# HOW ALICE FUZZES BUT MAY NOT EVEN KNOW IT 

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## Plan

Information paradox: (Why go beyond extremal BHs, AdS/CFT)

## Black hole complementarity

Fuzzballs
Firewalls
Unitarity, qubit models \& black holes/ fuzzballs
Fuzzball Complementarity

## Firewalls: Brings the debate back in focus

Fuzzballs<br>Mathur et. al., BDC et. al.



Susskind


Please no chairs!

Remnants
Ori Spacetime

Banks et. al.

## A history lesson: Information paradox

Semiclassical Gravity
Initial state: Shell + Unruh vacuum
Shell falls in making black hole
Tracing inside part of Unruh vacuum gives thermal state

Horizon Area measures

Entropy

Coarse Graining Entanglement Remnants

## A history lesson: Black Hole Complementarity

Idea
The observations of asymptotic and infalling observer do not commute

Postulates
Black hole S-matrix unitary
Semi-classical physics outside stretched horizon
Membrane for outside observer
Free fall for infalling observer
Note that I and 4 are in tension
Its not clear how to see Hawking radiation in the first place

## A history lesson: Black Hole Complementarity



Horizon

Stop gap procedure to reconcile GR and QM

## Entropy in radiation from burning bodies

## Page:

For typical state entropy rises and falls

For purity of final state entropy vanishing


## Entropy in radiation from burning bodies

## Mathur:

Traditional picture $\mathrm{S}_{\mathrm{BC}}=0$
Strong subadditivity

$$
S_{A B}+S_{B C} \geq S_{B}+S_{A B C}
$$

Subadditivity
Nice Slices


$$
\begin{aligned}
& S_{A B} \leq S_{B}+S_{A} \\
& S_{A B}>S_{A}
\end{aligned}
$$

Small corrections do not fix this
(Mathur, Avery)
Horizon cannot be information free


## Fuzzballs

True states of quantum gravity?
Infalling shell tunnels into fuzzball solutions
Fuzzballs have no horizon or singularity


No information paradox; just like a piece of coal
All 2-charge fuzzballs
Lunin+Mathur, Lunin+Maldacena, Skenderis+Taylor
Many 3-charge
Bena,Warner, Gimon, Giusto, Saxena, Levi, JMaRT
Hawking radiation from JMaRT
BDC+Mathur


Possibility of complementarity
Mathur
A large resistance to fuzzballs because black holes look good do they really?

## Fuzzballs Before I go further, what is the conjecture?

Radiation from the non-extremal fuzzball; BDC, Mathur; 2008
....the geometries of [13] have an 'ergoregion instability' ..... these decay modes are exactly the 'Hawking radiation' from these special microstates. ....such a result may seem surprising, because the instability of [14] is a classical instability, while one normally thinks of Hawking radiation as a weak, quantum process. But we will see that this difference arises simply because in the microstates under study we have a 'large number of CFT excitations in the same state'. Consider the radiation emitted by a gas of atoms. Each atom radiates independently, and if there are several different excited levels

## Black Holes, Black Rings and their Microstates; Bena, Warner; $\underline{2007}$

In fact, one should be careful and distinguish two variants of this conjecture. The weak variant is that the black hole microstates are horizon-sized stringy configurations that have unitary scattering, but cannot be described accurately using the supergravity approximation. These configurations are also sometimes called "fuzzballs." If the weak Mathur conjecture were true then the typical bulk microstates would be configurations where the curvature is Planck scale, and hence cannot be described in supergravity. The strong form of Mathur's conjecture, which is better defined and easier to prove or disprove, is that among the typical black hole microstates there are smooth solutions that can be described using supergravity.

## 3-charge geometries and their CFT duals; Giusto, Mathur, Saxena; 2005

If we consider extremal holes, and look at states where in the dual CFT we have many component strings in the same state, then we can have a good description of the geometry in classical supergravity.
Dual geometries for a set of 3-charge microstates; Giusto, Mathur, Saxena; $\underline{2004}$
Note that in the 3 -charge case (unlike the 2 -charge case) we do not expect the generic state to be well-described by a classical geometry; quantum fluctuations can be large. But there would still be special cases that are in fact well described by a classical metric, and we can gain insight by constructing these explicitly.

## Constructing 'hair' for the three charge hole; Mathur, Saxena, Srivastava; 2003

It is possible that the generic state is not well approximated by a classical configuration; what we do expect though on the basis of all that was said above is that the region where the different states depart from each other will be of the order the horizon size and not just a planck sized region near the singularity.

The last paper is the first paper to appear on 3-charge fuzzballs

## Fuzzballs Before I go further, what is the conjecture?

The conjecture says there are $O(1)$ corrections to the horizon.
Does not say anything about actual states being describable by SUGRA SUGRA was used because thats all we knew

Recent results of de Boer et. al. should be seen as critique of some techniques but one must not loose sight of the big picture.


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## AMPS: Two main arguments


I. Horizon cannot be pure state: not Unruh vacuum after Page time
2. Blue shifted quanta B

$$
\left.S_{B C} \neq 0 \quad\right\} \text { Alice burns }
$$

Entire argument in infalling frame
Should fuzzballers be happy or sad about this?


## AMPS: Review of assumptions

Assumptions:
Black hole S-matrix unitary
Semi-classical physics outside stretched horizon
Membrane for outside observer
Free fall for infalling observer

Further conjecture:
maybe before Page time also

## Some responses

Susskind: Firewall behind true horizon, cannot be before Page time, extension of singularity (no mechanism)


Bousso, Nomura (Harlow):Weaken postulate two - Observer complementarity

Mathur+Turton: Approx. Complementarity from fuzzballs

## My response: Back to basics

Requirements for Unitarity
I. Purity of final state
2. Invertibility
3. Linearity
4. Preservation of norm


Previous discussion focussed only on point I.


Introduce 'D' for $\left|\psi_{i}\right\rangle$
Do not need any slices for this argument

## EPR

Invertible $\left|\psi_{i}\right\rangle \rightarrow\left|\psi_{f}\right\rangle$
B should know about D

Hawking pair is like EPR:

$\left|\uparrow_{L}\right\rangle\left|\uparrow_{L}\right\rangle\left|\downarrow_{R}\right\rangle+\left|\downarrow_{L}\right\rangle\left|\uparrow_{R}\right\rangle\left|\Lambda_{R}\right\rangle$
$\rho_{\mathrm{R}}$ does not know about Alice


Alice
cannot communicate to
Bob


## No Bleaching, No information

Invertible $\left|\psi_{i}\right\rangle \rightarrow\left|\psi_{f}\right\rangle$
B should know about D

More formally


B should be a unitary map of a subsystem of D
Lack of quantum cloning means the said subsystem must be bleached
$B C$ cannot be in a special state in general

## Qubit models: "moving bit" model 1

Simplest evaporation model: move qubits from $x$ to $y$

$$
\begin{aligned}
& \left|\psi_{0}\right\rangle=\left|D_{n}^{x}\right\rangle \otimes \cdots \otimes\left|D_{1}^{x}\right\rangle=\prod_{j=n}^{1}\left|D_{j}^{x}\right\rangle \\
& \left|\psi_{i}\right\rangle=\prod_{j=n}^{i-1}\left|D_{j}^{x}\right\rangle \otimes \prod_{k=i}^{1}\left|D_{k}^{y}\right\rangle \\
& \left|\psi_{n}\right\rangle=\prod_{j=n}^{n}\left|D_{j}^{y}\right\rangle
\end{aligned}
$$

think of typical states: $D_{j}$ maximally entangled with $D_{!=j}$

Where are $B$ and $C$ ?
Turns out it is possible to introduce auxiliary variables at each step Have to trace over those in the end Avery

## Qubit models: "moving bit" model 2

Evaporation with auxiliary states

$$
\begin{aligned}
& \left|\psi_{0}\right\rangle=\left|D_{n}^{x}\right\rangle \otimes \cdots \otimes\left|D_{1}^{x}\right\rangle=\prod_{j=n}^{1}\left|D_{j}^{x}\right\rangle \\
& \left|\psi_{1}\right\rangle=\prod_{j=n}^{2}\left|D_{j}^{x}\right\rangle \otimes\left|d_{1}^{x}\right\rangle \otimes\left|c_{1}^{x}\right\rangle \otimes\left|B_{1}^{y}\right\rangle \\
& \left|\psi_{i}\right\rangle=\prod_{j=n}^{i+1}\left|D_{j}^{x}\right\rangle \otimes \prod_{k=1}^{i}\left(\left|d_{i}^{x}\right\rangle \otimes\left|c_{i}^{x}\right\rangle\right) \otimes \prod_{m=1}^{1}\left|B_{m}^{y}\right\rangle \\
& \left|\psi_{n}\right\rangle=\prod_{j=1}^{n}\left(\left|d_{i}^{x}\right\rangle \otimes\left|c_{i}^{x}\right\rangle\right) \otimes \prod_{m=1}^{n}\left|B_{m}^{y}\right\rangle
\end{aligned}
$$

To match previous model $\left|B_{i}^{y}\right\rangle=\left|D_{i}^{y}\right\rangle$
For unitarity auxiliary states have to be in fiducial form

$$
\left|d_{i}^{x}\right\rangle \otimes\left|c_{i}^{x}\right\rangle=|\phi\rangle \otimes|\phi\rangle \quad \forall i
$$

fiducial form $=$ bleaching $=$ taking information out=no quantum cloning

## Information in "moving bit" model

Using this simple "moving bit" model we see
Information leaves at every step, not just after Page time

$$
\left|B_{i}^{y}\right\rangle=\left|D_{i}^{x}\right\rangle
$$

$S_{B C}$ is not zero for any step

$$
\left|c_{i}^{x}\right\rangle=|\phi\rangle
$$

For 'typical' D, each bit B is maximally entangled with non-auxiliary part of BCD

Final state is pure $\left|\psi_{i}\right\rangle \rightarrow\left|\psi_{f}\right\rangle$ invertible


## Qubit models: more general models

$$
\rho_{0}=\left|\psi_{0}\right\rangle\left\langle\psi_{0}\right| \quad\left|\psi_{0}\right\rangle \in \operatorname{span}\left\{\left|\hat{q}_{1} \cdots \hat{q}_{n}\right\rangle\right\}
$$

$$
\begin{aligned}
\left\{\left|\hat{q}_{1} \hat{q}_{2} \cdots \hat{q}_{n+i}\right\rangle\left|q_{i} q_{i-1} \cdots q_{1}\right\rangle\right\} & \xrightarrow{C_{i}}\left\{\left|\hat{q}_{1} \hat{q}_{2} \cdots \hat{q}_{n+i} \hat{q}_{n+i+1}\right\rangle\left|q_{i+1} q_{i} q_{i-1} \cdots q_{1}\right\rangle\right\} \\
& \xrightarrow{\hat{U}_{i} \otimes U_{i}}\left\{\hat{\longrightarrow}\left|\hat{q}_{1} \hat{q}_{2} \cdots \hat{q}_{n+i} \hat{q}_{n+i+1}\right\rangle U\left|q_{i+1} q_{i} q_{i-1} \cdots q_{1}\right\rangle\right\} \\
\rho_{n}= & \operatorname{tr}_{a u x}\left[U\left(\rho_{a u x} \otimes \rho_{0}\right) U^{\dagger}\right]
\end{aligned}
$$

Physical motivation
$\left|\hat{q}_{1} \hat{q}_{2} \hat{q}_{3}\right\rangle_{\text {initial }}\left|\hat{1}_{4} \hat{0}_{5}\right\rangle_{\text {infalling }}\left|0_{2} 1_{1}\right\rangle_{\text {outgoing }}$


## Qubit models: more general models

$$
\begin{aligned}
\left\{\left|\hat{q}_{1} \hat{q}_{2} \cdots \hat{q}_{n+i}\right\rangle\left|q_{i} q_{i-1} \cdots q_{1}\right\rangle\right\} & \xrightarrow{C_{i}}\left\{\left|\hat{q}_{1} \hat{q}_{2} \cdots \hat{q}_{n+i} \hat{q}_{n+i+1}\right\rangle\left|q_{i+1} q_{i} q_{i-1} \cdots q_{1}\right\rangle\right\} \\
& \xrightarrow{\hat{U}_{i} \otimes U_{i}}\left\{\hat{U}\left|\hat{q}_{1} \hat{q}_{2} \cdots \hat{q}_{n+i} \hat{q}_{n+i+1}\right\rangle U\left|q_{i+1} q_{i} q_{i-1} \cdots q_{1}\right\rangle\right\}
\end{aligned}
$$

Linearity and norm preservation $\left(C_{i}\right)^{\dagger} C_{i}=I$

$$
\begin{aligned}
e^{\hat{c}^{\dagger} b^{\dagger}}|\hat{0} 0\rangle \longrightarrow\left|\varphi_{1}^{i}\right\rangle & =\frac{1}{\sqrt{2}}\left(\left|\hat{0}_{n+i+1}\right\rangle\left|0_{i+1}\right\rangle+\left|\hat{1}_{n+i+1}\right\rangle\left|1_{i+1}\right\rangle\right) \\
\left|\varphi_{2}^{i}\right\rangle & =\frac{1}{\sqrt{2}}\left(\left|\hat{0}_{n+i+1}\right\rangle\left|0_{i+1}\right\rangle-\left|\hat{1}_{n+i+1}\right\rangle\left|1_{i+1}\right\rangle\right) \\
\left|\varphi_{3}^{i}\right\rangle & =\left|\hat{0}_{n+i+1}\right\rangle\left|1_{i+1}\right\rangle \\
\left|\varphi_{4}^{i}\right\rangle & =\left|\hat{1}_{n+i+1}\right\rangle\left|0_{i+1}\right\rangle, \\
\left(C_{i}\right)^{\dagger} C_{i}=\hat{P}_{1}^{\dagger} \hat{P}_{1} & +\hat{P}_{2}^{\dagger} \hat{P}_{2}+\hat{P}_{3}^{\dagger} \hat{P}_{3}+\hat{P}_{4}^{\dagger} \hat{P}_{4}=\hat{I} .
\end{aligned}
$$

## Qubit models: more general models

Hawking model: $\quad C_{i}^{H}=\left|\varphi_{1}\right\rangle \otimes \hat{I}$

Burning Paper:

$n \log 2$

$$
\begin{aligned}
C_{i}=\left|\hat{0}_{n+i+1}\right\rangle\left|1_{i}\right\rangle \otimes & {\left[|\hat{1} \hat{0}\rangle\langle\hat{1} \hat{1}|+\frac{1}{\sqrt{2}}|\hat{0} \hat{0}\rangle\langle\hat{1} \hat{0}|+\frac{1}{\sqrt{2}}|\hat{0} \hat{0}\rangle\langle\hat{0} \hat{1}|\right]_{n+i-1, n+i} } \\
& +\left|\hat{0}_{n+i+1}\right\rangle\left|0_{i}\right\rangle \otimes\left[|\hat{0} \hat{0}\rangle\langle\hat{0} \hat{0}|+\frac{1}{\sqrt{2}}|\hat{1} \hat{0}\rangle\langle\hat{1} \hat{0}|-\frac{1}{\sqrt{2}}|\hat{1} \hat{0}\rangle\langle\hat{0} \hat{1}|\right]_{n+i-1, n+i} .
\end{aligned}
$$

Moving bit:

$$
C_{i}=|\hat{0} 0\rangle_{\text {pair }} \otimes|\hat{0}\rangle\left\langle\left.\hat{0}\right|_{\frac{n}{2}+i}+\mid \hat{0} 1\right\rangle_{\text {pair }} \otimes|\hat{0}\rangle\left\langle\left.\hat{1}\right|_{\frac{n}{2}+i} .\right.
$$



## Qubit models: more general models

A model like AMPS/ Susskind ( $\mathrm{S}_{\mathrm{BC}}=0$ before Page time):
Hawking before Page time $\quad C_{i}^{H}=\left|\varphi_{1}\right\rangle \otimes \hat{I} \quad i<\frac{n}{2}$
Moving bit acting on old infalling quanta after Page time

$$
\begin{aligned}
C_{i}= & |\hat{0} 0\rangle_{\text {pair }} \otimes|\hat{0}\rangle\left\langle\left.\hat{0}\right|_{\frac{n}{2}+i}+\mid \hat{0} 1\right\rangle_{\text {pair }} \otimes|\hat{0}\rangle\left\langle\left.\hat{1}\right|_{\frac{n}{2}+i} \quad i>\frac{n}{2}\right. \\
& \rightarrow \frac{\left.1 \hat{q}_{1} \cdots \hat{q}_{4}\right\rangle}{\sqrt{2}}\left|\hat{q}_{1} \cdots \hat{q}_{4}\right\rangle(|0 \hat{0}\rangle+|\hat{1} 1\rangle) \\
& \rightarrow \frac{1}{2}\left|\hat{q}_{1} \cdots \hat{q}_{4}\right\rangle(|\hat{0} \hat{0} 00\rangle+|\hat{0} \hat{1} 10\rangle+|\hat{1} \hat{0} 01\rangle+|\hat{1} \hat{1} 11\rangle) \\
& \rightarrow \frac{1}{2}\left|\hat{q}_{1} \cdots \hat{q_{4}} \hat{0}\right\rangle(|\hat{0} \hat{0} 000\rangle+|\hat{10} 010\rangle+|\hat{0} \hat{0} 101\rangle+|\hat{1} \hat{0} 111\rangle) \\
& \left.\rightarrow \frac{1}{2}\left|\hat{q}_{1} \cdots \hat{q}_{4} \hat{0} \hat{0} \hat{0}\right\rangle\right\rangle(|0000\rangle+|1010\rangle+|0101\rangle+|1111\rangle) .
\end{aligned}
$$

Each quanta after Page time maximally entangled with early system Information never came out

## Non-unitary!!!

## General lessons

If Hilbert space is preserved $S_{B C}=0$ for even a single step causes loss of unitarity

Zurek showed that irreversible black hole radiation $\quad S_{\text {rad }}=\frac{4}{3} S_{B H}$

$$
\text { reversible black hole radiation } \quad S_{\text {rad }}=S_{B H}
$$

Do not expect black hole properties to depend on how they form or evaporate.

In any case, effects of irreversibility should be spread out

## Unitarity requires horizon to not be in Unruh vacuum

## Fuzzballs: Falling into fuzzballs

Fuzzballs - atypical vs typical
Lots of experience with atypical: Ergoregion instability is Bose enhanced Hawking radiation (BDC,Mathur)

What about typical fuzzballs?


AMPS' argument in the near fuzz region: cannot talk about isolated quantum DAUNTING TASK! But only then can one talk about firewalls!

## Fuzzballs: Fuzzball Complimentarity (conjecture)



Can approximate region $\quad r<r_{*} \quad t<t_{e m} \sim G M$
inside horizon

near-horizon region
barrier
outgoing radiation

## $\xrightarrow{\Omega}$

B

High energy quanta E >>kT (asymptotic) see black hole
Typical quanta E ~ kT (asymptotic) do not see black hole

## Fuzzballs: Possibility of complementarity

We already have an example: AdS/CFT
Das+Mathur, Madlacena+Strominger, Lunin-Mathur, BDC+Mathur


## Fuzzballs: Possibility of complementarity

For this part we use AdS/CFT but expect lessons more general
Start with atypical state in CFT dual to infalling shell
Becomes typical

$$
\langle\psi| \hat{O}|\psi\rangle \approx \operatorname{Tr}(\rho \hat{O})=\frac{1}{\sum_{i} e^{-\frac{E_{i}}{k T}}} \sum_{k} e^{-\frac{E_{k}}{k T}}\left\langle E_{k}\right| \hat{O}\left|E_{k}\right\rangle
$$

Density matrices can be purified

Maldacena: Such entangled CFTs dual to eternal AdS black hole


## Fuzzballs: Possibility of complementarity

Using AdS/CFT each CFT state dual to (at least asymptotically) gravitational solution

Bulk

$$
|G\rangle_{\text {eternal }}=\frac{1}{\sqrt{\sum_{i} e^{-\frac{E_{i}}{k T}}} \sum_{k} e^{-\frac{E_{k}}{2 k T}}\left|g_{k}\right\rangle_{L} \otimes\left|g_{k}\right\rangle_{R}}
$$



# Fuzzballs: Possibility of complementarity 



$$
{ }_{R}\langle\psi| \hat{O}_{R}|\psi\rangle_{R} \approx\langle\Psi| \hat{O}_{R}|\Psi\rangle
$$

Bulk

$$
{ }_{R}\left\langle\psi_{g}\right| \hat{O}_{R}\left|\psi_{g}\right\rangle_{R} \approx\langle G| \hat{O}_{R}|G\rangle .
$$



## Fuzzballs: Possibility of complementarity

Conjecture: Similar story for observables capturing infall

$\mathrm{CFT}_{R}$

## Fuzzballs: Possibility of complementarity

Is Alice burning or fuzzing?

$B$ entangled with $D$ and $A$ both
Spacetime behind the horizon and the singularity are a short lived $t \sim M$ approximate description

Alice's encounter with $B$ is strong (AMPS) inconsistent to look at that in isolation


$$
\begin{gathered}
\text { Unruh radiation } \\
|0\rangle=\sum e^{-E / 2}|E\rangle_{L} \otimes|E\rangle_{R}
\end{gathered}
$$



## Fuzzballs: Possibility of complementarity

Is Alice burning or fuzzing?

Alice's encounter with BD is strong


A

Approximate complementarity arguments show the BD system should be replaced by temporary picture with horizon and singularity not just $D$


A

## Conclusions

In the fuzzball conjecture infalling Hawking quanta auxiliary
$S_{B C}$ is not zero if unitarity is to be preserved
Approximate complementarity different from Black hole complementarity
Inside of black hole only for $E \gg k T$
temporary description of emergent space from fuzz
Low energy quanta $E \sim k T$ part of fuzzball structure
carry out information, no free infall
such infalling quanta will undergo Brownian motion
Is Alice burning or fuzzing?
Alice! Alice! Who the $f^{* * *}$ is Alice?

