Topologically gauged "M2-brane" theories with 8 supersymmetries: some new results

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Talk based on:

- *"Topologically gauged superconformal Chern-Simons matter theories"* with Ulf Gran, Jesper Greitz and Paul Howe `arXiv:1204.2521 [hep-th]`,
- *“Three-dimensional N=8 superconformal gravity and its coupling to BLG M2 branes”* with Ulf Gran, `arXiv:0809.4478 [hep-th]`, in JHEP
- *"Aspects of topologically gauged M2-branes with six supersymmetries: towards a "sequential AdS/CFT"?* `arXiv:1203.5090 [hep-th]`

More references

See also

- *"Higgsing M2 to D2 with gravity: N=6 chiral supergravity from topologically gauged ABJM theory"*, with X. Chu, H. Nastase and C. Pagageorgakis, `arXiv:1012.5969 [hep-th]`, in JHEP
- *"Light-cone analysis of ungauged and topologically gauged BLG"*, `arXiv:0811.3388 [hep-th]`, in CQG
- *"D=3, N=8 conformal supergravity and the Dragon window"* with M. Cederwall and U. Gran, `arXiv:1103.4530 [hep-th]`, in JHEP

Some AdS/CFT literature on Neumann boundary conditions and repeated AdS boundaries:

- G. Compere and D. Marolf, `arXiv:0805.1902[hep-th]`
- S. de Haro, `arXiv:0808.2054[hep-th]`
- A.J. Amsel and G. Compere, `arXiv:0901.3609[hep-th]`
- T. Andrade and D. Marolf, `arXiv:1105.6337[hep-th]`
- T: Andrade and C. F. Uhlemann `arXiv:1111.2553[hep-th]`

Higher spin:

- M. Vasiliev, `arXiv:0106149[hep-th]`, `arXiv:1203.5554[hep-th]`,
- C-M Chang, S. Minwalla, T. Sharma and X. Yin, "ABJ Triality: from higher spin to strings", `arXiv:1207.4485[hep-th]`
- Sundborg (2001), Witten, Sezgin, Sundell, Klebanov, Polyakov,.....

Motivation

Three-dimensional conformal field theories are of interest in

- M-theory: M2-branes, AdS_4/CFT_3
- condensed matter: phase transitions, quantum critical points,...
- mathematics: 'monopole operators', 3d bosonization,

Here we will consider "Topologically Gauged "BLG" Theories":

- i.e. matter/Chern-Simons gauge theory with $\mathcal{N} = 8$ ("BLG") superconformal symmetry coupled to conformal supergravity
  - their Higgsing to chiral TMG supergravity (similar to higgsing from M2 to D2 branes)
  - comparison to similar results for ABJ(M) ($\mathcal{N} = 6$)
  - the possibility of a "sequential AdS/CFT"

Background for the discussion: classical

We want to understand systems of N conformal M2 branes with level k and 8 supersymmetries!
Examples of such field theories exist but relation to M2-branes tricky!

- BLG: standard classical picture [Bagger, Lambert], [Gustavsson]
  - parity symmetric $\mathcal{N} = 8$ superconformal Chern-Simons-matter theory
  - only with $SO(4) = SU(2) \times SU(2)$ gauge group (i.e. $N = 2$)
  - no $U(1)$ factor i.e. no center of mass coordinates
  - as a superconformal theory in 3d, any level k is possible

Reducing to 6 susy's the M2-brane connection is clear [ABJM]

- ABJ(M) is a quiver theory with possible gauge groups
  - $U_k(N) \times U_{-k}(N)$, for any k and any N
  - $SU_k(N) \times SU_{-k}(N)$, for any k and any N
    (for $N = 2$ and $k = 1, 2$ classically equivalent to BLG)
  - $SU_k(M) \times SU_{-k}(N) \times U(1)$, for any k and any M, N
  - $Sp(N) \times U(1)$, for any N

Background for the discussion: quantum

The quantum picture for $\mathcal{N} = 8$ is better:
monopole operators [ABJM, BKKS] can lead to enhanced symmetries for ABJM theories and relations to BLG,
this can be checked by comparing moduli spaces [Lambert et al]
and by using localization techniques to compute partition functions and indices [Kapustin et al]

- supersymmetry enhancement:
  Ex: ABJM $U_k(N) \times U_{-k}(N)$ has 2 extra susy's for $k = 1, 2$
- $U(1)$ enhancement
- parity enhancement

Motivation: some questions

Questions at the classical level:

- why is classical BLG restricted to only $SO(4)$?
- can CS(supergravity) help? (recall the role of CS(gauge)!)
- what would such a CS-gravity construction (=topological gauging) mean in M/string theory?
- in AdS/CFT?
- role of HS (higher spin)?

Questions at the quantum level:

- what aspects of CFT_2 can be taken over to CFT_3?
  - 3d bosonization? (recent speculations based on HS)
  - vertex operator algebras?

Outline

1. Brief review:
   - BLG: standard classical picture [Bagger, Lambert] [Gustavsson]
     - $\mathcal{N} = 8$ superconformal Chern-Simons-matter theory
     - only with $SO(4) = SU(2) \times SU(2)$ gauge group (i.e. $N = 2$)
2. Some new results for Chern-Simons-matter theories with 8 susy's
   - the topological gauging of the global symmetries [Gran,BN] [U. Gran, J. Greitz, P. Howe, BN]
   - higgsing to super-TMG (topologically massive supergravity)
   - the corresponding results in ABJ(M) [Chu, BN], [Chu, Nastase, BN, Papageorgakis]
3. Summary and some speculations on "Sequential AdS/CFT", Neumann b.c., etc [BN]

3-dim $\mathcal{N} = 8$ superconformal field theory : field content

The 3-dim BLG field content:

- scalars X^i_a
  - i: $SO(8)$ R-symmetry vector index
  - a: three-algebra index related to $[T^a, T^b, T^c] = f^{abc}{}_d T^d$ (structure constants f here antisymmetric in a, b, c)
- spinors ψ_a (2-comp Majorana)
  - with a hidden R-symmetry chiral spinor index (also real 8-dim),
- vector gauge potential $\tilde{A}_\mu{}^a{}_b = A_{\mu cd} f^{cda}{}_b$
  - conformal dimensions (deduced from their kinetic terms):
    - -1/2 for X^i_a
    - -1 for ψ_a
    - -1 for A_μ ("kinetic term" = Chern-Simons term) [Schwarz]

3-dim $\mathcal{N} = 8$ superconformal field theory: Lagrangian

The BLG Lagrangian

$$\begin{aligned}\mathcal{L} \quad = \quad & -\tfrac{1}{2}(D_\mu X^{ia})(D^\mu X^i{}_a) + \tfrac{i}{2}\bar{\Psi}^a\gamma^\mu D_\mu\Psi_a - \tfrac{i}{4}\bar{\Psi}_b\Gamma_{ij}X^i{}_c X^j{}_d\Psi_a f^{abcd} \\ & -V + \tfrac{1}{2}\varepsilon^{\mu\nu\lambda}\left(f^{abcd}A_{\mu ab}\partial_\nu A_{\lambda cd} + \tfrac{2}{3}f^{cda}{}_g f^{efgb}A_{\mu ab}A_{\nu cd}A_{\lambda ef}\right) \ ,\end{aligned}$$

where $D_\mu = \partial_\mu + \tilde{A}_\mu$ and the potential (a "single trace")

$$V = \tfrac{1}{12}(X^i{}_a X^j{}_b X^k{}_c f^{abcd})(X^i{}_e X^j{}_f X^k{}_g f^{efg}{}_d) \ .$$

- can introduce a (quantized) level k by rescaling $f^{abc}{}_d$, large k = weak coupling (but on orbifolds $k > 2$ unclear)
- no other free parameters!

BLG transformation rules

The BLG transformation rules for (global) $\mathcal{N} = 8$ supersymmetry are

$$\begin{aligned} \delta X_i^a &= i\epsilon\Gamma_i\Psi^a, \\ \delta\Psi_a &= D_\mu X_a^i \gamma^\mu \Gamma^i \epsilon + \tfrac{1}{6} X_b^i X_c^j X_d^k \Gamma^{ijk} \epsilon f^{bcd}{}_a . \end{aligned}$$

Demanding cancelation on the $(Cov.der.)^2$ terms in $\delta\mathcal{L}$ implies

$$\delta\tilde{A}_\mu{}^a{}_b = i\bar{\epsilon}\gamma_\mu\Gamma^i X_c^i \psi_d f^{cda}{}_b$$

Full susy needs the fundamental identity [Bagger, Lambert], [Gustavsson]

$$f^{abc}{}_g f^{efg}{}_d = 3 f^{ef[a}{}_g f^{bc]g}{}_d \,,$$

with the alternative but equivalent form [Gran, BN, Petersson]

$$f^{[abc}{}_g f^{e]fg}{}_d = 0 \,.$$

- one finite dim. realization, $\mathcal{A}_4$, with $SO(4)$ gauge symmetry (i.e. with levels $(k, -k)$) [Papadopoulos][Gauntlett,Gutowski]

BLG: more properties

- parity: interchanges the two quiver gauge fields for $SO(4) = SU(2) \times SU(2)$ with levels $(k, -k)$
- $k = 1, 2$ related to M2-branes via ABJM with *quantum* $U(1)$ sector [Lambert, Papageorgakis],[Bashkirov, Kapustin]
- The field equations for the Chern-Simons gauge field is

$$\tilde{F}_{\mu\nu}{}^{b}{}_{a} + \epsilon_{\mu\nu\rho}(X^i_c \partial^\rho X^i_d + \tfrac{i}{2}\bar{\Psi}_c \gamma^\rho \Psi_d) f^{cdb}{}_{a} = 0$$

i.e. it is not dynamical. In the light-cone gauge one can solve for the entire vector potential!

3-dim $\mathcal{N} = 8$ superconformal gravity

Introduce CS-gravity by gauging the global symmetries of the BLG theory

- Off-shell field content of 3-dim. $\mathcal{N} = 8$ conformal supergravity :

$$e_\mu{}^\alpha, \ \chi^i_\mu, \ B^{ij}_\mu, \ b_{ijkl}, \ \rho_{ijk}, \ c_{ijkl},$$

  but no lagrangian exists[Howe,Izquierdo,Papadopoulos,Townsend]
- On-shell Lagrangian = three Chern-Simons-like terms [Gran,BN] (compare $\mathcal{N} = 1$ [Deser,Kay(1983)], [van Nieuwenhuizen], and for any $\mathcal{N}$ [Lindström,Roček])

$$\mathcal{L} = \tfrac{1}{2}\epsilon^{\mu\nu\rho}Tr_\alpha(\tilde{\omega}_\mu\partial_\nu\tilde{\omega}_\rho + \tfrac{2}{3}\tilde{\omega}_\mu\tilde{\omega}_\nu\tilde{\omega}_\rho)$$

$$-ie^{-1}\epsilon^{\alpha\mu\nu}(\tilde{D}_\mu\bar{\chi}_\nu\gamma_\beta\gamma_\alpha\tilde{D}_\rho\chi_\sigma)\epsilon^{\beta\rho\sigma}-\epsilon^{\mu\nu\rho}Tr_i(B_\mu\partial_\nu B_\rho+\frac{2}{3}B_\mu B_\nu B_\rho),$$

- supercovariant spin connection: $\tilde{\omega}_{\mu\alpha\beta}(e_\mu{}^\alpha, \chi^i_\mu)$
- CS terms are of 3rd, 2nd and 1st order in derivatives, respectively

Symmetries of 3-dim $\mathcal{N} = 8$ superconformal gravity

The local symmetries are here

- 3-dim diff's and local $SO(8)$ R-symmetry
- local $\mathcal{N} = 8$ supersymmetry (f^ν is the spin 3/2 field strength)

$$\delta e_\mu{}^\alpha = i\bar{\epsilon}(x)\gamma^\alpha\chi_\mu, \;\; \delta\chi_\mu = \tilde{D}_\mu\epsilon(x),$$
$$\delta B_\mu^{ij} = -\tfrac{i}{2}\bar{\epsilon}(x)\Gamma^{ij}\gamma_\nu\gamma_\mu f^\nu,$$

- local scale invariance

$$\delta_\Delta e_\mu{}^\alpha = -\phi(x)e_\mu{}^\alpha, \;\; \delta_\Delta\chi_\mu = -\tfrac{1}{2}\phi(x)\chi_\mu, \;\; \delta_\Delta B_\mu^{ij} = 0,$$

- and local $\mathcal{N} = 8$ superconformal symmetry

$$\delta_S e_\mu{}^\alpha = 0, \;\; \delta_S\chi_\mu = \gamma_\mu\eta(x),$$

$$\delta_S B_\mu^{ij} = \tfrac{i}{2}\bar{\eta}(x)\Gamma^{ij}\chi_\mu.$$

Symmetries of 3-dim $\mathcal{N} = 8$ superconformal gravity: Fierz

A proof a la Deser-Kay requires some nice *Fierzing*! [Gran, BN]

Typical expressions that arise multiplying each other are

- the variation of supercovariant dual spin connection

$$\delta\tilde{\omega}^{*\alpha}_{\mu} = -2i(\bar{\epsilon}\gamma_{\mu}f^{\alpha} - \tfrac{1}{2}e_{\mu}{}^{\alpha}\bar{\epsilon}\gamma_{\nu}f^{\nu})$$

- and the triple dual of the Riemann tensor

$$\tilde{R}^{***}_{\mu} = i\bar{\chi}_{\nu}\gamma_{\mu}f^{\nu}$$

- which can be expanded in the Fierz basis (Gran's GAMMA is very useful here!)

$$\begin{array}{ll}
(-) & (\bar{\epsilon}\chi_{\mu})(\bar{f}_{\nu}f_{\rho})\epsilon^{\mu\nu\rho} = 0, \\
(-) & (\bar{\epsilon}\chi_{\alpha})(\bar{f}^{\beta}\gamma^{\alpha}f_{\beta}) = 0, \\
(1) & (\bar{\epsilon}\chi_{\alpha})(\bar{f}^{\alpha}\gamma^{\beta}f_{\beta}), \\
(2) & (\bar{\epsilon}\gamma^{\alpha}\chi_{\alpha})(\bar{f}^{\beta}f_{\beta}), \\
(3) & ....(\textit{about } 30 \textit{ basis elements})
\end{array}$$

Topologically gauged BLG theory

- This supergravity theory has no propagating degrees of freedom!
  - clear in the light-cone gauge: all non-zero field components (plus ∂_+ on them) can be solved for [BN]
    => "topologically gauged BLG"
- Conformal supergravity can be coupled to BLG by Noether methods
  - to order $(Cov.der.)^3$ and $(Cov.der.)^2$ [Gran,BN]
  - the full action now derived [Gran, Greitz, Howe, BN]
- or other methods like
  - demanding on-shell susy (as originally done for BLG) [Gran, Greitz, Howe, BN]
  - superspace [Gran, Greitz, Howe, BN],
    following "the Dragon window" in 3d [Cederwall, Gran, BN]
    see also [Howe, Izquierdo, Papadopoulos, Townsend]
    and recently [Greitz, Howe],
    [Kuzenko, Lindström, Targaglino-Mazzucchelli]

Topologically gauged BLG theory: details

Supersymmetry to order $(D_\mu)^2$ gives the conformal coupling $-\frac{e}{16}X^2\tilde{R}$:
(f^μ is the dual field strength of the spin 3/2 field χ_μ) [Gran, BN]

$$L^{top}_{BLG} \;=\; L^{conf}_{grav} + L^{cov}_{BLG}$$

$$+\frac{1}{\sqrt{2}} ie\bar{\chi}_\mu \Gamma^i \gamma^\nu \gamma^\mu \Psi^a \tilde{D}_\nu X^{ia} \quad (\text{"the supercurrent term"})$$

$$-\frac{i}{4}\epsilon^{\mu\nu\rho}\bar{\chi}_\mu \Gamma^{ij}\chi_\nu (X^i_a \tilde{D}_\rho X^j_a) + \frac{i}{\sqrt{2}}\bar{f}^\mu \Gamma^i \gamma_\mu \Psi_a X^i_a$$

$$-\frac{e}{16}X^2\tilde{R} + \frac{i}{4}X^2\bar{f}^\mu \chi_\mu$$

Topologically gauged BLG theory: more details

The extended transformation rules at order $(D_\mu)^2$ in δL are

$$
\begin{aligned}
\delta e_\mu{}^\alpha &= i\sqrt{2}\bar{\epsilon}\gamma^\alpha\chi_\mu\,, \\
\delta\chi_\mu &= \sqrt{2}\tilde{D}_\mu\epsilon, \\
\delta B_\mu^{ij} &= -\tfrac{i}{\sqrt{2}}\bar{\epsilon}\Gamma^{ij}\gamma_\nu\gamma_\mu f^\nu - \tfrac{i}{2\sqrt{2}}\bar{\chi}_\mu\Gamma^{k[i}\epsilon X_a^{j]}X_a^k + \tfrac{i}{16\sqrt{2}}\bar{\epsilon}\Gamma^{ij}\chi_\mu X^2 \\
&\quad -\tfrac{i}{16}\bar{\Psi}_a\Gamma^k\Gamma^{ij}\gamma_\mu\epsilon X_a^k - \tfrac{i}{2}\bar{\Psi}_a\gamma_\mu\Gamma^{[i}\epsilon X_a^{j]}, \\
\delta X_i^a &= i\epsilon\Gamma_i\Psi^a, \\
\delta\Psi_a &= (\tilde{D}_\mu X_a^i - \tfrac{1}{\sqrt{2}}\bar{\chi}_\mu\Gamma^i\Psi_a)\gamma^\mu\Gamma^i\epsilon + \tfrac{1}{6}X_b^iX_c^jX_d^k\Gamma^{ijk}\epsilon f^{bcd}{}_a, \\
\delta\tilde{A}_\mu{}^a{}_b &= i\bar{\epsilon}\gamma_\mu\Gamma^iX_c^i\Psi_d f^{cda}{}_b - \tfrac{i}{\sqrt{2}}\bar{\chi}_\mu\Gamma^{ij}\epsilon X_c^iX_d^j f^{cda}{}_b.
\end{aligned}
$$

Topologically gauged BLG theory: the transformation rules

The complete transformation rules with explicit coupling constants: the level parameter $\lambda = \frac{2\pi}{k}$ and the gravitational coupling g from rescaling of the trace ($\epsilon_m = A\epsilon_g$ and $A^2 = \frac{1}{2}$)

$$
\begin{aligned}
\delta e_\mu{}^\alpha &= i\bar{\epsilon}_g \gamma^\alpha \chi_\mu, \quad \delta\chi_\mu = \tilde{D}_\mu \epsilon_g, \\
\delta B^{ij}_\mu &= -\tfrac{i}{2e}\bar{\epsilon}_g \Gamma^{ij}\gamma_\nu\gamma_\mu f^\nu - \tfrac{ig}{4}\bar{\chi}_\mu \Gamma^{k[i}\epsilon_g X^{j]}_a X^k_a - \tfrac{ig}{32}\bar{\chi}_\mu \Gamma^{ij}\epsilon_g X^2 \\
&\quad - \tfrac{ig}{16}\bar{\Psi}_a \Gamma^{ijk}\gamma_\mu \epsilon_m X^k_a - \tfrac{3ig}{8}\bar{\Psi}_a \gamma_\mu \Gamma^{[i}\epsilon_m X^{j]}_a, \\
\delta X^i_a &= i\epsilon_m \Gamma^i \Psi_a, \\
\delta\Psi_a &= \gamma^\mu \Gamma^i \epsilon_m (\tilde{D}_\mu X^i_a - iA\bar{\chi}_\mu \Gamma^i \Psi_a) + \tfrac{\lambda}{6} X^i_b X^j_c X^k_d \Gamma^{ijk}\epsilon f^{bcd}{}_a \\
&\quad + \frac{g}{8}\Gamma^i \epsilon_m X^i_b X^j_b X^j_a - \frac{g}{32}\Gamma^i \epsilon_m X^i_a X^2, \ (NEW) \\
\delta\tilde{A}_\mu{}^a{}_b &= i\lambda \bar{\epsilon}_m \gamma_\mu \Gamma^i X^i_c \Psi_d\, \epsilon^{cda}{}_b - \tfrac{i\lambda}{2}\bar{\chi}_\mu \Gamma^{ij}\epsilon_g X^i_c X^j_d\, \epsilon^{cda}{}_b \\
&\quad + \frac{ig}{4}\epsilon_m \gamma_\mu \Gamma^i \psi_{[a} X^i_{b]} + \frac{ig}{8}\bar{\chi}_\mu \Gamma^{ij}\epsilon_g X^i_a X^j_b. \ (NEW)
\end{aligned}
$$

Topologically gauged BLG theory: the Lagrangian

The most interesting terms in L are

$$L = \frac{1}{g}L^{SUGRA}_{conf} + L^{BLG}_{cov} - \frac{e}{16}X^2R - V^{new}$$
$$+ \frac{ig}{64}e(\bar{\Psi}_a\Psi_aX^2 - 10\bar{\Psi}_a\Psi_bX^i_aX^i_b + 2\bar{\Psi}_a\Gamma^{ij}\Psi_bX^i_aX^j_b)$$

the scalar potential has a *single-trace* (st) contribution from L^{BLG}_{cov}

$$V^{st}_{BLG} = \frac{\lambda^2}{12}(X^i{}_aX^j{}_bX^k{}_c\,\epsilon^{abcd})(X^i{}_eX^j{}_fX^k{}_g\,\epsilon^{efg}{}_d)$$

and a new *triple-trace* (tt) term

$$V^{tt}_{new} = \frac{eg^2}{2\cdot 32\cdot 32}\Big((X^2)^3 - 8(X^2)X^j_bX^j_cX^k_cX^k_b + 16X^i_cX^i_aX^j_aX^j_bX^k_bX^k_c\Big)$$

- but *no* new *double-trace* terms as in the ABJ(M) case

Topologically gauged BLG theory: new properties

The above Lagrangian has also a new kind of parity non-symmetric Chern-Simons sector [Gran, Greitz, Howe, BN]

$$L_{CS(A)} = \frac{1}{a} L_{CS(A^L)} + \frac{1}{a'} L_{CS(A^R)}$$

where

$$a := \frac{g}{8} - \lambda, \quad a' := \frac{g}{8} + \lambda$$

This is easy to see from the form of $\delta\tilde{A}_\mu^{cd} = \delta A_\mu^{ab} f_{ab}{}^{cd}$ where gauging =>

$$f_{ab}{}^{cd} \to M_{ab}{}^{cd} = f_{ab}{}^{cd} - \frac{g}{4}\delta_{ab}^{cd} \quad (1)$$

Note: In superspace one starts from a non-parity invariant quiver Chern-Simons theory with independent level parameters a and a' which in the Noether and on-shell susy 3-algebra approaches arise as a level λ and a gravitational coupling constant (or level) g!
[Chu, Nastase, BN, Papageorgakis]

Topologically gauged BLG theory: new properties

New theories?

$$L_{CS(A)} = \frac{1}{a} L_{CS(A^L)} + \frac{1}{a'} L_{CS(A^R)}$$

where

$$a := \frac{g}{8} - \lambda, \;\; a' := \frac{g}{8} + \lambda$$

New theories arise as follows

- for $\lambda = 0$ the three-algebra indices can be extended arbitrarily: –> gauge group $SO(N)$ for any N
- even with non-zero λ there is an additional new $SO(3)$ theory for certain values of the parameters
- not parity symmetric but that is already so in the gravity sector!

3-dim $\mathcal{N} = 6$ superconformal field theory: ABJM basics

Stacks with more than two M2's require less susy than $\mathcal{N} = 8$ –>
ABJM (classical): $\mathcal{N} = 6$, any k and gauge group $U(N) \times U(N)$

- scalar fields now complex Z^A: in bifundamental of the gauge group (A-index a **4** of the R-symmetry $SU(4) \times U(1)$)
- the ABJM quiver version uses no three-algebra f symbols,
- but f can be reinstated [Bagger,Lambert]

3-dim $\mathcal{N} = 6$ superconformal field theory: more structure

It is natural to use [BN,Palmkvist]

- $f^{ab}{}_{cd}$ where $[ab]$ and $[cd]$
- $(Z^A_a)^* = Z^a_A$, $(\Psi_{Aa})^* = \Psi^{Aa}$

Then the ABJM theory is (with V on the next slide!)

$$\begin{aligned}
\mathcal{L} \;=\; & -\tfrac{1}{2}(D_\mu Z^A_a)(D^\mu \bar{Z}^a_A) + \tfrac{i}{2}\bar{\Psi}^{Aa}\gamma^\mu D_\mu \Psi_{Aa} \\
& -\tfrac{i}{2} f^{ab}{}_{cd}\bar{\Psi}^{Ad}\Psi_{Aa} Z^B_b \bar{Z}^c_B + i f^{ab}{}_{cd}\bar{\Psi}_{Aa}\Psi_{Bb} Z^B_c \bar{Z}^d_A \\
& -\tfrac{i}{4}\epsilon_{ABCD} f^{ab}{}_{cd}\bar{\Psi}^{Ac}\Psi^{Bd} Z^C_a Z^D_b - \tfrac{i}{4}\epsilon^{ABCD} f^{ab}{}_{cd}\bar{\Psi}_{Aa}\Psi_{Bb}\bar{Z}^c_C \bar{Z}^d_B \\
& -V + \tfrac{1}{2}\epsilon^{\mu\nu\lambda}\big(f^{ab}{}_{cd} A_{\mu b}^{d}\partial_\nu A_{\lambda a}^{c} + \tfrac{2}{3} f^{bd}{}_{gc} f^{gf}{}_{ae} A_{\mu b}^{a} A_{\nu d}^{c} A_{\lambda f}^{e}\big)\,,
\end{aligned}$$

and the fundamental identity (solutions classified by Palmkvist)

$$f^{e[a}{}_{dc} f^{b]d}{}_{gh} = f^{ab}{}_{d[g} f^{ed}{}_{h]c}\,.$$

Topologically gauged $\mathcal{N}=6$ superconformal ABJ(M)

The BLG type of new potential was found first for ABJ(M) [Chu, BN]

- The complete topologically gauged ABJM lagrangian has about 25 new terms including a new $U_R(1)$ CS gauge field
  - New scalar interaction terms (with explicit λ and g) :
    First: Recall the original ABJ(M) potential (single trace in 3-alg.)

$$V^{st}_{ABJ(M)}=\tfrac{2}{3}|\Upsilon^{CD}{}_{Bd}|^2,\ \ \Upsilon^{CD}{}_{Bd}=\lambda f^{ab}{}_{cd}Z^C_aZ^D_b\bar{Z}^c_B+\lambda f^{ab}{}_{cd}\delta^{[C}_BZ^{D]}_aZ^E_b\bar{Z}^c_E\,.$$

The new terms with one structure constant are (double trace)

$$V^{(dt)}_{new}=-\tfrac{g}{8}\lambda f^{ab}{}_{cd}|Z|^2Z^C_aZ^D_b\bar{Z}^c_CZ^d_D-\tfrac{g}{2}\lambda f^{ab}{}_{cd}Z^B_aZ^C_b(Z^D_e\bar{Z}^e_B)\bar{Z}^c_C\bar{Z}^d_D\,.$$

and without structure constant (triple trace)

$$V^{(tt)}_{new}=-g^2\big(\tfrac{5}{12\cdot 64}(|Z|^2)^3-\tfrac{1}{32}|Z|^2|Z|^4+\tfrac{1}{48}|Z|^6\big)\,.$$

  - also new Yukawa-like terms without structure constant

Higgsing of topologically gauged ABJM

Two observations:

- Higgsing to D2 branes leaves the theory at an AdS chiral point similar to the one of Li, Song, Strominger [Chu, BN]
- Scaling limits can be taken in different ways [Chu, Nastase, BN, Papageorgakis]

Several steps needed:

- introduce two parameters $\lambda = \frac{2\pi}{k}$ and g (via the triple product and the trace)
- expand the theory around a real VEV v: $Z^A = v\delta^{A4} + z^A$
- limits are taken in λ, g, v with various combinations kept fixed (including the gauge group): see below
- identify the six new ordinary supersymmetries: $Q_{AdS} = Q_{CFT} + S_{CFT}(\eta = ...\epsilon)$

Higgsing of topologically gauged ABJM: the chiral point

The appearance of the chiral point is seen from the scalar/gravitational terms [Chu, BN] (before introducing λ and g_M)

$$L^{ABJM}_{higgsed} = L_{CS(grav)} - \frac{e}{8}v^2 R - \frac{e}{256}v^6$$

- thus $\frac{v^2}{8} = \frac{1}{\kappa^2}$ and comparison to chiral gravity implies that $\mu = l^{-1} = \kappa^{-2}$, i.e. $\mu l = 1$
- The sign of the Einstein-Hilbert and cosmological terms are as in TMG, i.e., opposite to the signs used by Li, Song and Strominger => negative energy black holes, non-unitarity, etc (see Deser and Franklin)
    - these features are dictated by the sign of the ABJM scalar kinetic terms (via conformal invariance)!
    - introducing more parameters (levels) does not alter this conclusion

Higgsing of topologically gauged theories with 8 supersymmetries: the chiral points of the $SO(N)$ model

The appearance of the chiral point for $\mathcal{N}=8$ is seen from the terms

$$L^{SO(N)} = \frac{1}{g}L_{CS(grav)} - \frac{1}{16}X^2R - \frac{g^2}{2\cdot 32\cdot 32}((X^2)^3 - 8X^2X^4 + 16X^6)$$

where (a=1,2,...,N and i=1,2,..,8)

$$X^{ij} = X^i_aX^j_a,\;\; X^2 = tr(X^{ij}),\;\; X^4 = X^{ij}X^{ij},\;\; X^6 = X^{ij}X^{jk}X^{ki} \qquad (2)$$

Compare to the TMG version of Li, Song and Strominger:

$$L^{LSS} = \frac{1}{\kappa^2}(\frac{1}{\mu}L_{CS(grav)} - (R-2\Lambda)),\;\; \Lambda = -\frac{1}{l^2}$$

- one non-zero <X> => $\kappa^2\mu = g,\; \frac{v^2}{16} = \frac{1}{\kappa^2},\; \frac{2}{\kappa^2 l^2} = \frac{9g^2v^6}{2\cdot 32\cdot 32}$
- =>$\mu l = 1/3$??

Higgsing of topologically gauged theories with 8 supersymmetries: the chiral points of the $SO(N)$ model

$$L^{SO(N)} = \frac{1}{g} L_{CS(grav)} - \frac{1}{16} X^2 R - \frac{g^2}{2 \cdot 32 \cdot 32} ((X^2)^3 - 8X^2 X^4 + 16X^6)$$

where (a=1,2,...,N and i=1,2,..,8)

$$X^{ij} = X^i_a X^j_a, \;\; X^2 = tr(X^{ij}), \;\; X^4 = X^{ij} X^{ij}, \;\; X^6 = X^{ij} X^{jk} X^{ki} \qquad (3)$$

There are two critical points on the market:

- chiral AdS with $\mu l = 1$
- null-warped AdS with $\mu l = 3$

A more general set of VEV's are matrix VEV's: set $< X^i_a >= v\delta^I_A$ with $I, A = 1, 2, .., p$

- critical AdS for $p = 2$
- null-warped AdS for $p = 3, 6$
- Minkowski for $p = 4$

Higgsing of topologically gauged ABJM: the scaling limits

ABJM: After introducing λ and g_M (=g above) the relevant terms are schematically:

$$
\begin{aligned}
L \;=\; & \tfrac{1}{g_M^2} L_{CS(\omega)} - |Z|^2 R + \tfrac{1}{\lambda}(AdA + A^3) - DZD\bar{Z} \\
& -\lambda^2 V^{(1-trace)} - \lambda g_M^2 V^{(2-trace)} - g_M^4 V^{(3-trace)}
\end{aligned}
$$

- in the ABJM formulation (no "f") there are two gauge fields, A^L and A^R with opposite signs of the levels: for higgsing we need to
  - let A^+ and A^- be their sum and difference
  - then the gauge theory CS-term reads: $\frac{1}{\lambda} A^- F^+$ (with A^+ in D_μ)
  - the Z kinetic terms gives a term $v^2 (A^-)^2$
  - combining these two last steps gives a kinetic term $\frac{1}{v^2 \lambda^2}(F^+)^2$
- thus with $k \to \infty$ we need $v \to \infty$ to keep g_{YM} fixed,
- fixing also the cosm constant requires taking $g_M \to 0$
- gravitational coupling κ is prop. to $\frac{1}{v}$
- no subleading terms after taking the scaling limits

Higgsing of topologically gauged ABJM: the scaling limits

A second version of the scaling limit is obtained multiplying the whole action by g_M^2:

$$\begin{aligned} L &= L_{CS(\omega)} - g_M^2|Z|^2R + \frac{g_M^2}{\lambda}(AdA + A^3) - g_M^2 DZD\bar{Z} \\ &\quad -\lambda^2 g_M^2 V^{(1-trace)} - \lambda g_M^4 V^{(2-trace)} - g_M^6 V^{(3-trace)} \end{aligned}$$

Sending k to ∞ now leads to two fixed but tunable parameters:

- $g_{YM}^2 = \frac{v^2\lambda^2}{g_M^2}$ and $\kappa^2 = \frac{1}{v^2 g_M^2}$

The higgsed lagrangian reads

$$\begin{aligned} L &= L_{CS(\omega)} - (\frac{1}{\kappa^2} + \ldots)R + \frac{1}{g_{YM}^2}(F^+)^2 - \frac{1}{g_{YM}^2} D\tilde{z}D\bar{\tilde{z}} \\ &\quad -(\frac{1}{g_{YM}^2}(\tilde{z})^4 + subleading) - (\frac{1}{\kappa^2 g_{YM}^2}(\tilde{z})^3 + subleading) \\ &\quad -(\frac{1}{\kappa^6} + \frac{1}{\kappa^5 g_{YM}}(\tilde{z}) + \frac{1}{\kappa^4 g_{YM}^2}(\tilde{z})^2 + subleading) \end{aligned}$$

- "subleading" refers to higher powers of g_{YM}^{-1}

Summary

- $SO(N)$ gauge groups for any N possible in topologically gauged "free BLG"
- Topologically gauged theories exhibit spontaneous symmetry breaking to YM/matter theories coupled to chiral TMG gravity:
  - Chiral point in the "free BLG": critical AdS or null warped AdS!
  - Chiral point in the ABJ(M) case: critical AdS

Summary

- $SO(N)$ gauge groups for any N possible
  in topologically gauged "free BLG"
- Topologically gauged theories exhibit spontaneous symmetry breaking to YM/matter theories coupled to chiral TMG gravity:
  - Chiral point in the "free BLG": critical AdS or null warped AdS!
  - Chiral point in the ABJ(M) case: critical AdS
- "Sequencial AdS/CFT" ??: [BN] ([Marolf et al], [Vasiliev(2012)])

$$AdS_4^{N.b.c.}/TGCFT_3 \to CPAdS_3/CFT_2$$

  - $AdS_4^{N.b.c.}$ = AdS w/ Neumann b.c. ([de Haro][Compere, Marolf])
  - $TGCFT_3$ = top. gauged CFT_3
  - $CPAdS_3$ = 3d TMG at the Chiral Point
  - CFT_2 ??

Change of foliation in AdS_4 = 3d higgsing ? HS ?
AdS foliations, see [Andrade, Uhlemann (2011)]
and [Ohl, Uhlemann (2012)] (unitarity issues (for singletons))

Some relevant aspects of AdS4/CFT3: I

Start from the Fefferman-Graham metric [Compere,Marolf] [de Haro]

$$ds^2 = \frac{l^2}{r^2}(dr^2 + (\eta_{ij} + h_{ij}(r,x))dx^i dx^j) \tag{4}$$

- $h_{ij} = 0$ gives the AdS metric with its boundary at $r = 0$
- $h_{ij}(r,x)$ is the deviation from AdS allowed by Einstein's eq's

$$\bar{h}''_{ij} - \frac{2}{r}\bar{h}'_{ij} + \Box\bar{h}_{ij} = 0, \;\; (\bar{h}_{ij} = h^{TT}_{ij}, \; ' = \partial_r) \tag{5}$$

  - given in even and odd power series in the radial coordinate r

$$\bar{h}_{ij}(r,p) = (1 + ...)h^{(0)}_{ij}(p) + ((pr)^3 + ...)h^{(3)}_{ij}(p) \tag{6}$$

- both D and N b.c. are possible
- at the AdS boundary one may impose

$$\Box^{1/2}h^{(0)}_{ij} = \pm\epsilon_{ikl}\partial_k h^{(0)}_{lj} \tag{7}$$

Some relevant aspects of AdS4/CFT3:II

This corresponds to adding a grav. Chern-Simons term to the 3d boundary CFT: (below C_{ij} is the Cotton tensor)

- with D b.c.: $h^{(0)}$ is a fixed source and $< T_{ij} >= h^{(3)}_{ij} - C_{ij}(h^{(0)})$
- this defines the usual CFT, that is CFT_D

A second CFT, CFT_N, is obtained for Neumann b.c., i.e. $< T_{ij} >= 0$, or $h^{(3)}_{ij} = C_{ij}(h^{(0)})$: we need a dual pair $(\tilde{h}^{(0)}, < \tilde{T}_{ij} >)$

- Set $C_{ij}(h^{(0)}) =< \tilde{T}_{ij} >$ and solve for $\tilde{h}^{(0)}$ using the corresponding dual equation $< T_{ij} >= C_{ij}(\tilde{h}^{(0)})$
- can be done in momentum space [de Haro] or in light-cone [BN]
  - $h_{++} = -2\partial_-^{-3} T_{2-}$
  - $h_{+2} = -\partial_-^{-3} T_{--}$
  - $\partial_+ h_{++} = 2\partial_-^{-2} T_{2+} - 2\partial_-^{-3}(\partial_2 T_{22})$
  - $\partial_+ h_{+2} = \frac{1}{2}\partial_-^{-2} T_{22} - \partial_-^{-3}(\partial_2 T_{2-})$
  - $\partial_+^2 h_{+2} = -\partial_-^{-1} T_{++} + \partial_-^{-3}(\partial_2^2 T_{22})$

Some relevant aspects of AdS4/CFT3:III

Thus, we have the two dual CFT's of opposite parity both AdS/CFT-dual to the bulk theory:

- $T_{ij} = \tilde{C}_{ij}$ and $\tilde{T}_{ij} = C_{ij}$ in the two CFT's are like electric-magnetic variables
- S-duality between the two CFT's:[de Haro]
  - actually a Legendre transformation: $\tilde{W}(\tilde{h}) = W(h) + V(h, \tilde{h})$
  - with $V(h, \tilde{h}) = CS(h, \tilde{h})$ (= 2'nd variation of the CS functional)

3d boundary theories with supersymmetries

- $\mathcal{N} = 1$ susy: see [de Haro] [Amsel, Compere] [Becker et al]
- $\mathcal{N} = 6$ and $\mathcal{N} = 8$: This talk!