A toy model for the black hole interior

Juan Maldacena

Institute for Advanced Study

The holographic Way Conference

Nordita, Stockholm, Oct. 2012

A toy model for black holes and their interiors

- Simple toy model for thinking about the interior.
- Can be used to rephrase usual apparent paradoxes, information loss, firewall, etc.

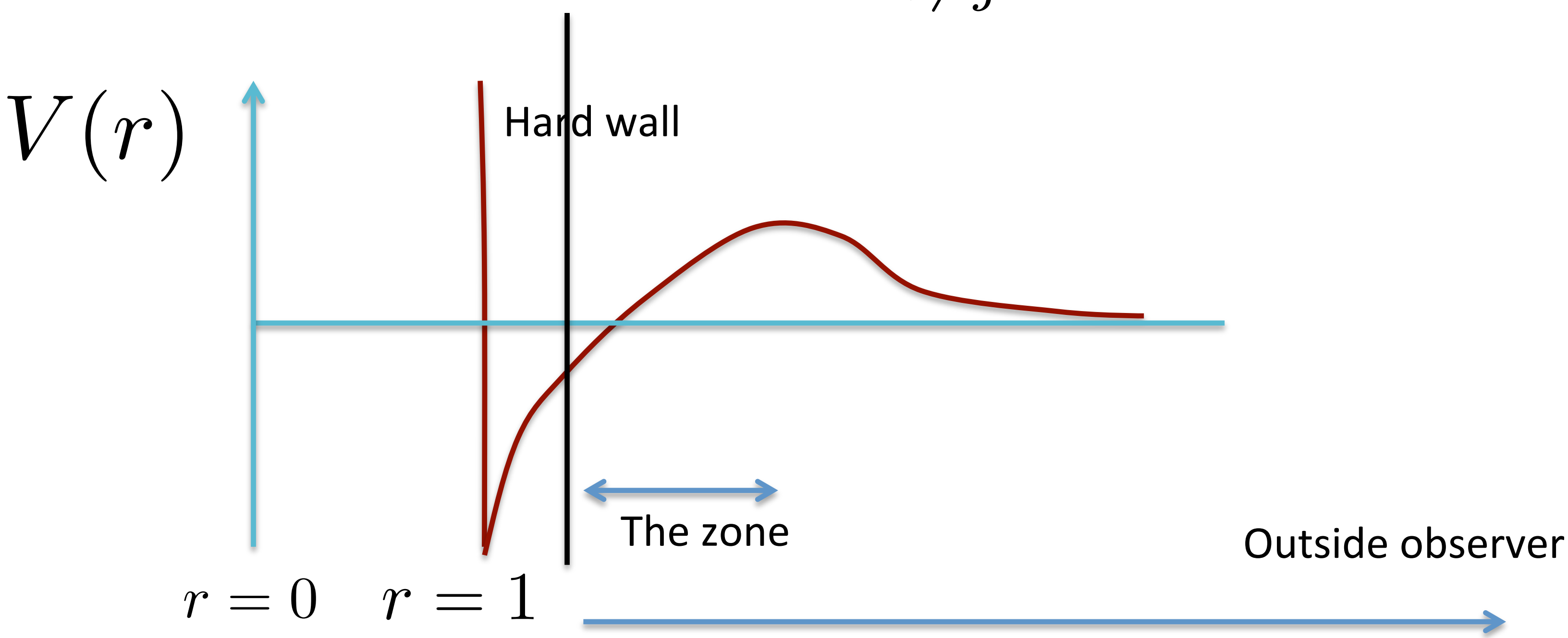
Features

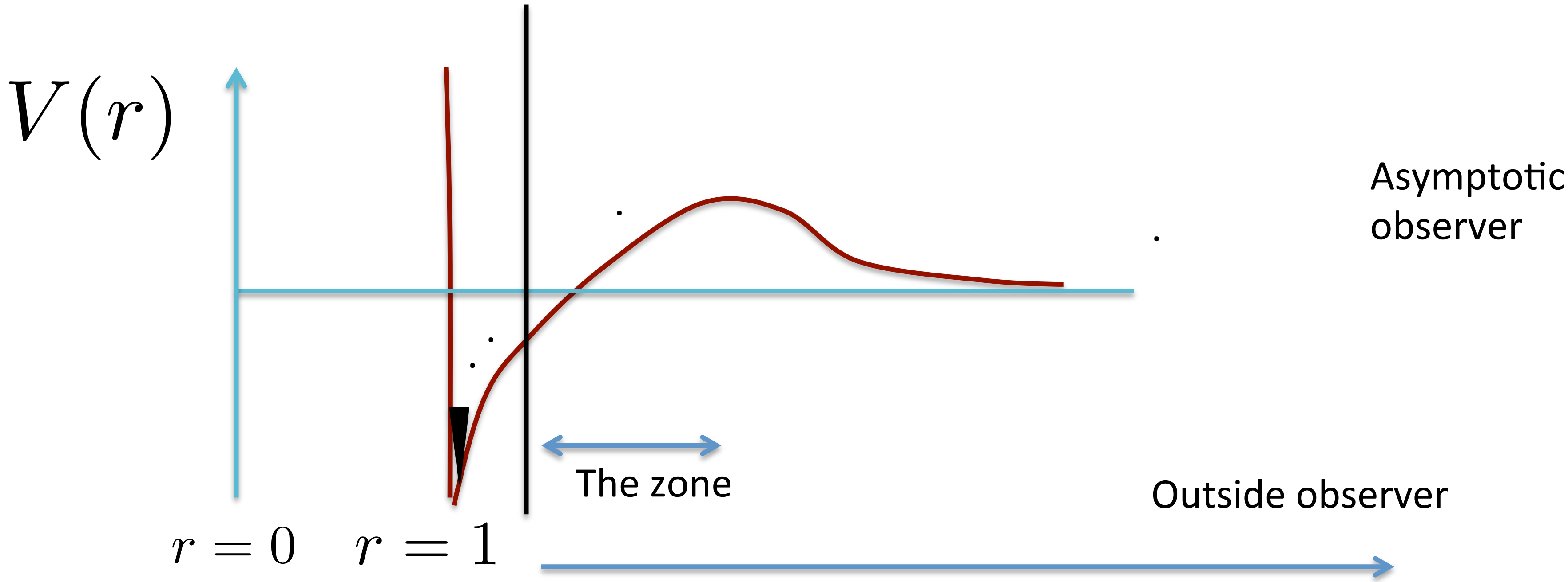
- Outside observer that can make precise measurements & recovers information
- A complete description of the system exists.
- Approximate interior modes.
- Approximate exterior modes (both, say 10 l_{pl} from the horizon)
- Entanglement between exterior and interior modes.
- In some sense, the interior modes are not accessible to the simple measurements made by the exterior observer. At the same time these modes are included in the full description of the system.

The model

- Non – relativistic particles in three spatial dimensions, interacting with an external potential V(r) and an interparticle potential:

$$v = \sum_{i \neq j} \frac{1}{d_{ij}}$$





Most particles are in the very deep region of the potential.
No particle is allowed to go to r<1.

We are at finite temperature, or we are in an excited state of the system.

Some particles are bouncing black and forth in the zone, and now and then some of them escape.

The asymptotic observer sees them coming out as ``Hawking radiation''.

We could adjust the features of the potential V so that as particles evaporate they continue to be able to escape, etc. V(r,N)

The interior

- → Mirror charges

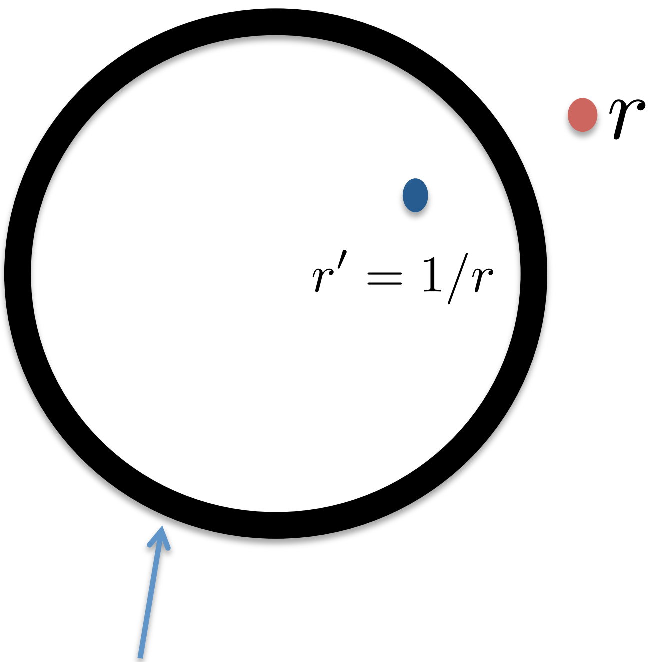
The surface of the sphere behaves like a conductor.

We then have mirror charges.

The mirror charges are an effective description of the surface degrees of freedom.

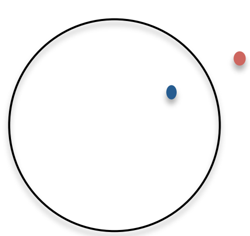
The position of the real charge and the mirror one are correlated. They are entangled. These are two independent degrees of freedom which are entangled. i.e. we can freeze the mirror charge (freezing the charges on the ``horizon'') and move the original charge Independently.

(no dynamical electric field)



Horizon charges

- We do not have an analog of ``the horizon as a smooth surface''.
- However, we do have an analog of an approximate interior and exterior Hilbert spaces.
- These are the Hilbert spaces for the charges and mirror charges.
- Most of the charges are concentrated at the surface (``horizon''). For such charges we cannot talk about mirror charges, but such charges do not appear in the interior or exterior Hilbert space.



- The correlation (in the positions of the charge and its mirror) continues to exist even for a thermal or mixed state.
- Here we are assuming that most of the free energy and entropy of the system comes from the ``Horizon'' charges and that they adjust, even at finite temperature to make sure that the mirror charge is present.

- We need that the free energy cost for moving the mirror charge away from the original charge is large:

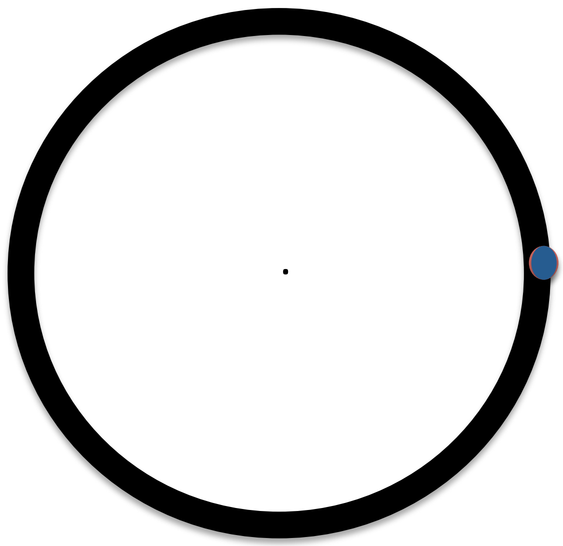
$$e^{-\epsilon/T} \ll 1$$

$$\lambda \ll r_s$$

- Complementarity (quantum membrane paradigm) → mirror charges are part of the ''Horizon'' degrees of freedom, or part of the full microscopic description.

- "UV – issues" when propagate the charge and mirror charge into the past until they are very close to the conductor.

Emission process



Particle falling into black hole

Information loss

Hawking
Mathur's ``theorem''

- In thermal ensemble, charges and mirror charges are correlated (are in a ``pure state'' with respect to their angular positions if we do the thermal average)
- When a charge is emitted → we lose the information about the correlation with the mirror charge. (Correlation in the angular position). If we do not observe the mirror charge!.
- It looks like entropy is increasing, since we are averaging over the position of the mirror particle.
- Solution: The correlation between the particle and its mirror is just that of a charge and the rest of the system (rest of horizon charges). Most microstates contain this correlation. The entropy does not increase.
- Notice that the ``purity'' of the charge and mirror is a feature of having done the thermal average. For particular microstates there is a slight preference for having the pair at particular angles, etc. This is what gets the information out.

Firewalls

Almheiri, Marolf, Polchinski, Sully

(Endorsed by Susskind & others)

- All of their four initial assumptions have a parallel here.
- Most are obvious.
- One requires a translation:

`` Nothing unusual happens for the observer falling in ''

→ translates into →

`` charges and mirror charges continue correlated as we expect "

(Nothing unusual = usual correlations between H_{in} and H_{out} = correlations between charges and mirror charges

Does the mirror continue to work when it is fully thermalized ?
Or it is in a particular typical microstate? Or is ``old” ?

Reasoning as in AMPS you would conclude that charges and mirror charges cease to be correlated for ``old’’ mirrors !.

Physical intuition and free energy arguments say that they are correlated.

Central issue here: the Horizon degrees of freedom and the zone ones, are living in a bigger Hilbert space, with log (dim) > S .

$$e^{-\epsilon/T} \ll 1$$

The states where the charges and their mirror charges are correlated look special to AMPS, but they are the generic situation for the generic microstate.

Conclusions

- This is a simple toy model that captures many conceptual features of black holes.
- But not all of them!
- Many of the arguments that are usually made, which do not use many of the specific features of black holes, have a translation to this simple model.
- Give us an intuitive picture of the phenomenon we are trying to reproduce.

For the future → The true model ?

- Large N (chromodynamics or non-abelian) version of the previous model ! . Color conductor!.

- Note that the charge in the previous model could also be identified with energy in the black hole case → Energy conductor…

- Radial direction as time, etc…

Problem with postulating a firewall

- Once the firewall forms, Hawking computation is invalid: temperature? Entropy ?

- In AdS/CFT we can easily make an ``old black hole'' (after half the initial entropy has been emitted), by coupling the AdS or CFT to an external system.

- In AdS, we expect that it continues to behave thermally even in old age.

Particle falling into black hole

Entanglement in de Sitter space

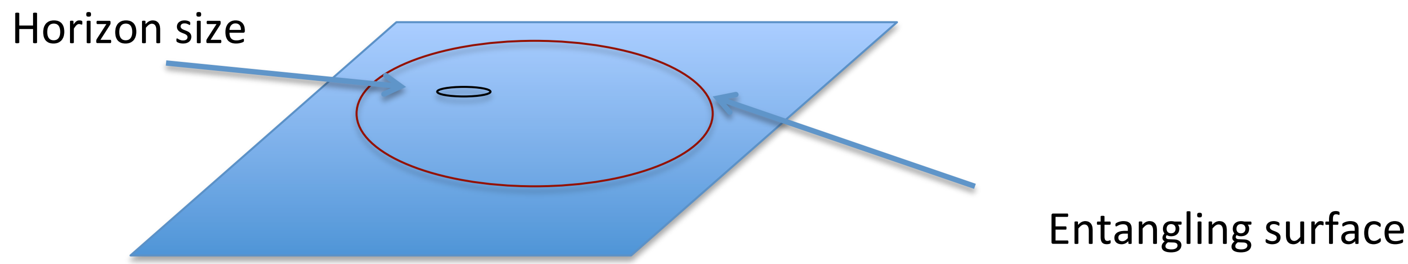
G. Pimentel & JM (to appear)

- Many studies for entanglement entropy in flat space.

- How does entanglement look like in de Sitter ?

- Also a single state : Bunch-Davies/Hartle-Hawking/Chernikov-Tagirov, etc…

- Here: Field theory in de-Sitter, no quantum gravity.



Consider a surface at late time, which is much larger than the Horizon size.

$$S = d_0 \frac{(\text{Area})}{\epsilon^2} + d_1 \log(\epsilon H) + \boxed{S_{\text{UV}-\text{Finite}}}$$

Concentrate on this piece

In flat space and for a massive field

$$S_{UV-finite} \sim (\mathrm{Area})$$

For flat space at finite temperature

$$S_{UV-finite} \sim (\mathrm{Volume})$$

In de-Sitter space

$$S_{UV-finite} \sim c_0(\mathrm{Area}) + c_1 \log{(\mathrm{Area})}$$

$$(c_1 \neq d_1)$$

Why ?

$$S_{UV-finite} \sim c_0(\mathrm{Area}) + c_1 \log(\mathrm{Area})$$

First argument:

- The entaglement entropy is invariant under de-Sitter isometries

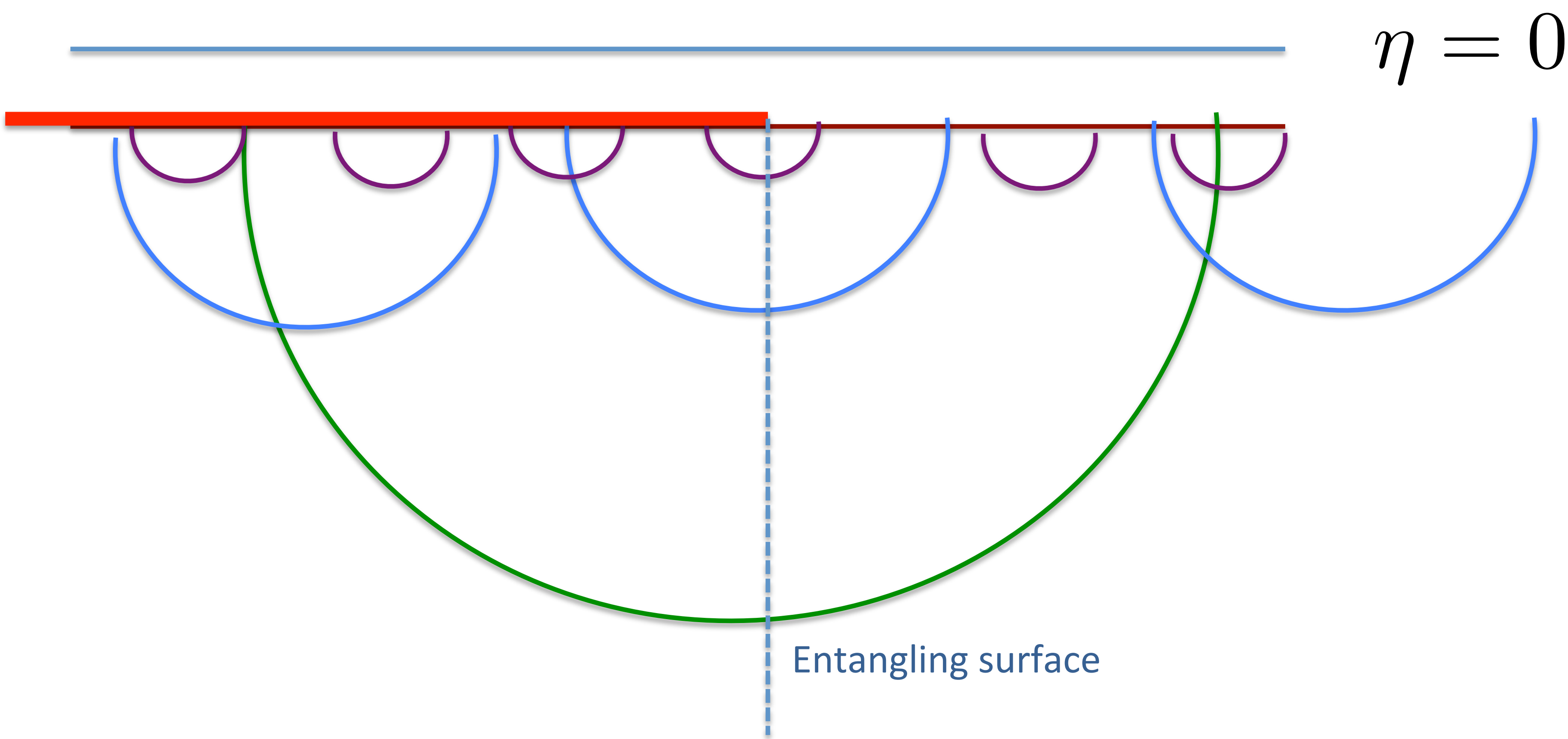
- The wavefunction in de-Sitter becomes essentially time independent outside of the horizon, up to local corrections in comoving coordinates (x):

$$ds^2 = \frac{-d\eta^2 + dx^2}{\eta^2}$$

→ The only time dependence an come through local terms

$$S = c_0 \frac{A_c}{\eta^2} + 2c_1 \log \eta + \mathrm{finite}$$

Long range entanglement contained here



In de Sitter → we create pairs of particles from the vacuum

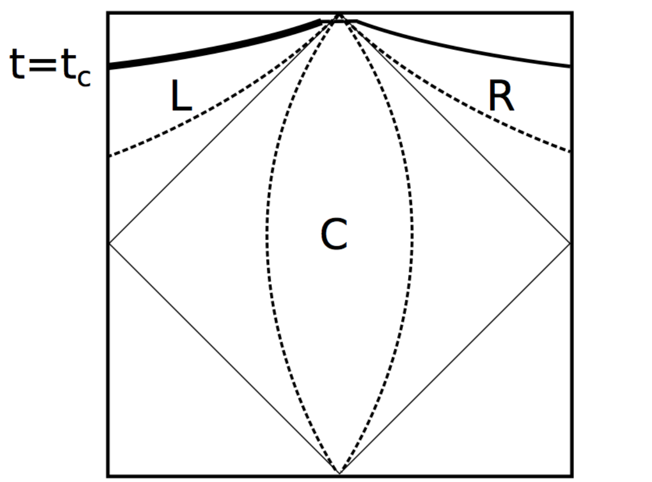
Particles contributing to the entanglement are those pairs which straddle across the entangling surface.

Constant number of pairs per e-folding → answer goes like

$$S_{pairs} \propto N_{eff} \propto \log \eta$$

Computation

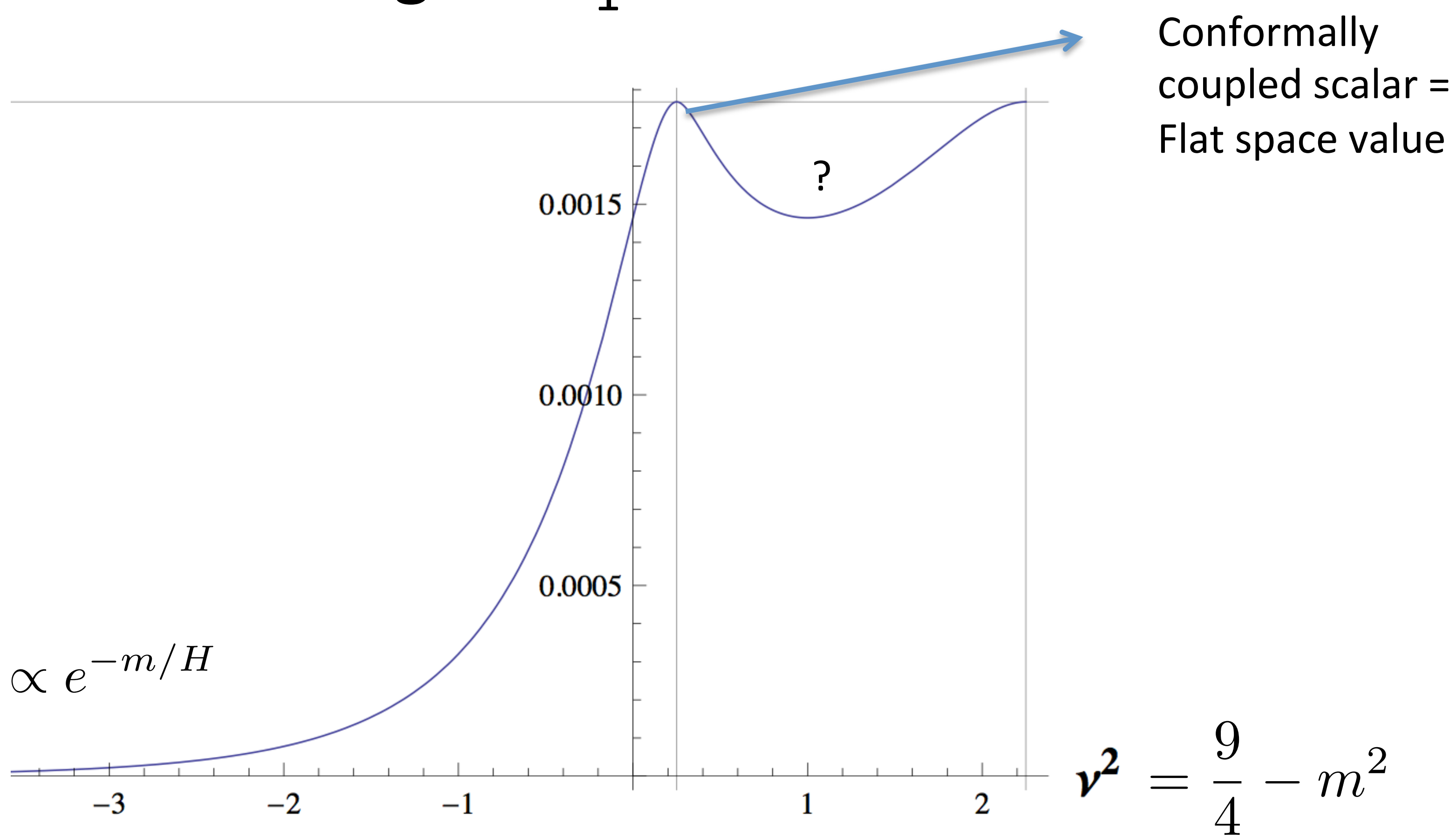
- Free massive scalar field in de Sitter.
- Choose some set of Hyperbolic coordinates



Find the expression for wavefunctions in terms of L and R wavefunctions using

Sasaki, Tanaka, Yamamoto

- Then compute the entanglement entropy and find the following for c_1



Holographic computations

- Use the conjectured Ryu-Takayanagi formula

Ryu, Takayanagi
Hubeny, Rangamani, Takayanagi

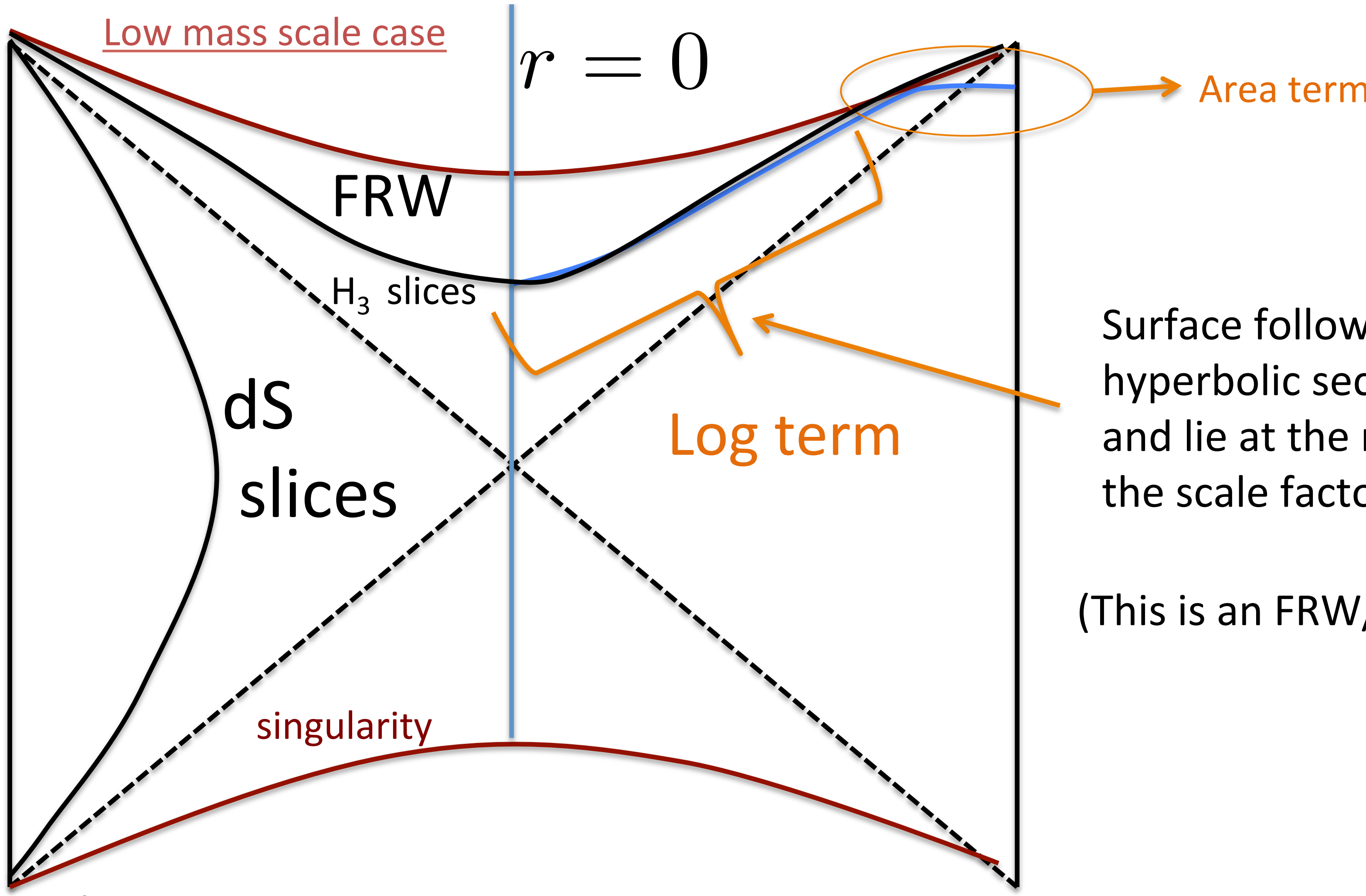
- Entaglement entropy = Area of minimal surface in the bulk.

- Gravity duals of non-conformal theories in de Sitter space.
- Look similar to Coleman de Luccia bubbles in AdS.

Strominger, JM, Hawking
Buchel, Langfelder Walcher,
Aharony, Fabinger, Horowitz, Silverstein,
Valasubramanian, Ross, Cai, Titchener,
Alishahiha, Karch, Tong, Larjo, Simon,
Hirayama, He, Rozali, Hutasoit, Kumar, Rafferty,
Marolf, Rangamani, Van Raamsdonk

- 2 Cases: - With horizon
  - No horizon

(which case we have depends on the parameters. Eg. low mass vs. high mass, m/H)



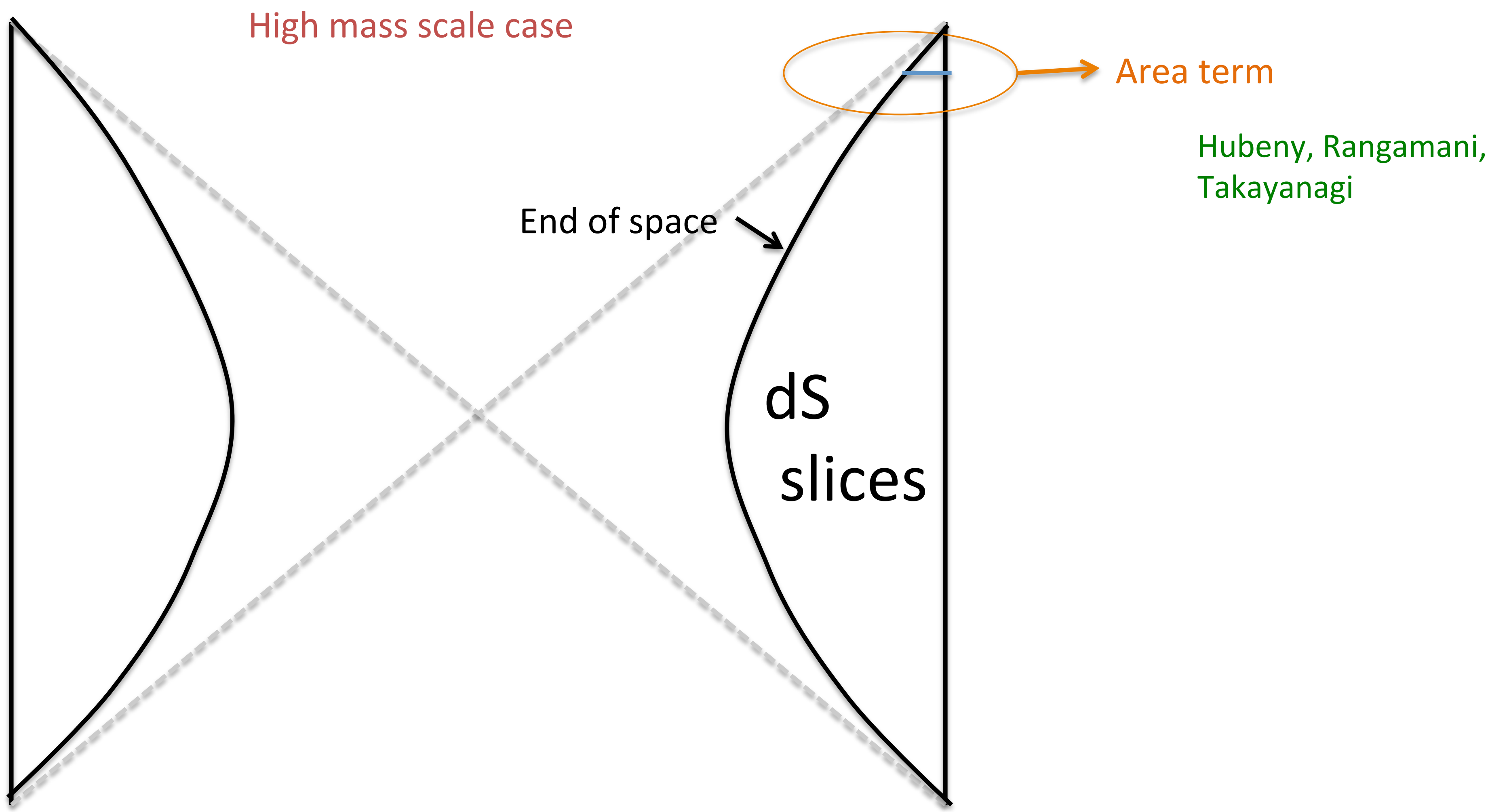
Surface follows the hyperbolic sections of FRW and lie at the maximum of the scale factor

(This is an FRW/CFT duality)

Conclusion:

The long range entanglement is coming from the region behind the bulk horizon (not to be confused with the cosmological horizon)

→ FRW region → related to transhorizon correlations



In this case the log term is zero (to order N^2).

There is an order 1 log term from bulk particles.

Conclusions

- Computed the entanglement entropy in de Sitter. And concentrated on the UV-finite piece.
- This finite piece contains a term proportional to the area and one proportional to the log .
- The log term encodes the entanglement of particles/fields across the surface.
- This entanglement is produced by the particle creation in the expansion of the universe.
- In holographic set ups, the log term is present at order N^2 , only when there is an FRW region.