Black Branes as Piezoelectrics

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The Holographic Way:
String Theory, Gauge Theory and Black Holes

Finding ways to describe and understand gravity

- Fluid/Gravity (inspired by AdS/CFT)
- Solid/Gravity (novel)

SETTING

This can be done by studying the dynamics and properties of

higher dimensional black branes

TOOL

Blackfold Approach

- Long wavelength effective theory.

  [Emparan, Harmark, Niarchos, Obers, 0910.1601]

- Directly derivable from the Einstein equations.

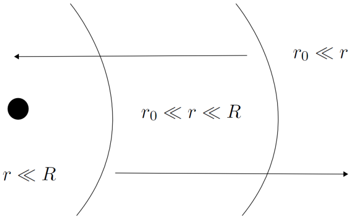
  [Camps, Emparan, 1201.3506]

Applicability

- Applicable when one has seperation of scales.
- Applicable when the fine-structure corrections dominates over backreaction corrections.
- Formalism can work for various backgrounds.

The effective dynamics of a black hole

Seperation of scales: $r_0 \ll R$.



'Near zone' $r \ll R$

$$ds^2 = ds^2_{\mathsf{sch.}} + \mathcal{O}\left(\frac{r}{R}\right)$$

'Far zone' $r \gg r_0$

$$ds^2 = ds^2_{\mathsf{background}} + \mathcal{O}\left(\frac{r_0}{r}\right)$$

'Matching zone'
$r_0 \ll r \ll R$

First step of matched asymptotic expansion.

THE BLACKFOLD APPROACH

With black p-branes the near zone is described by

$$ds^2 = \left(\eta_{ab} + \frac{r_0^n}{r^n} u_a u_b\right) d\sigma^a d\sigma^b + \frac{dr^2}{1 - \frac{r_0^n}{r^n}} + r^2 d\Omega_{(n+1)}^2 \,.$$

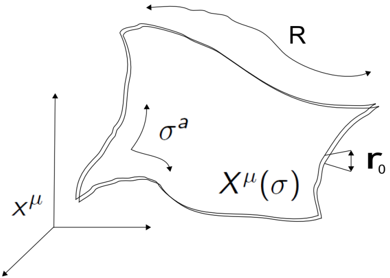
In $D = p + n + 3$ spacetime dimensions.

The thickness, velocity parameters, and embedding functions are promoted to collective fields over the worldvolume,

$$r_0 \to r_0(\sigma), \quad u_a \to u_a(\sigma), \quad X^\mu \to X^\mu(\sigma)\,,$$

with worldvolume coordinates σ^a.

THE BLACKFOLD APPROACH



A blackfold is a *black* p-brane with thickness r_0 wrapped over a submani*fold* $\mathcal{W}_{p+1}$ with characteristic length scale $R \gg r_0$ in the ambient space-time.

$$ds^2 = \left(\gamma_{ab}(X^\mu(\sigma)) + \frac{r_0^n(\sigma)}{r^n} u_a(\sigma) u_b(\sigma)\right) d\sigma^a d\sigma^b + \frac{dr^2}{1 - \frac{r_0^n(\sigma)}{r^n}} + r^2 d\Omega^2_{(n+1)} + \dots$$

THE BLACKFOLD APPROACH

The black brane is in the far region ascribed an effective stress-energy tensor which can be computed using the quasilocal Brown-York formalism on a surface at large r.

For the black p-brane the effective stress tensor is,

$$T^{ab} = \frac{\Omega_{(n+1)}}{16\pi G} r_0^n(\sigma) \left(n u^a(\sigma) u^b(\sigma) - \gamma^{ab}(\sigma) \right) + \dots \, ,$$

which at leading order is of the perfect fluid form,

$$T^{ab} = (\rho + P) u^a u^b + P \eta^{ab} \, .$$

THE BLACKFOLD APPROACH

The equation of motion of the blackfold

$$\nabla_\mu T^{\mu\nu} = 0\,,$$

Projecting along parrallel and orthogonal directions this leads to

$$\mathcal{D}_a T^{ab} = 0, \quad \textit{intrinsic equations } (\mathcal{D}_\tau m = 0)$$
$$T^{ab} {K_{ab}}^{\rho} = 0, \quad \textit{extrinsic equations } (m a^\mu = 0)$$

When the brane is carrying charge

$$\mathcal{D}_a J^a = 0, \quad \textit{intrinsic}$$

Extrinsic Curvature: ${K_{ab}}^{\rho} = {\perp^\rho}_\lambda \left(\partial_a \partial_b X^\lambda + \Gamma^\lambda_{\mu\nu} \partial_a X^\mu \partial_b X^\nu\right)$

Past successes of the blackfold approach:

- Uncovering new horizon topologies for black holes.

  [Emparan, Harmark, Niarchos, Obers, 0912.2352]

- Predicting the onset of GL-instabilities of black branes.

  [Camps, Emparan, Haddad, 1003.3636]

- New method for thermal probe branes.

  [Grignani, Harmark, Marini, Obers, Orselli, 1012.1494], [Niarchos, Siampos, 1205.1535 & 1206.2935]

BLACK BRANES AS FLUIDS AND SOLIDS

Goal: Show that black branes have properties of fluids and solids one has to do something with them

Two ways of perturbing the brane with derivative corrections

- Intrinsic perturbations along the worldvolume
- Extrinsic perturbations transverse to the worldvolume

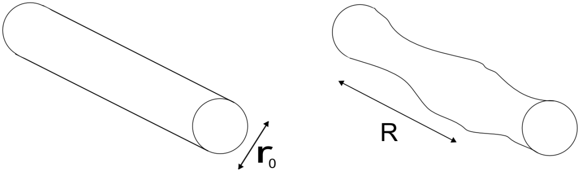
VISCOUS FLUIDS

First order derivative expansion

$$T_{ab} = \rho u_a u_b + P P_{ab} - \zeta \theta P_{ab} - 2\eta \sigma_{ab} + \mathcal{O}(\partial^2)$$

Viscosities

$$\text{shear:} \quad \eta = \frac{s}{4\pi}, \quad \text{bulk:} \quad \zeta = 2\eta \left(\frac{1}{p} - c_s^2 \right)$$



$r_0 \ll R$

[Camps, Emparan, Haddad, 1003.3636]

Various connections between fluid and gravity have been found

- Fluid/AdS-gravity correspondence
- Membrane paradigm

The blackfold approach naturally introduces perturbations of extrinsic nature

First order Dirac-delta function series

$$T^{\mu\nu} \sim T^{\mu\nu}_{(0)}\delta(r) + T^{\mu\nu}_{(1)\rho}\partial_\rho\delta(r) + \dots$$

and similar for the electric current,

$$J^{\mu} \sim J^{\mu}_{(0)}\delta(r) + J^{\mu\rho}_{(1)}\partial_\rho\delta(r) + \dots$$

SOLID/GRAVITY

Working out the physical intepretation of the components within $T_{(1)}^{\mu\nu\rho}$ and $J_{(1)}^{\mu\rho}$ takes some work, one finds two interesting contributions,

- Intrinsic transverse angular momenta.
- Dipole moment of worldvolume stress-energy.

When a current is present

- Worldvolume electic dipole moment.

SOLID/GRAVITY

The dipole moments are developed as the response of bending which are probed by considering finite thickness corrections.

Stress-energy dipole contribution

$$T^{\mu\nu\rho}_{(1)} \sim u^{\mu}_{a} u^{\nu}_{b} d^{ab\rho}$$

with

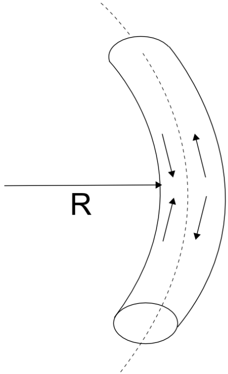
$$d^{ab\rho} = u^{a}_{\mu} u^{b}_{\nu} \perp^{\rho}{}_{\lambda} T^{\mu\nu\lambda}_{(1)}$$

and current dipole contribution

$$J^{\mu\rho}_{(1)} \sim u^{\mu}_{a} p^{a\rho}$$

with

$$p^{a\rho} = u^{a}_{\mu} \perp^{\rho}{}_{\lambda} J^{\mu\nu}_{(1)}$$



BLACK BRANES AS PIEZOELECTRICS

The dipole moment of worldvolume stress-energy $d^{ab\rho}$ and current $p^{a\rho}$ are not a priori constrained.

Under the expectation that bent black branes will behave like elastic solids and like piezoelectric material, one assume,

$$
\begin{aligned}
d^{ab\rho} &= \tilde{Y}^{abcd} K_{cd}{}^{\rho}\,, \\
p^{a\rho} &= \tilde{\kappa}^{abc} K_{bc}{}^{\rho}
\end{aligned}
$$

with elastic moduli $\tilde{Y}^{abcd}$ and piezoelectric moduli $\tilde{\kappa}^{abc}$.

MATCHED ASYMPTOTIC EXPANSION

Take the example of a black string
In order to construct *bent* string solutions one has to push the matched asymptotic expansion to next order in $1/R$.

- 0th (near/far); Boosted schwarzchild black string.
- 1st (far); Far field sourced by the effective stress tensor.
- 1st (near); Schwarzschild black string with $1/R$ corrections.

Boundary conditions are provided by matching with the far field and requiring regularity of the horizon.

Next: Read off the $1/R$ corrected stress tensor.

EXAMPLE

Black string carrying 0-brane charge in Einstein-Maxwell-Dilaton theory gives a realization of linear electroelasticity theory in gravitational physics.

$$\begin{aligned}
\tilde{Y}^{ttzz} &= -\frac{\Omega_{(n+1)} r_0^{n+2}}{16\pi G}\left(\frac{n^2(n+2)}{n+1}\sinh^2\alpha + n^2 + 3n + 4\right)\xi(n)\,,\\
\tilde{Y}^{tzzz} &= 0\,,\\
\tilde{Y}^{zzzz} &= \frac{\Omega_{(n+1)} r_0^{n+2}}{16\pi G}\,(3n+4)\,\xi(n)\,,
\end{aligned}$$

$$\begin{aligned}
\tilde{\kappa}^{tzz} &= -\frac{\Omega_{(n+1)} r_0^{n+2}}{8\pi G}(n+2)\xi(n)\,,\\
\tilde{\kappa}^{zzz} &= 0\,,
\end{aligned}$$

RESULT

The new response coefficients can be interpreted as black branes having piezoelectric behaviour.

This establish a realization between gravity and solids.

THANK YOU