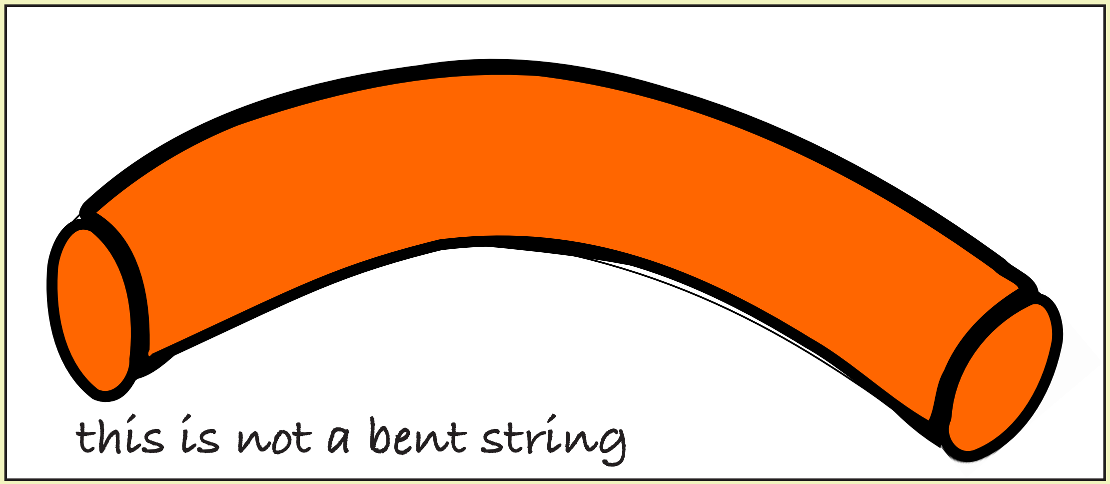
(Electro)Elasticity from Gravity



Jay Armas | NBI

BASED ON arXiv:1110.4835 J.Armas, J.Camps, T.Harmark, N.A.Obers

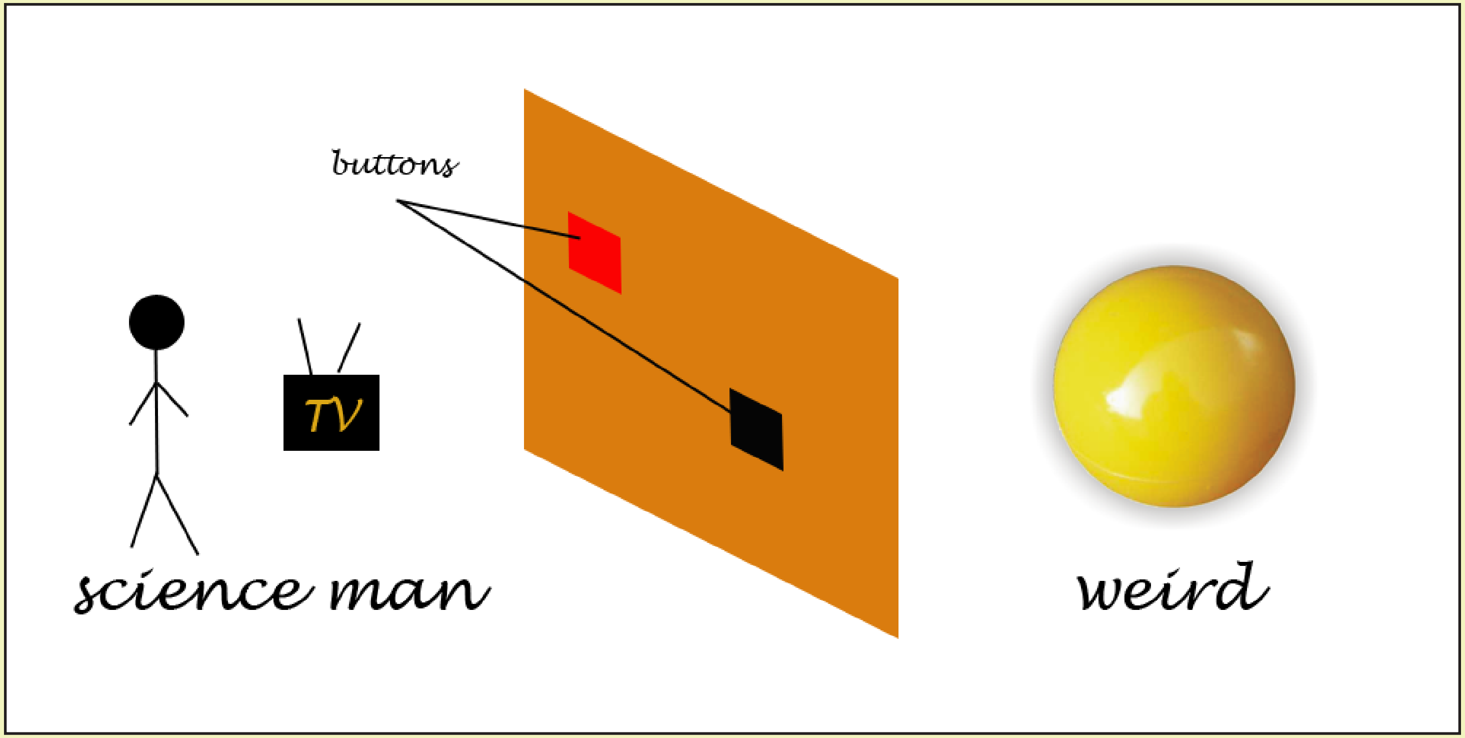
arXiv:1204.2127 J.Armas, J.Gath, N.A.Obers

arXiv:1204.5197 J.Armas, N.A.Obers

CHAPTER 1:

THE MATERIAL SCIENCE LABORATORY

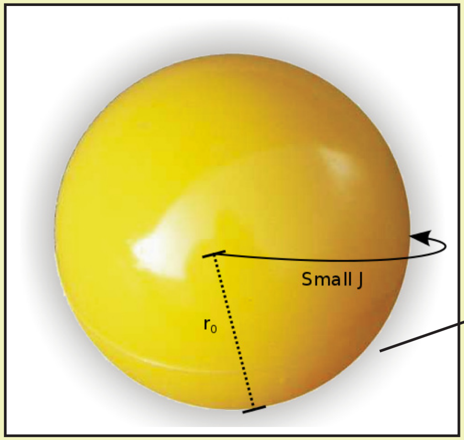
The Material Science Laboratory



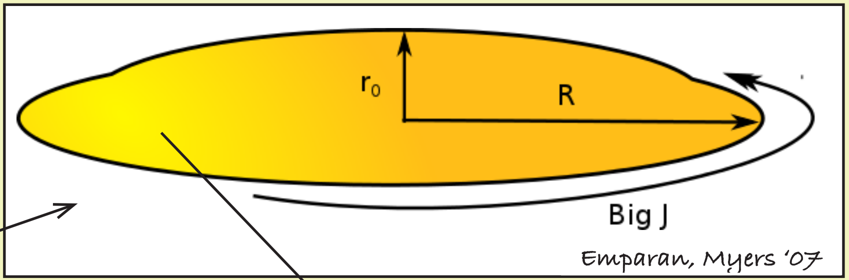
CHAPTER 2:

THE ART AND CRAFT OF BENDING BLACK BRANES

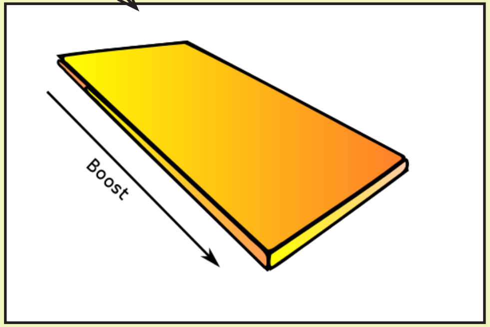
The Blackfold Approach (I): Myers Perry



Increase angular momentum



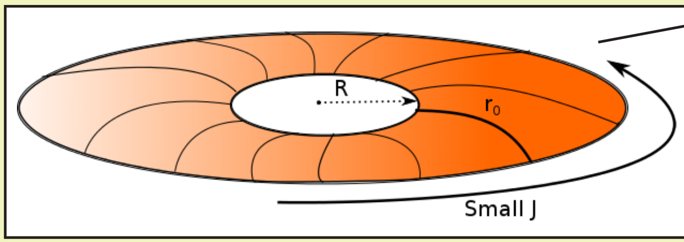
Zoom into horizon

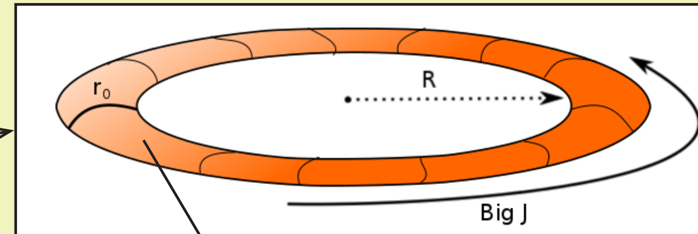


$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2_{D-4} + (d\sigma^2 + \sigma^2 d\phi^2)$$

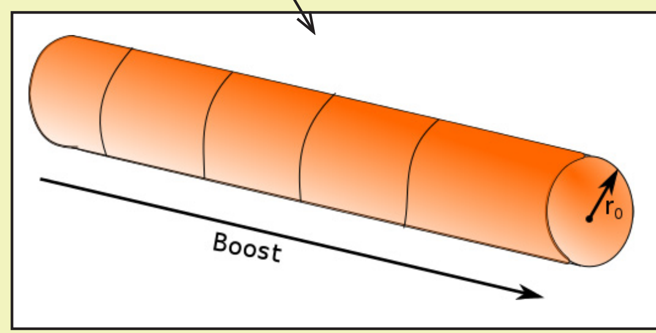
The Blackfold Approach (II): Black Ring

Increase angular momentum





Zoom into horizon



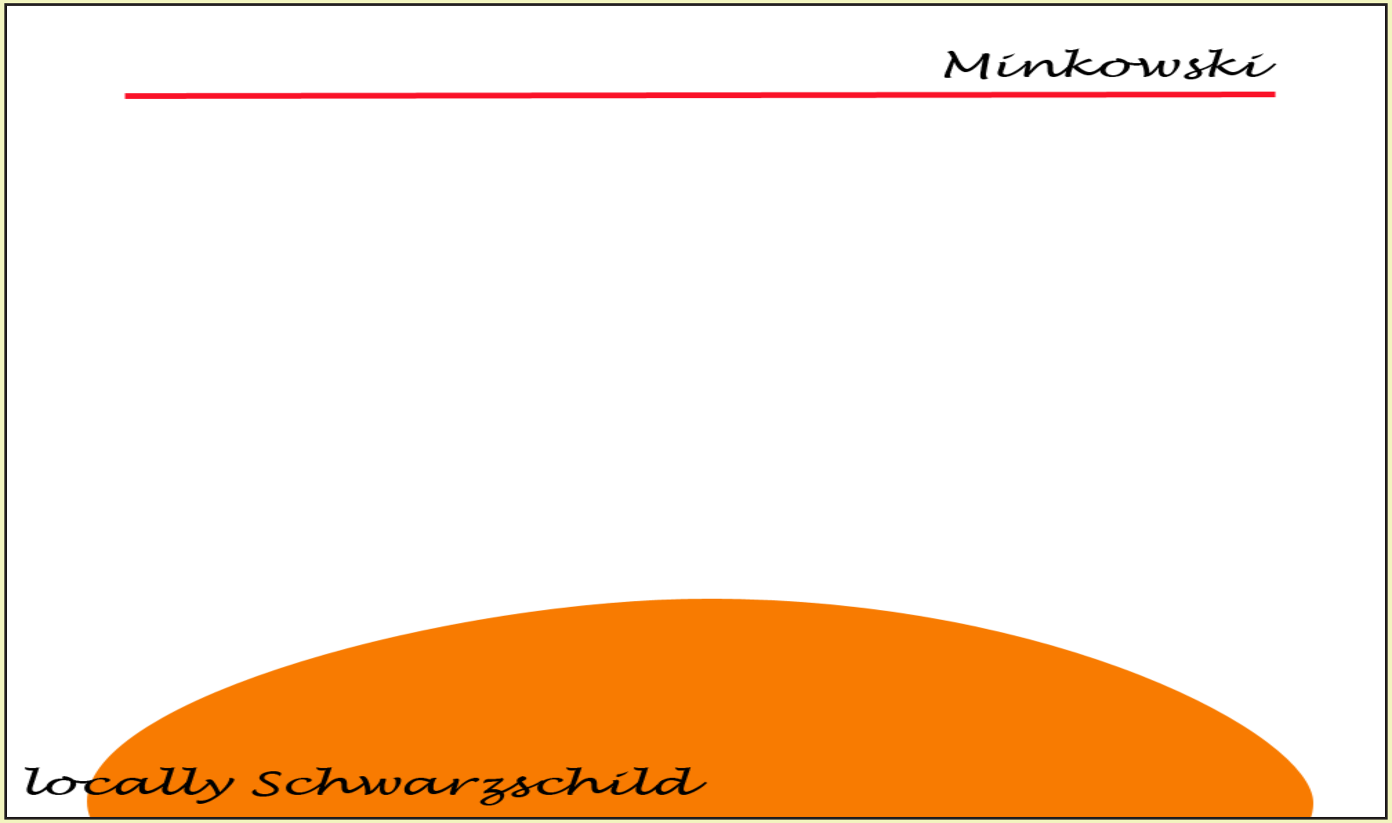
$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2_{D-3} + dz^2$$

The Blackfold Approach (III): Observation

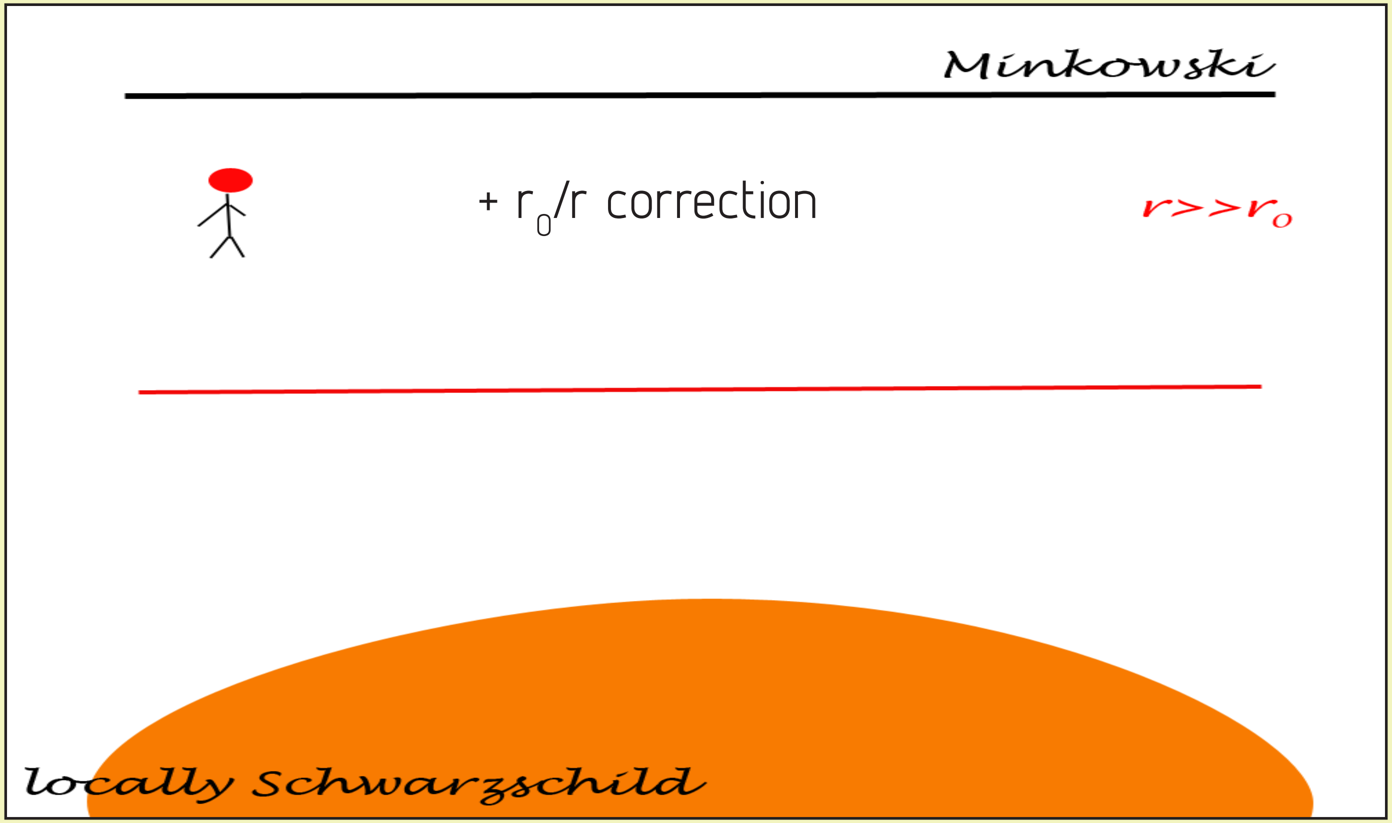
WE HAVE ABSOLUTELY NO IDEA OF WHAT KIND OF HIGHER DIMENSIONAL BLACK HOLES ARE OUT THERE BUT IT SEEMS TO BE A GENERAL FEATURE TO EXHIBIT A LIMIT WHERE THEY BECOME LOCALLY BLACK BRANES.

Emparan, Harmark Niarchos, Obers '10

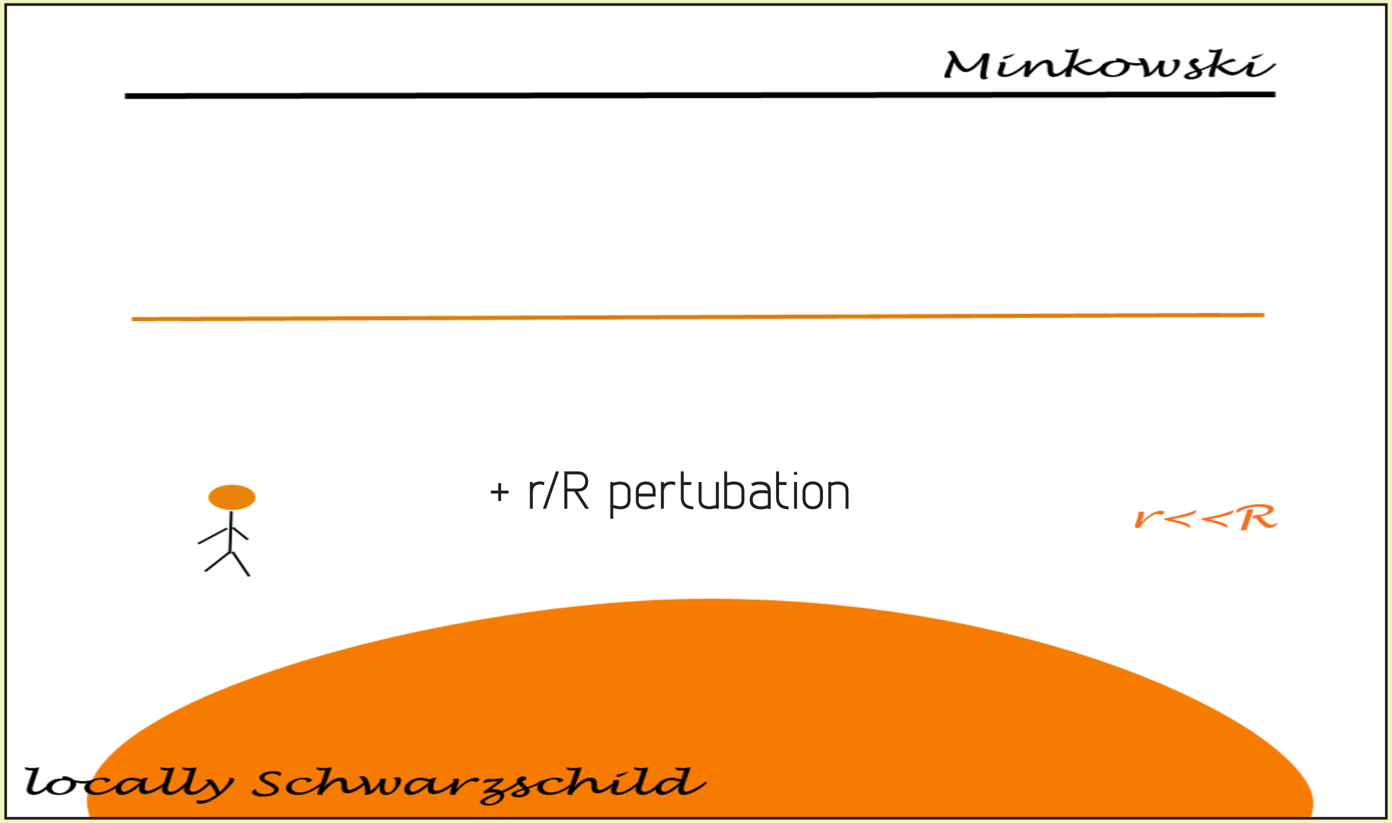
The Blackfold Approach (IV): Method



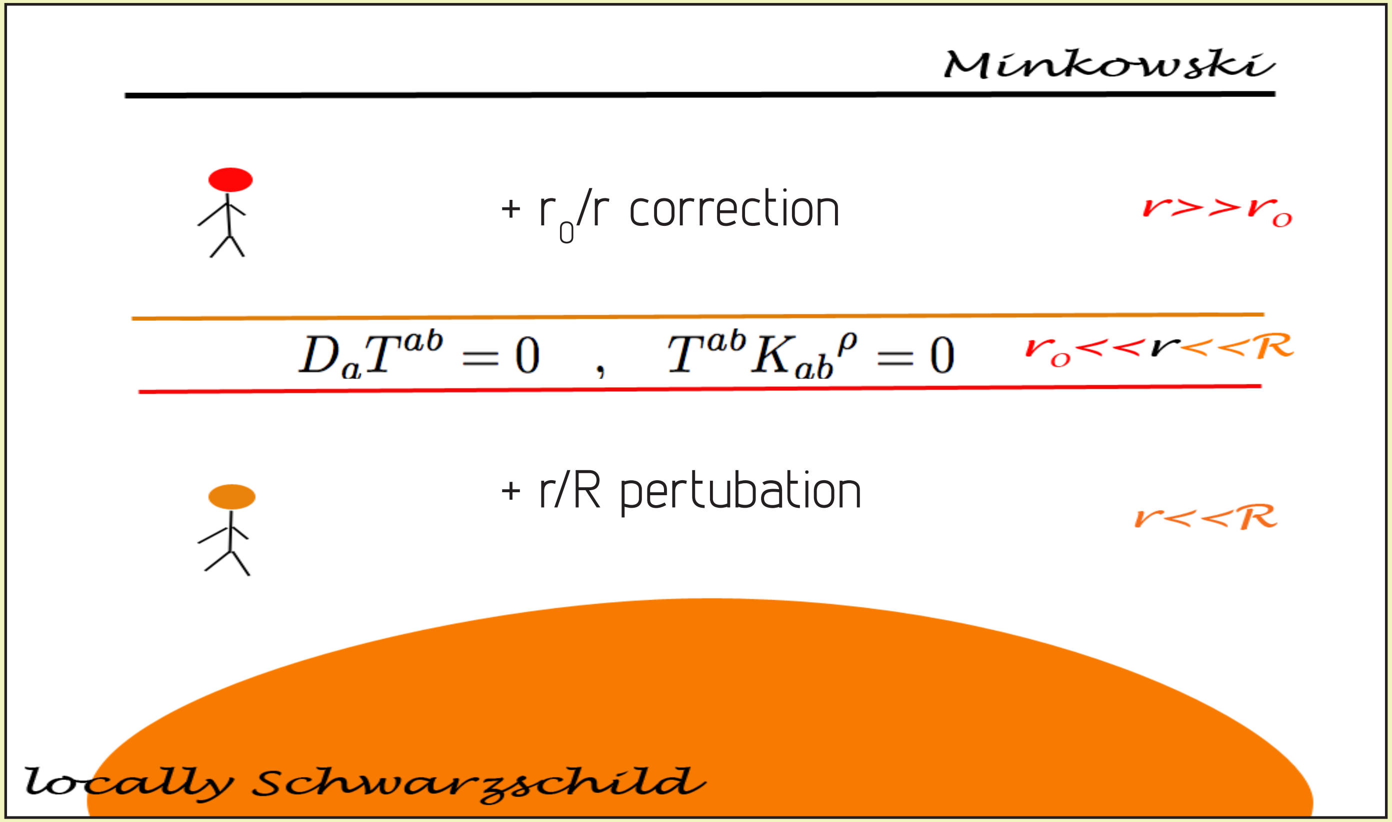
The Blackfold Approach (IV): Method



The Blackfold Approach (IV): Method



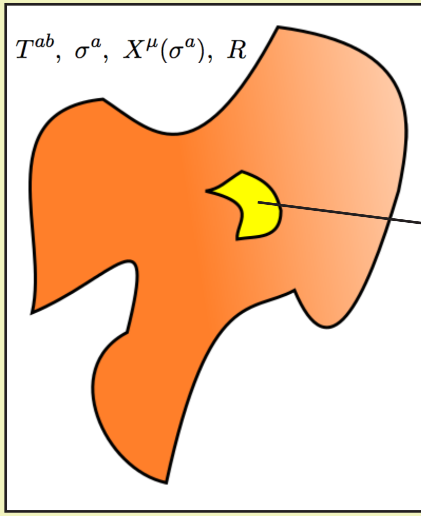
The Blackfold Approach (IV): Method



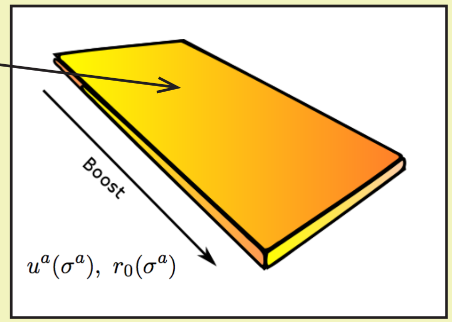
Emparan, Harmark Niarchos, Obers, Rodriguez '07

Camps, Emparan '12

The Blackfold Approach (IV): Method



$$\frac{r_0}{R} << 1$$



$$T^{ab} = \left((\epsilon + P)u^a u^b + P\gamma^{ab}\right)\delta_{\perp}^{n+2}(x^\rho - X^\rho(\sigma^a))$$

Emparan, Harmark Niarchos, Obers '10

$$\epsilon = -(n+1)P$$

If the object acts as a source to Einstein's equations then it must satisfy:

$$\nabla_\mu T^{\mu\nu} = 0$$

Projecting this in directions parallel and orthogonal to the worldvolume leads to:

$$D_a T^{ab} = 0$$

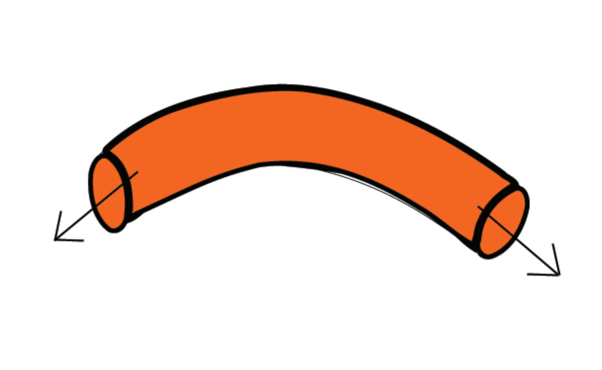
$$T^{ab} {K_{ab}}^{\rho} = 0$$

The Blackfold Approach (V): Elasticity

Integrate out the thickness:

$$F[X^\mu] = \frac{1}{2}\int_{vol} dV\, \sigma^{ab} U_{ab} \qquad \sigma^{ab} K_{ab}{}^{\rho} = 0$$

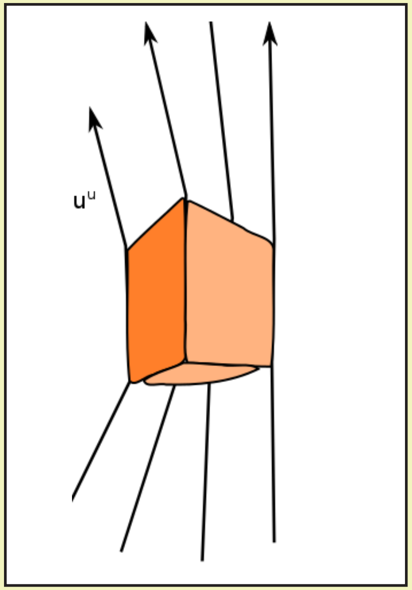
Landau & Lifshitz



A real 4D material can be described by a local particle density:

$$T^{ab} = \epsilon u^a u^b + \sigma^{ab}$$

$$\sigma^{ab} u_b = 0 \qquad \sigma^{ab} = P\left(\gamma^{ab} + u^a u^b\right)$$



The Blackfold Approach (V): Elasticity

We define the strain tensor as:

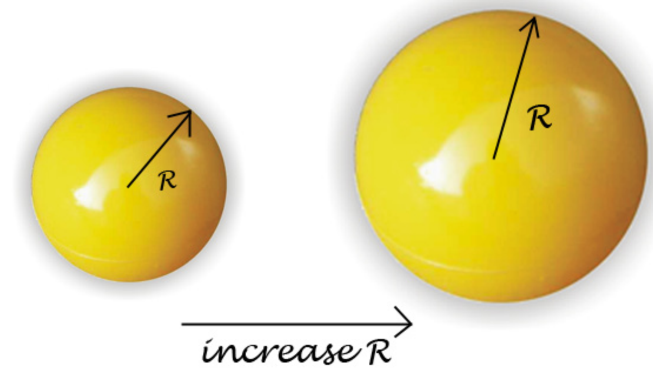
$$U_{ab} = -\frac{1}{2}\left(\gamma_{ab} - \bar{\gamma}_{ab}\right) \qquad dU_{ab} = N_\rho K_{ab}{}^\rho$$

Armas, Camps, Harmark, Obers '11

Hence the stress:

$$T^{ab} = \rho u^a u^b + \sigma^{ab}$$

$$\rho = \epsilon + P \quad , \quad \sigma^{ab} = P\gamma^{ab}$$



INTRINSIC FLUID DYNAMICS IMPLIES EXTRINSIC ELASTIC DYNAMICS

$$d\epsilon = \mathcal{T} ds \qquad \epsilon + P = \mathcal{T} s$$

$$u^a = \frac{k^a}{|k|} \quad , \quad k = \partial_t + \Omega_i \partial_{\phi_i}$$

$$s d\mathcal{T} = \sigma^{ab} dU_{ab} \quad dP = -P d\mathcal{V}$$

$$d\rho = \mathcal{T} ds + \sigma^{ab} dU_{ab}$$

Armas, Obers '12

The Blackfold Approach (V): Elasticity

Fluid dynamics fixes the properties:

$$T \quad , \quad \Omega_i$$

Elastic dynamics fixes the properties:

$$K \quad , \quad E^{abcd}$$

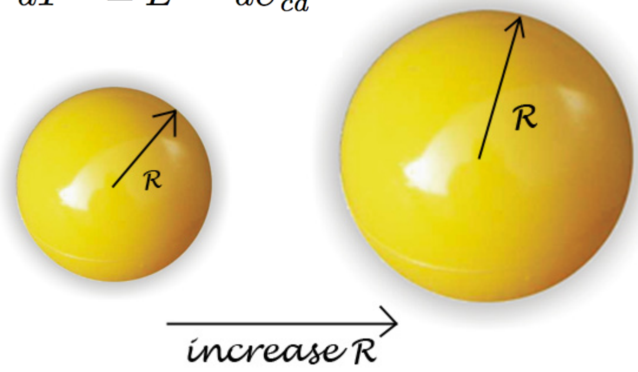
Modulus of hydrostatic compression:

$$K = \left(\frac{\partial P}{\partial \mathcal{V}}\right)_T = -P$$

Elasticity tensor:

$$E^{abcd} = 2\left(P\gamma^{a(c}\gamma^{d)b} - \left(\frac{\partial P}{\partial \gamma_{ab}}\right)\gamma^{cd} - 2\left(\frac{\partial^2 P}{\partial \gamma_{ab}\partial \gamma_{cd}}\right)\right)$$

$$dT^{ab} = E^{abcd} dU_{cd}$$



For neutral blackfolds:

$$P(\gamma_{ab}) = \left(\frac{n}{4\pi T}\right)|-\gamma_{ab}k^a k^b|^{\frac{n}{2}}$$

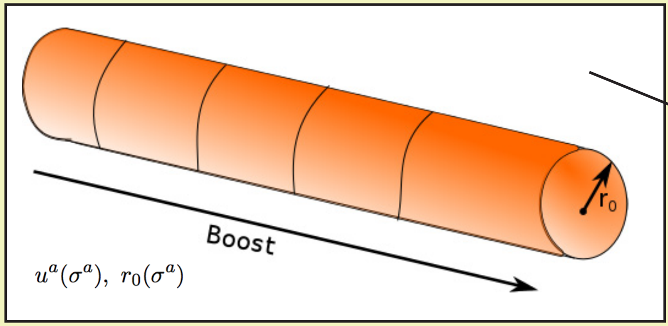
$$T^{ab} = 2\left(\frac{\partial P}{\partial \gamma_{ab}}\right) + \sigma^{ab}$$

$$E^{abcd} = P\left(2\gamma^{a(c}\gamma^{d)b} + n u^a u^b \gamma^{cd} - n(n-2)u^a u^b u^c u^d\right)$$

CHAPTER 3:

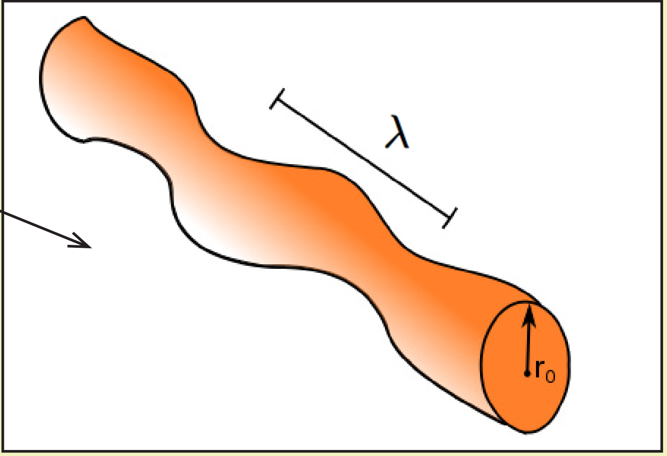
BLACK BRANES AS VISCOUS FLUIDS AND ELASTIC SOLIDS

Material Science (I): Viscosities



$$\frac{r_0}{\lambda} << 1$$

Small perturbation of intrinsic parameters



To first order in a derivative expansion:

$$T^{ab} = \left(\epsilon u^a u^b + P P^{ab} - 2\eta\sigma^{ab} - \zeta\vartheta P^{ab}\right)\delta^{n+2}(x^\rho - X^\rho)$$

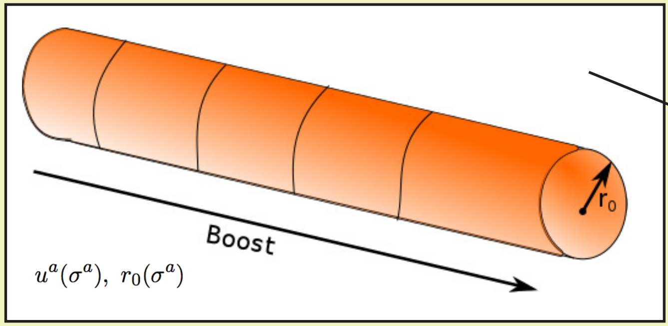
$$\eta = \frac{\Omega_{(n+1)} r_0^{n+1}}{16\pi G},\ \zeta = 2\eta\left(\frac{1}{p} + \frac{1}{n+1}\right)$$

Camps, Emparan, Haddad '10

The perturbed brane must satisfy:

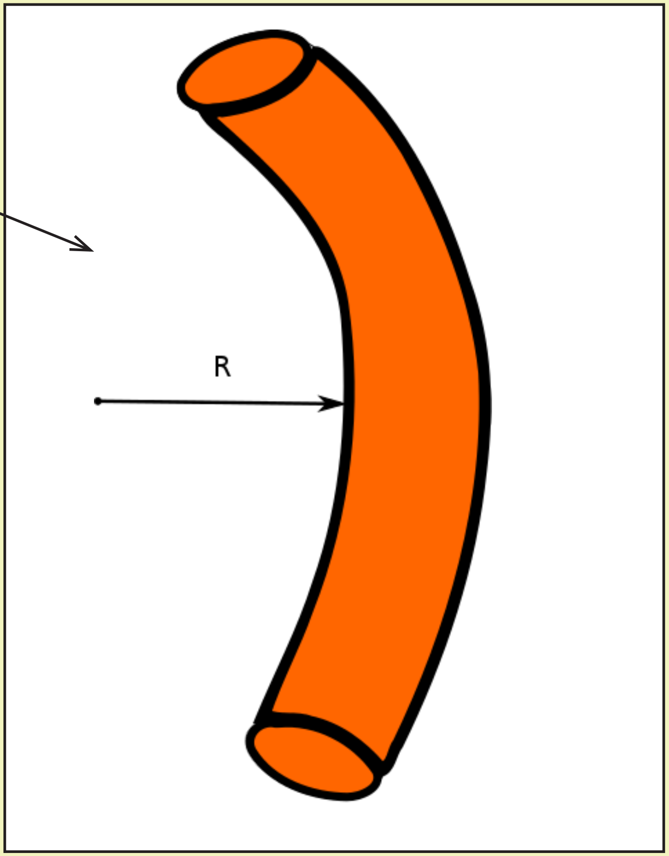
$$D_a T^{ab} = 0$$

Material Science (II): Young Modulus



$\frac{r_0}{R} << 1$

Small perturbation of extrinsic parameters



The equations of motion are modified to:

$$D_a T^{ab} = 0 \quad T^{ab} K_{ab}{}^{\rho} = 0$$

$$T^{ab} = \epsilon u^a u^b + P P^{ab} + d^{ab\rho} K_\rho$$

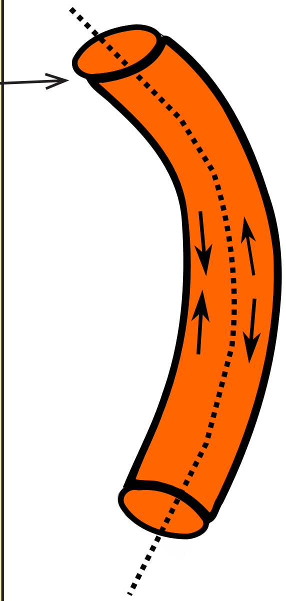
Armas, Camps, Harmark, Obers '11

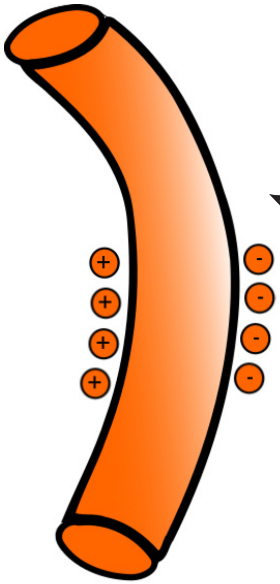
Material Science (II): Young Modulus

Bent rods develop a bending moment:

$$d_{ab}{}^{\hat{\rho}} = I\ Y_{ab}{}^{cd} K_{cd}{}^{\hat{\rho}}$$

Armas, Camps, Harmark, Obers '11

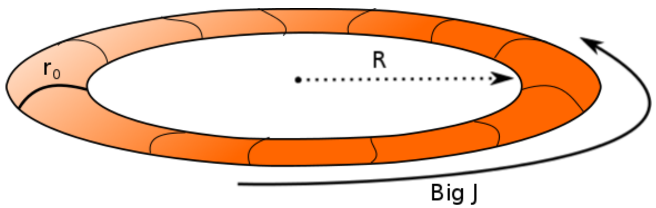




A p-brane develops dipoles of stress and requires a dipole contribution to stress-energy tensor:

$$T^{ab} = \left(\rho u^a u^b + \sigma^{ab} - d_{ab}{}^{\rho}\partial_\rho\right) \delta_\perp^{n+2}(x^\mu - X^\mu)$$

Material Science (II): Young Modulus



Emparan, Niarchos, Harmark, Obers, Rodriguez '07

The Young Modulus for Black Strings is:

$$Y^{ttzz} = Y^{zztt} = -\frac{\Omega_{(n)}(n+2)}{16\pi G r_0^2}(n^2+3n+4)\xi(n)$$

$$Y^{tztt} = Y^{tzzz} = 0$$

$$Y^{tttt} = Y^{zzzz} = \frac{\Omega_{(n)}(n+2)(n+4)}{16\pi G r_0^2}(3n+4)\xi(n)$$

Armas, Camps, Harmark, Obers '11

We can measure the dipole from an approximate analytic solution, in general we find the relation between stress and strain (Hookean Idealization):

$$d^{ab\rho}K_\rho = Y^{abcd}U_{cd}$$

$$U_{cd} = K_{cd}{}^{\rho}K_\rho$$

which can be measured in the linearized gravity regime far away from the source:

$$\bar{h}^{(D)}_{\mu\nu} = \frac{16\pi G\cos\theta}{\Omega_{(n+1)}r^{n+1}}d_{\mu\nu}{}^{\hat{\rho}}$$

Material Science (III): Elasticty tensor vs Young Modulus

Elasticity Tensor:

$$E^{abcd} = P\left(2\gamma^{a(c}\gamma^{d)b} + n u^a u^b \gamma^{cd} - n(n-2) u^a u^b u^c u^d\right)$$

Armas, Obers '12

Young Modulus:

$$\tilde{Y}^{abcd} = -P\, f(n)\left(\frac{1}{n+2}\gamma^{a(c}\gamma^{d)b} + 2u^{(a}\gamma^{b)(c}u^{d)} + \frac{3n+4}{n+2}u^a u^b u^c u^d\right)$$

Camps, Emparan '12

Material Science (IV): Piezoelectric Moduli

Charged dilatonic bent strings behave like piezoelectrics, developing electric dipoles:

$$J^a = \left(Qu^a - p^{a\rho}\partial_\rho\right)\delta_\perp^{n+2}(x^\mu - X^\mu)$$

assume:

$$p^{a\rho} = \tilde{k}^{abc}K_{cd}{}^\rho$$

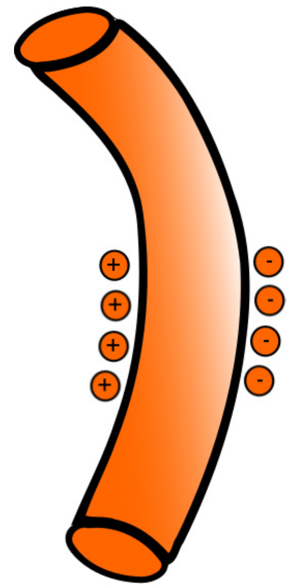
use:

$$\Box A_a = 16\pi G p_a{}^\rho \partial_\rho \delta(n+2)(r)$$

hence piezoelectric moduli:

$$\tilde{k}^{tzz} = -\frac{2(n+2)}{n+1}\xi(n) r_0^2 J^t_{(0)} \quad , \quad \tilde{k}^{zzz} = 0$$

Armas, Gath, Obers



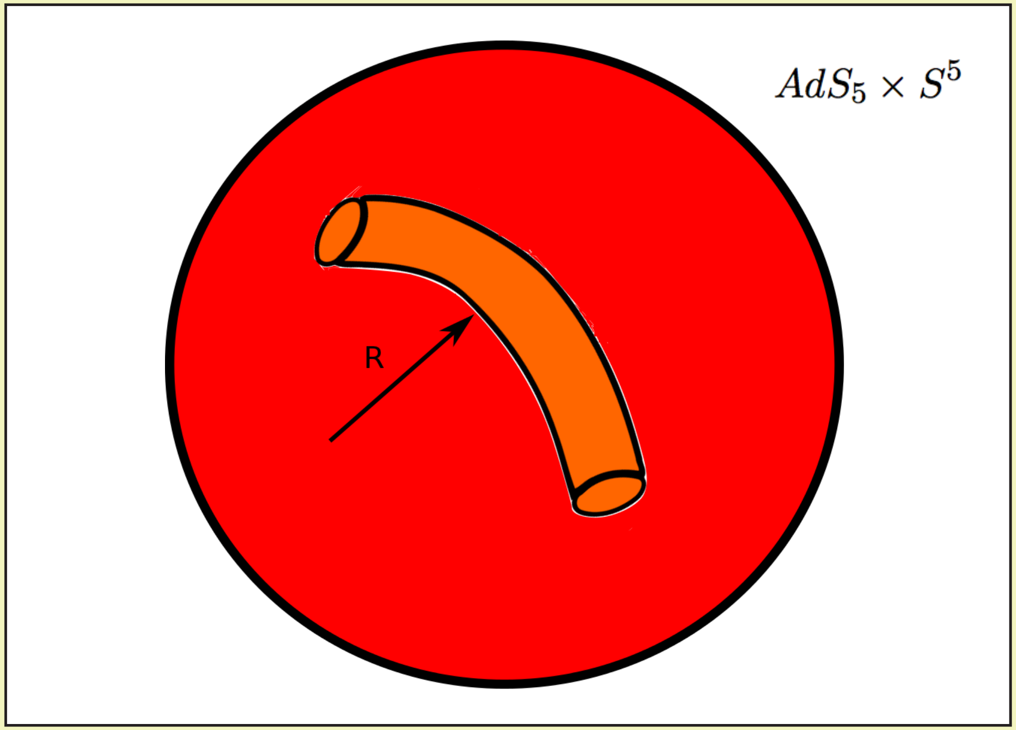
CHAPTER 4:

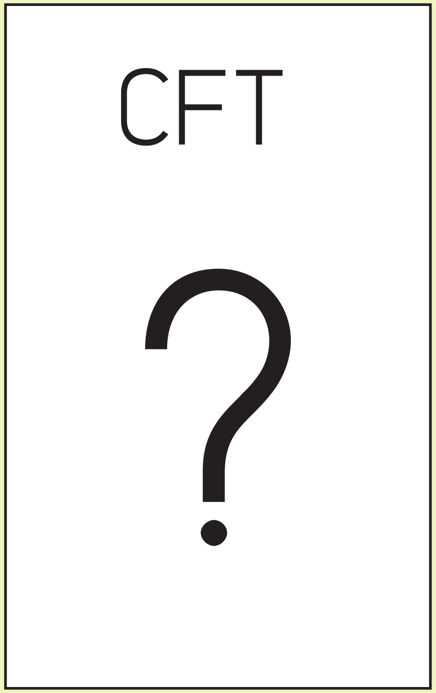
THE END

A Comment and food for thought

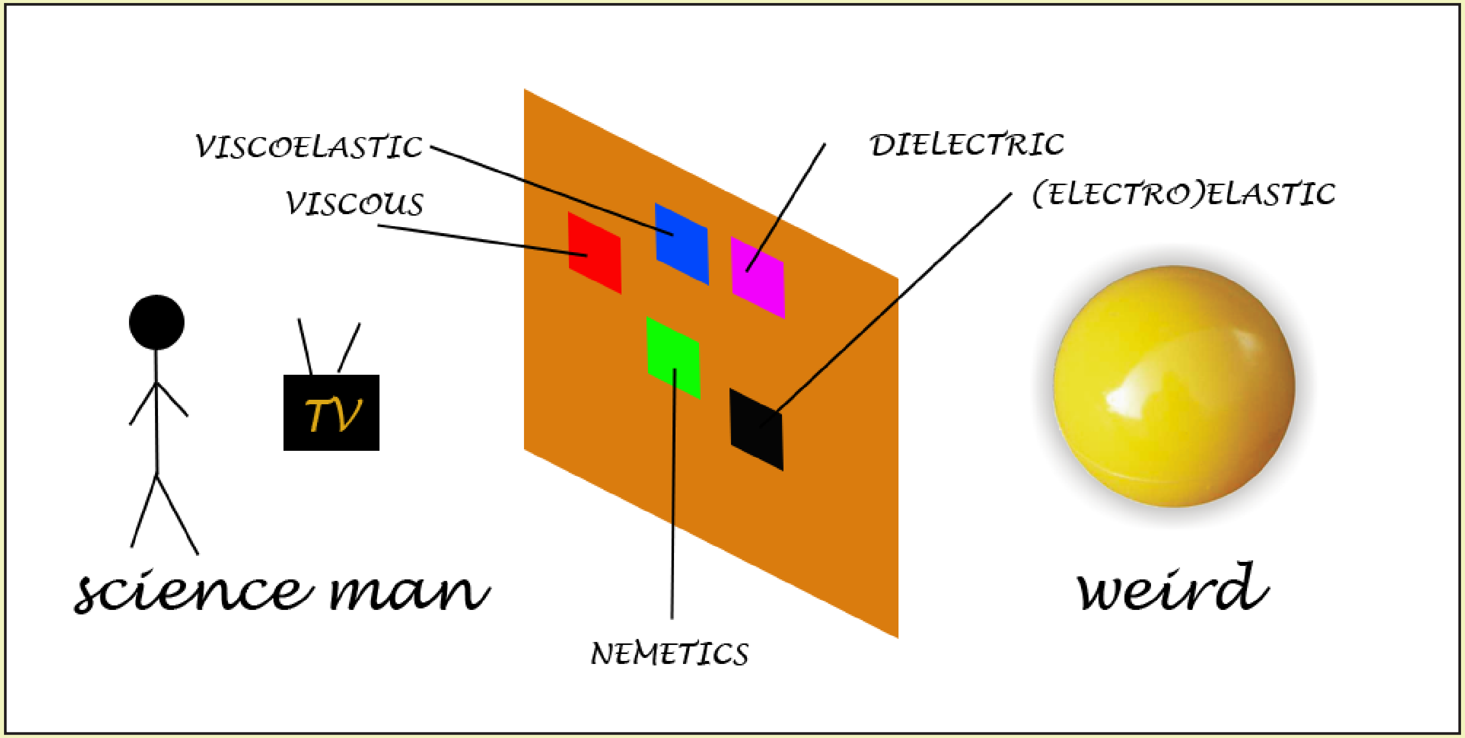
- A rather unusual use of jargon when speaking about black hole physics : elasticity, young modulus, stress, strain, piezo moduli, etc.

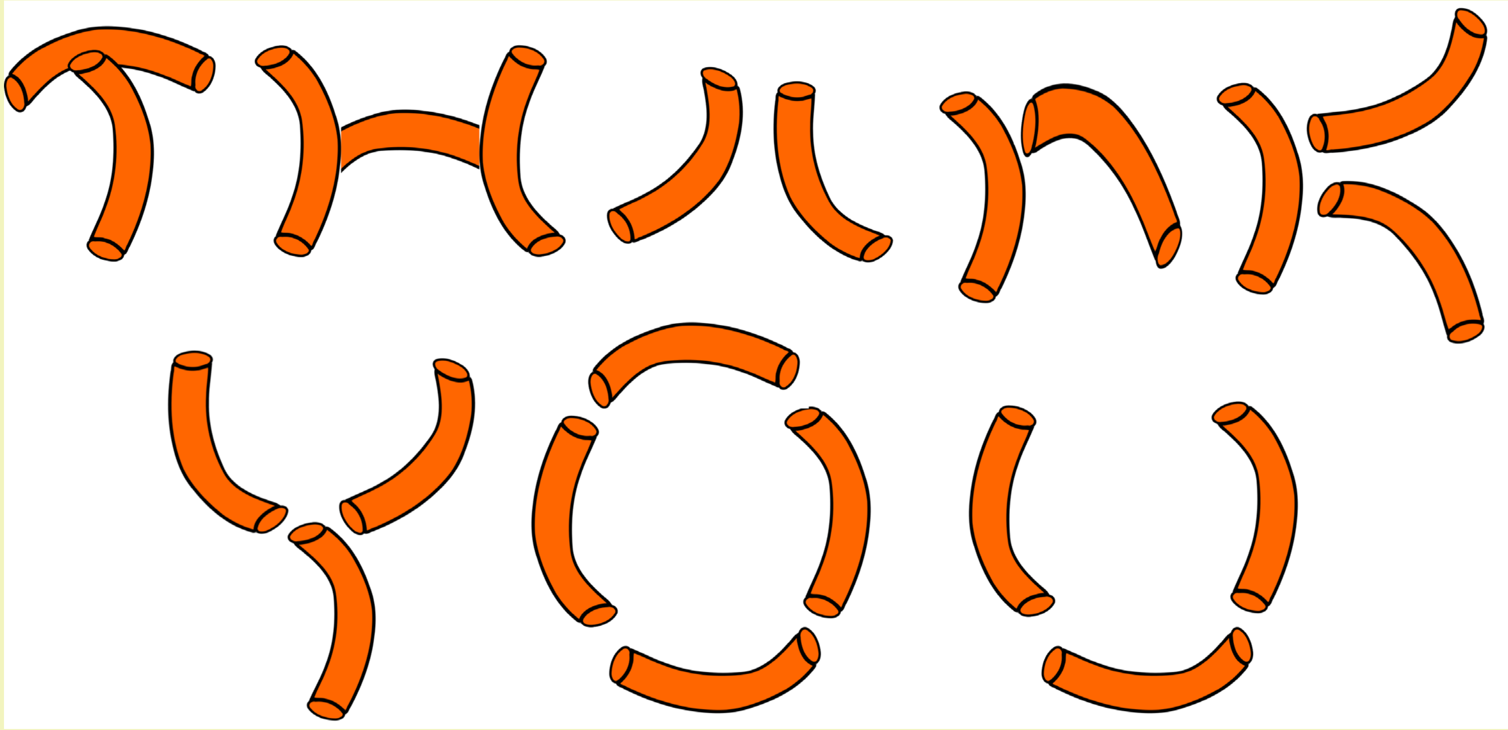
- Developing a theory of elasticity for black materials.





A Comment and food for thought




THANK
YOU