Does the Chern-Simons formulation provide the microscopic completion of BTZ black hole entropy?

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Why black holes in 3d gravity?

- 3d gravity non-trivial but comparatively simple
- No Hawking radiation 3d pure gravity – discrete black hole spectrum
- 3d quantum gravity may exist...
- May give non-stringy AdS/CFT

Does the Chern-Simons formulation provide the microscopic completion of BTZ black hole entropy?

Technical description in doubt, Witten: 'plausibly has some sort of relationship to three-dimensional Chern-Simons theory'.

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String theory microscopically completes supergravity with various brane configurations and so 'explains' many black hole entropies.

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String theory microscopically completes supergravity with various brane configurations and so 'explains' many black hole entropies – some part of the discussion has to be quantum.

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The notion of black holes presupposes classical horizons – some part of the discussion has to be classical.

Approach

- Follow Chern-Simons faithfully already at classical level
- Interpret 'horizon' in terms of Chern-Simons
- Geared towards semi-classical matching of states and entropy

Some unexpected consequences! New geometries to consider.

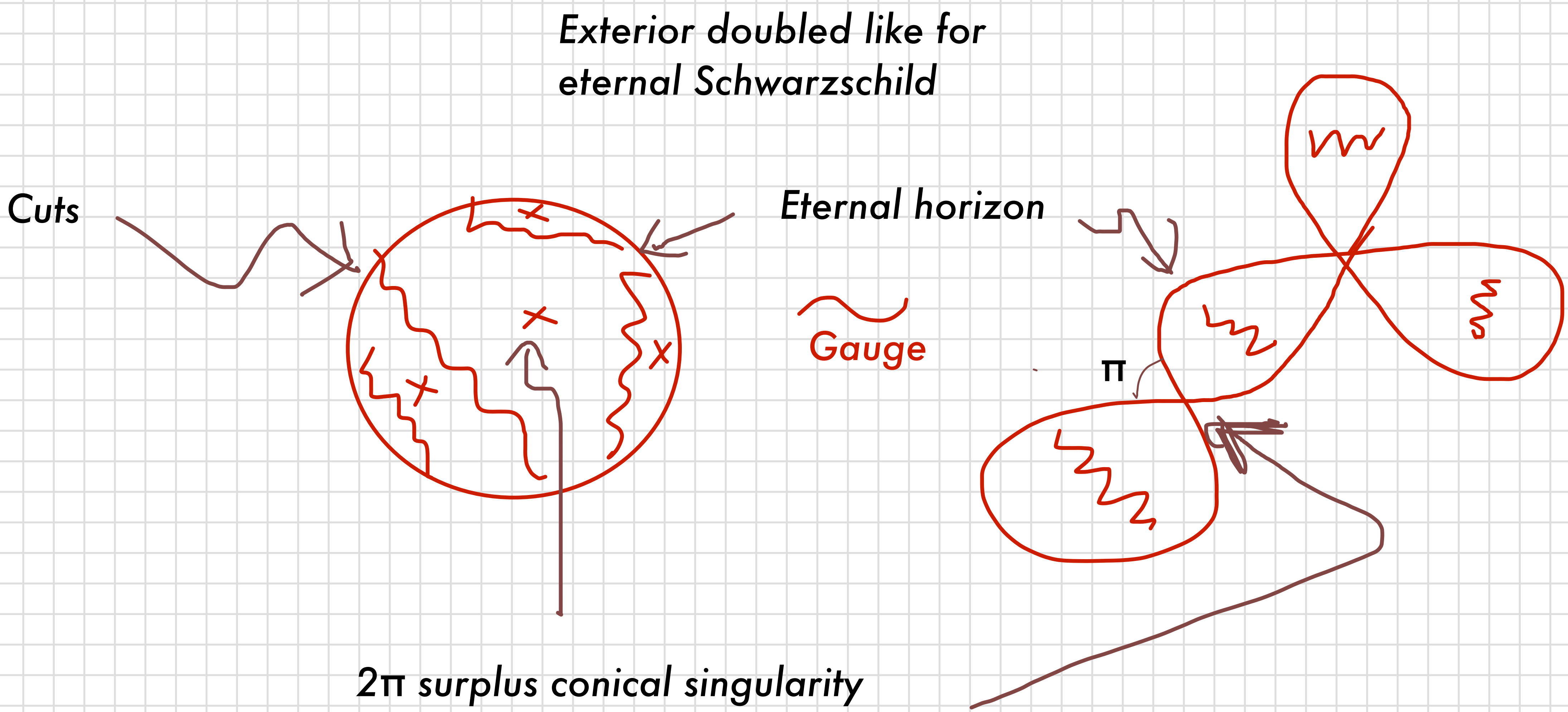
Plan of talk

- The picture
- Chern-Simons and 3d gravity
- Holographic intermission
- Microscopy and holography
- Black hole solution space
- Summary

The black hole picture that will follow

Indistinguishable exteriors

Stable tree interiors



Some similarity to fuzzballs

Mathur

As for fuzzballs classical geometries contain otherwise hidden information

In contrast to fuzzballs there is a horizon

As for fuzzballs we have solutions without causal singularities

In contrast to fuzzballs the geometry of the horizon contains information

Changing the classical description of gravity can change black hole geometry!

Classical descriptions of 3d gravity – Chern-Simons

Chern-Simons and 3d gravity: Basics

3d gravity is trivial

$$R_{\lambda\mu\nu\kappa} = g_{\lambda\nu} R_{\mu\kappa} - g_{\lambda\kappa} R_{\mu\nu} - g_{\mu\nu} R_{\lambda\kappa} + g_{\mu\kappa} R_{\lambda\nu} - \frac{1}{2}\left(g_{\lambda\nu} g_{\mu\kappa} - g_{\lambda\kappa} g_{\mu\nu}\right) R$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \lambda g_{\mu\nu} = 0$$

Trivial means solvable?

Classify all 3-manifolds... (Thurston)

Chern-Simons and 3d gravity: Deser-Jackiw-'t Hooft

Source-free regions have constant curvature

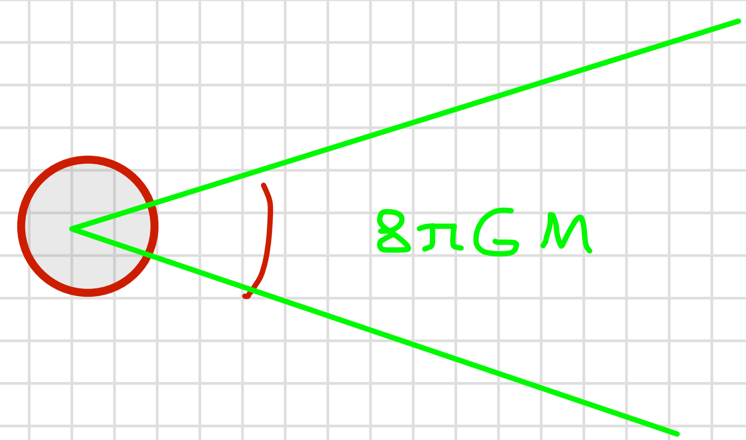
Sources only affect global geometry (conical singularities)

$$dl^2 = d\rho^2 + \rho^2 (d\theta')^2$$

$$0 \le \theta' \le 2\pi - 8\pi G m$$

deficit angle

$m \le \frac{1}{4G}$? $m = -\frac{1}{4G}$ double cover!



Chern-Simons and 3d gravity: Chern-Simons

Achúcarro-Townsend 1986; Witten 1988:

In 3d: translations 3 e^a_i

rotations 3 $\omega^a_i = \varepsilon^{abc}\omega^{bc}_i$

Vielbein formulation of gravity

$$g_{ij} = e^a_i e^b_j \eta_{ab} \quad \text{composite!}$$

Most ordinary metric concepts unobservable...

Defining vectors:

$$A^a_i = \omega^a_i + \frac{1}{l} e^a_i$$

$$A'^a_i = \omega^a_i - \frac{1}{l} e^a_i$$

$$S_{EH}[g] = S_{CS}[A] - S_{CS}[A'] \; ; \qquad S_{CS} = -\frac{k}{4\pi}\int \mathrm{Tr}\left[A\wedge dA + \frac{2}{3} A\wedge A\wedge A\right]$$

$$k = -\frac{l}{4G}$$

Eqs of motion: $F = dA + A\wedge A = 0$

$F' = dA' + A'\wedge A' = 0$

Chern-Simons and 3d gravity: BTZ

Bañados-Teitelboim-Zanelli 1993:

$$ds^2 = -\frac{1}{r^2 l^2}(r^2 - r_+^2)(r^2 - r_-^2)\,dt^2 + r^2 l^2 (r^2 - r_+^2)^{-1}(r^2 - r_-^2)^{-1} dr^2 + r^2\left(d\varphi - \frac{r_+ r_-}{r^2 l}\,dt\right)^2$$

$$M = \frac{r_+^2 + r_-^2}{8 G l^2} \qquad J = \frac{r_+ r_-}{4 G l}$$

Black hole solution!! For $|J| < Ml$.
3d gravity trivial and profound.

Inner and outer horizons. Singularities, but not curvature singularities.
Singular causal structure.

Just do it!

- *Calculate horizon area to find Bekenstein-Hawking entropy.*
- *Compare spectrum of "soluble" 3d quantum gravity.*

Chern-Simons and 3d gravity: The CS BTZ black hole

$$A^0 = \frac{1}{l}(r_+ - r_-)\,\mathrm{sh}\left(\rho - \alpha - \frac{\pi}{2}\right)\left(\frac{1}{l}dt + d\varphi\right)$$

$$A^1 = -\frac{1}{l}(r_+ - r_-)\,\mathrm{ch}\left(\rho - \alpha - \frac{\pi}{2}\right)\left(\frac{1}{l}dt + d\varphi\right)$$

$$A^2 = \frac{1}{l}d\rho$$

$$A'^0 = -\frac{1}{l}(r_+ + r_-)\,\mathrm{sh}\left(\rho - \alpha - \frac{\pi}{2}\right)\left(\frac{1}{l}dt - d\varphi\right)$$

$$A'^1 = \frac{1}{l}(r_+ + r_-)\,\mathrm{ch}\left(\rho - \alpha - \frac{\pi}{2}\right)\left(\frac{1}{l}dt - d\varphi\right)$$

$$A'^2 = -\frac{1}{l}d\rho$$

$$ds^2 = -\mathrm{sh}^2\left(\rho - \alpha - \frac{\pi}{2}\right)\left[r_+ dt - r_- d\varphi\right]^2 + d\rho^2 + \mathrm{ch}^2\left(\rho - \alpha - \frac{\pi}{2}\right)\left[r_- dt - r_+ d\varphi\right]^2$$

Outside the outer horizon.
This solution can be matched to similar forms between the horizons and in the interior. (Are these 'discontinuities' fundamental?)

Generalize solutions (Månsson,BS 2001):

$$\rho \to h(\rho, \varphi)$$

$$d\varphi \to f \;:\; df = \sum q_i\, \delta(\vec{x} - \vec{x}_i)\, dx \wedge dy$$

See also Coussaert-Henneaux 1995

(Quantum) gravity in asymptotic AdS backgrounds

Supposed to be dual to a conformal field theory (at infinity)

Holographic intermission: Brown-Henneaux 1986

Pure AdS gravity:

Asymptotic symmetries $\supset$ AdS group,

$\|$

2d conformal algebra with central extension $\quad c = \frac{3l}{2G} \quad \left(\lambda = -\frac{1}{l^2}\right)$

Precursor of AdS/CFT, but

- CFT not known, only symmetry
- Are conformal charges enough to determine bulk? (If so <-> Liouville)

Bañados 1999:

$$ds^2 = \frac{6}{c}T(z)\,dz^2 + \frac{6}{c}\bar{T}(\bar{z}) + \left(e^{2\rho} + \frac{36}{c^2}T(z)\bar{T}(\bar{z})\,e^{-2\rho}\right)dz\,d\bar{z} + d\rho^2$$

Holographic intermission: Wess-Zumino-Witten

The full strength of CS comes from the relation to 2d CFT pointed out by Witten

- Boundary degrees of freedom represented by WZW conformal field theory.
- Bulk states of the CS theory represented by conformal blocks of CFT.
- No bulk local degrees of freedom.

-

"*(2+1)-dimensional gravity as an exactly soluble system*"

Note another tempting analogy to AdS/CFT:

Boundary CFT from bulk gravitational theory.

Does boundary capture everything?

Holographic intermission: Cardy-Strominger AdS/CFT

Strominger 1998:

Brown-Henneaux conformal algebra with $c = \frac{3l}{2}$

Cardy used 2d modular invariance of unitary CFT to relate c to asymptotic density of states.

$$S_{BH_1} = \frac{Area}{4G} = \frac{\pi\sqrt{16GMl^2 + 2r_+^2}}{4G}$$

$$S_{BH_2} = 2\pi\sqrt{\frac{c n_R}{6}} + 2\pi\sqrt{\frac{c n_L}{6}} = \pi\sqrt{\frac{l(lM+J)}{2G}} + \pi\sqrt{\frac{l(lM-J)}{2G}}$$

$$S_{BH_2} = S_{BH_1} \quad !$$

But unknown how to find the CFT with the assumed properties from gravity...

Holographic intermission: Concrete CFT?

Chern-Simons –> sl(2,R) x sl(2,R) WZW model

Too few states, nonunitary, or not bounded from below or...

Chern-Simons –> sl(2,R) x sl(2,R) WZW model –> Liouville

Projection, so even fewer states...

Other?

Holographic intermission: Witten's monster?

Witten 2007:

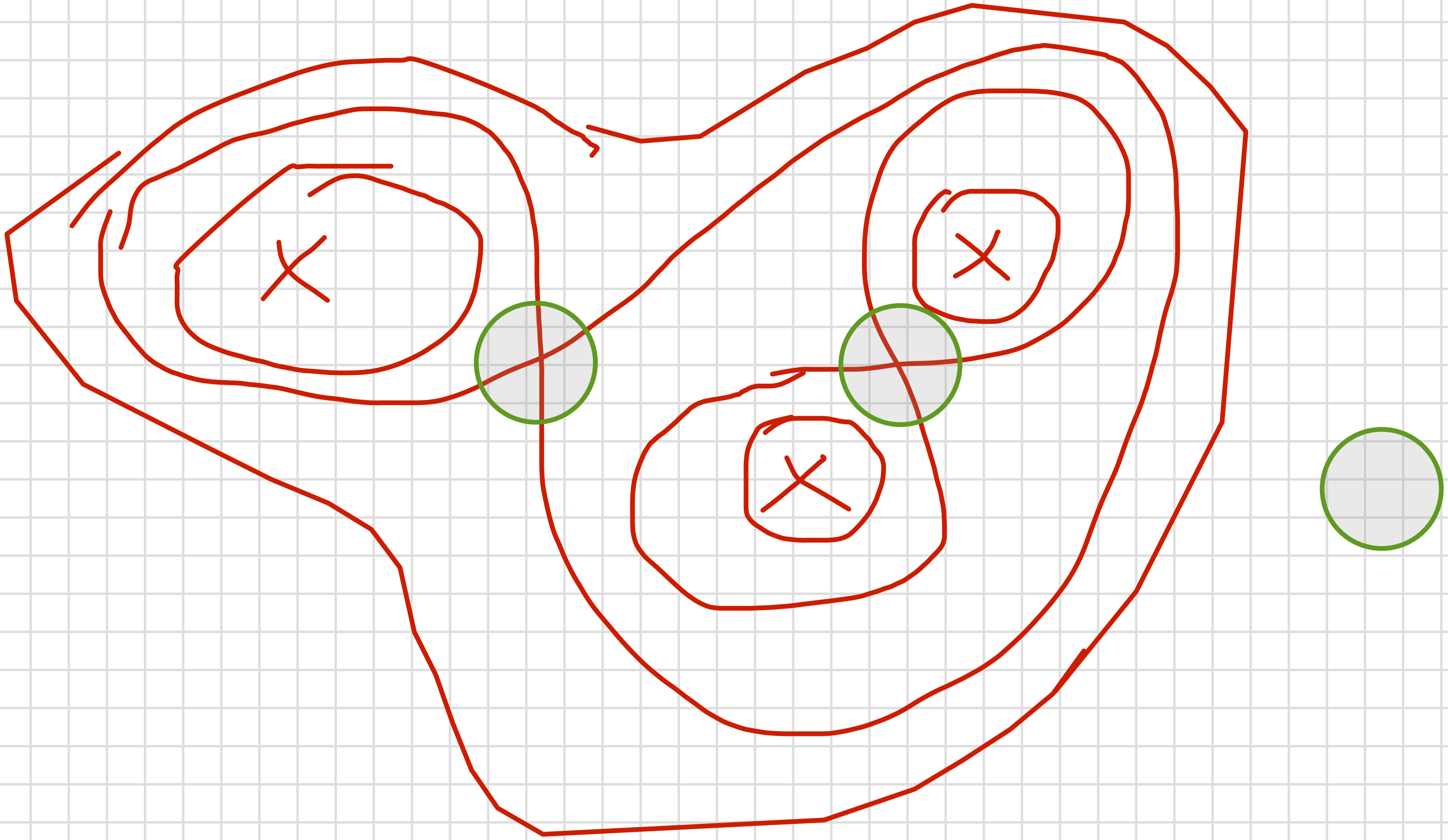
"But it has been pointed out (notably by N. Seiberg) that when we do know how to make sense of quantum gravity, we take the invertibility of the vierbein seriously."

"We can supplement the Chern-Simons action with an instruction to sum over three-manifolds, but it is not clear why we should do this."

Some Chern-Simons arguments and some CFT arguments lead Witten to a unique c=24 CFT without spin 1 symmetries (no massless vectors in the bulk, by AdS/CFT). It enjoys the finite but huge Monster symmetry... Entropy works, but other c values?

Can all microscopic information
be located at infinity?

Microscopy and holography: Multi-source black holes



Degenerate metrics

$dh = 0$

$f = 0$

Noted by Coussaert-Henneaux.
Clement pointed out that static singularities don't follow geodesics unless on horizon.

Not in free fall means accelerated means exchanges energy...

Månsson,BS: CS gauge transformation shifts singularities to horizon!

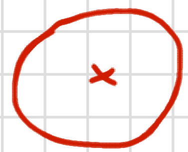
Can also identify critical points of f and h!

Microscopy and holography: Degenerate metrics

Locally around common critical points
of f and h:

$$ds^2 = [d(xy)]^2 + \frac{1}{4}[d(x^2-y^2)]^2 = (x^2+y^2)(dx^2+dy^2)$$

Manifestly flat metric, but



$$\text{Circumference}^2 \neq 4\pi \,\text{Area}$$
$$= 8\pi \,\text{Area}$$

Double cover:
2π surplus angle
conical singularity

- Degenerate metrics allowed in CS give special conical singularities
- Typically one more branch point per additional source
- Not ok in metric gravity, but automatic in CS!
- Does CS provide "state completion" to gravity?

Microscopy and holography: Twisted solutions

Fjelstad-Hwang 2002

$$\tilde{A} = g^{-1}dg + g^{-1}Ag; \quad g = e^{-sx^2 T_0}, \quad x^2 = \varphi$$

$$\tilde{A}^0_\mu = A^0_\mu - s\delta_{\mu,2}, \quad \tilde{A}^\pm = e^{\mp isx^2} A^\pm_\mu, \quad A^\pm \equiv iA^1 \mp A^2$$

Proper gauge transformation, ie g periodic if $\frac{s}{2} \in \mathbb{Z}$.

$$s = \frac{1}{|k|}, \frac{2}{|k|}, \ldots, 2 - \frac{1}{|k|}$$

map unitary representations unitary representation but are NOT gauge transforms.
New twisted solutions!

$$\begin{aligned}\Delta(ds^2) \sim & -4(s-s')\,\mathrm{sh}\rho\,(r_+ dt - r_- d\varphi)\,d\varphi + (s-s')^2 d\varphi^2 \\ & - 4\sin^2\left(\tfrac{s-s'}{2}\varphi\right)\left[d\rho^2 - (r_+^2 - r_-^2)\,\mathrm{ch}^2\rho\,(d\varphi^2 - dt^2)\right] \\ & - 4\sin(s-s')\varphi\,\mathrm{ch}\rho\,(r_- dt - r_+ d\varphi)\,d\rho\end{aligned}$$

Note that $s = s'$ leaves metric invariant.
Typically not periodic $\varphi \to \varphi + 2\pi$

!Not Brown-Henneaux boundary conditions!

Microscopy and holography

Do not only study boundary at infinity

Cf Maldacena's original discussion of duality: microscopic D-branes relate to geometry near horizon

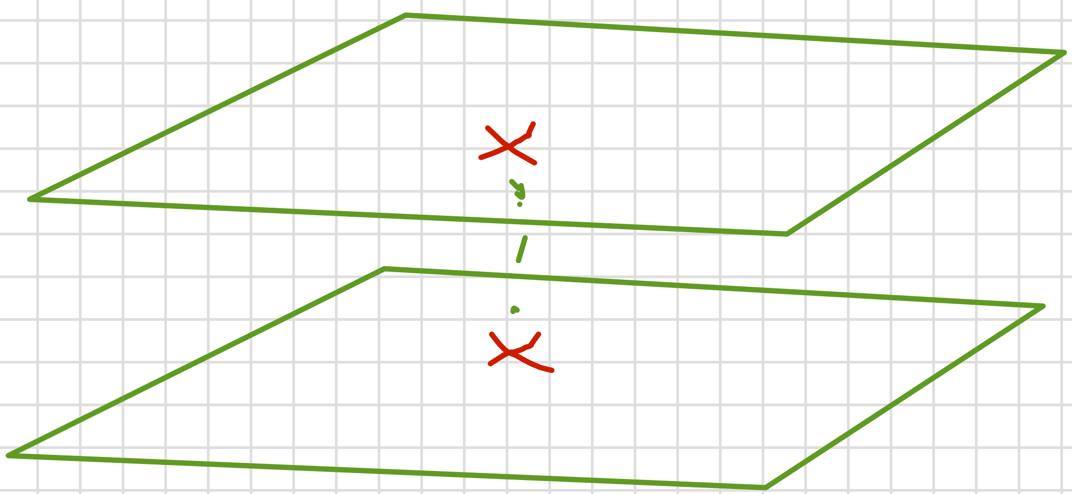
Horizon is crucial for establishing dictionary

Black hole details unknown
to asymptotic infinity

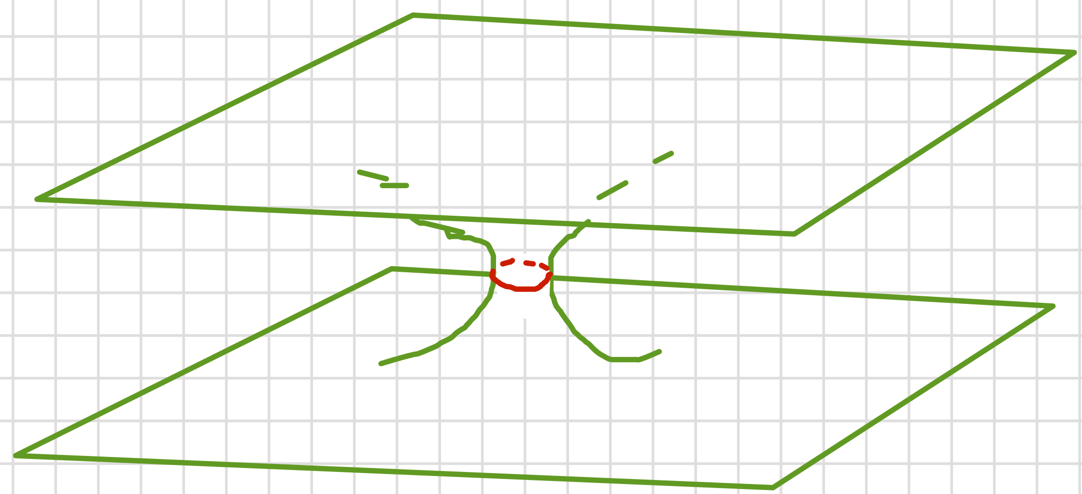
Black hole solution space: replace sources with handles

We do not want arbitrary sources inside horizon.

Different topologies could be ok.



Source

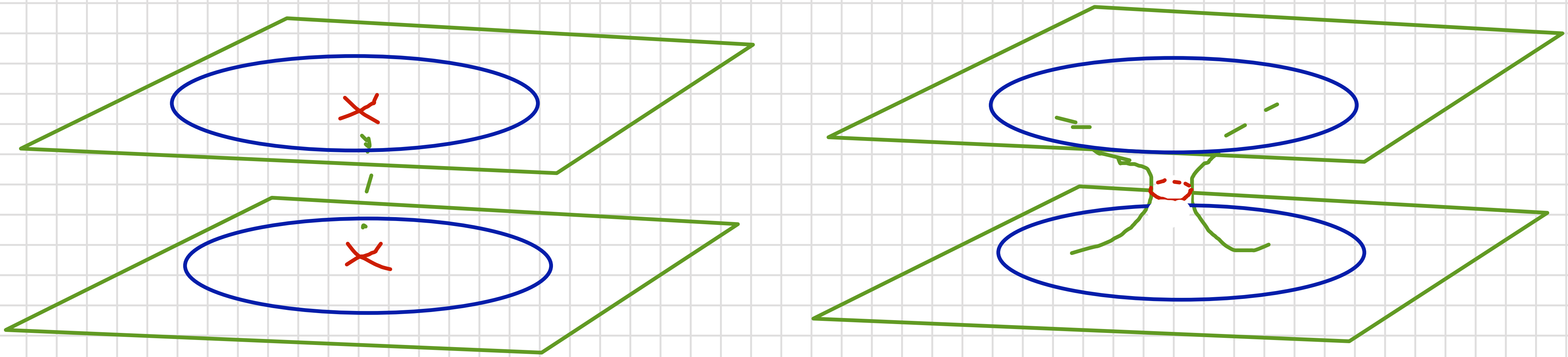


Holonomy

Black hole solution space: a genus 1 black hole

An eternal black hole has two asymptotic regions

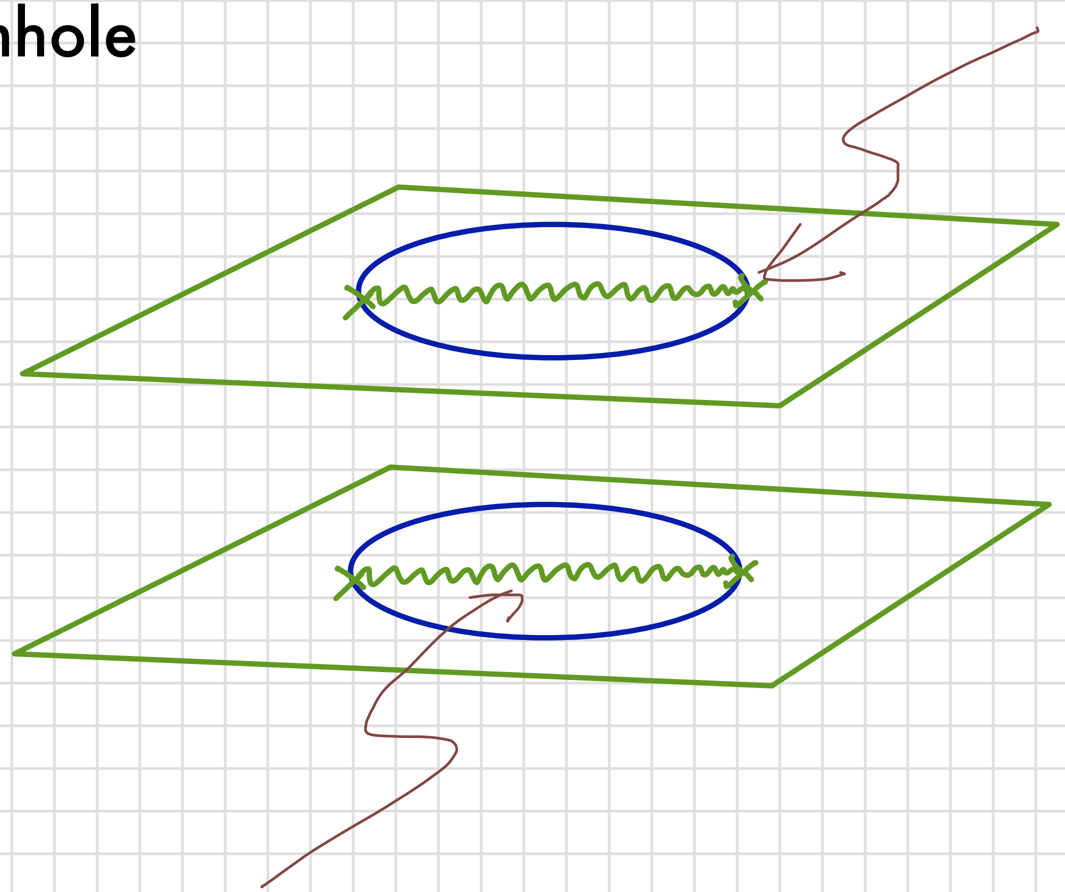
Can we replace the source/singularity with a wormhole?



Recall: Chern-Simons does not mind double covers…

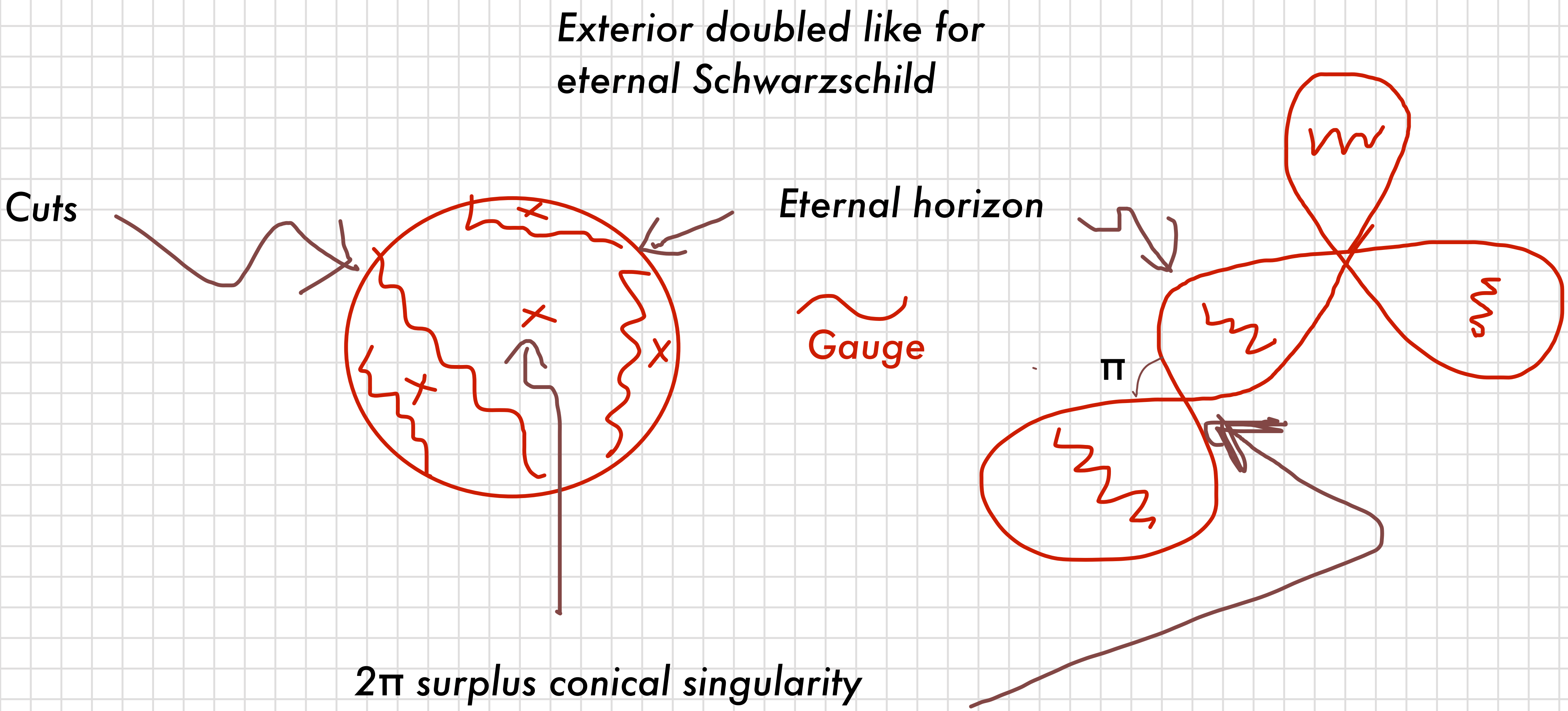
Black hole solution space: cutting and pasting genus 1

The static wormhole



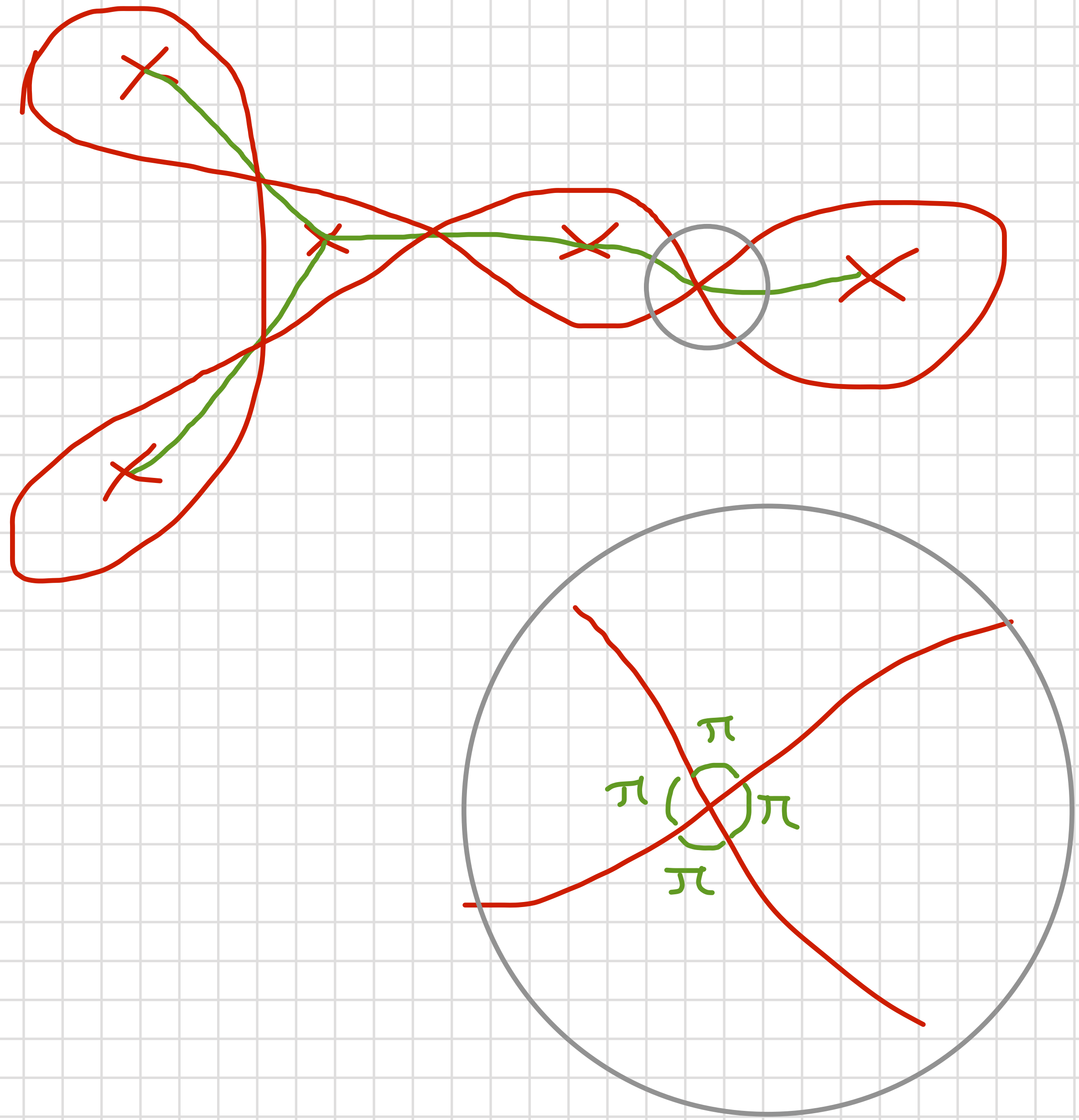
Black hole solution space: higher genus

Solutions are characterized by holonomies around the branch cuts



Black hole solution space: Counting trees

Branch points on single horizon:



TREE STRUCTURE!

Assume all N sub horizons have unit area. Then total area

$$A \sim aN$$

and by tree maths, number of trees

$$d(N) \sim e^{cN}$$

$$S = \log d(N) = cN = \frac{c}{a} A \ !$$

Summary and future

Conclusions

- Do not only study boundary at infinity
- The horizon is the interface to microscopic information
- Changing the classical description of gravity can change black hole geometry

To do

- Geodesic condition completely translated to CS
- All meaningful geometries?
- Space of solutions/global diffeomorphisms?
- Volume of phase space/semi-classical density of states?