The Holographic Way: String Theory, Gauge Theory and Black Holes

Thermal Brane Probes

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- **G. G., T. Harmark, A. Marini, N. A. Obers and M. Orselli**, "Heating up the BIon", JHEP **1106**, 058 (2011) [arXiv:1012.1494 [hep-th]]
- **G. G., T. Harmark, A. Marini, N. A. Obers and M. Orselli**, "Thermodynamics of the hot BIon", Nucl. Phys. B **851**, 462 (2011) [arXiv:1101.1297 [hep-th]]
- **G. G., T. Harmark, A. Marini, N. A. Obers and M. Orselli**, "Thermal string probes in AdS and finite temperature Wilson loops", [arXiv:1201.4862 [hep-th]]
- **G. G., T. Harmark, A. Marini and M. Orselli**, "Thermal Dirac-Born Infeld Action at Weak and Strong Coupling", [arXiv:1210.xxxx [hep-th]]

Outline

Motivations and Overview

- Subject of talk: New method to describe D-brane and F-string probes in thermal backgrounds in the AdS/CFT correspondence.
  - F-string probes $\Longrightarrow$ Wilson loops.
  - F-string probes $\Longrightarrow$ Quark-gluon plasma (*e.g.* energy loss of a heavy quark, drag force...).
  - D7-brane probes $\Longrightarrow$ Flavors
  - D3-brane/D5-brane probes $\Longrightarrow$ Wilson loops in large sym/antisym rep
  - D3-brane probes on S^3 in AdS_5 or S^5 $\Longrightarrow$ Large operators in gauge theory (Giant gravitons)
- Holographic duals to finite temperature gauge theory
  $AdS_5 \Longrightarrow$ BH in AdS_5
- We need to put also the probe branes and strings at finite temperature
- $\Longrightarrow$ Method of this talk can be used to find new finite temperature effects in holography.

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Zero-temperature backgrounds

D-brane and string probes

1. The action for the string world-sheet is the usual Nambu-Goto action

$$I_{\mathrm{NG}} = \frac{1}{2\pi l_s^2} \int d\tau d\sigma \sqrt{\gamma}$$

$\gamma = \det \gamma_{ab}$.

2. Low energy effective theory for a single D-brane is DBI action (valid for $g_s \ll 1$)

$$I_{\mathrm{DBI}} = -T_{\mathrm{D_p}} \int_{\mathrm{w.v.}} d^p\sigma \sqrt{-\det(\gamma_{ab} + 2\pi l_s^2 F_{ab})} - T_{\mathrm{D_p}} \int_{\mathrm{w.v.}} e^F \wedge C_{(4)}$$

$$\gamma_{ab} = g_{\mu\nu} \partial_a X^\mu \partial_b X^\nu$$

γ_{ab} is the induced metric on the string or brane world-sheet, $g_{\mu\nu}$ the spacetime metric, F_{ab} the two-form field strength living on the D-brane, $C_{(4)}$ is the pull-back to the world-volume of the RR-four form gauge field of the background, T_{Dp} the Dp-brane tension and l_s the string length.

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- Equations of motion for any probe string or brane $\Longrightarrow$ Carter equation

$$\boxed{K_{ab}{}^{\rho} T^{ab} = J \cdot F^{\rho}}$$

  - T_{ab} is the world-sheet energy-momentum tensor for the string,
  - $K_{ab}{}^{\rho}$ is the extrinsic curvature given by the embedding geometry
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- The Carter equation gives the same equation of motion of the DBI and NG actions once the energy momentum tensor is computed as usual by varying the DBI or NG action with respect to the world-volume metric γ_{ab} *i.e.*

$$T^{ab} = \frac{2}{\sqrt{\gamma}} \frac{\delta I_{\rm DBI,NG}}{\delta \gamma_{ab}}$$

where we defined the determinant $\gamma = -\det(\gamma_{ab})$.

- For example for D3 branes we find

$$T^{ab} = -\frac{T_{\rm D3}}{2} \frac{\sqrt{-\det(\gamma + 2\pi l_s^2 F)}}{\sqrt{\gamma}} \left[((\gamma + 2\pi l_s^2 F)^{-1})^{ab} + ((\gamma + 2\pi l_s^2 F)^{-1})^{ba} \right]$$

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Finite-temperature backgrounds

- This is for D-branes or string in zero-temperature string theory backgrounds
- What about the DBI and Nambu-Goto actions as probes of finite temperature backgrounds?

Standard method employed in the last 14 years:

Use the Wick rotated classical DBI or NG action in Euclidean background

1. Take a thermal background
2. go to Euclidean sector
3. put the Wick rotated classical DBI action in this background
4. solve the EOM's setting the temperature of the brane to be equal to that of the background as a boundary condition

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Finite Temperature Backgrounds

- In the applications of the DBI or NG action as a probe of thermal backgrounds the D-brane or string are treated as if the temperature of the background does not affect the physics on the brane or on the string

$\longrightarrow$ the EM tensor that enters in the Carter equation is the same as in the zero-temperature case.

$\Longrightarrow$ the EOM's Wick rotated back to Lorentzian signature are given by $T^{ab}K_{ab}{}^{\sigma} = J \cdot F^{\sigma}$ where T^{ab} is the zero temperature EM tensor

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In more detail $\longrightarrow$ Consider putting a D3-brane without electric and magnetic fluxes on the brane in a thermal background.

The extremal EM tensor is (locally) Lorentz invariant, thus in appropriate coordinates

$$T_{ab} = -T_{\mathrm{D3}}\eta_{ab}$$

But since the thermal background is like a heat bath the electromagnetic DOFs on the brane gets heated up.

For a single D3-brane at small temperatures this is described by $\mathcal{N} = 4$ SYM (QED + SUSY): It is a gas of photons + superpartners on the brane

$$T_{ab} = -T_{\mathrm{D3}}\eta_{ab} + T_{ab}^{(\mathrm{NE})} \,, \qquad T_{00}^{(\mathrm{NE})} = \rho \,, \qquad T_{ii}^{(\mathrm{NE})} = p \,, \quad i = 1, 2, 3$$

Equation of state:

$$\rho = 3p = \frac{\pi^2}{2} T^4$$

Note also: For non-zero temperature the local Lorentz invariance on the brane is broken

We conclude:

The Euclidean DBI probe method is not accurate! Clearly the EM tensor is changed for $T > 0$ and has the form of a gas of gluons.

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Finite-temperature backgrounds

- The thermal background should act as a heat bath for the D-brane probe and the system should attain thermal equilibrium with the D-brane probe gaining the same temperature as the background
- the DOFs living on the brane change the EM tensor of the brane and thus in turn change the EOMs that one should solve for the probe brane.
- challenge $\longrightarrow$ we do not know what replaces the DBI action, which is a low energy effective action for a single extremal D-brane at weak string coupling, when turning on the temperature.
- effective description of D-branes: strongly coupled regime $\longrightarrow$ in the regime of a large number N of coinciding D-brane probes in terms of a supergravity solution in the bulk when $g_s N \gg 1$, g_s being the string coupling.
- weakly coupled regime:$\longrightarrow$ regime in which $g_s N \ll 1$ so that the D-branes are not sufficiently heavy to backreact on the background geometry $\longrightarrow$ Marini's talk.
- Using the supergravity description one can determine the EM tensor for the D-brane in the regime of large N, for any value of the temperature.

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- The supergravity EM tensor enables to write down the EOMs given by the Carter equation for a non-extremal D-brane probe in the regime of large N.
- new method $\longrightarrow$ replace the DBI action by another description that can describe N coincident non-extremal D-branes probing a thermal background such that the probe is in thermal equilibrium with the background.
- method based on the blackfold approach $\longrightarrow$ general framework to describe black branes in the probe approximation

  [Emparan, Harmark, Niarchos, Obers (2009)]
- action principle $\longrightarrow$ for stationary blackfolds the extrinsic blackfold equations $\equiv$ Carter equations can be obtained from an action which is proportional to the Gibbs free energy.

$$\mathcal{F} = M - TS$$

  where $\mathcal{F} = \mathcal{F}(T, N, k)$ is the free energy appropriate for the ensemble where the temperature T and number of D-branes N and F-strings k are fixed. the total mass M and entropy S are found by integrating the energy density T^{00} and the entropy density S over the D-brane worldvolume.
- for stationary blackfolds the extrinsic equations are equivalent to requiring the first law of thermodynamics.

details

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- action principle $\longrightarrow$ for stationary blackfolds the extrinsic blackfold equations $\equiv$ Carter equations can be obtained from an action which is proportional to the Gibbs free energy.

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  where $\mathcal{F} = \mathcal{F}(T, N, k)$ is the free energy appropriate for the ensemble where the temperature T and number of D-branes N and F-strings k are fixed. the total mass M and entropy S are found by integrating the energy density T^{00} and the entropy density S over the D-brane worldvolume.
- for stationary blackfolds the extrinsic equations are equivalent to requiring the first law of thermodynamics.

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Applications

- BIon solution $\Longrightarrow$ the first example of a solution of the DBI action that contains full non-linear dynamics of the DBI action $\Longrightarrow$ new phenomena introduced.
- thermal generalization of the string solution dual to a rectangular Wilson loop that provides the energy of a quark anti-quark pair of $\mathcal{N} = 4$ SYM in 4D at finite temperature
- Thermal fundamental string probes: a new term in the potential between static quarks which for sufficiently low temperatures is the leading correction to the Coulomb force potential.
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Outline

1. Motivations and Overview
2. BIon
3. Wilson loops at finite temperature: standard method
4. Wilson loop at finite temperature: new method
5. Physics of the rectangular Wilson loop
6. Charges in a hot plasma wind
7. Summary and Outlook

Outline

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D-brane at zero temperature

- Callan and Maldacena exploited for the first time the non-linearity of the DBI action to show how a F-string "dissolves" into a D-brane. Consider a D3!
- D3-brane in flat background

$$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) + \sum_{i=1}^{6} dx_i^2$$

Worldvolume coordinates: $\tau = t$, $\sigma \equiv \sigma^1 = r$, $\sigma^2 = \theta$, $\sigma^3 = \phi$

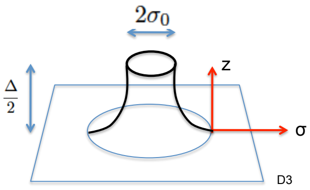
DBI Lagrangian in presence of an electric field $E(\sigma)$ along σ direction and with only one excited transverse coordinate, say $z(\sigma) = x^1(\sigma)$

$$\mathscr{L}_{\rm CM} = -T_{\rm D3}\int d^3\sigma\sqrt{1 - E(\sigma)^2 + z'(\sigma)^2}$$

EOM: canonical momentum density

$k \equiv 4\pi\sigma^2\Pi(\sigma) = \delta L/\delta(\partial_\tau A_1)$; $\kappa \equiv \frac{kT_{\rm F1}}{4\pi N T_{\rm D3}}$

$$\partial_\sigma\left[\sigma^2\frac{z'(\sigma)\left(1+\frac{\kappa^2}{\sigma^4}\right)}{\sqrt{\left(1+z'(\sigma)^2\right)\left(1+\frac{\kappa^2}{\sigma^4}\right)}}\right] = 0$$



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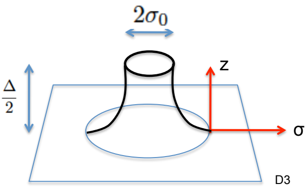

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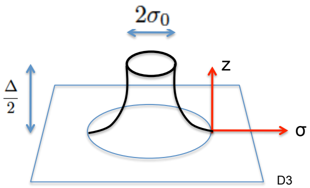

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Callan and Maldacena - Spike solution

Solution of the EOM $\Rightarrow$ BIon solution

$$z(\sigma) = \int_\sigma^\infty \frac{B}{\sqrt{r^4 - (B^2 - \kappa^2)}} dr = \int_\sigma^\infty \frac{\sqrt{\sigma_0^4 + \kappa^2}}{\sqrt{r^4 - \sigma_0^4}} dr$$

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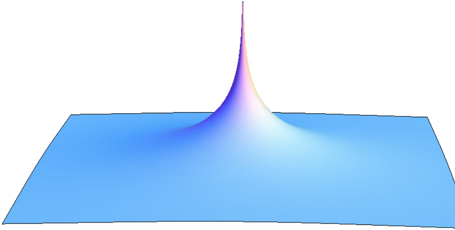
If $\sigma_0 = 0 \quad \Rightarrow \quad z(\sigma) = \frac{\kappa}{\sigma}$ Spike



The energy of the spike $\propto$ energy of the F-string; the proportionality factor is $k = 4\pi T_{D3}\kappa \Rightarrow k$ F-string attached to the brane

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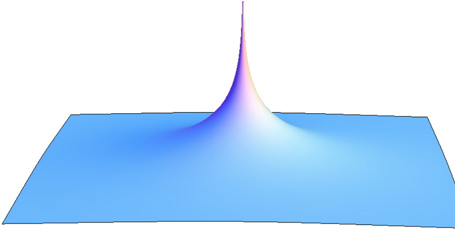

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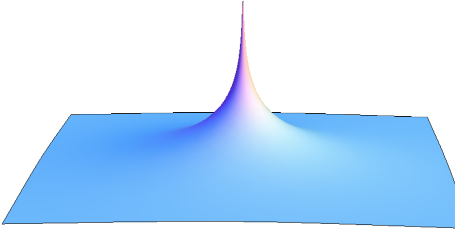

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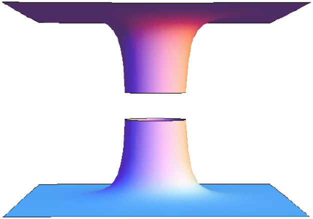
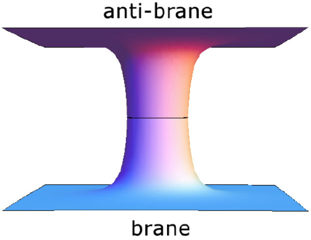
When $\sigma_0 > 0 \Rightarrow$ the brane cannot end at σ_0 because of charge conservation $\Rightarrow$ we attach a mirror of the solution $\Rightarrow \tilde{z}(\sigma) = 2z(\sigma_0) - z(\sigma) \Rightarrow$ "brane-antibrane" configuration





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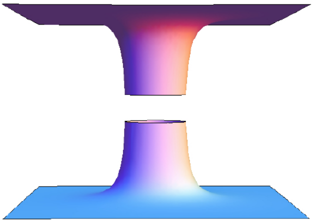

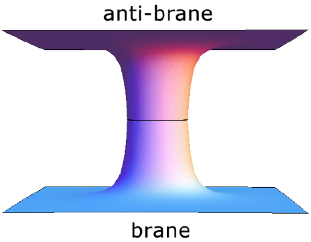

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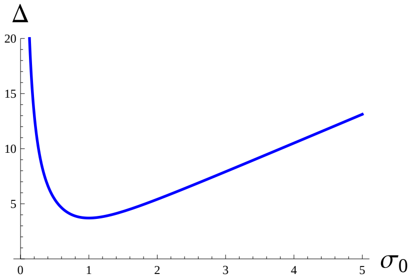
Define Δ the distance between the two branes

$$\Delta \equiv 2z(\sigma_0) = \frac{2\sqrt{\pi}\Gamma(\frac{5}{4})\sqrt{\sigma_0^4 + \kappa^2}}{\Gamma(\frac{3}{4})\sigma_0}$$

Δ reaches a minimum for $\sigma_0 = \kappa$



Δ as a function of σ_0 for $\kappa = 1$

Fixing Δ and the number of strings (*i.e.* κ) there exist two solution for σ_0

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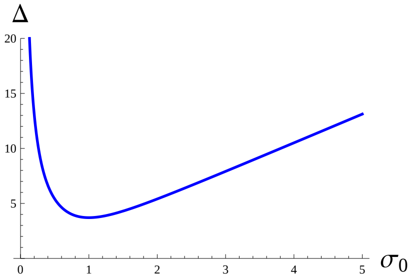

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How to heat up the BIon?

D3-brane DBI action $\Rightarrow$ $S = -T_{\rm D3} \int d^4\sigma \sqrt{-\det(\gamma_{ab} + 2\pi l_s^2 F_{ab})}$

The general EOM is a generalization of the geodesic equation for point particles $\Rightarrow$

Carter equation

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Energy-momentum tensor

- This EM tensor can be obtained from the supergravity solution describing non extremal black-branes (we are using the *blackfold method*)

For a system of N D3-brane at finite temperature

$$ds^2 = D^{-\frac{1}{2}} H^{-\frac{1}{2}}(-fdt^2 + dx_1^2) + D^{\frac{1}{2}} H^{-\frac{1}{2}}(dx_2^2 + dx_3^2) + D^{-\frac{1}{2}} H^{\frac{1}{2}}(f^{-1}dr^2 + r^2 d\Omega_5^2)$$

$$f = 1 - \frac{r_0^4}{r^4} \quad H = 1 + \frac{r_0^4 \sinh^2\alpha}{r^4} \quad D^{-1} = \cos^2\zeta + \sin^2\zeta H^{-1}$$

$$T^{00} = -\gamma^{00}\frac{\pi^2}{2}T_{\mathrm{D3}}^2 r_0^4(5 + 4\sinh^2\alpha) \quad T^{11} = -\gamma^{11}\frac{\pi^2}{2}T_{\mathrm{D3}}^2 r_0^4(1 + 4\sinh^2\alpha)$$

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Small temperatures

$$T^{00} = NT_{\mathrm{D3}} + \frac{3\pi^2}{8}N^2T^4\,, \quad T^{ii} = \gamma^{ii}(-NT_{\mathrm{D3}} + \frac{\pi^2}{8}N^2T^4)\,, \quad i = 1,2,3$$

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Hot Blon

In a flat background the Carter equation becomes

$$\frac{z''(\sigma)}{z'(\sigma)(1+z'(\sigma)^2)} = -\frac{2}{\sigma}\frac{1+4\cos^2\zeta(\sigma)\sinh^2\alpha(\sigma)}{1+4\sinh^2\alpha(\sigma)}$$

This equation can be derived from the following action

$$\mathcal{F} = M - TS = \frac{2T_{\mathrm{D3}}^2}{\pi T^4}\int_{\sigma_0}^{\infty} d\sigma\sqrt{1+z'(\sigma)^2}F(\sigma)$$

where $F(\sigma) = \sigma^2\frac{4\cosh^2\alpha - 3}{\cosh^4\alpha}$ and the temperature and entropy density are

$$T = \frac{1}{\pi r_0 \cosh\alpha}\ , \quad \mathcal{S} = 2\pi^3 T_{\mathrm{D3}}^2 r_0^5 \cosh\alpha$$

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Constraints and bounds

- Beyond the EOM we also have further constraints that follow from charge conservation of both D3-brane and F-string charge

- The number of D3-branes in the bound state is N

$$\cos\zeta\, r_0^4 \cosh\alpha \sinh\alpha = \frac{N}{2\pi^2 T_{\mathrm{D3}}}$$

- imposing that we have k F-strings gives

$$\frac{k}{N} = \frac{T_{\mathrm{D3}}}{T_{\mathrm{F1}}} \int_{V_{23}} d\sigma^2 d\sigma^3 \sqrt{\gamma_{22}\gamma_{33}} \tan\zeta \Longrightarrow \kappa \equiv \frac{k T_{\mathrm{F1}}}{4\pi N T_{\mathrm{D3}}} = \sigma^2 \tan\zeta$$

- Consequently

- $\exists$ maximum temperature $T_{\mathrm{bnd}} \equiv \left(\frac{4\sqrt{3}T_{\mathrm{D3}}}{9\pi^2 N}\right)^{1/4} \Rightarrow$ define $\bar{T} \equiv \frac{T}{T_{\mathrm{bnd}}}$
- $\exists$ minimum $\sigma \Rightarrow \sigma \geq \sigma_{\mathrm{min}} \equiv \sqrt{\kappa}\left(\bar{T}^{-8} - 1\right)^{-1/4}$

Constraints and bounds

- Beyond the EOM we also have further constraints that follow from charge conservation of both D3-brane and F-string charge

- The number of D3-branes in the bound state is N

$$\cos\zeta\, r_0^4 \cosh\alpha \sinh\alpha = \frac{N}{2\pi^2 T_{\mathrm{D3}}}$$

- imposing that we have k F-strings gives

$$\frac{k}{N} = \frac{T_{\mathrm{D3}}}{T_{\mathrm{F1}}} \int_{V_{23}} d\sigma^2 d\sigma^3 \sqrt{\gamma_{22}\gamma_{33}} \tan\zeta \Longrightarrow \kappa \equiv \frac{k T_{\mathrm{F1}}}{4\pi N T_{\mathrm{D3}}} = \sigma^2 \tan\zeta$$

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The solution is

$$z(\sigma) = \int_{\sigma}^{\infty} dr \left(\frac{F(r)^2}{F(\sigma_0)^2} - 1 \right)^{-\frac{1}{2}}$$

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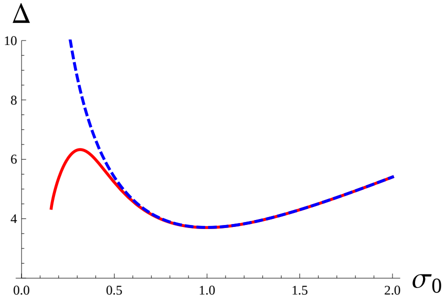

— $\Delta(\sigma_0)$ for $\kappa = 1$ and $\bar{T} = 0.4$

- - Callan and Maldacena ($\bar{T} = 0$)

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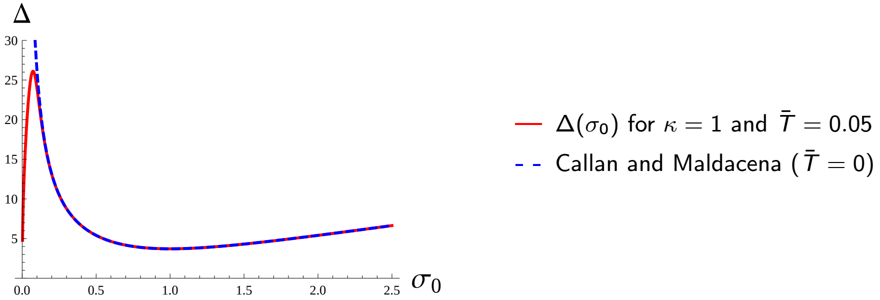

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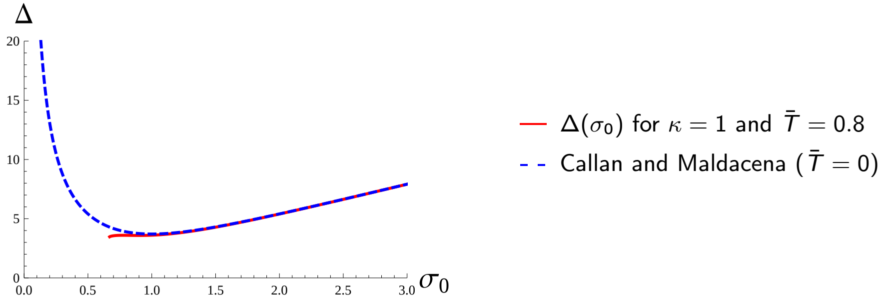

Analysis of the solution

The main differences with the zero temperature case are:

- $\exists$ lower bound on σ_0 which avoids the possibility to have a spike solution
- Three different phases for a fixed values of Δ

Figure: The free energy $\mathcal{F}$ versus Δ for $\bar{T} = 0.4$ and $\kappa = 1$.

- Computing the free energy one finds that the energetically favored phase is the one between the maximum and the minimum of Δ
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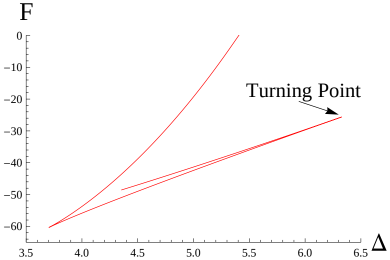

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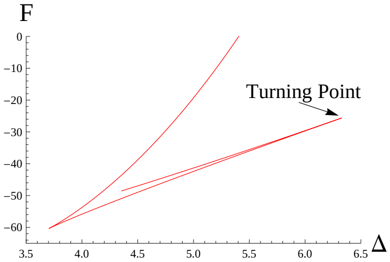

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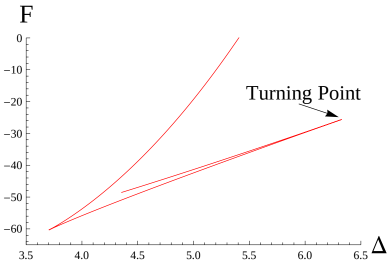

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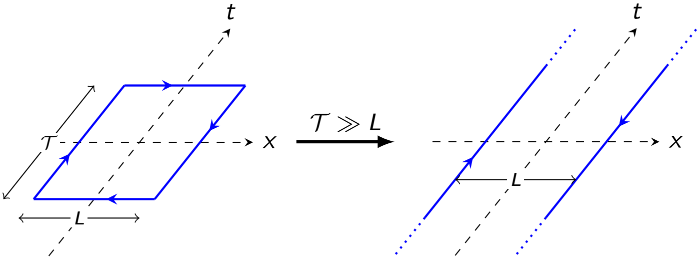
Outline

Wilson loops at finite temperature

- The Wilson loops in $\mathcal{N} = 4$ SYM include a coupling to the scalar fields

$$W(\mathcal{C}) = \frac{1}{N} \mathrm{Tr} \mathcal{P} \exp \left[\oint \left(A_\mu \dot{x}^\mu + |\dot{x}| \theta^I \Phi_I \right) ds \right]$$

- The potential between very massive quarks is necessary to compute Wilson loops in the $U(N)$ theory. For a pair of antiparallel lines (rectangular Wilson loop)



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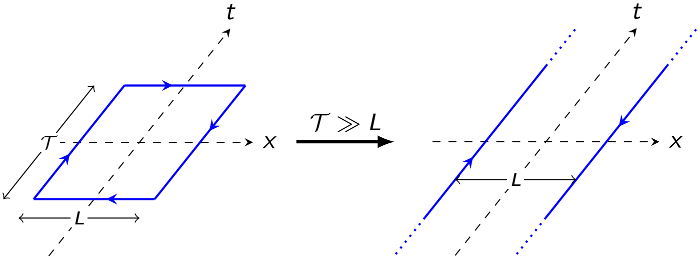


Rectangular WL with $\mathcal{T} \gg L$

- This configuration corresponds to a static quark-antiquark ($q\bar{q}$) pair separated by a distance L

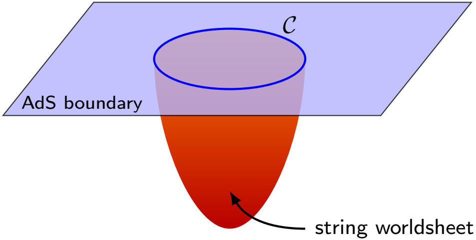
- The potential energy $V_{q\bar{q}}(L)$ of the pair is related to the expectation value of the WL
$$\langle W(\mathcal{C})\rangle \approx \exp[-\mathcal{T}\, V_{q\bar{q}}(L)]$$

- Explicit calculations at weak and strong couplings give

$$V_{q\bar{q}}(L,\lambda) = \begin{cases} -\frac{\lambda}{4\pi L} + \frac{\lambda^2}{8\pi^2 L}\ln\frac{T}{L} + \dots, & \lambda \ll 1; \\ -\frac{4\pi^2\sqrt{\lambda}}{\Gamma\left(\frac{1}{4}\right)^4 L}\left(1 - \frac{1.3359\ldots}{\sqrt{\lambda}} + \dots\right), & \lambda \gg 1 \end{cases}$$

Wilson loop at strong coupling

- In the AdS/CFT correspondence a Wilson loop is dual to a fundamental string (F-string) probe with its world-sheet extending into the bulk of Anti-de Sitter space (AdS) and ending at the location of the loop on the boundary of AdS [Rey, Yee(1998); Maldacena(1998)] .



Wilson loop at strong coupling

For $N \gg 1$, $\lambda \gg 1$ $\longrightarrow$ $\langle W(\mathcal{C})\rangle = e^{-S(\mathcal{C})}$

$S(\mathcal{C})$ = classical string action $\longrightarrow$ extremize the Nambu-Goto action for the string worldsheet

$$S_{\rm NG} = \frac{1}{2\pi l_s^2}\int d\tau d\sigma \sqrt{\gamma} \qquad \gamma_{ab} = g_{\mu\nu}\partial_a X^\mu \partial_b X^\nu$$

$\gamma = \det \gamma_{ab}$, γ_{ab} is the induced metric on the string world-sheet and $g_{\mu\nu}$ the spacetime metric

- However the classical NG action provides an effective description of the F-string probe only at zero temperature
- Standard method at finite temperature: Wick rotated classical Nambu-Goto action in Euclidean background

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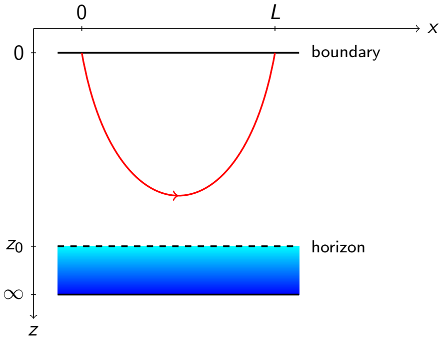
Background and induced metric

- Rectangular Wilson loop in $\mathcal{N} = 4$ SYM on $S^1 \times \mathbb{R}^3$, the S^1 meaning that it is at finite temperature.
- Finite temperature background $\longrightarrow$ we want to probe AdS black hole in the Poincaré patch times a five-sphere.
- The metric of this background is

$$ds^2 = \frac{R^2}{z^2}(-f\, dt^2 + dx^2 + dy_1^2 + dy_2^2 + f^{-1}dz^2) + R^2 d\Omega_5^2\ , \quad f(z) = 1 - \frac{z^4}{z_0^4}$$

where R is the AdS radius, the boundary of AdS is at $z = 0$ and the event horizon is at $z = z_0$. The temperature of the black hole as measured by an asymptotic observer is $T = 1/(\pi z_0)$.

- Ansatz for the embedding of the F-string probe: $t = \tau \equiv \sigma^0$, $z = \sigma \equiv \sigma^1$, $x = x(\sigma)$
- Static string probe between the point charge Q at $(x, y_1, y_2) = (0, 0, 0)$ and the point charge $\bar{Q}$ at $(x, y_1, y_2) = (L, 0, 0)$ on the boundary of AdS at $z = 0$.



The induced metric and redshift factor for this embedding are

$$\gamma_{ab} d\sigma^a d\sigma^b = \frac{R^2}{\sigma^2}\left[-f\, d\tau^2 + \left(f^{-1} + x'^2\right) d\sigma^2\right] \ , \quad R_0(\sigma) = \frac{R}{\sigma}\sqrt{f(\sigma)}$$

- The redshift factor R_0 induces a local temperature $T_{\rm local} = T/R_0$ which is the temperature that the static string probe locally is subject to, T is the global temperature of the background space-time as measured by an asymptotic observer Law (1930).

- With this embedding the Nambu-Goto action becomes

$$S = \frac{R^2}{2\pi l_s^2}\int \frac{d\sigma}{\sigma^2}\sqrt{1+f(\sigma)x'(\sigma)^2}$$

- x is a cyclic variable, its conjugate momentum is conserved

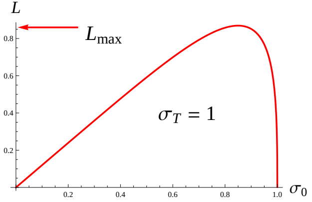
$$\frac{f(\sigma)x'(\sigma)}{\sigma^2\sqrt{1+f(\sigma)x'(\sigma)^2}} = const = \frac{\sqrt{f(\sigma_0)}}{\sigma_0^2}$$

- x can now be expressed as a function of σ

$$x(\sigma) = \frac{\sigma_0\sqrt{\sigma_T^4-\sigma_0^4}}{\sigma_T^2}\int_0^{\sigma/\sigma_0}\frac{y^2dy}{\sqrt{(1-y^4)\left(1-\frac{\sigma_0^4}{\sigma_T^4}y^4\right)}} \qquad \sigma_T = \frac{1}{\pi T}$$

The integration constant σ_0 can be related to L, the distance between the quark and the anti-quark

$$L = \frac{2\sigma_0\sqrt{\sigma_T^4-\sigma_0^4}}{\sigma_T^2}\int_0^1\frac{y^2dy}{\sqrt{(1-y^4)\left(1-\frac{\sigma_0^4}{\sigma_T^4}y^4\right)}}$$
$$= \frac{(2\pi)^{3/2}\sigma_0\sqrt{\sigma_T^4-\sigma_0^4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{5}{4};\frac{\sigma_0^4}{\sigma_T^4}\right)}{\sigma_T^2\Gamma\left(\frac{1}{4}\right)^2}$$

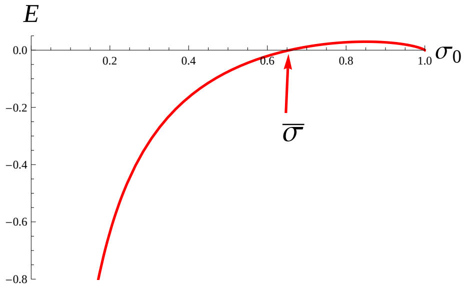


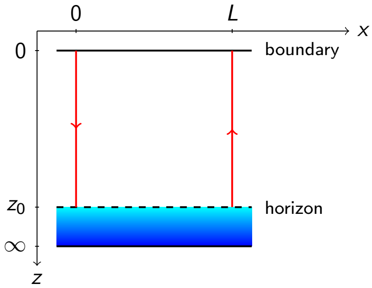
There is a maximum length L_{max}!

Energy

To obtain a finite result from the action S computed on the solution, subtract the (infinite) mass of the quarks which corresponds to two strings stretched between the boundary at $z = 0$ and the horizon at $z = z_0 \Longrightarrow \sigma = \sigma_T$

$$E = \frac{\sqrt{2\pi} R^2 \left(\sigma_0^4 - \sigma_T^4\right) {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{1}{4}; \frac{\sigma_0^4}{\sigma_T^4}\right)}{l_s^2 \sigma_0^4 \sigma_T^4 \Gamma\left(\frac{1}{4}\right)^2} + \frac{R^2}{\pi l_s^2 \sigma_T}$$

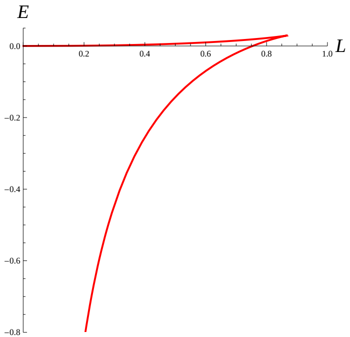


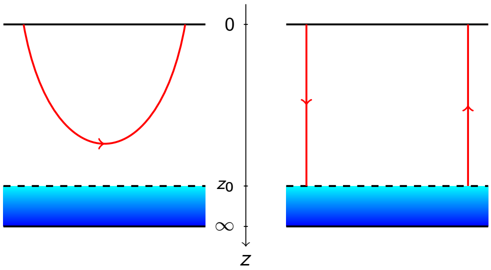


There is a critical $\overline{\sigma}$ at which the energy of the string configuration is the same as that of a pair of free quark and anti-quark with asymptotically zero force between them.

$\overline{\sigma}$ is reached before the L reaches its maximal value L_{max} value.

Static energy $E(L,T)$ between the "quark" and the "anti-quark" $\Longrightarrow$ eliminate σ_0 in terms of L and T.





Small temperatures $\Longrightarrow$ solve perturbatively for σ_0 as a function of L and get (the first term is the zero temperature Maldacena result)

$$E = -\frac{\sqrt{\lambda}}{L}\left(\frac{4\pi^2}{\Gamma\left(\frac{1}{4}\right)^4} + \frac{3}{160}(LT)^4\Gamma\left(\frac{1}{4}\right)^4 + O\left((LT)^8\right)\right)$$

Physical picture: for a given temperature T we encounter two region with different behavior. For $L \ll 1/T \Longrightarrow$ Coulomb like potential. For $L \gg 1/T$ the quarks become free due to screening by the thermal bath.

Outline

NG string as a probe of thermal backgrounds

Equations of motion for any probe brane ⟶ Carter equation

extrinsic curvature

worldsheet EM tensor

$$K_{ab}{}^{\rho}\, T^{ab} = J \cdot F^{\rho}$$

possible external forces

- For a string probe ⟶ NG EOM ≡ Carter eq with T^{ab} the same EM tensor as in the zero-temperature (extremal) case
  - NG action does not take into account the thermal excitations of the string

NG is not accurate for the description of F-string probes in thermal backgrounds. For an AdS black hole the classical NG action is an increasingly inaccurate description as one approaches the event horizon since by the Tolman Law the local temperature for a static string probe is redshifted towards infinity.

- Thermal equilibrium ⟶ the string probe gains the same temperature as the background
- String DOF's are "heated up" by the temperature of the background ⟶ this changes the EM tensor

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Thermal F-string probe

New method

Replace the Energy-momentum tensor in the Carter equation with that of a thermal F-string

Thermal F-string probe $\Longrightarrow$ SUGRA solution of k coincident black F-strings in type IIB SUGRA in 10D Minkowski space.

Energy-momentum tensor

$$T_{00} = A r_0^6 (7 + 6\sinh^2\alpha)\ , \qquad T_{11} = -A r_0^6 (1 + 6\sinh^2\alpha)$$

with $A = \Omega_7/(16\pi G)$.

Temperature

$$\frac{3}{2\pi r_0 \cosh\alpha}$$

Entropy density and charge

$$\mathcal{S} = 4\pi A r_0^7 \cosh\alpha\ , \qquad Q = k T_{\rm F1} = 6 A r_0^6 \cosh\alpha \sinh\alpha$$

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Embedding of the F-string

Static F-string probe between Q and $\bar{Q}$

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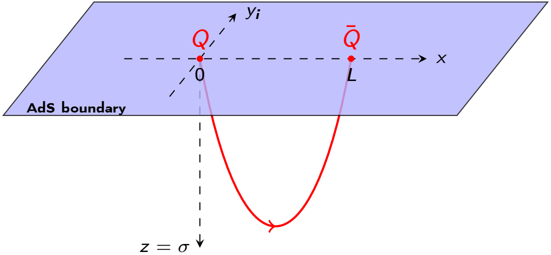

- The induced metric γ_{ab} for this embedding is

$$\gamma_{ab} d\sigma^a d\sigma^b = \frac{R^2}{\sigma^2}\left[-f\, d\tau^2 + \left(f^{-1} + x'^2\right) d\sigma^2\right]$$

Embedding of the F-string

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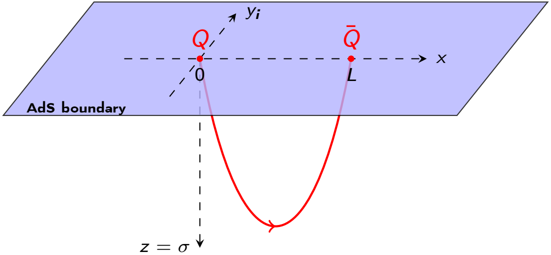

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Local temperature and thermal equilibrium

- Redshift factor R_0

$$R_0(\sigma) = \sqrt{-\gamma_{00}} = \frac{R}{\sigma}\sqrt{f(\sigma)}$$

- Local temperature $T_{\rm local}$: temperature the static string probe locally is locally subject to (Tolman law);
  $T \longrightarrow$ global temperature of the background space-time as measured by an asymptotic observer

[Tolman (1930); Tolman, Ehrenfest (1930)]

$$T_{\rm local} = \frac{T}{R_0}$$

- Thermal equilibrium between the F-string probe and the background

$$\frac{T}{R_0} = \frac{3}{2\pi r_0 \cosh\alpha}$$

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Action principle and EOM

For a stationary blackfold $\longrightarrow$ action = free energy

Free energy for a SUGRA F-string probe in a general background with redshift factor R_0

$$\mathcal{F} \equiv M - TS = A \int dV_{(1)} R_0 r_0^6 (1 + 6 \sinh^2 \alpha)$$

With the setup defined before the free energy for the thermal F-string probe

$$\mathcal{F} = A \left(\frac{3}{2\pi T} \right)^6 \int d\sigma \sqrt{1 + f(\sigma) x'(\sigma)^2} G(\sigma) \,, \quad G(\sigma) \equiv \frac{R^8}{\sigma^8} f(\sigma)^3 \frac{1 + 6 \sinh^2 \alpha(\sigma)}{\cosh^6 \alpha(\sigma)}$$

Varying with respect to $x(\sigma)$ this gives the EOM

$$\left(\frac{f(\sigma) x'(\sigma)}{\sqrt{1 + f(\sigma) x'(\sigma)^2}} G(\sigma) \right)' = 0$$

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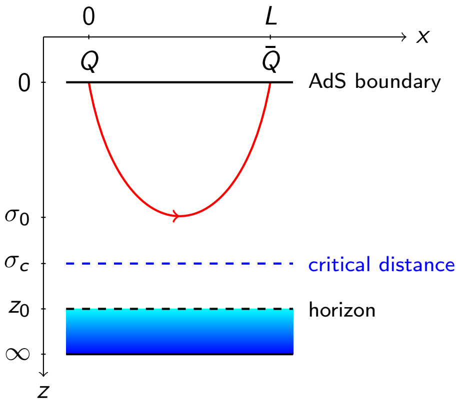
Solution

Boundary conditions: $x(0) = 0$ and $\lim_{\sigma \to \sigma_0} x'(\sigma) = \infty$

general solution $\longrightarrow$ $$x'(\sigma) = \left(\frac{f(\sigma)^2 G(\sigma)^2}{f(\sigma_0) G(\sigma_0)^2} - f(\sigma) \right)^{-\frac{1}{2}}$$



Introduce the dimensionless coordinate $\hat{\sigma} = \pi T \sigma$

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with $f(\hat{\sigma}) = 1 - \hat{\sigma}^4$, $\hat{\sigma}_0 = \pi T \sigma_0$,

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- LT depends only on the dimensionless quantities κ and $\hat{\sigma}_0$
- The equation for κ enforces the charge conservation and can be used to find $\alpha(\hat{\sigma})$
- In terms of the gauge theory variables k, λ and N, κ can be written

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"Screened" solution

Another type of solution → $x'(\sigma) = 0$ trivially solves the EOM

- Disconnected configuration → two straight strings stretching from the charges Q and $\bar{Q}$ towards the horizon
- In the gauge theory side this corresponds to two Polyakov loops, *i.e.* Wilson lines along the thermal circle
- Pair of free charges Q and $\bar{Q}$ → screened pair

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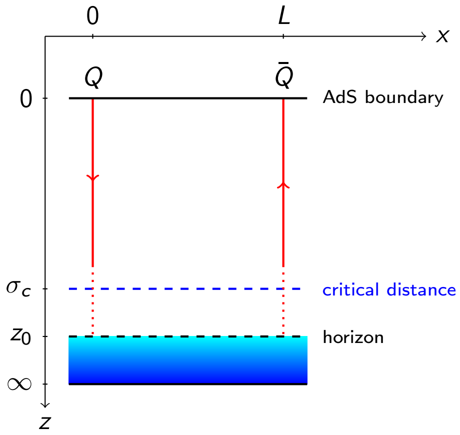

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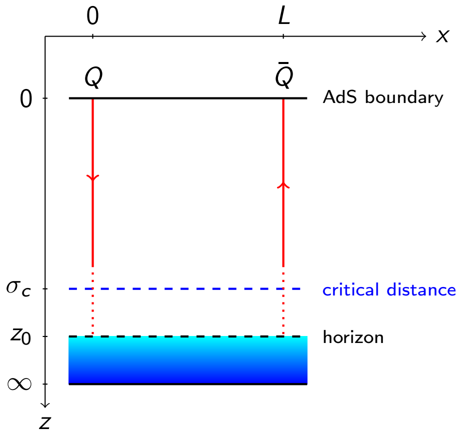

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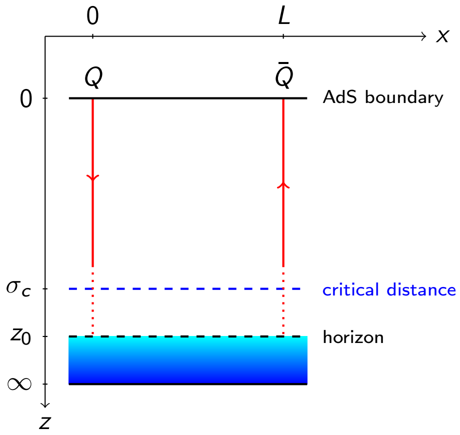

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Critical distance From the equation for κ and the fact that $\sinh\alpha/\cosh^5\alpha \leq 2^4/5^{5/2}$ for a given κ the equation can only be satisfied provided

$$\hat{\sigma} \leq \hat{\sigma}_c \quad \text{with } \hat{\sigma}_c^2 = \sqrt{1 + \frac{5^{5/3}}{2^{14/3}}\kappa^{\frac{2}{3}}} - \frac{5^{5/6}}{2^{7/3}}\kappa^{\frac{1}{3}}$$

- $\hat{\sigma}_c \leq 1 \longrightarrow$ one reaches the critical distance $\hat{\sigma}_c$ before reaching the horizon
- Physical interpretation: F-string probe is in thermal equilibrium with the background
  - The SUGRA F-string has a maximal temperature for a given k
  - Tolman law $\longrightarrow$ the local temperature goes to infinity as one approaches the black hole
- This is a qualitatively new effect which means that the probe description breaks down beyond the critical distance $\hat{\sigma}_c$

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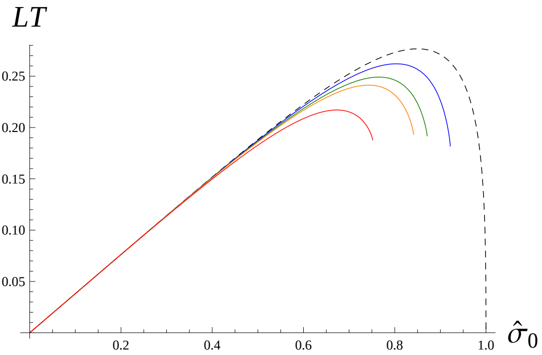
Outline

LT as a function of $\hat{\sigma}_0$

Plot of LT as a function of $\hat{\sigma}_0$ for various values of κ:



$\kappa = 0.01$ (blue line)
$\kappa = 0.001$ (red line)
$\kappa = 0.0001$ (green line)
extremal probe (black dashed line)

- For a given κ there exists solutions for $\hat{\sigma}_0 \in [0, \hat{\sigma}_c(\kappa)]$
- LT always shows a maximum $(LT)_{\max}$
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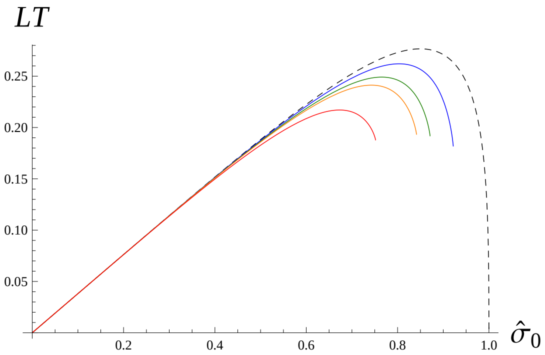

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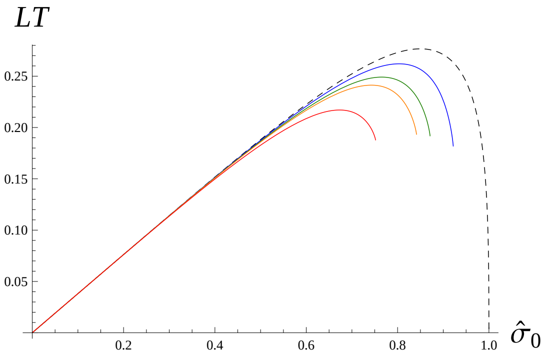

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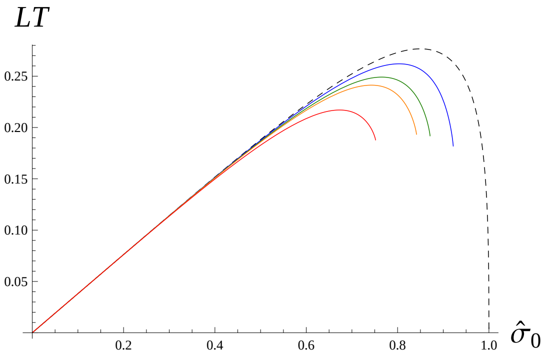

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The extremal F-string probe corresponds to $\kappa = 0$

$$LT|_{\kappa=0} = \frac{2\sqrt{2\pi}}{\Gamma\left(\frac{1}{4}\right)^2}\hat{\sigma}_0\sqrt{1-\hat{\sigma}_0^4}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{5}{4};\hat{\sigma}_0^4\right)$$

$\longrightarrow$ matches with the result found using the NG string probe

[Brandhuber et al. (1998); Rey et al. (1998)]

LT for small $\hat{\sigma}_0 = \pi T \sigma_0$ for $\kappa \ll 1$

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- Leading term $\longrightarrow$ probing the zero temperature AdS background

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The SUGRA F-string probe induces both qualitative and quantitative differences

- Qualitative difference
  - extremal probe ⟶ two solutions available for given $LT < (LT)_{\max}$
  - thermal F-string probe ⟶ only one solution for $LT < LT|_{\hat{\sigma}_c}$

extremal probe

thermal probe $\kappa = 0.01$

- Quantitative differences
  - in the dependence of LT on $\hat{\sigma}_0$
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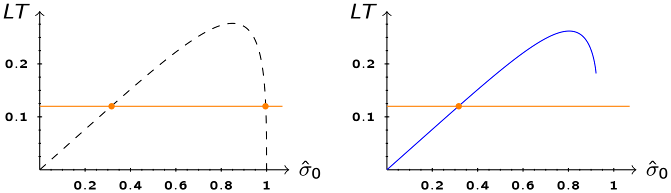

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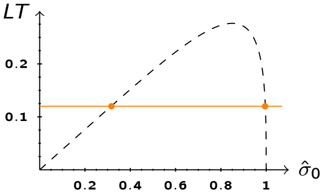
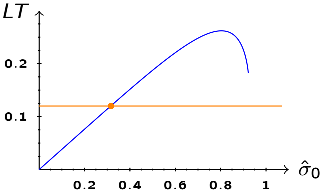


extremal probe



thermal probe $\kappa = 0.01$

- Quantitative differences
  - in the dependence of LT on $\hat{\sigma}_0$
  - $(LT)_{\rm max}$ receives a $O(\sqrt{\kappa})$ correction for small κ ⟶ $1/N$ effect missed by the less accurate extremal F-string probe

Free energy

The free energy of the string extended between Q and $\bar{Q}$

$$\mathcal{F} = \sqrt{\lambda} kT \int_0^{\hat{\sigma}_0} \frac{d\hat{\sigma}}{\hat{\sigma}^2}(1-X)\sqrt{1+fx'^2}\ , \quad X \equiv 1 - \tanh\alpha - \frac{1}{6\cosh\alpha\sinh\alpha}$$

→ It is divergent near $\hat{\sigma} = 0$ → UV divergence in the gauge theory

Regularization → subtract the free energy of two Polyakov loops (has the same UV divergences)

To do this in a controlled way, introduce an infrared cutoff at $z = \sigma_{\rm cut}$ near the event horizon with $\sigma_{\rm cut} \leq \sigma_c$

$$\mathcal{F}_{\rm sub} = \sqrt{\lambda} kT \int_0^{\hat{\sigma}_{\rm cut}} \frac{d\hat{\sigma}}{\hat{\sigma}^2}(1-X)$$

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Regularized free energy

The difference $\Delta\mathcal{F} = \mathcal{F} - \mathcal{F}_{\rm sub}$ is the regularized free energy and it can be written as

$$\Delta\mathcal{F} = \mathcal{F}_{\rm loop} - 2\mathcal{F}_{\rm charge}$$

with

$$\mathcal{F}_{\rm loop}(T, L, k, \lambda) = \sqrt{\lambda} k T \left(-\frac{1}{\hat{\sigma}_0} + \int_0^{\hat{\sigma}_0} d\hat{\sigma} \frac{(1-X)\sqrt{1+f x'^2} - 1}{\hat{\sigma}^2} \right)$$

$$\mathcal{F}_{\rm charge}(T, k, \lambda, \sigma_{\rm cut}) = -\frac{1}{2}\sqrt{\lambda} k T \left(\frac{1}{\hat{\sigma}_{\rm cut}} + \int_0^{\hat{\sigma}_{\rm cut}} d\hat{\sigma} \frac{X}{\hat{\sigma}^2} \right) \overset{\kappa\to 0}{\simeq} -\frac{1}{2}\sqrt{\lambda} k T$$

- $\mathcal{F}_{\rm loop}$ ⟶ regularized free energy of the rectangular Wilson loop
- $\mathcal{F}_{\rm charge}$ ⟶ regularized free energy for each of the Polyakov loops

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Q-$\bar{Q}$ potential

Regularized free energy of the rectangular Wilson loop for small LT

$$\mathcal{F}_{\text{loop}} = -\frac{\sqrt{\lambda}k}{L}\left(\frac{4\pi^2}{\Gamma(\frac{1}{4})^4} + \frac{\Gamma(\frac{1}{4})^4}{96}\sqrt{\kappa}(LT)^3 + \frac{3\Gamma(\frac{1}{4})^4}{160}(LT)^4 + \cdots\right)$$

- Leading term $\longrightarrow$ well-known Coulomb force potential found by probing AdS in the Poincaré patch with an extremal F-string

  [Rey, Yee (1998); Maldacena (1998)]

- For $\kappa = 0 \longrightarrow$ same results found by using an extremal F-string probe in the AdS black hole background

  [Brandhuber et al. (1998); Rey et al. (1998)]

- Higher order term $\sim \sqrt{\kappa}(LT)^3 \longrightarrow$ "new term"
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$$LT \ll \frac{\sqrt{k}\lambda^{1/4}}{N}$$

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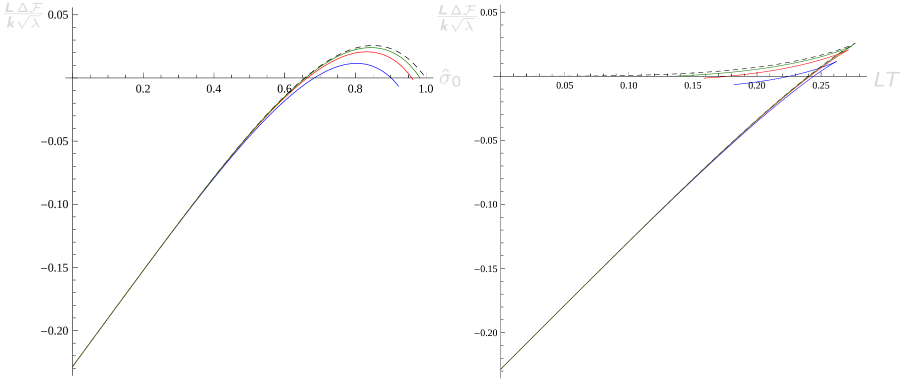
Debye screening

- $\Delta\mathcal{F} < 0 \longrightarrow$ Wilson loop is thermodynamically preferred
- $\Delta\mathcal{F} > 0 \longrightarrow$ Polyakov loop is thermodynamically preferred
- $\Delta\mathcal{F} = 0 \longrightarrow$ phase transition $\longrightarrow$ Debye screening of the Q-$\bar{Q}$ pair



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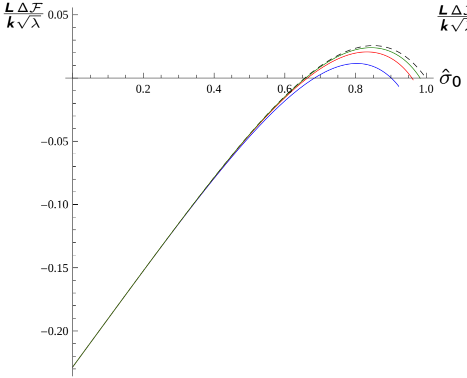

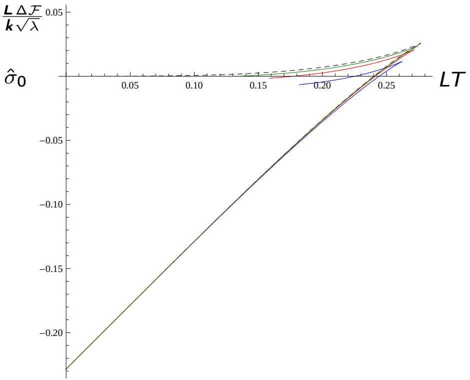

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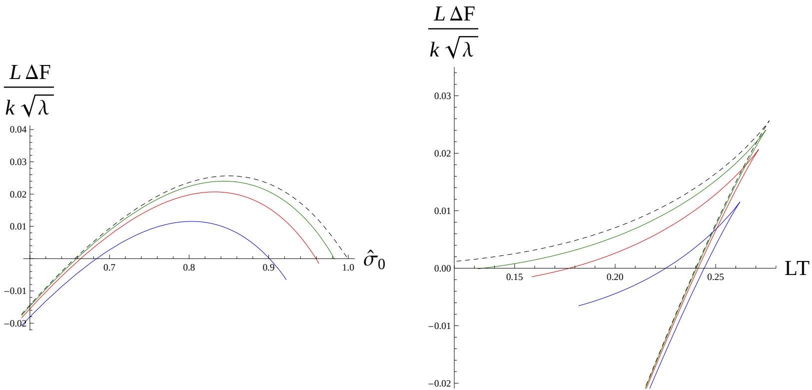

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- $\hat{\sigma}_0|_{\Delta\mathcal{F}=0}$ and $(LT)|_{\Delta\mathcal{F}=0}$ $\longrightarrow$ onset for charge screening
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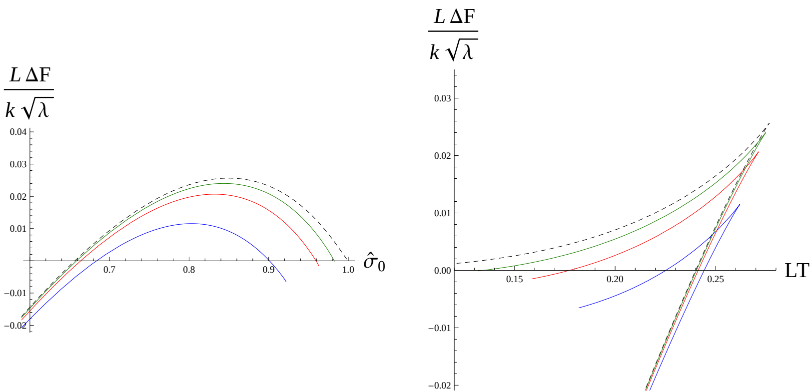

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The SUGRA F-string probe induces both qualitative and quantitative differences

- Qualitative difference
  - Using the thermal F-string probe the pair is less easily screened compared to the extremal F-string probe
    $\longrightarrow$ increasing κ the onset moves to higher values of LT and $\hat{\sigma}_0$
- Quantitative differences
  - For a finite value of LT in the small κ limit $\Delta\mathcal{F}$ receives $\sqrt{\kappa}$ corrections to the extremal probe results
  - Onset of charge screening for small κ

$$(LT)|_{\Delta\mathcal{F}=0} \simeq 0.240038 + 0.0379706\sqrt{\kappa}$$

  - For the gauge theory $\longrightarrow$ $\sqrt{k}\lambda^{1/4}/N$ correction to
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Outline

Charges in a hot plasma wind

Problem

A charge Q moving through the $\mathcal{N}=4$ SYM hot plasma with constant velocity v along the $\tilde{x}$ direction.

- Dual picture: open string attached to that charge which is located at the boundary of AdS.
- System studied with the standard approach in which the string profile is derived from the EOM's derived from the Nambu-Goto action [Gubser(2006),Herzog et al.(2006),Chernicoff et al.(2006)] .

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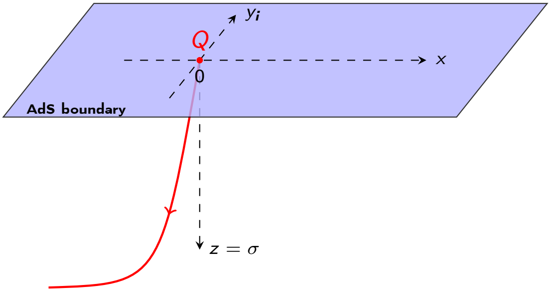

- Stationary ansatz for the embedding of the F-string probe

$$\tilde{t} = \tilde{\tau} \equiv \tilde{\sigma}^1 \ , \quad \tilde{z} = \tilde{\sigma} \equiv \tilde{\sigma}^2 \ , \quad \tilde{x} = \tilde{x}(\tilde{\tau}, \tilde{\sigma}) = v\tilde{\tau} + \xi(\tilde{\sigma}) \ , \quad \tilde{y}_1 = \tilde{y}(\tilde{\sigma})$$

  embedding of the string in the rest frame of the plasma, the charges and the string move with velocity v;
- the formula for the free energy $M - TS$ is valid only if the string is at rest. To use it we should switch to the string rest frame.
- In this frame the system that we consider is that of a charge Q at rest subject to the hot wind of the plasma.

Lorentz boost from the plasma rest frame $\tilde{X}^\mu$ to the string rest frame X^μ

$$t = \gamma\left(\tilde{t} - v\tilde{x}\right) \ , \quad x = \gamma\left(\tilde{x} - v\tilde{t}\right) \ , \quad y_1 = \tilde{y}_1 \ , \quad y_2 = \tilde{y}_2 \ , \quad z = \tilde{z}$$

where $\gamma = 1/\sqrt{1-v^2}$. In this frame the AdS-BH metric becomes

$$ds^2_{\rm AdS} = \frac{R^2}{z^2}\left[\gamma^2(v^2-f)dt^2 + \gamma^2(1-v^2 f)dx^2 + dy_1^2 + dy_2^2 + f^{-1}dz^2 + 2\gamma^2 v(1-f)dtdx\right]$$

with $f(z) = 1-(z/z_0)^4$, $z_0 = \tilde{z}_0$ and the stationary ansatz for the embedding of the F-string probe translates into

$$t = \tau \equiv \sigma^1 \ , \quad z = \sigma \equiv \sigma^2 \ , \quad x = x(\sigma) \ , \quad y_1 = y(\sigma)$$

The induced metric g_{ab} and redshift factor R_0 for this embedding are

$$g_{ab}d\sigma^a d\sigma^b$$
$$= \frac{R^2}{\sigma^2}\left[\gamma^2(v^2-f)d\tau^2 + \left(f^{-1} + \gamma^2(1-v^2 f)x'^2 + y'^2\right)d\sigma^2 + 2\gamma^2 v(1-f)x' d\tau d\sigma\right]$$

$$R_0(\sigma) = \frac{R}{\sigma}\gamma\sqrt{f(\sigma)-v^2}$$

Redshift

- The redshift factor R_0 induces a local temperature $T_{\rm local} = T/R_0$ the string probe locally is subjected to it, T being the global temperature of the background space-time as measured by an asymptotic observer.
- Imposing

$$\boxed{T/R_0 = 3/(2\pi r_0 \cosh\alpha)}$$

ensures that the probe is in thermal equilibrium with the background.

- It is convenient to rescale σ to the dimensionless variable $\hat{\sigma}$

$$\sigma \to \hat{\sigma} = \frac{\sigma}{z_0} = \pi T \sigma$$

- the redshift factor vanishes at $\hat{\sigma} = \hat{\sigma}_v \equiv \gamma^{-1/2} \longrightarrow$ at this point the local temperature of the string diverges.

It has never been taken into account the fact that one cannot describe the system for $\hat{\sigma} > \hat{\sigma}_v$, since at this point the temperature of the string is infinite.

- Write the charge constraint in terms of a new dimensionless parameter κ defined by

$$\kappa \equiv \frac{2^5 k T_{\mathrm{F1}}}{3^7 A R^6} = \gamma^6 \left(\frac{f(\hat{\sigma}) - v^2}{\hat{\sigma}^2} \right)^3 \frac{\sinh \alpha(\hat{\sigma})}{\cosh^5 \alpha(\hat{\sigma})}$$

  instead of using k.
- In terms of the gauge theory variables k, λ and N, κ can be written

$$\kappa = \frac{2^7}{3^6} \frac{k\sqrt{\lambda}}{N^2}$$

  using the AdS/CFT dictionary for the AdS radius $R^4 = \lambda l_s^4$ and the string coupling $4\pi g_s = \lambda/N$ with λ being the 't Hooft coupling of $\mathcal{N} = 4$ SYM theory.

The free energy for the thermal F-string probe with the chosen ansatz for the embedding

$$\mathcal{F} = \left(\frac{3}{2}\right)^6 A R^8 \pi T \int d\hat{\sigma} \Lambda(x', y', \hat{\sigma}) G(\hat{\sigma})$$

$$G(\hat{\sigma}) \equiv \frac{\gamma^7 \left(f(\hat{\sigma}) - v^2\right)^3}{\hat{\sigma}^8} \frac{1 + 6\sinh^2\alpha(\hat{\sigma})}{\cosh^6\alpha(\hat{\sigma})}$$

$$\Lambda(x', y', \hat{\sigma}) = \frac{\sigma^2\sqrt{-\det(g_{ab})}}{\gamma R^2}$$
$$= \sqrt{1 - v^2\left(y'(\hat{\sigma})^2 + \frac{1}{f(\hat{\sigma})}\right) + f(\hat{\sigma})\left(y'(\hat{\sigma})^2 + \frac{x'(\hat{\sigma})^2}{\gamma^2}\right)}$$

The free energy is independent of both x and y $\longrightarrow$ their corresponding momenta

$$\pi_x = \frac{\partial\left(\Lambda(x', y', \hat{\sigma})G(\hat{\sigma})\right)}{\partial x'} = \frac{f(\hat{\sigma})G(\hat{\sigma})x'(\hat{\sigma})}{\gamma^2\Lambda(x', y', \hat{\sigma})},$$
$$\pi_y = \frac{\partial\left(\Lambda(x', y', \hat{\sigma})G(\hat{\sigma})\right)}{\partial y'} = \frac{(f(\hat{\sigma}) - v^2)G(\hat{\sigma})y'(\hat{\sigma})}{\Lambda(x', y', \hat{\sigma})}$$

are constant.

Critical distance

Imposing the suitable boundary conditions it is then straightforward to write the corresponding solutions.
From the equation for κ and the fact that $\sinh\alpha/\cosh^5\alpha$ is bounded from above with maximal value $2^4/5^{5/2}$ we see that for a given κ the equation can only be satisfied provided $\hat{\sigma} \leq \hat{\sigma}_c$ with $\hat{\sigma}_c$ given by

$$\hat{\sigma}_c^2 = \frac{1}{\gamma}\sqrt{1 + \frac{5^{5/3}}{2^{14/3}\gamma^2}\kappa^{\frac{2}{3}}} - \frac{5^{5/6}}{2^{7/3}\gamma^2}\kappa^{\frac{1}{3}}$$

Note that $\hat{\sigma}_c \leq \hat{\sigma}_v$ and the equality holds for $\kappa = 0$.

details

Single charge Q

- Consider now a single charge moving in the plasma rest frame along the trajectory $\tilde{z} = 0$, $\tilde{x} = v\tilde{t}$, $\tilde{y}_1 = \tilde{y}_2 = 0$.
- In the string rest frame the charge will sit at $(z, x, y_1, y_2) = (0, 0, 0, 0)$;
- boundary condition $\longrightarrow$ one end of the string coincides with the charge *i.e.* $x(0) = 0$ and $y(0) = 0$
- symmetry considerations $\longrightarrow$ the $\hat{\sigma}$ dependence of the y coordinate has to be trivial, namely $y(\hat{\sigma}) = 0$, which then implies $\pi_y = 0$.
- Solving the equation giving π_x for x' we get

$$x'(\hat{\sigma}) = \pm \frac{\pi_x \gamma \hat{\sigma}^8 \sqrt{f(\hat{\sigma}) - v^2}}{f(\hat{\sigma})\sqrt{f(\hat{\sigma})\gamma^{12}(f(\hat{\sigma}) - v^2)^6(1 + 6\sinh^2(\alpha))^2 \cosh^{-12}(\alpha) - \pi_x^2 \hat{\sigma}^{16}}}$$

x' has to be chosen negative since the physical configuration is that of the string is pushed back with respect to the charge in the bulk and not the one in which the string precedes the charge.

- In the "standard" approach based on the Nambu-Goto action the request of having a solution which describes a string stretching up to the horizon was used to fix the momentum π_x

  [Gubser(2006),Herzog(2006)]

- only one value of π_x which makes x' real and analytic in the whole range of $\hat{\sigma}$ from the boundary to the horizon.
- follow the same path $\longrightarrow$ impose that the square roots in the numerator and in the denominator of x' have their zeros at the same value of $\hat{\sigma}$.
- Looking at the square root at the numerator we see that this results to be at $\hat{\sigma}$ equals to

$$\boxed{\hat{\sigma}_v = \left(1 - v^2\right)^{1/4}}$$

- π_x should be fixed by imposing a boundary condition at $\hat{\sigma}_v$.
- But, $\hat{\sigma}_v$ is always larger that the critical distance, $\boxed{\hat{\sigma}_v \geq \hat{\sigma}_c}$,
- equality holds only for $\kappa = 0$.

The method breaks down at distances strictly smaller than $\hat{\sigma}_c$ and thus for larger values of $\hat{\sigma}$ it does not allow to say anything about the solution

$\longrightarrow$ we cannot impose any condition to fix π_x and we cannot even know if the solution can reach the horizon.

Going a little beyond the validity of the blackfold approach we can focus on the $\kappa = 0$ case and apply the procedure described above: One can then fix π_x to

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- The procedure can be applied only in the case $\kappa = 0$. In fact we cannot even obtain the small κ corrections to the previous result since as soon as one turns on κ the value of $\hat{\sigma}_v$ at which one imposes the condition becomes unreachable
- Even if one forgets this fact and tries to repeat the procedure it turns out that it is impossible to satisfy the condition that fixes π_x at higher order in the small κ expansion.
- There exists also the trivial solution with $\pi_x = 0$ which correspond to a straight string $x(\hat{\sigma}) = 0$.
- for this solution $-g$ vanishes at $\hat{\sigma}_v$ and becomes negative for $\hat{\sigma}_v < \hat{\sigma} < 1$; $\longrightarrow$ this profile was considered not physical

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- for this solution $-g$ vanishes at $\hat{\sigma}_v$ and becomes negative for $\hat{\sigma}_v < \hat{\sigma} < 1$; $\longrightarrow$ this profile was considered not physical

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- However, since our approach breaks down before one can reach $\hat{\sigma}_v$, this motivation does not hold anymore and this solution is indeed acceptable.

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Summary

- New method to describe thermal probes in finite temperature backgrounds

  $\longrightarrow$ it keeps into account the fact that for extended probes the internal degrees of freedom should be in thermal equilibrium with the background

- Application of this method to the study of Wilson loops in finite temperature $\mathcal{N} = 4$ SYM using thermal F-string probes in the AdS black hole background

  $\longrightarrow$ there are both qualitative and quantitative differences with respect to the "standard approach" based on the Nambu-Goto action

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Outlook

- Examine holographic aspects of quark-gluon plasma physics
  - *e.g.* energy loss of a heavy quark moving through the plasma

[Herzog, Karch, Kovtun, Kozcaz, Yaffe (2006); Gubser (2006)]

- Higher branes $p > 4$ do not have a well defined energy momentum tensor at strong coupling (instabilities and no extremal limit) $\longrightarrow$ study thermal corrections to the DBI at weak coupling $\longrightarrow$ Marini's talk on Thursday!

- Revisit the thermal generalization of the Wilson loop in higher representations, in the regime where it involves a "blown-up" version:
  - D3-brane (symmetric representation)
  - D5-brane (antisymmetric representation)

$\longrightarrow$ interesting in view of the discrepancies between gauge theory and gravity results found for the symmetric representation

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Blackfold

- The idea behind the new method for non-extremal D-branes probing thermal backgrounds comes from the so-called blackfold approach which has been developed in the study of higher-dimensional black holes.
- the blackfold approach provides a general description of black holes in a regime in which the black hole approximately is like a *black* brane curved along a submani*fold* of a background space-time (hence the name "blackfold").
- this regime entails in particular that the thickness of the black p-brane (horizon scale, say r_0) is much smaller than the length scale of the embedding geometry (curvature length scale say R) $\longrightarrow$ $r_0 << R$.
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- general idea: one integrates out the brane thickness scale and obtains an effective description in terms of the large scales over which the brane varies, such as the scale of the embedding geometry $\longrightarrow$ in $r_0 << R$ regime: make an effective description of the geometry with an effective Energy-Momentum tensor $T^{ab} \longrightarrow$ EM tensor of effective fluid
- Effective theory at large scale R: Dynamics governed by conservation of T^{ab} with the dynamical principle being the conservation of the EM tensor.
- The EOMs resulting from this consist of the conservation of the EM tensor on the world-volume along with the extrinsic EOMs for the blackfold of the form $T^{ab}K_{ab}{}^{\sigma} = J \cdot F^{\sigma}$: Carter Equation

$\longrightarrow$ Leading order in r_0/R: Probe approximation

$\longrightarrow$ Higher orders in r_0/R: Backreaction can systematically be taken into account

- To leading order the brane can be regarded as a probe brane that does not backreact on the background geometry.
- if the distance to the brane is of order the brane thickness scale one would observe a backreaction to the surrounding geometry from the brane.
- integrating out the brane thickness scale $\longrightarrow$ the probe approximation implies that the backreaction from the brane is sufficiently small to be neglible for the large scale physics described by the extrinsic EOMs .
- *i.e.* only when considering corrections to the probe approximation one should start taking into account the backreaction of the brane on the background geometry when computing the extrinsic curvature tensor $K_{ab}{}^{\rho}$.

$\longrightarrow$ one can parallel the probe approximation in the blackfold approach with the probe approximation that the DBI action assumes and the only difference in the extrinsic EOMs is that one should replace the DBI EM tensor with that of the fluid EM tensor for the black brane.

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Validity of the probe approximation 1.

- The probe approximation means that we should be able to piece the string probe together out of small pieces of SUGRA F-strings in hot flat ten-dimensional space-time. For this to work, the local length scale of the string probe, *i.e.* the "thickness" of the string, should be much smaller than the length scale of each of the pieces of F-string in hot flat space.
- We want to consider the branch of the SUGRA F-string connected to the extremal F-string $\Longrightarrow$ the thickness of the F-string is the charge radius

$$r_c = r_0(\cosh\alpha \sinh\alpha)^{1/6}$$

- Using this one finds $r_c \propto \kappa^{1/6} R$.
- In the AdS black hole background R and $z_0 \propto 1/T$ are the two length scales in the metric. Since we need to require that the sizes of the pieces of F-string should be smaller than the length scales of the metric we need that $r_c \ll R$ and $r_c \ll 1/T$. This gives the conditions

$$\boxed{\kappa \ll 1\,, \quad RT \ll \kappa^{-\frac{1}{6}}}$$

- Given that $\kappa \ll 1$, the condition $RT \ll \kappa^{-1/6}$ is easily fulfilled as it gives a weak upper bound on how high the asymptotic temperature T can be.
- The condition $\kappa \ll 1 \implies$ critical distance $\hat{\sigma}_c$ is very close to the horizon: $1 - \hat{\sigma}_c \propto \kappa^{1/3}$ for small κ.
- The regime of validity of the SUGRA F-string is $1 \ll k \ll N$ and $\lambda^2 k \gg N^2$. Instead, the probe approximation requires $\kappa \ll 1$. We see that this is consistent with the regime of validity of the SUGRA F-string provided that $\lambda \ll N^2$. This translates to $g_s \ll N$ which is trivially satisfied since we assume weak string coupling $g_s \ll 1$.
- A relevant quantity is the local temperature $T_{\rm local} = T/R_0$. Need that the local temperature varies over sufficiently large length scales such that we can regard the probe locally as a SUGRA F-string in hot flat space of temperature $T_{\rm local} \implies$ we need in particular that $r_c T'_{\rm local}(\sigma)/T \ll 1$, *i.e.* that the variation of the local temperature is small over the length scale of the F-string probe.

$$\boxed{\frac{r_c T'_{\rm local}}{T} = \frac{r_c}{R\sqrt{f}} + \frac{2r_c(\pi T\sigma)^4}{Rf^{3/2}} \ll 1}$$

- As a check we can see that for $\sigma \gg z_0$ the condition $r_c T'_{\rm local}/T \ll 1$ reduces to $r_c \ll R$.
- Considering instead the near horizon region $z_0 - \sigma \ll z_0$ we find in the $\kappa \to 0$ limit that $r_c T'_{\rm local}/T \ll 1$ requires $z_0 - \sigma \gg \kappa^{1/9} z_0$. When we reach

$$\boxed{z_0 - \sigma \sim \kappa^{1/9} z_0}$$

the probe approximation breaks down. For $\sigma \simeq \sigma_c$ the probe approximation is not valid, indeed $r_c T'_{\rm local}/T \sim \kappa^{-1/3}$.
- The probe approximation in fact breaks down before we reach the critical distance $\sigma = \sigma_c$ where the local temperature reaches the maximal possible temperature of the SUGRA F-string.
- We should also consider the extrinsic curvature of the solution. Since we already took into account the variation of the background, the easiest way to analyze the extrinsic curvature is to neglect the derivatives of the metric. Doing this we should require $z_0 - \sigma \gg \kappa^{1/3} z_0$. However, this is already guaranteed by the stronger condition $z_0 - \sigma \gg \kappa^{1/9} z_0$ found above.

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Validity of the probe approximation 2.

- The validity of the probe approximation for the SUGRA F-string requires that the local length scale of the string probe, *i.e.* the "thickness" of the string, should be much smaller than the length scale of each of the pieces of F-string in hot flat space.
- we want to consider the branch of the SUGRA F-string connected to the extremal F-string the thickness of the F-string is the charge radius $r_c = r_0(\cosh\alpha \sinh\alpha)^{1/6}$.
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- The conditions $\kappa \ll 1$ and $RT \ll \kappa^{-1/6}$ are not sufficient to ensure the validity of the probe approximation.
- the analysis of the length scales of the background is only valid away from the near horizon region $z_0 - \sigma \ll z_0$.
- In the near horizon region $z_0 - \sigma \ll z_0$ further analysis is required. A relevant quantity to consider is the local temperature $T_{\rm local} = T/R_0$. In order for the probe approximation to be valid we need that the local temperature varies over sufficiently large length scales such that we can regard the probe locally as a SUGRA F-string in hot flat space of temperature $T_{\rm local}$.
- we need in particular that $r_c T'_{\rm local}(\sigma)/T \ll 1$, *i.e.* that the variation of the local temperature is small over the length scale of the F-string probe

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- The conditions $\kappa \ll 1$ and $RT \ll \kappa^{-1/6}$ are not sufficient to ensure the validity of the probe approximation.
- the analysis of the length scales of the background is only valid away from the near horizon region $z_0 - \sigma \ll z_0$.
- In the near horizon region $z_0 - \sigma \ll z_0$ further analysis is required. A relevant quantity to consider is the local temperature $T_{\rm local} = T/R_0$. In order for the probe approximation to be valid we need that the local temperature varies over sufficiently large length scales such that we can regard the probe locally as a SUGRA F-string in hot flat space of temperature $T_{\rm local}$.
- we need in particular that $r_c T'_{\rm local}(\sigma)/T \ll 1$, *i.e.* that the variation of the local temperature is small over the length scale of the F-string probe

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- Check $\longrightarrow$ $\sigma \ll z_0$ the condition $r_c T'_{\rm local}/T \ll 1$ reduces to $r_c \ll R$.
- regions far from the boundary $\longrightarrow$ $\sigma_v - \sigma \ll \sigma_v$, with $\sigma_v = \gamma^{-1/2} z_0$, we find in the $\kappa \to 0$ limit that $r_c T'_{\rm local}/T \ll 1$ requires $\sigma_v^4 - \sigma^4 \gg \kappa^{1/9}\sigma_v^4$. Thus, when we reach $\sigma_v^4 - \sigma^4 \sim \kappa^{1/9}\sigma_v^4$, the probe approximation breaks down.
- For $\sigma \simeq \sigma_c$ the probe approximation is not valid, indeed $r_c T'_{\rm local}/T \sim \kappa^{-1/3}$.

the probe approximation breaks down before reaching the critical distance $\sigma = \sigma_c$ where the local temperature reaches the maximal possible temperature of the SUGRA F-string.

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