

SuperBayeS

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IFIC

In collaboration with:

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Outline

- The problem: analysis pipeline for SUSY phenomenology
- The SuperBayeS package
- Parameter inference: present results and future prospects in the CMSSM

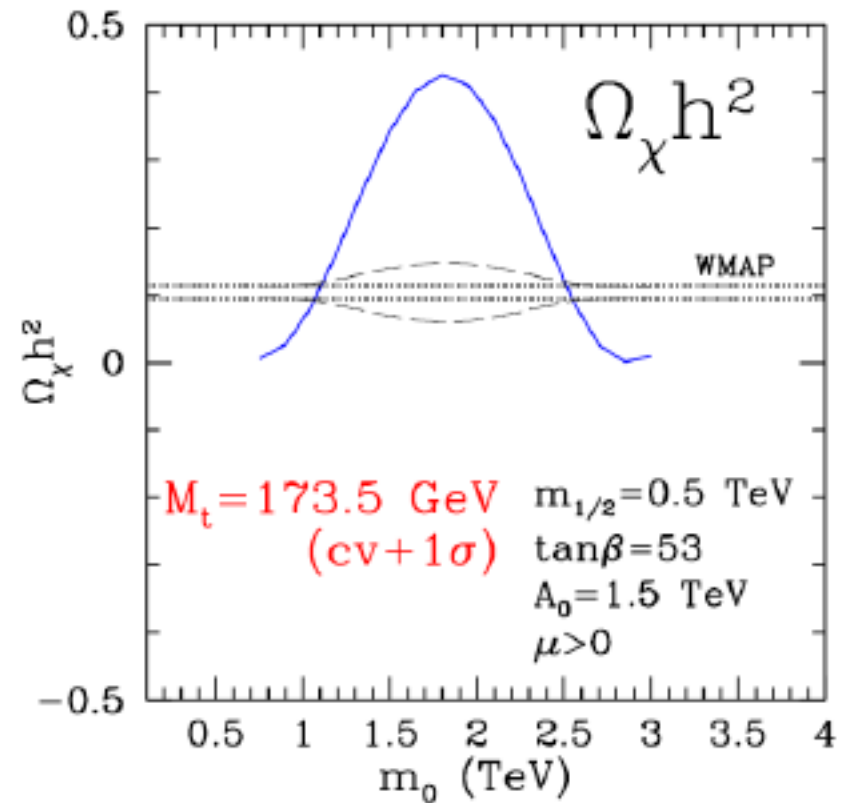
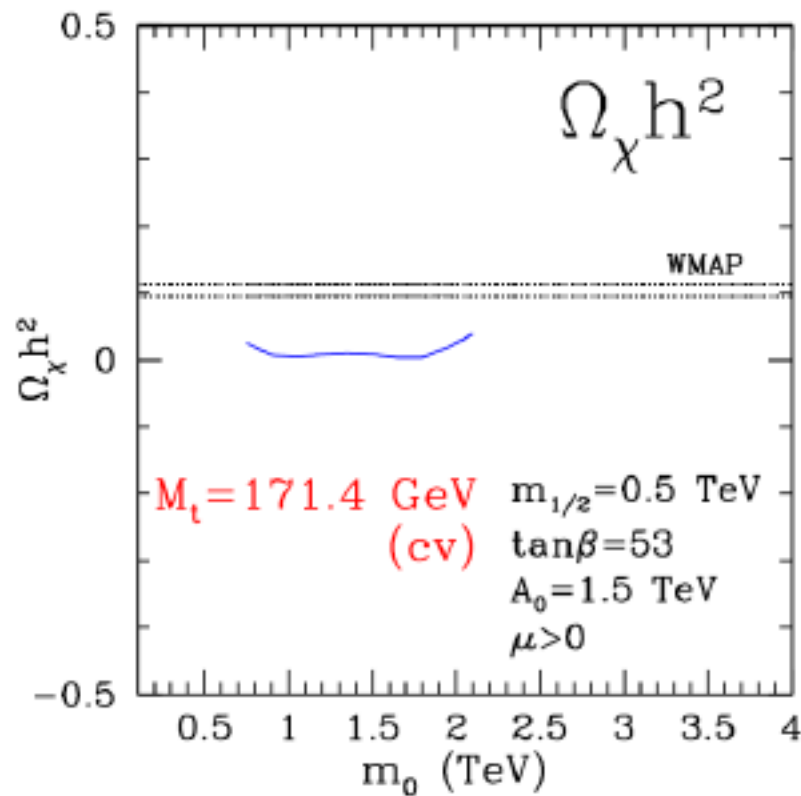
The model & data

- The general Minimal Supersymmetric Standard Model (MSSM): 105 free parameters!
- Need some simplifying assumption: i.e. the Constrained MSSM (CMSSM) reduces the free parameters to just 5 variables
- Present-day data: collider measurements of rare processes, CDM abundance (WMAP), sparticle masses lower limits, EW precision measurements. Soon, LHC sparticle spectrum measurements
- Astrophysical direct and indirect detection techniques might also be competitive: neutrino (IceCUBE), gamma-rays (Fermi), antimatter (PAMELA), direct detection (XENON, CDMS, Eureka, Zeplin)
- Goal: inference of the model parameters but it is difficult problem

Why is this a difficult problem?

- **Inherently 8-dimensional**: reducing the dimensionality oversimplifies the problem. Nuisance parameters (in particular m_t) cannot be fixed!
- **Likelihood discontinuous** and multi-modal due to physicality conditions
- RGE connect input parameters to observables in highly non-linear fashion: **only indirect (sometimes weak) constraints on the quantities of interest** ($\rightarrow$ prior volume effects are difficult to keep under control)
- **Mild discrepancies between observables** (in particular, $g-2$ and $b \rightarrow s\gamma$) tend to pull constraints in different directions

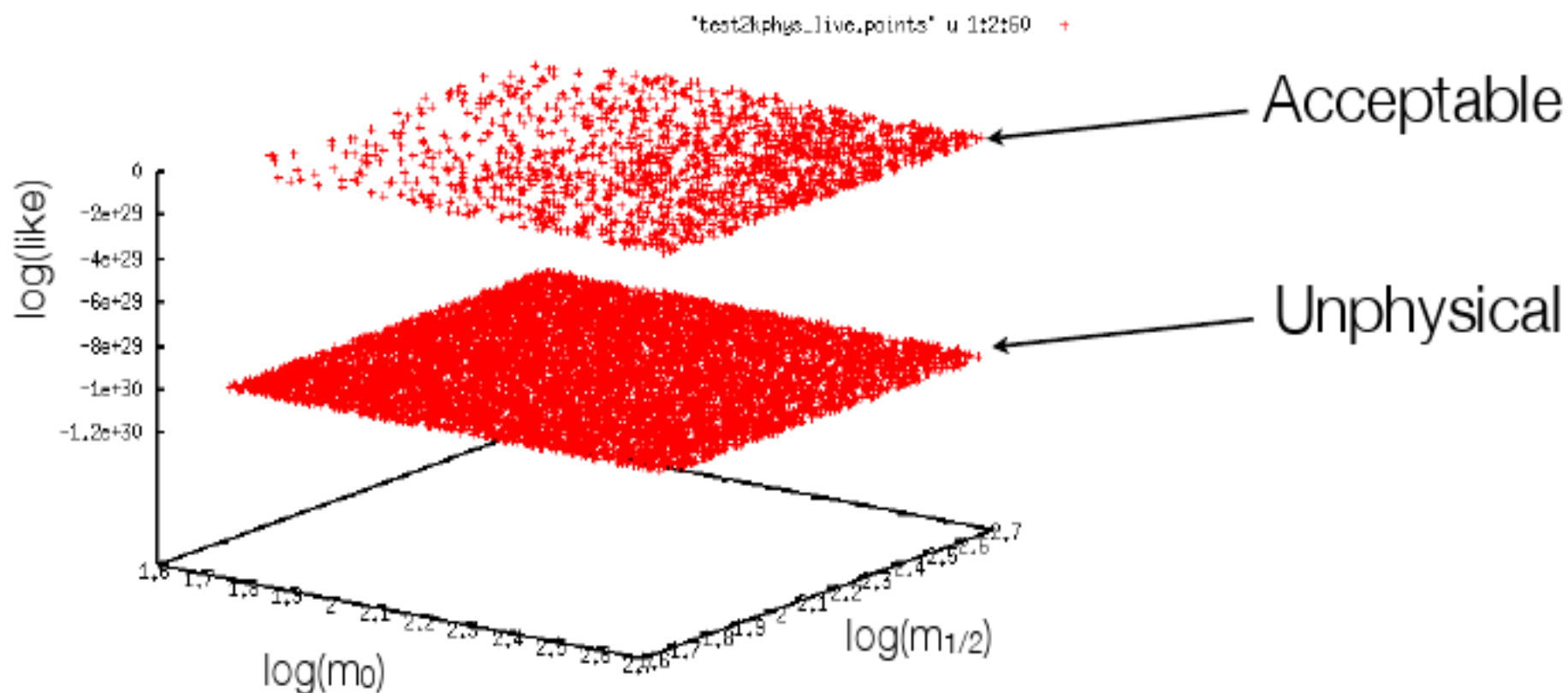
Impact of nuisance parameters



Roszkowski et al (2007)

The accessible “surface”

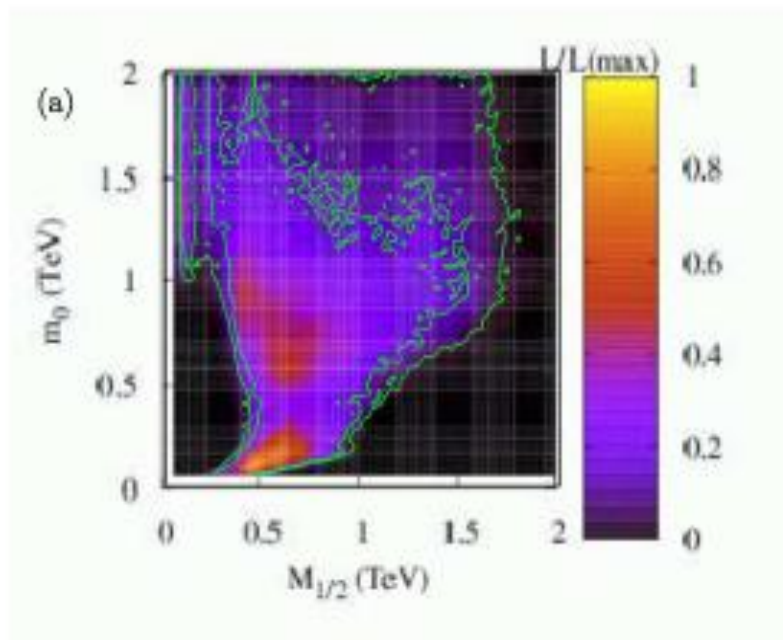
Scan from the prior with no likelihood except
physicality constraints



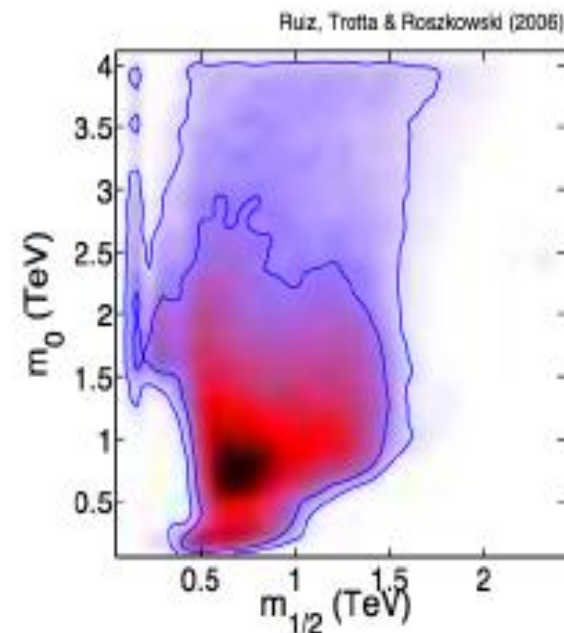
Bayesian parameter inference

The Bayesian approach

- Bayesian approach led by two groups (early work by Baltz & Gondolo, 2004):
- Ben Allanach (DAMPT) et al (Allanach & Lester, 2006 onwards, Cranmer, and others)
- RdA, Roszkowski & Roberto Trotta (2006 onwards)
SuperBayeS public code (available from: superbayes.org)
+ Feroz & Hobson (MultiNest), + Silk (indirect detection), + de los Heros (IceCube), + Casas et al. (Naturalness) + Bertone et al. (pmssm)



Allanach & Lester (2006)



Ruiz de Austri, Roszkowski & RT (2006)

Bayes' theorem

posterior

likelihood

prior

$$P(H|d, I) = \frac{P(d|H, I)P(H|I)}{P(d|I)}$$



evidence

H: hypothesis

d: data

I: external information

- **Prior**: what we know about **H** (given information **I**) before seeing the data
- **Likelihood**: the probability of obtaining data **d** if hypothesis **H** is true
- **Posterior**: our state of knowledge about **H** after we have seen data **d**
- **Evidence**: normalization constant (independent of **H**), crucial for model comparison

...

- Ignoring the prior and identifying

$$p(\theta_i|\text{data}) \equiv p(\text{data}|\theta_i)$$

- implicitly amounts to

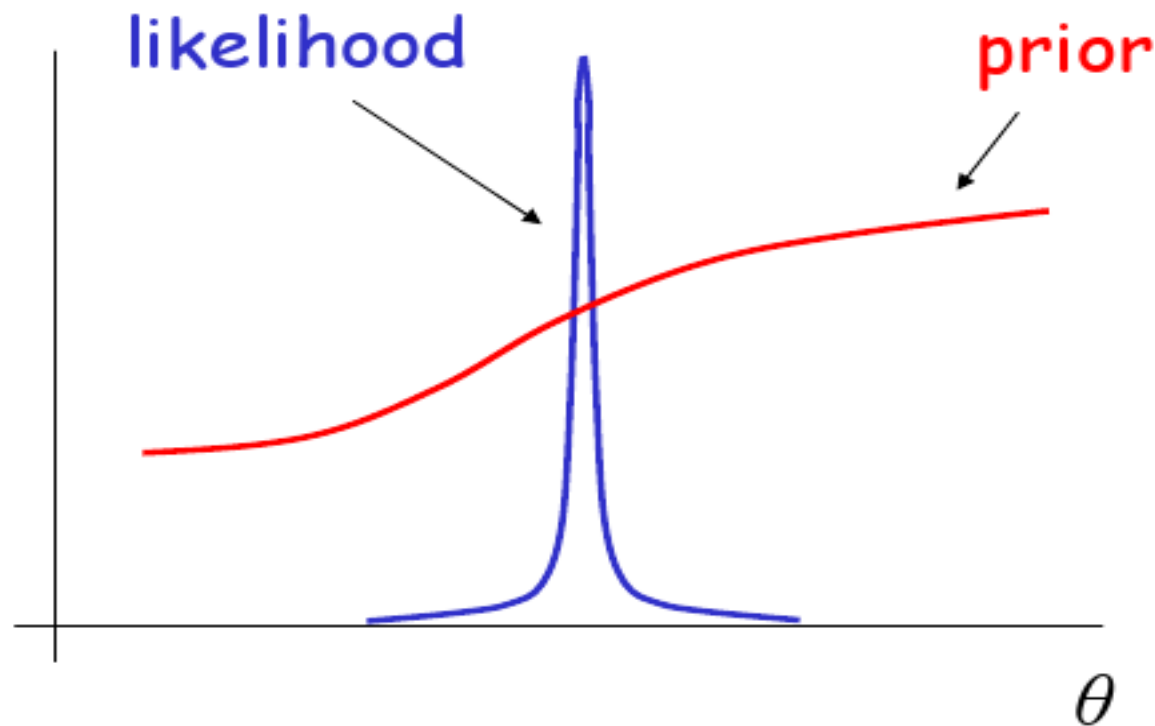
$$p(\theta_i) = \text{const.} \equiv \textit{“flat”}$$

- But e. g. $\theta_i \longrightarrow \theta_i^2$

$$\Rightarrow \textit{“flat”} \longrightarrow \textit{“non-flat”}$$

But

If data are good enough to select a small region of $\{\theta\}$ then the prior $p(\theta)$ becomes irrelevant



$$p(\theta_i|\text{data}) \equiv p(\text{data}|\theta_i)$$

Priors

- There is a vast literature on priors: Jeffreys', conjugate, non-informative, ignorance, reference, ...
- In simple problems, “good” priors are dictated by symmetry properties
- Flat: All values of θ equally probable

$$p(\theta) = \text{const.}$$

- Logarithmic: All magnitudes of θ equally probable

$$p(\ln \theta) = \text{const.}$$

$$\hookrightarrow p(\theta) \propto \frac{1}{\theta}$$

Key advantages

- **Efficiency:** computational effort scales $\sim N$ rather than k^N as in grid-scanning methods. Orders of magnitude improvement over previously used techniques.
- **Marginalisation:** integration over hidden dimensions comes for free
Suppose we have θ_i and are interested in $p(\theta_1|\text{data})$

$$p(\theta_1|\text{data}) = \int d\theta_2 \cdots d\theta_N p(\theta_i|\text{data})$$

- **Inclusion of nuisance parameters:** simply include them in the scan and marginalise over them. Notice: nuisance parameters in this context must be well constrained using independent data.
- **Derived quantities:** probabilities distributions can be derived for any function of the input variables (crucial for DD/ID/LHC predictions).

Analysis pipeline

SCANNING ALGORITHM

4 CMSSM parameters

$$\theta = \{m_0, m_{1/2}, A_0, \tan\beta\}$$

(fixing $\text{sign}(\mu) > 0$)

4 SM “nuisance
parameters”

$$\Psi = \{m_t, m_b, \alpha_S, \alpha_{EM}\}$$

Data:

Gaussian likelihoods
for each of the Ψ_j
($j=1\dots 4$)

RGE

Non-linear
numerical
function

via SoftSusy 2.0.18
DarkSusy 4.1
MICROMEGAS 2.2
FeynHiggs 2.5.1
Hdecay 3.102

Observable
quantities
 $f_i(\theta, \Psi)$

CDM relic abundance
BR's
EW observables
g-2
Higgs mass
sparticle spectrum
(gamma-ray, neutrino,
antimatter flux, direct
detection x-section)

Likelihood = 0

↑ NO

Physically acceptable?
EWSB, no tachyons,
neutralino CDM

↓ YES

Joint likelihood function

Data:

Gaussian likelihood
(CDM, EWO, g-2, $b \rightarrow sy$, ΔM_{B_s})
other observables have
only lower/upper limits

Posterior Samplers

- MCMC: A **Markov Chain** is a list of samples $\theta_1, \theta_2, \theta_3, \dots$ whose density reflects the (unnormalized) value of the posterior
- Crucial property: a **Markov Chain converges to a stationary distribution**, i.e. one that does not change with time. In our case, the posterior
- Different algorithms: MH, Gibbs... all need a proposal distribution => **difficult to find a good one in complex problems**
- Nested: New technique for efficient evidence evaluation (and posterior samples) (Skilling 2004)
- MultiNest: Also an extremely efficient sampler for multi-modal likelihoods !
Feroz & Hobson (2007), RT et al (2008), Feroz et al (2008)

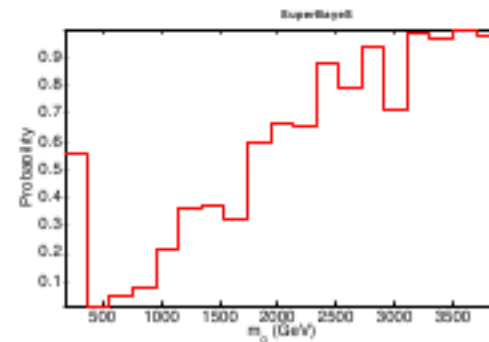
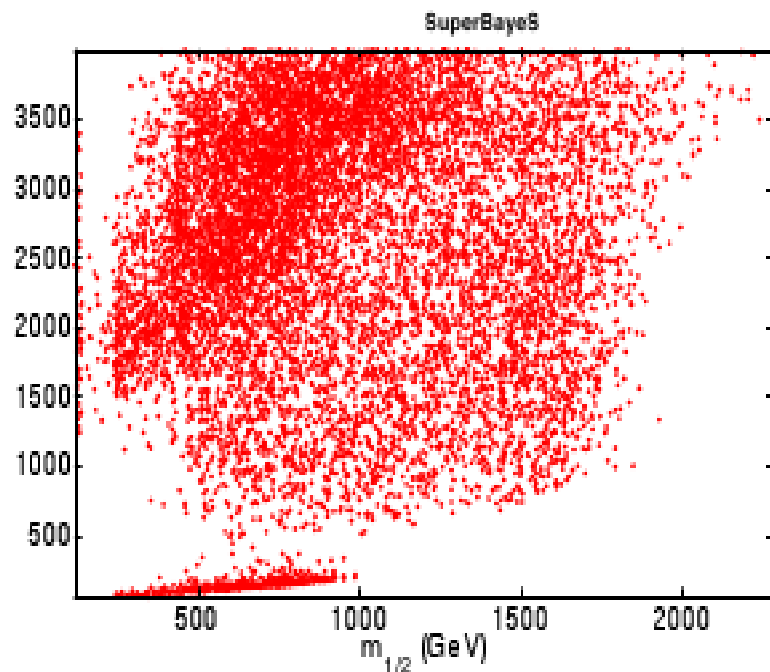
The SuperBayeS package (superbayes.org)

- Supersymmetry Parameters Extraction Routines for Bayesian Statistics
- Implements the CMSSM, but can be easily extended to the general MSSM
- Currently linked to SoftSusy 2.0.18, DarkSusy 4.1, MICROMEAS 2.2, FeynHiggs 2.5.1, Hdecay 3.102. **New release (v 1.5)**
- Includes up-to-date constraints from all observables
- Bayesian MCMC, MULTI-MODAL NESTED SAMPLING or grid scan mode.
- MULTI-MODAL NESTED SAMPLING (Feroz & Hobson 2008), efficiency increased by a factor 200. **A full 8D scan now takes 3 days on a single CPU** (previously: 6 weeks on 10 CPUs)
- Fully parallelized, MPI-ready, user-friendly interface a la cosmomc (thanks Sarah Bridle & Antony Lewis),
- SuperEGO: SuperBayeS Enhanced Graphical Output as a MATLAB graphical user interface for statistical analysis and plotting

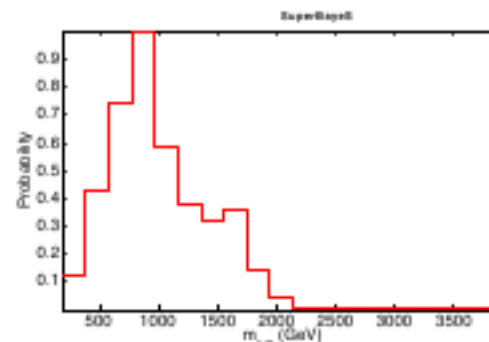
MCMC estimation

- Marginalisation becomes trivial: create bins along the dimension of interest and simply count samples falling within each bins ignoring all other coordinates
- Examples (from superbayes.org) :

2D distribution of samples from joint posterior

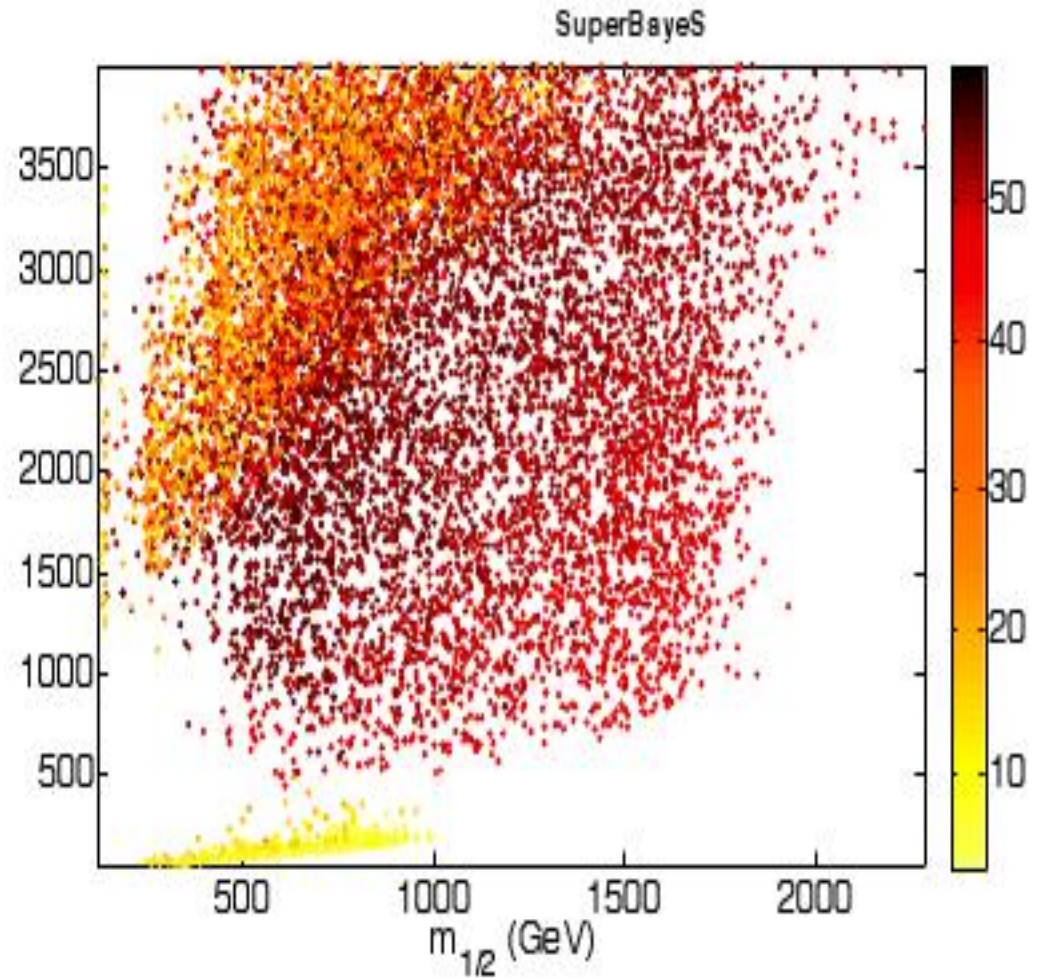
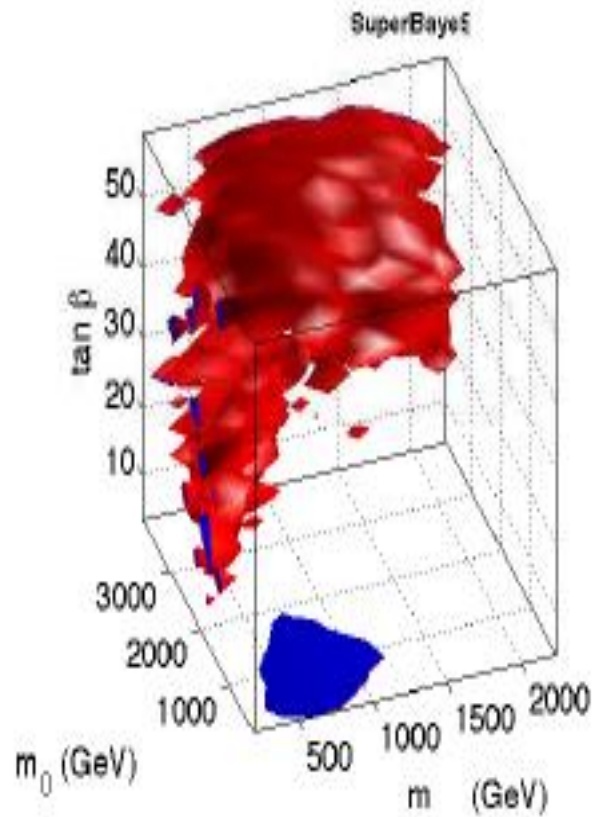


1D marginalised
posterior
(along y)



1D marginalised
posterior
(along x)

SuperEGO



Global CMSSM constraints

The CMSSM

Assuming Universal boundary conditions at M_{GUT}

- Gaugino masses

$$M_1 = M_2 = M_3 = m_{1/2}$$

- Scalar masses

$$m_{H_d}^2 = m_{H_u}^2 = M_L^2 = M_R^2 = M_Q^2 = M_D^2 = M_U^2 = m_0^2$$

- Trilinear couplings

$$A_u = A_d = A_l = A_0$$

- Higgs vev ratio

$$\tan\beta = v_u/v_d$$

- μ^2 from EWSB

flat priors: CMSSM parameters

$$50 \text{ GeV} < m_0 < 4 \text{ TeV}$$

$$50 \text{ GeV} < m_{1/2} < 4 \text{ TeV}$$

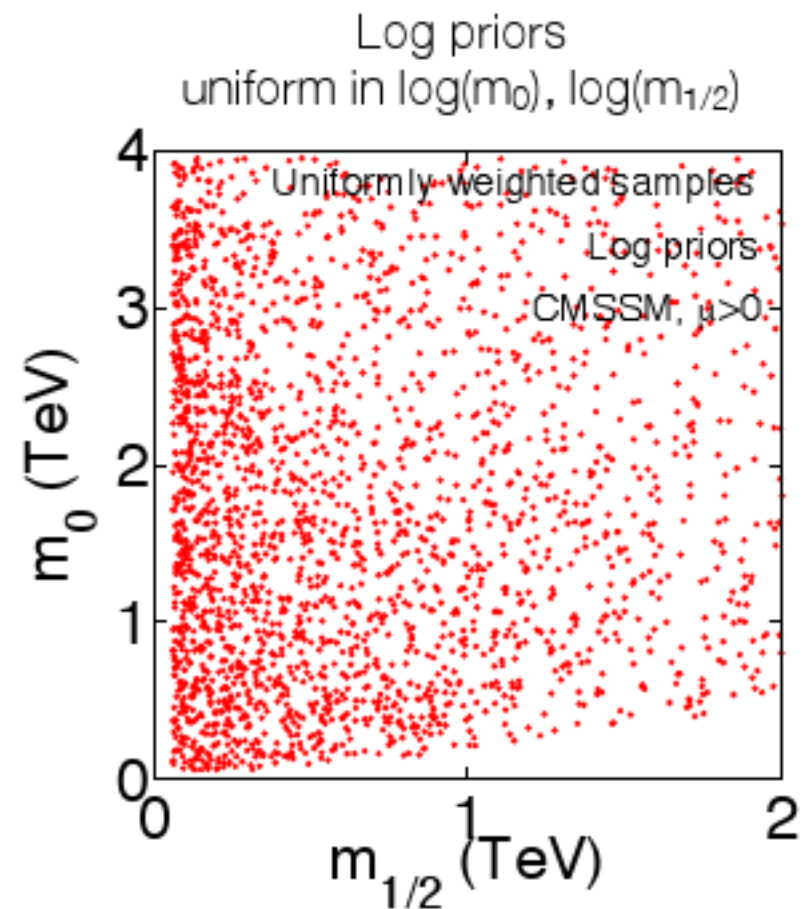
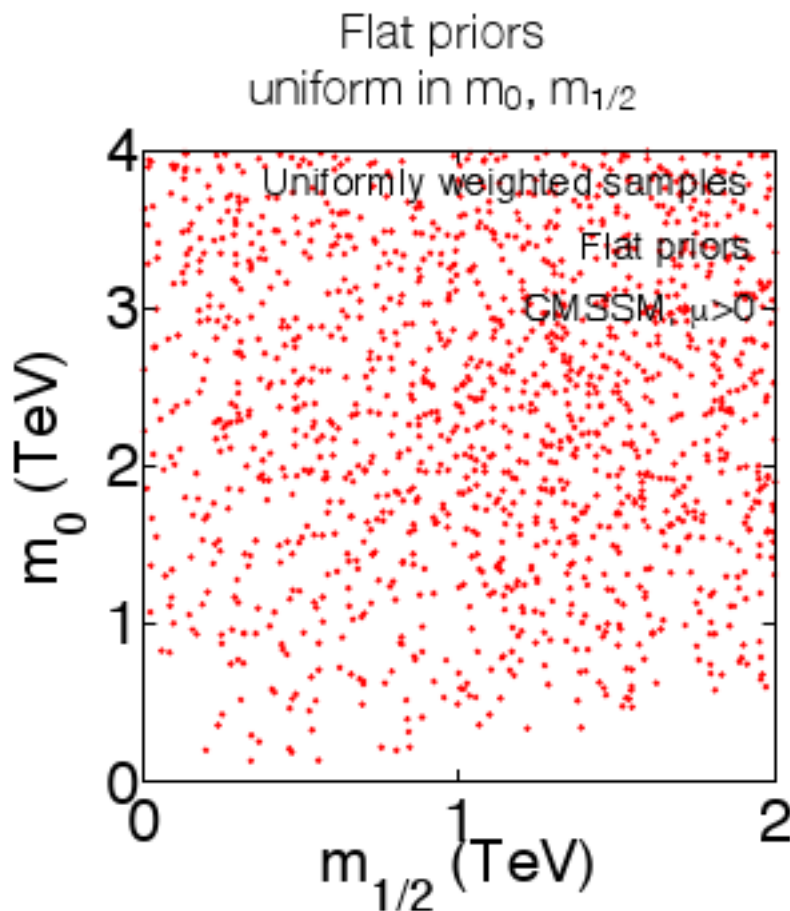
$$|A_0| < 7 \text{ TeV}$$

$$2 < \tan\beta < 62$$

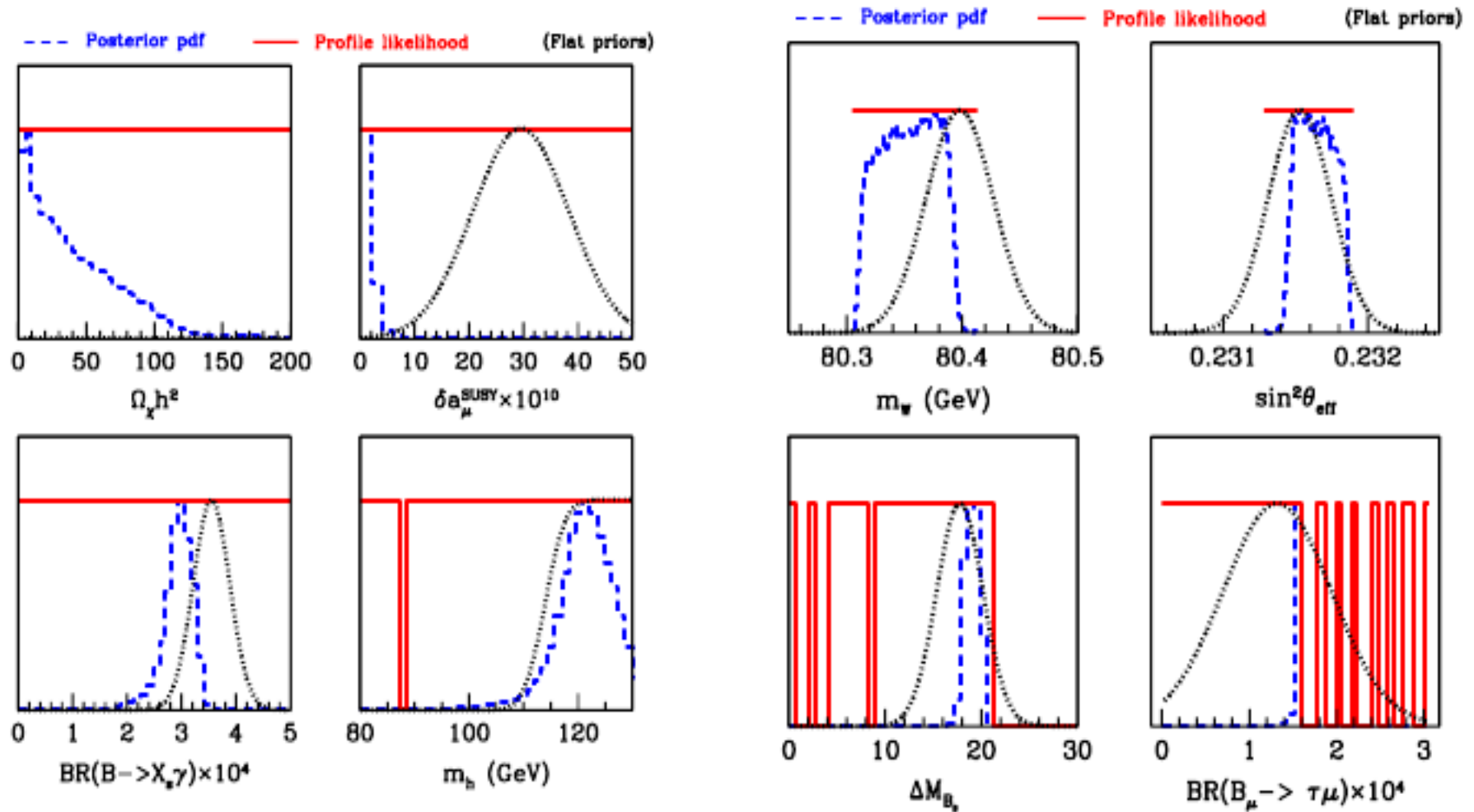
Samples from priors only

- No data in the likelihood, non-physical points discarded priors

$$p(\mathcal{F}) = p(m) \left| \frac{dm}{d\mathcal{F}} \right| \Rightarrow \text{flat prior on log means } p(m) \propto m^{-1}$$



Priors distributions from observables



Priors are **quite informative** regardless the quantities being constrained !!!

Data included

Indirect observables

Observable	Mean value	Uncertainties		ref.
	μ	σ (exper.)	τ (theor.)	
M_W	80.398 GeV	25 MeV	15 MeV	[30]
$\sin^2 \theta_{\text{eff}}$	0.23153	16×10^{-5}	15×10^{-5}	[30]
$\delta a_\mu^{\text{SUSY}} \times 10^{10}$	29.5	8.8	1.0	[31]
$BR(\bar{B} \rightarrow X_s \gamma) \times 10^4$	3.55	0.26	0.21	[32]
ΔM_{B_s}	17.77 ps^{-1}	0.12 ps^{-1}	2.4 ps^{-1}	[33]
$BR(\bar{B}_u \rightarrow \tau \nu) \times 10^4$	1.32	0.49	0.38	[32]
$\Omega_\chi h^2$	0.1099	0.0062	$0.1 \Omega_\chi h^2$	[34]
	Limit (95% CL)		τ (theor.)	ref.
$BR(\bar{B}_s \rightarrow \mu^+ \mu^-)$	$< 5.8 \times 10^{-8}$		14%	[35]
m_h	$> 114.4 \text{ GeV}$ (SM-like Higgs)		3 GeV	[36]
ζ_h^2	$f(m_h)$ (see text)		negligible	[36]
$m_{\tilde{q}}$	$> 375 \text{ GeV}$		5%	[25]
$m_{\tilde{g}}$	$> 289 \text{ GeV}$		5%	[25]
other sparticle masses	As in table 4 of ref. [6].			

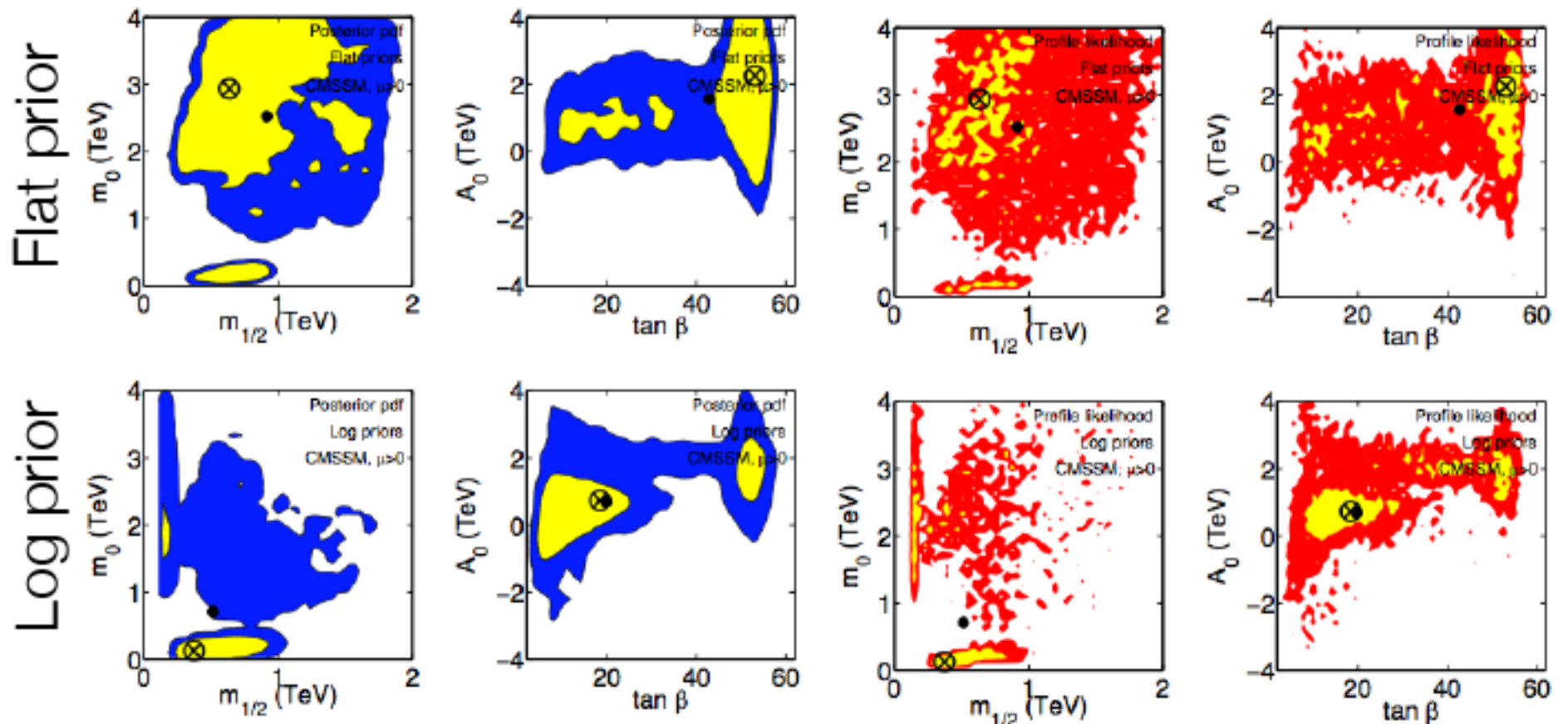
SM parameters

SM (nuisance) parameter	Mean value μ	Uncertainty σ (exper.)	Ref.
M_t	172.6 GeV	1.4 GeV	[24]
$m_b(m_b)^{\overline{MS}}$	4.20 GeV	0.07 GeV	[25]
$\alpha_s(M_Z)^{\overline{MS}}$	0.1176	0.002	[25]
$1/\alpha_{\text{em}}(M_Z)^{\overline{MS}}$	127.955	0.03	[26]

2D posterior vs profile likelihood

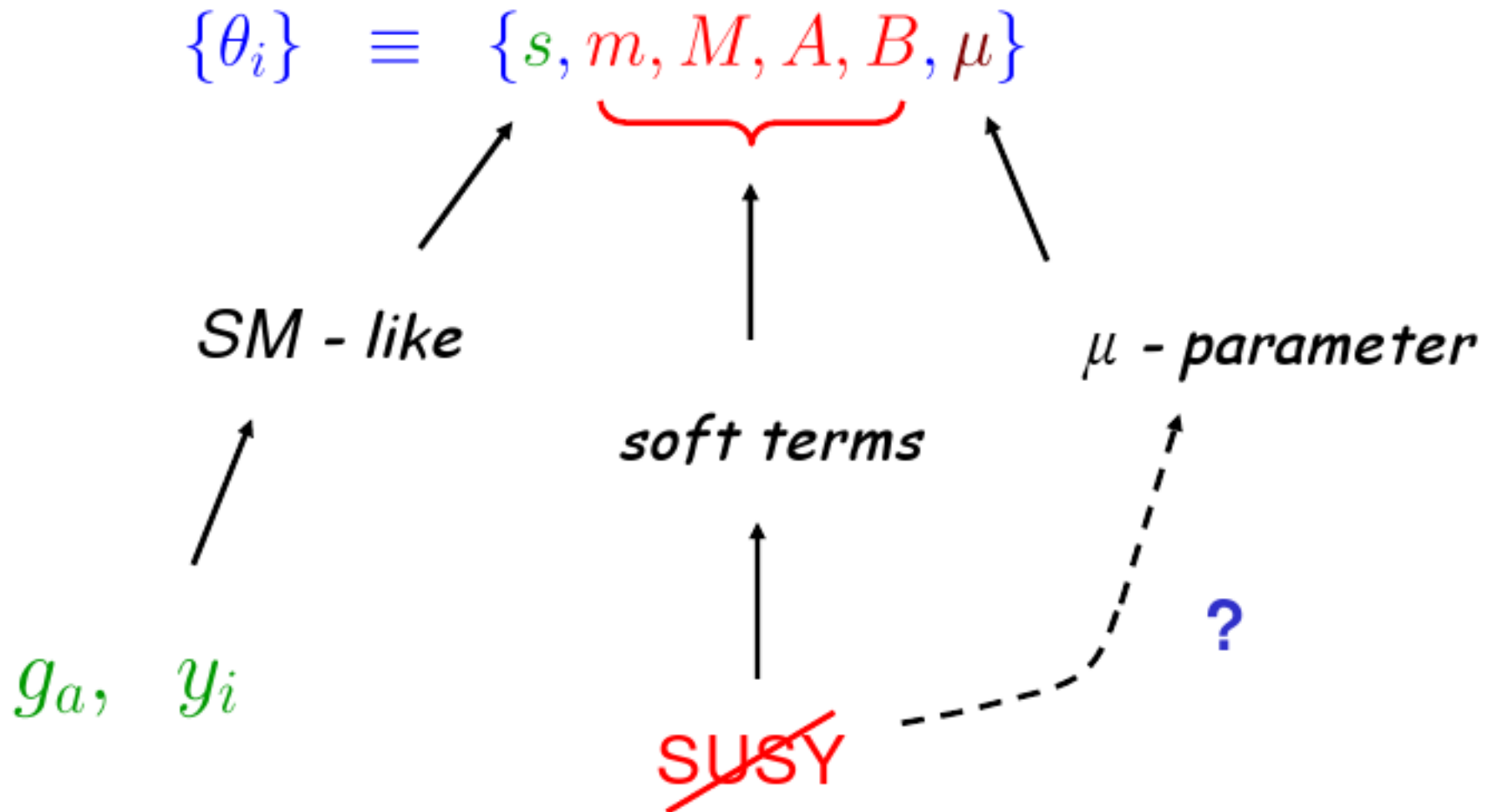
Posterior

Profile likelihood



$$D_{KL} = \int d\theta P(\theta|d) \ln \frac{P(\theta|d)}{P(\theta)} = -\ln P(d) - \langle \chi^2/2 \rangle$$

Towards a more refine analysis



Universality at High Scale supported for FCNC constraints

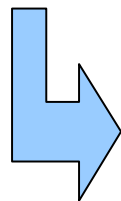
Goal

- Scan the model

$$\{\theta_i\} = \{s, m, M, A, B, \mu\}$$

evaluating

$$p(\theta_i|\text{data}) \propto p(\text{data}|\theta_i) p(\theta_i)$$



Forecast map for LHC

Bayesian and Naturalness

- Recall an usual assumption

$$\underbrace{m, M, A, B, \mu}_{(\equiv M_{\text{soft}})} \text{ should be } \lesssim \mathcal{O}(TeV)$$

- In order to get a

Natural Electroweak symmetry Breaking
(with no fine-tunings)

...

$$V(H_1, H_2) = m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 - 2B\mu H_1 H_2 \\ + \frac{1}{8}(g^2 + g'^2) (|H_1|^2 - |H_2|^2)^2$$

$$M_Z^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} - 2\mu^2$$

$$\sin 2\beta = \frac{2\mu}{B} (m_{H_1}^2 + m_{H_2}^2 + 2\mu_{\text{low}}^2)$$

Unnatural fine-tuning
unless $M_{\text{soft}} \lesssim \mathcal{O}(\text{TeV})$

...

- Instead solving μ^2 in terms of M_Z and the other soft-terms, treat M_Z as another exp. Data
- Approximate the likelihood as

$$\mathcal{L} = N_Z e^{-\frac{1}{2} \left(\frac{M_Z - M_Z^{\text{exp}}}{\sigma_Z} \right)^2} \mathcal{L}_{\text{rest}}$$

$$\simeq \delta(M_Z - M_Z^{\text{exp}}) \mathcal{L}_{\text{rest}}$$

Likelihood associated to
the other observables

...

- Use M_Z to marginalize μ

$$p(s, m, M, A, B | \text{data}) = \int d\mu p(s, m, M, A, B, \mu | \text{data})$$

$$\simeq \mathcal{L}_{\text{rest}} \left[\frac{d\mu}{dM_Z} \right]_{\mu_Z} p(s, m, M, A, B, \mu_Z)$$

$$p(s, m, M, A, B | \text{data}) = 2 \mathcal{L}_{\text{rest}} \frac{\mu_Z}{M_Z} \frac{1}{c_\mu} p(s, m, M, A, B, \mu_Z)$$

$$c_\mu = \frac{\partial \ln M_Z^2}{\partial \ln \theta_i} \quad (\text{Ellis et al, Barbieri-Gudice measure of fine-tuning})$$

$c^{-1} \sim$ Probability of cancellation between the various contributions to get $M_Z \sim \mathcal{O}(90\text{GeV})$

At practical level

- Besides, we have done a similar analysis for the fermion masses

$$y_t \longrightarrow m_t$$

- And traded

$$B \longrightarrow \tan \beta$$

Putting all the pieces together

$$\{\mu, y_t, B\} \xrightarrow{J} \{M_Z, m_t, \tan \beta\}$$

$$p(m_t, m, M, A, \tan \beta | \text{data}) = J|_{\mu=\mu_Z} p(y_t, m, M, A, B, \mu_Z) \mathcal{L}_{\text{rest}}$$

$$p_{\text{eff}}(m_t, m, M, A, \tan \beta)$$

$$p_{\text{eff}}(m_t, m, M, A, \tan \beta) \propto \underbrace{\left[\frac{E}{R_\mu^2} \right] \frac{y}{y_{\text{low}}} \frac{t^2 - 1}{t(1 + t^2)} \frac{B_{\text{low}}}{\mu_Z}}_J p(m, M, A, B, \mu = \mu_Z)$$

model-independent part !

still undefined

It contains the fine-tuning penalization

It penalizes large $\tan \beta$

Finally

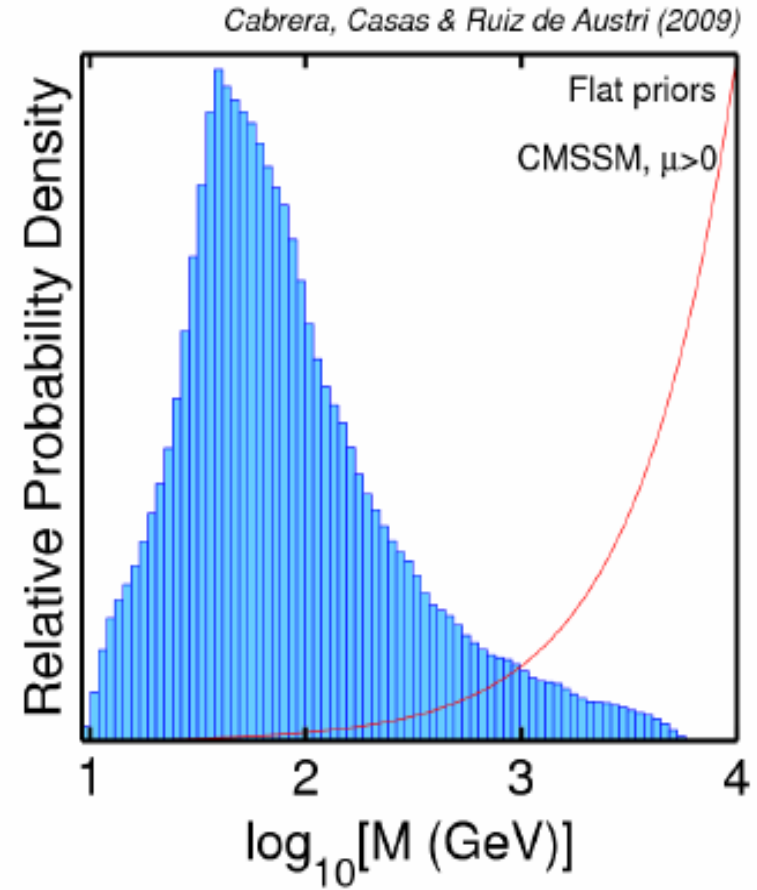
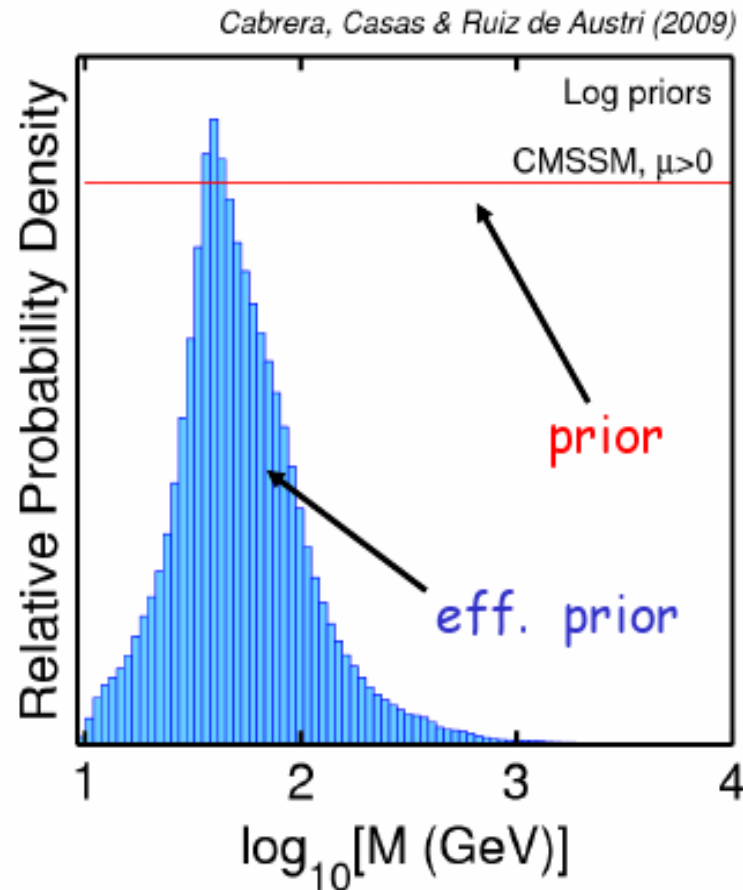
- For the prior

$$p(m, M, A, B, \mu)$$

we take the two basic possibilities:

flat

logarithmic



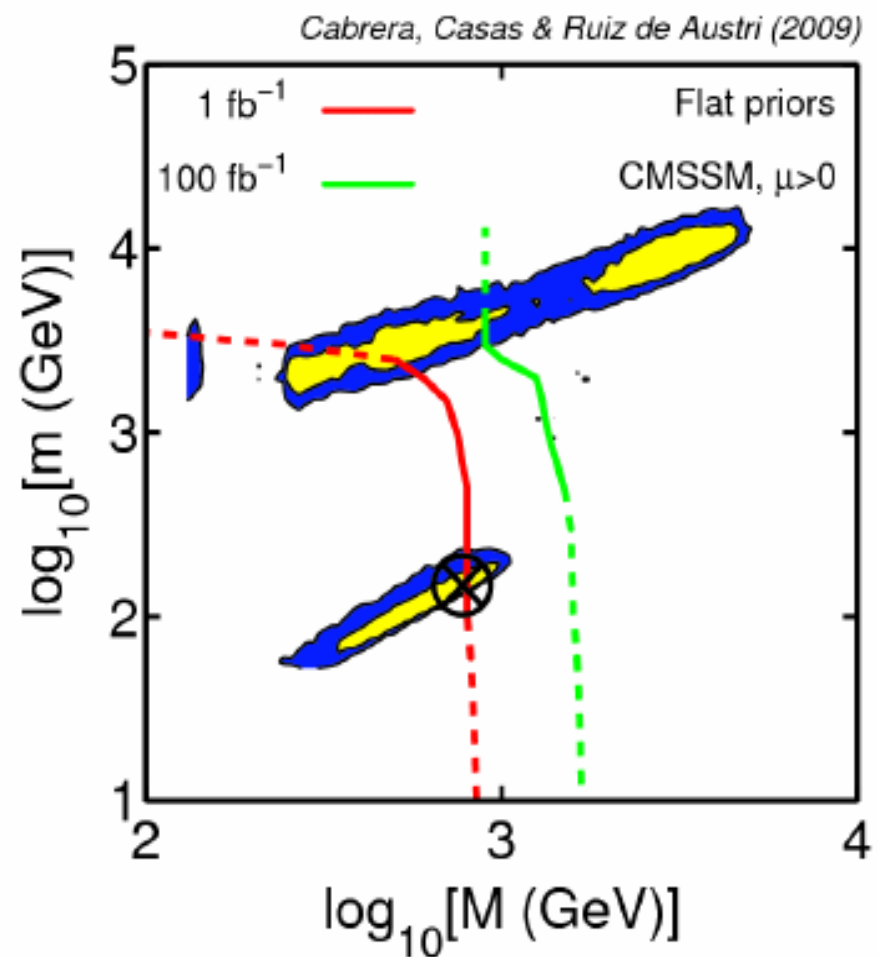
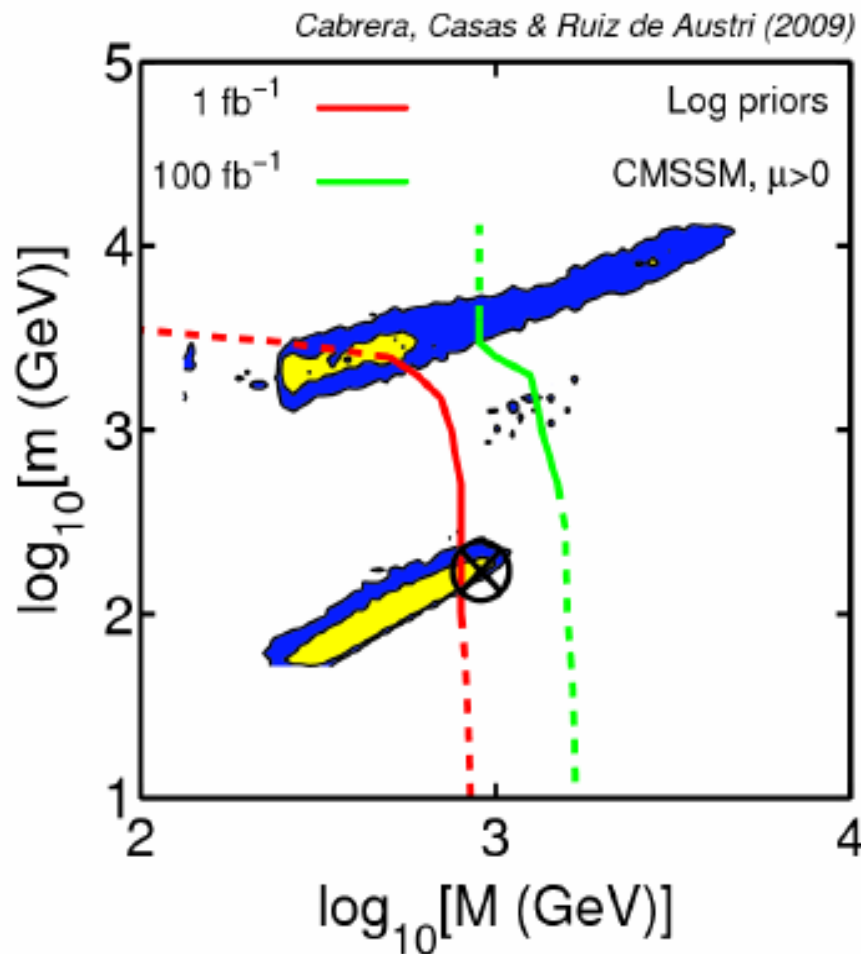
M_Z^{exp} brings SUSY to the LHC region

- We may vary M_{soft} up to M_X the results do not depend on the range chosen
- This suggests that large soft-masses are disfavoured

Data Included

Observable	Mean value	Uncertainties	
	μ	σ (exper.)	τ (theor.)
M_W	80.398 GeV	27 MeV	15 MeV
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$\delta a_\mu^{\text{SUSY}} \times 10^{10} (e^+e^-)$	29.5	8.8	2.0
$\delta a_\mu^{\text{SUSY}} \times 10^{10} (\tau)$	14.0	8.4	2.0
ΔM_{B_s}	17.77 ps ⁻¹	0.12 ps ⁻¹	2.4 ps ⁻¹
$BR(\overline{B} \rightarrow X_s \gamma) \times 10^4$	3.52	0.33	0.3
$\frac{BR(B_u \rightarrow \tau \nu)_{\text{MSSM}}}{BR(B_u \rightarrow \tau \nu)_{\text{SM}}}$	1.28	0.38	0
$\Delta_{0-} \times 10^2$	3.6	2.65	0
$\frac{BR(B \rightarrow D \tau \nu)}{BR(B \rightarrow D e \nu)} \times 10^2$	41.6	12.8	3.5
R_{l23}	1.004	0.007	0
$BR(D_s \rightarrow \tau \nu) \times 10^2$	5.7	0.4	0.2
$BR(D_s \rightarrow \mu \nu) \times 10^3$	5.8	0.4	0.2
$\Omega_\chi h^2$	0.1099	0.0062	$0.1 \Omega_\chi h^2$
	Limit (95% CL)		τ (theor.)
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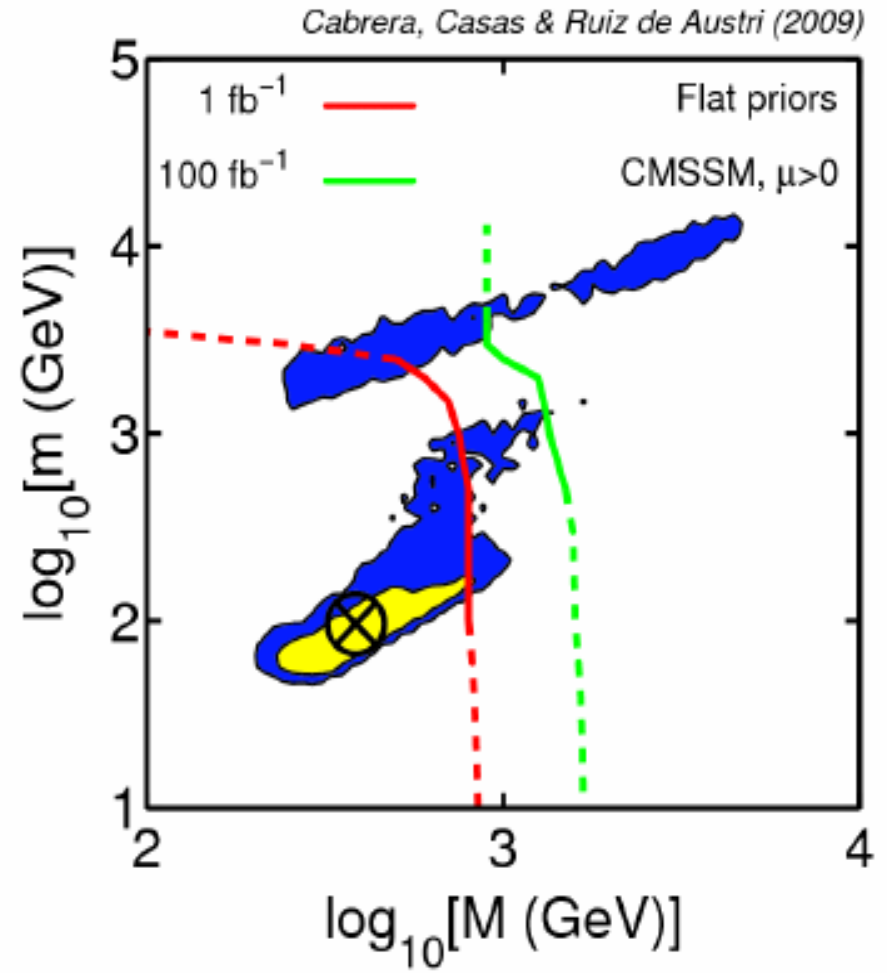
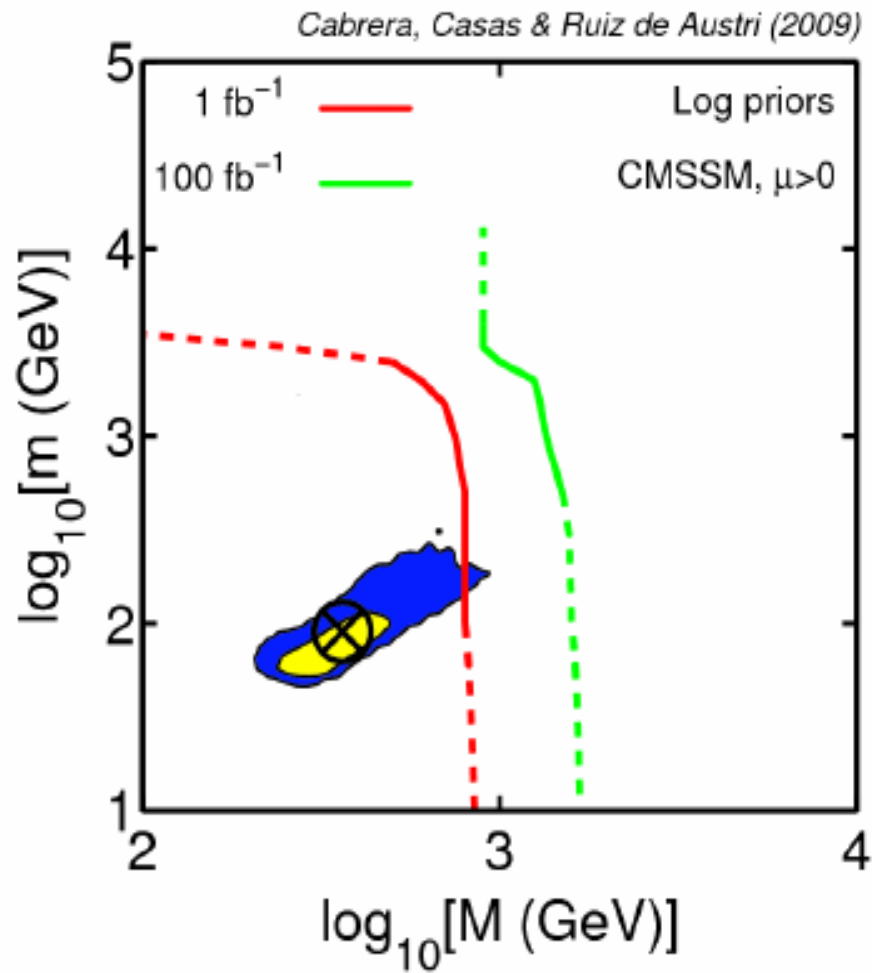
Adding Ω_{DM} [and not g-2]



Events with more or equal to 2 jets [Baer et al 0907.1922]

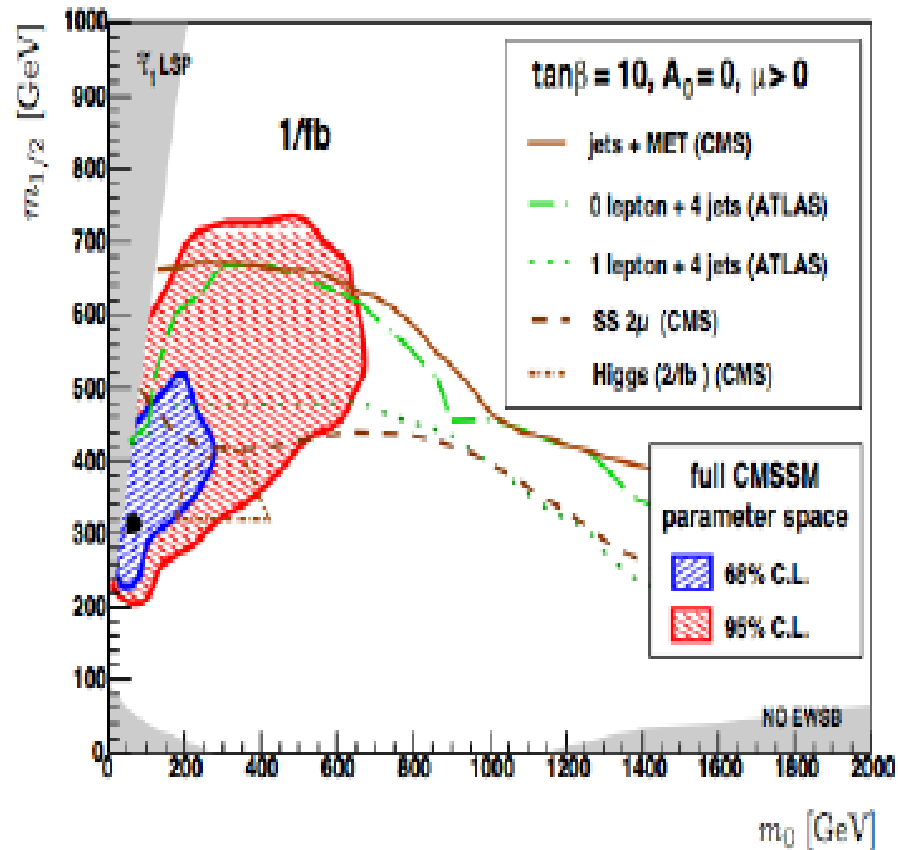
(LHC contours at 14 TeV C.M)

Adding all

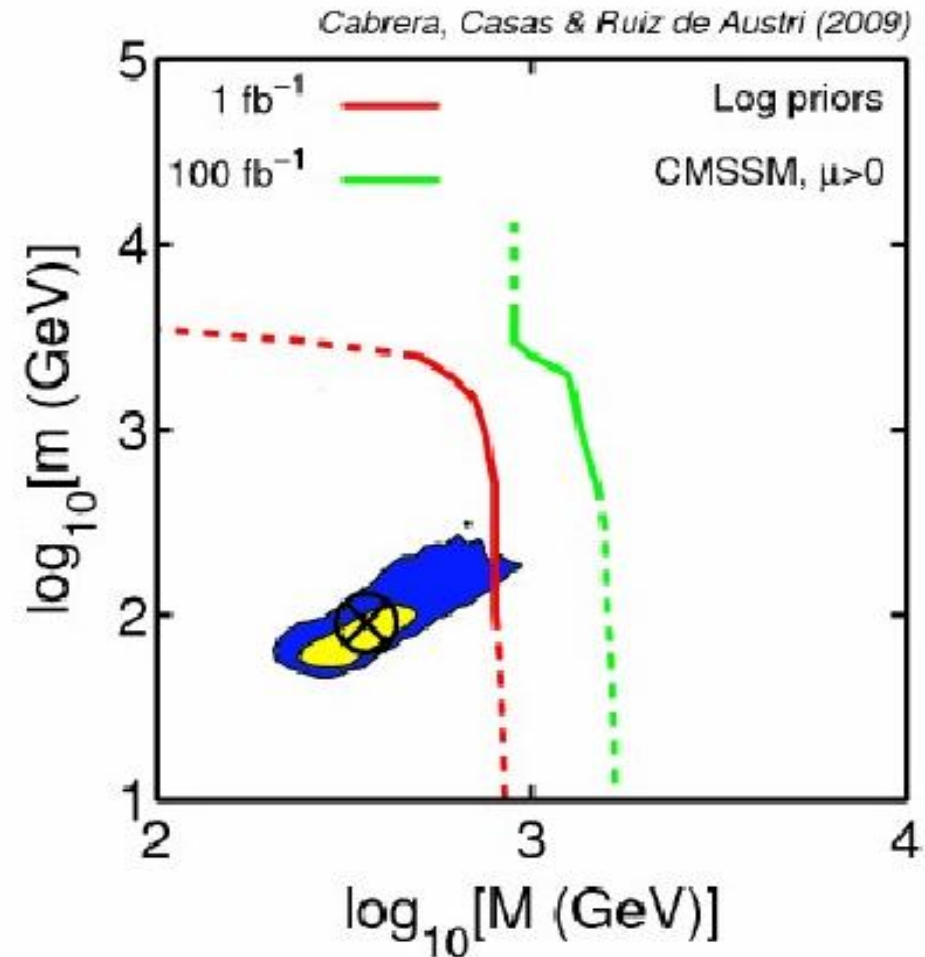


Comparison with Likelihood based inference

Buchmueller et al. (2009)



Cabrera et al. (2009)

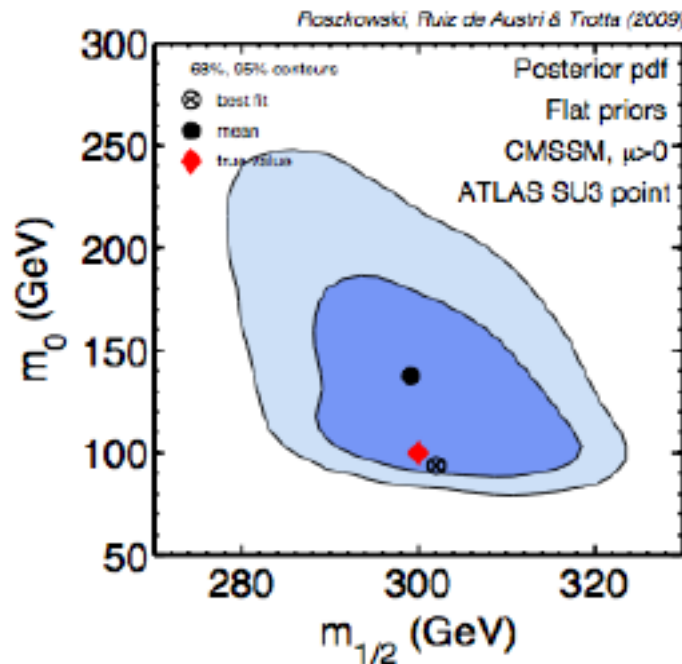


ATLAS will solve the prior dependency

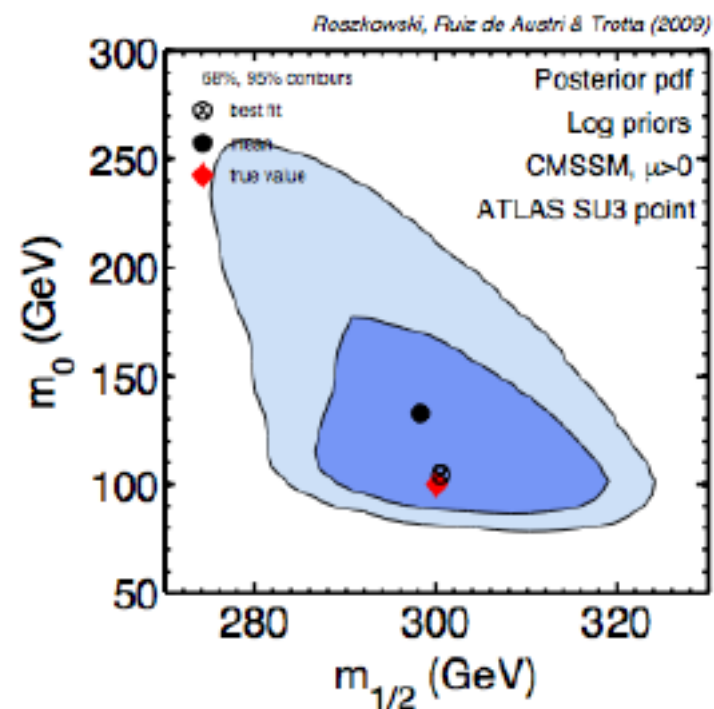
- Projected constraints from ATLAS, (dilepton and lepton+jets edges, 1 fb⁻¹ luminosity)

$$\theta = \{m_{\chi_1^0}, m_{\chi_2^0} - m_{\chi_1^0}, \tilde{m}_{\tilde{l}} - m_{\chi_1^0}, \tilde{m}_{\tilde{q}} - m_{\chi_1^0}\}.$$

Flat prior

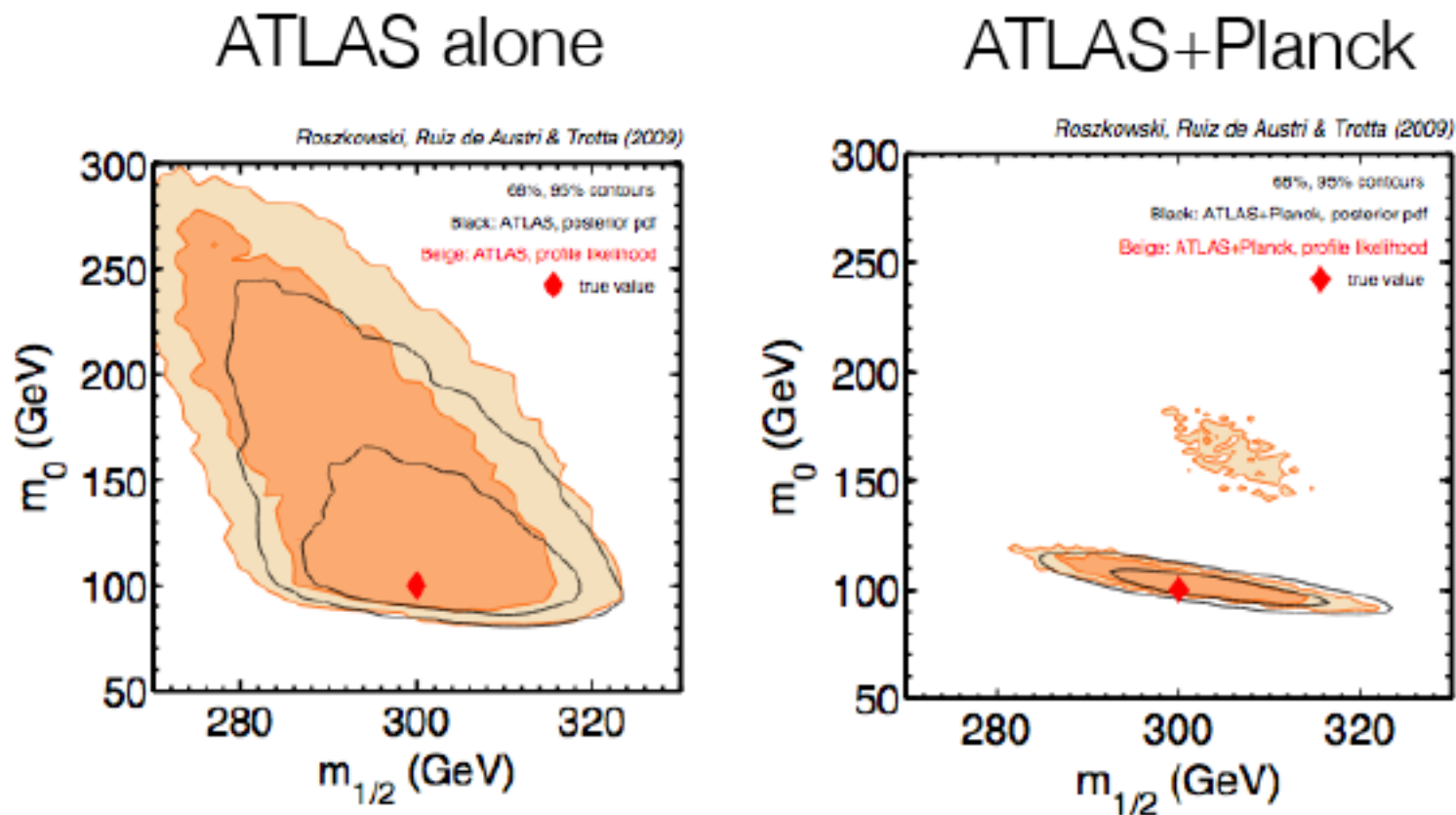


Log prior



Residual dependency on the statistics

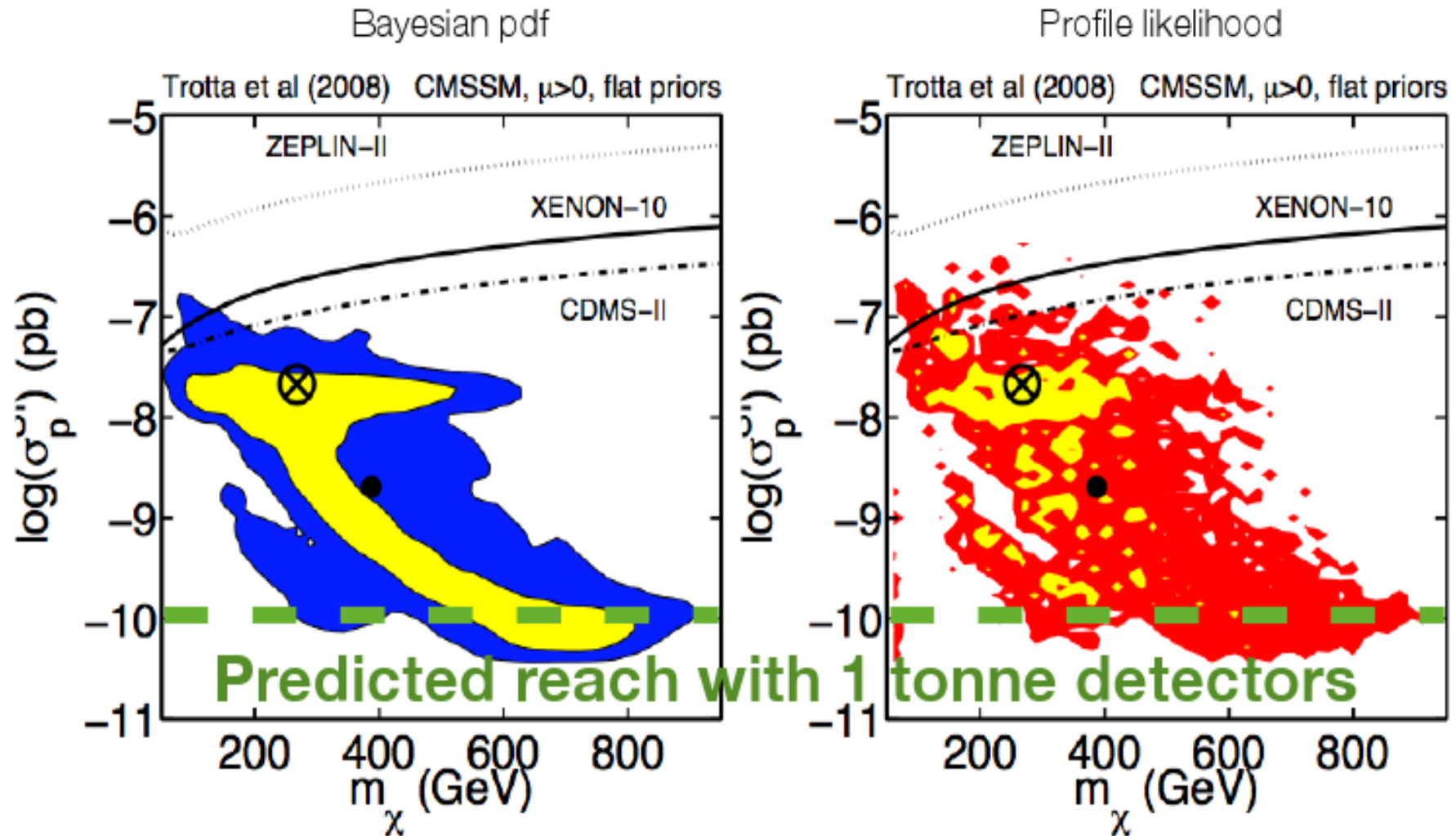
- **Marginal posterior** and **profile likelihood** will remain somewhat discrepant using ATLAS alone. Much better agreement from ATLAS+Planck CDM determination.



Direct and indirect detection prospects

Direct detection prospects

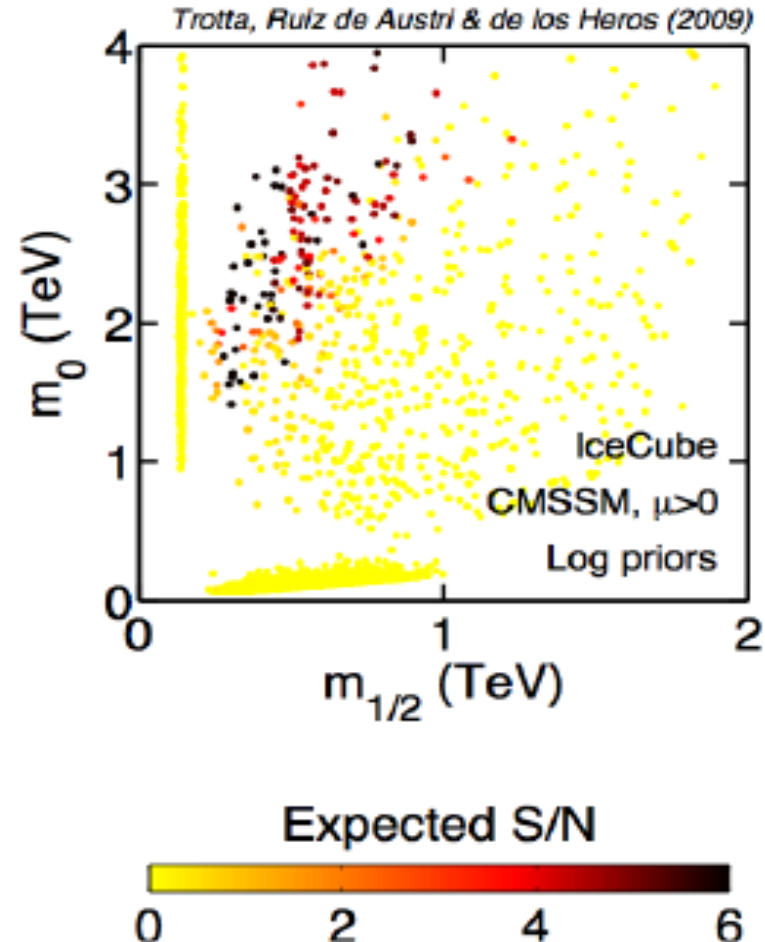
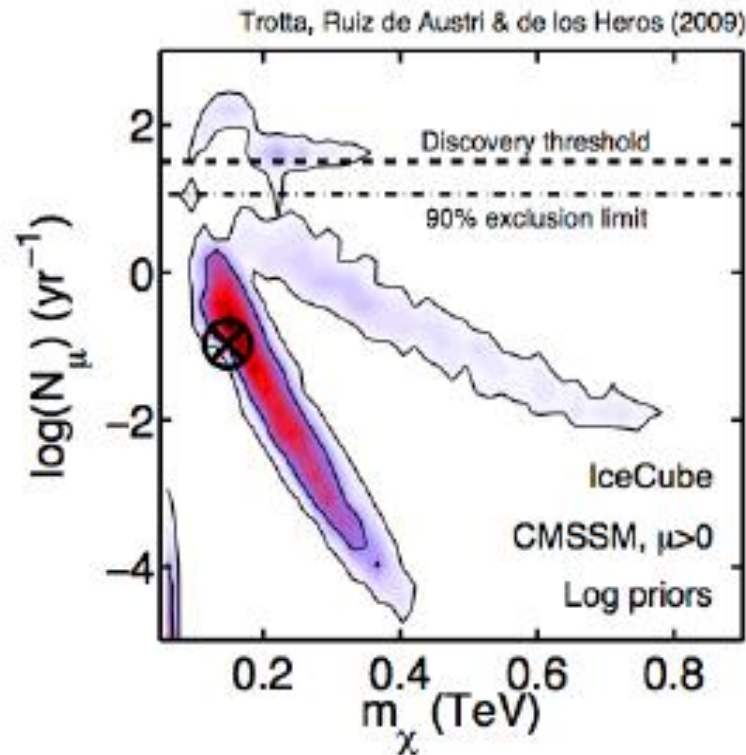
R. Trotta, F. Feroz, M.P. Hobson, R. Ruiz de Austri and L. Roszkowski, 0809.3792



Direct detection experiments will probe most of the **favoured** region of the CMSSM

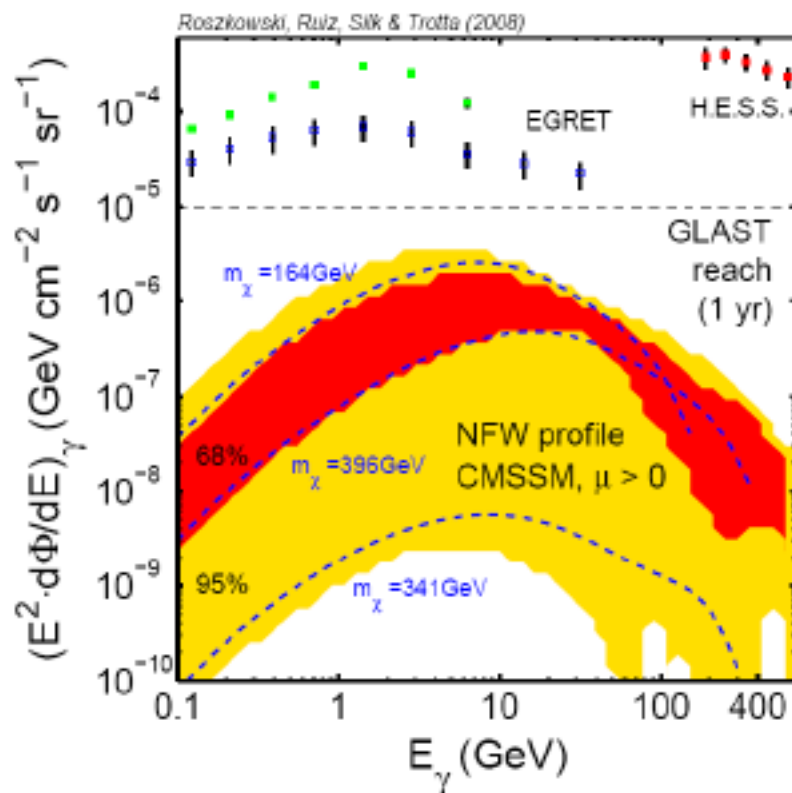
Neutrinos from WIMP annihilations in the Sun

- In the context of the CMSSM, the final configuration of IceCube (with 80 strings) has between 2% and 12% probability of achieving a 5-sigma detection



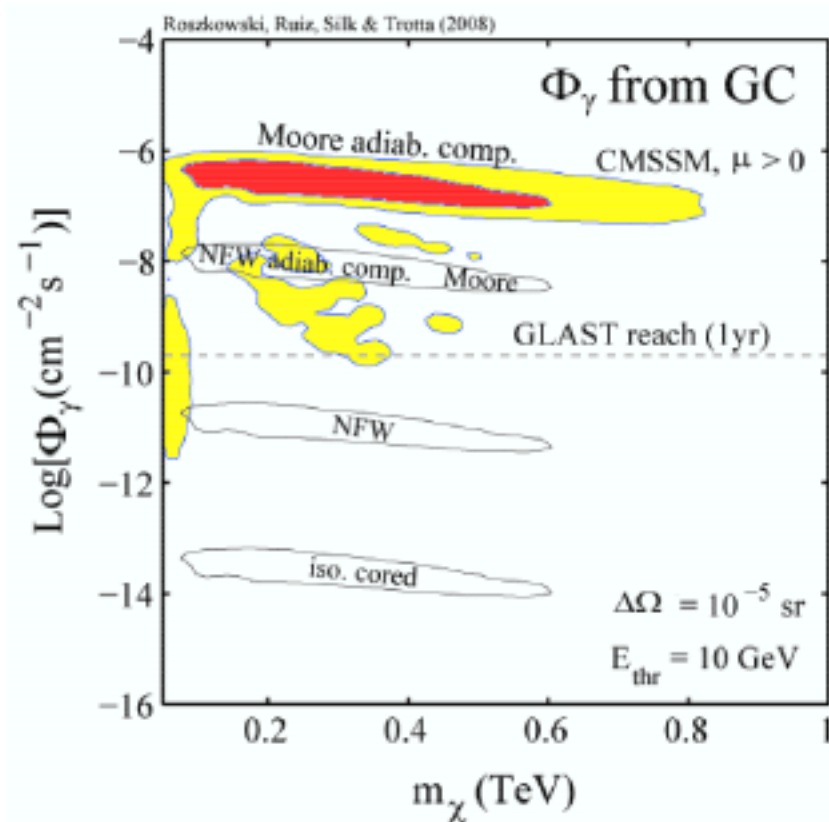
Predictions for Fermi in the CMSSM

Predicted gamma-ray spectrum
probability distribution from the galactic
center at Fermi resolution

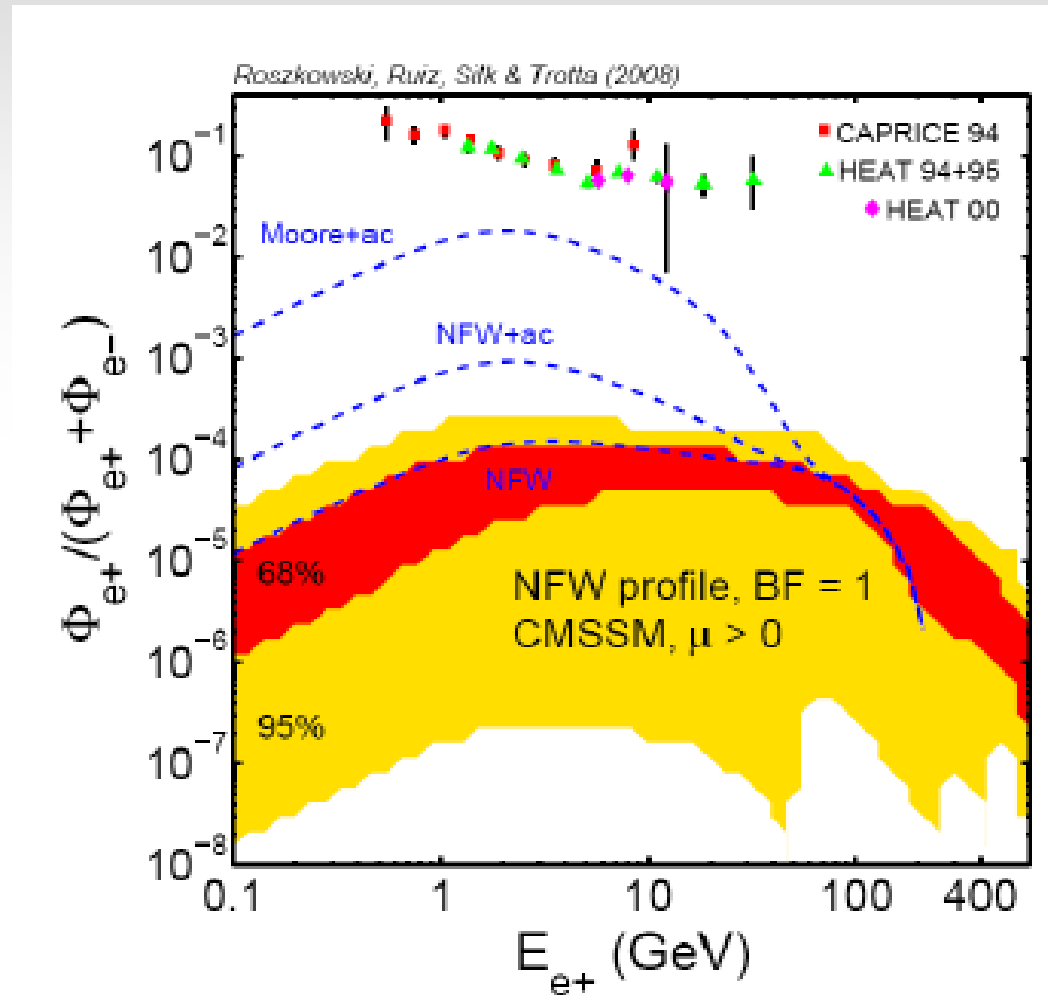


Roszkowski, Ruiz, Silk & RT (2008)

Predicted gamma-ray flux above 10 GeV
at Fermi resolution



Predictions for the positrons spectrum



Conclusions

- SUSY phenomenology provides a timely and challenging problem for parameter inference and model selection. A considerably harder problem than cosmological parameter extraction!
- **Bayesian advantages**: higher efficiency, inclusion of nuisance parameters, predictions for derived quantities, model comparison
- CMSSM only a case study: **Bayesian** analysis naturally **penalizes fine-tunings**. The exp. value of M_Z brings the relevant parameter space to the low-energy region ($\sim$ **accessible to LHC**). The results are quite **stable** under changes of the initial prior (logarithmic or flat) or in the ranges of the parameters
- Currently, even the CMSSM is somewhat underconstrained: **ATLAS+Planck** will take us to “**statistics nirvana**”
- CMSSM neutralino dark matter: direct detection **possible** by the **end of the decade**.

Bayesians address the question everyone is interested in by using assumptions no-one believes, while **Frequentists** use impeccable logic to deal with an issue of no interest to anyone” (L. Lyons)

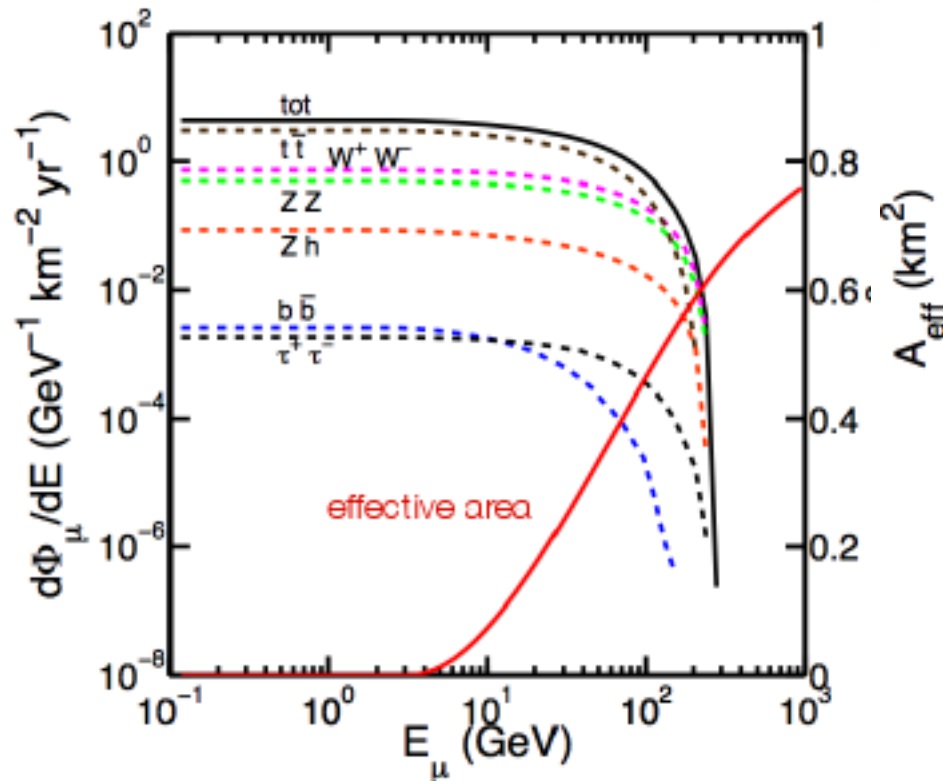
THANKS !!!

Backups

Bias from assuming the wrong final state

Muon events rate =
$$N_\mu = \int_{E_\mu^{\text{th}}}^{+\infty} dE_\mu A_\mu^{\text{eff}}(E_\mu) \frac{d\Phi_\mu}{dE_\mu}$$

↑ effective area ↑ total spectrum - sum over final states



Total events = 34 1/yr

BR's for dominant channels	
$\text{BR}(\chi\chi \rightarrow b\bar{b})$	2.1×10^{-3}
$\text{BR}(\chi\chi \rightarrow t\bar{t})$	0.717
$\text{BR}(\chi\chi \rightarrow W^+W^-)$	0.162
$\text{BR}(\chi\chi \rightarrow ZZ)$	0.094
Muon events fraction for IceCube	
$N_{b\bar{b}}/N_\mu$	5×10^{-5}
$N_{t\bar{t}}/N_\mu$	0.46
$N_{W^+W^-}/N_\mu$	0.30
N_{ZZ}/N_μ	0.21

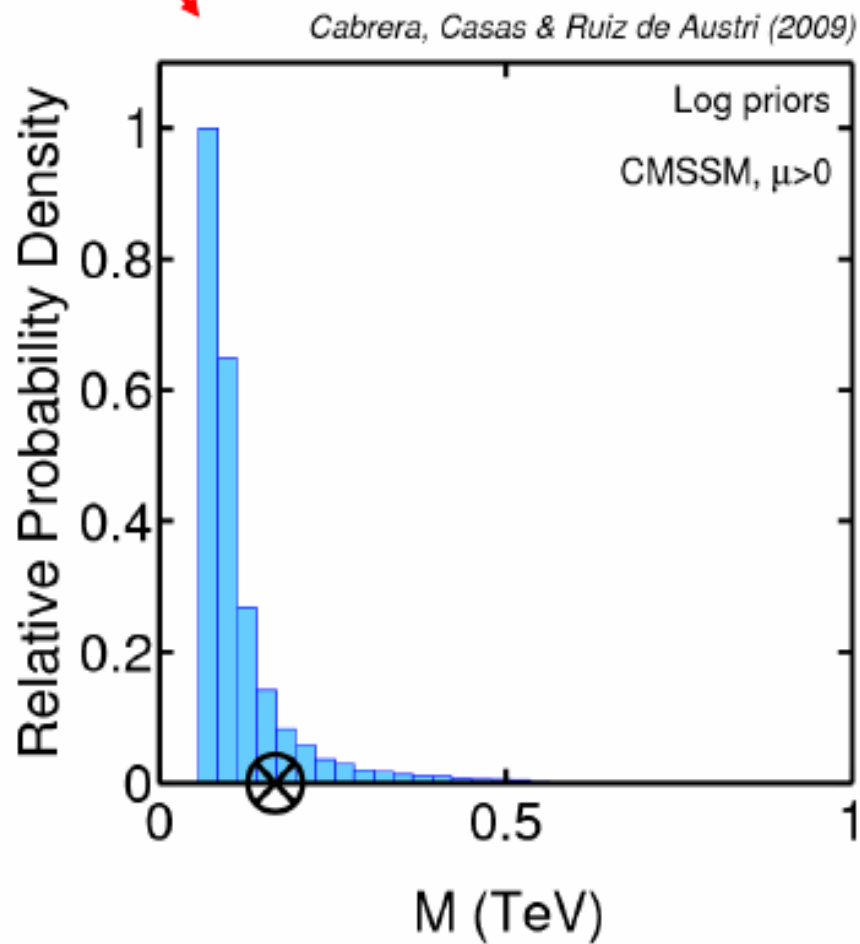
Events/yr assuming only 1 channel

0.8

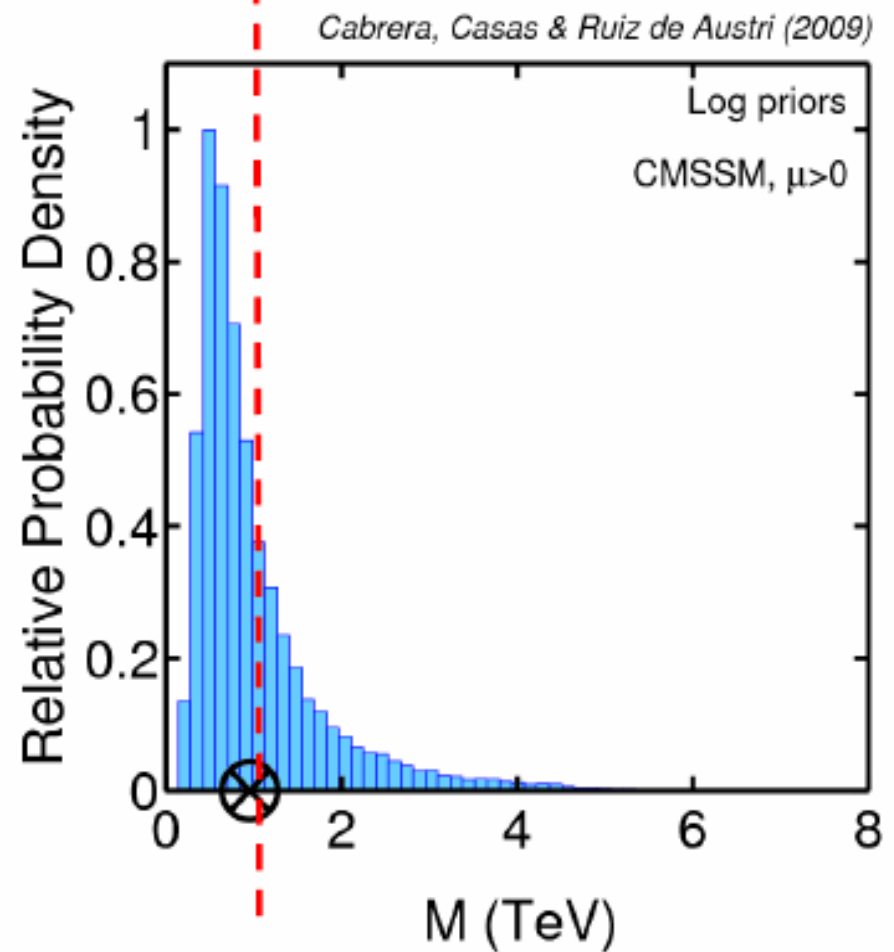
64

EW and B-phys. and limits on particle masses

just with M_Z^{exp}

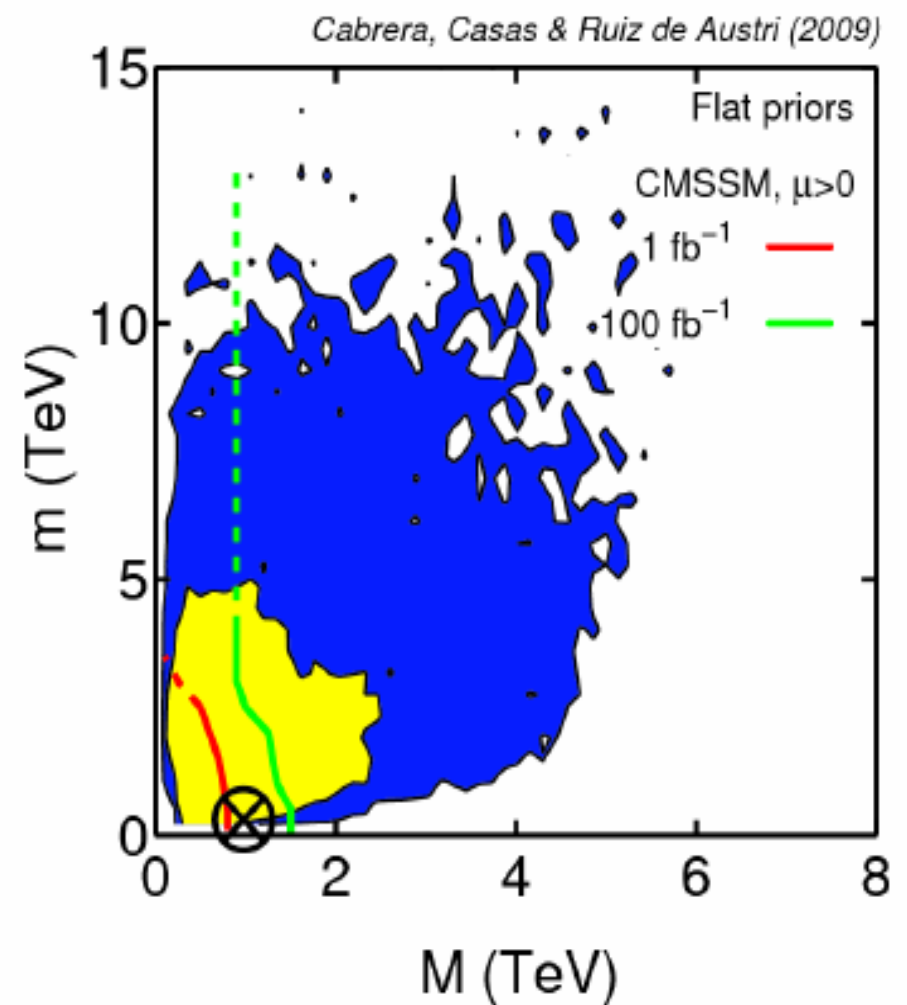
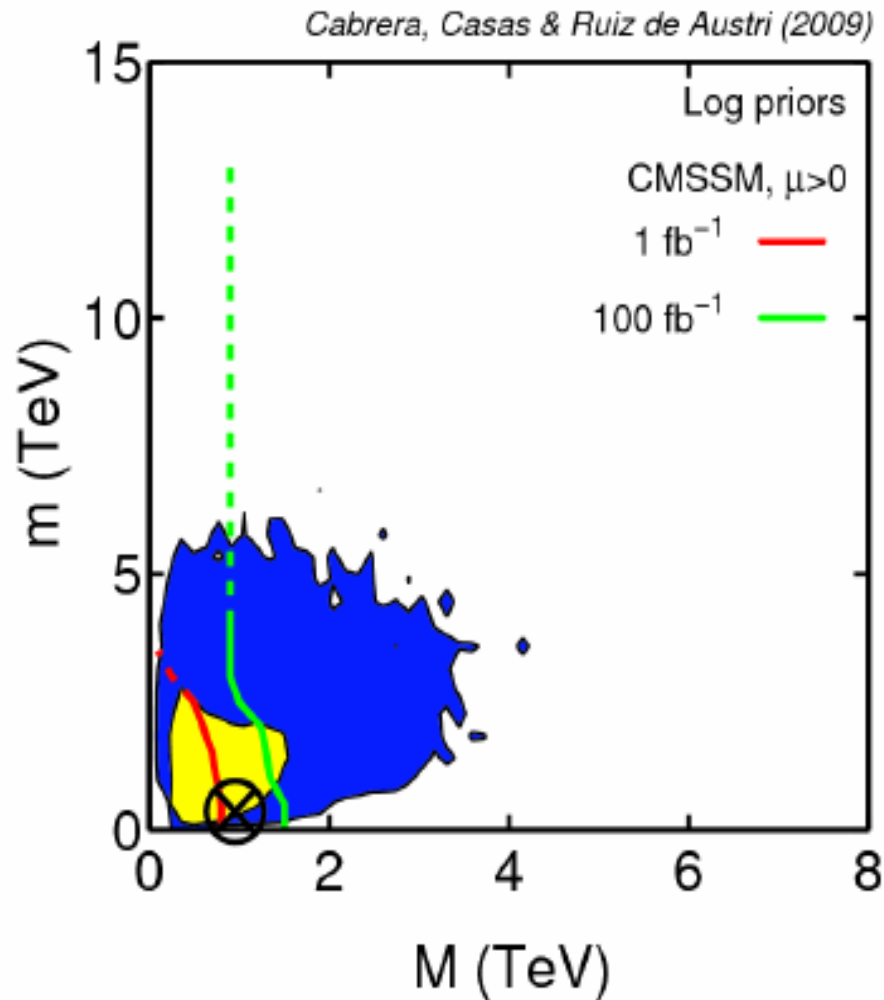


$\sim 100 \text{ GeV}$



$\sim 1 \text{ TeV}$

2D



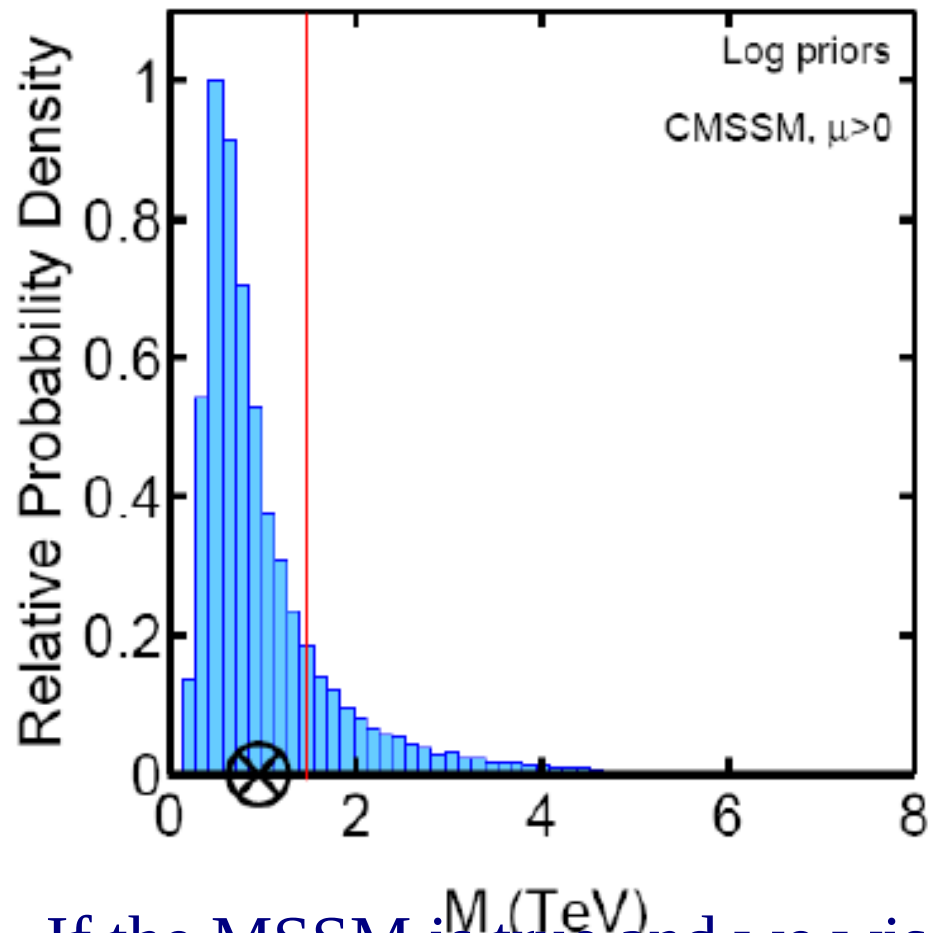
Events with more or equal to 2 jets [Baer et al 0907.1922]

(LHC contours at 14 TeV C.M.)

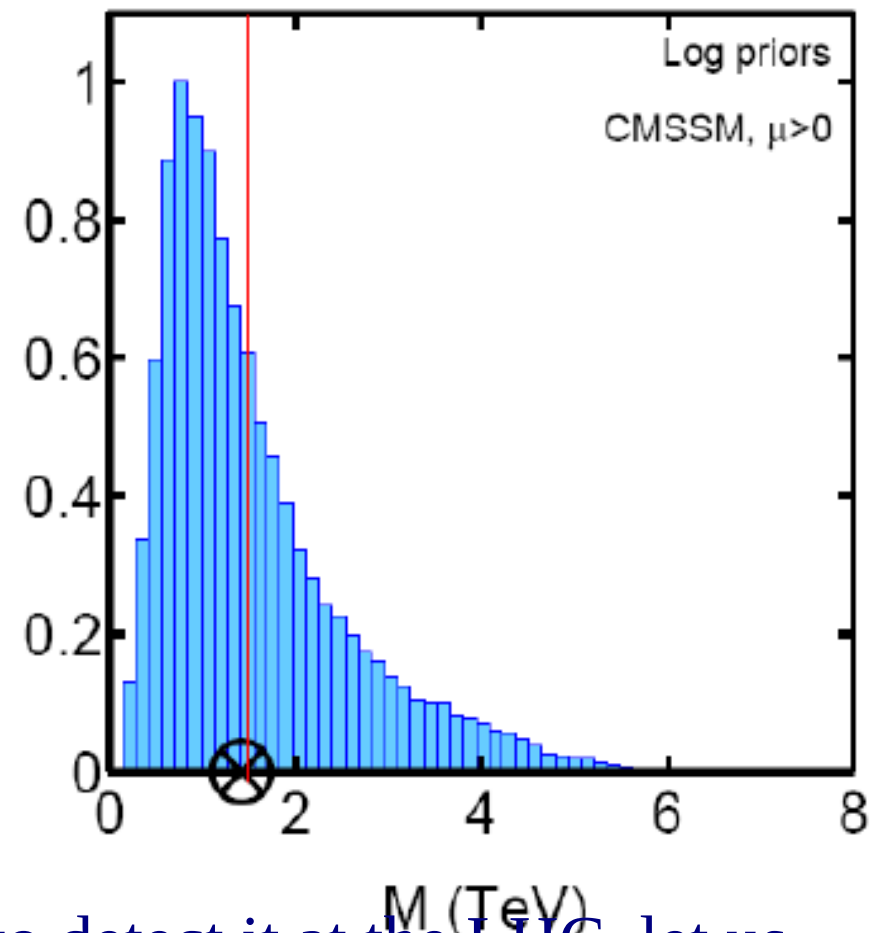
$$m_h > 114 \text{ GeV}$$

$$m_h > 120 \text{ GeV}$$

Cabrera, Casas & Ruiz de Austri (2009)



Cabrera, Casas & Ruiz de Austri (2009)



If the MSSM is true and we wish to detect it at the LHC, let us **hope** that the Higgs mass is **close** to the present exp. **limit**

Experimental Constraints

We have considered 3 groups of exp. Constraints

E.W. and B-physics observables,
and limits on particle masses

Constraints from $(g-2)_\mu$

Constraints from Dark Matter
abundance

Nested

- New technique for efficient evidence evaluation (and posterior samples) (Skilling 2004)

- Define $X(\lambda) = \int_{L(\theta) > \lambda} \pi(\theta) d\theta$
- Write inverse $L(X)$, i.e. $L(X(\lambda)) = \lambda$
- Evidence becomes one-dimensional integral

$$E = \int L(\theta) \pi(\theta) d\theta = \int_0^1 L(X) dX$$

- Suppose can evaluate $L_j = L(X_j)$ where

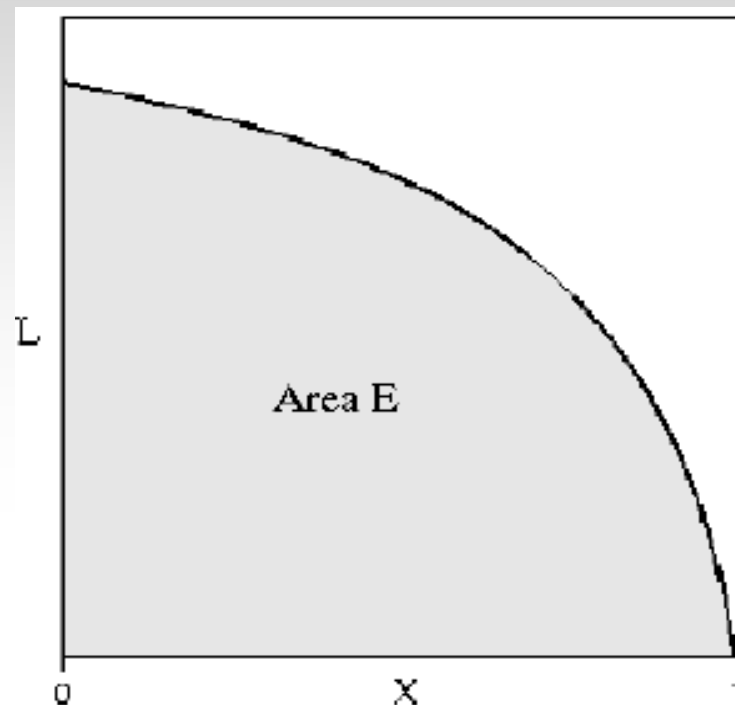
$$0 < X_m < \dots < X_2 < X_1 < 1$$

- $\Rightarrow$ estimate E by any numerical method:

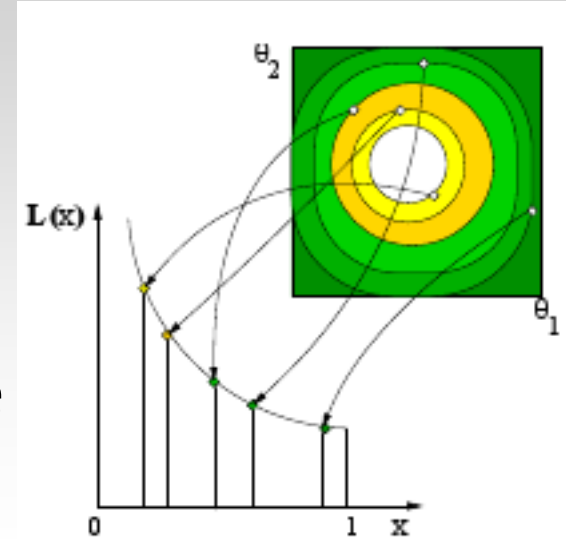
- $w_j = \frac{1}{2}(X_{j-1} - X_{j+1})$ for trapezium r

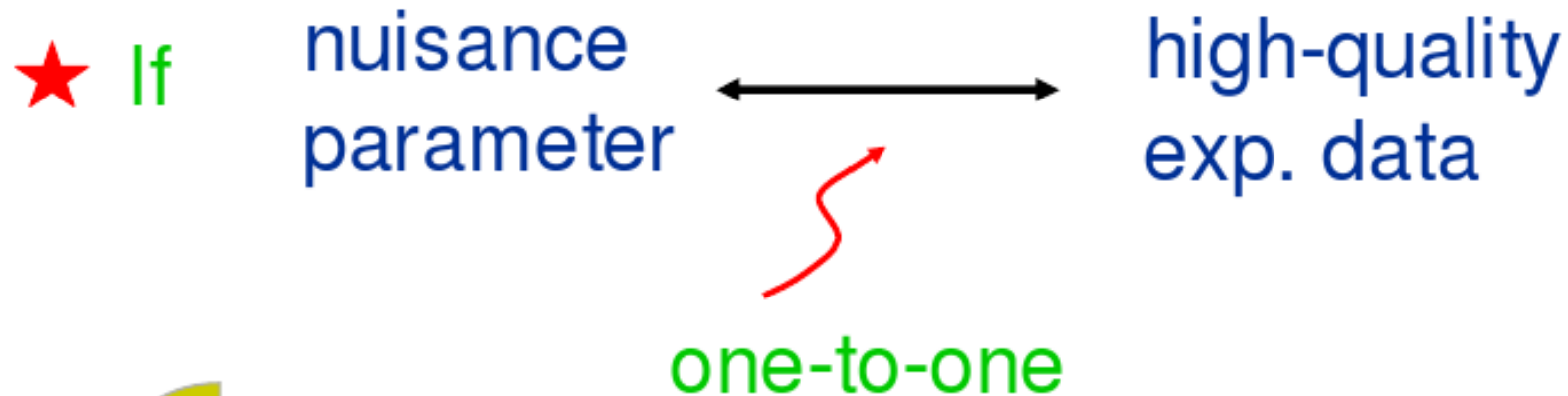
- Posterior as a by product

$$E = \sum_{j=1}^m L_j w_j$$



- 1. Set $i = 0$; initially $X_0 = 1, E = 0$
- 2. Sample N points $\{\theta_j\}$ randomly from $\pi(\theta)$ and calculate their likelihoods
- 3. Set $i \rightarrow i + 1$
- 4. Find point with lowest likelihood value (L_i)
- 5. Remaining prior volume $X_i = t_i X_{i-1}$ where
- 6. Increment $\Pr(t_i|N) = N t_i^{N-1}$;
- 7. Remove lowest point $E \rightarrow E + L_i w_i$
- 8. Replace with new point sampled from $\pi(\theta)$ within hard-edged region
- 9. If $L(\theta) > L_i$ (where some tolerance)
 $L_{\max} X_i < \alpha E$
 $\Rightarrow E \rightarrow E + X_i \sum_{j=1}^N L(\theta_j)/N$





Then, the parameter is easily eliminated (without leaving any footprint)

E.g. g, g', g_3

(their prior is irrelevant)

★ This is **not** the case for the Yukawa couplings, y_i , in the **MSSM**

$$\text{E.g. } m_t = \frac{1}{\sqrt{2}} y_t v_2 = \frac{1}{\sqrt{2}} y_t v \sin \beta$$

↑
Derived
quantity

Two different points in the **MSSM**-
par. space will have in general
different y_t . Thus the relative
probability depends upon $p(y_t)$.

(something ignored in
previous literature)

★ Thus the marginalization of y_i leaves a footprint in the pdf

$$\text{Likelihood} \sim \delta(m_t - m_t^{\text{exp}}) \delta(m_b - m_b^{\text{exp}}) \dots$$

Take $m_t = \frac{1}{\sqrt{2}} y_t^{\text{low}} v s_\beta, \quad m_b = \frac{1}{\sqrt{2}} y_b^{\text{low}} v c_\beta, \quad \dots$

(with $y_i^{\text{low}} = R_i y_i$)



$$\int [dy_t dy_b \dots] p(y, m, M, A, B | \text{data}) \sim p(y) \left| \frac{dy_t}{dm_t} \right| \left| \frac{dy_b}{dm_b} \right| \dots = p(y) s_\beta^{-1} c_\beta^{-1} \dots$$

Normally people just take y_i “as needed” to reproduce m_i and forget about.

This equivaless to take

$$p(y_i) \propto \frac{1}{y_i} \quad (\text{log prior})$$

reasonable....

$$y_e \sim 10^{-6}, \quad y_t \sim 1$$

In order to write a sensible prior for $\{m, M, A, B, \mu\}$ one has to consider the dynamical origin of these parameters: ~~SUSY~~

They typically go like $\sim \frac{F}{\Lambda} \equiv M_S$

A particular soft term, say A , receives several $\mathcal{O}\left(\frac{F}{\Lambda}\right)$ contributions (dep. on the details of ~~SUSY~~)

So, it is reasonable to expect

$$-qM_S \leq B \leq qM_S$$

$$-qM_S \leq A \leq qM_S$$

$$0 \leq m \leq qM_S$$

$$0 \leq M \leq qM_S$$

$$0 \leq \mu \leq qM_S$$

$$q = \mathcal{O}(1)$$

with “*flat*” probability

$$p(\Lambda) = \frac{1}{2M_S}, \quad \text{etc.}$$

Recall: M_S is the scale of ~~SUSY~~ in the observable
sector

In principle M_S can have any value, say

$$M_S^0 \leq M_S \leq M_X, \quad M_S^0 \sim 10 \text{ GeV}$$

with

$$\text{flat: } p(M_S) = N_{M_S} \quad \text{or} \quad \text{log: } p(M_S) = N_{M_S} \frac{1}{M_S}$$

probability density

Log. prior:

$$\begin{aligned} p(m, M, A, B, \mu) &= \int_{\max\{m, M, |A|, |B|, \mu, M_S^0\}}^{M_X} p(m, M, \mu, A, B) p(M_S) dM_S \\ &\propto \frac{1}{[\max\{m, M, |A|, |B|, \mu, M_S^0\}]^5} \\ &\quad \text{(neglecting } \frac{1}{M_X^5} \text{ terms)} \end{aligned}$$

For a particular parameter, say M :

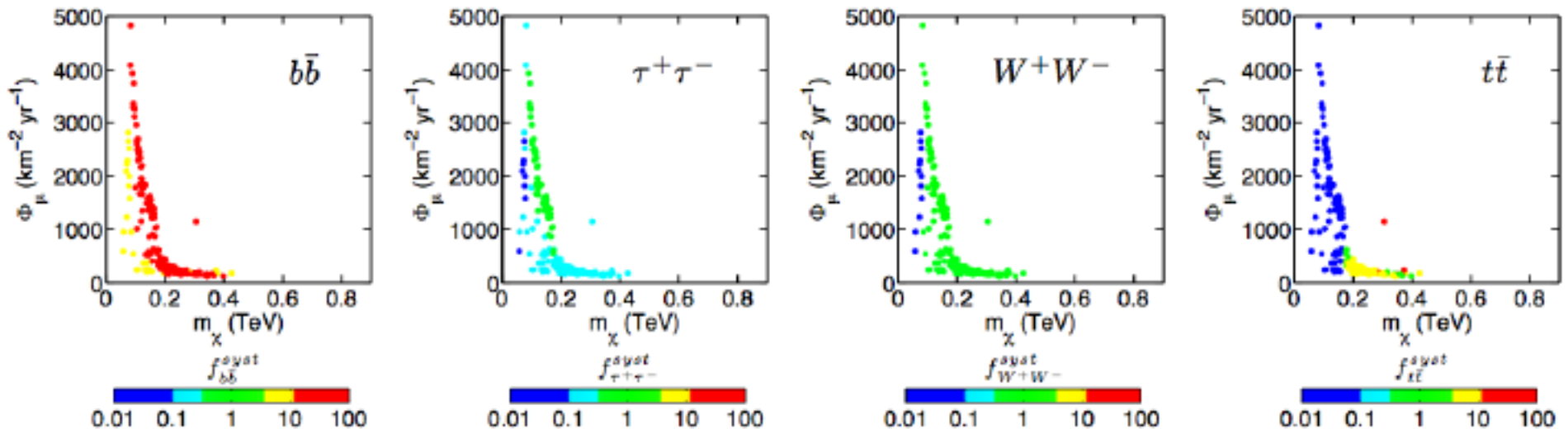
$$\mathcal{P}(M) \propto \frac{1}{\max\{M, M_S^0\}}$$

Bias from assuming the wrong final state

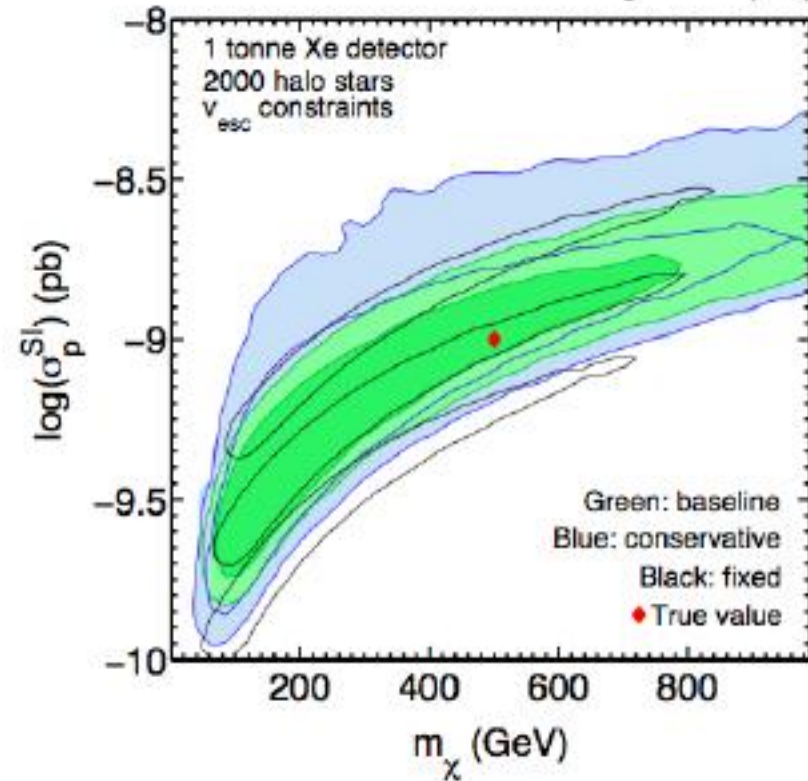
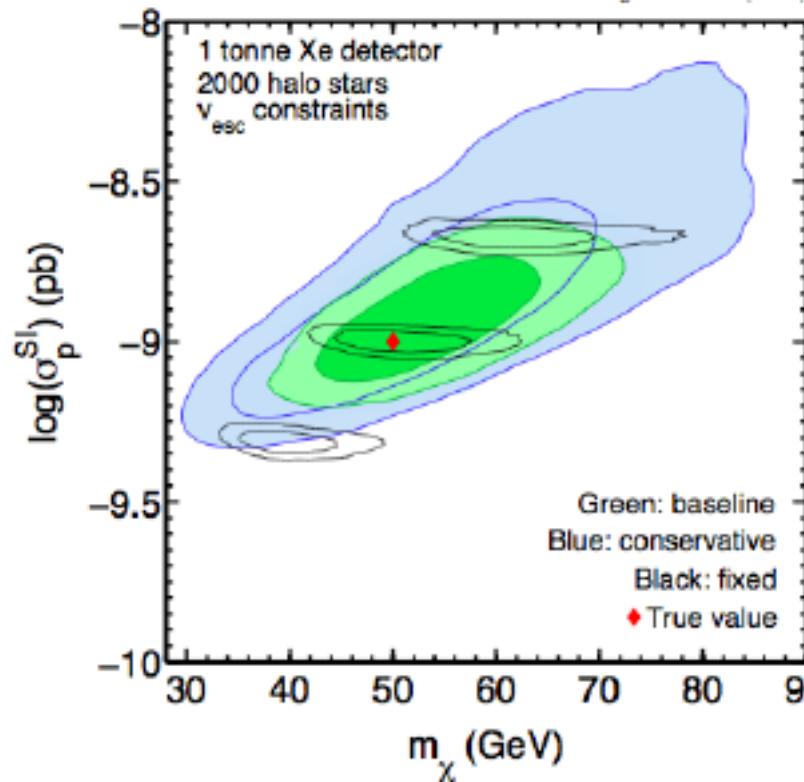
- In general, the systematic error from assuming only 1 dominating channel is given by

$$f_i^{\text{syst}} = \frac{\text{BR}(\chi\chi \rightarrow i)}{N_i/N_\mu},$$

Systematic bias from assuming single-channel domination (IceCube)



The importance of modeling the MW



2) can lead to a 15 sigma bias in the reconstructed σ

On going

- Neural Networks: computation reduced from 3 days to a few minutes
- Other SUSY scenarios and UED
- Analysis of Fermi data as external members of the DM group: DMBayes
- Cosmic rays determination in collaboration with Galprop team
- IceCube DM group collaboration
- Zeplin collaboration

Model independent reconstruction

$$\frac{d\Phi_\gamma}{dE_\gamma} \propto \sum_i \frac{\langle \sigma_i v \rangle}{m_\chi^2} \frac{dN_\gamma^i}{dE_\gamma} \int \rho_\chi^2 dl$$

- Assuming the wrong final state can lead to severe bias in the reconstructed WIMP properties
- Branching ratios must be estimated simultaneously!

