Thermal DBI at weak and strong coupling

The Holographic Way:
String Theory, Gauge Theory and Black Holes
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Outline

Overview

Subject of the talk

Study of the effective action for D-branes in thermal backgrounds
$\longrightarrow$ "Thermal Dirac-Born-Infeld (DBI) action"

[G. Grignani, T. Harmark, A.M., M. Orselli: arXiv:1210.xxxx]

We consider D-branes with constant electric and magnetic fields

- First correction to the DBI action for small temperatures at weak coupling ($g_s N \ll 1$)
  - one-loop quantum correction
- For D3-branes: Compare this correction with that at strong coupling ($g_s N \gg 1$)
  - SUGRA description (blackfold approach)

Outline

Brane probes in thermal backgrounds

- Standard method to describe brane probes in thermal backgrounds uses the DBI action
- Equations of motion for any probe brane $\longrightarrow$ Carter equation

extrinsic curvature

worldsheet EM tensor

$$K_{ab}{}^{\rho}\, T^{ab} = J \cdot F^{\rho}$$

possible external forces

- For a D-brane probe $\longrightarrow$ DBI EOM $\equiv$ Carter eq with T^{ab} the same EM tensor as in the zero-temperature (extremal) case
- Brane DOF's are "heated up" by the temperature of the background $\longrightarrow$ this changes the EM tensor
- DBI is not accurate for the description of D-brane probes in thermal backgrounds

Thermal D-brane probe

1. Strong coupling description
   - method proposed in [G. Grignani, T. Harmark, A.M., N. Obers, M. Orselli: arXiv:1012.1494, arXiv:1101.1297 and arXiv:1201.4862]
   - based on the blackfold approach [R. Emparan, T. Harmark, V. Niarchos, N. Obers: arXiv:0910.1601, arXiv:0902.0427, ...]
     - SUGRA black brane to describe the thermal brane probe
   - non-perturbative in the temperature
   - valid for $g_s N \gg 1$ $\longleftrightarrow$ different regime w.r.t. that usually considered for probes
   - cannot be used to describe thermal Dp-brane with $p \geq 5$
     - thermodynamic instability [T. Harmark, V. Niarchos, N. Obers: arXiv:hep-th/0701022; ...]
     - no consistent extremal limit
2. Weak coupling description
   - compute the one-loop correction to the DBI action
   - valid for $g_s N \ll 1$ $\longleftrightarrow$ regime usually considered for probes
   - valid only for small temperature

Outline

D3 with a uniform electric field

Super-DBI action for D3-branes in flat background [M. Aganagic, C. Popescu, J. H. Schwarz: arXiv:hep-th/9612080]

$$I_{\mathrm{SD3}} = -T_{\mathrm{D3}} \int d^4\sigma \sqrt{-\det\left(\eta_{ab} + \partial_a X^i \partial_b X^i - 2\bar{\psi}\Gamma_a \partial_b \psi + F_{ab}\right)}$$

Expand the fields around a classical configuration

$$A_a = A_a^{(\mathrm{cl})} + \tilde{A}_a\,, \qquad X^i = X^i_{(\mathrm{cl})} + \tilde{X}^i \qquad \psi = \psi_{(\mathrm{cl})} + \tilde{\psi}$$

Classical fields $\longrightarrow$ $X^i_{(\mathrm{cl})} = $ const, $\psi_{(\mathrm{cl})} =$ const and $A_a^{(\mathrm{cl})}$ such that

$$F_{ab}^{(\mathrm{cl})} = \partial_a A_b^{(\mathrm{cl})} - \partial_b A_a^{(\mathrm{cl})} = \begin{pmatrix} 0 & E_1 & 0 & 0 \\ -E_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{with } E_1 = \text{const}$$

Super-DBI expansion

Expand the SDBI action up to the quadratic order in the fields

$$I_{\mathrm{SD3}} \simeq I_0 + I_{2,A} + I_{2,X} + I_{2,\psi}$$

where

$$I_0 = I_{\mathrm{SD3}}\left[A^{(\mathrm{cl})}, X^i_{(\mathrm{cl})}, \psi_{(\mathrm{cl})}\right]$$

$$I_{2,A} \sim \int dt\, d^3r \left[-E_x^2 + B_x^2\left(1-E_1^2\right)^2 + \left(B_y^2 + B_z^2 - E_y^2 - E_z^2\right)\left(1-E_1^2\right)\right]$$

$$I_{2,X} \sim \int dt\, d^3r\, X^i\left[-\partial_0^2 + \partial_1^2 + \left(1-E_1^2\right)\left(\partial_2^2+\partial_3^2\right)\right]X^i$$

$$I_{2,\psi} \sim \int dt\, d^3r\, \bar{\psi}\left[-\left(\Gamma_0 + \Gamma_1 E_1\right)\partial_0 + \left(\Gamma_1 + \Gamma_0 E_1\right)\partial_1 + \left(1-E_1^2\right)\left(\Gamma_2\partial_2 + \Gamma_3\partial_3\right)\right]\psi$$

One-loop partition function $\mathcal{Z}$

Perform a Wick rotation $t \to i\tau$, $I_2 \to I_2^E$

Partition function factorizes $\longrightarrow$ $\mathcal{Z} = \mathcal{Z}_A \mathcal{Z}_X \mathcal{Z}_\psi$

$$\mathcal{Z}_A = \int \mathcal{D}A_a \, \delta(h) \det\left(\frac{\delta h}{\delta \chi}\right) e^{-I^E_{2,A}}$$

Gauge fixing $\longrightarrow$ $h\left[A, \partial A\right] = 0$
χ is the gauge transformation parameter $A_a \to A'_a = A_a + \partial_a \chi$

$$\mathcal{Z}_X = \int \mathcal{D}X^i \, e^{-I^E_{2,X}} \qquad \mathcal{Z}_\psi = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \, e^{-I^E_{2,\psi}}$$

periodic boundary conditions for A_a and X^i and anti-periodic for ψ in the τ direction with period $\beta = 1/T$

One-loop free energy

Gauge field contribution: Choosing $h = A_3 = 0$

$$I_{2,A}^E \sim \int_0^\beta d\tau \int d^3r \mathbf{A}^T \mathcal{M} \mathbf{A}$$

where $\mathbf{A} = (A_0, A_1, A_2)^T$ and $\mathcal{M}$ is the matrix

$$\begin{pmatrix} -\partial_1^2 - \left(\partial_2^2 + \partial_3^2\right)\left(1 - E_1^2\right) & \partial_0\partial_1 & \partial_0\partial_2\left(1 - E_1^2\right) \\ \partial_0\partial_1 & -\partial_0^2 - \left(\partial_2^2 + \partial_3^2\right)\left(1 - E_1^2\right) & \partial_1\partial_2\left(1 - E_1^2\right) \\ \partial_0\partial_2\left(1 - E_1^2\right) & \partial_1\partial_2\left(1 - E_1^2\right) & -\left(1 - E_1^2\right)\left(\partial_0^2 + \partial_1^2 + \partial_3^2\left(1 - E_1^2\right)\right) \end{pmatrix}$$

$$\mathcal{Z}_A = \frac{\det\,\partial_3}{\sqrt{\det \mathcal{M}}} = \prod_{\boldsymbol{k}} \prod_{n=-\infty}^{\infty} \left[\left(\frac{2\pi n}{\beta}\right)^2 + f(\boldsymbol{k})\right]^{-1}$$

where $f(\boldsymbol{k}) = k_1^2 + \left(1 - E_1^2\right)\left(k_2^2 + k_3^2\right)$

$$\mathcal{G}_A = -\frac{1}{\beta}\log \mathcal{Z}_A = \frac{2V_3}{\beta}\int \frac{d^3k}{(2\pi)^3}\left[\frac{1}{2}\beta\sqrt{f(\boldsymbol{k})} + \log\left(1 - e^{-\beta\sqrt{f(\boldsymbol{k})}}\right)\right]$$

One-loop free energy

Scalar fields contribution: $\mathcal{Z}_X = \prod_{\boldsymbol{k}} \prod_{n=-\infty}^{\infty} \left[\left(\frac{2\pi n}{\beta} \right)^2 + f(\boldsymbol{k}) \right]^{-3} = \mathcal{Z}_A^3$

$$\mathcal{G}_X = 3\mathcal{G}_A$$

Fermionic contribution: $\mathcal{Z}_\psi = \prod_{\boldsymbol{k}} \prod_{n=-\infty}^{\infty} \left[\left(\frac{\pi(2n+1)}{\beta} \right)^2 + f(\boldsymbol{k}) \right]^{4}$

$$\mathcal{G}_\psi = -\frac{8V_3}{\beta} \int \frac{d^3k}{(2\pi)^3} \left[\frac{1}{2}\beta\sqrt{f(\boldsymbol{k})} + \log\left(1 + e^{-\beta\sqrt{f(\boldsymbol{k})}}\right) \right]$$

One-loop free energy: $\mathcal{G}_1 = \mathcal{G}_A + \mathcal{G}_X + \mathcal{G}_\psi$

$$\mathcal{G}_1 = \frac{8V_3}{\beta} \int \frac{d^3k}{(2\pi)^3} \left[\log\left(1 - e^{-\beta\sqrt{f(\boldsymbol{k})}}\right) - \log\left(1 + e^{-\beta\sqrt{f(\boldsymbol{k})}}\right) \right]$$

$$= -\frac{T^4\pi^2V_3}{6\left(1 - E_1^2\right)} \longleftrightarrow \left\{ \begin{array}{l} \text{one-loop effective action:} \\ \text{first temperature correction of the DBI} \end{array} \right.$$

D3 with electric and magnetic fields

Electric and magnetic field on the D3: $\boldsymbol{E} = (E_1, 0, 0)$, $\boldsymbol{B} = (B_1, 0, B_3)$

$$F_{ab}^{(\mathrm{cl})} = \begin{pmatrix} 0 & E_1 & 0 & 0 \\ -E_1 & 0 & B_3 & 0 \\ 0 & -B_3 & 0 & B_1 \\ 0 & 0 & -B_1 & 0 \end{pmatrix}$$

One-loop free energy:

$$\begin{aligned} \mathcal{G}_1 &= -\frac{T^4 \pi^2 V_3}{6} \frac{1 - \boldsymbol{E}^2 + \boldsymbol{B}^2 - (\boldsymbol{E} \cdot \boldsymbol{B})^2}{\left(1 - \boldsymbol{E}^2\right)^2} \\ &= -\frac{T^4 \pi^2 V_3}{6} \left[\frac{1 + \boldsymbol{B}^2}{1 - \boldsymbol{E}^2} + \frac{\boldsymbol{E} \times \boldsymbol{B}}{\left(1 - \boldsymbol{E}^2\right)^2} \right] \end{aligned}$$

Generalization to Dp-branes

Super-DBI action for a Dp-brane

$$I_{\mathrm{SD}p} = -T_{\mathrm{D}p} \int d^{p+1}\sigma \sqrt{-\det\left(\eta_{ab} + \partial_a X^i \partial_b X^i - 2\bar{\psi}\Gamma_a \partial_b \psi + F_{ab}\right)}$$

Electric and magnetic field on the Dp-brane:

$$F^{(\mathrm{cl})}_{ab} = \begin{pmatrix} 0 & E & & & & \\ -E & 0 & B_1 & & & \\ & -B_1 & 0 & & & \\ & & & \ddots & & \\ & & & & 0 & B_{p-1} \\ & & & & -B_{p-1} & 0 \end{pmatrix}$$

One-loop free energy:

$$\mathcal{G}_1 = -\frac{16 V_p T^{p+1} \Gamma(p)\, \zeta\left(p+1, \frac{1}{2}\right)}{4^p \pi^{\frac{p}{2}} \Gamma\left(\frac{p}{2}\right)} \frac{\left(-\det(\eta + F)^{(\mathrm{cl})}\right)}{\left(1 - E^2\right)^{\frac{p+1}{2}}}$$

Outline

D3 with a uniform electric field

F1-D3 brane bound state in string frame metric

$$ds^2 = \frac{1}{\sqrt{DH}}\left[-f dt^2 + dx_1^2 + D(dx_2^2 + dx_3^2)\right] + \frac{\sqrt{H}}{\sqrt{D}}\left[f^{-1}dr^2 + r^2 d\Omega_5^2\right]$$

with

$$H = 1 + \frac{r_0^4 \sinh^2\alpha}{r^4} \qquad f = 1 - \frac{r_0^4}{r^4} \qquad D^{-1} = \cos^2\theta + \sin^2\theta H^{-1}$$

- Dilaton: $e^{2\phi} = D^{-1}$
- Kalb-Ramond 2-form: $B_{(2)} = \sin\theta(H^{-1} - 1)\coth\alpha\, dt \wedge dx^1$
- R-R 2-form: $A_{(2)} = \tan\theta(H^{-1}D - 1)dx_2 \wedge dx_3$
- R-R 4-form: $A_{(4)} = \cos\theta D(H^{-1} - 1)\coth\alpha\, dt \wedge dx^1 \wedge dx^2 \wedge dx^3$

F1-D3 thermodynamics

- D3 and F1 charges:

$$(\mathrm{D3}) \qquad N = 2\pi^2 T_{\mathrm{D3}} \cos\theta r_0^4 \cosh\alpha \sinh\alpha$$
$$(\mathrm{F1}) \qquad kT_{\mathrm{F1}} = 2\pi^2 T_{\mathrm{D3}}^2 V_2 \sin\theta r_0^4 \cosh\alpha \sinh\alpha$$

- From the Kalb-Ramond field $B_{\mu\nu}$ we read off the corresponding world-volume field strength

$$F_{01} = T_{\mathrm{F1}} \sin\theta \tanh\alpha \equiv T_{\mathrm{F1}} E$$

- Temperature: $T = \dfrac{1}{\pi r_0 \cosh\alpha}$
- Mass and entropy:

$$M = \frac{\pi^2}{2} T_{\mathrm{D3}}^2 V_3 r_0^4 (5 + 4\sinh^2\alpha), \quad S = 2\pi^3 T_{\mathrm{D3}}^2 V_3 r_0^5 \cosh\alpha$$

Helmholtz free energy

- Helmholtz free energy $\mathcal{F} = M - TS$

$$\mathcal{F} = \frac{\pi^2}{2} T_{\mathrm{D3}}^2 V_3 r_0^4 (1 + 4\sinh^2\alpha)$$

$$d\mathcal{F} = -SdT + \omega dN + \mu dk$$

- We need a quantity that depends on $E \sim \mu$

$$\mu = V_1 F_{01}$$

- Legendre transform of $\mathcal{F}$

$$\mu k = 2\pi^2 T_{\mathrm{D3}}^2 V_3 \sin^2\theta r_0^4 \sinh^2\alpha$$

Gibbs free energy

- Gibbs Free energy $\mathcal{G} = M - TS - \mu k$

$$d\mathcal{G} = -SdT + \omega dN - kd\mu$$

$$\mathcal{G} = \frac{\pi^2}{2} T_{\mathrm{D3}}^2 V_3 r_0^4 (1 + 4\cos^2\theta \sinh^2\alpha)$$

- Expanding $\mathcal{G}$ for small temperature

$$\mathcal{G} = N T_{\mathrm{D3}} V_3 \sqrt{1 - E^2} \left[1 - \frac{\pi^2 N}{8 T_{\mathrm{D3}}} \frac{T^4}{(1 - E^2)^{3/2}} - \frac{\pi^4 N^2}{32 T_{\mathrm{D3}}^2} \frac{T^8}{(1 - E^2)^3} + \cdots \right]$$

Comparison between weak and the strong coupling

$$\frac{\mathcal{G}_{1,\text{strong}}}{\mathcal{G}_{1,\text{weak}}} = \frac{3}{4}$$

Comparison of the entropies

- Entropy is independent of the ensembles unlike the free energy
- One needs only the leading order behavior of $\cosh\alpha$ and $\sin\theta$ for small T

Entropy of the SUGRA solution $S = 2\pi^3 V_3 T_{\rm D3}^2 r_0^5 \cosh\alpha = \dfrac{2T_{\rm D3}^2 V_3}{\pi^2 T^5 \cosh^4\alpha}$

Charge quantization: $\dfrac{\pi^4 N^2 T^8}{4T_{\rm D3}^2 \cos^2\theta}\cosh^6\alpha - \cosh^2\alpha + 1 = 0$

At the leading order for $T \to 0$ $\longrightarrow$ $\cosh^2\alpha = \dfrac{2T_{\rm D3}\cos\theta}{\pi^2 N T^4}, \quad E = \sin\theta$

$$\longrightarrow \qquad S_{\textsf{strong}} = \frac{\pi^2}{2} V_3 N^2 \frac{T^3}{1-E^2}$$

Compare with the entropy at weak coupling (with $N=1$)

$$S_{\textsf{weak}} = -\left(\frac{\partial \mathcal{G}_{1,\textsf{weak}}}{\partial T}\right)_E = \frac{2\pi^2}{3} V_3 \frac{T^3}{1-E^2}$$

D3 with parallel electric and magnetic fields

- SUGRA solution for the (F1,D1)-D3 brane bound state

Entropy for small $T \longrightarrow S_{\text{strong}} = \frac{\pi^2}{2} V_3 N^2 T^3 \frac{1+B^2}{1-E^2}$

- At weak coupling (with $N = 1$) when $\boldsymbol{E} \times \boldsymbol{B} = 0$

$$S_{\text{weak}} = -\left(\frac{\partial \mathcal{G}_{1,\text{weak}}}{\partial T}\right)_{E,B} = \frac{2\pi^2}{3} V_3 T^3 \frac{1+B^2}{1-E^2}$$

- For D3 with $\boldsymbol{E} \times \boldsymbol{B} = 0$

$$\frac{S_{\text{strong}}}{S_{\text{weak}}} = \frac{3}{4}$$

Outline

Conclusions

- Weak coupling: First finite temperature correction to the DBI action for Dp-branes with electric and magnetic fields
- Strong coupling: Leading order correction in temperature for the effective action for D3-branes
- The famous factor of $3/4$ between the strongly and weakly coupled regimes for D3-branes with $\boldsymbol{E} = \boldsymbol{B} = 0$ extends to the full non-linear DBI regime, at least for parallel $\boldsymbol{E}$ and $\boldsymbol{B}$
- Quite possibly this could also extend more generally to when $\boldsymbol{E} \times \boldsymbol{B} \neq 0$