The D3-D7 Holographic Dual of Relativistic Materials in 3D

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THE HOLOGRAPHIC WAY, Nordita, October 17,

Nordita, October 17, 2012

Outline

1. D3-D7
2. Graphene as a realization of SO(2,3) CFT
3. D3-D7 as a planar expansion of graphene

Relativistic fermions in 2+1-dimensions

We want to model systems of SO(2,1) symmetric (relativi
fermions in 2+1-dimensions

$$S = \int d^3x \sum_i \bar{\psi}_i(x) i\gamma^\mu \partial_\mu \psi_i(x) + \text{ interactions}$$

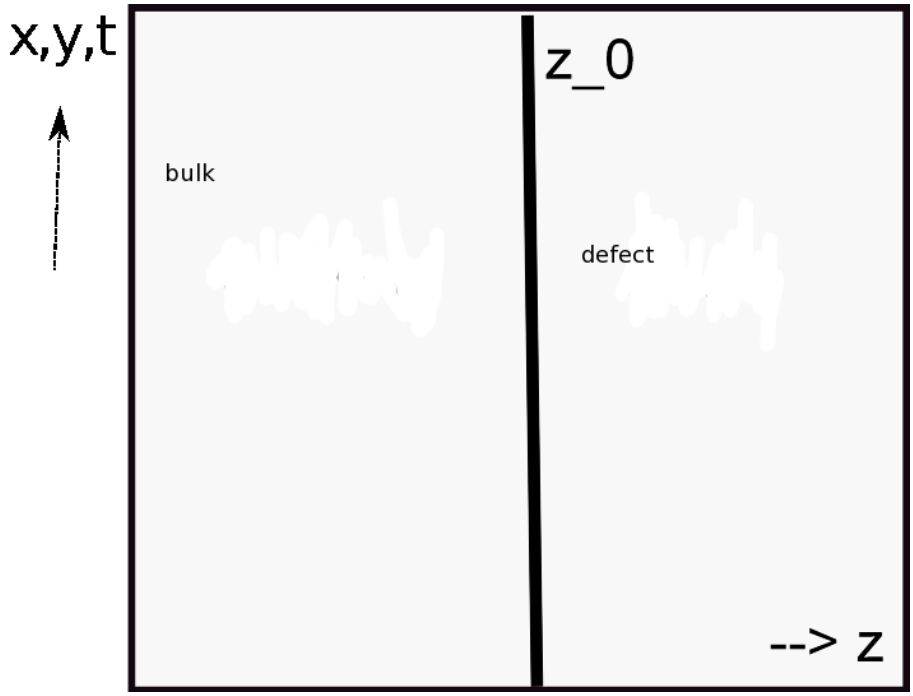
$\psi_i(x)$ is a 2-component spinor of $SO(2,1)$

C P T:

Fermion mass $m\bar{\psi}_i\psi_i$ breaks parity and time reversal sym
or flavor symmetry

$$\mu_{ij} = \langle \bar{\psi}_j(x)\bar{\psi}_i(x)\rangle$$

Defect field theory

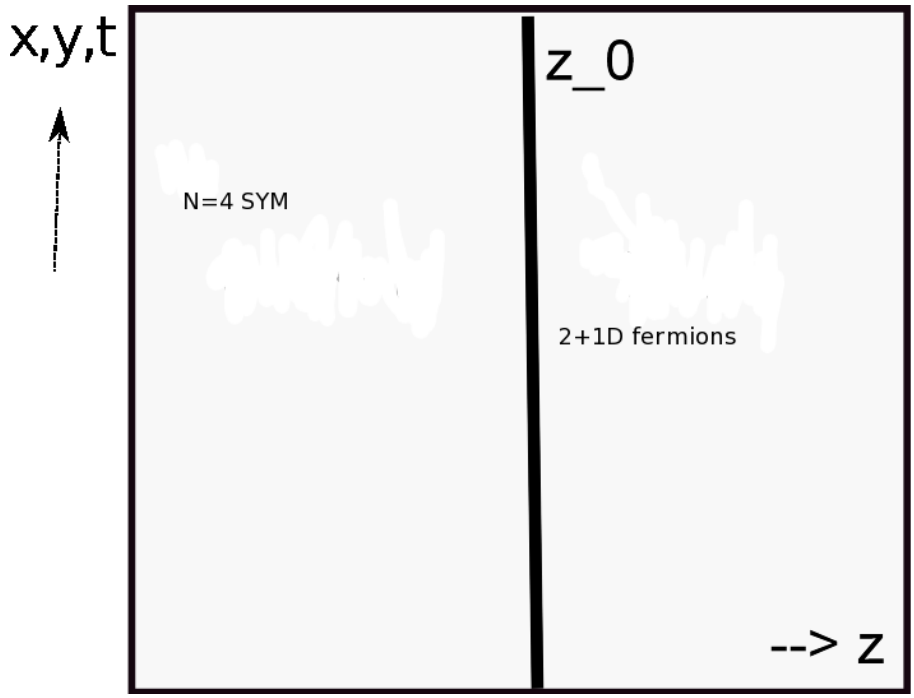


2-component fermions inhabit 2+1-dimensional subspace

Interact by exchanging degrees of freedom of field theory

3+1-dimenisonal bulk

Defect conformal field theory



2-component massless fermions inhabit 2+1-dimensional s
Interact by exchanging gluons, etc. of $\mathcal{N} = 4$ super Yang
theory in bulk
Solve in large N planar limit and at strong coupling $\lambda =$

D3-D7 brane system

	0	1	2	3	4	5	6	7	8	9
$D3$	X	X	X	X	O	O	O	O	O	O
$D7$	X	X	X	O	X	X	X	X	X	O

brane extends in directions X
brane sits at single point in directions O

$\#ND = 6$ system – no supersymmetry – no tachyon – onl
modes of 3-7 strings are in R-sector and are 2-component
(N_7 flavors and N_3 colors).
Mass = separation in x_9-direction.

$$S = \int d^3x \sum_{\sigma=1}^{N_7} \sum_{\alpha=1}^{N_3} \bar{\psi}_\alpha^\sigma [i\gamma^\mu \partial_\mu - m] \psi_\alpha^\sigma + \text{interactior}$$

$N_3 \to \infty$, $\lambda = 4\pi g_s N_3$ fixed $\to$ replace D3's by $AdS_5 \times S^5$
probe D7-branes in $AdS_5 \times S^5$

D3-D7 system

	0	1	2	3	4	5	6	7	8	9
$D3$	X	X	X	X	O	O	O	O	O	O
$D7$	X	X	X	O	X	X	X	X	X	O

Probe D7-brane has geometry $AdS_4 \times S^4$

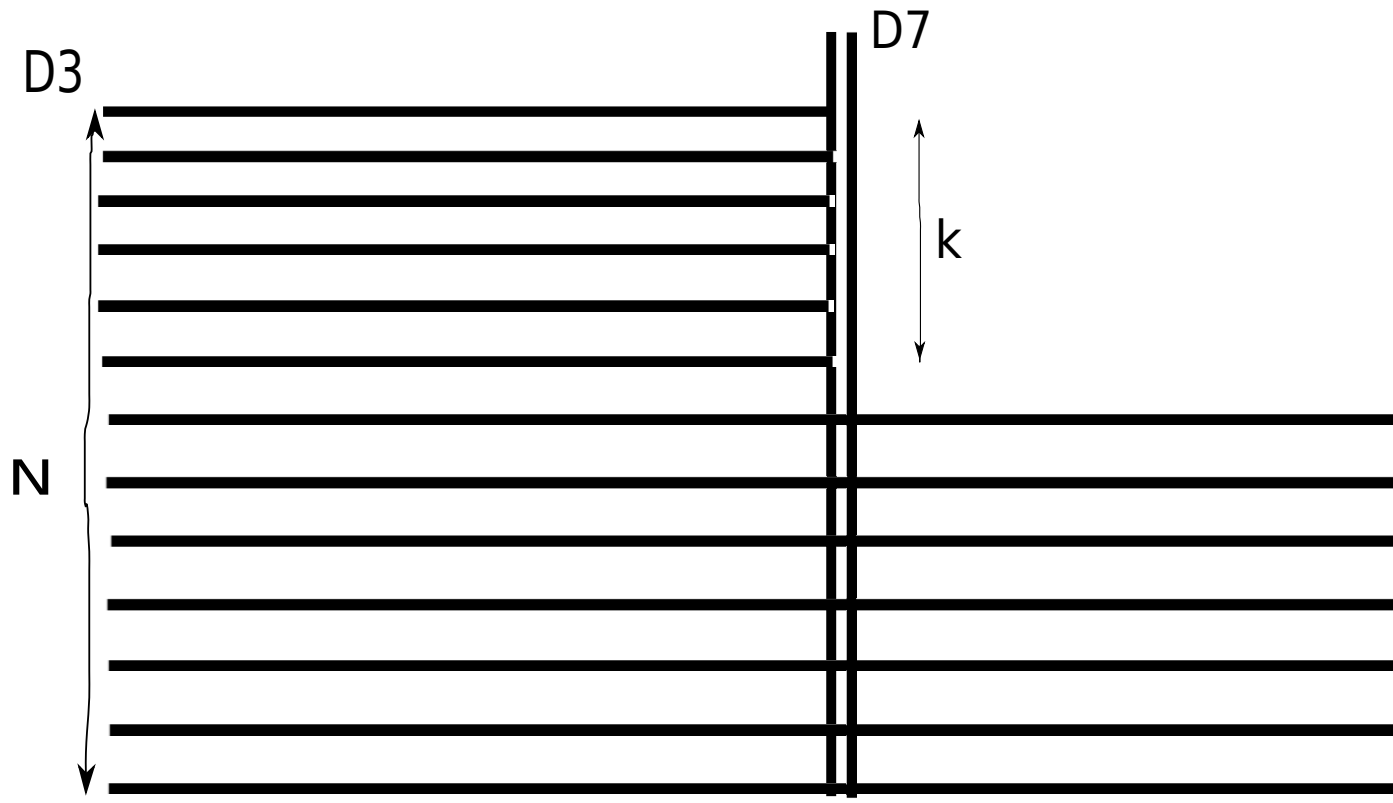
S. J. Rey, Talk at Strings 2007;
Prog. Theor. Phys. Suppl. 177, 128 (2009) arXiv:0
D-brane construction of graphene
This embedding is unstable.
Fluctuations violate BF bound for AdS_4
D. Kutasov, J. Lin, A.Parnachev, arXiv:1107.2324

D3-D7 system

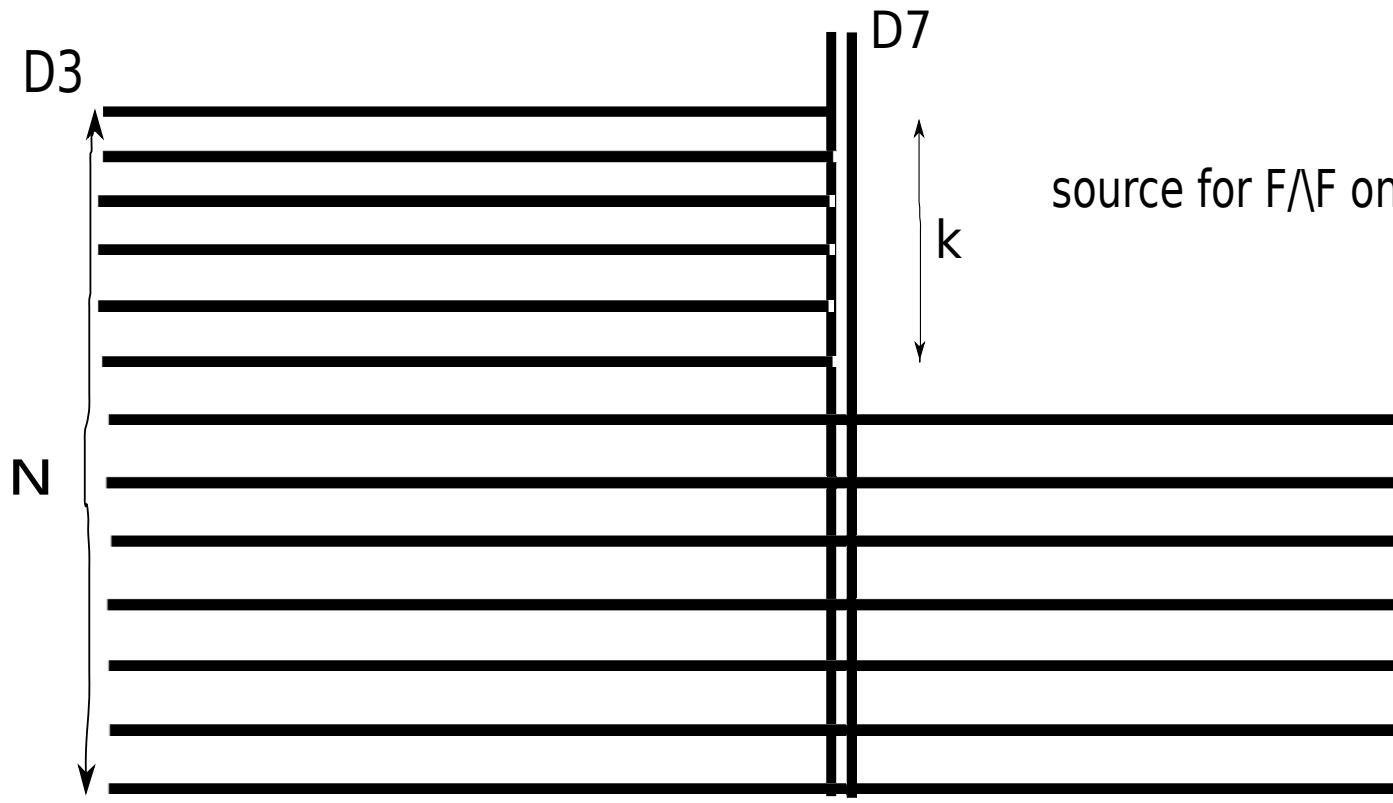
	0	1	2	3	4	5	6	7	8	9
$D3$	X	X	X	X	O	O	O	O	O	O
$D7$	X	X	X	O	X	X	X	X	X	O

R. C. Myers and M. C. Wapler, JHEP 0812, 115 (
[arXiv:0811.0480 [hepth]].
Stabilize by putting instanton bundle on S^4.
O. Bergman, N. Jokela, G. Lifschytz and M. Lippe
JHEP 1010 (2010) 063 [arXiv:1003.4965 [hep-th]].
Probe brane geometry $AdS_4 \times S^2 \times S^2$ with **U(1) fluxes**
2-spheres
Stable when f or $\tilde{f}$ large enough.
Discrete symmetries, P and C: $f = \tilde{f}$.
J.Davis, H.Omid and G.S., arXiv:1107.4397 [hep-t

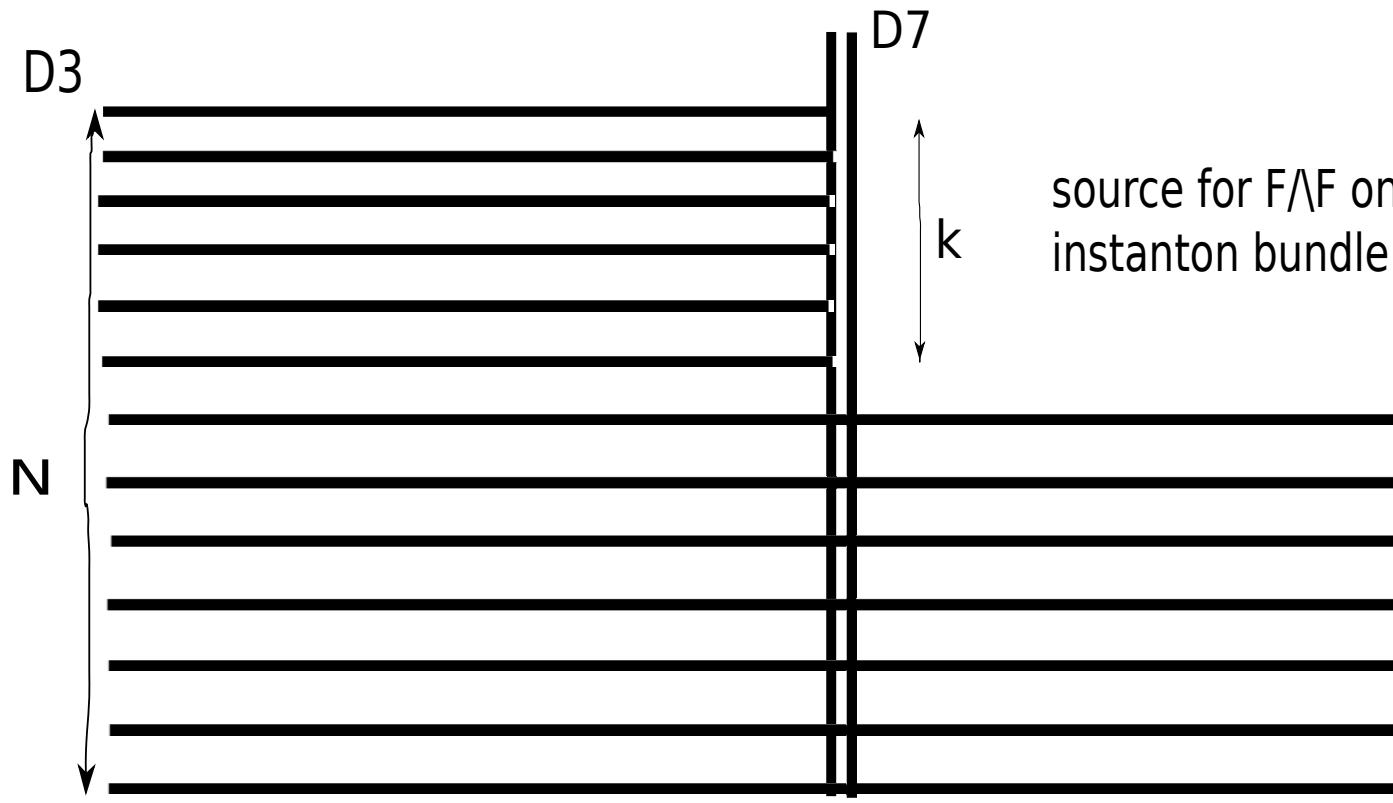
D3-branes ending on D7-branes



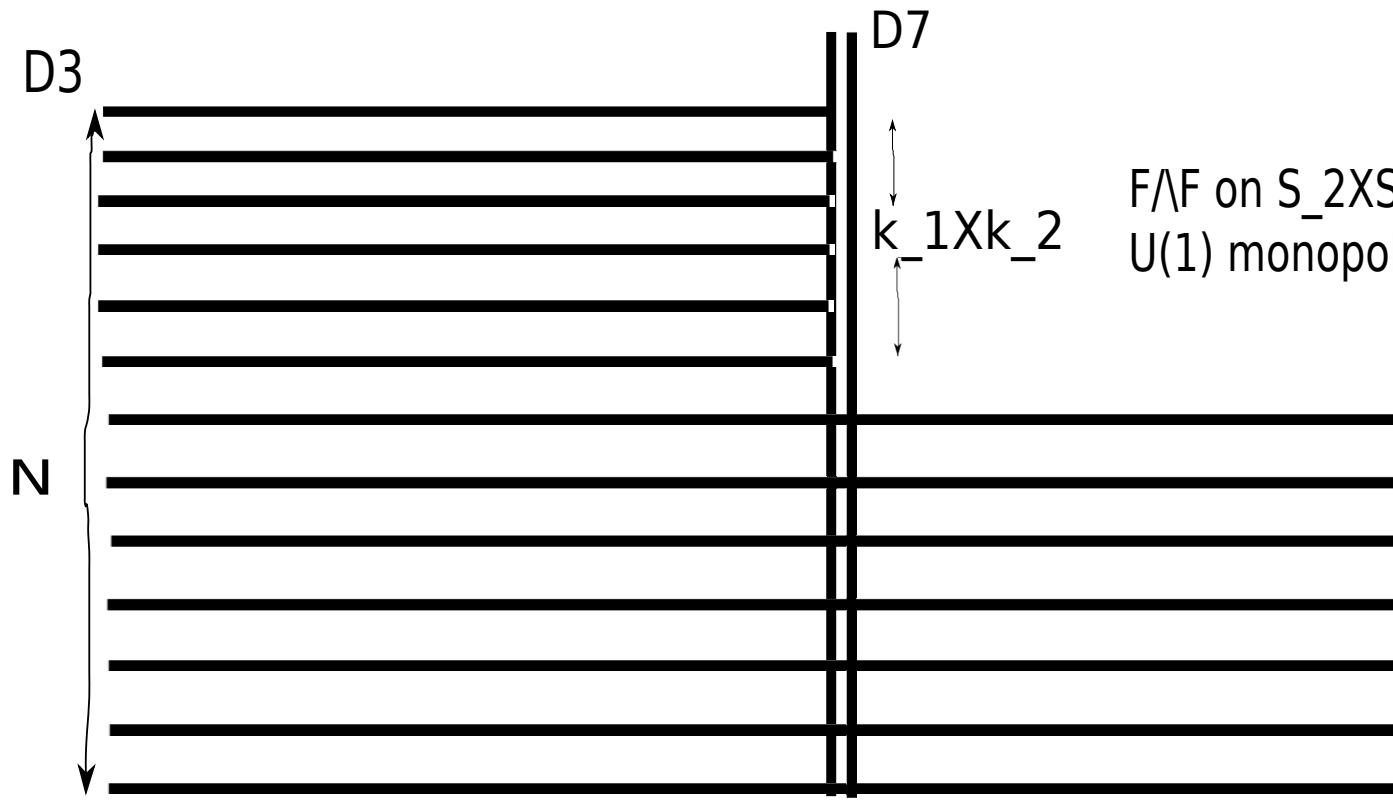
D3-branes ending on D7-branes



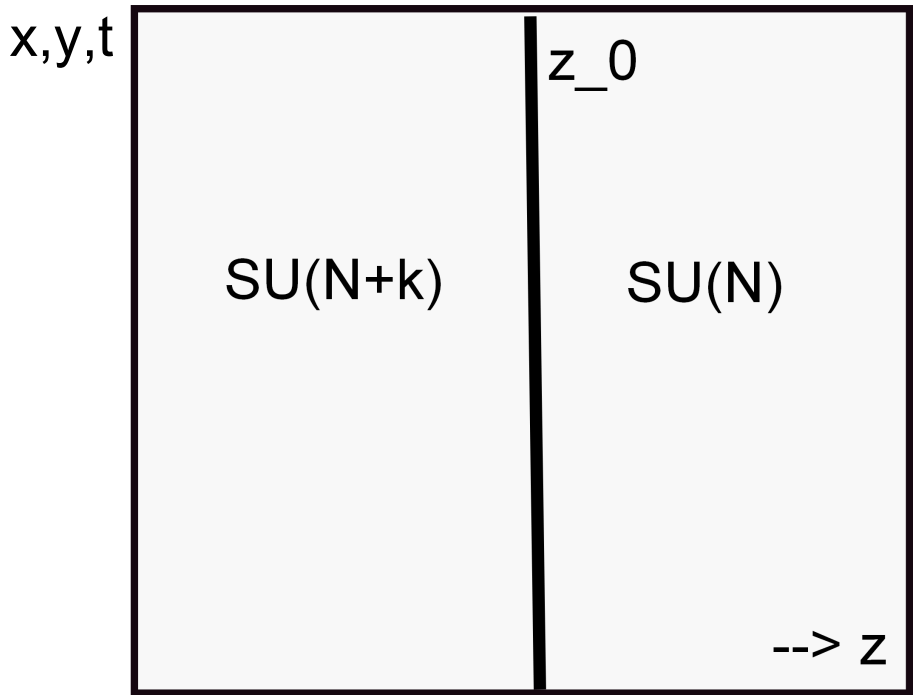
D3-branes ending on D7-branes



D3-branes ending on D7-branes



Defect conformal field theory



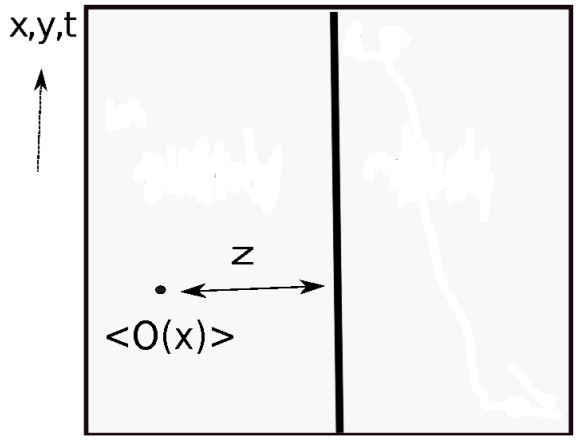
2-component massless fermions inhabit 2+1-dimensional s

$z = z_0$

Bulk contains $\mathcal{N} = 4$ super Yang-Mills theory

Rank of gauge group changes by $k = k_1 k_2$ at defect

One-point function in defect CFT



Bulk primary operator one-point function

$$\langle \mathcal{O}_\Delta(x) \rangle = \frac{c_\Delta}{|z|^\Delta}$$

Compute c_Δ for chiral primary operators

K.Nagasaki, S.Yamaguchi, arXiv:1205.1674 for D3-

C.Kristjansen, GWS, D.Young, to appear for D3-D

small parameter (like BMN)

$$\frac{\lambda}{k_1^2 + k_2^2} \quad , \quad \sqrt{\lambda} << k_1, k_2 << N$$

Chiral Primary operator

$$\mathcal{O}_{\Delta I}(x) \equiv \frac{(8\pi^2)^{\frac{\Delta}{2}}}{\lambda^{\frac{\Delta}{2}}\sqrt{\Delta}} C_I^{i_1 i_2 \ldots i_\Delta} \operatorname{Tr}\left(\phi_{i_1}(x)\phi_{i_2}(x)\ldots\phi_{i_\Delta}(x\right.$$

$$\sum_{i_1 i_2 \ldots i_\Delta} C_I^{i_1 i_2 \ldots i_\Delta} C_J^{i_1 i_2 \ldots i_\Delta} = \delta_{IJ}$$

$$\langle \mathcal{O}_{\Delta I}(x)\mathcal{O}_{\Delta' I'}(y)\rangle = \frac{\delta_{\Delta\Delta'}\delta_{II'}}{|x-y|^{2\Delta}}$$

At each Δ, $\exists$ a unique chiral primary operator $\mathcal{O}_\delta$ with $SO(3) \times SO(3)$ symmetry which can have a 1-point funct

$$\langle \mathcal{O}_\Delta(x)\rangle = \frac{c_\Delta}{|z|^\Delta}$$

Classical solution of $\mathcal{N}$=4 SYM

$A_\mu = 0$, $\psi = 0$,

$$\nabla^2 \phi_i - \sum_{j=1}^{6} [\phi_j, [\phi_j, \phi_i]] = 0 \ , \ \phi_i \nabla \phi_i = 0$$

$$\phi_i(z) = -\frac{1}{z}(t_i^{k_1} \otimes 1_{k_2 \times k_2}) \oplus 0_{(N-k_1 k_2)\times(N-k_1 k_2)}, \quad i =$$

$$\phi_i(z) = -\frac{1}{z}(1_{k_1 \times k_1} \otimes t_i^{k_2}) \oplus 0_{(N-k_1 k_2)\times(N-k_1 k_2)}, \quad i =$$

plug into $\mathrm{Tr}\,(\phi_{i_1}(x)\phi_{i_2}(x)\ldots\phi_{i_\Delta}(x))$

$$N >> k_1, k_2 >> \sqrt{\lambda}$$

Classical solution of $\mathcal{N}$=4 SYM

Semiclassical computation of chiral primary operator yiel

$$\begin{aligned}\langle \mathcal{O}_\Delta(x)\rangle &= \frac{k_1 k_2}{\sqrt{\Delta}}\left(\frac{2\pi^2(k_1^2+k_2^2)}{\lambda}\right)^{\Delta/2} Y_\Delta(\psi)\ \frac{1}{|z|^\Delta},\\ \langle \mathcal{O}_\Delta(x)\rangle &= 0, \qquad (z>0).\end{aligned}$$

where Δ is even ψ in

$$ds^2 = d\psi^2 + \cos^2\psi d\Omega_2^2 + \sin^2\psi d\tilde{\Omega}_2^2$$

is

$$\psi = \arctan\left(\frac{k_2}{k_1}\right)$$

String theory computation: Small parameter is $\sqrt{\frac{\lambda}{k_1^2+}}$

The supergravity dual of the chiral primary operators is s

$$
\begin{aligned}
h_{\mu\nu}^{AdS_5} &= -\frac{2\Delta(\Delta-1)}{\Delta+1} g_{\mu\nu}^{AdS_5} s + \frac{4}{\Delta+1} \nabla_\mu \nabla_\nu s, \\
h_{\alpha\beta}^{S^5} &= 2\Delta g_{\alpha\beta}^{S^5} s, \\
a_{\mu\nu\rho\sigma}^{AdS_5} &= 4i\sqrt{g^{AdS_5}} \epsilon_{\mu\nu\rho\sigma\omega} \nabla^\omega s,
\end{aligned}
$$

s is replaced by its bulk-to-boundary propagator, correspo

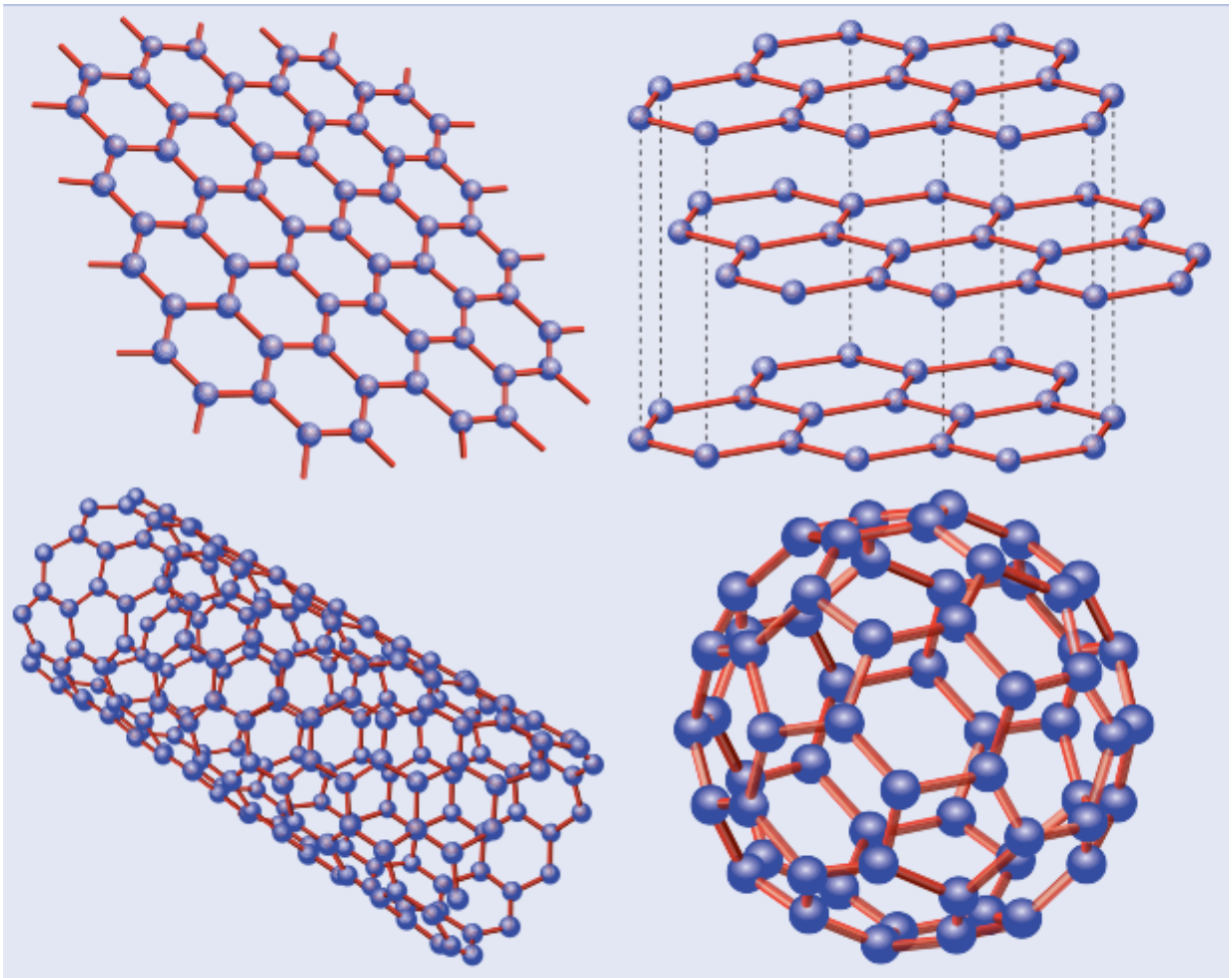
a delta-function source s_0 on the boundary at $(t, x, y, z) =$

$$
s \to \frac{\Delta+1}{2^{2-\Delta/2} N \sqrt{\Delta}} \frac{Y_\Delta(\psi)}{r^\Delta \left(t^2 + x^2 + y^2 + z^2 + 1/r^2\right)^\Delta}
$$

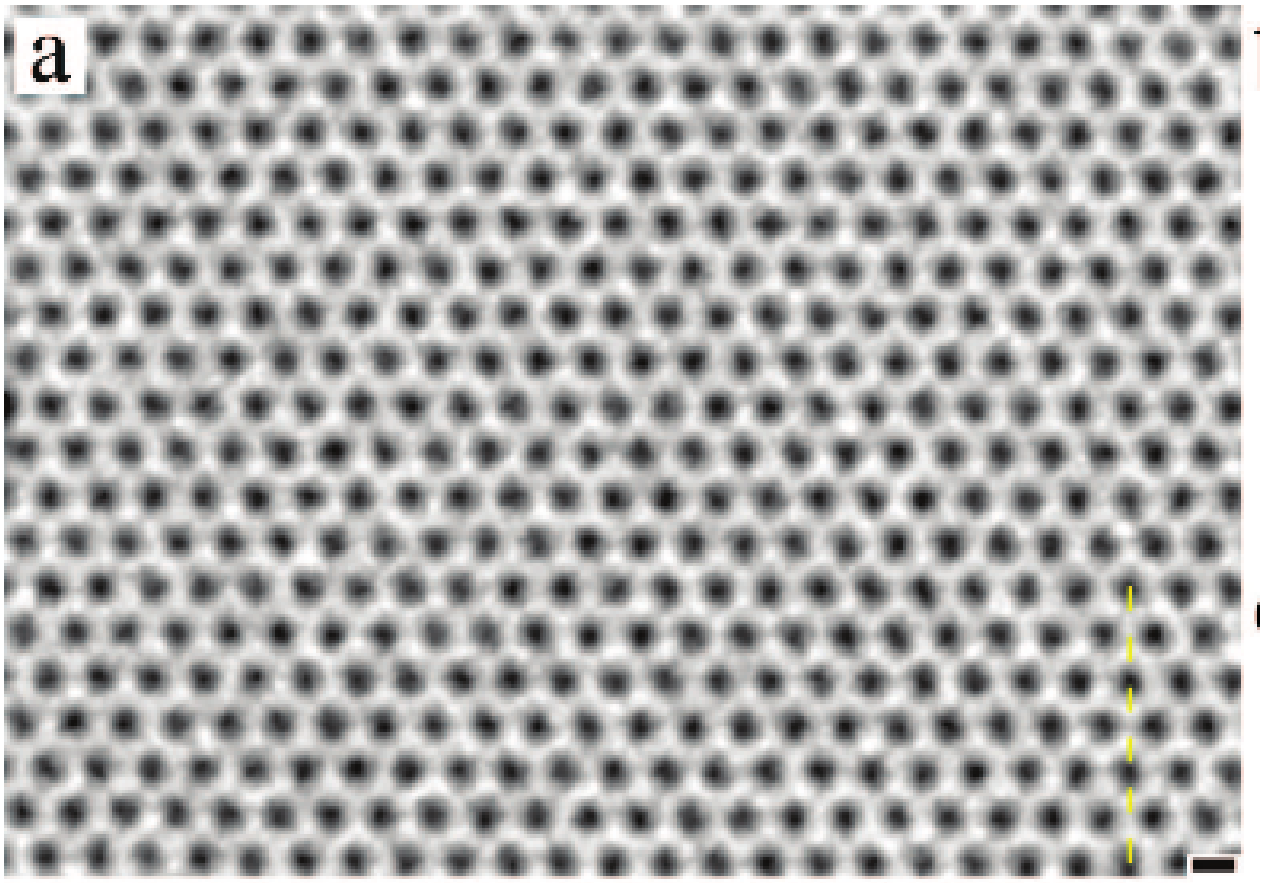
Compute $\langle \mathcal{O}_\Delta \rangle = -\delta S_{\rm DBI} - \delta S_{\rm WZ}$

$$\begin{aligned}
\langle \mathcal{O}_\Delta(x) \rangle &= \frac{k_1 k_2}{\sqrt{\Delta}} \left(\frac{2\pi^2 (k_1^2 + k_2^2)}{\lambda} \right)^{\Delta/2} Y_\Delta \left[\arctan(\frac{k_2}{k_1}) \right] \\
\langle \mathcal{O}_\Delta(x) \rangle &\sim \left(\frac{(k_1^2 + k_2^2)}{\lambda} \right)^{-\Delta/2}, \qquad (z > 0).
\end{aligned}$$

Graphene is a 2-dimensional array of carbon atom with a hexagonal lattice

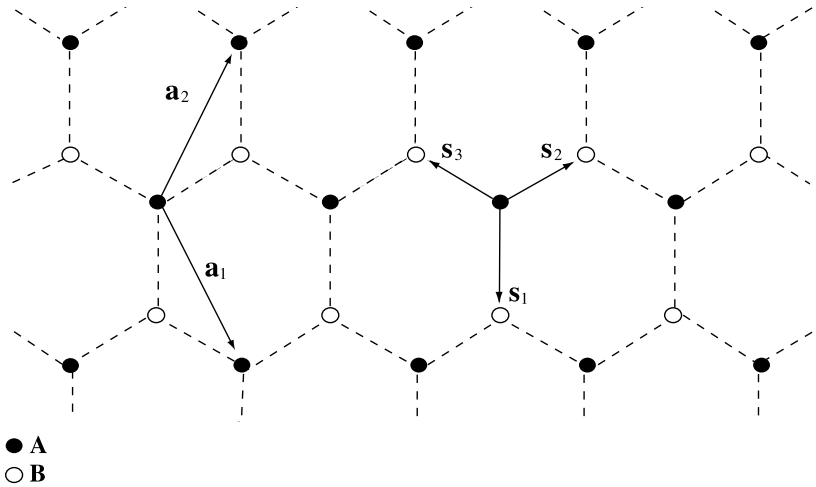


TEAM Electron Microscope image



Jannik C. Meyer, C. Kisielowski, R. Erni, Marta D. Rosse
Crommie, and A. Zettl, Nano Letters 8, 3582 (2008).

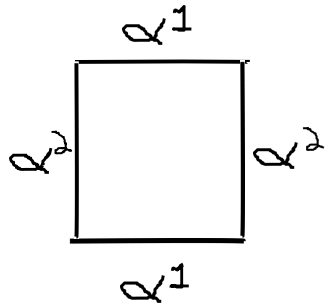
Tight-binding model



hexagonal lattice = two triangular sub-lattices $\vec{A}$ and $\vec{B}$ c
by vectors $\vec{s}_1, \vec{s}_2, \vec{s}_3$.

$$H = \sum_{\vec{A},i} \left(t \; b^{\dagger}_{\vec{A}+\vec{s}_i} a_{\vec{A}} + t^* \; a^{\dagger}_{\vec{A}} b_{\vec{A}+\vec{s}_i} \right) \quad , \quad t \sim 2.7ev \quad |\vec{s}_i| \sim$$

$$H = \int d^2x \psi^\dagger i\vec{\alpha}\cdot\vec{\nabla}\psi \;\rightarrow\; \sum_x \frac{i}{2}\Big[\psi^\dagger(x)\alpha^a\psi(x+\hat{e}_a) - \psi^\dagger(x+\hat{e}$$



For a square lattice

$$\prod_{\text{plaquette}} \alpha^a = -1$$

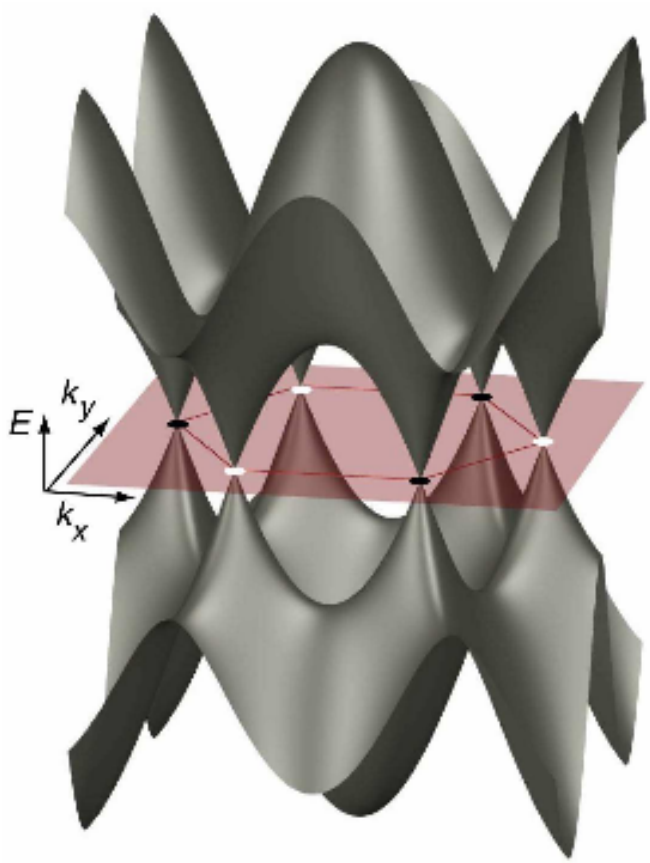
Gauge transform can diagonalize α^a to U(1) valued link v
with $\frac{1}{2}$ magnetic flux quantum in each plaquette.

$$\prod_{\text{hexagon}} \alpha^a = 1$$

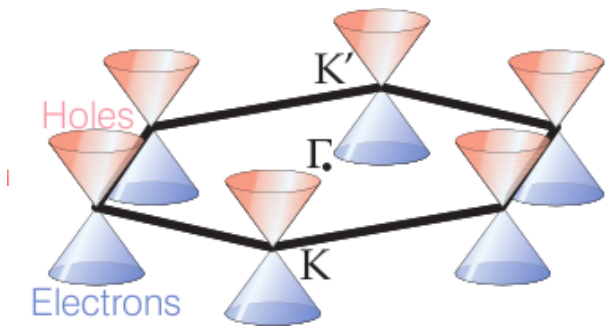
graphene tight-binding model = staggered fermions

Band structure of graphene

Tight binding model with one electron per site of 2D hexa
lattice has valence and conduction bands



Linearize spectrum near degeneracy points



$$E(k) = \hbar v_F |\vec{k}|$$

$v_F \sim 10^6 m/s \sim c/300$, good up to $\sim 1ev$

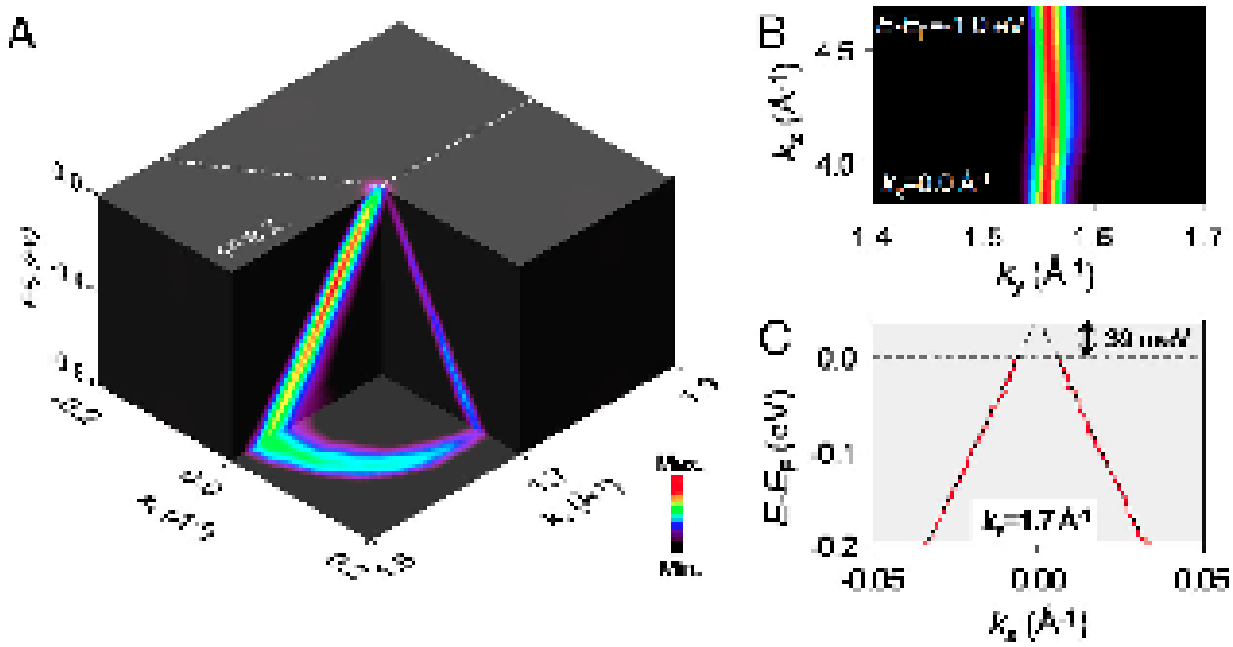
$$H_{\text{Dirac}} = \hbar v_F \begin{bmatrix} 0 & k_x - ik_y & 0 & 0 \\ k_x + ik_y & 0 & 0 & 0 \\ 0 & 0 & 0 & k_x + ik_y \\ 0 & 0 & k_x - ik_y & 0 \end{bmatrix} \begin{bmatrix} \psi \\ \psi \\ \psi \\ \psi \end{bmatrix}$$

2 valleys $\times$ 2 spins = U(4) symmetry
with 2-component massless fermions

$$L = \sum_{k=1}^{4} \bar{\psi}_k i\gamma^\mu \partial_\mu \psi_k$$

Electron dispersion relation with ARPES

D.A. Siegel et. al. PNAS,1100242108



Graphene with Coulomb interaction $\quad V(r) = \frac{e^2}{4\pi r}$

$$S = \int d^3x \sum_{k=1}^{4} \bar{\psi}_k \left[\gamma^t (i\partial_t - A_t) + v_F \vec{\gamma} \cdot (i\vec{\nabla} - \vec{A})\right] \psi_k$$

$$+ \frac{1}{4e^2} \int dt d^2x \left[F_{0i} \frac{1}{2\sqrt{\partial_t^2 - c^2\nabla^2}} F_{0i} - F_{ij} \frac{c^2}{2\sqrt{\partial_t^2 - c}}\right.$$

- The interaction is not relativistic since speeds of light
different

- This theory is strongly coupled: the graphene fine str
constant is larger than one,

$$\alpha_{\text{graphene}} = \frac{\frac{e^2}{4\pi\lambda}}{\hbar v_F/\lambda} = \frac{e^2}{4\pi\hbar v_F} = \frac{e^2}{4\pi\hbar c}\frac{c}{v_F} \approx \frac{300}{137}\,,\ e^2 =$$

Electron dispersion relation with ARPES

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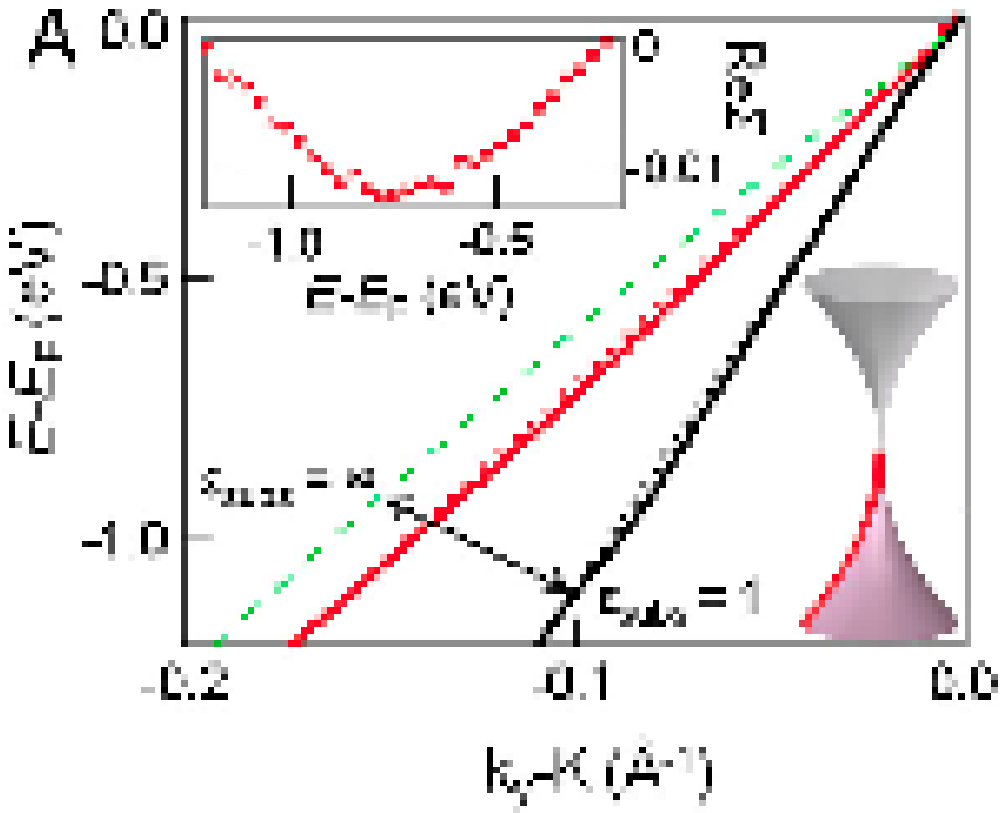


AC Conductivity of Neutral Graphene

$\omega >> k_B T$ RG improved two-loop correction

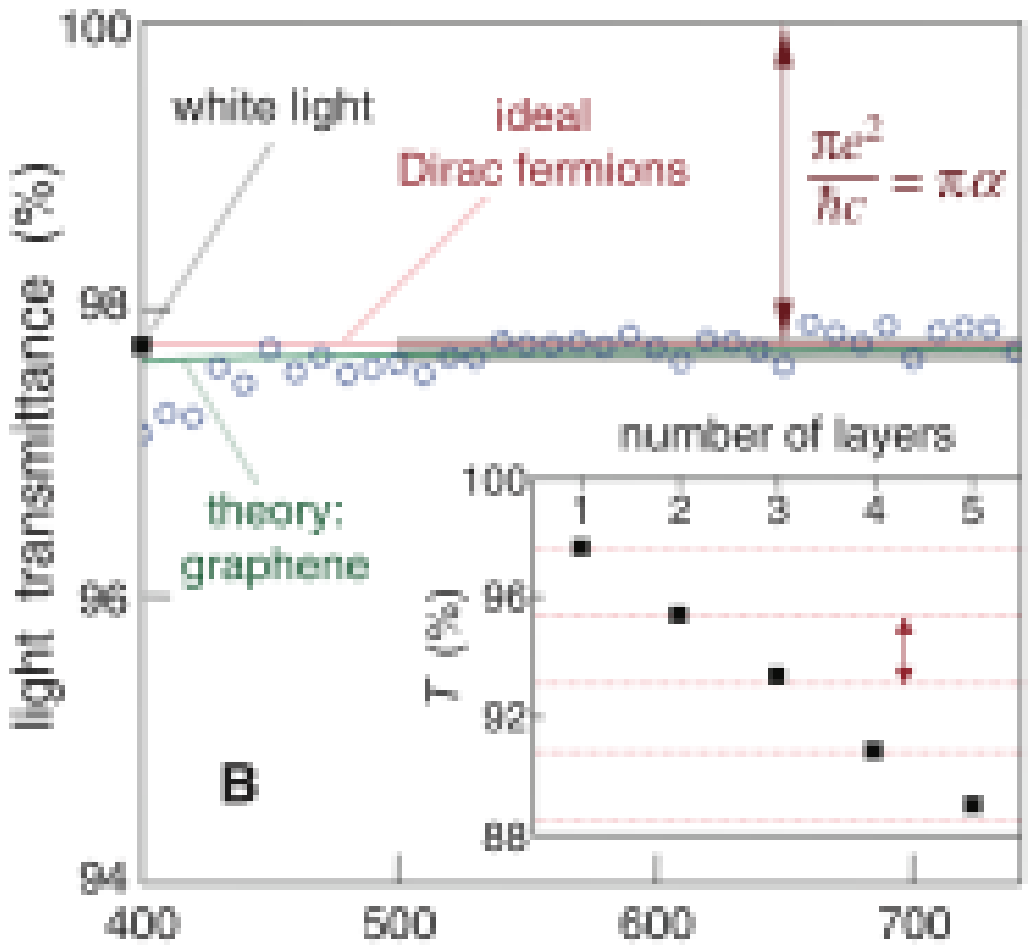
$$\sigma(\omega) = \frac{e^2}{4\hbar}\left(1 + \mathcal{C}\frac{e^2}{4\pi\hbar v_F}\frac{1}{\left(1 + \frac{e^2}{4\pi\hbar v_F}\frac{1}{4}\ln(\Lambda/\omega)\right)}\right)$$

Theory $\mathcal{C} = \frac{11-3\pi}{6} \sim .26$

V. Juricic et.al. Phys. Rev. B 82, 235402 (2010)

Experiments $\mathcal{C} = 0\pm?$

R. Nair et.al., Science 320, 1308 2008.



Nordita, October 17, 2012

“Dirac cones reshaped by interaction effects in sus
graphene”

D.C.Elias, R.V.Gorbachev, A.S.Mayorov, S.V.Morozov,
A.A.Zhukov, P.Blake, L.A.Ponomarenko, I.V.Grigorieva,
K.S.Novoselov, F.Guinea, A.K.Geim

Nature Physics 7, 701704 (2011) doi:10.1038/nphys2049
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(February, 2012)

Large N approximation

$$S = \int dt d^2x \sum_{k=1}^{N} \bar{\psi}_k \left[\gamma^t (i\partial_t - A_t) + v_F \vec{\gamma} \cdot (i\vec{\nabla} - \vec{A}) \right] \psi$$

$$+ \frac{1}{4e^2} \int dt d^2x \left[F_{0i} \frac{1}{2\sqrt{\partial_t^2 - c^2\nabla^2}} F_{0i} - F_{ij} \frac{c^2}{2\sqrt{\partial_t^2 - c}}\right.$$

In this large N, we integrate out fermions to get effective

$$S = \frac{N}{32} \int dt d^2x \left[F_{0i} \frac{1}{2\sqrt{\partial_t^2 - v_F^2 \nabla^2}} F_{0i} - v_F^2 \; F_{ij} \frac{1}{2\sqrt{\partial_t^2 -}}\right.$$

$$+ \frac{1}{4e^2} \int dt d^2x \left[F_{0i} \frac{1}{2\sqrt{\partial_t^2 - c^2\nabla^2}} F_{0i} - F_{ij} \frac{c^2}{2\sqrt{\partial_t^2 - c}}\right.$$

Relativistic at infinite N with speed of light v_F.
AC conductivity $\sigma(\omega) = \frac{e^2 N}{16} + \mathcal{O}(1)$.

DBI + WZ Action for D7-Brane

$$L = N_7 T_7 \left[-\sqrt{-\det(g + 2\pi\alpha' F)} + F \wedge F \wedge \omega^{(4)} \right.$$

AdS_5 coordinates and 4-form

$$\frac{dS^2}{\sqrt{\lambda}\alpha'} = r^2(dt^2 + dx^2 + dy^2 + dz^2) + \frac{dr^2}{r^2} + d\psi^2 + \sin^2\psi d^2\Omega_2 +$$

$$\omega^{(4)} = \lambda\alpha'^2 r^4 dt \wedge dx \wedge dy \wedge dz + \lambda\alpha'^2 \frac{c(\psi)}{2} d\Omega_2 \wedge d$$

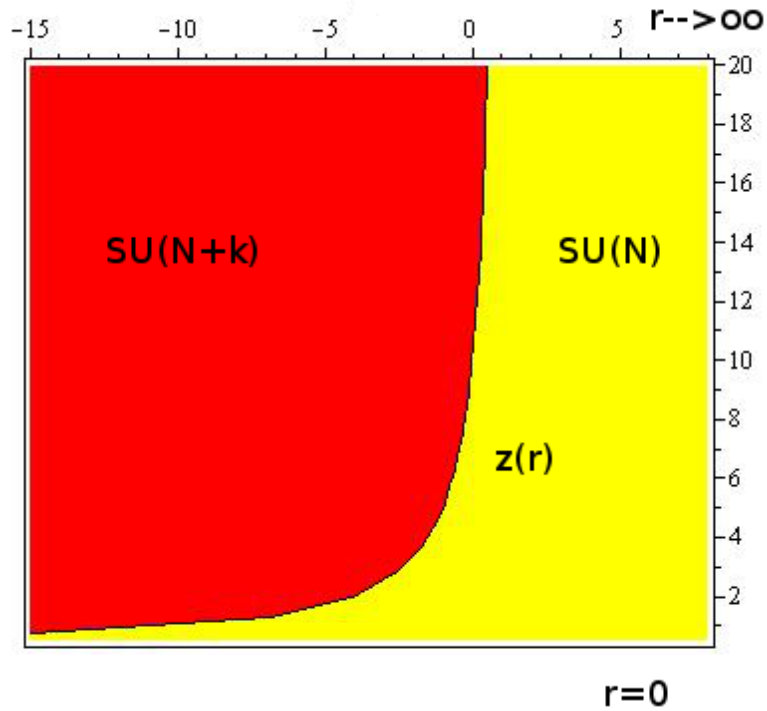
$\exists$ C P symmetric solution with D7-metric $AdS_4 \times S^2 \times S$

$$\frac{ds^2}{\sqrt{\lambda}\alpha'} = r^2(-dt^2 + dx^2 + dy^2) + \frac{dr^2}{r^2}\frac{(1+f^2)^2}{1+2f^2} + \frac{1}{2}d^2\Omega_2$$

$$\psi = \frac{\pi}{4}\ ,\ z(r) = -\frac{f^2}{\sqrt{1+2f^2}}\frac{1}{r}$$

$$F = \frac{n_D}{2}(d\Omega_2 + d\tilde{\Omega}_2)\ ,\ n_D = \sqrt{\lambda} f\ ,\ \left(f^2 > \frac{23}{50}\right)$$

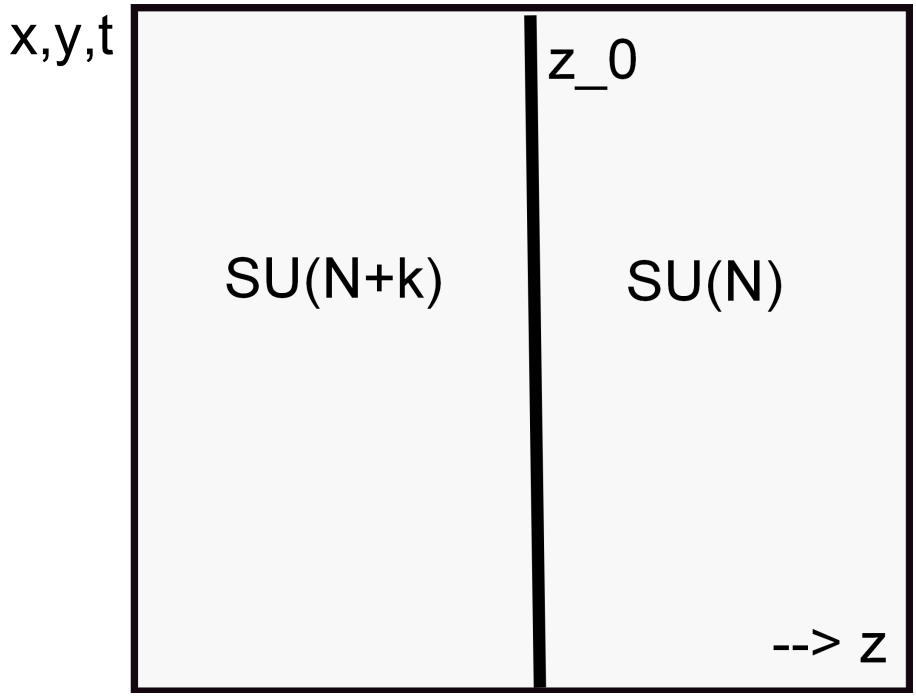
Defect conformal field theory



$k = n_D^2$ where $n_D = \sqrt{\lambda} f$

$$z(r) = -\frac{f^2}{\sqrt{1+2f^2}} \frac{1}{r}$$

Defect conformal field theory



$$\left\langle \bar{\Psi}\gamma_\mu\Psi \;\; \bar{\Psi}\gamma_\nu\Psi \right\rangle = \frac{N_3 N_7}{2\pi^2}\frac{(f^2+1)}{q}\left(q^2 g_{\mu\nu} - q_\mu q_\nu\right)$$

$\sigma(\omega) = \frac{2\lambda}{\pi^2\hbar}(1+f^2)$, compare with free field value $\sigma(\omega) =$

Charge Density

$\exists$ a solution ($q \sim \rho$)

$$ds^2 = \sqrt{\lambda}\alpha' \left[r^2(-dt^2 + dx^2 + dy^2) + \frac{dr^2}{r^2} \frac{(1+f^2)^2 + q^2/}{1 + 2f^2 + q^2/r} \right.$$

$$F = \frac{\sqrt{\lambda}}{2\pi} \frac{q}{(1+2f^2)r^4 + q^2} dt \wedge dr + \frac{n_D}{2}(d\Omega_2 + d\tilde{\Omega}_2)$$

$$\boxed{\rho = \frac{8\pi\sqrt{2f^2+1}\ N}{\Gamma^4[\frac{1}{4}]}\mu^2 \sim .065 N\mu^2 \quad \text{with } f^2 = 1/}$$

Free field theory: $\rho = \frac{N}{(2\pi)^2} \int d^2k\theta(\mu - |k|) = \frac{N}{4\pi}\ \mu^2 \sim .080$

Debye mass

$$\boxed{M_D = \frac{d}{d\mu}\rho = \frac{16\pi\sqrt{2f^2+1}\ N}{\Gamma^4[\frac{1}{4}]}\mu}$$

Diamagnetism

$$M \sim -\frac{\partial}{\partial B} F$$

$$F \sim B^{\frac{3}{2}} \quad , \quad M \sim \sqrt{B}$$

$$M = -\frac{(f^2+1)^{\frac{1}{2}} 3N}{4(2f^2+1)^{\frac{1}{4}}\sqrt{(2\pi)^5\lambda}} \mathcal{B}[\tfrac{1}{4},\tfrac{1}{4}]\sqrt{B} \sim -\frac{(0.06)}{\sqrt{\lambda}} N\sqrt{B}$$

For free fields

$$F = -4\frac{|B|}{2\pi}\sum_{n=1}^{\infty}\sqrt{2|B|n} = -4\sqrt{\frac{1}{2\pi^2}}|B|^{\frac{3}{2}}\zeta(-1/2)$$

$$M = 4\sqrt{\frac{9}{8\pi^2}}\zeta(-1/2)\sqrt{|B|} \sim -(4)(0.07)\sqrt{|B}$$

AC conductivity

$$\sigma(\omega) \simeq \frac{3e^2}{\pi^2\hbar}$$

Debye screeming length

$$\mu L_D \simeq e \quad , \quad e \simeq 5$$

Diamagnetism

$$M \simeq -(0.24)e\sqrt{B}$$

Free fermions:

$$\sigma(\omega) = \frac{e^2}{4\hbar}$$

$$\mu L_D \simeq 1.6$$

$$M \simeq -0.28\sqrt{B}\ \mathrm{sign}(B)$$

WIP: Plasmon frequency, Thermodynamics, Heat transpo

Conclusions

- D7-D3 system as strongly coupled 2+1-dimensional re
  fermions
- Conformal field theory at strong coupling
- Graphene as a strongly coupled quantum fluid
  - AC conductivity
  - Screening length versus chemical potential
  - Magnetization density
  - Experimental signatures of strong coupling, confor
    symmetry?