Hydrodynamics of charged black branes

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in progress

Connection between black objects and fluid dynamics

map between gravity and fluid dynamics

Long distance limit of
Einstein equation of gravity ⟺ Relativistic Navier-Stokes equation of fluid dynamics

Ex.
Fluid/gravity Correspondence:

equations of asymptotically AdS gravity reduce to relativistic Navier Stokes equations in a long wavelength expansion

Blackfolds:

develop an effective worldvolume theory to study the dynamics of black branes on length scales larger than the thickness of the brane

- new approach to find higher-dimensional black holes and understand their properties
- useful for the analysis of dynamical, non-stationary situations, in particular of the Gregory-Laflamme instability of black branes.

study the dissipative effect of a Dp-brane at finite temperature

The dynamics of the brane is expressed in terms of its effective stress-energy tensor (the Brown-York quasilocal stress-energy tensor)

$$T^{(ql)}_{\mu\nu} = \frac{1}{8\pi G}\left(\Theta_{\mu\nu} - \tilde{h}_{\mu\nu}\Theta\right)$$

- computed in asymptotic flat region.

The blackfold equations:

$$D_\mu T^{\mu\nu} = 0$$

Intrinsic equations: energy-momentum conservation on the worldvolume.

Is it possible to extend this method for charged black object?

We are able to construct an effective worldvolume theory for black D-brane or charged black object.

Which are the differences between neutral and charged case?

Black p-folds with only p-brane charge

p-brane current is conserved ⇒ Charge is constant along the worldvolume

The charge is not a collective variable of the fluid
but is a parameter of the equation of state of the fluid.

The dynamical variables are such as the neutral case.

Effective theory for a charged dilatonic black p-brane

Setup:

$$I = \frac{1}{16\pi G}\int d^D x\sqrt{-g}[R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2(n+1)!}e^{a\phi}F^2_{(n+1)}]$$

$$n = D - p - 3 \qquad a^2 = 4 - \frac{2(p+1)n}{D-2}$$

black p-brane:

$$ds^2 = \triangle_-^{\frac{n}{D-2}}(-\frac{\triangle_+}{\triangle_-}dt^2 + \sum_{i=1}^{p} dx_i^2) + \triangle_-^{-\frac{(p+1)}{D-2}}(dr^2\frac{\triangle_-^{\frac{3n-2}{n}}}{\triangle_+} + r^2\triangle_-^{\frac{2}{n}}d\Omega^2_{n+1})$$

where:

$$\triangle_+ = 1 - \frac{r_0^n}{r^n} \qquad \triangle_- = 1 - \frac{r_-^n}{r^n}$$

It’s useful to change the frame:

$$\bar{g}_{\mu\nu} = e^{\frac{a\phi}{n}} g_{\mu\nu}$$

$$ds^2 = -\triangle_-^{-\frac{2}{n}} \triangle_+ dt^2 + \triangle_-^{-\frac{2}{n}+1} \sum_{i=1}^{p} dx_i^2 + \frac{dr^2}{\triangle_+ \triangle_-} + r^2 d\Omega_{n+1}^2$$

Since we don’t want to deform the shape of the Sn+1, it’s convenient to perform a reduction along Sn+1

The reduction ansatz for the D dimension metric:

$$ds_D^2 = ds_{p+2}^2 + e^{2\varphi(x)} d\Omega_{n+1}^2$$

The metric and the dilaton of a Dp-brane in E.F. ingoing coordinates

$$ds^2_{p+2} = -\triangle_-^{-\frac{2}{n}}\triangle_+ u_a u_b d\sigma^a d\sigma^b + \triangle_-^{-\frac{2}{n}+1} P_{ab} d\sigma^a d\sigma^b - 2\triangle_-^{-\frac{n+2}{2n}} u_a dr d\sigma^a$$

$$e^{\varphi} = r$$

After the reduction on S^{n+1}, the effective action becomes

$$I_{p+2} = \frac{\Omega_{n+1}}{16\pi G}\int d^{p+2}x\sqrt{-g}e^{\gamma\phi+(n+1)\varphi}[R + n(n+1)(e^{-2\varphi} + (\partial\varphi)^2) + \\ + 2\gamma(n+1)\partial_\mu\phi\partial^\mu\varphi - \frac{1}{2}\omega(\partial\phi)^2 + \alpha Q^2 e^{-2(n+1)\varphi}]$$

The metric and the ... coordinates

$$ds^2_{p+2} = -\triangle_-^{-\frac{2}{n}}\triangle_+ \ldots \sigma^b - 2\triangle_-^{-\frac{n+2}{2n}} u_a dr d\sigma^a$$

$$e^{\varphi} = r$$

Φ : dilaton

φ : radius of the sphere

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Fluctuations:

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The solution is characterized by:

- the horizon radius r_o
- worldvolume velocity u^a

} Collective coordinates

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$$\triangle_- = 1 - \frac{r_-^n}{r^n}$$

We can express r_- in terms of r_o using the relation with the charge:

$$Q \propto r_-^n r_0^n$$

dynamics that keeps the charge fixed

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We promote these collective coordinates to functions of the worldvolume coordinates σ^a

$$\begin{aligned} ds^2_{p+2} = & - \triangle_-(\sigma)^{-\frac{2}{n}}\triangle_+(\sigma) u_a u_b d\sigma^a d\sigma^b + \triangle_-(\sigma)^{-\frac{2}{n}+1} P_{ab} d\sigma^a d\sigma^b + \\ & - 2\triangle_-(\sigma)^{-\frac{n+2}{2n}} u_a dr d\sigma^a + \ldots \end{aligned}$$

$$ds^2_{p+2} = -\underline{\triangle_-(\sigma)^{-\frac{2}{n}}\triangle_+(\sigma)}u_a u_b d\sigma^a d\sigma^b + \underline{\triangle_-(\sigma)^{-\frac{2}{n}+1}}P_{ab}d\sigma^a d\sigma^b +$$
$$- 2\underline{\triangle_-(\sigma)^{-\frac{n+2}{2n}}}u_a dr d\sigma^a + \ldots$$

We need to add some terms so that the total metric is still a solution of the fields equations

$$\epsilon f_{\mu\nu} dx^\mu dx^\nu$$

ε is a derivatives-counting parameter

At leading order in derivatives, the fluctuating metric takes the form

$$ds^2_{p+2} \to ds^2_{p+2} + \epsilon f_{ab} d\sigma^a d\sigma^b + 2\epsilon f_{ar} dr d\sigma^a + O(\epsilon^2)$$

the correcting functions can be decomposed into SO(p)-algebraically-irreducible terms

$$f_{ab} = \theta u_a u_b \, \mathsf{s}_1(r) + \frac{1}{p}\theta P_{ab} \, \mathsf{s}_2(r) + a_{(a} u_{b)} \, \mathsf{v}_1(r) + \sigma_{ab} \, \mathsf{t}(r)$$
$$f_{ar} = \theta u_a \, \mathsf{s}_3(r) + a_a \, \mathsf{v}_2(r)$$

Where **s** , **v**, **t** correspond to SO(p) scalars, vectors and tensors

$$\theta = \nabla_a u^a$$

$$a_a = u^b \nabla_b u_a$$

$$\sigma_{ab} = P_a{}^c P_b{}^d \nabla_{(c} u_{d)} - \frac{\theta}{p} P_{ab}$$

SO(p) tensors: shear-channel contributions $\propto \sigma_{ab}$

shear viscosity

SO(p) vectors: acceleration-channel contributions $\propto a_a$

SO(p) scalars: expansion-channel contributions $\propto \theta$

Bulk viscosity

At first order in derivative expansion,
the stress-energy tensor of the effective fluid

$$T_{ab} = \rho u_a u_b + P P_{ab} - \zeta \theta P_{ab} - 2\eta \sigma_{ab} + O(\partial^2)$$

correction to the metric at first order $f_{\mu\nu}$

↓

dissipative effects:

the shear and bulk viscosities.

What do we expect :

- Get the same result obtained using the fluid/gravity correspondence but now with an asymptotic flat brane
- Check that the intrinsic dynamics of charged black brane becomes the same as the fluid dynamics of AdS charged black branes in the decoupling limit
- Recover the neutral case
- Understand GL instabilities in long wavelength regime

Charged dilatonic black p-brane

Fluid perturbations at first order in derivatives using the blackfold approach.

Study the effective dynamics when the perturbation wavelength λ is long, $\lambda \gg r_o$ where r_o is horizon thickness.

Compute the boundary stress tensor with the corresponding viscous transport coefficients.

Application:
dissipative effect of a Dp-brane at finite temperature.

Future directions:

- Black p-brane with 0-brane charge
  (isotropic case)
- Black p-brane with q-brane charge, where q $\leq p$
  (anisotropic case)

Thank you

for your attention!