Benasque workshop on Gravity (stringy and hi-d)

July 14-26 2013

Black Brane Fluid Flows

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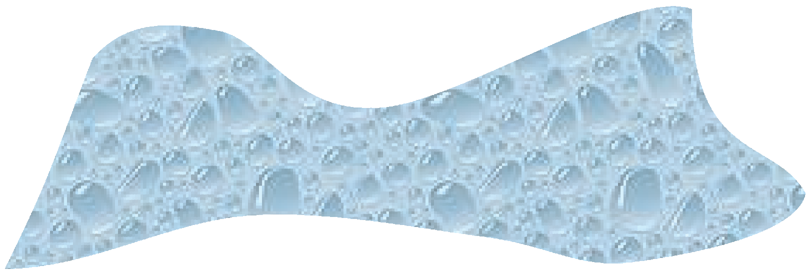
w/ M. Martínez 1205.5646
w/ V. Hubeny + M. Rangamani, in progress

Black Hole dynamics

- Black holes are not passive sinks, but highly *dynamical* objects
- Concepts and techniques from other areas of physics apply to them:
  - Effective field theories of dynamics
  - Thermodynamics
  - Hydrodynamics
  - Elasticity theory

Black Hole dynamics

- Full dynamics of bhs is very complicated
- Effective theory for long-wavelength fluctuations
- In D>4 black holes exhibit both
  - hydrodynamical behavior (fluid)
  - elastic behavior (solid)



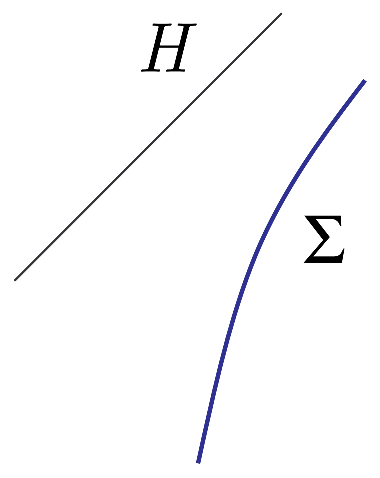
Effective theories for black objects

- Take spacetime w/ horizon H
- Choose some timelike surface Σ outside horizon

1st, 2nd fundamental forms on Σ

$h_{\mu\nu}$: induced metric

$\Theta_{\mu\nu}$: extrinsic curvature



Effective theories for black objects

- $h_{\mu\nu}, \Theta_{\mu\nu}$
- Stress tensor: Brown-York quasilocal

$$8\pi G T_{\mu\nu} = \Theta_{\mu\nu} - \Theta\ h_{\mu\nu}$$

- Automatically conserved:

Gauss-Codacci = ‘momentum constraints’

$$R^{r\nu} = 0 \rightarrow \nabla_{\mu} T^{\mu\nu} = 0$$

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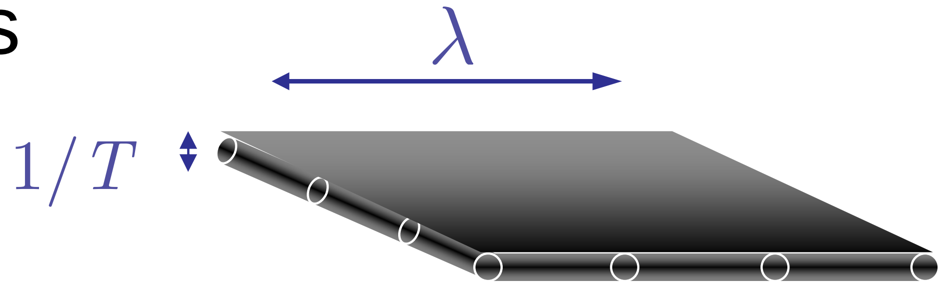
- Is this already the hydrodynamic theory?

Effective theories for black objects

- Not yet: must specify long-wavelength regime
- $\lambda \gg 1/T$ (T measured on Σ)

  This can exclude horizon fluctuations in some directions



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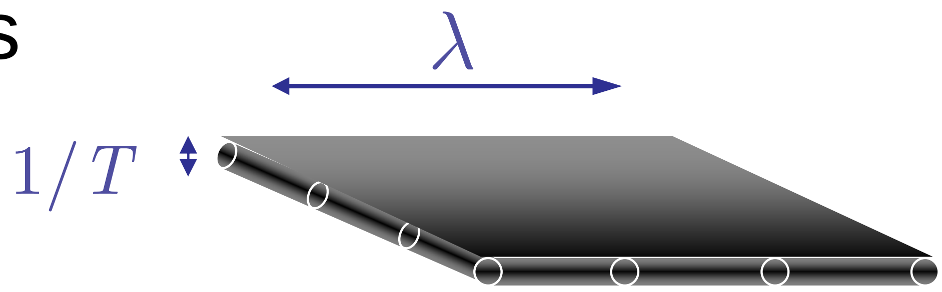
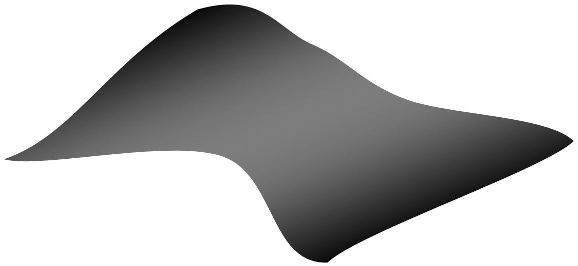


- Is this now the hydrodynamic theory?

Effective theories for black objects

- Not necessarily: *it is* a long-wavelength effective theory, but maybe not only hydro
- E.g., a worldvolume may have elastic dynamics



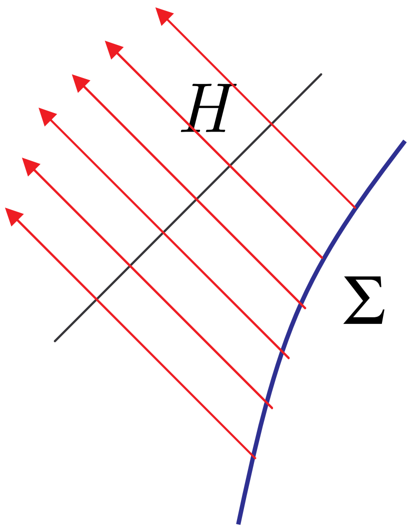
this is *extrinsic* dynamics, orthogonal to worldvolume: $X^\mu(\sigma)$

Fluids from gravity

- **Hydro**: *long-wavelength* fluctuations *parallel* to horizon
- characterized by velocity u^a
  - $|u|^2 = -1$ on Σ, $|u|^2 = 0$ on H
- Expand in worldvol derivatives of u^a

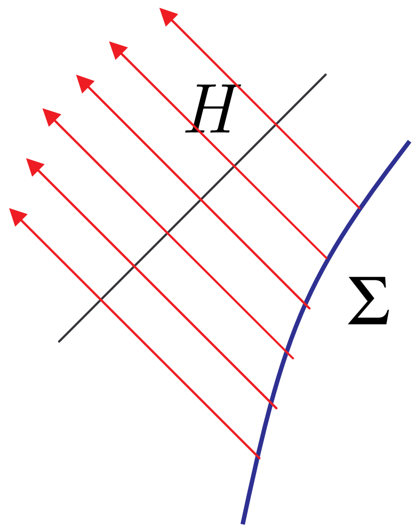
Fluids from gravity

- $\nabla_\mu T^{\mu\nu} = 0$: constraint eqs
- Remaining Einstein eqs then uniquely solved in 'inside region' (bulk) with fixed $h_{\mu\nu}$ (Dirichlet bc) and regularity at horizon



Fluids from gravity

- Have integrated inside d.o.f.'s
- Fluid theory is the effective theory, in terms of collective dof's, for the dynamics of inside



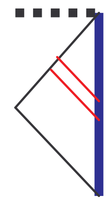
Fluids from gravity

- Could use this eff theory to couple to outside (*if any*) via effective $T_{\mu\nu}$
  - must let metric on Σ fluctuate

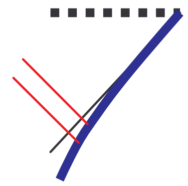
- This is then similar to *membrane paradigm*
  - but in the latter *no dof's are integrated*
  - no equations solved, *only bdry data*

Fluids from gravity

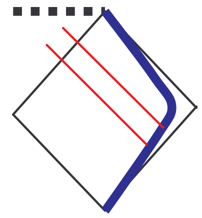
- Very general:
  - timelike surface Σ
  - long-wavelength, intrinsic fluctuations
- Examples
  - AdS black branes (fluid/gravity corresp)



  - near-horizon (Rindler dual fluids)



  - Asympflat vacuum black branes



Camps+RE+Haddad 2010

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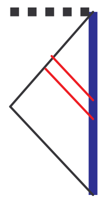
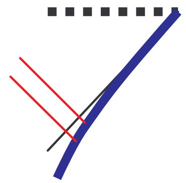
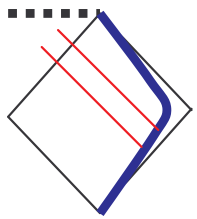
  - – AdS black branes (fluid/gravity corresp) 
  - – near-horizon (Rindler dual fluids) 
  - – **Asympflat vacuum black branes** 

Camps+RE+Haddad 2010

Changing surface, changing fluid

- Given a black object, each surface Σ gives a different fluid

- Reminiscent of RG flow
  - but we integrate dof's from horizon (IR) to surface, not from asymptopia (UV) to surface

- Related work:
  - *Bredberg et al, Compere et al, Eling+Oz...*
  - *Brattan et al* (AdS black brane)

Changing surface, changing fluid

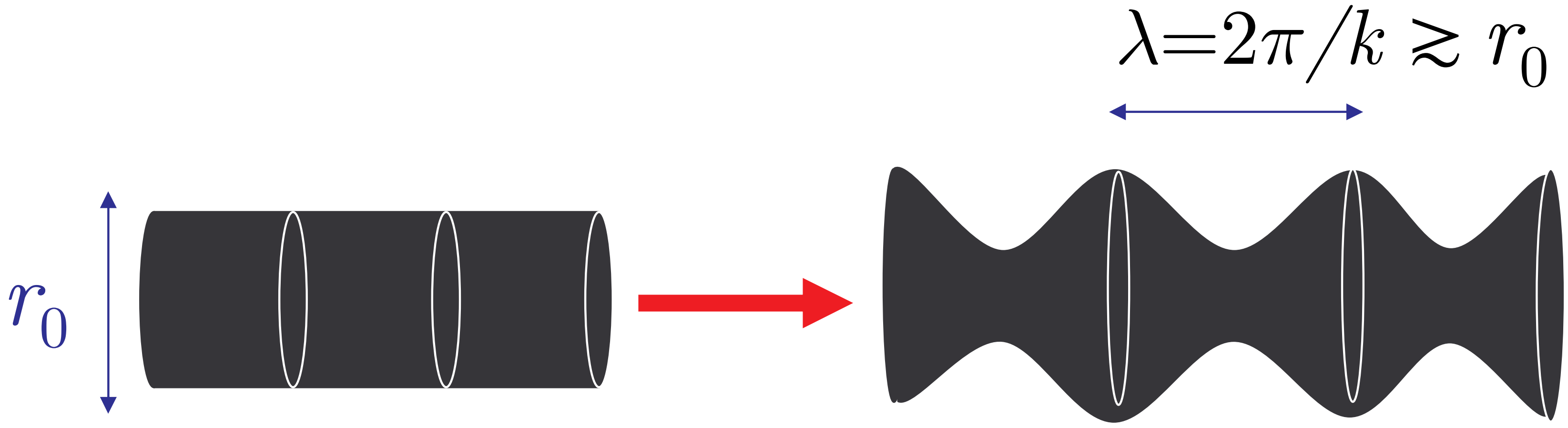
- For AF black branes, as we move Σ we can observe interesting phenomena:

1. Black brane instability *On*/*Off*

2. Nested fluids:

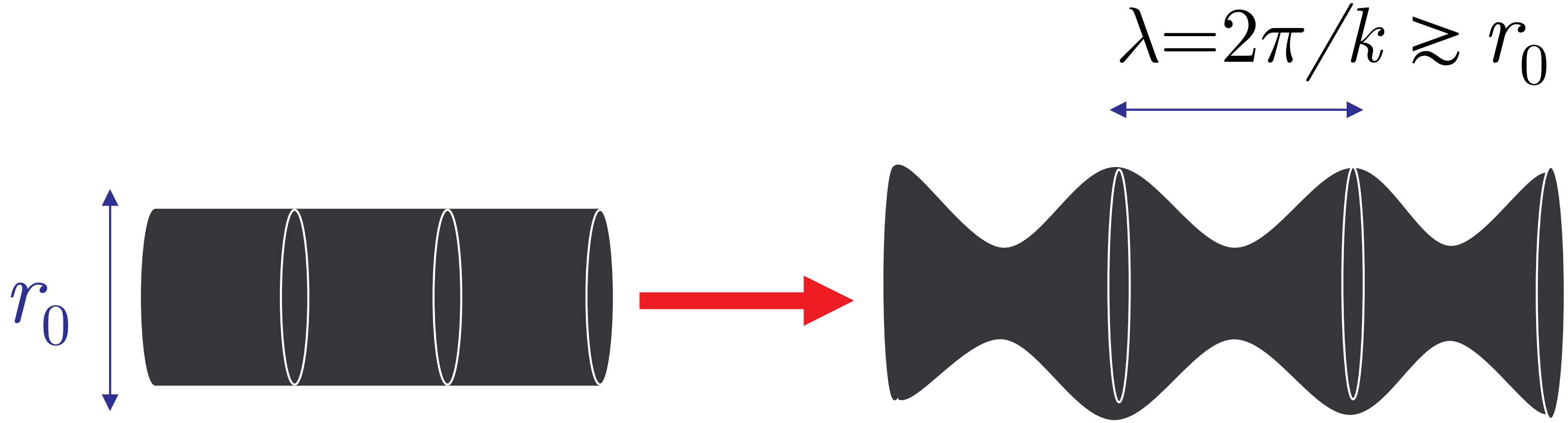
   Blackfold > Fluid/gravity > Rindler fluid

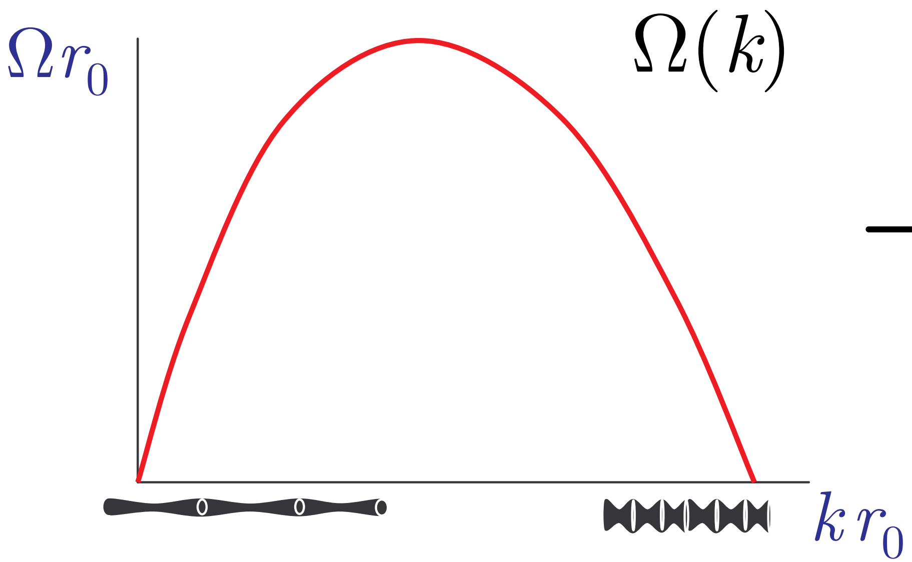
Gregory-Laflamme instability



$$\delta r_0 \sim e^{\Omega t + ikz}$$

Gregory-Laflamme instability

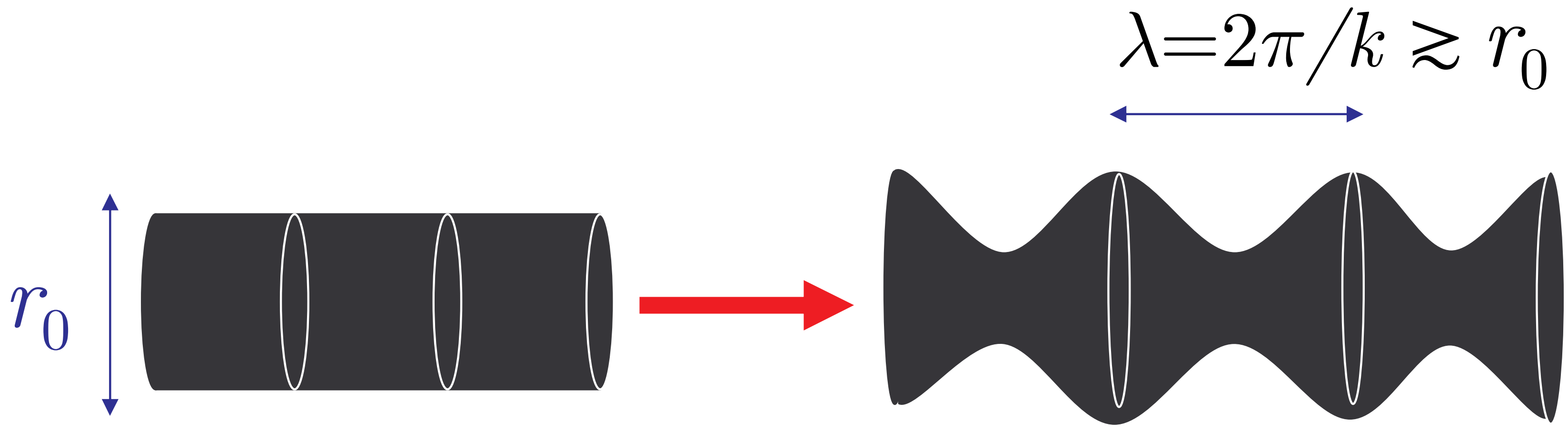


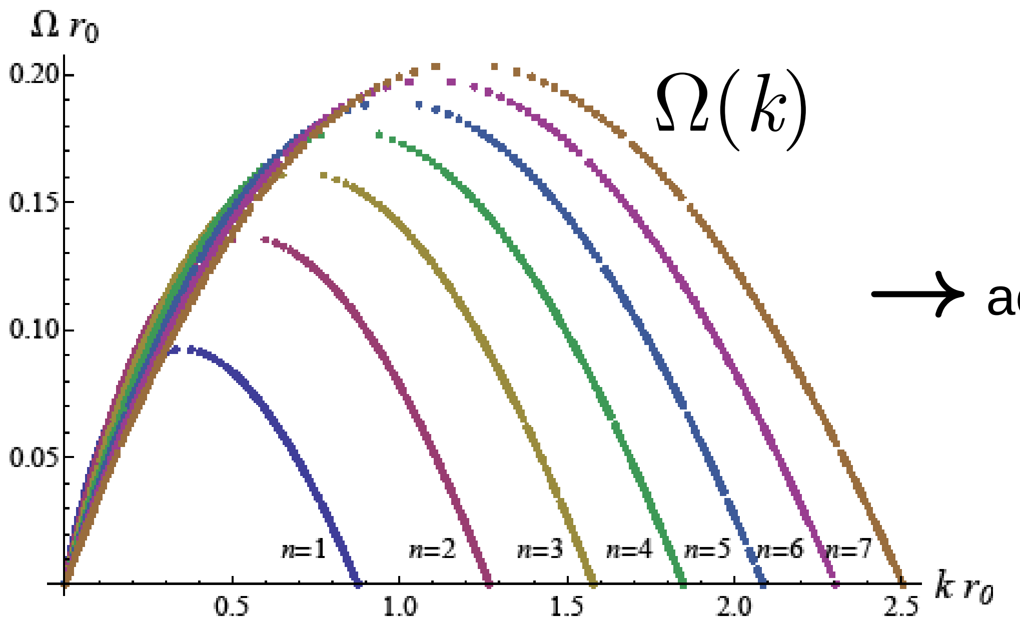


$$\delta r_0 \sim e^{\Omega t + ikz}$$

$\longrightarrow$ must be computed numerically from linearized perturbation

Gregory-Laflamme instability





$$\delta r_0 \sim e^{\Omega t + ikz}$$

$\longrightarrow$ actual numerical results (*Figueras*)

$(D = n + 4)$

Gregory-Laflamme instability

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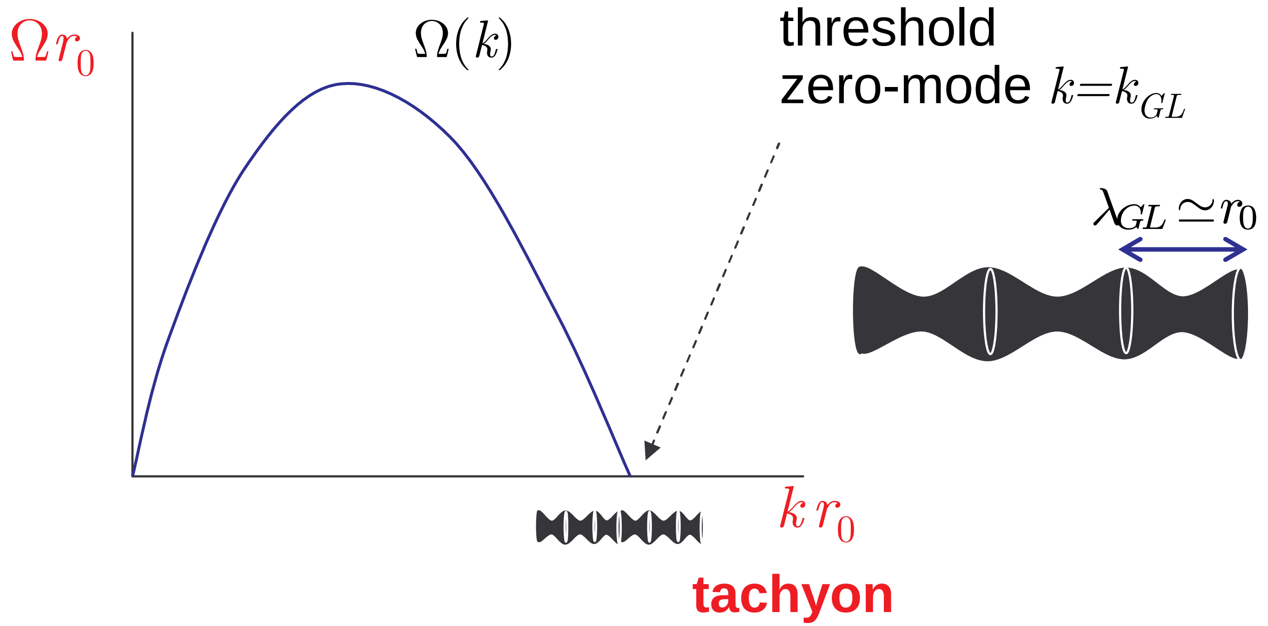

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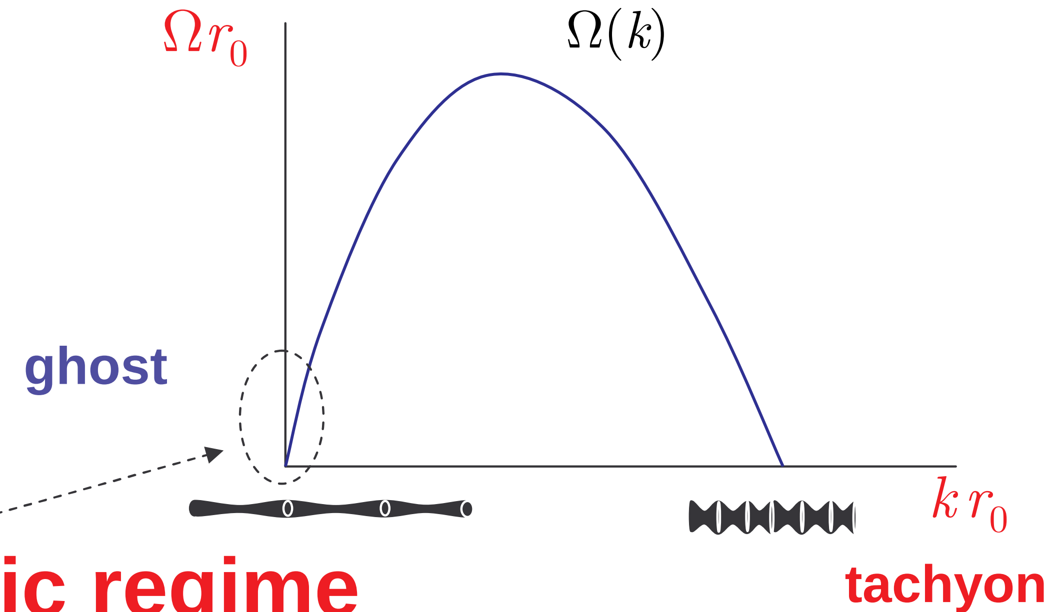
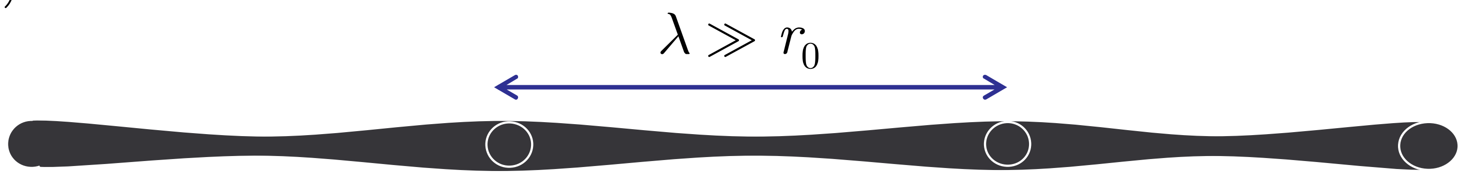


Hydrodynamic regime

$$\Omega \simeq 0, \; k \simeq 0$$



RE+Harmark+Niarchos+Obers

Black string effective 1+1 fluid

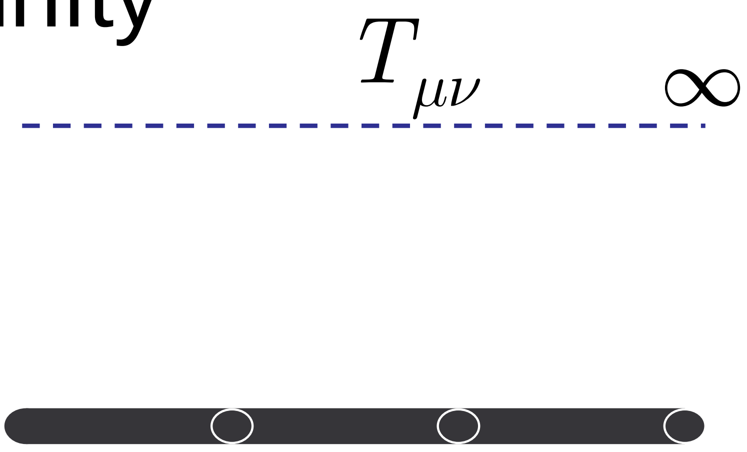
- Black string in $D=4+n$

$$ds^2 = -\left(1 - \frac{r_0^n}{r^n}\right) dt^2 + dx^2 + \frac{dr^2}{1 - \frac{r_0^n}{r^n}} + r^2 d\Omega_{n+1}$$

- Stress-energy at asymp infty

$$\varepsilon = -(n+1)P$$

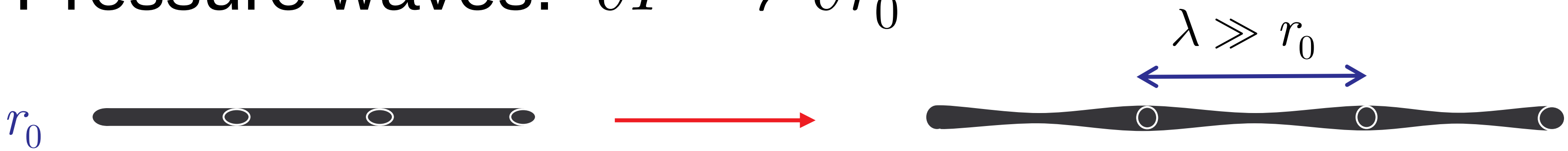
$$P = -{r_0}^n$$



Gregory-Laflamme from fluid dynamics

- Effective fluid $\begin{cases} \varepsilon = -(n+1)P \\ P = -r_0{}^n \end{cases}$ $(n = D-4)$

  Pressure waves: $\delta P \to \delta r_0$



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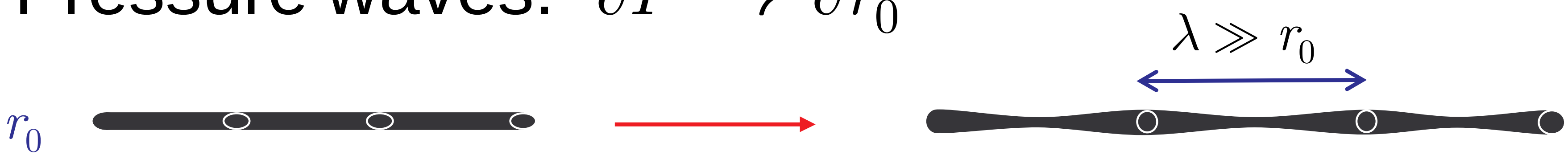

- Sound velocity $v_s{}^2 = dP/d\varepsilon = -1/(n+1) < 0$

Unstable

Gregory-Laflamme from fluid dynamics

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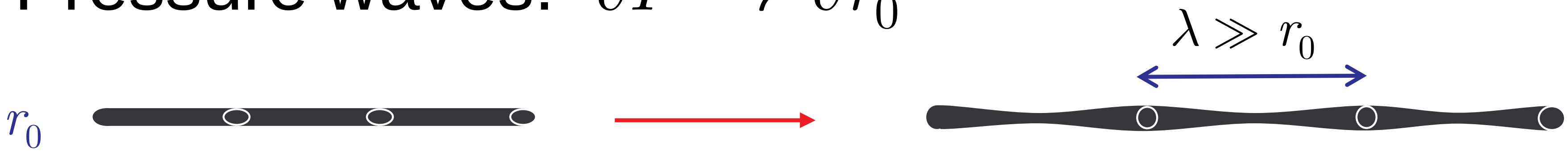
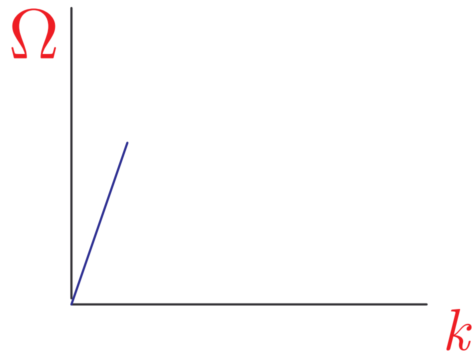


- Sound velocity $v_s{}^2 = dP/d\varepsilon = -1/(n+1) < 0$

$$\Omega = \sqrt{-v_s^2}\, k + O(k^2)$$

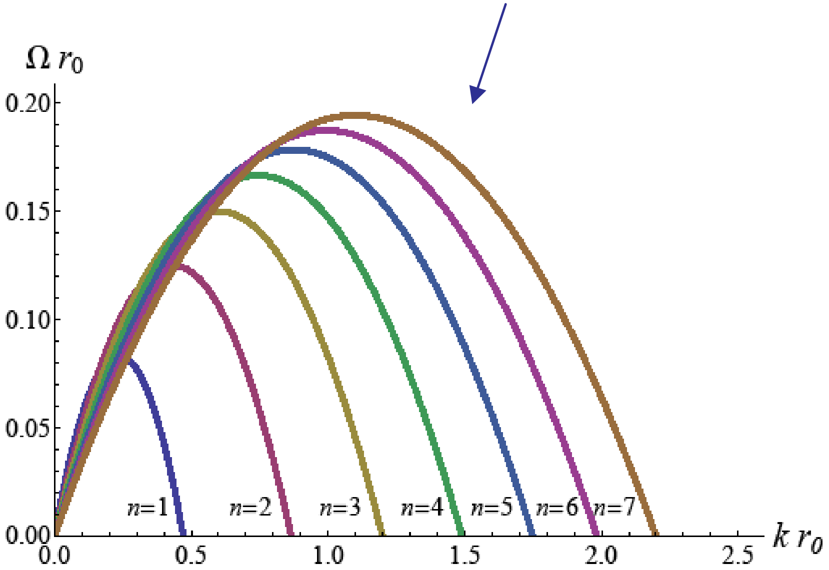


Almost effortless

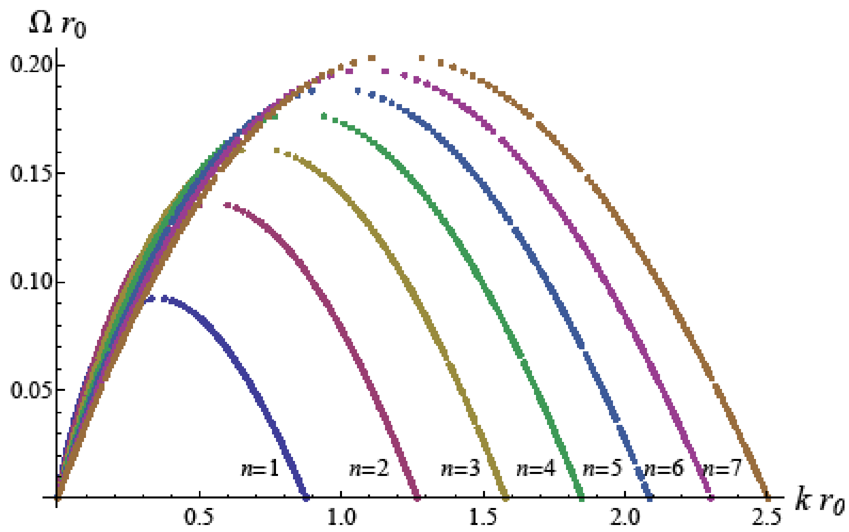
Viscous damping of sound

- Viscosity allows to compute next order, k^2

$$\Omega = \frac{k}{\sqrt{n+1}}\left(1 - \frac{n+2}{n\sqrt{n+1}}kr_0\right) \qquad (n = D-4)$$



Viscous 1+1 fluid

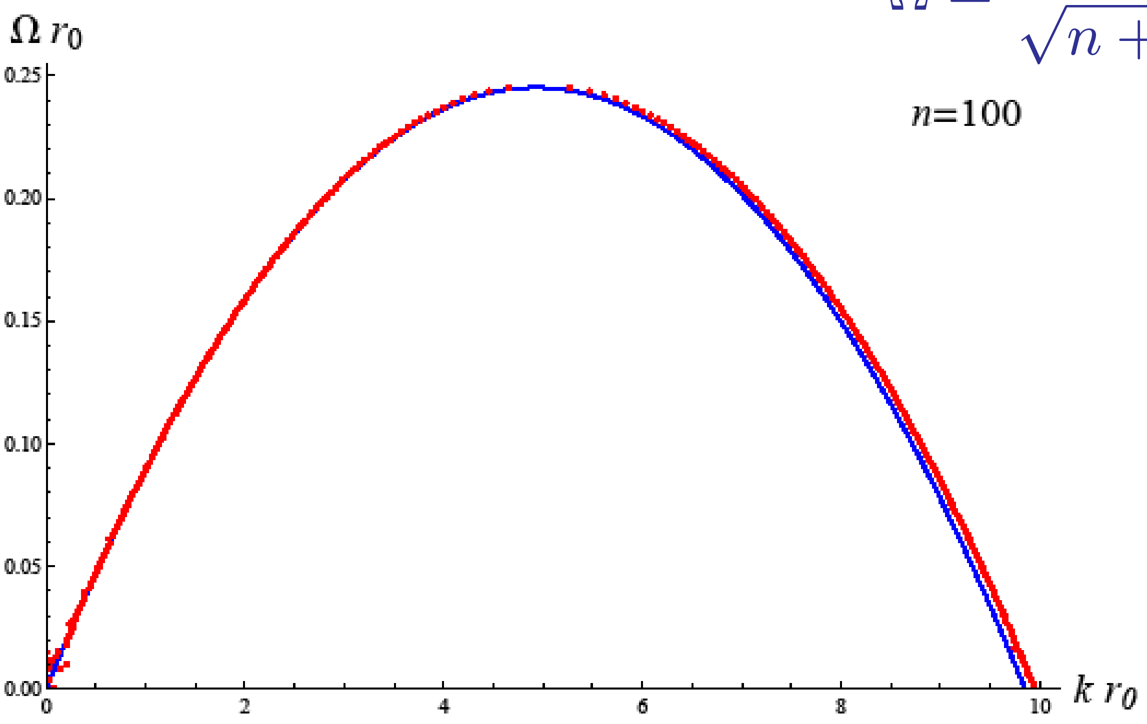


Numerical Gregory-Laflamme

Viscous damping of sound

- Agreement is impressively good for large n

$$\Omega = \frac{k}{\sqrt{n+1}}\left(1 - \frac{n+2}{n\sqrt{n+1}} k r_0\right)$$

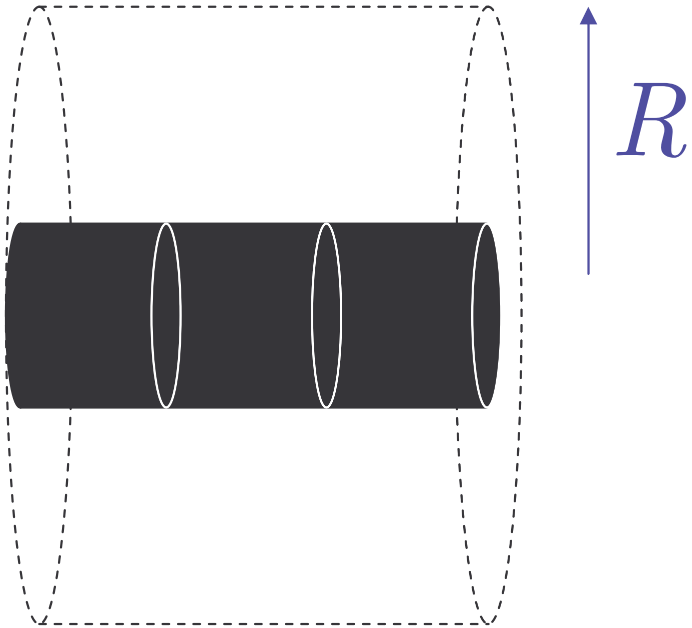


$$T = \frac{n}{4\pi r_0}$$

large n
$\rightarrow$ small thermal length

Framing the ghost

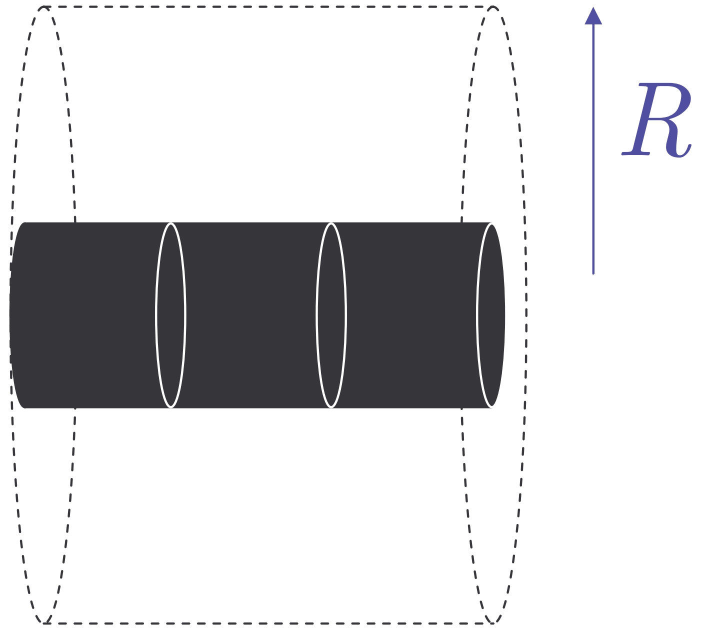
Cavity: fix the metric on a cylinder at finite R



$$\varepsilon(R),\, P(R),\, T(R)$$

Framing the ghost

Cavity: fix the metric on a cylinder at finite R



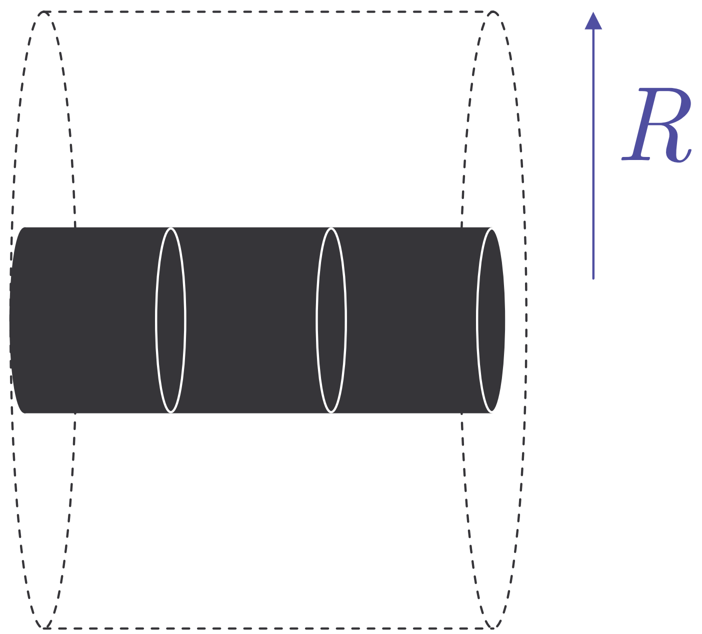
Thermo instability disappears at critical $R = R_c$

$$C_v(R < R_c) > 0$$

$$\varepsilon(R),\ P(R),\ T(R)$$

Framing the ghost

Cavity: fix the metric on a cylinder at finite R



Thermo instability disappears at critical $R = R_c$

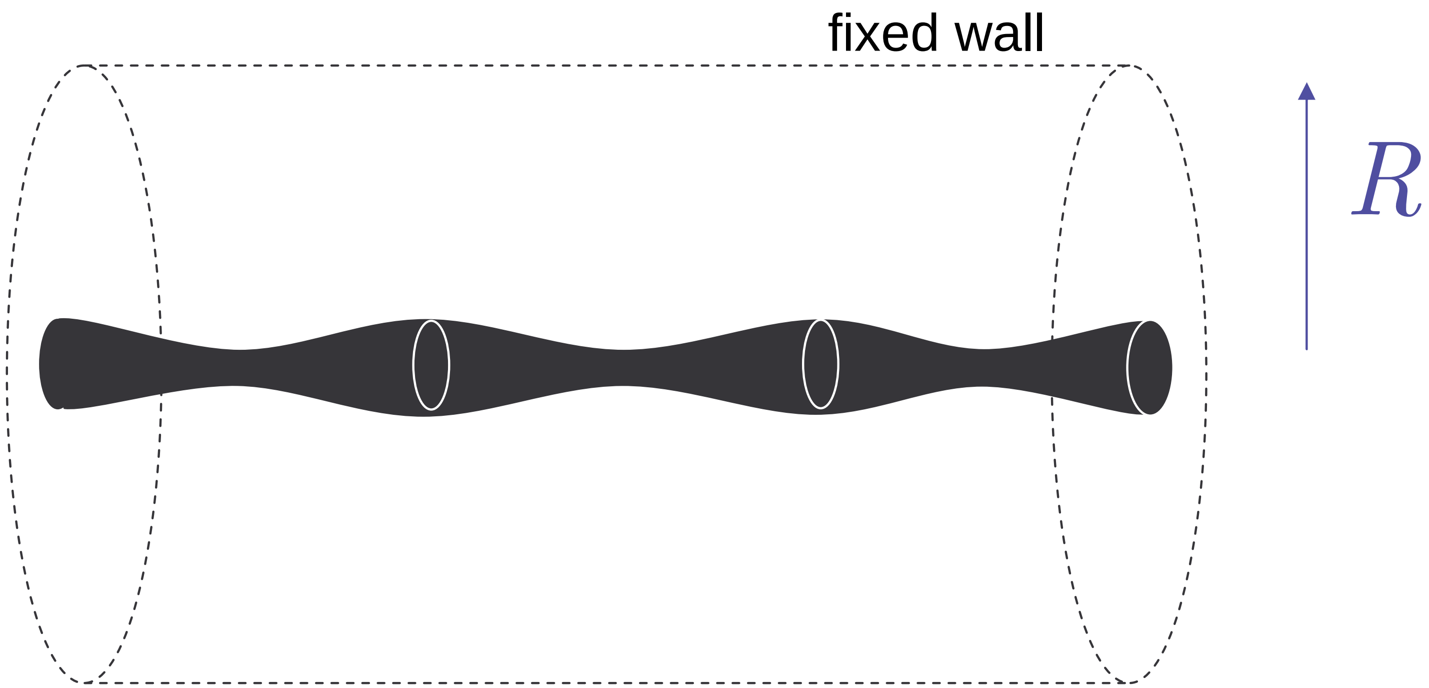
$$C_v(R < R_c) > 0$$

Zero-mode tachyon does disappear at $R = R_c$

Gregory+Ross

Framing the ghost: hydro view

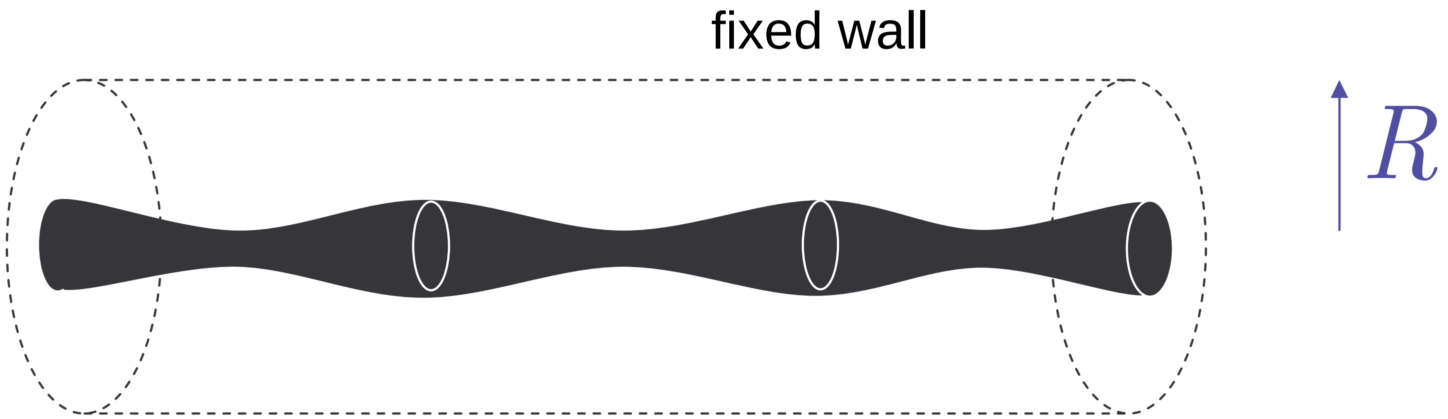
- Solve for fluctuating black brane in cavity, in hydro regime



$$\varepsilon(R),\ P(R),\ T(R),\ \eta(R),\ \zeta(R)$$

Framing the ghost: hydro view

- Solve for fluctuating black brane in cavity, in hydro regime

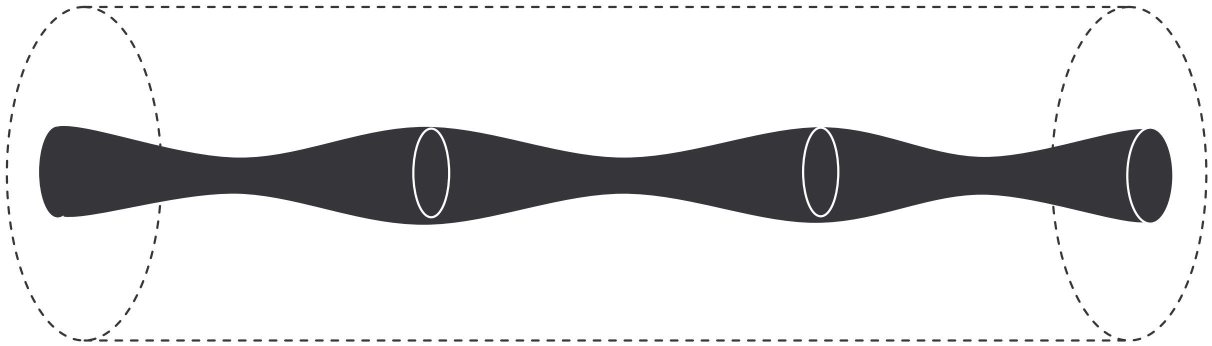


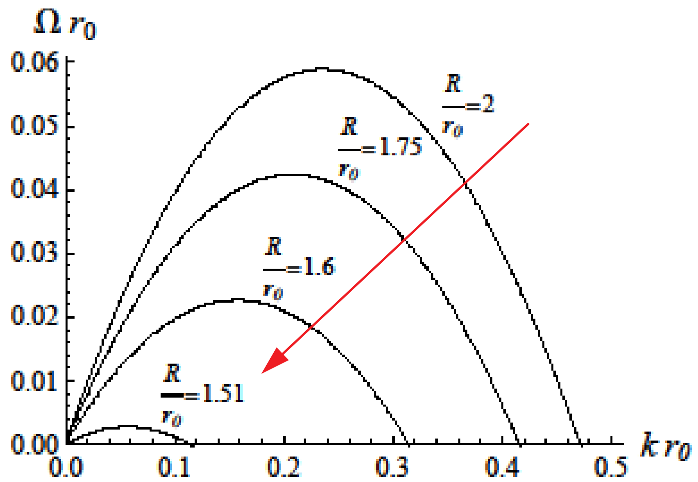
- Sound speed v_s^2 **grows** as R shrinks

  → rigidity increases (redshift on wall hindered)

Framing the ghost: hydro view

- Solve for fluctuating black brane in cavity – hydro regime





- Sound speed v_s^2 **grows** as R shrinks

→ rigidity increases (redshift on wall hindered)

→ instability weakens and disappears at R_c

Correlated stability (aka Gubser-Mitra)

- Transparent in this approach:

local thermo instability

$$\Updownarrow$$

hydrodynamic instability

(***ghost***, not tachyon)

  - Simple proof:

$$v_s{}^2 = \mathrm{d}P/\mathrm{d}\varepsilon = s\,\mathrm{d}T/\mathrm{d}\varepsilon = s/C_v$$

- Can argue ghost $\Rightarrow$ tachyon (but not $\Leftarrow$)

Viscosity: no running with R

- Compute stress tensor at finite R for fluctuating p-brane & extract viscosities

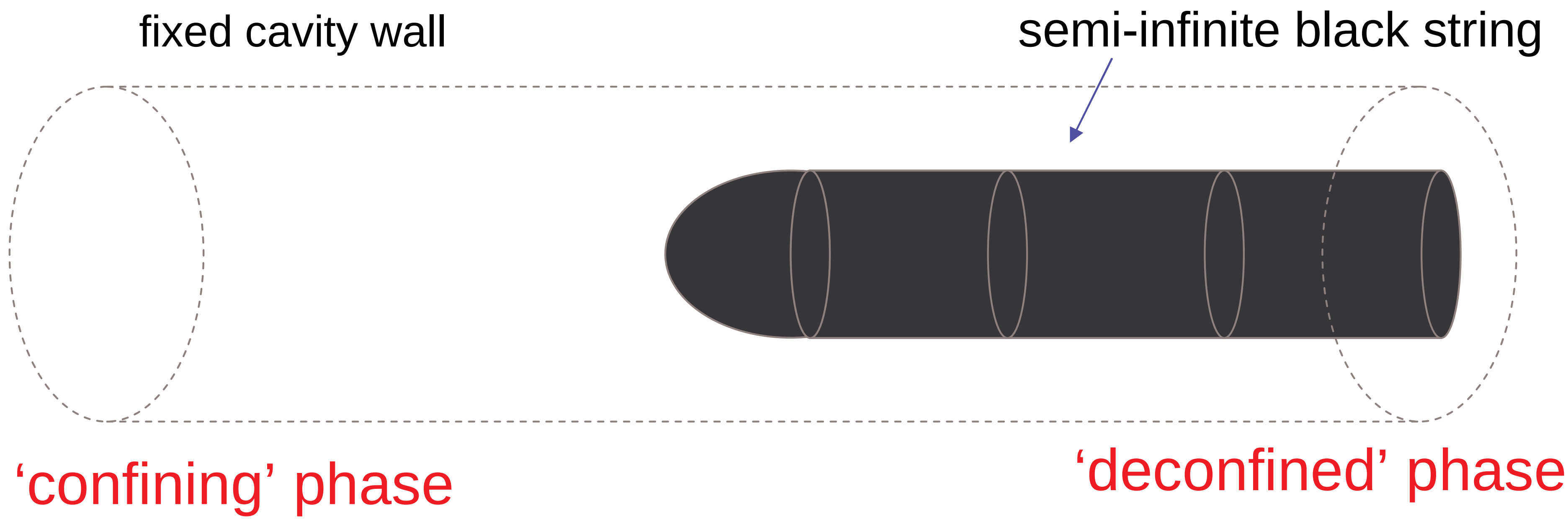
$$\eta(R) = \frac{s}{4\pi}$$

$$\zeta(R) = \frac{s}{2\pi}\left(\frac{1}{p} - v_s^2(\infty)\right)$$

no dependence on R

Black fingers

- ‘Confining/deconfining’ equilibrium

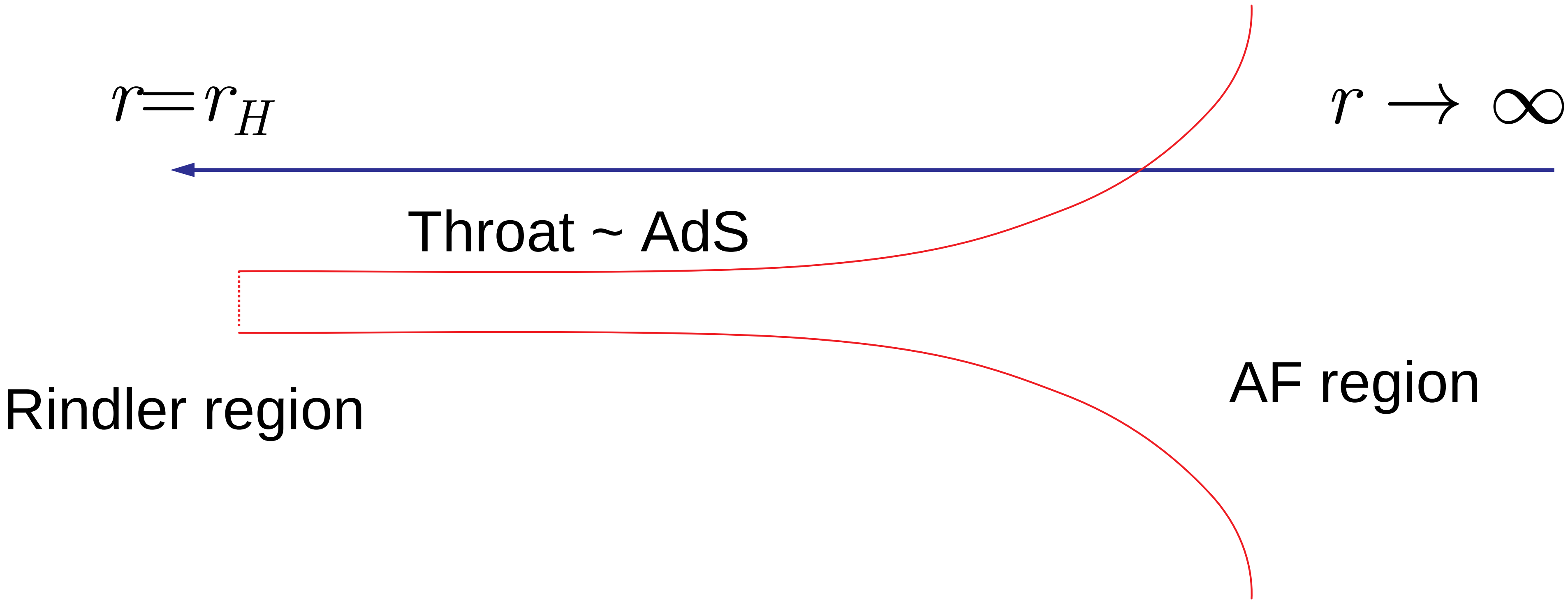


(*numerical w.i.p.*)

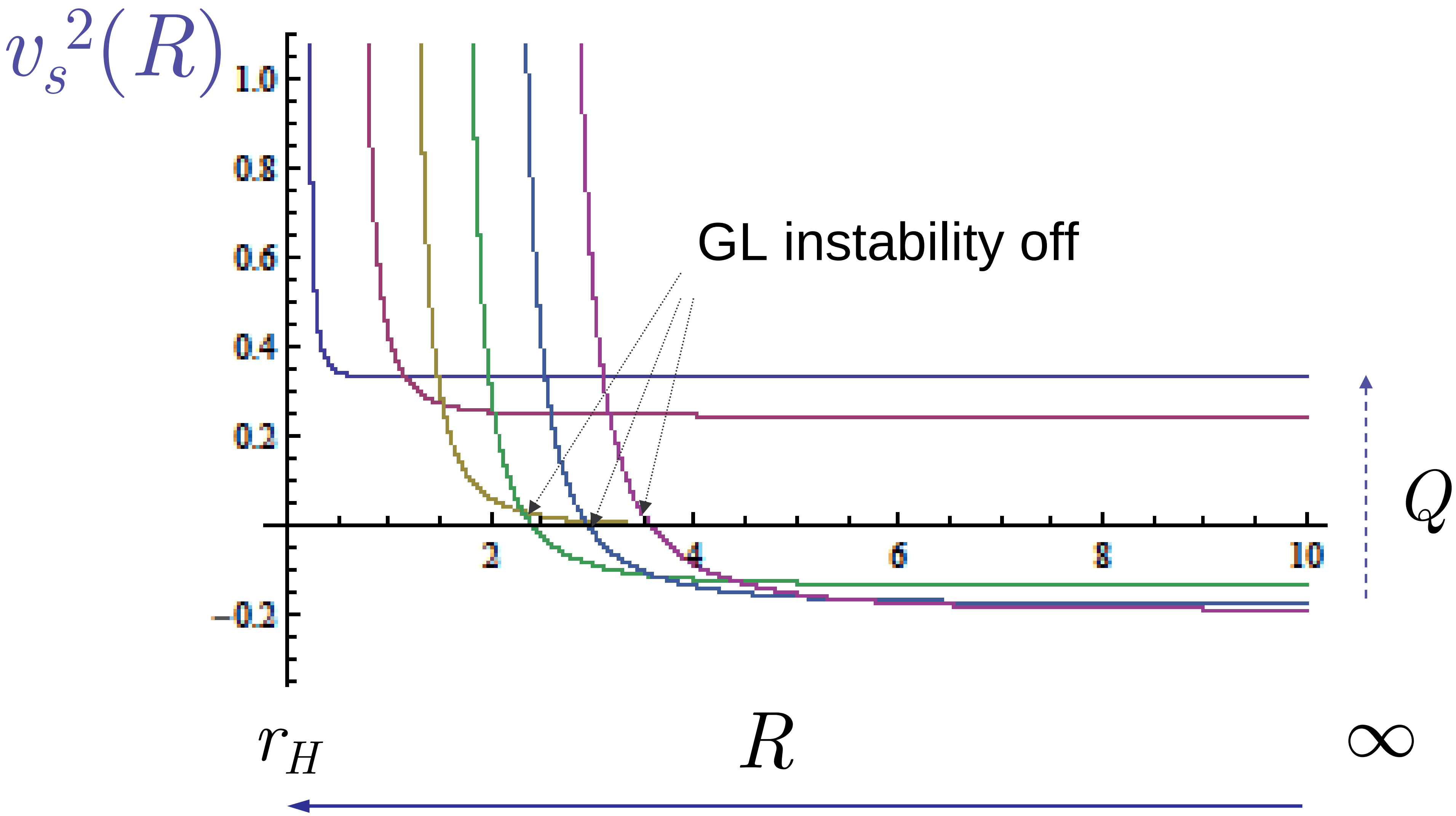
Stable solution in vacuum gravity (w/ wall)

Nested fluids

- Non-extremal D3 branes
- Flow from asymptotic infty down to horizon



Sound from the throat



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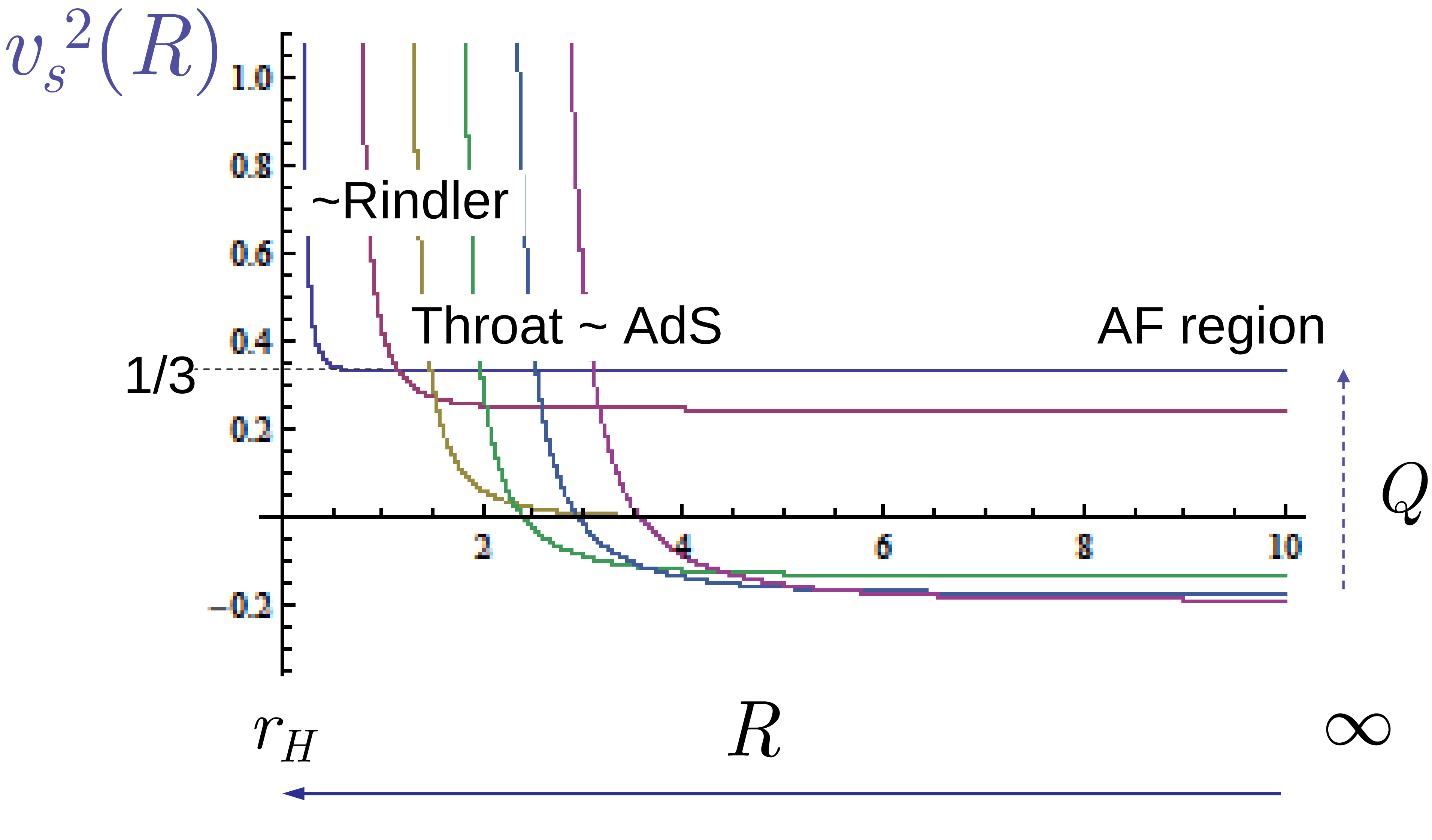

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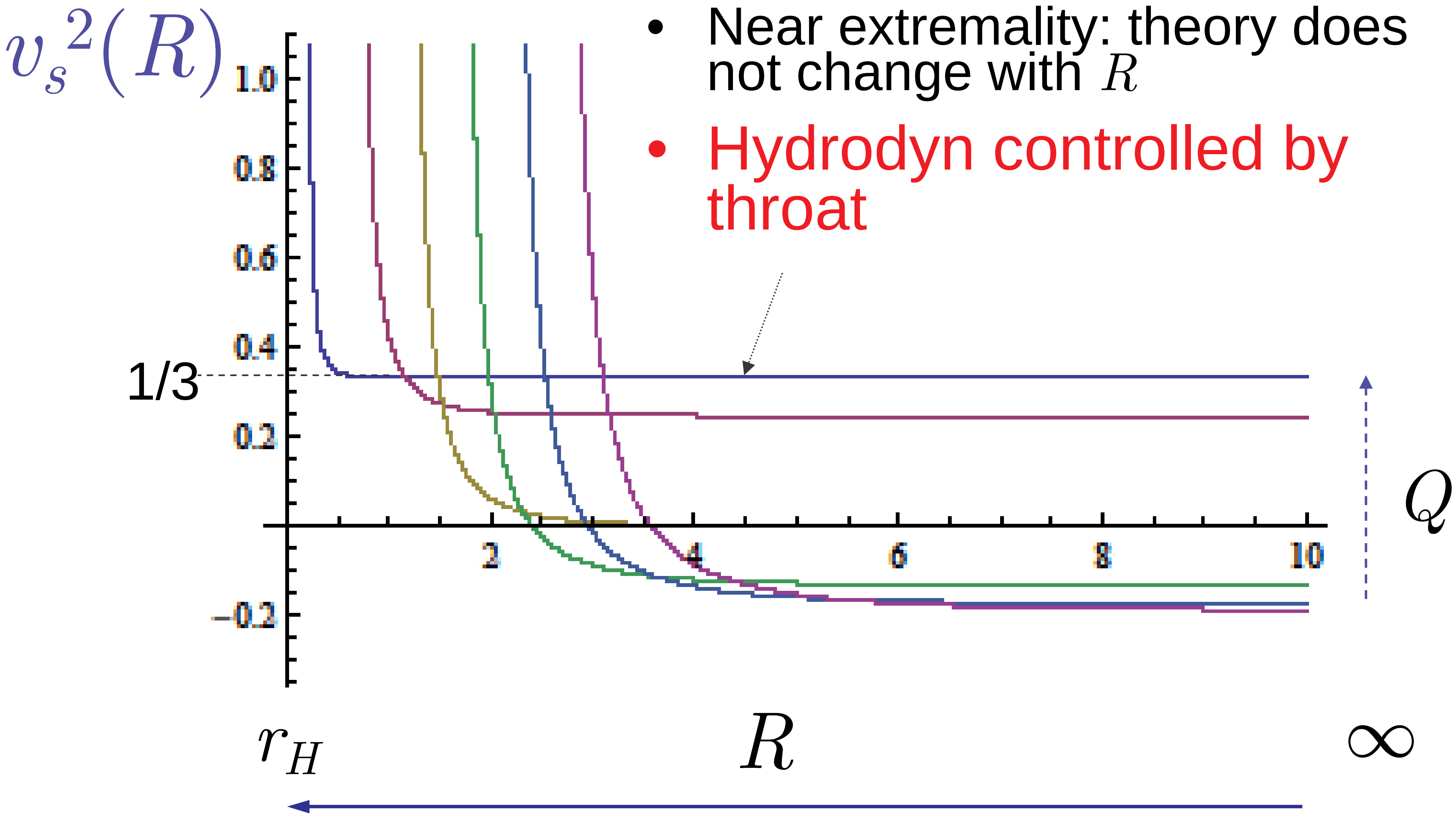

- Near extremality: theory does not change with R
- Hydrodyn controlled by throat

To the horizon

- Near-extremality approx breaks down: hydro effective theory *does run*
- Go over to Rindler fluid descriptions:
  - sound speed diverges
  - effective fluid is incompressible
- Throat to horizon: done by *Brattan+Camps+Loganayagam+Rangamani*
- Blackfold > Fluid/gravity > Rindler fluid