Recovering bulk QFT from boundary CFT

A. Hamilton, D. Kabat, G. Lifschytz, D. Lowe 2005 and 2006

D. Kabat, G. Lifschytz, D. Lowe   1102.2910

D. Kabat ,G. Lifschytz, S. Roy, D. Sarkar   1204.3914

D. Kabat and G. Lifschytz   to appear

The Holographic way, Nordita October 2012

Introduction

- What are the operators in Q.G , How local can they be ?. Requirement of Diff. invariance suggests that local bulk observables are absent other than at infinity.
- How does local physics emerge, are there more observables, for instance relational observables?
- AdS/CFT makes it clear, the full set of observables are boundary observables.
- Can we, and how do we, recover bulk physics?, or rather mimic bulk physics.
- Once we fix a gauge completely the remaining fields are observables.

Free field construction

- At infinite N, one can use free field approach, and identify bulk creation and annihilation operators with some Fourier mode of boundary operator. (Banks,Douglas,Horowitz,Martinec, Balasubramanian, Kraus, Lawrence,Trivedi., Bena)

$$ds^2 = \frac{R^2}{Z^2}\left(-dT^2 + |dX|^2 + dZ^2\right)$$

$$\phi(z,x) = \int dx'\, K(x'|z,x)\phi_0(x')$$

$$\phi(t,\mathbf{x},z) = \int_{|\omega|>|\mathbf{k}|} d\omega d^{d-1}k\, a_{\omega\mathbf{k}} e^{-i\omega t} e^{i\mathbf{k}\cdot\mathbf{x}} z^{d/2} J_\nu(z\sqrt{\omega^2 - |\mathbf{k}|^2})$$

$$a_{\omega\mathbf{k}} = \frac{2^\nu\Gamma(\nu+1)}{(2\pi)^d(\omega^2 - |\mathbf{k}|^2)^{\nu/2}} \int dt d^{d-1}x\, e^{i\omega t} e^{-i\mathbf{k}\cdot\mathbf{x}} \phi_0(t,\mathbf{x})$$

(HKLL 2006)

$$K_{\rm Poincare} = 2c_{d\Delta} \lim_{Z\to 0} \left(\sigma(T,Z,X|P)Z\right)^{\Delta-d} \theta(\text{spacelike})$$

$$\sigma(T,X,Z|T',X',Z') = \frac{1}{2ZZ'}\left(Z^2 + Z'^2 + |X-X'|^2 - (T-T')^2\right)$$

- A convenient expression for the smearing function in terms of complexified coordinates. For poincare patch

$$\phi(t,\mathbf{x},z) = \frac{\Gamma(\Delta - \frac{d}{2}+1)}{\pi^{d/2}\Gamma(\Delta-d+1)} \int\limits_{t'^2+|\mathbf{y}'|^2<z^2} dt' d^{d-1}y' \left(\frac{z^2 - t'^2 - |\mathbf{y}'|^2}{z}\right)^{\Delta-d} \phi_0(t+t', \mathbf{x}+i\mathbf{y}')$$

- Found smearing function for global AdS
- Can in principal find smearing function for BH space time including inside the horizon (see also HMPS).
- Explicit construction for BTZ was done. Seems to need the smearing at complex coordinate.
- The two point function of such operators reproduce bulk two point functions. Including coincident and light-cone singularities., including inside the horizon.
- Position in bulk specified by the region smeared on the boundary.

- In holographic gauge (KLRS, Heemskerk)

$$zA_\mu(t,\mathbf{x},z) = \frac{1}{\mathrm{vol}(S^{d-1})}\int_{t'^2+|\mathbf{y}'|^2=z^2} dt' d^{d-1}y'\, j_\mu(t+t',\mathbf{x}+i\mathbf{y}')$$

$$zA_\mu(t,\mathbf{x},z) = \frac{\Gamma(\Delta-d/2+1)}{\pi^{d/2}\Gamma(\Delta-d+1)}\int_{t'^2+\mathbf{y}'^2<z^2} dt' d^{d-1}y' \left(\frac{z^2-t'^2-\mathbf{y}'^2}{z}\right)^{\Delta-d} A^0_\mu(t+t',x+i\mathbf{y}')$$
$$+\frac{z\Gamma(\Delta-d/2+1)}{2\pi^{d/2}\Gamma(\Delta-d+2)}\int_{t'^2+\mathbf{y}'^2<z^2} dt' d^{d-1}y' \left(\frac{z^2-t'^2-\mathbf{y}'^2}{z}\right)^{\Delta-d+1} \partial_\mu A^0_z(t+t',x+i\mathbf{y}')$$

$$z^2 h_{\mu\nu}(t,\mathbf{x},z) = \frac{1}{\mathrm{vol}(B^d)}\int_{t'^2+|\mathbf{y}'|^2<z^2} dt' d^{d-1}y'\, T_{\mu\nu}(t+t',\mathbf{x}+i\mathbf{y}')$$

Including interactions

What is wrong with these operators beyond the free field approximation. Inside three point functions and higher, this operator does not commute at bulk space like separation.

$$
\begin{aligned}
<\phi_i(x,z)\mathcal{O}_j(y_1)\mathcal{O}_k(y_2)> &= c\frac{1}{(y_1-y_2)^{\Delta_j+\Delta_k-\Delta_i}}\left[\frac{z}{z^2+(x-y_1)^2}\right]^{\Delta_j+\Delta_i-\Delta_k} \\
&\times \left[\frac{z}{z^2+(x-y_2)^2}\right]^{\Delta_k+\Delta_i-\Delta_j} f(\chi) \qquad (67)
\end{aligned}
$$

$$\chi = \frac{[(x-y_1)^2+z^2][(x-y_2)^2+z^2]}{z^2(y_2-y_1)^2}$$

$$f(\chi) = \left(\frac{\chi}{\chi-1}\right)^{\Delta_0} F(\Delta_0, \Delta_0-\frac{d}{2}+1, \Delta_i-\frac{d}{2}+1, \frac{1}{1-\chi})$$

Bulk construction (KLL)

- From bulk point of view we want to constructing Heisenberg picture operators In a perturbative expansion, but not the usual one in causal free fields

$$S = \int d^2x\sqrt{-g}\left(-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{3}\lambda\phi^3\right)$$

$$\nabla\phi = \lambda\phi^2$$

$$\phi = \phi^{(0)} + \phi^{(1)} + \phi^{(2)} + \cdots$$

$$\nabla\phi^{(0)} = 0$$
$$\nabla\phi^{(1)} = \lambda\left(\phi^{(0)}\right)^2$$
$$\nabla\phi^{(2)} = 2\lambda\phi^{(0)}\phi^{(1)}$$
$$\vdots$$

$$\phi^{(1)}(x) = \int d^2x'\sqrt{-g}\,G(x|x')\,\lambda\left(\phi^{(0)}(x')\right)^2$$

$$G(T,Z|T',Z') = \frac{1}{2}\theta(Z-Z')\theta(Z-Z'-|T-T'|)$$

$$\phi^{(1)}(T,Z) = \frac{\lambda R^2}{8}\int_0^Z \frac{dZ'}{(Z')^2}\int_{T-(Z-Z')}^{T+(Z-Z')} dT' \int_{T'-Z'}^{T'+Z'} dT_1 dT_2 : \mathcal{O}(T_1)\mathcal{O}(T_2):$$

- In this construction a space like Greens function is very useful.(HKLL,HMPS)
- Pros: In this way we can write bulk fields in perturbation theory, using the CFT operators, that reproduces in perturbation theory any bulk correlation function. micro-causality properties will be those of the bulk theory in holographic gauge.
- Cons: Need to know bulk action. Not clear how to use knowledge of CFT input (for instance unitarity, finite N information etc).
- Note: This construction by itself does not constitute holography. Holography is the statement that the boundary data is that of a unitary CFT.

CFT approach (KLL)

Scalar case: We build up the bulk scalar by requiring bulk micro-causality and correct transformation under AdS isometries.

- On CFT side add higher dimension smeared primary operators who's 3-point function is non vanishing.

$$\phi_i(Z,X,T) = \int K_{\Delta_i}(Z,X,T|X',T')\mathcal{O}_i(X',T') + \sum_n d_n \int K_{\Delta_n}(Z,X,T|X',T')\mathcal{O}_n(X',T')$$

- Existence of appropriate primary operators as multi trace operators with derivatives is tied to the 1/N expansion. We fix the coefficients (of order 1/N) by requiring micro-causality. The bulk result is the formal sum of the CFT higher dimension operators.
- One can suppress the non commutativity $[\phi(t,\mathbf{x},z),\mathcal{O}(0)] \sim \left(\frac{t^2-|\mathbf{x}|^2}{z^2}\right)^{\Delta_{\max}}$
- This is exponentialy small other than near bulk light-cone, giving seemingly non-locality of order $\delta S \sim R/\Delta_{\max}$

Gauge field case (a prelude to gravity): Requirement of micro causality has to be refined since Gauss law forbids it. i.e conserved charges are integrals over the boundary.

- Let us start as in the scalar case, but with a non conserved current

$$< \; j_\mu(x)\phi_1(y_1)\phi_2(y_2) >= \frac{1}{(y_1-y_2)^{\Delta_1+\Delta_2-\Delta+1}(y_1-x)^{\Delta+\Delta_1-\Delta_2-1}(y_2-x)^{\Delta+\Delta_2-\Delta_1-1}}\left(\frac{(y_1-x)_\mu}{(y_1-x)^2}-\frac{(y_2-x)_\mu}{(y_2-x)^2}\right)$$

This can be written as

$$\left(\frac{1}{\Delta_2+1-\Delta_1-\Delta}\frac{\partial}{\partial_{(y_1-x)_\mu}}-\frac{1}{\Delta_1+1-\Delta_2-\Delta}\frac{\partial}{\partial_{(y_2-x)_\mu}}\right)$$
$$\left[\frac{1}{(y_1-y_2)^{\Delta_1+\Delta_2-\Delta+1}(y_1-x)^{\Delta+\Delta_1-\Delta_2-1}(y_2-x)^{\Delta+\Delta_2-\Delta_1-1}}\right]$$

This is basically the 3-point function of three scalar primaries with dimensions $\Delta, \Delta_1, \Delta_2 + 1$

- So when we smear $\phi_1(y_1)$ we get the same result as in the scalar case which can be made causal by addition of smeared higher dimension scalar operators.
- For instance the first one is

$$\frac{1}{\Delta - d + 1}\partial^\mu j_\mu \phi_2(y_1) - \frac{1}{\Delta_2} j_\mu \partial^\mu \phi_2(y_1)$$

- Conserved current: This is not possible for a conserved current (consistent with Ward identity). This is good since the bulk space-like singularity gives a non zero commutator consistent with Gauss law.
- However just the second term in enough to cure micro-causality

For a 3-point with $\partial_\mu j_\nu - \partial_\nu j_\mu$

- Since it is a non primary scalar, this makes the bulk operator transform differently under AdS transformation. But this transformation matches the expected one since in holographic gauge the charged bulk scalar is secretly

$$e^{i\int_{(\mathbf{x},z)}^{(\mathbf{x},0)} A_z\,dz}\phi(\mathbf{x},z)$$

- This correction seems to be enough to make the bulk scalar commute with the boundary scalar.
- Thus even in this case one can construct the bulk scalar using a refined micro-causality requirement.
- One can also compute the corrected definition of the bulk field strength to make it local inside this 3-point function.

Comments on background independence”

- The bulk operators we constructed are a propagation of the boundary data by the radial Hamiltonian on a fixed background. This can be done for any background.

$$\mathcal{O}(x,t) \to e^{-\int_0^z H_r} \mathcal{O}(x,t) e^{\int_0^z H_r}$$

- For each background the radial Hamiltonian is different and the smearing functions are different. The background is fixed by the expectation value of the energy momentum tensor (and other operators) of the boundary theory in the particular state of the CFT.
- One can instead imagine using the Wheeler-DeWitt radial constraint to propagate the boundary data, including the energy momentum tensor (metric). In a large N approximation, the leading solution for the metric can be plugged for the matter fields equations (including gravitons), to give the bulk operators for them, on a fixed background.
- For this to work one needs that in the CFT deviations from the expectation value, in the particular state obey 1/N factorization. This suggest which CFT states will look semiclassical.
- There should be a natural gauge theory operation whose large N limit gives this picture.

Conclusions and Future

- We can construct CFT operators in 1/N perturbation theory whose CFT expectation value gives the gauge fixed correlation function of the dual bulk theory.
- Still need to work out the interacting graviton case from CFT perspective, but expect to be similar to gauge field case.
- Re-interpreting bulk divergencies.
- Better understand finite N trace relationship and their implications on this construction.
- Are any of the non localities (perturbative or non perturbative) larger near or inside the BH horizon.
- What is the natural CFT language for the bulk operators.
- Role of string states.