Holographic tachyon: strongly coupled fermions and walking technicolor

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October 23, 2012

Preview

AdS/CFT and, more generally, gauge/gravity duality is a great tool for studying strongly coupled field theories. This talk is devoted to the holographic models based on Tachyon DBI. Will focus on two applications:

- Strongly interacting fermions at finite temperature and density
- Holographic technicolor model of EW symmetry breaking

Based on work with D. Kutasov and J. Lin (arXiv:1107.2324; 1201.4123) and with M. Goykhman (arXiv:1210.????)

Outline

NJL model

There are examples where quantum effects lead to the breaking of conformal symmetry. Consider NJL model in 2+1 dimensions with the UV cutoff Λ:

$$\mathcal{L}=\bar{\psi}^i\gamma^\mu\partial_\mu\psi_i+\frac{g^2}{N}(\bar{\psi}^i\psi_i)^2=\bar{\psi}^i\gamma^\mu\partial_\mu\psi_i-\sigma\bar{\psi}^i\gamma^\mu\partial_\mu\psi_i-\frac{N\sigma^2}{2g^2}$$

Integrating out fermions produces effective potential for σ:

$$\frac{1}{N}V_{eff}=\frac{\sigma^2}{2g^2}+\mathrm{tr}\log(\gamma^\nu\partial_\mu+\sigma)$$

NJL model

It is technically convenient to compute the derivative of the effective potential:

$$\frac{V'_{eff}}{N} = \frac{\sigma}{g^2} - \frac{\sigma}{\pi^2}\left(\Lambda - \sigma \arctan(\frac{\Lambda}{\sigma})\right)$$

$\sigma = 0$ is always a solution of $V'(\sigma) = 0$ but a second solution appears for $g > g_c^2 = \pi^2/\Lambda$. Note that $V''(0)/N = 1/g^2 - 1/g_c^2$ so the solution with $\langle\sigma\rangle \neq 0$ is preferred for $g^2 > g_c^2$.
In the symmetry breaking solution, the fermion mass is

$$M \sim \left(\frac{1}{g_c^2\Lambda} - \frac{1}{g^2\Lambda}\right)\Lambda$$

In the limit $M \ll \Lambda$ the UV cutoff decouples.

Fermion matter at finite density

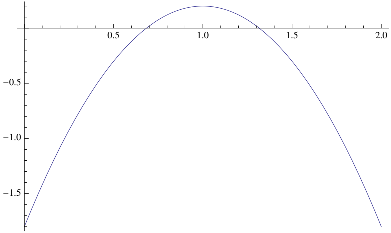
Lattice simulations is a great tool for studying QCD and other strongly interacting systems. However strongly interacting fermions at finite density (generally) suffer from the sign problem, and therefore even equilibrium physics is hard to understand. The non-equilibrium quantities (like viscosity) are hard to compute.

The holographic tachyon model has its roots in the D-brane system which describes strongly interacting fermions. Hence, it is natural to study it at finite temperature and density. We will see that the are situations with critical points in the phase diagram, connecting 1st and 2nd order phase transition lines.

Miransky (BKT) scaling

Consider the situation where β-function has the form

$$\beta(g;\alpha) = (\alpha - \alpha_*) - (g - g_*)^2$$



α is an external parameter. In the AdS/CFT context, $\alpha - \alpha_* \sim m^2 - m^2_{BF}$.

Miransky (BKT) scaling

In AdS/CFT, the flow between the two fixed points is driven by the double-trace operator. When the two zeroes merge at $m^2 = m^2_{BF}$, this operator becomes marginal.

Small $\alpha - \alpha_* < 0$ results in "walking behavior" where the system sits close to RG fixed point for a long RG time. There is a BKT-like separation of scales:

$$M \simeq \Lambda \exp\left(-\frac{c}{\sqrt{\alpha_* - \alpha}}\right)$$

because $\log M \sim \int d \log \mu \sim \int \frac{dg}{\beta(g;\alpha)}$

QCD etc

QCD in Veniziano limit ($N_f \sim N_c \gg 1$) is believed to have a conformal phase transition as the number of flavors is decreased significantly below $11/2N_c$. It is believed to happen when the fermion bilinear $\bar{\psi}^i \psi_j$ acquires large anomalous dimension $\gamma = 1$ so that the "double trace" operator becomes marginal, just like in the AdS/CFT example.

However investigating this in detail is beyond the regime of applicability of perturbation theory. We can change the parameters of the system to push it into the regimes where one can maintain control.

Technicolor

Electroweak symmetry $SU(2) \times U(1)$ is broken to $U(1)$, which results in W and Z gauge bosons acquiring masses (and the photon stays massless). One possibility is to break it spontaneously by the higgs vev υ_{EW}.

Technicolor models: consider a QCD-like theory with 2 massless Dirac fermions (techniquarks). The UV symmetry $SU(2)_L \times SU(2)_R$ is broken to the diagonal subgroup $SU(2)_{diag}$ by chiral condensate. If we identify $SU(2) \times U(1) \subset SU(2)_L \times SU(2)_R$ with electroweak symmetry and gauge it, we end up with (dynamical) electroweak symmetry breaking.

Technicolor

Problems with simple technicolor models: FCNC. Standard model fermions acquire masses via irrelevant operators

$$\frac{1}{\Lambda_{ETC}^2}\left[(\bar\psi_{SM}\gamma^\mu\psi_{TC})(\bar\psi_{SM}\gamma^\mu\psi_{TC})+(\bar\psi_{SM}\gamma^\mu\psi_{SM})(\bar\psi_{SM}\gamma^\mu\psi_{SM})\right]$$

Then e.g. $m_s \sim 4\pi \upsilon_{EW}^3/\Lambda_{ETC}^2$ implies $\Lambda_{ETC} \sim 50\,TeV$. But this gives rise to the FCNC term which is $\mathcal{O}(10^5)$ larger than experimentally observed.

Walking technicolor

Walking technicolor was invented to solve this problem. We have seen examples where the theory is nearly conformal for a long RG time and the UV and IR scales are widely separated and the condensate has $\gamma = 1$. In this case the SM quark masses are enhanced via

$$m_{SM} \sim 4\pi \left(\frac{\Lambda_{ETC}}{\upsilon_{EW}}\right)^{\gamma} \frac{\upsilon_{EW}^3}{\Lambda_{ETC}^2}$$

So one can raise the value of Λ_{ETC} to avoid large FCNC and still have acceptable quark masses. There have been arguments for the existence of a very light "technidilaton" ($m \sim 125$ GeV?).

Holographic walking technicolor

It is natural to model strongly interacting ($\gamma = 1$) walking regime in holography. The holographic tachyon model is well suited for this. But...

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Holographic walking technicolor

It is natural to model strongly interacting ($\gamma = 1$) walking regime in holography. The holographic tachyon model is well suited for this. But...

- It has no parametrically light "technidilaton"
- However the lightest technimeson can be made an order of magnitude lighter than the next one.

D3-D7 system

Consider N D3 branes stretched along 0123 directions and a D7 brane stretched along 012 45678. The low energy dynamics is described by defect fermions coupled to CFT:

$$\mathcal{S} = \mathcal{S}_{N=4} + \int d^3x(i\bar{\psi}\gamma^\mu\mathcal{D}_\mu\psi + g\bar{\psi}\phi^9\psi)$$

Giving vev to ϕ^9 corresponds to giving a bare mass to ψ. We will consider situation where the bare mass vanishes. There might be dynamical mass generation if in the vacuum $\phi^9 \neq 0$; at small 't Hooft coupling this does not happen.

DBI action and EOM

At large 't Hooft coupling $\lambda \gg 1$ we are instructed to consider $AdS_5 \times S^5$ space $[r^2 = \rho^2 + (x^9)^2]$

$$ds^2 = r^2 dx_\mu dx^\mu + r^{-2}\left(d\rho^2 + \rho^2 d\Omega_4^2 + (dx^9)^2\right)$$

The D7 brane embedding is specified by $x^9 = f(\rho)$, giving the DBI action

$$S_{D7} \sim \int d^3x \int d\rho \frac{\rho^4}{\rho^2 + f(\rho)^2}\sqrt{1 + f'(\rho)^2}$$

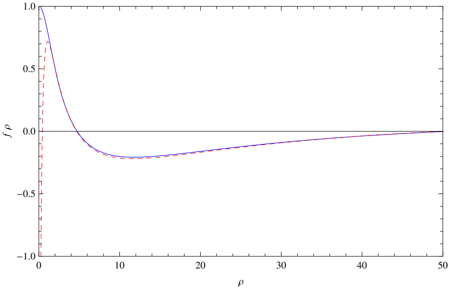
EOM for $f(\rho)$ are second order, non-linear differential equation. In the UV region, it gets linearized and can be solved analytically.

Solutions of EOM

Namely, for $\rho \gg f(\rho)$,

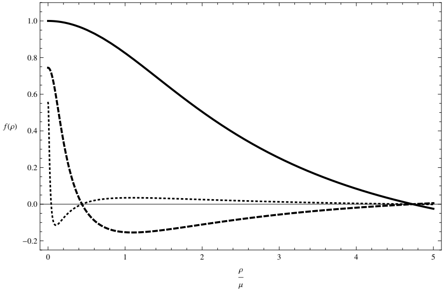
$$f(\rho) \approx A\mu^{3/2}\rho^{-1/2}\sin(\frac{\sqrt{7}}{2}\log\rho/\mu+\varphi)$$

One can also solve the full EOM numerically starting from $f'(0)=0$.



Solutions of EOM

To proceed, impose Dirichlet boundary conditions at the UV cutoff Λ. Solutions are labeled by the number of nodes in $(0, \Lambda)$.



In particular, $f_0(0) \sim \Lambda$. One can study spectrum of excitations around $f_n(\rho)$. There are n tachyons. Solution $f_0(\rho)$ is energetically preferred and does not have tachyons living on it. Masses $\sim \Lambda$.

Comments

- In the large momenta regime the lagrangian for $f(\rho)$ is that for the tachyon (significantly) below the BF bound. We follow its condensation.

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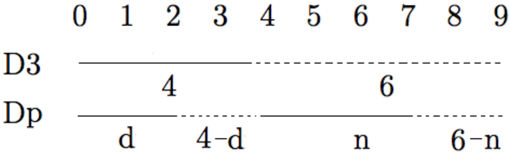
- This requires a UV cutoff Λ. The vacuum of the theory $f(\rho)$ has a mass gap of order Λ
- Presumably there is a phase transition at $\lambda \sim 1$.
- It is desirable to separate the scale of meson masses from Λ. See below.

The setup

Consider Dp-brane which shares d spacetime dimensions with the D3-branes, and is extended in $n = p + 1 - d$ additional spatial directions:



The nonsupersymmetric D3-D7 corresponds to $(d, n) = (3, 5)$.
Supersymmetric D3-D7 is $(d, n) = (4, 4)$.

D3-Dp system: BKT limit

Consider strong coupling regime, $\lambda \gg 1$; set $d = 3$ and write $AdS_5 \times S^5$ metric as

$$ds^2 = \left(\frac{r}{L}\right)^2 dx_\mu dx^\mu + \left(\frac{L}{r}\right)^2 \left(d\rho^2 + \rho^2 d\Omega_{n-1}^2 + df^2 + f^2 d\Omega_{5-n}^2\right)$$

The resulting DBI action is

$$S_{Dp} = \int d^d x \int d\rho \left(\frac{\rho^2 + f^2}{L^2}\right)^{\frac{d-n}{2}} \rho^{n-1} \sqrt{1 + f'^2}$$

Again, the EOM are nonlinear, but linearize in the large momentum limit.

D3-Dp system: BKT limit

The $n = 5$ case was studied before; there was a tachyon well below the BF bound and no separation of scales. The tachyon mass is controlled by

$$\kappa = \lim_{\lambda \to \infty} \kappa(\lambda) = n - d - \left(\frac{d-2}{2}\right)^2$$

In $d = 3$ case the critical value of n is $n_c = 13/4$. Tune $n = n_c + \epsilon$ so that $\kappa \ll 1$. In the large momentum regime

$$f(\rho) = A\mu \left(\frac{\mu}{\rho}\right)^{\frac{1}{2}} \sin\left(\sqrt{\kappa} \ln \frac{\rho}{\mu} + \phi\right)$$

D3-Dp system: BKT limit

The dynamically generated scale is

$$f(0) = \mu \simeq \Lambda \exp\left(-\frac{\pi}{\sqrt{\kappa}}\right)$$

Note that for $\Lambda \gg \rho \gg \mu$

$$f(\rho) = \mu \left(\frac{\mu}{\rho}\right)^{\frac{1}{2}} \left(C_1 \ln \frac{\rho}{\mu} + C_2\right)$$

One can find C_1, C_2 numerically by integrating full EOM at $\kappa = 0$. The resulting values of A, ϕ are obtained via

$$A = \frac{C_1}{\sqrt{\kappa}}, \qquad \phi = \frac{C_2}{C_1}\sqrt{\kappa}$$

Comments

We focus on the BKT limit ($\kappa \to 0$) where there is a parametric separation between the UV cutoff and the physical scale.

- The ground state is given by integrating EOM at $\kappa = 0$ starting from $f(0) = \mu; f'(0) = 0$. The action is negative compared to that of the trivial $f = 0$ solution.

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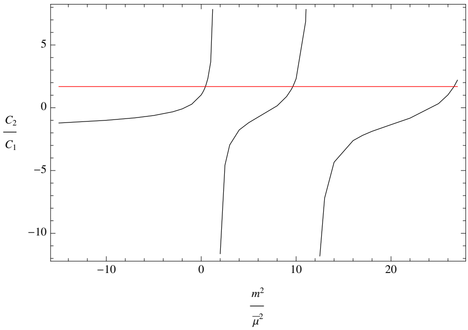
- The ground state is given by integrating EOM at $\kappa = 0$ starting from $f(0) = \mu; f'(0) = 0$. The action is negative compared to that of the trivial $f = 0$ solution.
- At finite κ there were other solutions labeled by a number of nodes. They disappear in the BKT limit.

Spectrum

We can study the spectrum of mesonic excitations by considering small fluctuations around the background solution. The scalar mesons come from the fluctuations of $f(x^\mu, \rho)$. Boundary conditions at the UV cutoff fix the ratio of C_1/C_2.



Spectrum

The spectrum of the scalar mesons is given by

$$m^2/\mu^2 \approx 0.44, 9.65, 26.63, 51.35, 84, \cdots$$

While that of the vector mesons is

$$m^2/\mu^2 \approx 3.08, 15.12, 34.87, 62.32, 97.46, 140.31, \ldots$$

Note that there are no tachyons. There is a light "dilaton", but it is not parametrically light.

Action of Dp brane in $AdS_5 \times S^5$ as Tachyon DBI

The action for the Dp brane can be written in the TDBI form

$$\int d^{d+1}x V(T)\sqrt{-G} = \int d^{d+1}x \sqrt{-g}\, V(T)\sqrt{1 + g^{MN}\partial_M T \partial_N T},$$

where g_{MN} is the AdS metric,

$$G_{MN} = g_{MN} + \partial_M T \partial_N T$$

is the induced metric and the tachyon field is related to the coordinates as

$$\rho = r\cos T, \qquad f = r\sin T$$

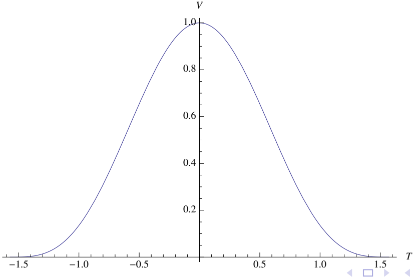
The tachyon potential is fixed to be $V(T) = (\cos T)^{n-1}$.

Hard wall from TDBI

The tachyon potential has a maximum at $T = 0$,

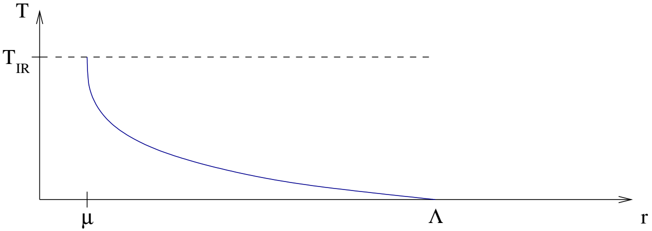
$$V(T) \approx 1 + \frac{m^2}{2} T^2 + \dots$$

Mass can be tuned to the BF value $m^2 = -d^2/4$ in the BKT limit. Potentials with a vacuum at $T_{IR} < \infty$ give rise to hard wall models.



Hard wall from TDBI

In the T, r coordinates the vacuum solution is described by a nontrivial profile $T_0(r)$ with an excised IR part of AdS. The conformal symmetry is dynamically broken by the solution. The effective metric for the small fluctuations ("mesons") contains a hard wall which leads to KK spectrum $m_n \sim n$.



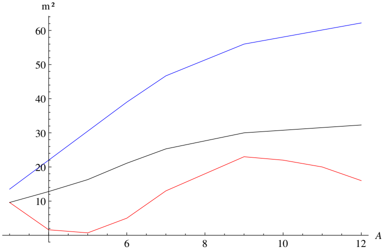
Soft wall from TDBI

An alternative scenario is realized for the potentials with $T_{IR} = \infty$. Consider $V(T) = \exp(-\beta T^2/2)$. This gives rise to $T_0 \simeq r^{-\frac{\beta}{d}}$ which dynamically generates a soft wall type potential for the mesons.

The high energy behavior of the meson spectrum is determined by the IR (large r) behavior of the effective metric and is given by $m_n^2 \sim n^{(1-\beta/d)}$. Linear confinement, $m_n^2 \sim n$ is realized for $V(T) \simeq -dT^2 \log T$.

Low lying mesons

Consider the following soft wall potential (with $A > 2$) $V(T) = (1 + (A-2)T^2)e^{-AT^2}$. The lightest meson masses^2 (with the lowest rescaled by a factor of 10)



No parametrically light meson, but factor of 10 suppression!

Finite temperature from holographic TDBI

At finite temperature we use the TDBI action in the AdS-Schwarzschild background

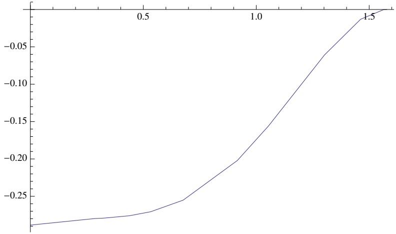
$$\int dr r^{d-1} V(T) \sqrt{1 + F(r)(r\partial_r T)^2}$$

where $F(r) = 1 - (r_h/r)^d$.

We can study solutions and construct the phase diagram. Interestingly, we can interpolate between the first and the second order phase transitions by changing the tachyon potential.

Soft wall at finite temperature

There are solutions which "intersect the horizon" at $r = r_h$ and are labeled by the values of $T_h \equiv T(r_h)$. One can compute r_h/μ and δS.



Free energy (with $\mathcal{F}(r_h = 0)$ subtracted) vs. r_h/μ.

Soft wall at finite temperature

Two interesting regimes are

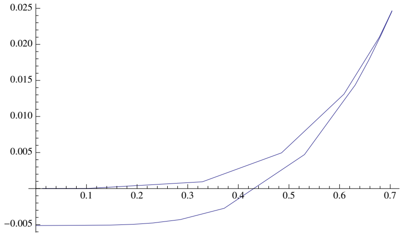
- Small temperatures, $r_h/\mu \ll 1$
- Small T

In the small r_h/μ region, the solution approximates $r_h = 0$ solution in the $r \gg r_h$ region, but "crosses the horizon" at $r = r_h$. This implies dissipation at low temperatures.

Small T regime corresponds to critical point where the 2nd order phase transition between the $T \neq 0$ and $T = 0$ solutions occurs. EOM becomes linear as $T \to 0$. Deviations from linearity are governed by the $\mathcal{O}(T^4)$ term in tachyon potential.

Soft wall at finite temperature

Depending on the value of the coefficient in the $\mathcal{O}(T^4)$ term, one can have either 1st or second order phase transition:



Phase diagram for "hard wall" potential is similar.

Soft wall at finite temperature

The value of critical temperature that corresponds to $T = 0$ can be determined from the linearized EOM. Generally,

$$T_0 = \frac{1}{r^{d/2}} \left[c_1(T_h) \log r/r_h + c_2(T_h) \right]$$

But this must be equal to $T_0 = \frac{1}{r^{d/2}} \left[C_1 \log r/\mu + C_2 \right]$ and therefore

$$\log \mu/r_h = \frac{C_2}{C_1} - \frac{c_2(T_h)}{c_1(T_h)} \to \log \mu/r_h^*, \quad T_h \to 0$$

One can also show that $\partial \mathcal{F}/\partial r_h > 0$. Hence, if $\partial \mathcal{F}/\partial T_h > 0$ then we have a first order phase transition (and $\partial T_h/\partial r_h > 0$), while $\partial \mathcal{F}/\partial T_h < 0$ implies the second order phase transition (and $\partial T_h/\partial r_h < 0$). This depends on the aT^4 term in $V(T)$.

Finite temperature and density

$$S_{TDBI} = -\int_{r_h}^{\infty} dr \int d^d x r^{d-1} V(T) \sqrt{1 + r^2 \dot{T}^2 F - \dot{A}_0^2}$$

From equation of motion for A_0 we obtain

$$\dot{A}_0^2 = \frac{\hat{d}^4 (1 + r^2 F \dot{T}^2)}{r^{2(d-1)} V^2 + \hat{d}^4}$$

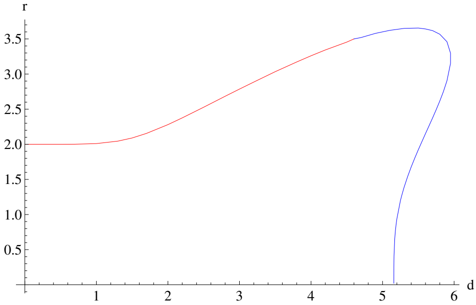
Substituting back and subtracting the action for $T = 0$ we get $(\tilde{r} = r/\hat{d}^{\frac{2}{d-1}})$, $\hat{d}^2$= "density":

$$\mathcal{F}(r_h, \hat{d}) = \hat{d}^{8/3} \int_{\tilde{r}_h}^{\infty} d\tilde{r} \int d^4 x \tilde{r}^6 \left(\frac{V^2}{\sqrt{1 + \tilde{r}^6 V^2}} \sqrt{1 + \tilde{r}^2 F T'^2} - \frac{1}{\sqrt{1 + \tilde{r}^6}} \right)$$

Finite temperature and density

Similarly to the $\hat{d} = 0$ we can determine the critical point from

$$\frac{\hat{d}^{2/3}}{\mu} = \exp\left(-\frac{C_2}{C_1} + \frac{c_2}{c_1}\left(\frac{r_h/\mu}{\hat{d}^{2/3}/\mu}, T_h \to 0\right)\right)$$



Finite temperature and density

Let us expand $V(T)$ and write terms that are $\mathcal{O}(T^2)$ and $\mathcal{O}(T^4)$

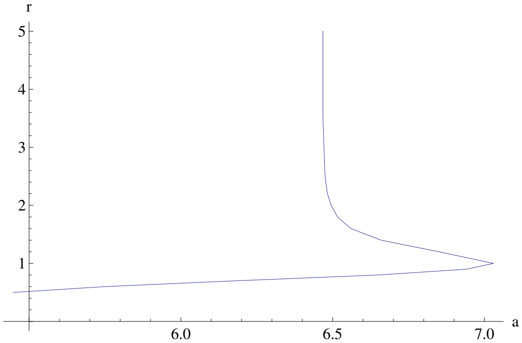
$$\mathcal{F}(r_h, \hat{d}) = \mathcal{F}_2(r_h, \hat{d}) + \mathcal{F}_4(r_h, \hat{d}) + \cdots$$

Expand $T = T_1 + T_3$ where $T_n = \mathcal{O}(T_h^n)$. Then, using equations of motions gives

$$\mathcal{F}(T) \simeq -\int drT_1 P_3(T_1) = -\mathcal{F}_4(T_1) = \tilde{a}(a, \tilde{r}_h) T_h^4$$

so whenever $\tilde{a}(a, \tilde{r}_h) > 0$ we have a 1st order phase transition, and second order otherwise.

Finite temperature and density



On the right - 2nd order; on the left - 1st order.

S-parameter

Consider a model which has an $SU(2) \times SU(2)$ symmetry dynamically broken do the diagonal $SU(2)$. Embed electroweak symmetry $SU(2) \times U(1) \subset SU(2) \times SU(2)$ and gauge it. The resulting model describes EW symmetry breaking.

An important observable is S-parameter

$$S = -4\pi(\Pi'_{VV}(q^2) - \Pi'_{AA}(q^2))|_{q^2=0}$$

We will compute it as a function of A for the tachyon potential $V(T) = (1 + (A-2)T^2)e^{-AT^2}$.

S-parameter

Global $SU(2) \times SU(2)$ symmetry in the boundary theory is introduced by gauging bi-fundamental tachyon in the bulk:

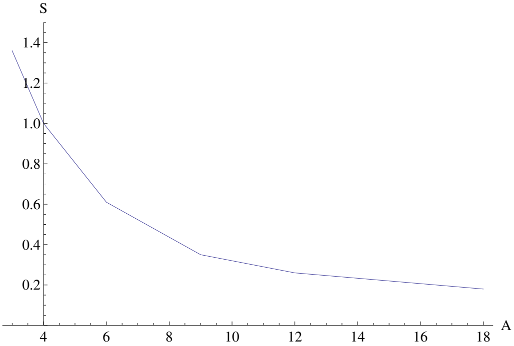
$$S = -\int d^4x dr \,\mathrm{tr}\, V(|T|) \left(\sqrt{-G^{(L)}} + \sqrt{-G^{(R)}} \right)$$

where $G^{(R)}_{MN} = G_{MN} + F^{(R)}_{MN}$, $G_{MN} = g_{MN} + (D_{(M} T)^{\dagger} D_{N)} T$, $G^{(R)} = \det\, G^{(R)}_{MN}$ and similarly for the left; covariant derivative of tachyon is given by

$$D_M T = \partial_M T + i A^{(L)}_M T - i T A^{(R)}_M$$

S-parameter

Action quadratic in $A^{(L)}, A^{(R)}$ can be used to compute vector and axial meson masses and the S-parameter



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- Dynamical symmetry breaking generates both hard and soft wall models.
- Holographic TDBI at finite temperature and density describes both 1st and 2nd order phase transitions with possible critical points
- Compute technimeson masses and S-parameter; no parametric supression of the lowest mass, but can get a factor of 10 difference.

THE END

Thank you!