Superconducting instabilities of R-charged black branes

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Based on [1207.3086] with Benjamin Withers

Motivation

- Hairy black holes are commonplace in Anti-de Sitter space.
- Charged, asymptotically AdS black holes can develop charged scalar hair below a critical temperature $\rightarrow$ holographic superconductors: [Gubser; Hartnoll, Herzog & Horowitz]
- Consistent embeddings in supergravity: [Gubser, Herzog, Pufu & Tesileanu; Gauntlett, Sonner & Wiseman]
- Results of a broader sweep depend sensitively on the truncation: [Denef & Hartnoll; Donos & Gauntlett; Aprile, Roest & Russo]

$\star$ Are there any generic features in the landscape of consistent holographic superconductors?

This talk

Aim: Delve deeper into a wider family of consistent truncations.

Outline:

1. Truncation and the normal phase
2. Fluctuations and marginal modes
3. Open questions

Truncation

- Take the bosonic sector of $\mathcal{N} = 8$ $SO(6)$ gauged supergravity in 5D: [Pernici, Pilch & van Nieuwenhuizen; Gunaydin, Romans & Warner]
- Truncate down to the metric, 15 gauge fields and 20 scalars that saturate the Breitenlohner-Freedman bound.
- Write the adjoint $SO(6)$ indices in matrix form: $(A_\mu)_{ij}$ and T_{ij} with $\det T = 1$.

Action

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left(R + \frac{1}{8} \mathrm{tr}\, T^{-1} F_{\mu\nu} T^{-1} F^{\mu\nu} \right.$$
$$\left. - \frac{1}{4} \mathrm{tr}\, T^{-1} \left(D_\mu T \right) T^{-1} \left(D^\mu T \right) - V \right)$$

$$V = \tfrac{1}{2} \left(2\, \mathrm{tr}\, T^2 - \left(\mathrm{tr}\, T \right)^2 \right), \quad D_\mu T = \nabla_\mu T + [A_\mu, T]$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$$

Normal phase

Three-charge family of asymptotically AdS$_5$ black branes:
[Behrndt, Cvetic & Sabra]

$$T = T_0 \equiv \mathrm{diag}(X_1 \mathbb{1}, X_2 \mathbb{1}, X_3 \mathbb{1})$$
$$A = A_0 dt \equiv \mathrm{diag}(A_1 \sigma, A_2 \sigma, A_3 \sigma) dt$$

with $X_1 X_2 X_3 = 1$, $\sigma \equiv i\sigma_2$ and

$$ds^2 = -H^{-2/3} f dt^2 + \frac{H^{1/3}}{f} dr^2 + r^2 H^{1/3} d\vec{x}_3^2$$
$$X_I = \frac{H^{1/3}}{H_I}, \quad A_I = \sqrt{\frac{r_+^4 H(r_+)}{q_I}} \left(\frac{1}{H_I(r)} - \frac{1}{H_I(r_+)} \right)$$
$$H_I = 1 + \frac{q_I}{r^2}, \quad H = H_1 H_2 H_3, \quad f(r) = -\frac{r_+^4 H(r_+)}{r^2} + r^2 H(r)$$

Multiple black brane branches

Work in the grand canonical ensemble with generic ratios of the chemical potentials

$$\mu_I = \frac{\sqrt{q_I(1+q_1)(1+q_2)(1+q_3)}}{1+q_I}$$

Gibbs potential density

$$\omega \equiv \frac{\Omega}{\mathrm{vol}_3} = -\frac{(1+q_1)(1+q_2)(1+q_3)}{16\pi G_5}, \quad \hat{\omega} \equiv \frac{16\pi G_5\,\omega}{(\mu_1^2+\mu_2^2+\mu_3^2)^2}$$

and temperature

$$T = \frac{2+q_1+q_2+q_3-q_1q_2q_3}{2\pi\sqrt{(1+q_1)(1+q_2)(1+q_3)}}, \quad \hat{T} \equiv \frac{T}{\sqrt{\mu_1^2+\mu_2^2+\mu_3^2}}$$

Local thermodynamic stability

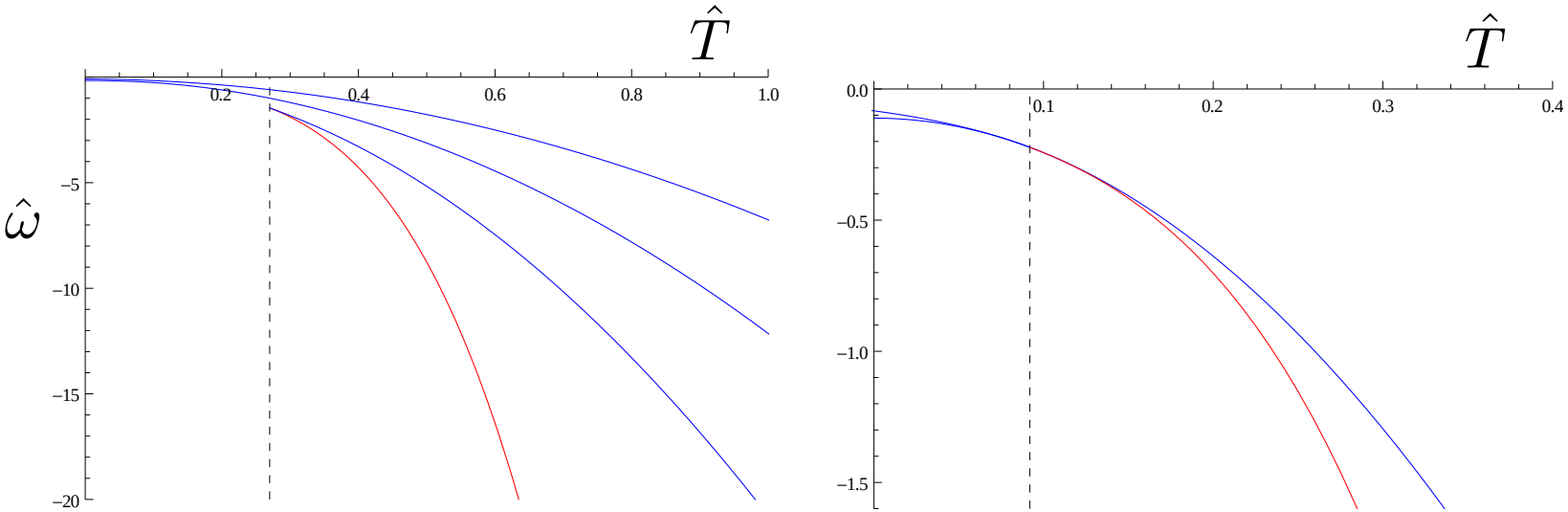
- This black brane family is not thermodynamically stable everywhere: [Chamblin, Emparan, Johnson & Myers; Cvetic & Gubser; Cai & Soh; Harmark & Obers]
- Necessary condition for local thermodynamic stability:

$$\det \mathcal{H} \propto 2 - q_1 - q_2 - q_3 + q_1 q_2 q_3 > 0, \quad \mathcal{H} = \frac{\partial^2 \varepsilon}{\partial s \partial \rho_I}$$

- Note: not a sufficient condition.

Examples

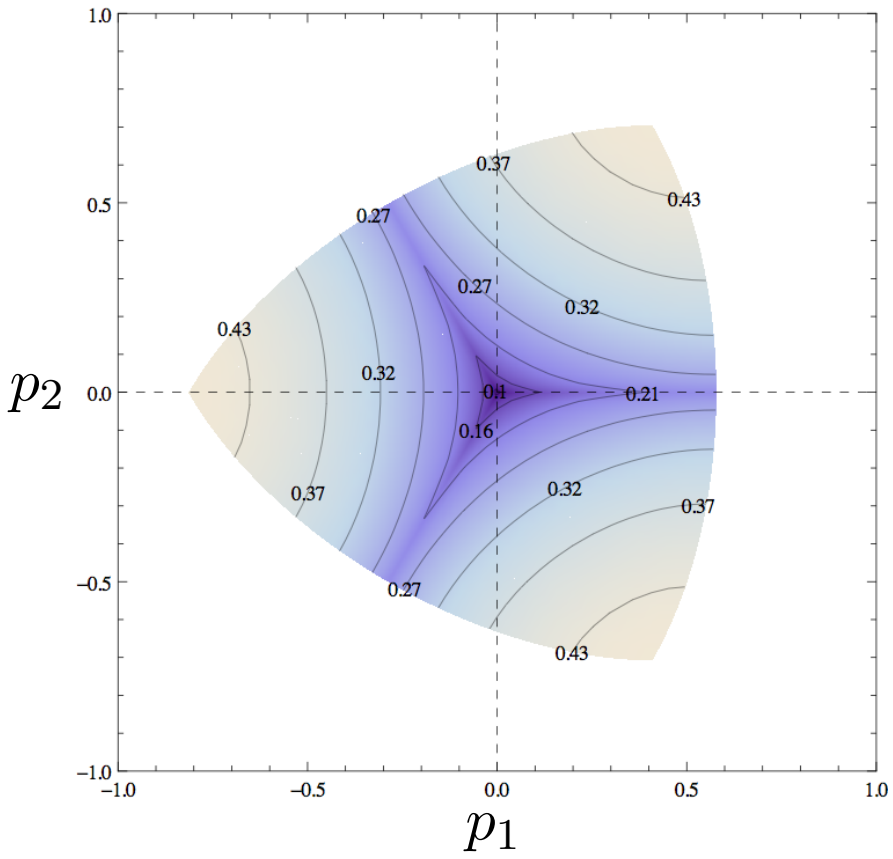
Gibbs free energy density for the various black brane branches that exist at fixed generic μ_I (left) or at $\mu_1 = \mu_2 = \mu_3$ (right):



- At fixed μ_I we find a minimum temperature for the existence of a thermodynamically stable dominant branch of solutions.
- Equal-charge brane (RN) is thermodynamically unstable and subdominant below a minimum temperature!

Visualise minimum temperature surface

Minimum temperature $\hat{T}$ for which the dominant branch is thermodynamically stable:



$$p_1 \equiv \frac{\mu_1 + \mu_2 - 2\mu_3}{\sqrt{6}\sqrt{\mu_1^2 + \mu_2^2 + \mu_3^2}}, \quad p_2 \equiv \frac{\mu_1 - \mu_2}{\sqrt{2}\sqrt{\mu_1^2 + \mu_2^2 + \mu_3^2}}$$

Marginal modes

- Seek time-independent solutions that are regular at the horizon and normalisable at the AdS boundary.
- Fluctuations of the gauge field at the boundary are required to vanish so that the chemical potentials are not perturbed.
- Such marginal modes indicate new branches of superconducting and normal black brane solutions.
- Consider general μ_I.

Class of fluctuations

- Fluctuate the 15 gauge fields and 20 scalar fields

$$T = T_0 e^{\chi(r)}\,, \quad \mathrm{tr}\chi = 0\,; \quad A = (A_0 + \alpha(r))dt$$

- Background is block diagonal $\Rightarrow$ fluctuations in different 2×2 blocks do not couple.
- These do not source metric fluctuations if

$$\mathrm{tr}\,\chi_{II} = \mathrm{tr}\,\sigma\alpha_{II} = 0 \quad \text{general } q_I$$

$$\sum_I \mathrm{tr}\,\chi_{II} = \sum_I \mathrm{tr}\,\sigma\alpha_{II} = 0 \quad \text{two/three equal } q_I$$

Class of fluctuations

- Work with linear combinations. For scalar fluctuations:

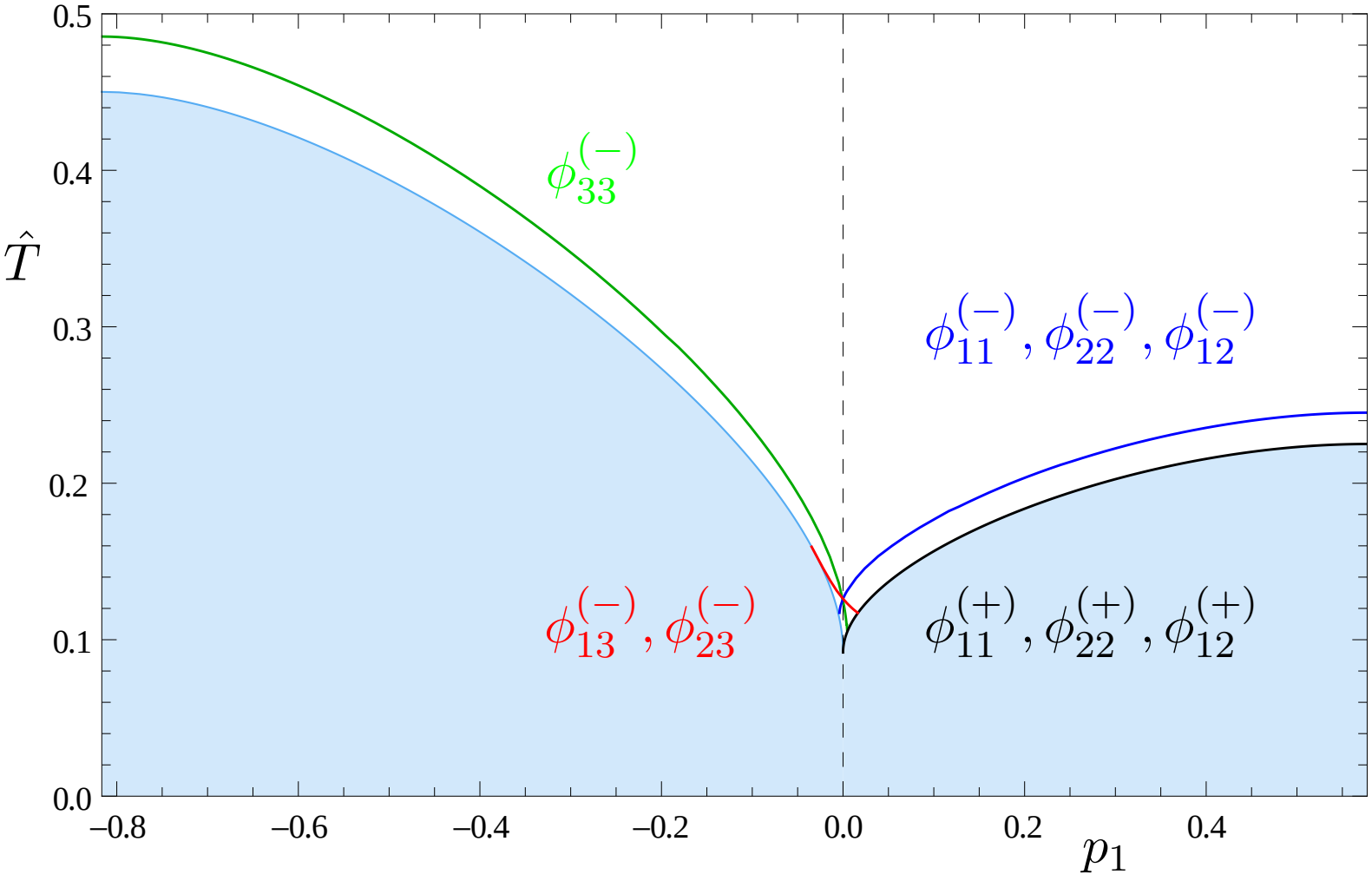
	$\phi_{IJ}^{(-)}$	$\phi_{IJ}^{(+)}$
$I = J$	charged	neutral
$I \neq J$	charged	charged

- Various special cases of diagonal charged modes considered by [Aprile, Roest & Russo].
- All other modes couple to gauge field fluctuations in general.
- For neutral diagonal modes, need to include other blocks with two/three equal q_I.
- When $q_I = q_J$, $\phi_{IJ}^{(\pm)}$ satisfies same EOM as $\phi_{II}^{(\pm)}$.

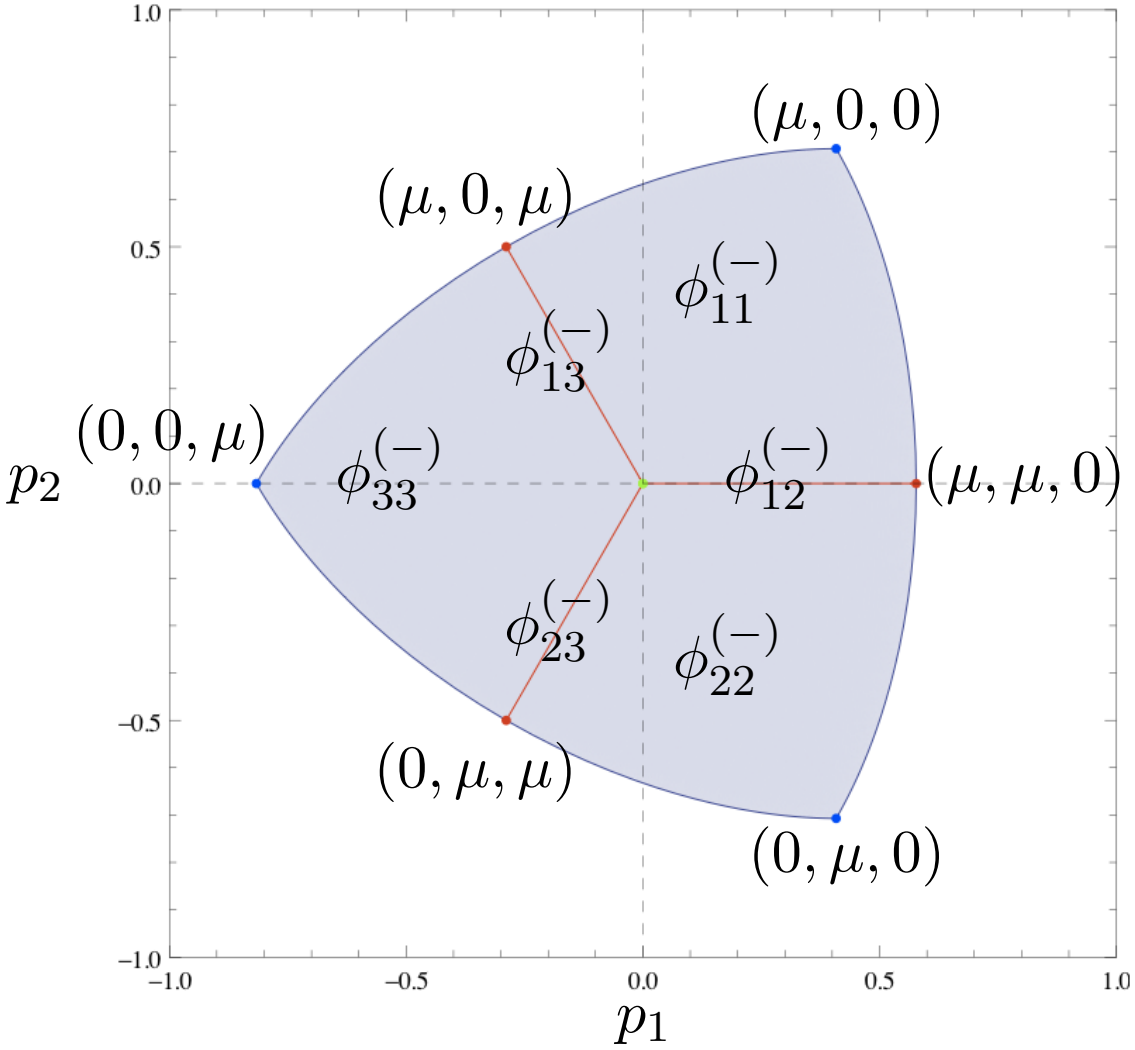
Sector $(-)$: results

- Superconducting branch *always* exists in the region of the normal phase that is thermodynamically stable.
- Off-diagonal modes have lower critical temperatures than diagonal modes, except along loci of equal chemical potential for which they have the same temperature.

Sector $(-)$: slice at $\mu_1 = \mu_2$



Sector $(-)$: full scan



Sector $(+)$ and Gubser-Mitra instabilities

- Only find marginal $(+)$ modes along loci of equal chemical potential, for which these modes are all neutral.
- Interpret as zero modes moving within family of black branes.
- Find analytically, *e.g.*

$$\frac{\delta X_1}{X_1} = -\frac{\delta X_2}{X_2} = \frac{1}{1+r^2}, \quad \delta X_3 = 0$$

$$\delta A_1 = -\delta A_2 = \frac{\sqrt{(1+q_3)}(r^2-1)}{(1+r^2)^2}, \quad \delta A_3 = 0$$

$$\hat{T} = \frac{1}{\pi\sqrt{2(1+4q_3+q_3^2)}}$$

- Can populate entire minimum temperature surface, involving gravitational fluctuations in general.
- Anticipate a true instability associated with these modes.

Open questions

This system:

- Nonlinear construction of branches and comparison of their free energy with that of the normal phase.
- Do the condensate curves for off-diagonal modes bend the right way and are they preferred? Are spatially modulated phases preferred?

Problems with consistent truncations:

- A subdominant solution in a given theory can appear as a dominant solution in a consistent truncation of that theory.
- Work in the full 10D or 11D theory, or keep all KK modes?
- What is the true ground state?