Probing Higher Spin Black Holes

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Introduction

- Higher spin gravity is an (apparently) consistent theory that sits “midway” between low energy field theory and string theory (Vasiliev)
  - infinite towers of fields
  - nonlocal dynamics
  - huge enlargement of gauge symmetry

- Extra symmetry provides soluble examples of AdS/CFT correspondence (Klebanov/Polyakov; Gaberdiel/Gopakumar; …)
  - can we gain insight into the big problems of quantum gravity?

3D HS Gravity

- Extension of Chern-Simons formulation of ordinary gravity with $\Lambda < 0$ (Achucarro, Townsend / Witten)

$$\text{vielbein} \quad e^a_\mu \ , \qquad \text{spin connection} \quad \omega^a_\mu = \tfrac{1}{2}\epsilon^a{}_{bc}\omega^{bc}_\mu$$

SL(2,R) x SL(2,R) gauge fields $A = (\omega^a + \frac{1}{l}e^a)J_a \ , \quad \overline{A} = (\omega^a - \frac{1}{l}e^a)J_a$

$$[J_a, J_b] = \epsilon_{ab}{}^c J_c$$

$$R_{\mu\nu} = \frac{1}{l^2}g_{\mu\nu} \quad \Longleftrightarrow \quad \begin{array}{l} dA + A \wedge A = 0 \\ d\overline{A} + \overline{A} \wedge \overline{A} = 0 \end{array}$$

$$S = \frac{k}{4\pi}\int \mathrm{Tr}(AdA + \tfrac{2}{3}A^3) - \frac{k}{4\pi}\int \mathrm{Tr}(\overline{A}d\overline{A} + \tfrac{2}{3}\overline{A}^3) \qquad k = \frac{l}{4G} = \frac{c}{6}$$

- Replacing SL(2) by a larger algebra that contains SL(2) yields a higher spin gravity theory
- Ordinary 3D gravity is a consistent subsector

- example: SL(3) describes ordinary gravity coupled to a massless spin-3 field (Campoleonii et. al.)

$$g_{\mu\nu} \sim \mathrm{Tr}(\mathrm{e}_\mu \mathrm{e}_\nu) \ , \quad \varphi_{\alpha\beta\gamma} \sim \mathrm{Tr}(\mathrm{e}_\alpha \mathrm{e}_\beta \mathrm{e}_\gamma)$$

$$e \sim A - \overline{A}$$

Gauge symmetry includes coord. transformations under which $g_{\mu\nu}$ and $\varphi_{\alpha\beta\gamma}$ transform as tensors, as well as spin-3 gauge transformations under which $g_{\mu\nu}$ transforms in novel way

e.g. Ricci scalar not gauge invariant

- Just as SL(2) gravity has asymptotic Virasoro symmetry, HS theories have asymptotic W-algebras containing higher spin currents

  (Henneaux/Rey; Campoleoni et. al.)

- Pure HS theory contains no propagating degrees of freedom.

Matter

- Attempt to introduce scalar matter via equation

$$dC + AC - C\overline{A} = 0$$

  - For gauge group SL(2) (or other finite dimension group) this leads to an unfamiliar finite dimensional system
  - For appropriate infinite dimensional gauge group get Klein-Gordon equation when evaluated in AdS

- Above construction appears in linearized version of full nonlinear Vasiliev equations (Prokushkin, Vasiliev)
  - Mass of scalar, and its interactions, fixed by gauge symmetry

Higher spin gauge algebra: hs(λ)

- Introduce $y_{1,2}$ and the Moyal product:

$$f(y_\alpha) * g(y_\beta) = e^{i\epsilon^{\alpha\beta}\partial_\alpha \partial'_\beta} f(y_\alpha) g(y'_\beta)\Big|_{y'=y}$$

$$[y_1, y_2]_* = 2i$$

- Elements of hs(1/2) are symmetric, even degree polynomials $V^s_m \sim y_1^{s+m-1} y_2^{s-m-1}$
  - SL(2) generated by:

$$L_1 = -\frac{i}{4} y_1^2 \ , \quad L_0 = -\frac{i}{4} y_1 y_2 \ , \quad L_{-1} = -\frac{i}{4} y_2^2$$

- General case of hs(λ) obtained from deformed commutator: $[y_1, y_2]_* = 2i(1 + \nu k), \quad \lambda = (1+\nu)/2$

hs(N) = SL(N)

Scalar wave equation

- AdS:

$$A = e^{\rho} V_1^2 dz + V_0^2 d\rho$$
$$\overline{A} = e^{\rho} V_{-1}^2 d\overline{z} - V_0^2 d\rho$$
$$\Rightarrow \quad ds^2 = d\rho^2 + e^{2\rho} dz d\overline{z}$$

$$V_{\pm1}^2 = L_{\pm1} \ , \quad V_0^2 = L_0$$

- Scalar equation: $dC + AC - C\overline{A} = 0$

$$C = \sum_{ms} C_m^s(x^\mu) V_m^s$$

- plug in and solve recursively. Lowest component is constrained to obey:

$$[\nabla^2 - (\lambda^2 - 1)] C_0^1 = 0$$

KG equation with: $m^2 = \lambda^2 - 1$

- Higher spin deformation of background leads to higher derivative scalar wave equation

e.g.:

$$A = e^{\rho} V_1^2 dz + V_0^2 d\rho - \eta e^{2\rho} V_2^3 d\bar{z}$$

$$\overline{A} = e^{\rho} V_{-1}^2 d\bar{z} - V_0^2 d\rho$$

$$\Rightarrow \quad ds^2 = d\rho^2 + e^{2\rho} dz d\bar{z} \ , \quad \varphi_{\bar{z}\bar{z}\bar{z}} \sim \eta e^{4\rho}$$

  - scalar equation: $[\nabla^2 - 4\eta e^{-2\rho}\partial^3 - (\lambda^2 - 1)]C_0^1 = 0$

- Solution of generalized wave equation leads to AdS 3-point correlation functions:

$$\langle \mathcal{O}_{\pm}(z_1)\overline{\mathcal{O}}_{\pm}(z_2)J^{(s)}(z_3)\rangle = \frac{(-1)^{s-1}}{2\pi}\frac{\Gamma(s)^2}{\Gamma(2s-1)}\frac{\Gamma(s\pm\lambda)}{\Gamma(1\pm\lambda)}\left(\frac{z_{12}}{z_{13}z_{23}}\right)^s \langle \mathcal{O}_{\pm}(z_1)\overline{\mathcal{O}}_{\pm}(z_2)\rangle$$

$\mathcal{O}_- \sim$ standard quantization

$\mathcal{O}_+ \sim$ alternate quantization

(Ammon, P.K., Perlmutter)

Duality

- Gaberdiel and Gopakumar conjecture duality between bulk hs[λ] theory and W_N minimal model CFT

$$\frac{SU(N)_k \oplus SU(N)_1}{SU(N)_{k+1}} \qquad k, N \to \infty \ , \qquad \lambda = \frac{N}{k+N} \quad \text{fixed}$$

$$c \sim N(1-\lambda^2)$$

- various checks based on symmetries, spectrum, etc. e.g.:

$$\langle \mathcal{O}_\pm(z_1)\overline{\mathcal{O}}_\pm(z_2)J^{(s)}(z_3)\rangle = \frac{(-1)^{s-1}}{2\pi}\frac{\Gamma(s)^2}{\Gamma(2s-1)}\frac{\Gamma(s\pm\lambda)}{\Gamma(1\pm\lambda)}\left(\frac{z_{12}}{z_{13}z_{23}}\right)^s \langle \mathcal{O}_\pm(z_1)\overline{\mathcal{O}}_\pm(z_2)\rangle$$

- much of the challenge in proving duality involves defining the bulk theory at the fully quantum, non-perturbative level

Black Holes

- Black hole solutions carrying higher spin charge have been constructed (Gutperle, P.K; P.K., Perlmutter)

- Solutions contribute to generalized partition functions $Z(\beta,\mu_i) = \mathrm{Tr}\left[e^{-\beta(H+\sum_i \mu_i Q_i)}\right]$

- Partition function extracted from bulk matches that of dual CFT in high temperature regime

(P.K., Perlmutter; Gaberdiel, Hartman, Jin)

- Main subtlety involves interpretation of spacetime solution. Non-obvious causal structure due to higher spin gauge invariance

Building HS Black Holes

- BTZ:

$$A = (e^\rho L_1 - \tfrac{2\pi}{k} e^{-\rho}\mathcal{L}L_{-1})dx^+ + L_0 d\rho$$
$$\overline{A} = -(e^\rho L_{-1} - \tfrac{2\pi}{k} e^{-\rho}\overline{\mathcal{L}}L_1)dx^- - L_0 d\rho$$

$$\Rightarrow \quad BTZ \quad \mathcal{L} = \frac{M-J}{4\pi} \quad \overline{\mathcal{L}} = \frac{M+J}{4\pi}$$

- Now add in spin-3 chemical potential. Ward identity analysis establishes:

$$A_- \ \sim \ \mu e^{2\rho} W_2 + \cdots$$

chiral spin-3 chemical potential

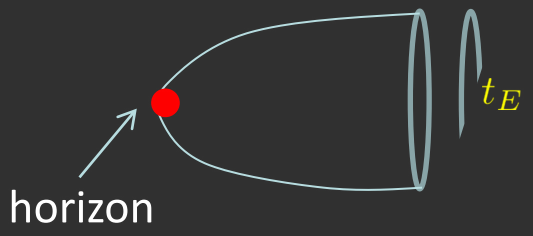
spin-3 generator

- Expect this to induce spin-3 charge: $A_+ \sim e^{-2\rho}\mathcal{W}W_{-2}$

- In hs(λ) case expect infinite number of charges to be induced, due to nonlinear symmetry algebra

Smoothness conditions

- ordinary gravity: relation between (M,Q) and their conjugate potentials (T,μ) fixed by smoothness at Euclidean horizon



- Inapplicable for HS black holes, since even existence of event horizon is a gauge dependent statement. Need a new gauge invariant condition

Thermodynamics

$$Z(\tau,\alpha) = \mathrm{Tr}\left[e^{4\pi^2 i(\tau\mathcal{L}+\alpha\mathcal{W})}\right]$$

- holonomy condition implies integrability:

$$\frac{\partial\mathcal{L}}{\partial\alpha} = \frac{\partial\mathcal{W}}{\partial\tau}$$

$$\ln Z(\tau,\alpha) = \frac{i\pi k}{2\tau}\left[1 - \frac{4}{3}\frac{\alpha^2}{\tau^4} + \frac{400}{27}\frac{\lambda^2-7}{\lambda^2-4}\frac{\alpha^4}{\tau^8} - \frac{1600}{27}\frac{5\lambda^4-85\lambda^2+377}{(\lambda^2-4)^2}\frac{\alpha^6}{\tau^{12}} + \cdots\right]$$

valid for: $\tau \to 0, \quad \alpha \to 0, \quad \frac{\alpha}{\tau^2} \quad \text{fixed}$

- first few terms found to agree with CFT computation based on modular properties of partition function

(Gaberdiel, Hartman, Jin)

Holonomy condition

- gauge invariant information captured by holonomies of CS gauge fields

$$H = \mathrm{P}e^{\oint A} \; , \qquad \overline{H} = \mathrm{P}e^{\oint \overline{A}}$$

- We demand that holonomy around Euclidean time circle should be in center of gauge group

- Gives precisely enough information to fix all charges in terms of the potentials, and allows for a consistent thermodynamic interpretation

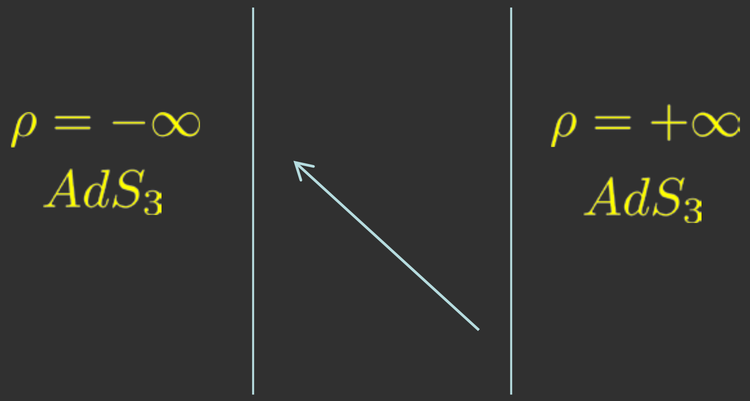
Causal structure

- Metric for non-rotating case takes form

$$ds^2 = d\rho^2 - F(\rho)dt^2 + G(\rho)d\phi^2$$

$F(\rho),\ G(\rho)\ >\ 0$ no event horizon!

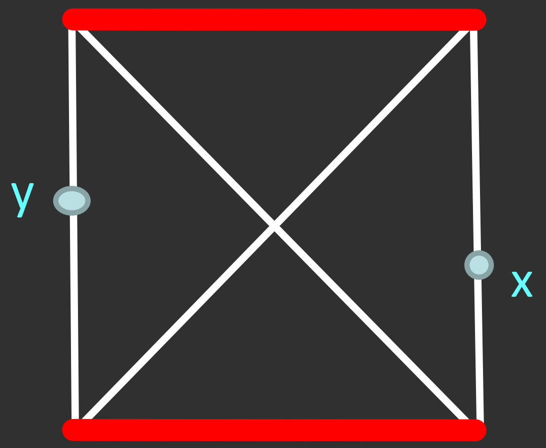
traversable wormhole:



- But when holonomy conditions are obeyed, one can find a true black hole metric somewhere on this gauge orbit (Ammon, Gutperle, P.K., Perlmutter)

Probing causual structure

- Our “black hole” metrics either look like traversable wormholes or black holes, depending on choice of gauge
- To map out physical causal structure we can compute AdS/CFT two-point functions of probe scalars, and look for lightcone singularities



- Black hole causal structure: G(x,y) nonsingular

Scalar two-point function

- Elegant approach to obtaining scalar bulk-boundary propagator: start from propagator at $A = \overline{A} = 0$, then gauge transform to physical solution c.f. (Giombi/Yin)

no spacetime dependence

starting point: $A = \overline{A} = 0 \ , \quad C = c$

gauge transform: $A = g^{-1} \star dg \ , \quad \overline{A} = \overline{g}^{-1} \star d\overline{g}$

$$C = g^{-1} \star c \star \overline{g}$$

bulk solution: $G(x^M) = \mathrm{Tr}(\mathrm{C})$

- purely algebraic procedure

bulk-boundary propagator

- Construct universal A=0 gauge propagator
- highest weight state of hs(λ): $V^s_{m>0} \star c_\pm = 0$

$$V^2_0 \star c_\pm = -\underset{\uparrow}{\left(\tfrac{1\pm\lambda}{2}\right)} c_\pm$$

dimension of CFT operator

λ=1/2: $V^s_m = \left(\frac{-i}{4}\right)^{s-1} y_1^{s+m-1} y_2^{s-m-1}$

highest weight states: $c_- = e^{-iy_1y_2}$, $c_+ = y_1 * e^{-iy_1y_2} * y_2$

These give bulk-boundary propagator with source point at origin along boundary

Constructing physical gauge propagator now becomes exercise in star product gymnastics

Examples

- Pure AdS: $G_-(\rho, z, \overline{z}) = \mathrm{Tr}\left[\mathrm{e}^{\frac{\mathrm{i}}{4}\mathrm{z}\mathrm{e}^{\rho}\mathrm{y}_1^2} * \mathrm{e}^{-\mathrm{i}\mathrm{y}_1\mathrm{y}_2} * \mathrm{e}^{-\frac{\mathrm{i}}{4}\overline{\mathrm{z}}\mathrm{e}^{-\rho}\mathrm{y}_2^2}\right]$

$$= \left(\frac{e^{-\rho}}{e^{-2\rho}+|z|^2}\right)^{1/2}$$

- AdS with spin-3 deformation $\varphi_{\overline{z}\overline{z}\overline{z}} \sim \mu e^{4\rho}$

$$G_-(\rho, z, \overline{z}) = \mathrm{Tr}\left[\mathrm{e}^{\frac{\mathrm{i}}{4}\mathrm{z}\mathrm{e}^{\rho}\mathrm{y}_1^2 - \frac{\mu\overline{\mathrm{z}}\mathrm{e}^{2\rho}}{16}\mathrm{y}_1^4} * \mathrm{e}^{-\mathrm{i}\mathrm{y}_1\mathrm{y}_2} * \mathrm{e}^{-\frac{\mathrm{i}}{4}\overline{\mathrm{z}}\mathrm{e}^{-\rho}\mathrm{y}_2^2}\right]$$

$$= \left(\frac{e^{-\rho}}{e^{-2\rho}+|z|^2}\right)^{1/2} \times \sqrt{\frac{2y}{\pi}}\, e^{y} K_{1/4}(y)$$

$$y = -\frac{(1+z\overline{z}e^{2\rho})^2}{8\mu\overline{z}^3 e^{4\rho}}$$

- yields scalar 2-point function for CFT deformed by dimension (3,0) operator

Black holes: thermal periodicity

- establish general conditions for scalar propagator to exhibit thermal periodicity

define: $$\Lambda = A_z(\rho)z + A_{\bar{z}}(\rho)\bar{z}$$ $$\overline{\Lambda} = \overline{A}_z(\rho)z + \overline{A}_{\bar{z}}(\rho)\bar{z}$$

(gauge parameters for stationary background)

- scalar master field propagator: $C(\rho, z, \bar{z}) \propto e^{-\Lambda} \star c \star e^{\overline{\Lambda}}$
- holonomies: $H = e^{2\pi(\tau A_z + \bar{\tau} A_{\bar{z}})}$; $\overline{H} = e^{2\pi(\tau \overline{A}_z + \bar{\tau}\overline{A}_{\bar{z}})}$

if $\overline{H} = H$ and H lies in center of gauge group, then:

$$C(\rho, z + 2\pi\tau, \bar{z} + 2\pi\bar{\tau}) = C(\rho, z, \bar{z})$$

- for known black holes at λ =1/2: $H = -2\pi i \delta^{(2)}(y)$

propagator for HS black hole

- Black holes of hs(λ) theory only known perturbatively in hs chemical potential
- Similarly, propagator must be worked out perturbatively. Divergences would indicate change in causal structure

first order correction:

$$\frac{i\alpha e^{\rho/2}}{16\tau^2}\Big[\cosh^2(2\overline{Z})\left(-4(Z+\overline{Z})(\cosh(4Z)-2)-\sinh(4Z)\right) \\ +4e^{2\rho}\tau\overline{\tau}\sinh(4\overline{Z})\left(-4(Z+\overline{Z})\sinh(4Z)+2(\cosh(4Z)-1)\right) \\ -(4e^{2\rho}\tau\overline{\tau})^2\sinh^2(2\overline{Z})\left(4(Z+\overline{Z})(\cosh(4Z)+2)-3\sinh(4Z)\right)\Big] \\ \times\left(\cosh(2Z)\cosh(2\overline{Z})+4e^{2\rho}\tau\overline{\tau}\sinh(2Z)\sinh(2\overline{Z})\right)^{-5/2},$$

$$Z=\frac{iz}{4\tau}$$

- only singularities are on light cone for single sided correlator.

Open issues

- subleading corrections to entropy
- phase structure

(Banerjee et. al.

- effect of light states
- black holes formed from collapse?
- higher dimensions