Hairy black holes and solitons in global AdS_5

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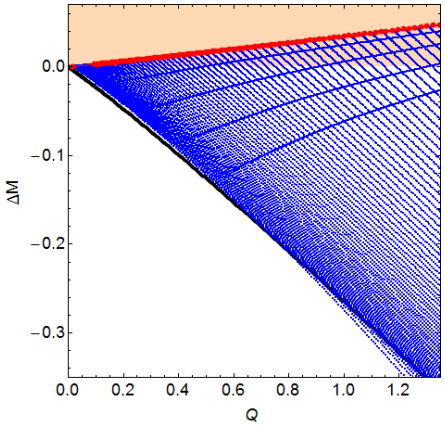
1112.4447: with O.J.C. Dias, P. Figueras, S. Minwalla, P. Mitra, J.E. Santos

The Holographic Way, Nordita, 16 October 2012



Motivation

- Holography: gravitational phase diagrams in AdS describe state structure of dual CFT.
- Poincaré patch of $AdS_d \leftrightarrow$ CFT on M^{d-1}.
  Global $AdS_d \leftrightarrow$ CFT on $S^{d-2} \times R$.
- Interesting toy model: AdS Einstein-Maxwell theory with minimally coupled scalar field. Scalar coupling e is free parameter.
- Phase structure of planar solutions: holographic superconductors.
  (Gubser 08, Hartnoll, Herzog, Horowitz '08, Horowitz, Roberts '09)
- Phase structure in global AdS?

Goal

- To obtain complete phase diagram in global AdS_5 of solutions*,** to

$$S = \int d^5x\sqrt{-g}\left[\frac{1}{2}(R+12) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |D\phi|^2\right]$$
$$D_\mu = \nabla_\mu - ieA_\mu$$

  * static, spherically symmetric ** discard infinite # of them
  Parameter space: scalar coupling e, mass M, charge Q.

- Builds on study for "small" solutions: hairy black holes and solitons, apart from Reissner-Nordström-AdS black holes.
  (Basu, Bhattacharya, Bhattacharyya, Loganayagam, Minwalla, Umesh '10)

- Methods: numerics and perturbation theory.

- Expected: matches "small" solutions, matches planar phase diagram.
  Not expected: very intricate!

Outline

- RN-AdS black holes and their instabilities
- Hairy black holes
- Solitons
- Phase diagram
- Conclusion

RN-AdS black holes and their instabilities

Reissner-Nordström-AdS solution

- Solution

$$ds^2 = -V(r)\,dt^2 + \frac{dr^2}{V(r)} + r^2\,d\Omega_3^2\,,$$

$$V(r) = \left(1 - \frac{R^2}{r^2}\right)\left(1 + r^2 + R^2 - \frac{2}{3}\frac{\mu^2 R^2}{r^2}\right)\,,$$

$$A_t(r) = \mu\left(1 - \frac{R^2}{r^2}\right)\,, \qquad \phi = 0\,.$$

Parameters: R is radius of event horizon, μ is chemical potential.

- Asymptotic charges

$$M = \frac{3\,\pi}{8}\,R^2\left(1 + R^2 + \frac{2}{3}\,\mu^2\right)\,, \qquad Q = \frac{\pi}{2}\,\mu\,R^2\,.$$

- Regular extremal limit, with near-horizon geometry $AdS_2 \times S^3$.

- Stability? No-hair theorem fails in AdS.

RN-AdS instabilities I: near-horizon instability

- Low temperature instability, better seen at extremality.
  Extremality: near-horizon geometry is $AdS_2 \times S^3$.
  Instability if AdS_2 BF bound is violated: $m^2 \ell_{2D}^2 < -1/4$.

- Our minimally-coupled scalar ϕ acquires effective mass $e^2 A^2$.
  BF bound is violated if

$$e^2 > \frac{2(1+3R^2)^2}{3R^2(1+2R^2)} = 3 + \frac{1}{2R^2} + \mathcal{O}(1/R^4)\,.$$

  (monotonic, decreasing with R)
  Large extremal black holes unstable first. Small black holes safe.

- Endpoint is hairy black hole, which has higher entropy.

- The only instability of "large" black holes: extremal black holes with $R \geq 1$ are stable otherwise. (Dias, Figueras, RM, Reall, Santos '10)

RN-AdS instabilities II: superradiant instability

- Superradiant scattering: wave $e^{-i\omega t}$ incident upon charged black hole is amplified if $\omega < e\mu$. AdS box leads to instability.

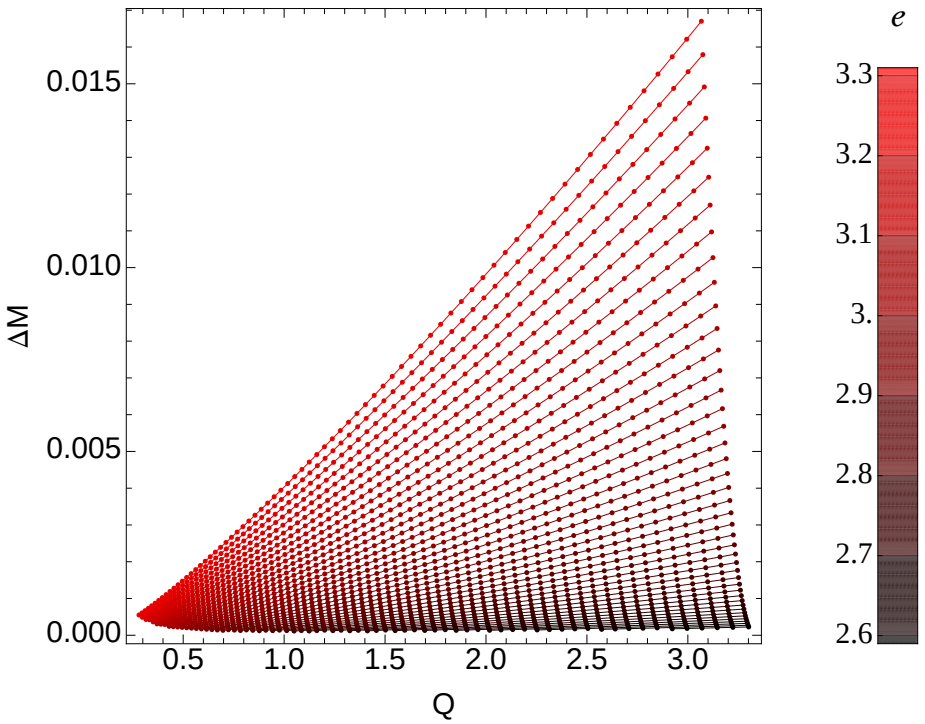
- Threshold for small black holes:
  - lowest normal mode in AdS_5 is $\omega = 4$.
  - assume extremal black hole is most unstable: $\mu^2 = \frac{3}{2}$.
  - then
$$\omega < e\mu \qquad \Rightarrow \qquad e^2 > \frac{32}{3}\,.$$

- Onset gives bifurcation to hairy black hole.
  Traditional superradiance: onset $\hat{\phi} \sim e^{-ie\mu t}$, $\hat{A}(r=\infty) = 0$.
  Gauge transformation: ϕ static, $A = \hat{A} + \mu\, dt$, $A(r=\infty) = \mu\, dt$.

- Instability of small black holes is not near-horizon type.

RN-AdS instabilities III: parameter space

- Three regions: $\boxed{e^2 < 3 \qquad 3 < e^2 < \frac{32}{3} \qquad e^2 > \frac{32}{3}}$
  - $e^2 < 3$: RN-AdS bhs stable.
  - $3 < e^2 < \frac{32}{3}$: large RN-AdS bhs unstable close to extremality.
  - $e^2 > \frac{32}{3}$: RN-AdS bhs of any size unstable close to extremality.



Onset of instability:
solve $D^2\,\phi = 0$

(lowest mode only)

$$\Delta M \;\equiv\; M - M^{\text{extremal}}(Q)$$

Hairy black holes

Hairy black holes: methods

- Need to solve EOMs: coupled 2nd-order ODEs for $f(r),\ A_t(r),\ \phi(r)$.

  $ds^2 = -f(r)dt^2 + g(r)dr^2 + r^2 d\Omega_3^2, \quad g(r)$ determined by others.

- Boundary conditions: regularity at horizon, asymptotically AdS.
  At AdS boundary, $A_t(r) \to \mu,\ r^4\phi(r) \to \epsilon$.

- Complementary methods

  - Perturbatively for $R \ll 1$, $|e^2 - 32/3| \ll 1$.

  - Numerically: Newton's method on Chebyshev grid,
    coord $y = 1 - R^2/r^2$, $y \in [0, 1]$.
    Use as seed the onset mode (last slide).

Hairy black holes: non-interacting model

- Matched asymptotic expansion: three regions
  - near region: close to horizon.
  - intermediate region: small black hole in flat space.
  - far region: perturbation of global AdS.

- Agrees with simple non-interacting model! (cf. Basu et al '10)
  Thermodynamically, hairy black hole = RN-AdS + soliton.

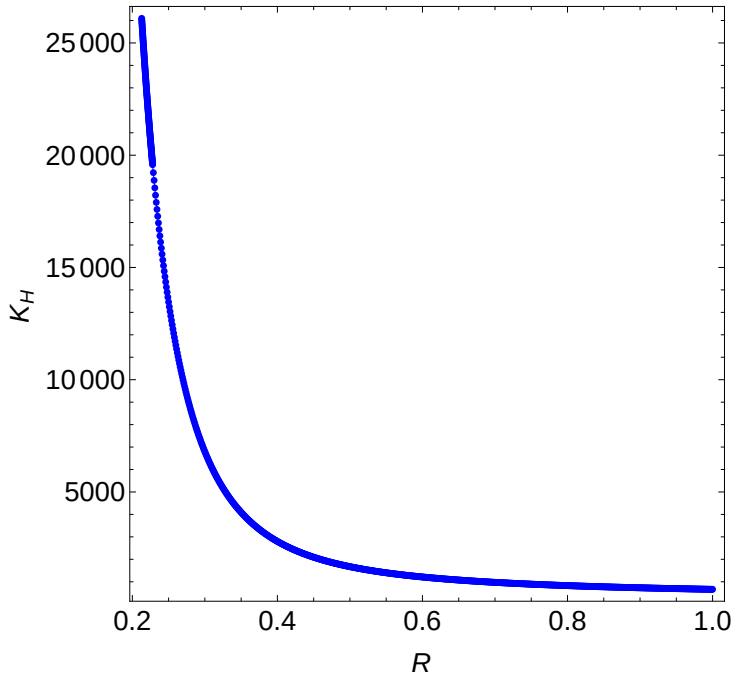
$$S(M,Q) = S_{\text{RN-AdS}}(M - M_{\text{sol}}, Q - Q_{\text{sol}})$$

  Extremize S and use 1st law: get $\mu_{\text{RN-AdS}} = \mu_{sol}$.

- Perturbation theory was crucial guide for numerics!

Hairy black holes: numerical results I

- For $e^2 < 3$, no hairy black holes (RN-AdS stable).
- For $3 < e^2 < \frac{32}{3}$, find hairy black holes, but minimum size.

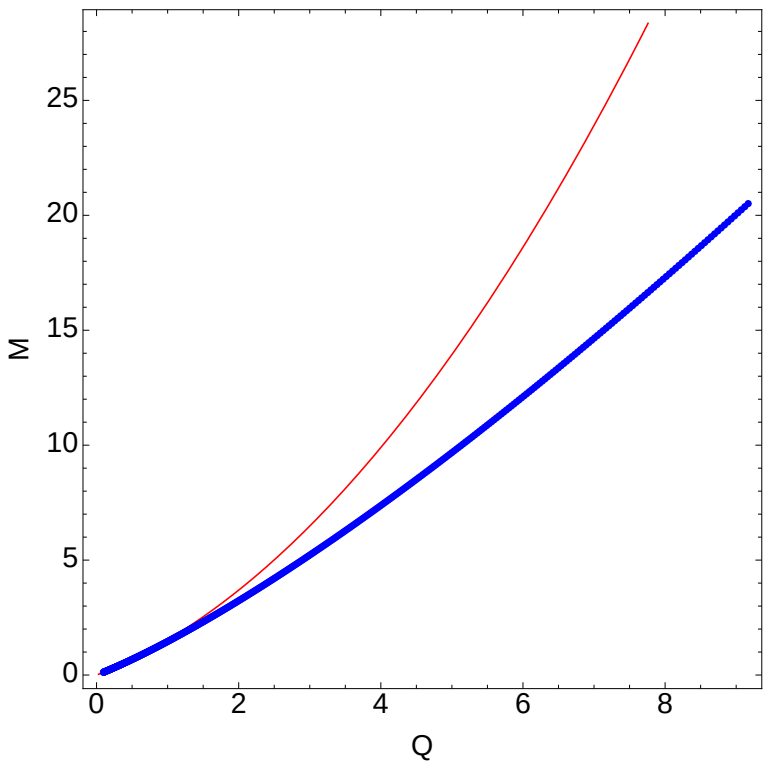


Fix scalar vev ϵ, decrease radius R:
reach singular solution as $T \to 0$,
cf. no regular extremal hairy black hole.
(Fernandez-Gracia, Fiol '09)

$(\epsilon = 0.5\,, \quad e = 3.2)$

Hairy black holes: numerical results II

- For $e^2 > \frac{32}{3}$, find hairy black holes of any size.



Blue - numerical data.
Red - perturbative analysis.
Agreement for small charge.

$(\epsilon = 0.1\,, \quad e = 3.33)$

Fix charge Q, decrease R:

- $Q \leq Q_c(e^2)$: reach soliton at $R = 0$, infinite temperature limit.
- $Q > Q_c(e^2)$: reach singular solution, zero temperature limit.

(see plot later)

Solitons

Solitons: methods

- System to solve is the same.
  Interior boundary conditions change: regularity at origin.

- Complementary methods

  - Perturbatively for $\epsilon \ll 1$: done in Basu et al '10.

$$\phi(r) = \sum_{n=0}^{\infty} \epsilon^{2n+1}\, \phi_{2n+1}(r)\,, \qquad \text{with} \quad \phi_1(r) = \frac{1}{(r^2+1)^2}.$$

    ϕ_1 is lowest normal mode of AdS (we ignore excited states).
    Similar expansions for $A_t(r)$ and metric functions.

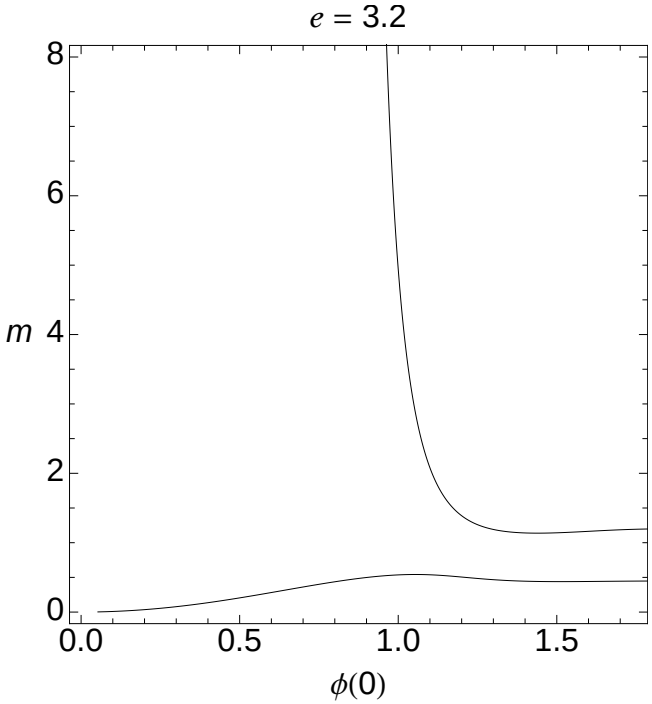
  - Numerically: Newton's method on Chebyshev grid,
    coord $y = r^2/(1+r^2)$, $y \in [0,1]$.
    Use as seed the perturbative result.

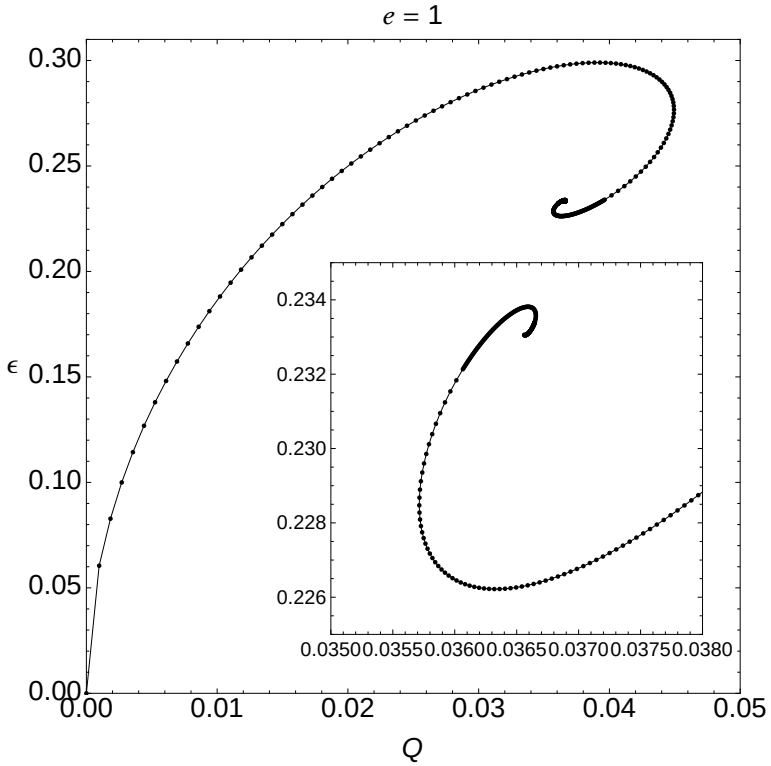
Solitons: numerical results I

For $e^2 < \frac{32}{3}$, find 1-parameter branch of solutions connected to AdS vacuum.

Exists up to maximum charge.

Self-similar behaviour at end point, plane (ϵ, Q).





Second branch connected to zero-temperature limit of planar hairy black hole. (Horowitz, Roberts '09)

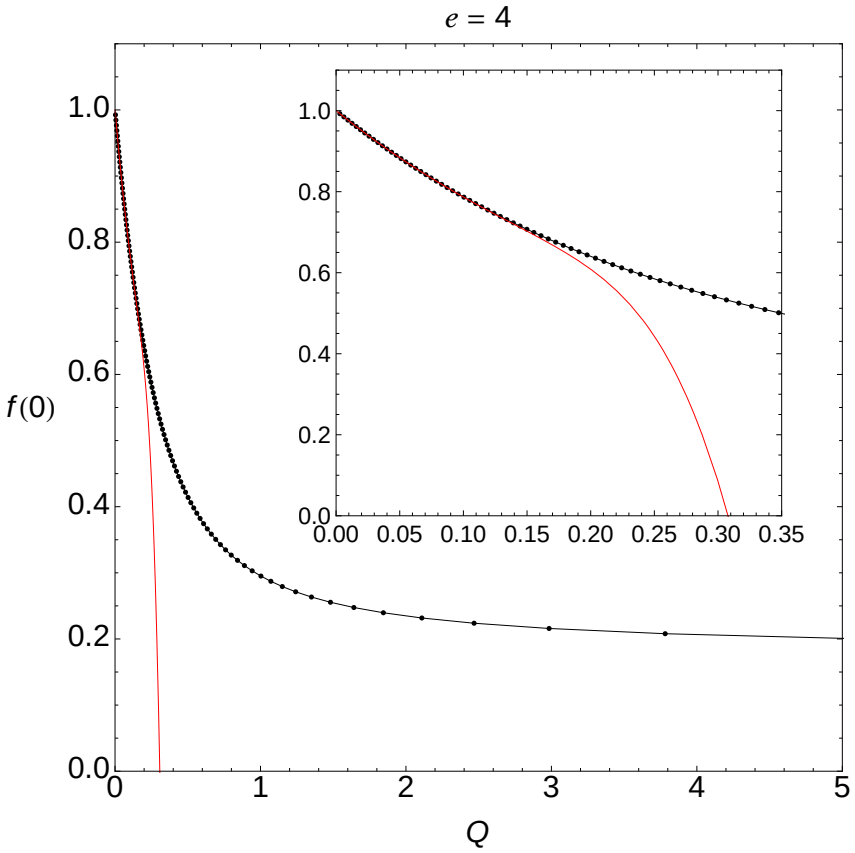
Found by Gentle, Rangamani, Withers '11.

Self-similar behaviour at end point.

Solitons: numerical results II

For $e^2 > \frac{32}{3}$, find 1-parameter branch of solutions connected both to AdS vacuum and to planar limit.

Black - numerical data.
Red - perturbative analysis.

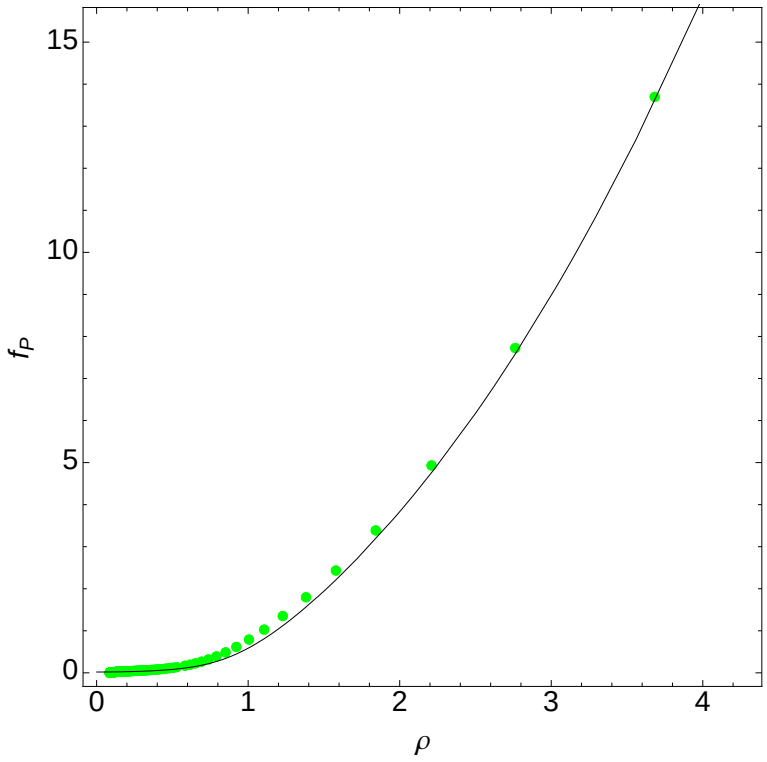


There are also bubbles in parameter space, connected neither to AdS vacuum nor to planar limit. (Gentle, Rangamani, Withers '11)

Solitons: numerical results III

We saw that, for $e^2 > \frac{32}{3}$, solitons of arbitrarily large charge exist.

Planar limit: new coords $r = L\,\rho\,,\ t = \tau/L$. Take limit $L \to \infty$.



Define:

$$f_P(\rho) = \lim_{L\to\infty} \frac{f(\rho L)}{L^2}\,,\ g_P(\rho) = \lim_{L\to\infty} L^2\, g(\rho L)\,,$$

$$A_P(\rho) = \lim_{L\to\infty} \frac{A(\rho L)}{L}\,,\ \phi_P(\rho) = \lim_{L\to\infty} \phi(\rho L)\,.$$

Planar solution:

$$ds_P^2 = -f_p(\rho)d\tau^2 + g_p(\rho)d\rho^2 + \rho^2 d\vec{x}_3^2\,,$$

$$A_P = A_{P\tau}(\rho)d\tau\,, \quad \phi_P = \phi_P(\rho)\,.$$

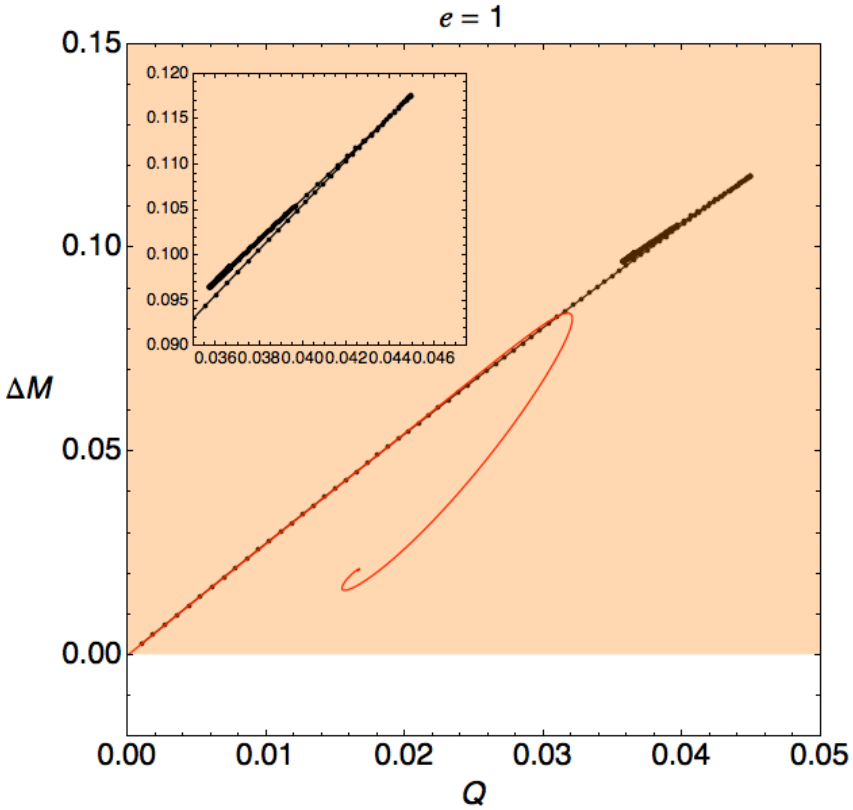
Black: solitons.

Green: "extremal" hairy bhs.

Same planar limit!

Phase diagram

Phase diagram: $e^2 < 3$

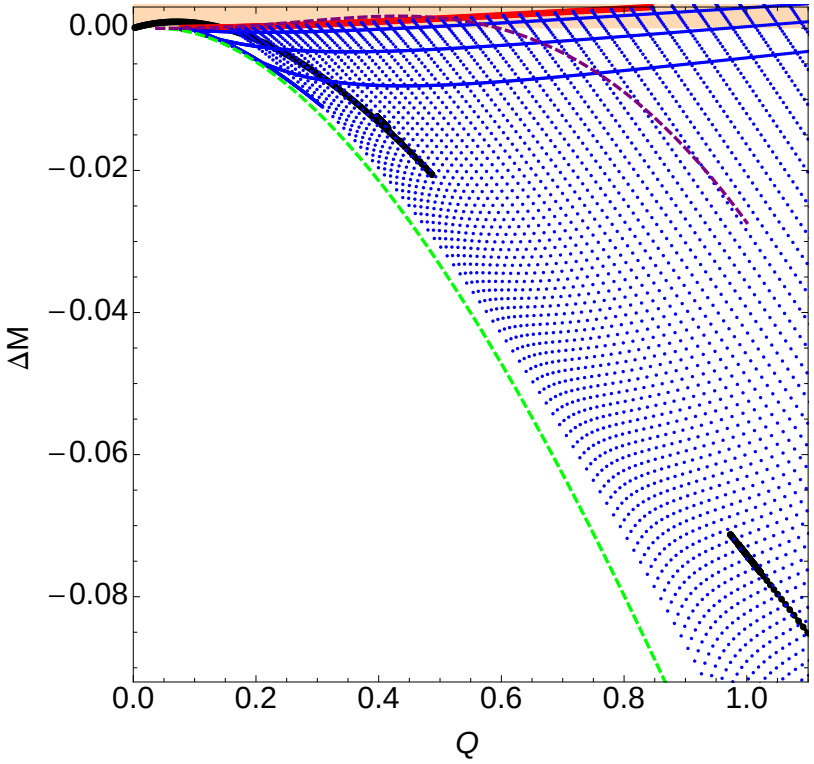


$$\Delta M \equiv M - M^{\text{extremal}}_{\text{RN-AdS}}(Q)$$

$\Delta M > 0$ - RN-AdS black holes.
Black - solitons, numerical.
Red - solitons, perturbative.

No hairy black holes.
Solitons have self-similar behaviour.
(2nd soliton branch not represented)

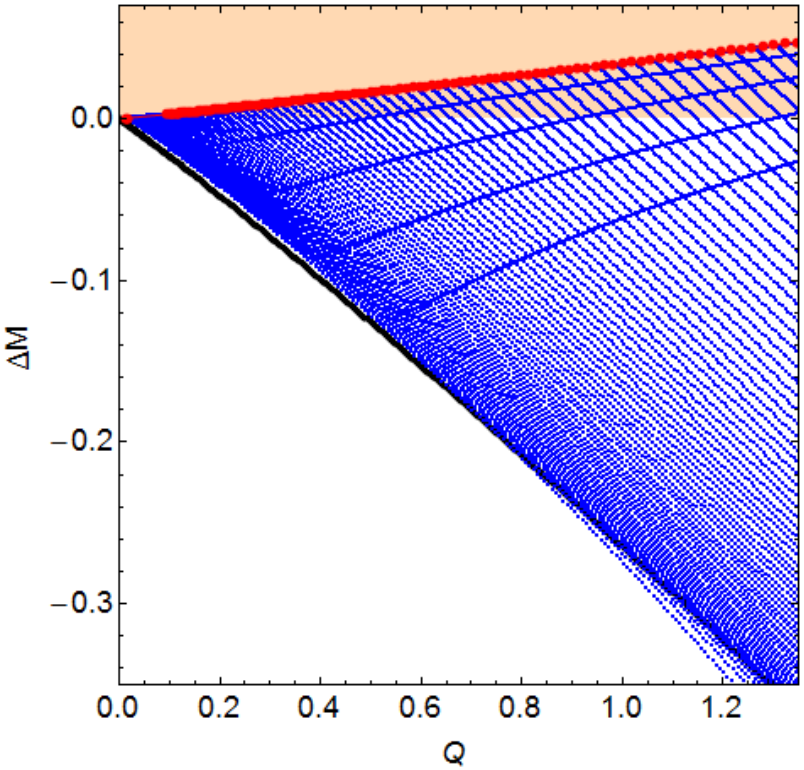
Phase diagram: $3 < e^2 < 32/3$



$\Delta M > 0$ - RN-AdS black holes.
Black - solitons, numerical.
Blue - hairy black holes, numerical.
Green - hairy bound, perturbative.
Red - merger, numerical.
Purple - merger, perturbative.
Grid - vev ϵ versus R

Hairy black holes not small,
have singular extremal limit.
Solitons have self-similar behaviour.

Phase diagram: $e^2 > 32/3$



$\Delta M > 0$ - RN-AdS black holes.
Black - solitons, numerical.
Blue - hairy black holes, numerical.
Red - merger, numerical.
Grid - vev ϵ versus R

Hairy bhs and solitons of any charge. Connected at small Q, then hairy bhs have singular extremal limit: non-analytic lower bound at $Q_c(e^2)$.

Conclusion

- Considered Einstein-Maxwell theory with minimally coupled scalar in global AdS.
- Constructed hairy black holes and solitons.
- Intricate phase diagram!
  - Coincidence at $e^2 = 32/3$: small hairy black holes allowed <u>and</u> meeting of soliton branches.
  - Lowest mass limit of hairy black holes: soliton for $Q < Q_c(e^2)$ when $e^2 > 32/3$; singular solution otherwise.
- In type IIB SUGRA truncation, relevant for vacuum structure of $\mathcal{N} = 4$ SYM at finite $SO(6)$ charge density.