Quantum phase transitions in charged holographic matter

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Preliminaries

- Holography is a powerful tool for examining strongly coupled quantum field theories. Can we apply it to learn about strongly coupled systems in condensed matter?
- A focus: place a field theory at some temperature T and chemical potential μ for global $U(1)$
- What kinds of phases (& transitions) are possible?
- What is the phase diagram (T, μ, B, etc.)?
- Construct *all* AAdS black brane solutions at $T, \mu, \ldots$, and find lowest free energy

'Universal' normal phase

- Minimal bulk ingredients for finite T and μ for global $U(1)$:

$$S_{\text{bulk}} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left(R + 24 - \frac{1}{4}F^2\right)$$

- Admits RN black brane solutions

$$ds^2 = -g(r)dt^2 + \frac{dr^2}{g(r)} + r^2(dx^2+dy^2), \qquad F = -\phi'(r)dt\wedge dr$$
$$g(r) = 4r^2 - \left(4r_+^2 + \frac{\mu^2}{4}\right)\frac{r_+}{r} + \frac{\mu^2 r_+^2}{4r^2}, \qquad \phi = \mu\left(1 - \frac{r_+}{r}\right)$$

- Field theory charge given by boundary electric flux $\langle Q\rangle = \int_{\mathbb{R}^2} *F$
- Here all flux comes from horizon. Gauss' law $d*F = 0$ $\implies \langle Q\rangle = \int_{\mathcal{H}} *F = \mu r_+ vol_2$.

'Universal' normal phase: $T = 0$

- At $T = 0$ RN-AdS$_4$ becomes AdS$_2 \times \mathbb{R}^2 \to$AdS$_4$ domain wall
- Residual entropy density, $s \neq 0$
- Provides a diagnostic for instabilities, through the violation of the AdS$_2$ BF-bound, e.g.
  - Superconductivity / superfluidity m^2_{eff} for linearised charged scalar in AdS$_2$ [Gubser][Hartnoll, Herzog, Horowitz]
  - Spatial modulation where lightest m^2_{eff} occurs at wavenumber $k \neq 0$ for Fourier decomp. in $\mathbb{R}^2$. Spatially modulated UV CFT [Nakamura, Ooguri, Park] [Donos, Gauntlett] [Bergman, Jokela, Lifschytz, Lippert]
- Heated up: new branch of black brane solutions emerging at some T_c (here with reduced symmetry)
- Nonlinearly construct the branch to check the free energy

Comments on theories

- But instabilities can depend on the ingredients
- **'Top-down'** – supergravity approach
  - know the field theory
  - lots of fields $\leftrightarrow$ lots of new phases (problem?)
  - truncations can miss things
- **'Bottom-up'** – phenomenological approach
  - tailor a theory to examine a particular phenomenon
  - some phases still hard to access (e.g. spatial modulation)
  - may not have a field theory dual
  - lots of freedom (e.g. superfluid at low temperatures)

Back to RN:

- RN-AdS is not robust at low T. In many holographic settings the $T = 0$ limit will be cloaked by e.g. superconducting phase.
- But, geometries with $\int_{\mathcal{H}} *F$ represent physically distinct bulk source of flux... physically distinct field theory features?
- Natural association of geometries with $\int_{\mathcal{H}} *F$ with to 'fractionalised' phases of matter [Sachdev]
- What physically distinct ways in the bulk of producing finite charge density? Do they give distinct field theory features?

A taxonomy of charge

- Model with pseudo-scalar σ, charged scalar χ, single $U(1)$.

$$S = \int d^4x\sqrt{-g}\left(R - \frac{1}{2}(\partial\sigma)^2 - \frac{\tau(\sigma)}{4}F^2 - h(\sigma,|\chi|)\,|D\chi|^2 - V(\sigma,|\chi|)\right) + \frac{1}{2}\int \vartheta(\sigma)F\wedge F\,,$$

- Distinct contributions to charge density.
  $d(\tau * F) = *j_\chi + d(\vartheta F)$
  1. IR electric flux (fractionalised, as in RN AdS example)
  2. Bulk charged matter. e.g.
     - bosons $j_\chi \neq 0$ (U(1)-breaking) [Hartnoll, Herzog, Horowitz],
     - fermions e.g. in fluid approximation [Hartnoll, Tavanfar]
  3. IR magnetic $\vartheta F \wedge F$-contributions
     - Applied magnetic field e.g. [D'Hoker, Kraus]
     - Spontaneous IR contributions, e.g. spatially modulated phases e.g. [Donos, Gauntlett]

M-theory

- Example of a model which has all these types of phases and competition between them. IR flux will not contribute to the physics.
- Model of this form – consistent truncation of M-theory on an arbitrary SE_7. Every solution in $D = 4$ can be lifted to infinite class of solutions in $D = 11$. [Gauntlett, Kim, Varela, Waldram][Gauntlett, Sonner, Wiseman]
- Apply magnetic field B as an external dial, construct grand-canonical phase diagram
- Start at $T = 0$ ...

Dyonic pseudo-scalar $\mathrm{AdS}_2 \times \mathbb{R}^2$ families

- $\mathrm{AdS}_2 \times \mathbb{R}^2$ families, with $\sigma = \sigma_0$

$$\begin{aligned} ds^2 &= L^2 ds^2\,(AdS_2) + ds^2(\mathbb{R}^2)\,, \\ F &= -EL^2 \mathrm{Vol}(AdS_2) + B\,\mathrm{Vol}(\mathbb{R}^2)\,, \end{aligned}$$

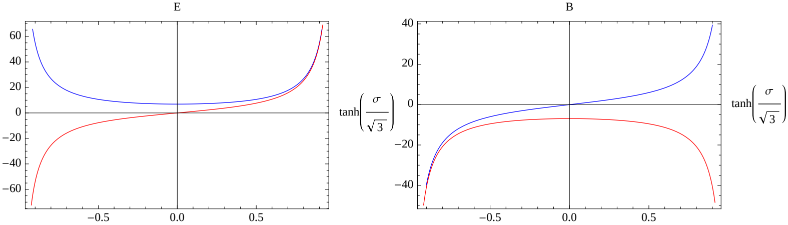
- Solutions defined by algebraic conditions

$$E^2 + B^2 = -\frac{2V}{\tau}, \qquad \frac{\tau'}{2}(E^2 - B^2) - \vartheta' EB - V' = 0\,,$$

with the AdS_2 radius given by $L^{-2} = -V$.

Dyonic pseudo-scalar $AdS_2 \times \mathbb{R}^2$ families

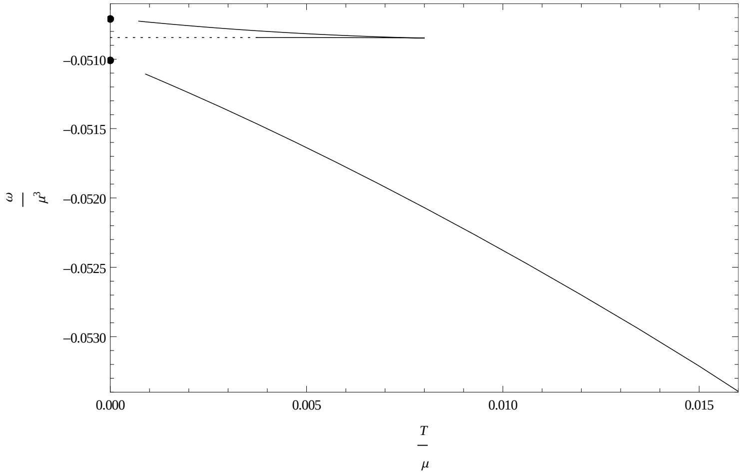
- Electric-family and Magnetic-family



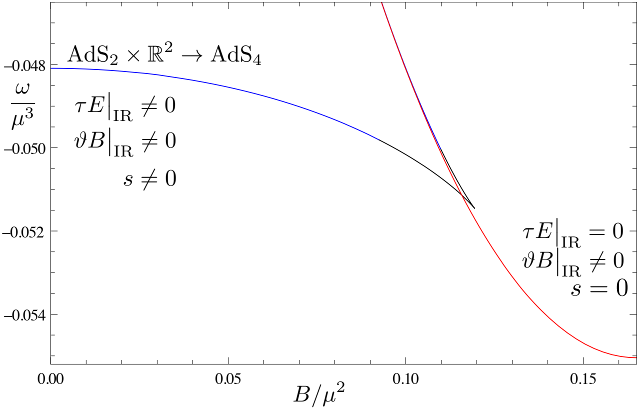
- These will form the IR end of $AdS_2 \times \mathbb{R}^2 \rightarrow AdS_4$ domain walls
- Correspond to the IR of $T = 0$ limit of dyonic black holes with σ-hair

Dyonic black holes at fixed $\frac{B}{\mu^2}$

- Construct branches of black hole solutions at some fixed $\frac{B}{\mu^2}$ which connect with the dyonic $AdS_2 \times \mathbb{R}^2 \to AdS_4$'s at $T = 0$.
- Can be more than one...



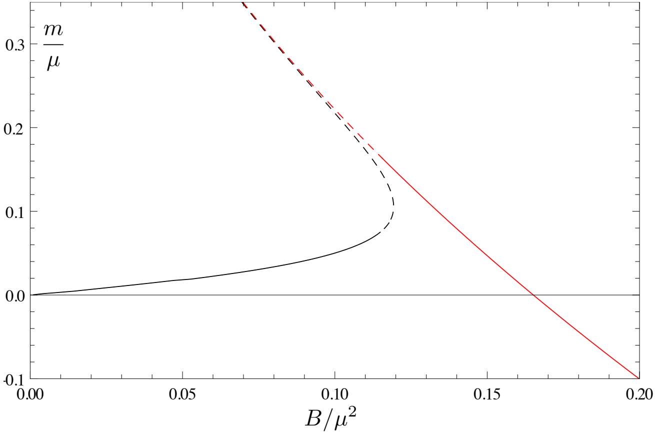
General $\frac{B}{\mu^2}$ at $T = 0$



- ——— UV flux sourced only through $\vartheta F \wedge F$ (in fact, $q = B$)

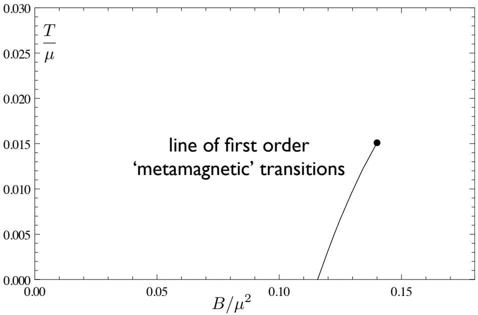
Metamagnetism at $T = 0$

- Magnetisation at $T = 0$, $m \equiv -\frac{\partial w}{\partial B}\Big|_{\mu,T}$



- First order 'metamagnetic' transition
- Other examples [D'Hoker, Kraus], [Lifschytz, Lippert]

Metamagnetism at $T \neq 0$



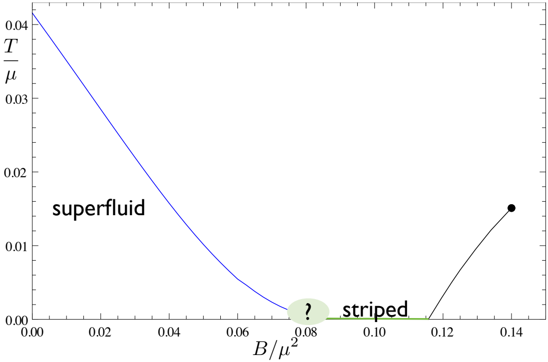
- Phase structure observed in a variety of strongly correlated metals
  - Line of first order transitions terminating at some $T^*(B^*)$
  - Magnetisation jumps
  - No broken symmetries
- Here: consequence of $\vartheta(\sigma) F \wedge F$ term

What about symmetry breaking? – AdS_2 stability

- Look at fluctuation spectrum in each $AdS_2 \times \mathbb{R}^2$ as indicator of instability.
- Spatially modulated phases:
  - Look at finite k plane wave instabilities
  - All fluctuations couple (metric, σ, A) except χ
  - The spectrum of scaling dimensions for CFT dual to AdS_2 is independent of σ_0, up to the rescaling $k \to \sqrt{\cosh(\sigma_0/\sqrt{3})}k$
  - BF bound violated for approx $k \in (5.94, 6.48)$
- Superconductors
  - Perturbative formation of superfluid droplets in $AdS_2 \times \mathbb{R}^2$
  - Only the low B/μ^2 domain walls are unstable
- Expect a competition between these phases – requires solving pde's

Putting it all together : a phase diagram

- We can construct the line of critical temperatures for the superfluid droplets
- Striped instabilities at finite T technically out of reach



- Ground state entropy avoided

Summary

- Considered ‘gravitationally distinct’ ways of sourcing charge density.
- In part motivated by basic idea that distinct sources of flux (bulk) $\rightarrow$ distinct phases of CFT (c.f. IR flux and fractionalised phases)
- Competing phases in M-theory: superfluids, spatial modulation
- Avoidance of ground state entropy
- Metamagnetic transition from parity-violating term (generic?)