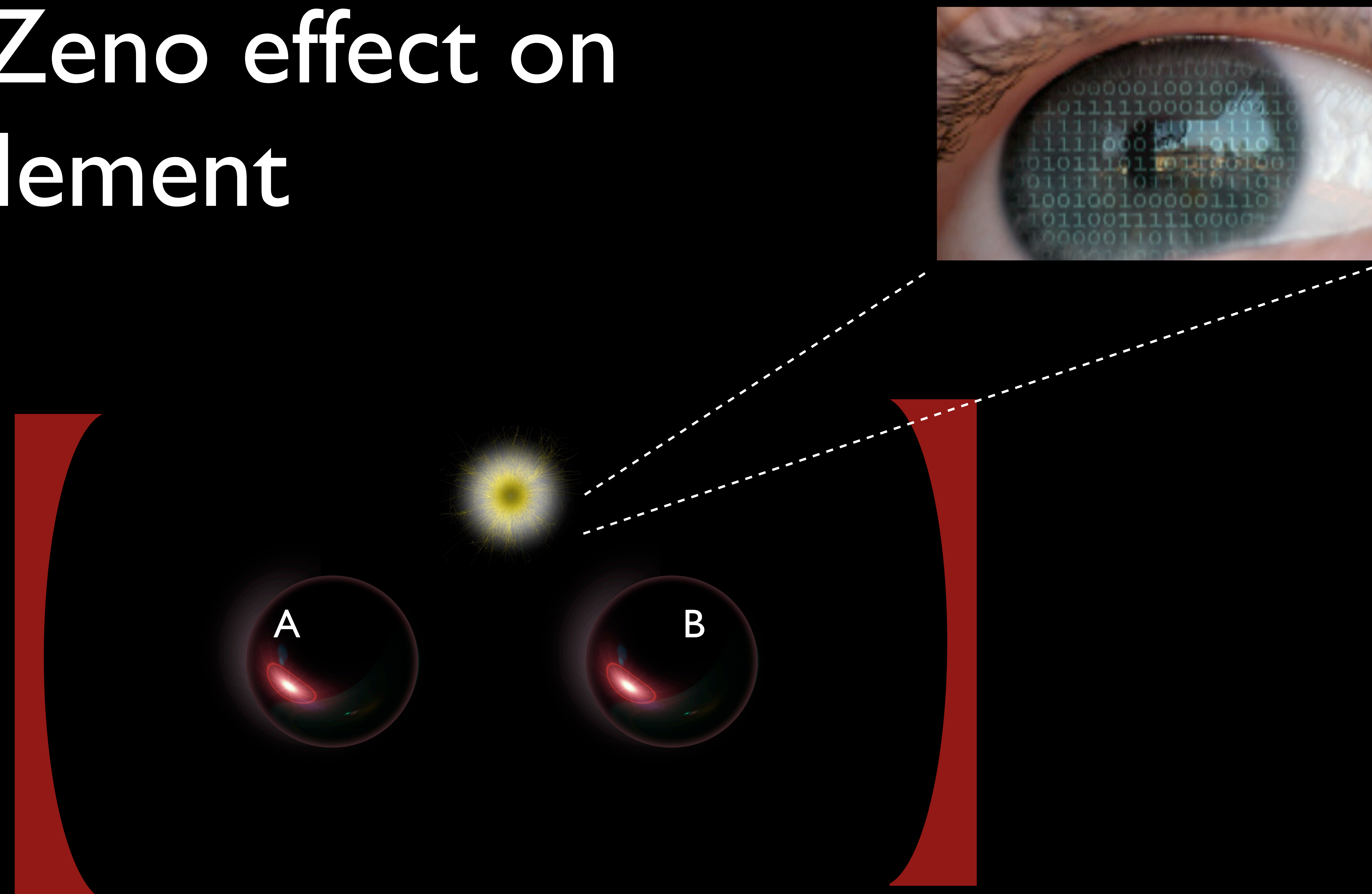
S. Maniscalco, F. Francica, R. L. Zaffino, N. Lo Gullo, and F. Plastina, Phys. Rev. Lett. 100, 090503 (2008)

Entangled atoms in a cavity



$$|e\rangle_A|g\rangle_B + |g\rangle_A|e\rangle_B$$

Quantum Zeno effect on the entanglement



Quantum Zeno Effect is **difficult** to observe in the experiments

Non-**M**arkovian environment

J. Piilo, S. Maniscalco, and K.-A. Suominen, Phys. Rev. Lett 100, 180402 (2008)

ARE REALLY
ALL
QUANTUM PROPERTIES DESTROYED BY THE ENVIRONMENT

OPEN QUANTUM SYSTEMS AND ENTANGLEMENT

www.openq.fi

OQS&E

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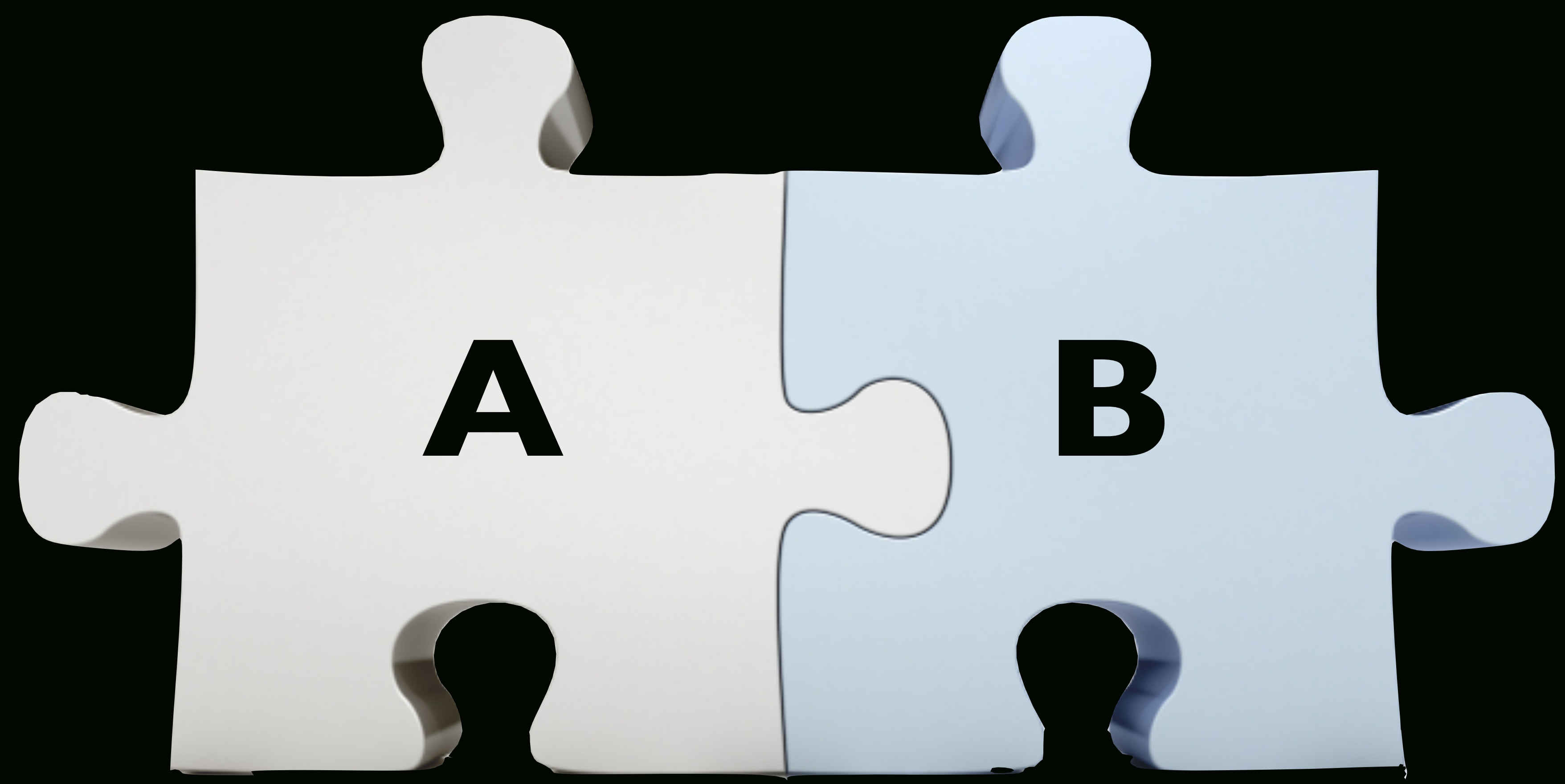
(April-September)
Francesco Francica
(March-April)

A new way of looking at quantum correlations

QUANTUM DISCORD

CLASSICAL INFORMATION THEORY

Mutual information is a measure of *correlations*



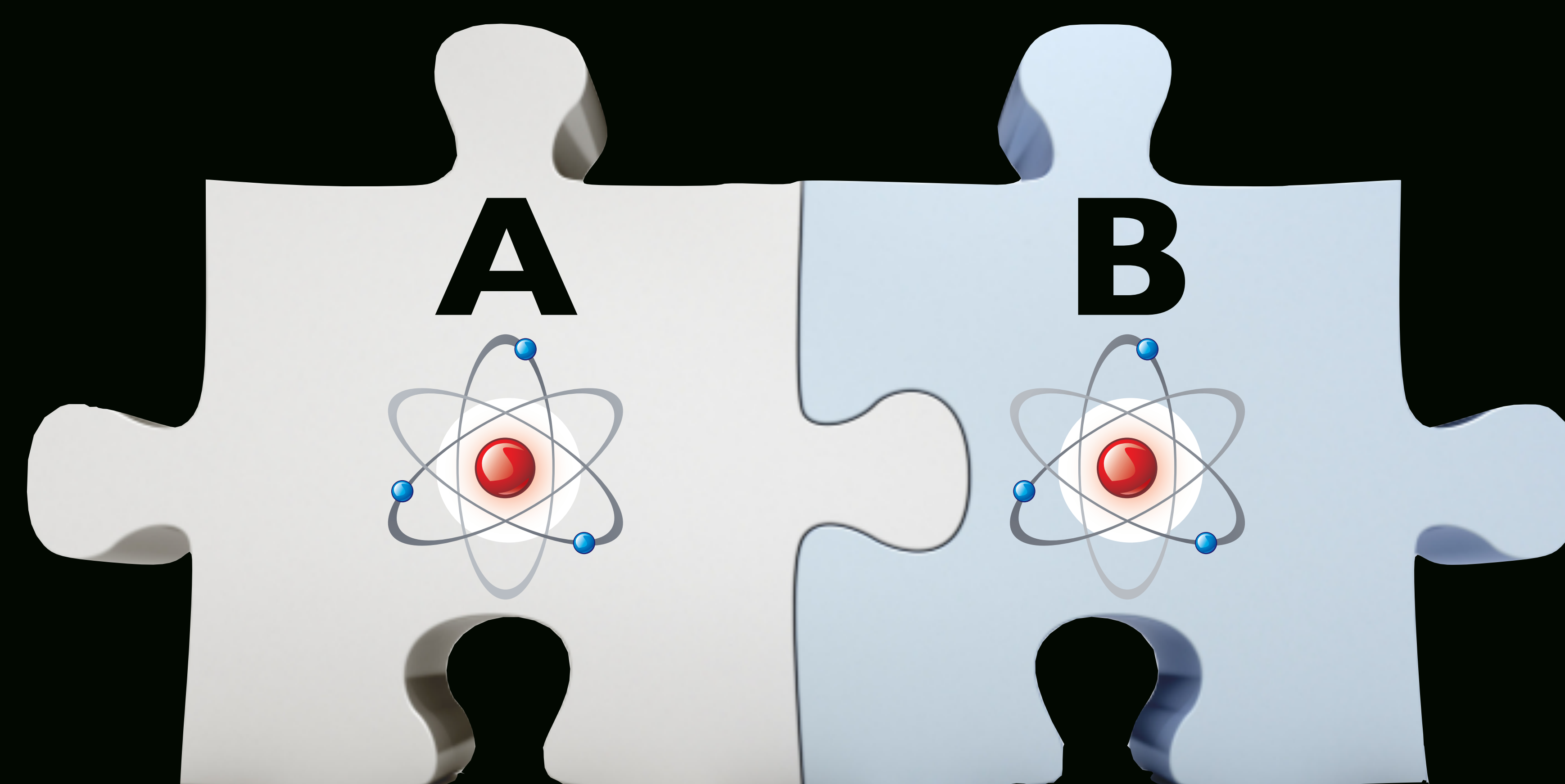
$$I(A:B) = H(A) + H(B) - H(A,B)$$

$$H(X) = -\sum_{x \in X} p(x) \log p(x)$$

Shannon entropy

QUANTUM INFORMATION THEORY

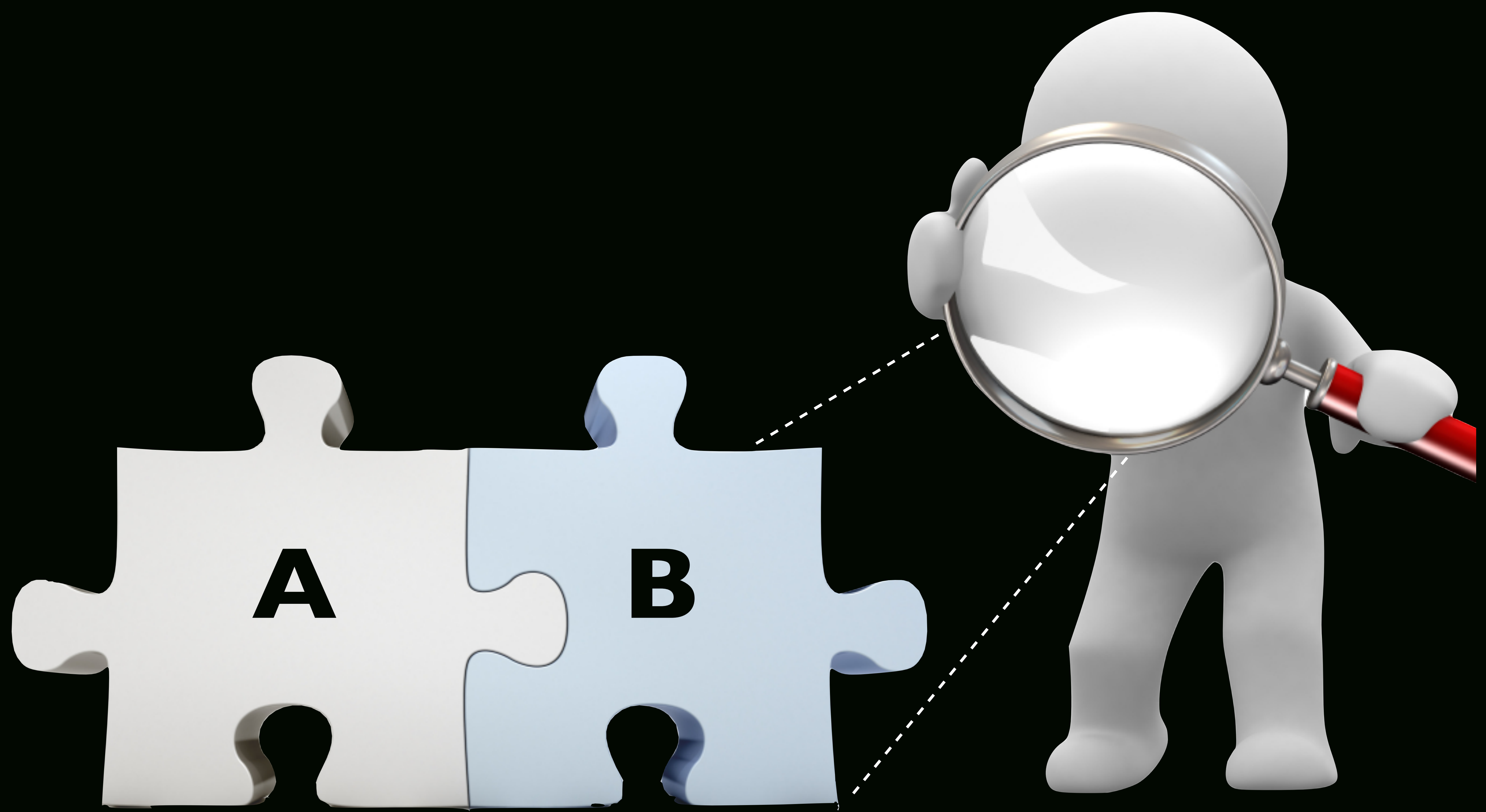
Quantum ~~*Mutual*~~ *information* is a measure of Quantum ~~*correlations*~~



$$\mathcal{I}(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$$

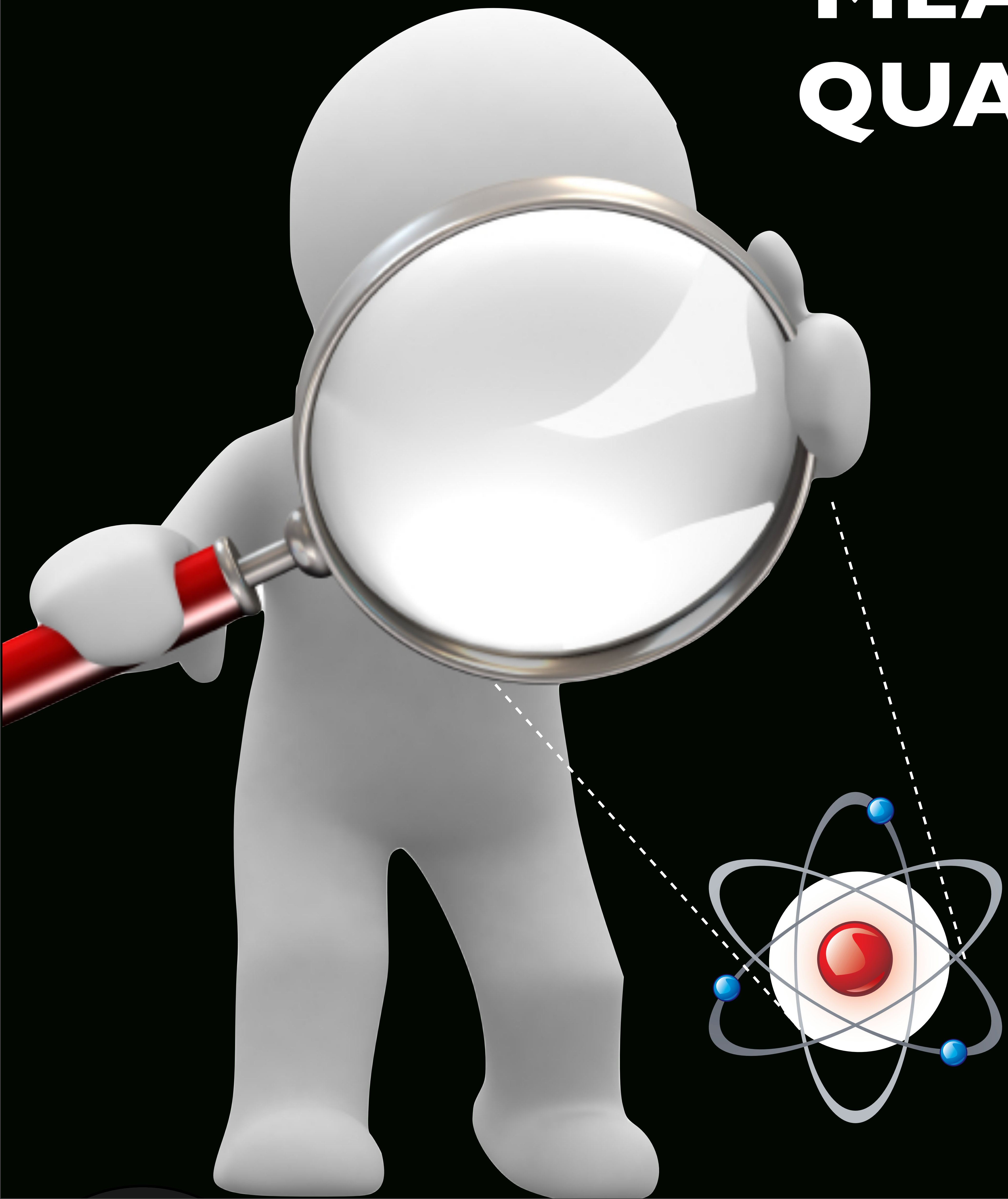
$$S(\rho) = -Tr\{\rho \log \rho\}$$ von Neumann entropy

CLASSICAL INFORMATION THEORY

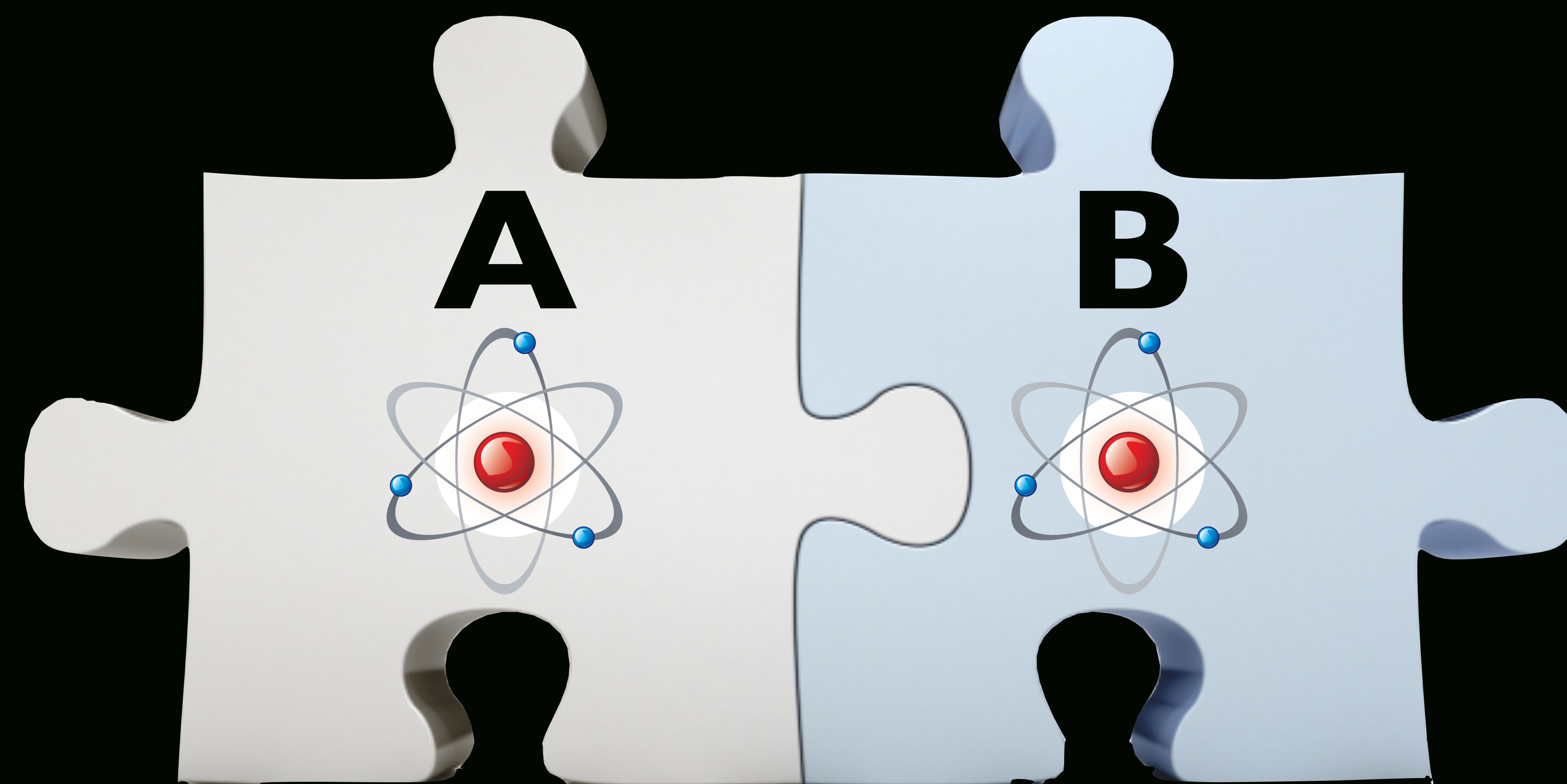


$$I(A:B) = H(A) + H(B) - H(A,B)$$

MEASUREMENTS AFFECT QUANTUM SYSTEMS



QUANTUM INFORMATION THEORY

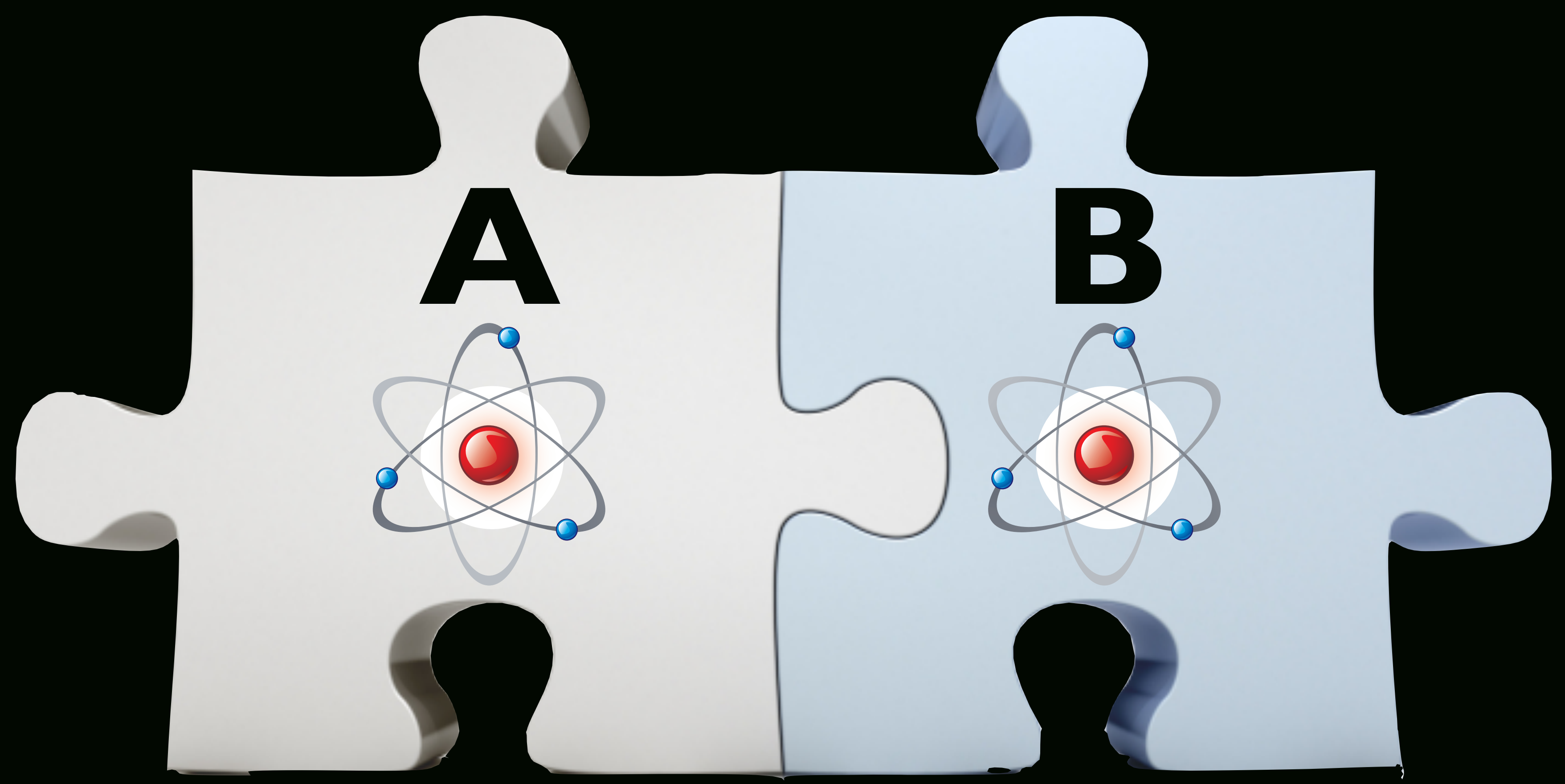


Classical correlations

$$\mathcal{C}(\rho_{AB}) = \quad [S(\rho_A) - S(\rho_{AB}|\{\Pi_k\})]$$

conditional entropy

QUANTUM INFORMATION THEORY

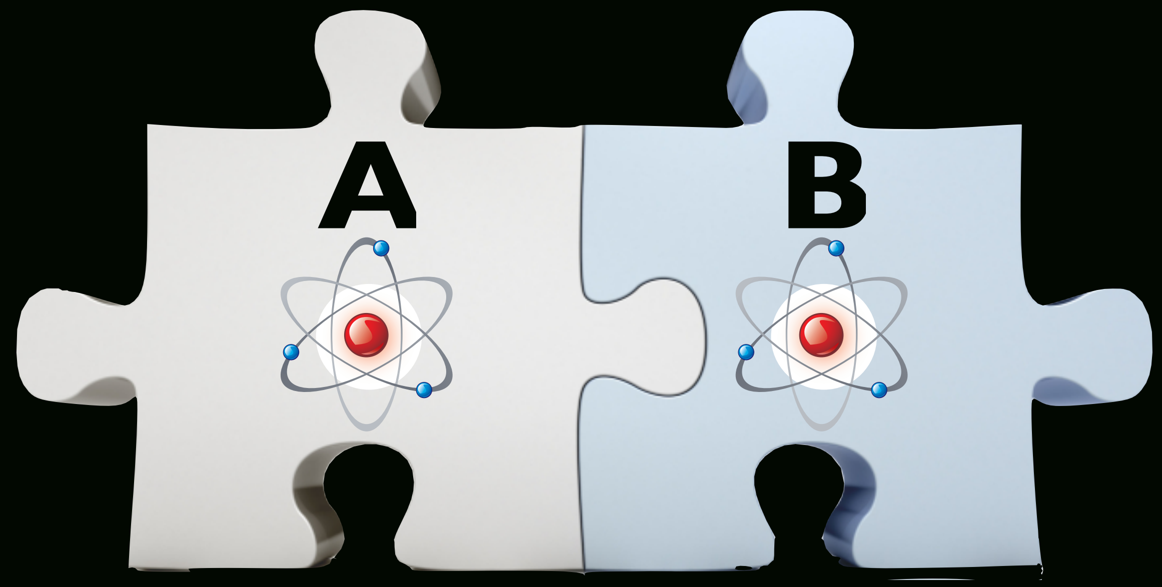


Classical correlations

$$\mathcal{C}(\rho_{AB}) = \max_{\{\Pi_k\}}[S(\rho_A) - S(\rho_{AB}|\{\Pi_k\})]$$

conditional entropy

Classically correlated states



$$\rho_{cl} = \sum_{kl} p_{kl} |k\rangle\langle k| \otimes |l\rangle\langle l| \qquad p_{kl} \neq p_k p_l$$

$$\mathcal{I}(\rho_{AB}) = \mathcal{C}(\rho_{AB})$$

QUANTUM DISCORD

Quantum Correlations

$$\mathcal{D}(\rho_{AB}) = \mathcal{I}(\rho_{AB}) - \mathcal{C}(\rho_{AB})$$

L. Henderson and V. Vedral, J. Phys. A: Math. Gen. 34, 6899 (2001)
H. Olivier and W. H. Zurek, Phys. Rev. Lett. 88, 017901 (2001)

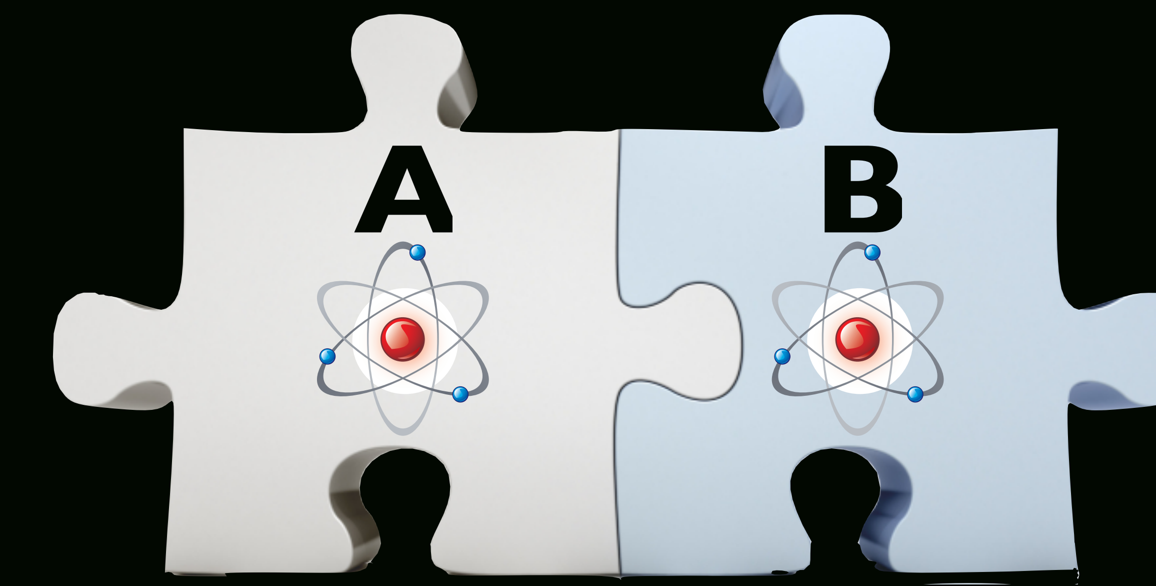
ENTANGLEMENT

=

DISCORD

FOR PURE STATES

FOR MIXED STATES



$$\rho_{AB} = 1/2(|0\rangle\langle 0|_A \otimes |0\rangle\langle 0|_B + |1\rangle\langle 1|_A \otimes |+\rangle\langle +|_B$$

$$|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$$

ENTANGLEMENT = 0

$$\mathcal{D}(\rho_{AB}) \neq 0$$

A. Datta, A. Shaji, and C. Caves, Phys. Rev. Lett. 100, 050502 (2008)

HOW **SENSITIVE** IS QUANTUM DISCORD TO THE ENVIRONMENT

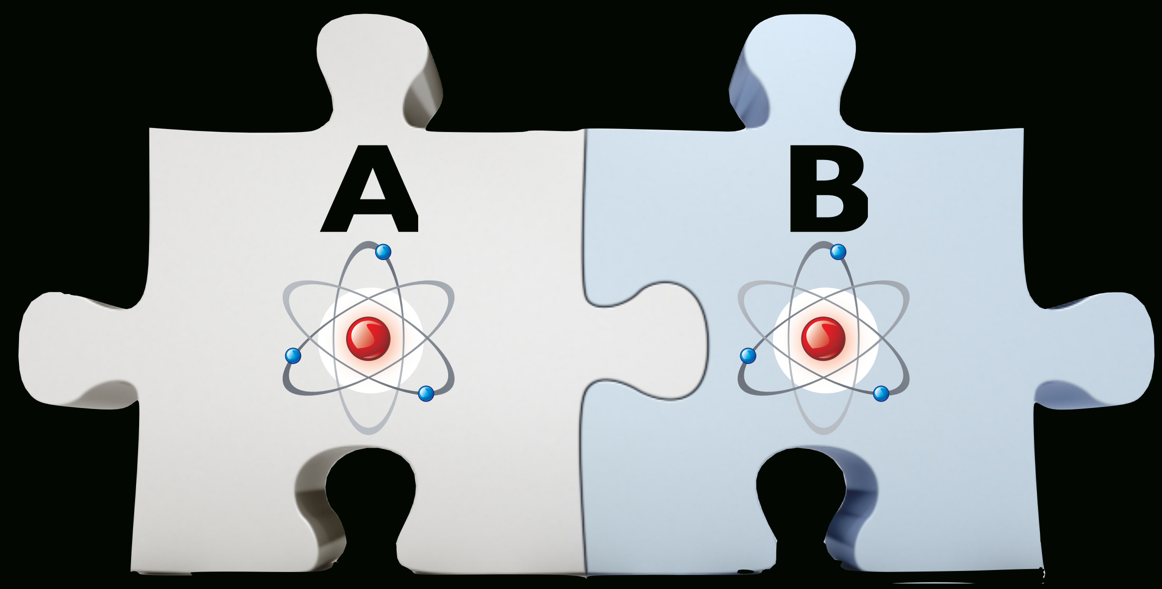
QUBIT 1

phase flip noise

QUBIT 2

phase flip noise

INITIAL STATES



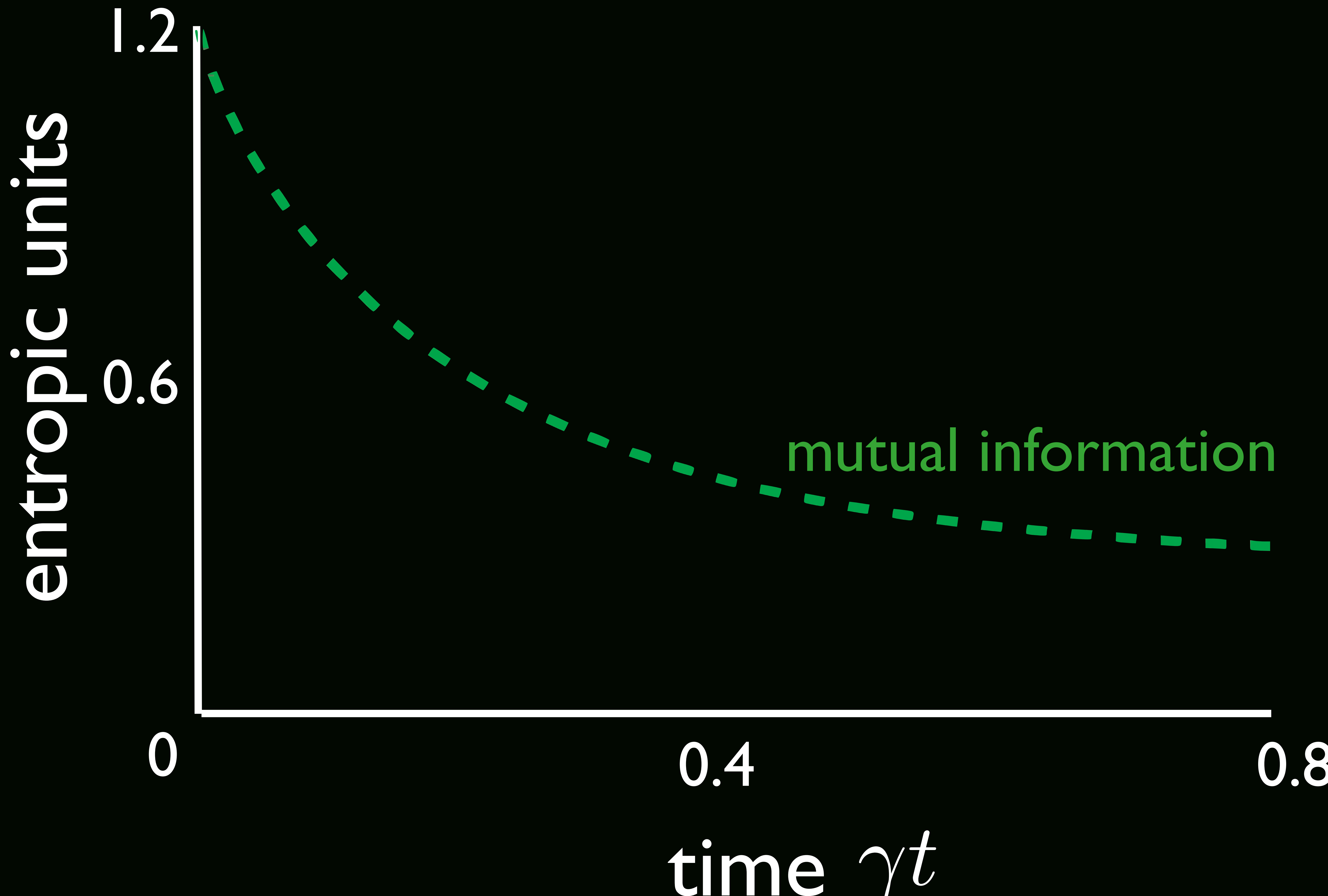
$$\rho_{AB} = \frac{(1+c_3)}{2}|\Psi^{\pm}\rangle\langle\Psi^{\pm}| + \frac{(1-c_3)}{2}|\Phi^{\pm}\rangle\langle\Phi^{\pm}|$$

$$|c_3| < 1$$

$$\begin{aligned} |\Psi^{\pm}\rangle(t) &= (|00\rangle \pm |11\rangle)/\sqrt{2} \\ |\Phi^{\pm}\rangle(t) &= (|01\rangle \pm |10\rangle)/\sqrt{2} \end{aligned}$$

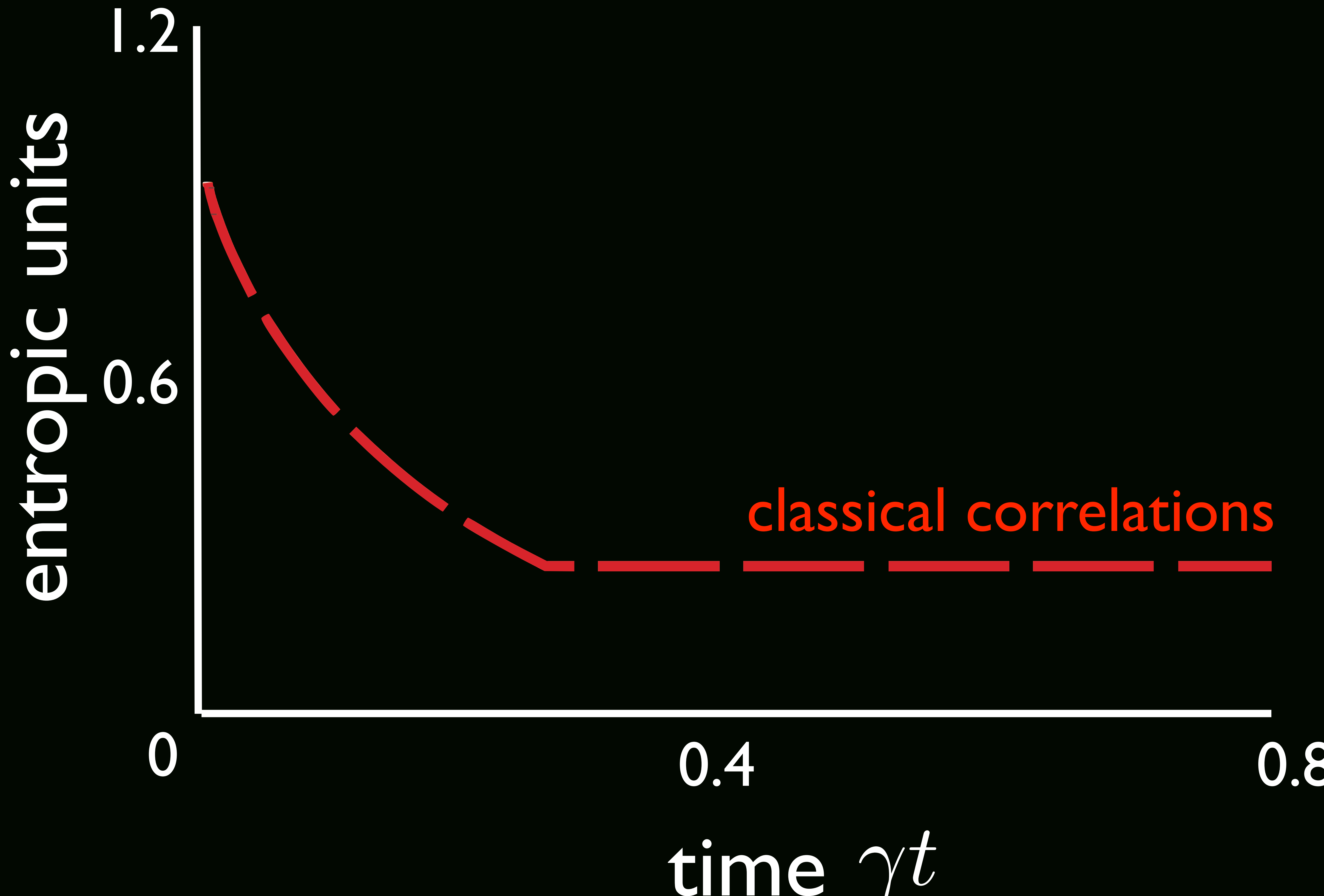
Bell states

DYNAMICS



L. Mazzola, J. Piilo, and S. Maniscalco, arXiv: 1001.5441 (2010), in press in Phys. Rev Lett.

DYNAMICS



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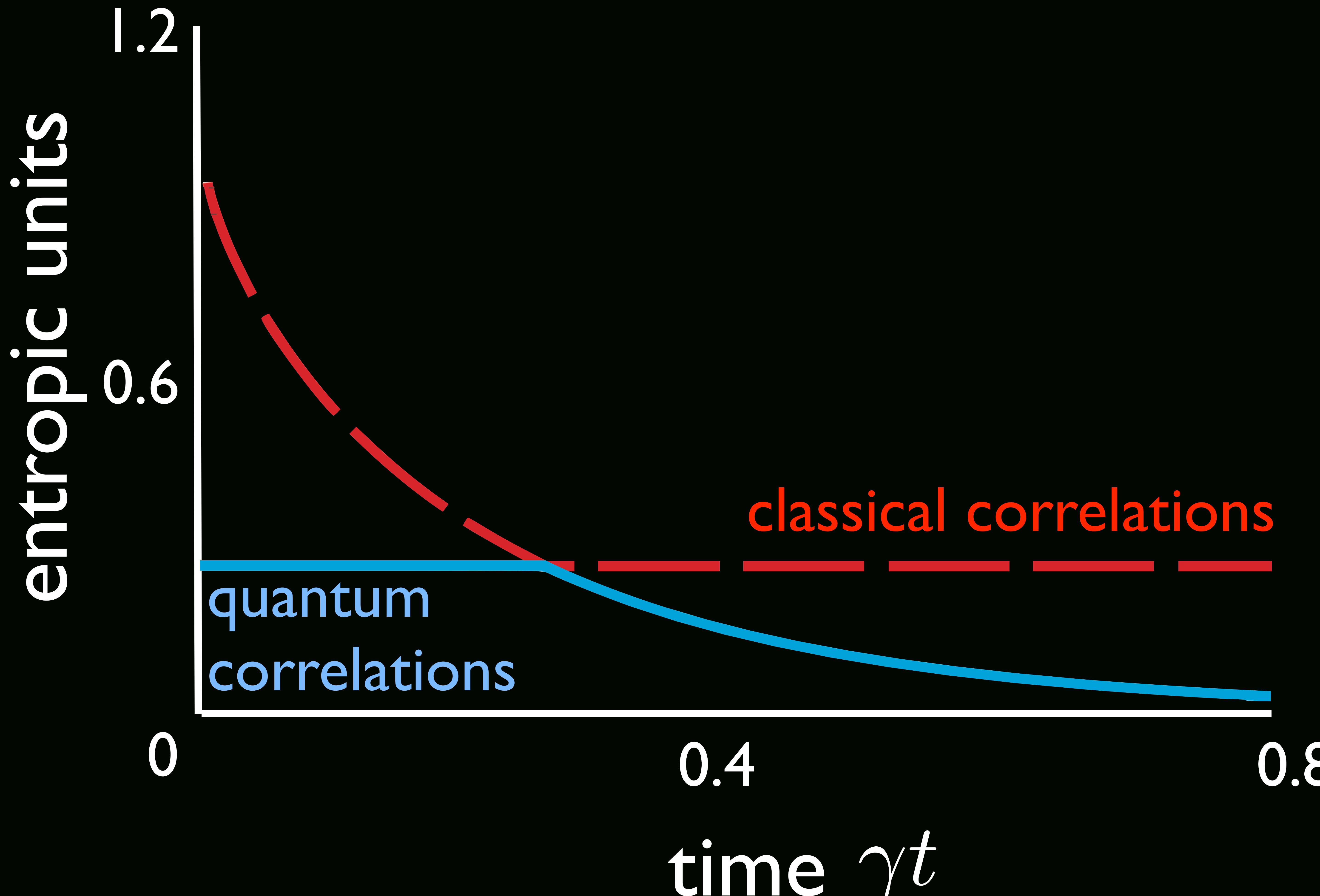
DYNAMICS



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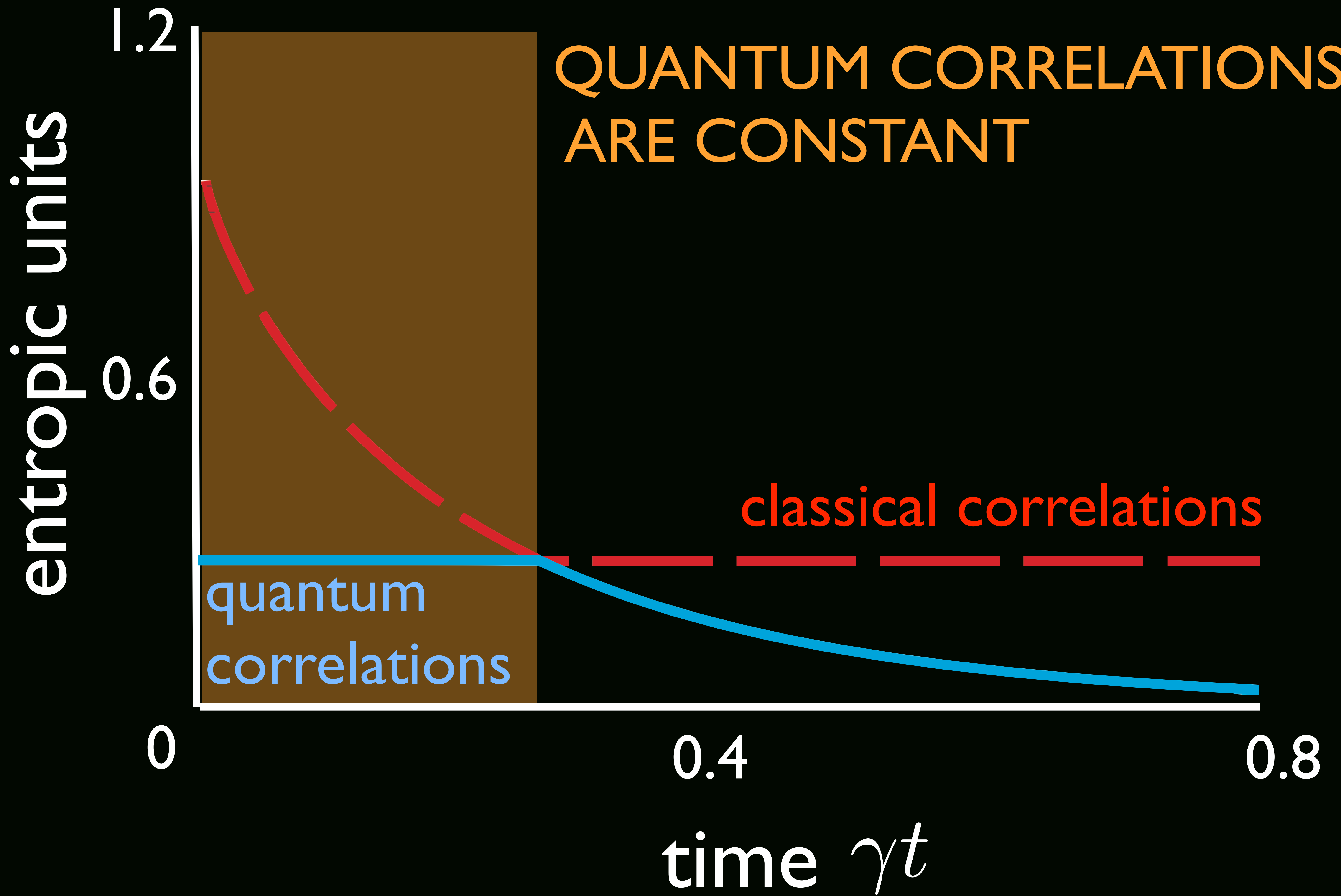
DYNAMICS



L. Mazzola, J. Piilo, and S. Maniscalco, arXiv: 1001.5441 (2010), in press in Phys. Rev Lett.

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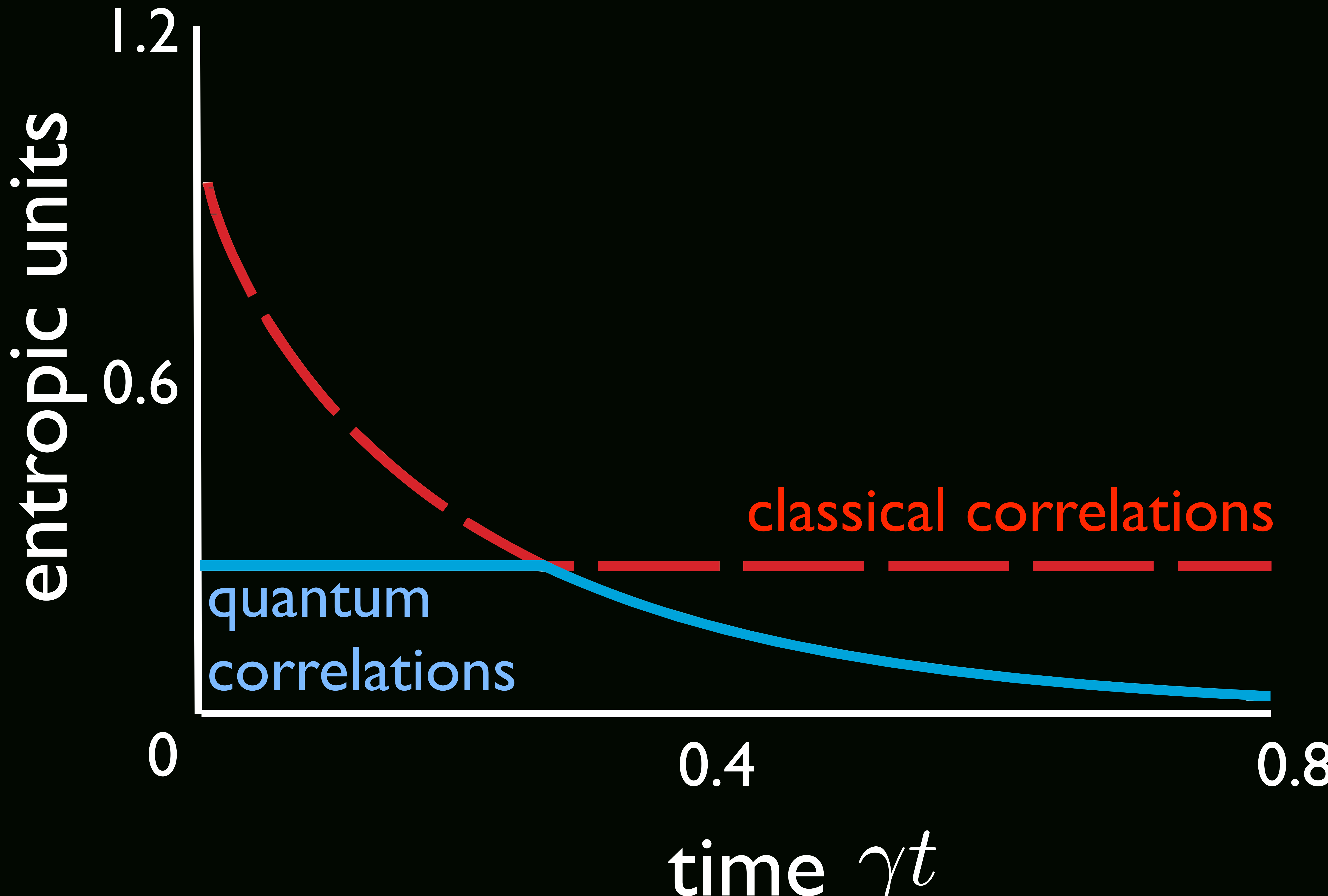

L. Mazzola, J. Piilo, and S. Maniscalco, arXiv: 1001.5441 (2010), in press in Phys. Rev Lett.

The quantum to classical transition is **blocked** until a fixed time instant

$$\bar{t} = -\ln(|c_3|)/(2\gamma)$$

what happens?

MUTUAL INFORMATION

$$\mathcal{I}[\rho_{AB}(t)] = \sum_{j=1}^{2} \frac{1+(-1)^j c_3}{2} \log_2[1+(-1)^j c_3] + \sum_{j=1}^{2} \frac{1+(-1)^j e^{-2\gamma t}}{2} \log_2[1+(-1)^j e^{-2\gamma t}]$$

CLASSICAL CORRELATIONS

$$\mathcal{C}[\rho_{AB}(t)] = \sum_{j=1}^{2} \frac{1+(-1)^j \chi(t)}{2} \log_2[1+(-1)^j \chi(t)]$$

$$\chi(t) = \max\{e^{-2\gamma t}, |c_3|\}$$

MUTUAL INFORMATION

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Classical Correlations

CLASSICAL CORRELATIONS

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MUTUAL INFORMATION

$$\mathcal{I}[\rho_{AB}(t)] = \sum_{j=1}^{2} \frac{1+(-1)^j c_3}{2} \log_2[1+(-1)^j c_3]$$

Discord

$$+ \sum_{j=1}^{2} \frac{1+(-1)^j e^{-2\gamma t}}{2} \log_2[1+(-1)^j e^{-2\gamma t}]$$

Classical Correlations

CLASSICAL CORRELATIONS

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MUTUAL INFORMATION

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Classical Correlations

$$+ \sum_{j=1}^{2} \frac{1+(-1)^j e^{-2\gamma t}}{2} \log_2[1+(-1)^j e^{-2\gamma t}]$$

Discord

CLASSICAL CORRELATIONS

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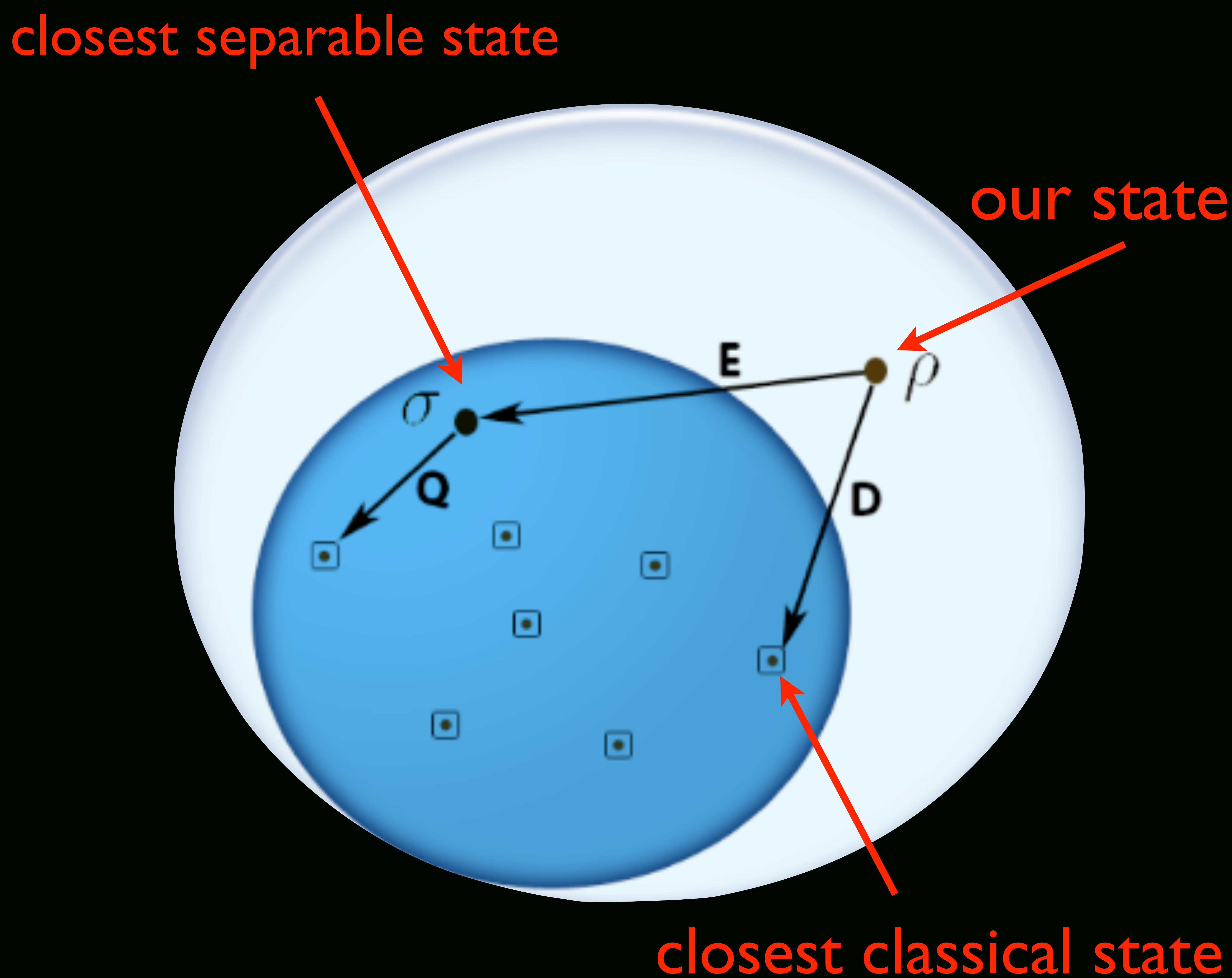
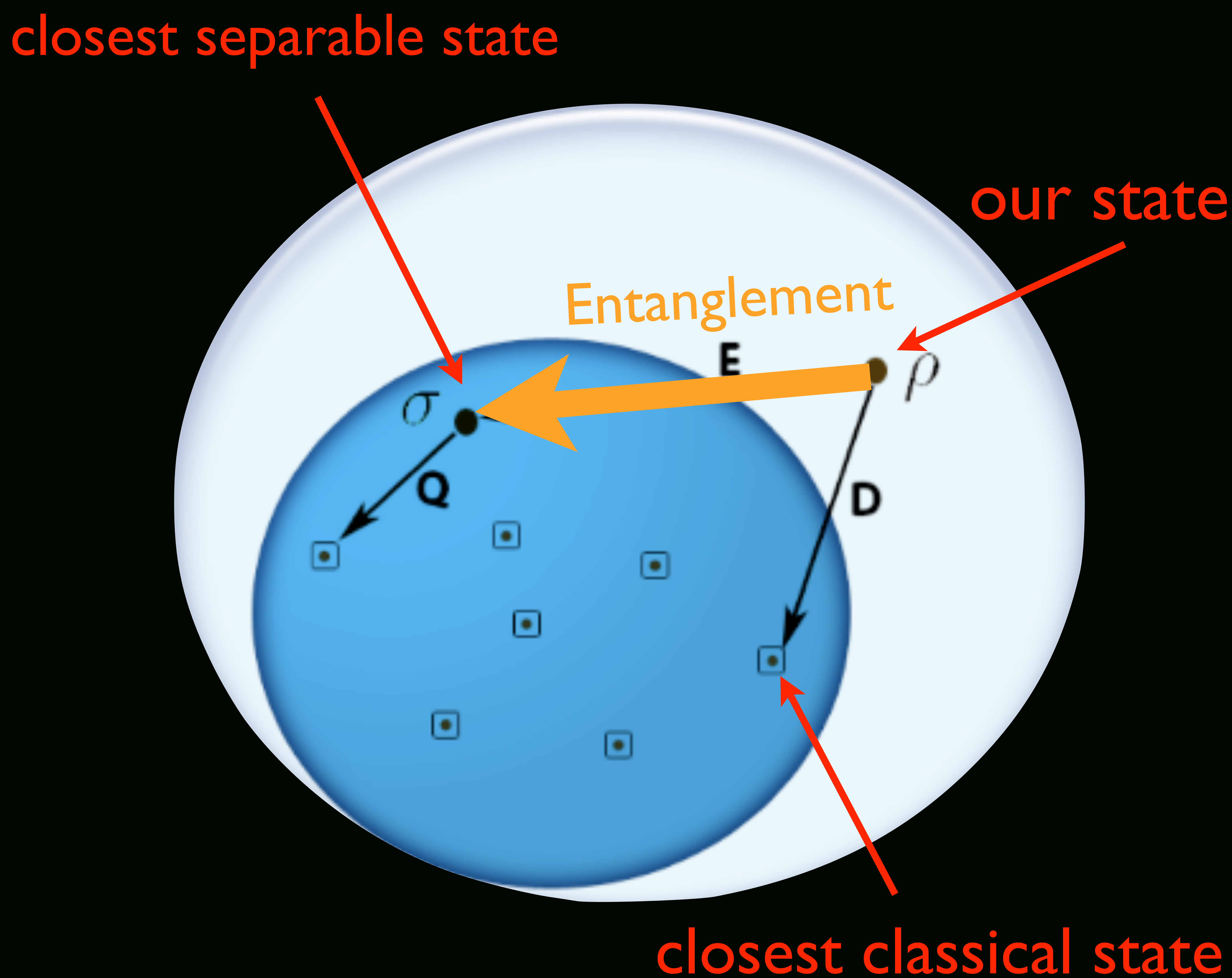


closest separable state
our state
E
ρ
σ
Q
D
closest classical state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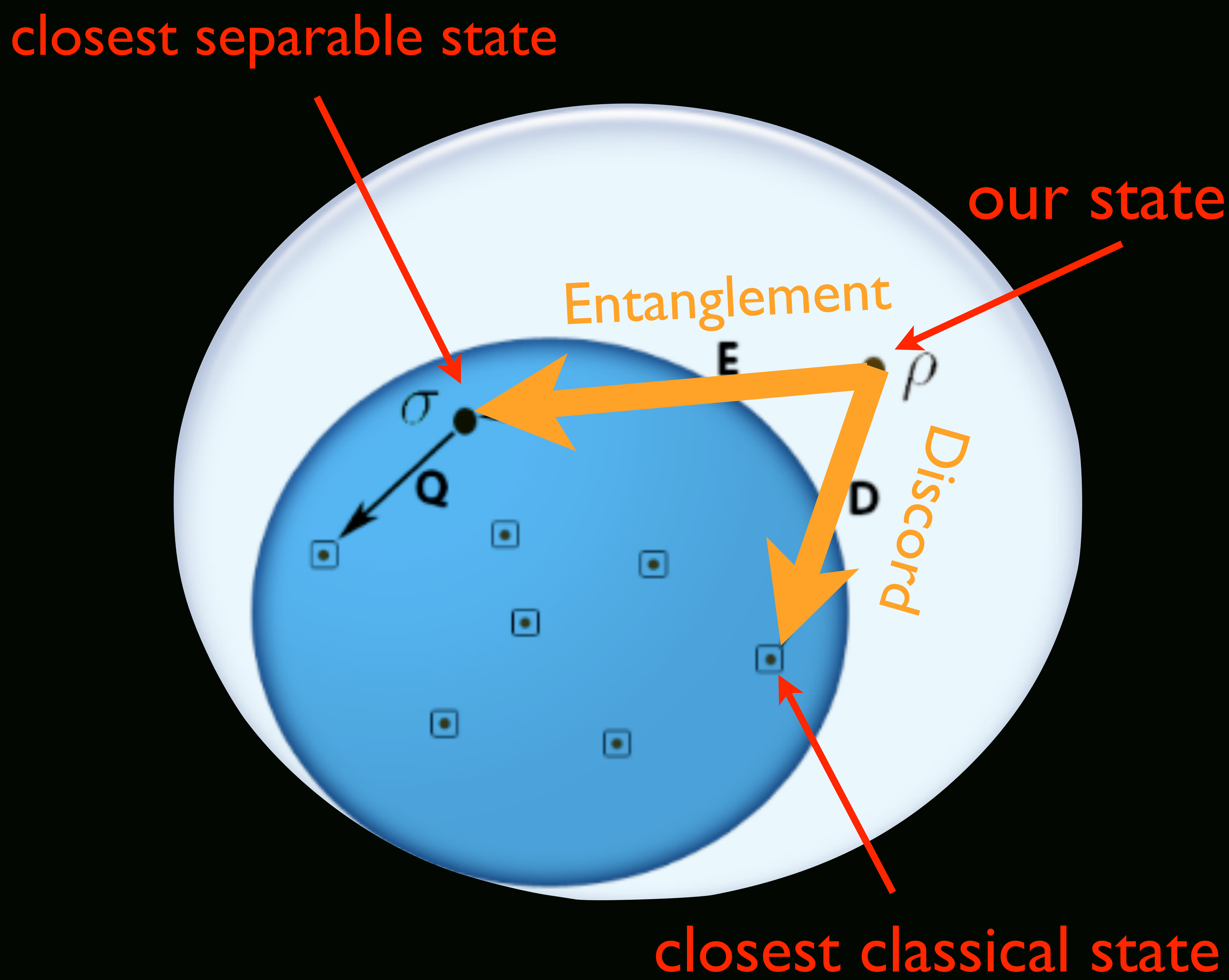
closest separable state
our state
Entanglement
E
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Q
D
Discord
closest classical state

$t = \bar{t}$

our state at time t: $\rho(t)$

time ➡

its closest classical state

$$\rho_{\rm cl}(t < \bar{t}) = \frac{1 + e^{-2\gamma t}}{4} \left(|\Psi^+\rangle\langle\Psi^+| + |\Phi^+\rangle\langle\Phi^+| \right) + \frac{1 - e^{-2\gamma t}}{4} \left(|\Phi^-\rangle\langle\Phi^-| + |\Psi^-\rangle\langle\Psi^-| \right)$$

$t = \bar{t}$

our state at time t: $\rho(t)$

time ➡

its closest classical state

$$\begin{aligned}\rho_{\mathrm{cl}}(t < \bar{t}) &= \frac{1+e^{-2\gamma t}}{4}\left(|\Psi^+\rangle\langle\Psi^+| + |\Phi^+\rangle\langle\Phi^+|\right) \\ &+ \frac{1-e^{-2\gamma t}}{4}\left(|\Phi^-\rangle\langle\Phi^-| + |\Psi^-\rangle\langle\Psi^-|\right)\end{aligned}$$

$$t = \bar{t}$$

our state at time t: $\rho(t)$

time ➡

its closest classical state

$$\rho_{\rm cl}(t<\bar{t}) = \frac{1+e^{-2\gamma t}}{4}\left(|\Psi^+\rangle\langle\Psi^+| + |\Phi^+\rangle\langle\Phi^+|\right) + \frac{1-e^{-2\gamma t}}{4}\left(|\Phi^-\rangle\langle\Phi^-| + |\Psi^-\rangle\langle\Psi^-|\right)$$

our state at time t: $\rho(t)$

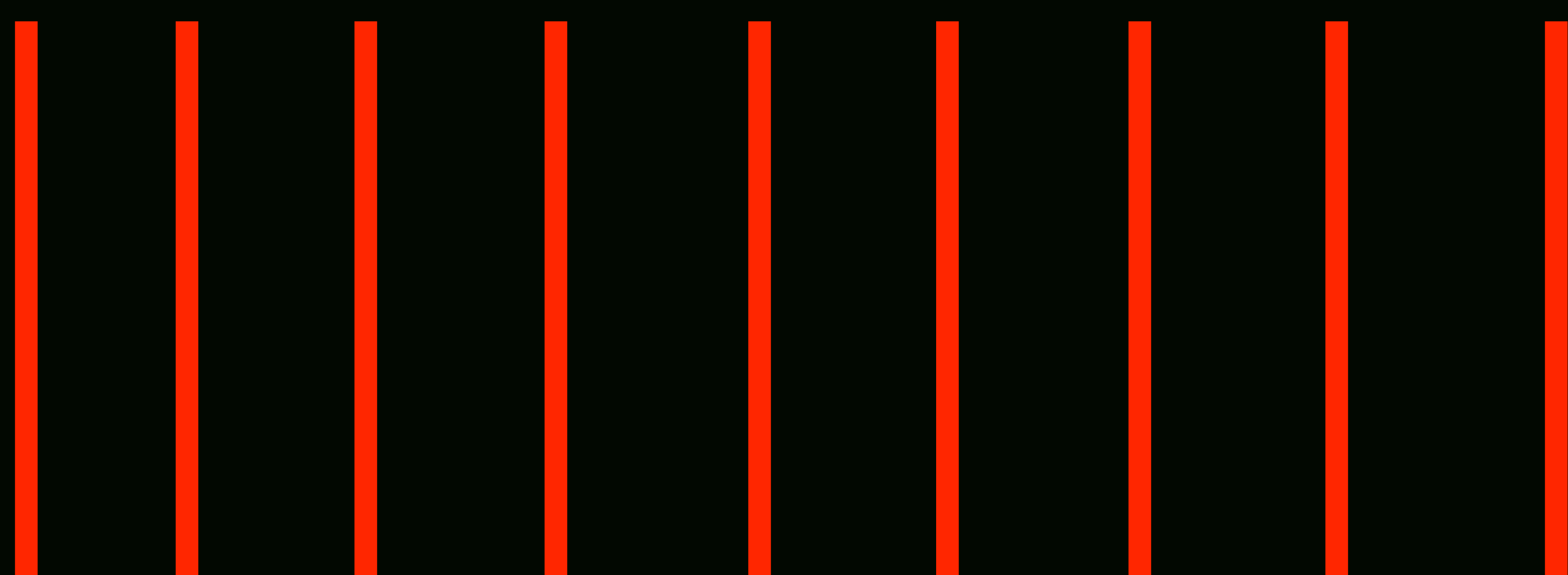
its closest classical state

$$\rho_{\rm cl}(t>\bar{t}) = \frac{1+c_3}{4}\left(|\Psi^+\rangle\langle\Psi^+| + |\Psi^-\rangle\langle\Psi^-|\right) + \frac{1-c_3}{4}\left(|\Phi^-\rangle\langle\Phi^-| + |\Phi^+\rangle\langle\Phi^+|\right)$$

$t = \bar{t}$

our state at time t: $\rho(t)$

time →



its closest classical state

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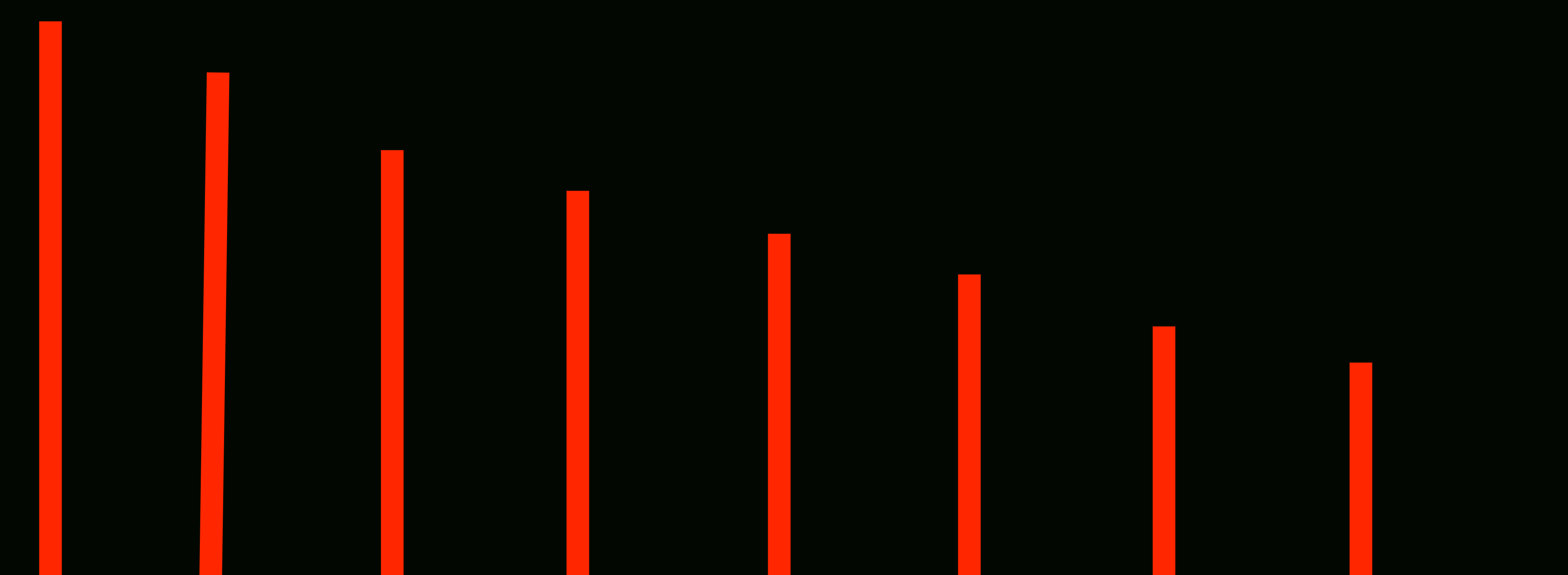
our state at time t: $\rho(t)$

time →



its closest classical state

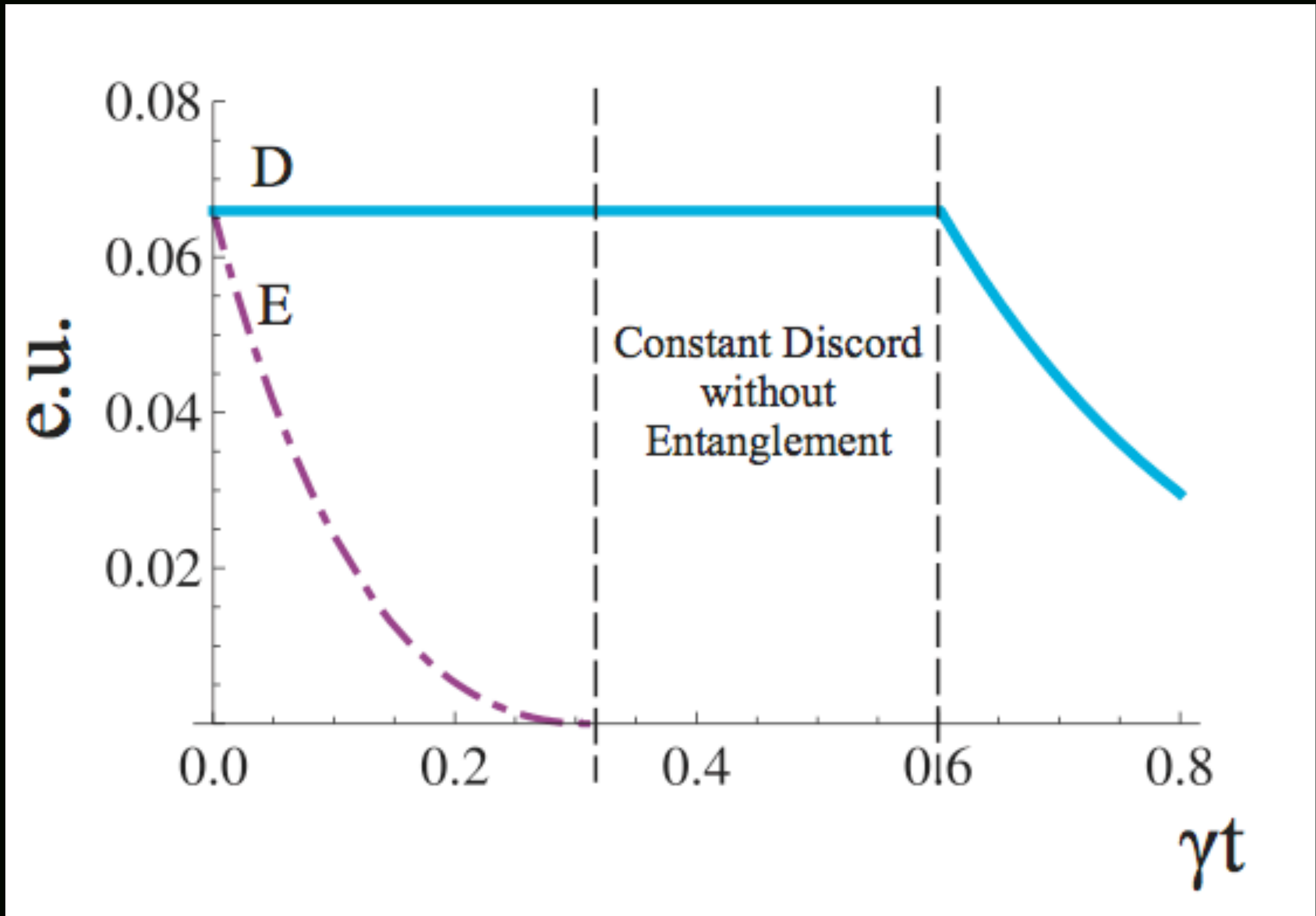
$$\rho_{\rm cl}(t>\bar{t}) \;=\; \frac{1+c_3}{4}\left(|\Psi^+\rangle\langle\Psi^+|+|\Psi^-\rangle\langle\Psi^-|\right) + \frac{1-c_3}{4}\left(|\Phi^-\rangle\langle\Phi^-|+|\Phi^+\rangle\langle\Phi^+|\right)$$

PHYSICAL EXPLANATION

ENTANGLEMENT

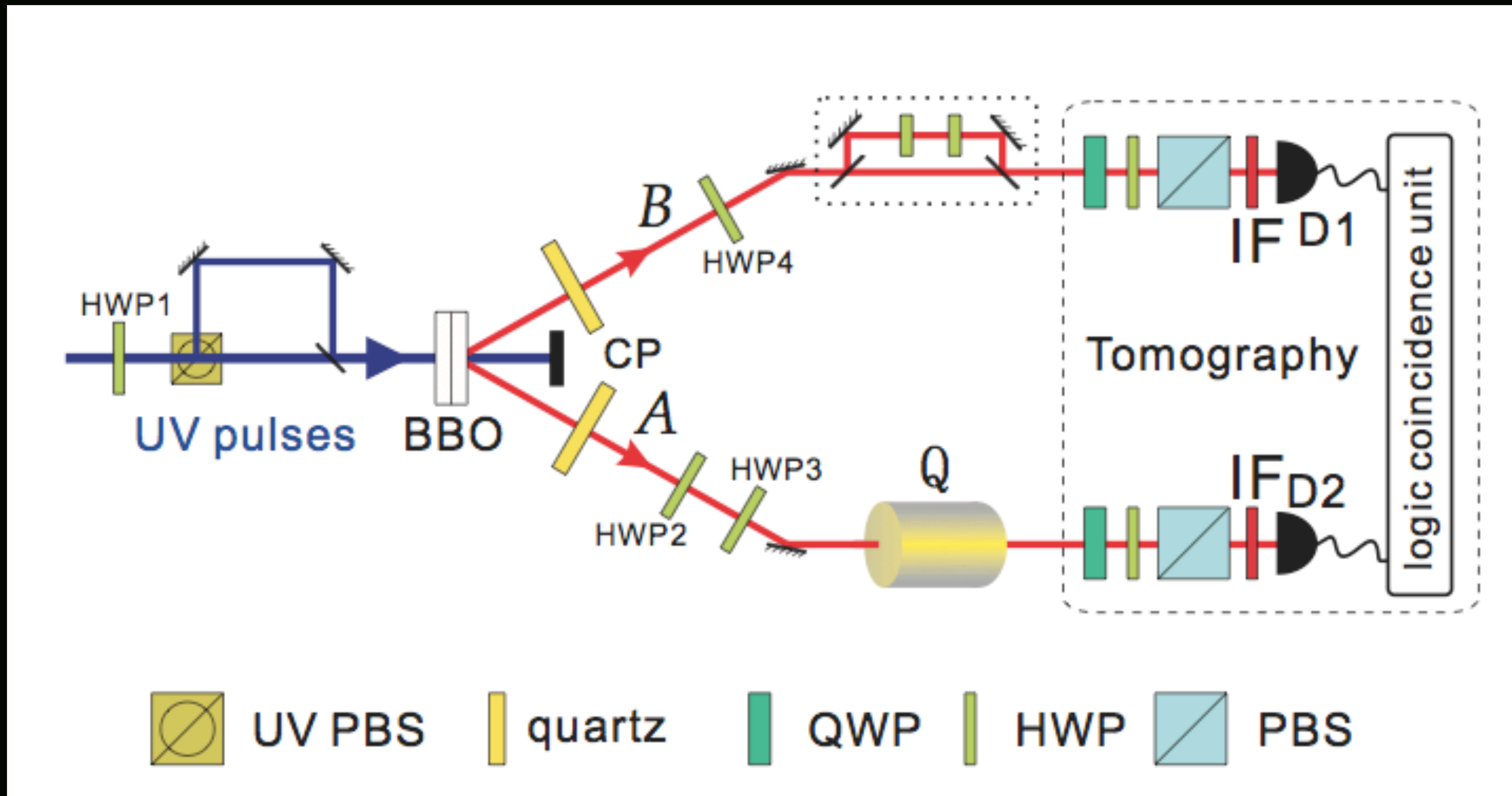
DISCORD

ENTANGLEMENT

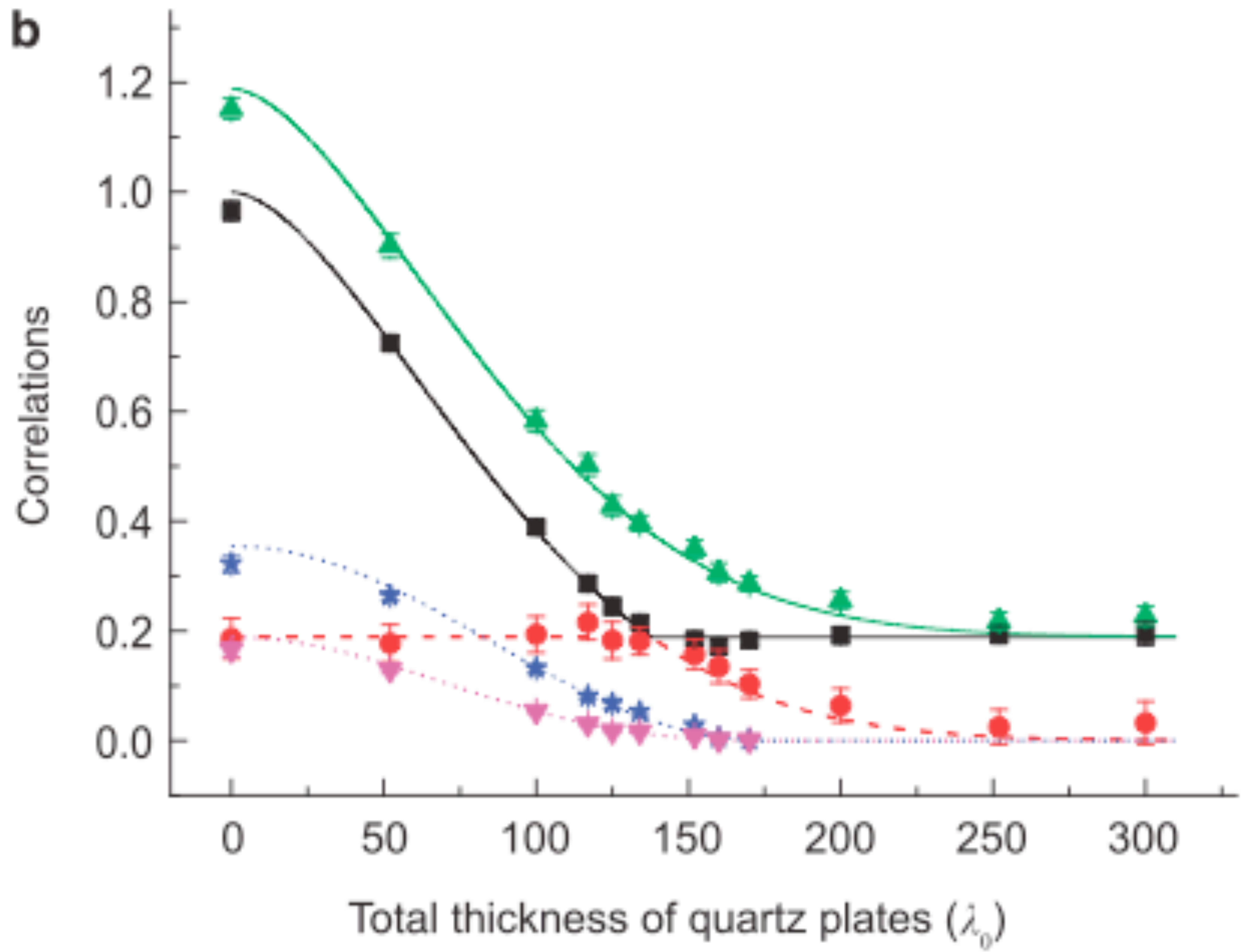


DISCORD

EXPERIMENTS



Jin-Shi Xu et al., Nature Communications 1, 7 (2010)



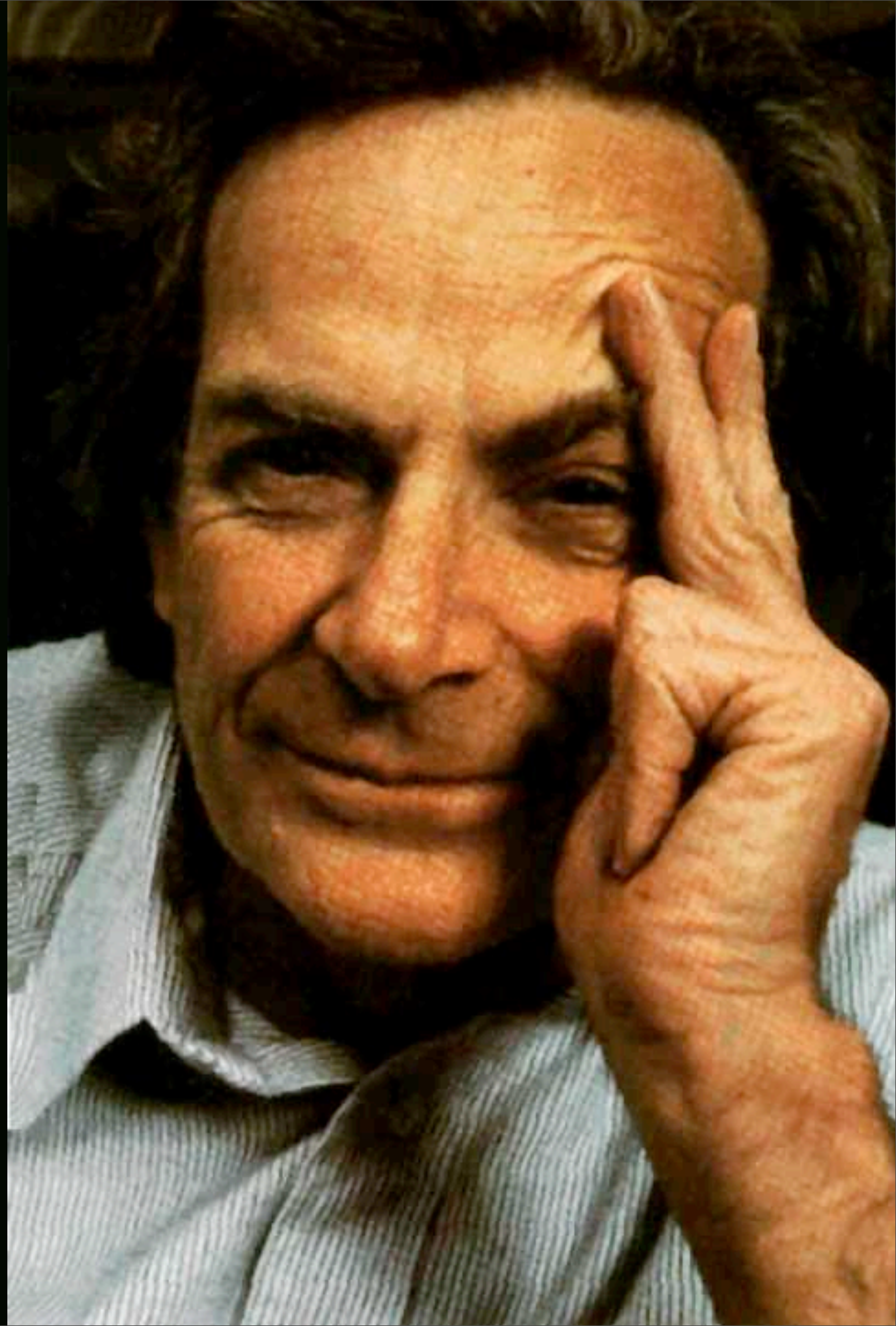
b
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Correlations
0
50
100
150
200
250
300
Total thickness of quartz plates (λ_0)


many questions

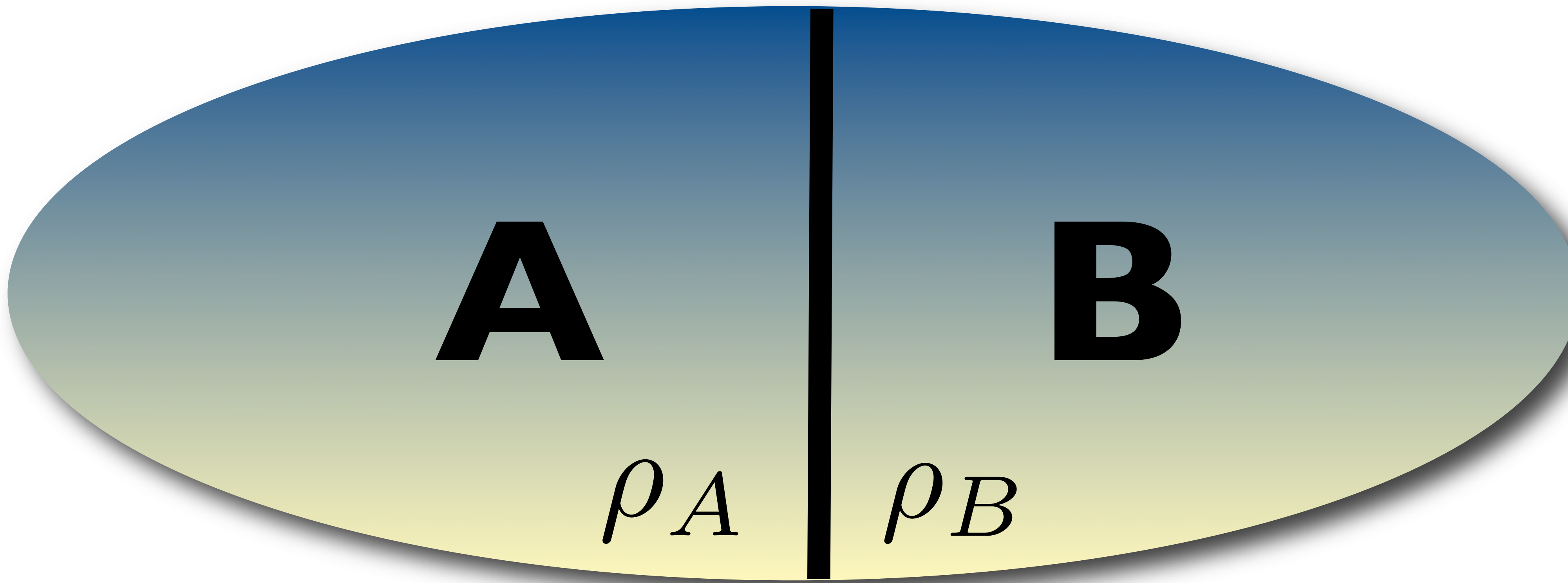
The first evidence of a **quantum property** that is dynamically **not affected by the environment** for a long interval of time http://www.youtube.com/watch?v=jrk3GbJU0k0&feature=related

"I think Nature's imagination is so much greater than man's, she's never gonna let us relax"

Richard Feynman



Classical Correlations



measurement on B: Π_k

measurement outcome k

probability p_k

$$p_k = Tr_{AB}(\rho_{AB}\Pi_k)$$

state of A after the measurement

$$\rho_k = Tr_B(\Pi_k \rho_{AB} \Pi_k)/p_k$$

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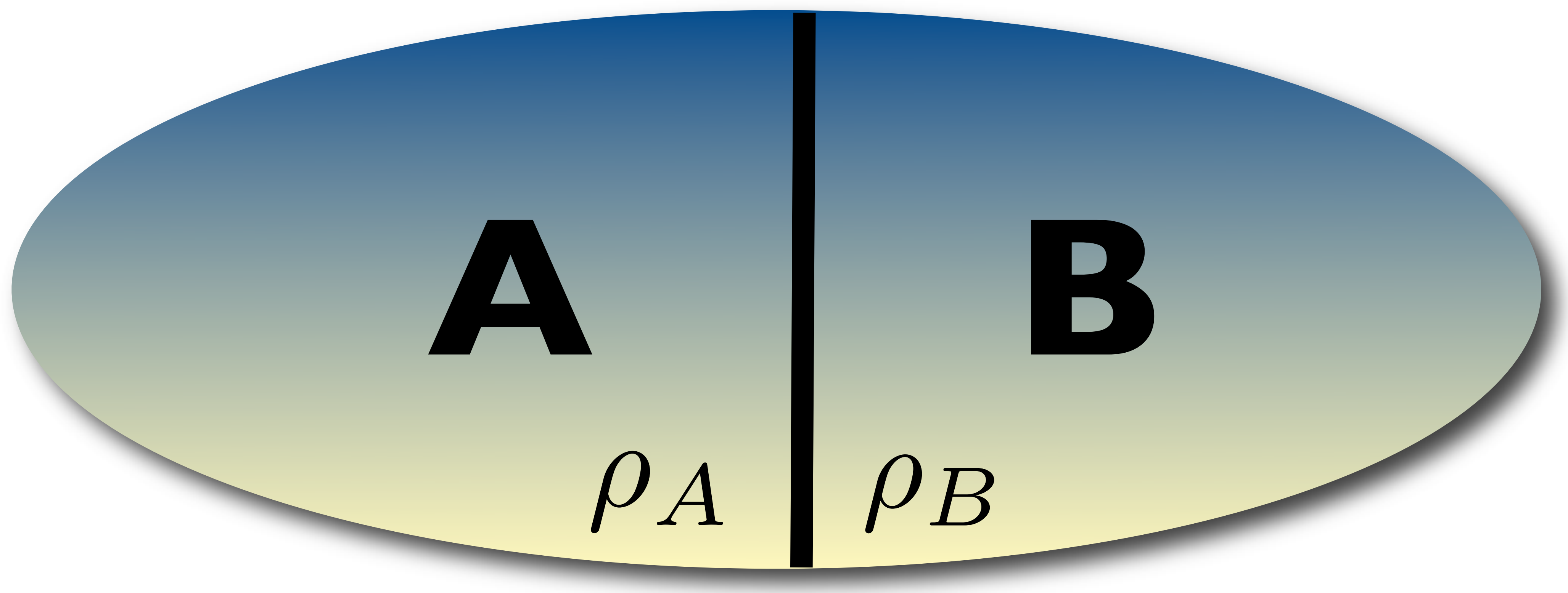

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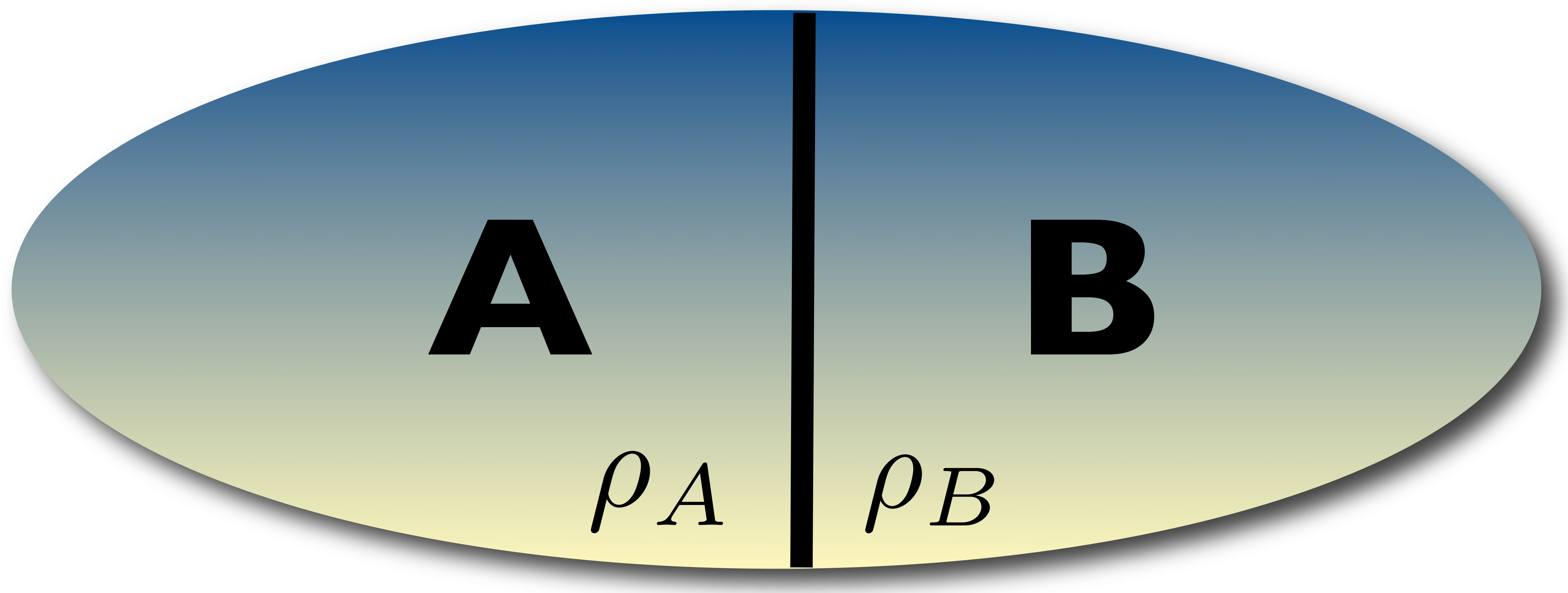

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Classical Correlations

$$\mathcal{C}(\rho_{AB}) = \max_{\{\Pi_k\}}[S(\rho_A) - S(\rho_{AB}|\{\Pi_k\})]$$

Non-classically correlated separable states

$$\sum_i p_i \rho_A^i \otimes \rho_B^i$$

separable state

Non-classically correlated separable states

$$\sum_i p_i \rho_A^i \otimes \rho_B^i$$

separable state

$|k\rangle$ set of orthogonal states of A

$|l\rangle$ set of orthogonal states of B

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subset of the separable states

$p_{kl} \neq p_k p_l$

$$\sum_{kl} p_{kl} |k\rangle\langle k| \otimes |l\rangle\langle l|$$

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classically correlated states

$\Rightarrow$ $\mathcal{I} = \mathcal{C}$

example of separable state with non-classical (quantum) correlations

$$\rho_{AB} = \frac{1}{2}\left(|0\rangle\langle 0|_A \otimes |0\rangle\langle 0|_B + |1\rangle\langle 1|_A \otimes |+\rangle\langle +|_B\right)$$

$\Downarrow$

$\mathcal{I} \neq \mathcal{C}$ $\quad \mathcal{D} \neq 0$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

Dynamics of quantum correlations

in presence of the environment

SYSTEM: 2 qubits

ENVIRONMENT: non-dissipative independent environments

J. Maziero, L. C. Celeri, R. M. Serra, and V. Vedral, Phys. Rev. A 80, 044102 (2009).

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bit, bit-phase and phase flips

j=1 j=2 j=3

$$\mathcal{L}[\rho_{A(B)}] = \gamma[\sigma_j^{A(B)} \rho_{A(B)} \sigma_j^{A(B)} - \rho_{A(B)}]/2$$

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initial states: class of states with maximally mixed marginals

$$\rho_{AB} = \frac{1}{4}\left(\mathbf{1}_{AB} + \sum_{i=1}^{3} c_i \sigma_i^A \sigma_i^B\right)$$

Werner states

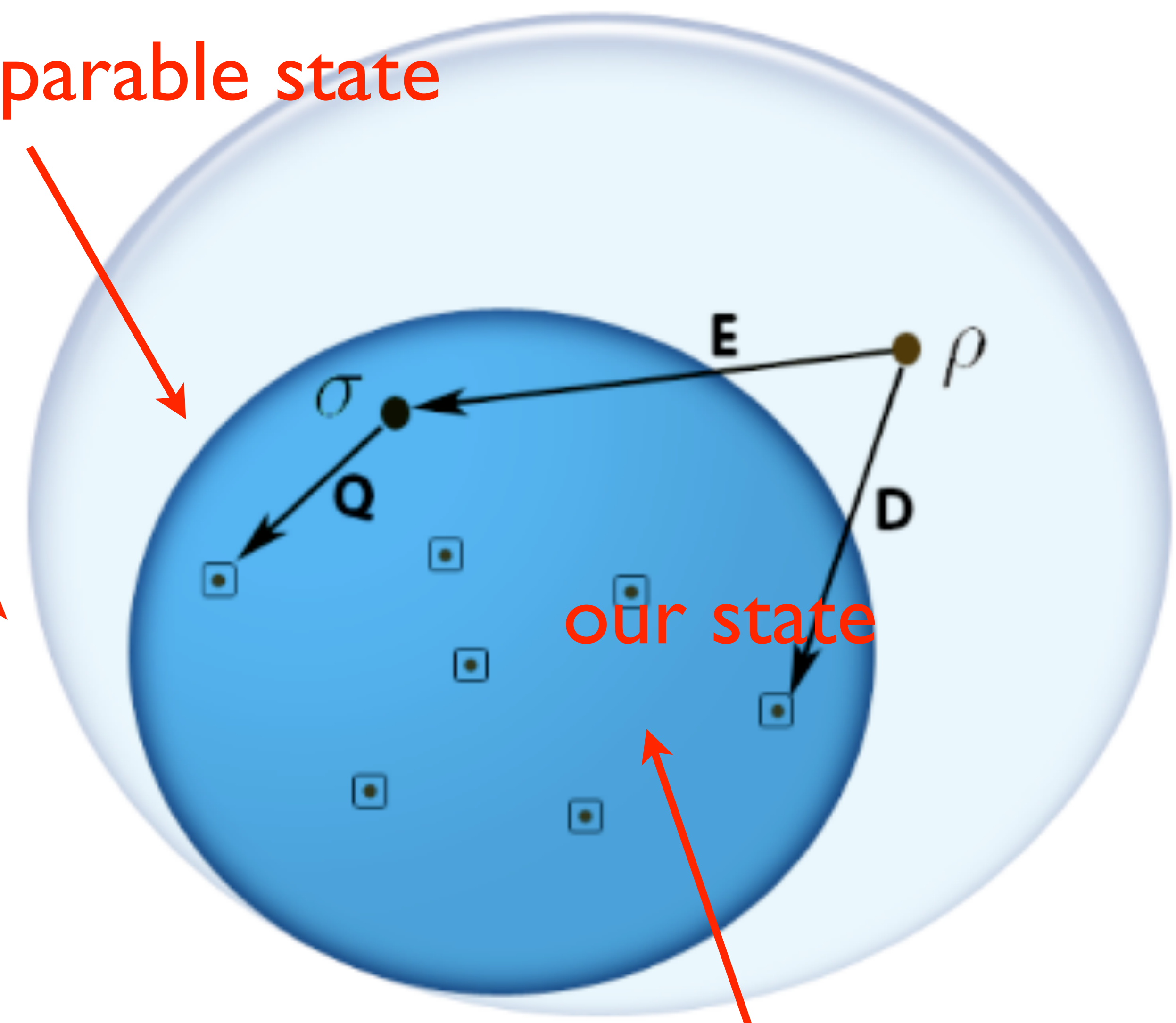
$$|c_1| = |c_2| = |c_3| = c$$

Bell states

$$|c_1| = |c_2| = |c_3| = 1$$

J. Maziero, L. C. Celeri, R. M. Serra, and V. Vedral, Phys. Rev. A 80, 044102 (2009).

Correlations as distances



RELATIVE ENTROPY

$$S(x||y) \equiv -Tr(x \log y) - S(x)$$

Entanglement E

distance to the closest separable state

$$E = \min_{\sigma \in \mathcal{S}} S(\rho||\sigma)$$

Discord D

distance to the closest classical state

$$D = \min_{\chi \in \mathcal{C}} S(\rho||\chi)$$

Dissonance Q

distance of the closest separable state to its closest classical state

$$Q = \min_{\chi \in \mathcal{C}} S(\sigma||\chi)$$

K. Modi, T. Patarek, W. Son, V. Vedral, and M. Williamson, arXiv:0911.5417, in press in PRL

Open Questions

L. Mazzola, J. Piilo, and S. Maniscalco, arXiv: 1001.5441 (2010).

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- Is it possible to exploit the class of initial states displaying such a property to perform quantum computation or communication tasks without any disturbance from the noisy environment for long enough intervals of time?

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- Which is the most general class of states (and dynamical maps) exhibiting a sudden transition from classical to quantum decoherence?

- Which are the physical mechanisms that forbid the loss of quantum correlations at the initial times and that allow only quantum correlations to be lost after the transition time

L. Mazzola, J. Piilo, and S. Maniscalco, arXiv: 1001.5441 (2010).