Determination of SUSY parameters with Fittino

- Motivation
- Technology
- SUSY constraints from existing measurements
- LHC + ILC prospects
- Open ends…

Klaus Desch

University of Bonn

The Fittino (core) crew:

Philip Bechtle (DESY), Peter Wienemann (Bonn)
Gabriel Alvarez (Bonn), Felix Buehrer (Freiburg), Bjorn Sarazzin (DESY), Matthias Hamer (Göttingen), Takanori Kono (Hamburg), Xavier Prudent (Dresden), Mathias Uhlenbrock (Bonn) +friends, contributors, users…

Motivation

Standard Model (although very successful) challenged by:

Theory: fine tuning / hierarchy problem

Observation: Higgs still missing
no explanation for dark matter
some (small) deviations of measurements e.g. $(g\text{-}2)_\mu$
neutrino masses?

New Physics expected at the TeV scale by many

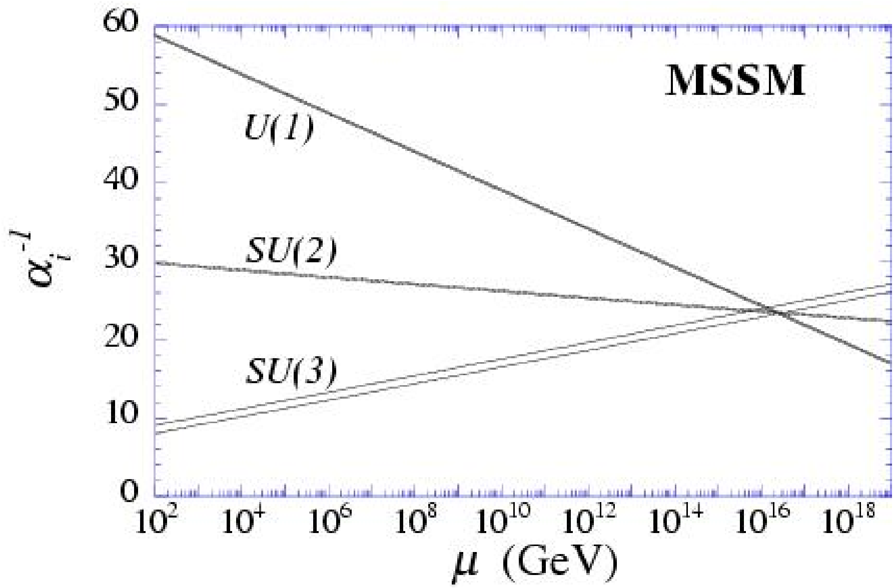
Due to our ignorance, New Physics models usually come with many free parameters – need to be measured!

No matter what we will discover – the „inverse problem" needs technology to be solved

Take Supersymmetry (SUSY) (at least) as a show-case

SUSY is still good!

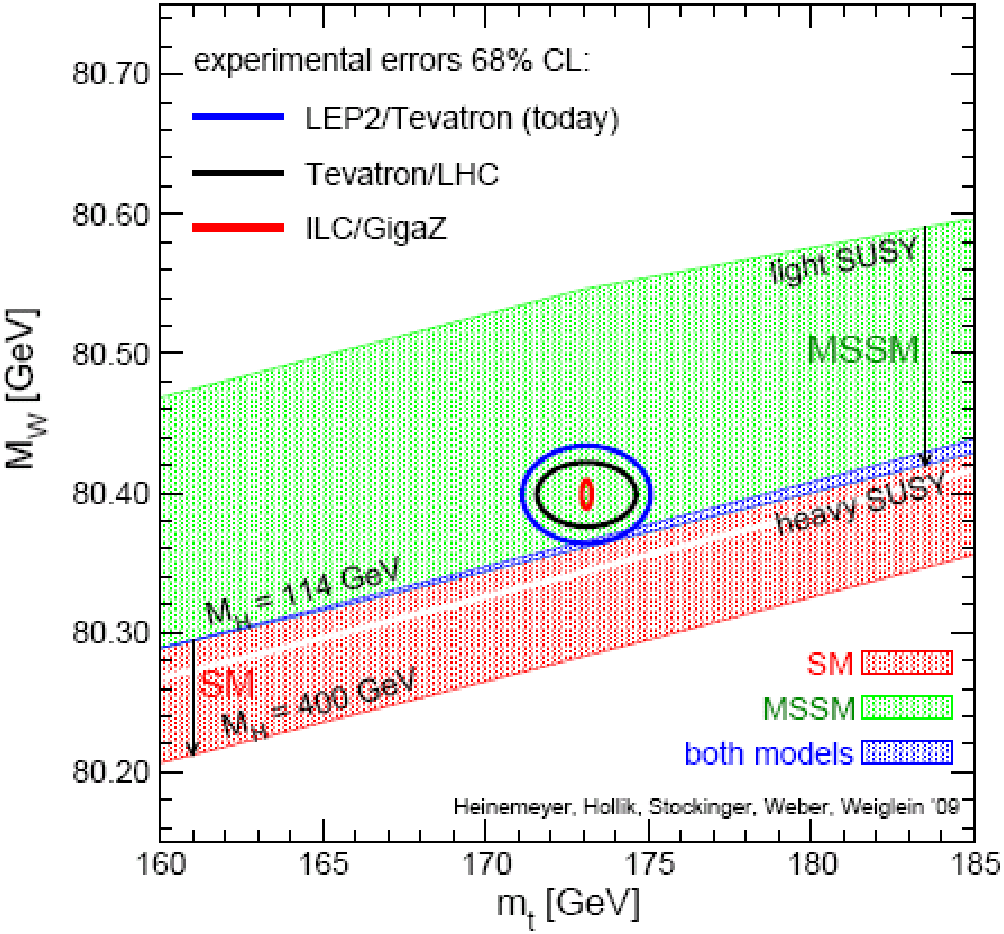
“Experiments within the next five to ten years will enable us to decide whether supersymmetry at the weak interaction scale is myth or reality” **H.P.Nilles, 1984**



This is achieved for $\sin^2\theta_W^{SUSY} = 0.2335(17)$

Experiment: $\sin^2\theta_W^{exp.} = 0.2315(02)$

SUSY is still good!



Precision measurements very consistent with SUSY

Technique

Fittino „History“

Fittino started in back in 2003

- initially to assemble Linear Collider prospects into a common frame

- soon after first (crude) attempt to incorporate LHC prospects

- inputs now:

  „Low energy“ constraints (~ up to LEP)

  Dark Matter (cosmological constraints – only CDM up to now)

  LHC prospects (kinematic edges)

  ILC prospects

- not yet (but in preparation):

  Astrophysical constraints (Direct & Indirect DM search)

  Expected rates from LHC (→ Ben O´Leary´s talk)

  Tevatron exclusions (?)

Fittino „History“

Reconstruction of fundamental SUSY parameters with ~~SFitter~~ Fittino

Philip Bechtle
Peter Wienemann
DESY

3rd Meeting of the EuroGDR “Supersymmetry”
December 11-13, 2003

SUSY parameter determination with SFITTER

Remi Lafaye, Tilman Plehn, Dirk Zerwas

3rd EuroGDR meeting in Orsay

December 11, 2003

Resources

Results in this talk mainly based on

Eur.Phys.J.C 66 **(2010) 215 [arXiv:0907.2589 [hep-ph]]**

New results „in the pipeline" preview:

Fittino-Workshops

https://indico.desy.de/conferenceDisplay.py?confId=2301 (Oct 09)

https://indico.desy.de/conferenceDisplay.py?confId=2741 (Feb 10)

SUSY/BSM Fit group of Helmholtz-Alliance „Physics at the Terascale"

https://indico.desy.de/conferenceDisplay.py?confId=3079 (July 10)

Documentation (could be in better shape ☹)

http://www-flc.desy.de/fittino/

Code: svn://pi.physik.uni-bonn.de/fittino/tags

Technique

Measured observables M_i and errors $cov(M_i,M_j)$

Predicted observables O_i (P_j)

Model Parameters P_j

The job (simple): Find P_j which match M_i best and their uncertainties

1. Minimize

$$\chi^2 = (M - O(P))^T \; cov^{-1} \; M \; (M - O(P)) + \text{(soft) limits.}$$

Standard: Gradient-based (MINUIT) unreliable for weakly constrained systems an/or complex P-spaces

Better minimization: simulated annealing, ...

2. Estimate uncertainties

Standard: assume parabolic χ^2-surface: not correct for complex P-space

Better: a) Intelligent Scan: Markov Chains
b) Rigorous Frequentist approach: Toy Fits

Using Fittino

theory codes interfaced via SLHA2

main workhorses so far: SPHENO (Porod),
Mastercode (Buchmüller et al) + micrOMEGAs

SUSPECT + SOFTSUSY

optional: HiggsBounds (Williams et al)

Steering through free format ASCII file

Experimental inputs which can be specified:
- measured masses
- branching ratios
- kinematic edges of various types
- cross sections x BRs
- „Low energy“ observables $(g-2)_\mu$, …
- Ωh^2
- nuisance parameters (m_Z, m_{top}, G_F, …)

including errors + correlations

Very Short Example Input File

```
#####################################################
###           Fittino input file                  ###
#####################################################
massh0                   115.237 GeV +- 0.05 GeV +- 0.5 GeV # comment
massTop                  178.0   GeV +- 0.3  GeV
correlationCoefficient   massh0 massTop 0.05                # quatsch
# etc
edge  1 massNeutralino1 massNeutralino2      263.5 GeV +- 1.2 GeV alias 1
sigma ( ee -> Neutralino1 Neutralino2, 1000.,0.8,0.6 ) 7.678 fb +- 2.0 fb
BR ( h0 -> Bottom Bottom~ )   0.8033 +- 0.01 +-(lumiErr) 0.05
BR ( h0 -> Charm Charm~   )   0.05   +- 0.02 +-(lumiErr) 0.01
nofit cos2PhiL                                         0.62865 +- 0.0005
# etc
fitParameter  TanBeta            10.0
fixParameter  Mu                 358.6 GeV
universality  MSelectronR MSmuR
# etc
LoopCorrections        on
CalcPullDist           off
```

Parameter estimation: Toy Fits with Simulated Annealing

„Classical" χ^2-minimization

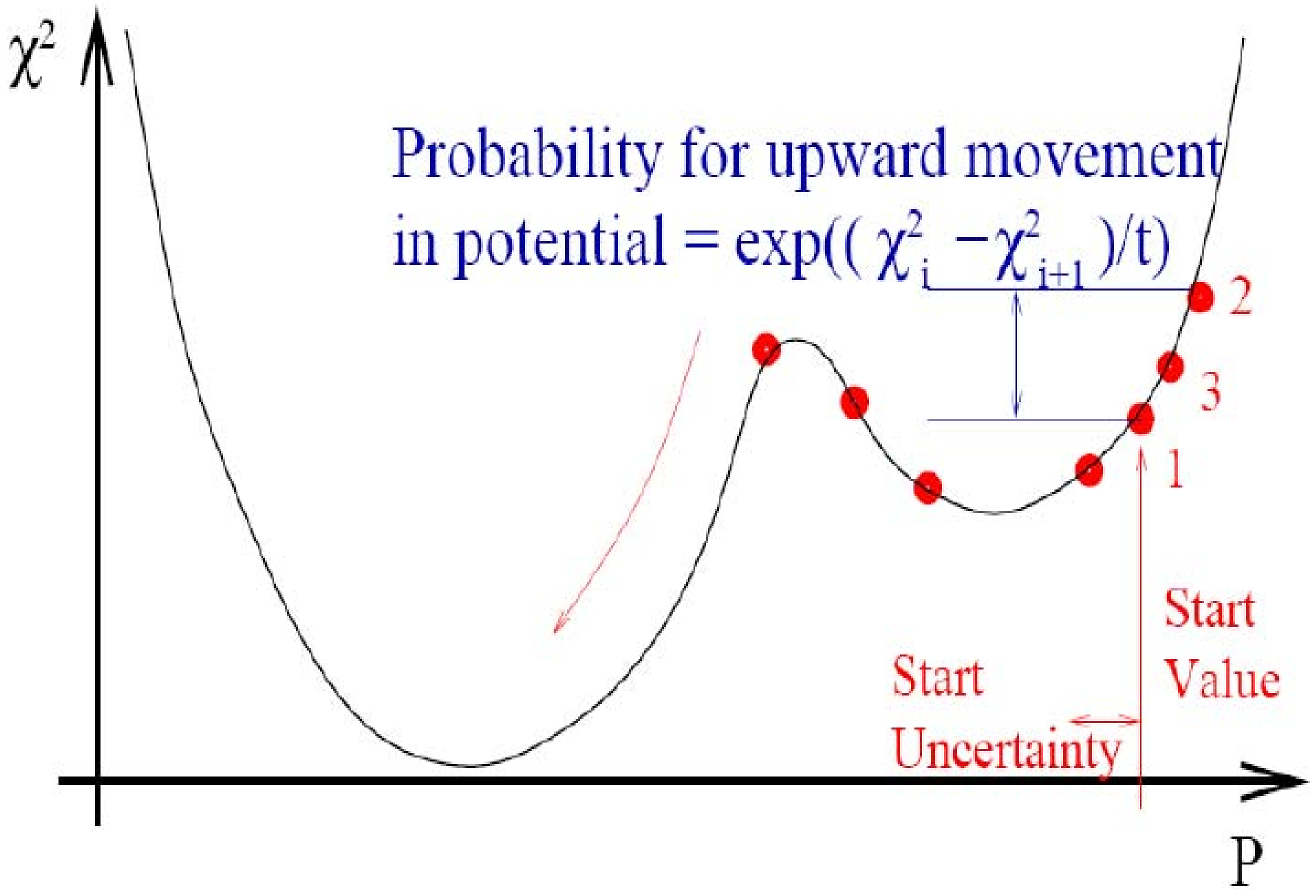
Minimum search aided by „simulated annealing" algorithm
→ avoid getting stuck in secondary minima

Error estimation (brute force):
Repeat fits with input observables randomly smeared acc. to full covariance matrix
→scatter of parameters allows for error estimate (incl. correlation and non-gaussianities)

Pro: robust, built-in quality control: χ^2-distribution
can be fast (apart from toy-fitting)

Con: will not generally find two seperated minima if they are far apart

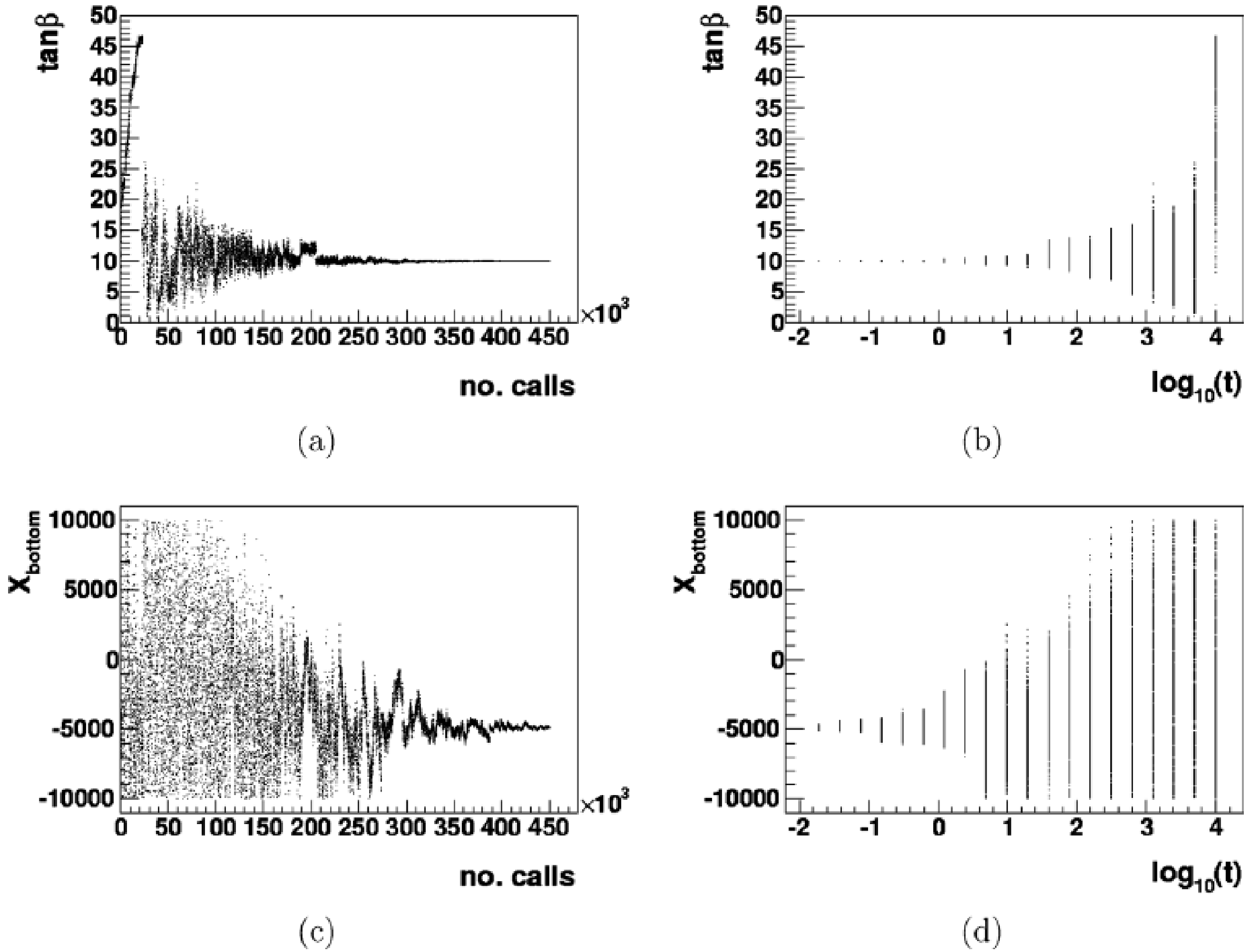
Parameter estimation: Toy Fits with Simulated Annealing



Very similar to Markov-Chain –
„t“-dependent proposal distribution

Parameter estimation: Toy Fits with Simulated Annealing

Simulationed annealing at work:



Parameter scan: Markov Chain Monte Carlo

Sequence of points p_i in p-space with associated likelihood $-2\ln L=\chi^2$

Choose new point randomly (acc. to proposal density) and add to chain if $L(p_{n+1}) > L(p_n)$ Otherwise accept with prob. $L(p_{n+1})/L(p_n)$

Properties:

- efficient scan of „important" p-space
- sampling density ~ posterior likelihood

Pro:

- Bayesian (sampling density ~ L) + Frequentist (look for minimum of $-2\ln L=\chi^2$ within sampled points)
- Discover non-contiguous regions of high L

Con:

- CPU intensive
- No built-in quality control of best fit

Markov Chain Monte Carlo: „burn-in"

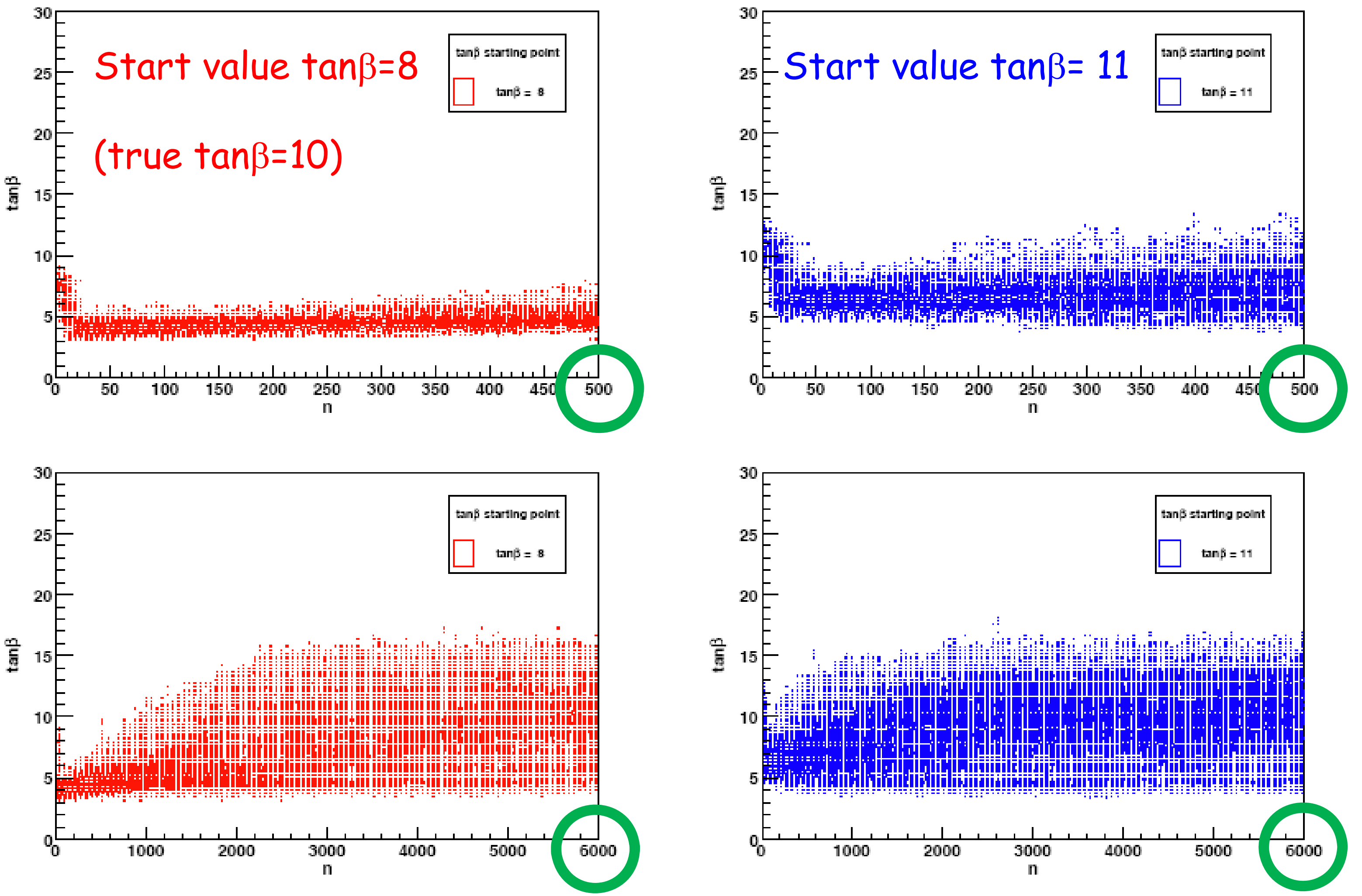


Fig. 1: $\tan\beta$ sampling behaviour of Markov chain using LE and LHC measurements for an integrated luminosity of 1 fb^{-1} within mSUGRA for two different starting points.

Need "sufficient" chain length essential for unbiased results

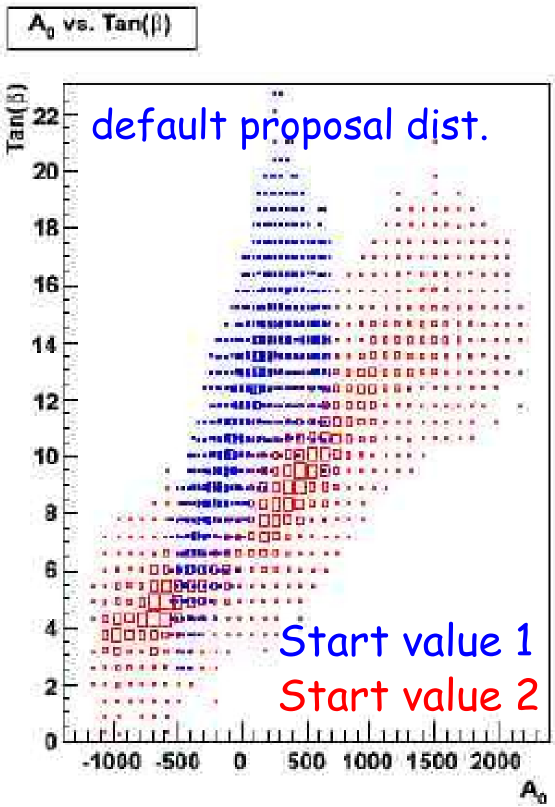
Parameter estimation: Markov Chain Monte Carlo

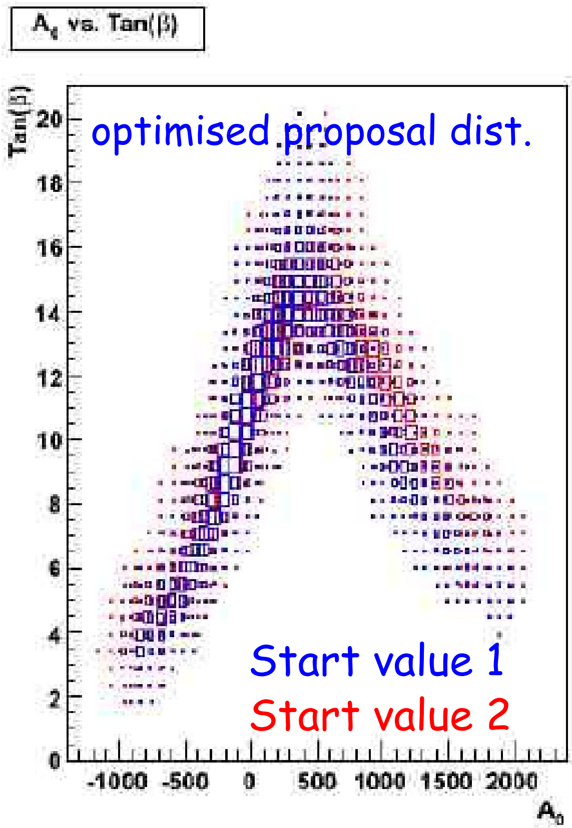
X.Prudent

Parameter analysis is CPU intensive

Optimised algorithm settings are crucial → green Fittino

Example: Optimised proposal density distribution for Markov chain





Short chain length!

Constraints from existing measurements

The full(?) list

Observable	Experimental Value	Uncertainty stat	Uncertainty syst
$\mathcal{B}(B \to s\gamma)/\mathcal{B}^{SM}(B \to s\gamma)$	1.117	0.076	0.096
$\Delta m_{B_s}/\Delta m_{B_s}^{SM}$	1.11	0.01	0.32
$\mathcal{B}(B_s \to \mu\mu)/SM$	$< 4.7\times10^{-8}$		0.02×10^{-8}
$\mathcal{B}(B \to \tau\nu)/SM$	1.15	0.40	
$\mathcal{B}(B_s \to s\ell\ell)/SM$	0.99	0.32	
$\mathcal{B}(K \to \ell\nu)/SM$	1.008	0.014	
$a_\mu^{exp} - a_\mu^{SM}$	30.2×10^{-10}	8.8×10^{-10}	2.0×10^{-10}
m_W	80.399	0.025	0.010
$\sin^2\theta_{eff}$	0.2324	0.0012	
Γ_Z	2.4952	0.0023	0.001
R_l	20.767	0.025	
R_b	0.21629	0.00066	
R_c	0.1721	0.003	
$A_f(bb)$	0.0992	0.0016	
$A_f(bc)$	0.0707	0.0035	
A_b	0.923	0.020	
A_c	0.670	0.027	
A_l	0.1513	0.0021	
m_h	> 114.4		3.0
Ωh^2	0.1099	0.0062	0.012
A_τ	0.1465	0.0032	
$A_f(bl)$	0.01714	0.00095	
σ_{had}	41.540	0.037	
$\Delta\epsilon_K^{exp}/\Delta\epsilon_K^{SM}$	0.92	0.14	
$\mathcal{B}(K \to \pi\nu\bar{\nu})$	< 4.5	1.0×10^{-10}	
$\mathcal{B}(B_d \to \ell\ell)$	$< 2.3\times10^{-8}$		0.001×10^{-8}
$(\Delta m_{B_s}/\Delta m_{B_s}^{SM})/(\Delta m_{B_d}/\Delta m_{B_d}^{SM})$	1.09	0.01	0.16

EW precision observables (weak dependence on SUSY directly but infuence on numerical values of the other predictions)

Let´s take it at face value and put it all together

Theory predictions taken conveniently from „Mastercode“ (compilation of many codes)

[Buchmüller et al, 2008] (all „puclic“ expect m_W (at 2-loop SUSY))

Comparison with other codes (SPHENO) + Updated measurements ongoing

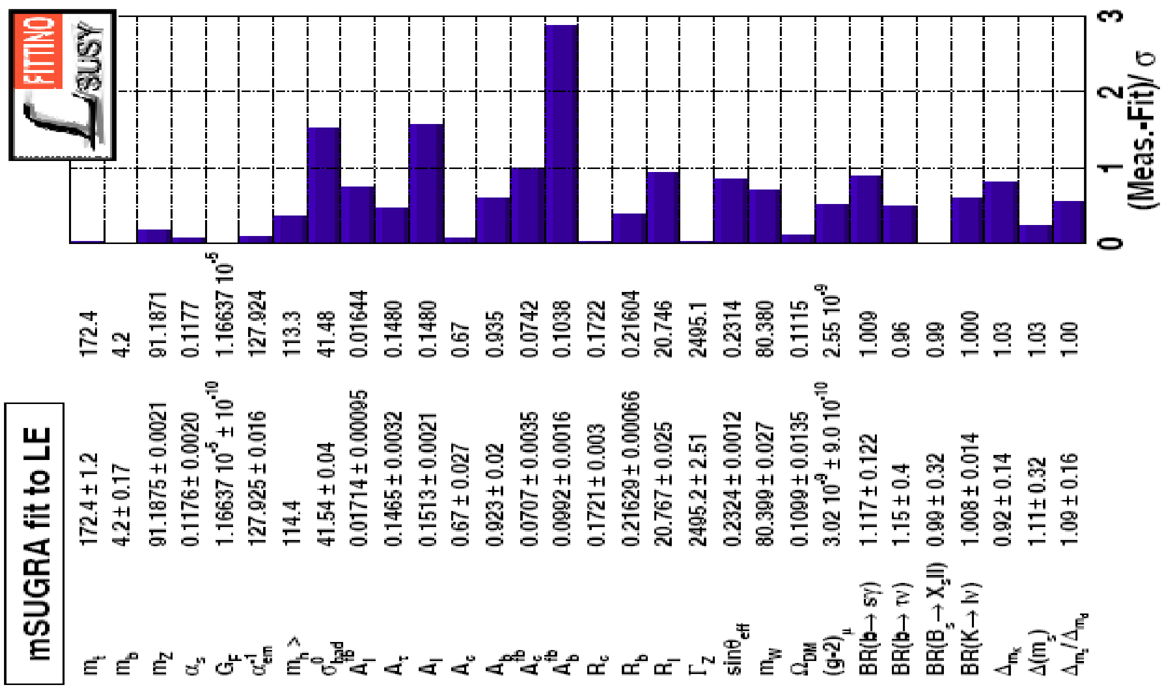
Best fit of LE („low energy“) data - mSUGRA

sign(μ)=+1	„best fit“	1σ error
$M_{1/2}$ (GeV)	331.5	± 86.6
M_0 (GeV)	76.2	+79.2 -29.1
tanβ	13.2	± 7.2
A_0 (GeV)	383.8	± 647

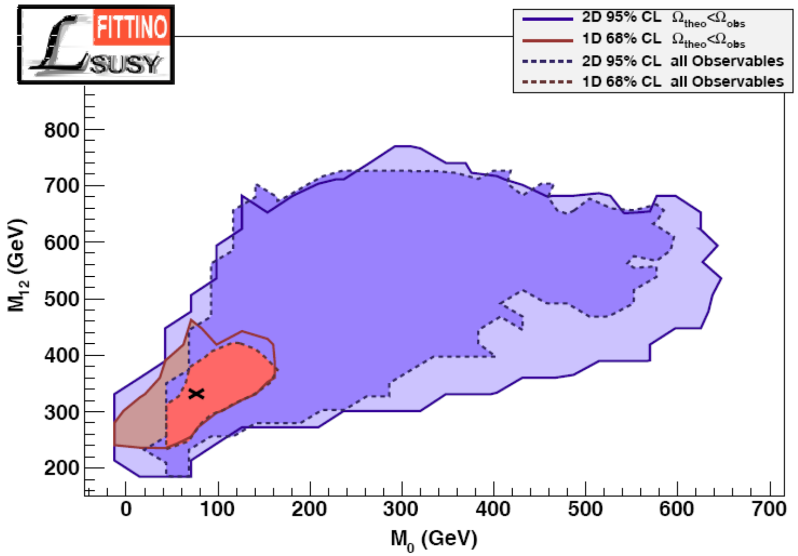
Prob(χ^2) = 54.4%

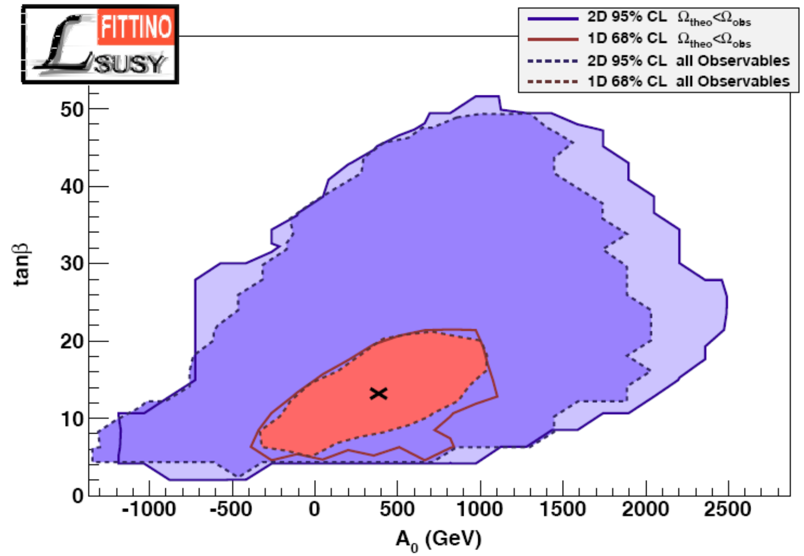
Prob(χ^2) = 9.4% for sign(μ) = -1

(Prob(χ^2)$_{SM}$ ≈ 17%)



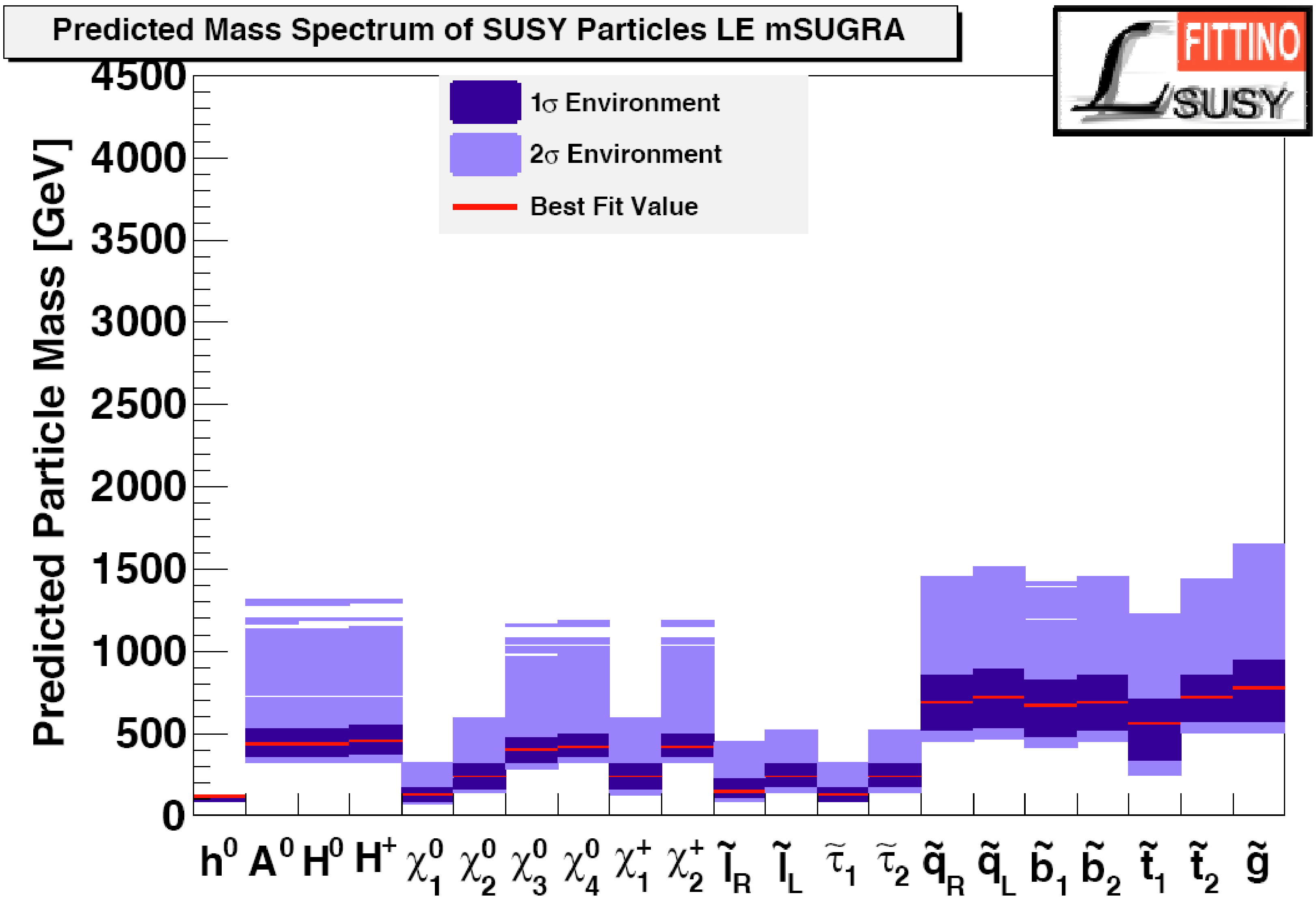
Best fit of LE data - mSUGRA - 2D projections



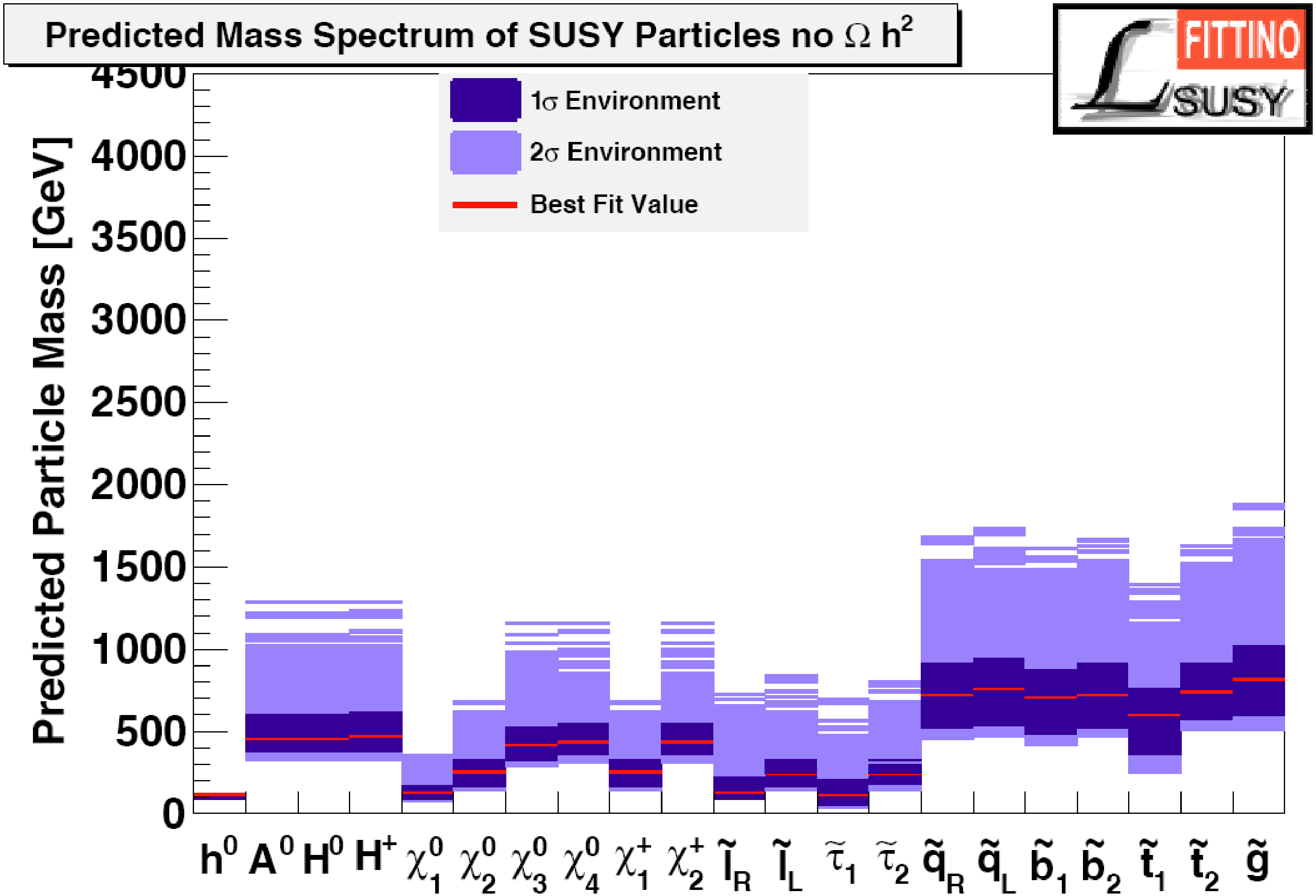


- Fits performed using Markov Chain MC (with profile LH interpreation)
- rather strong constraints on sparticle mass scales $M_{1/2}$ and M_0
- how does this translate into the sparticle mass spectrum?
- what are the most sensitive observables?

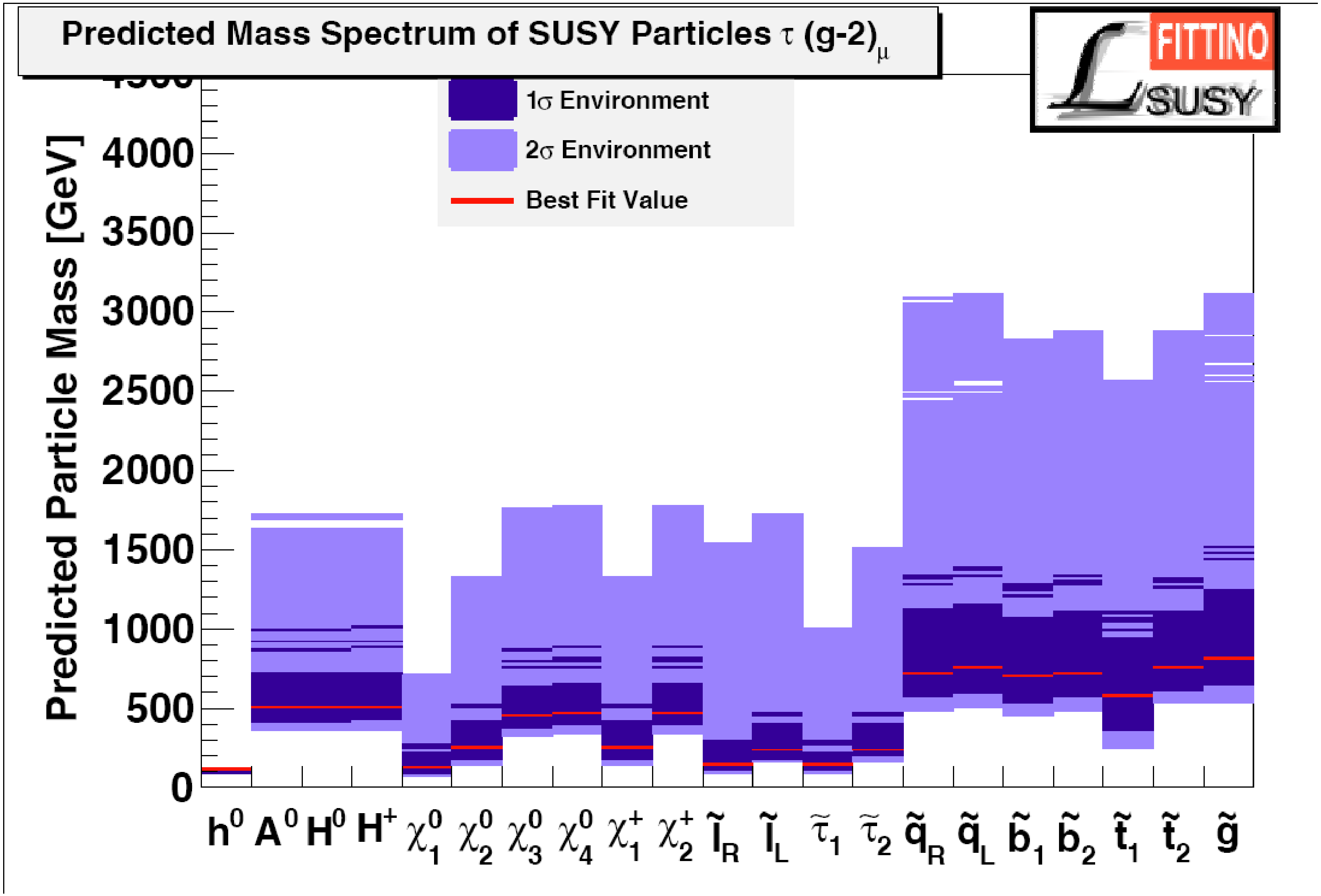
Predicted mass spectrum from LE observables



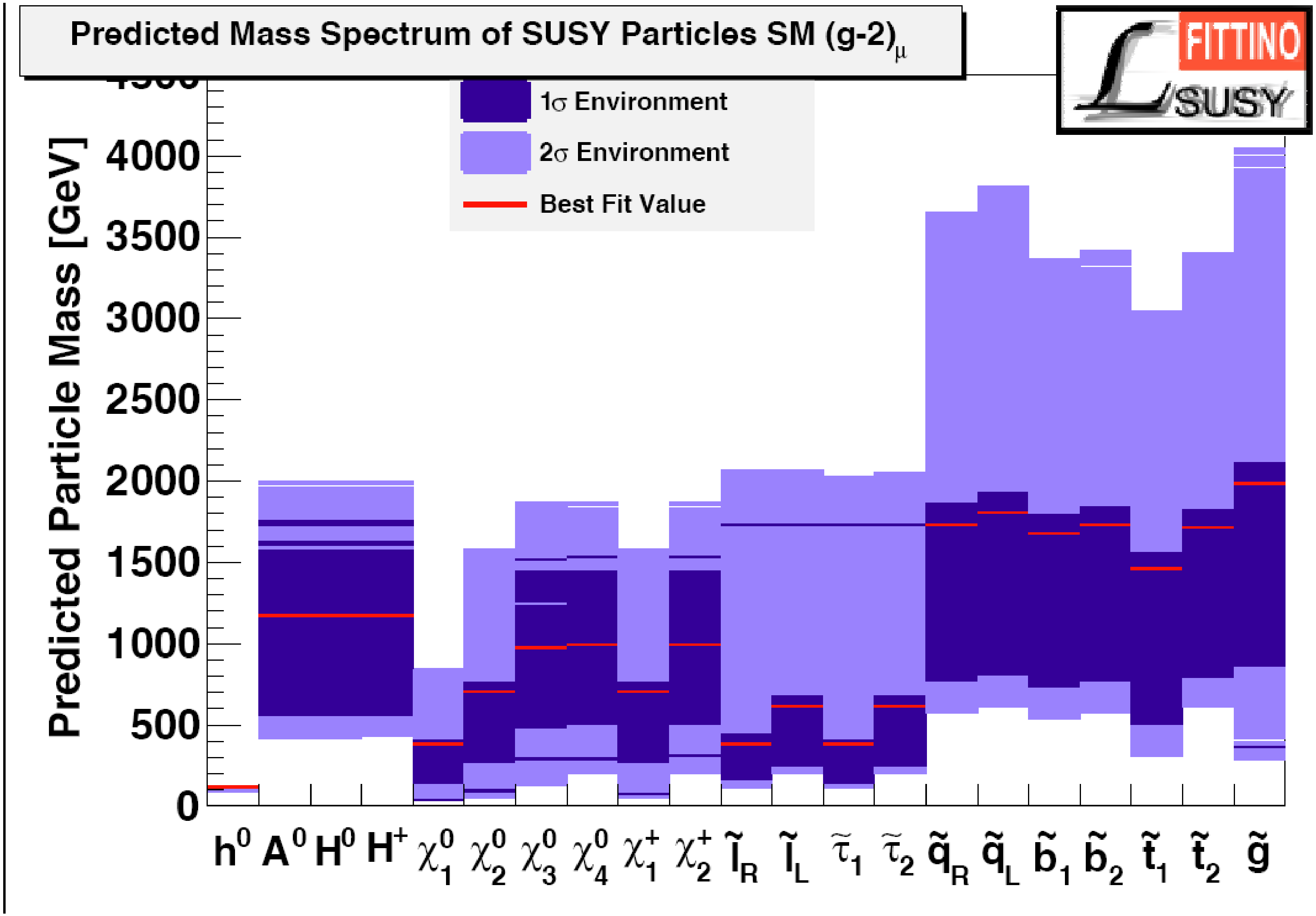
Without Ωh^2



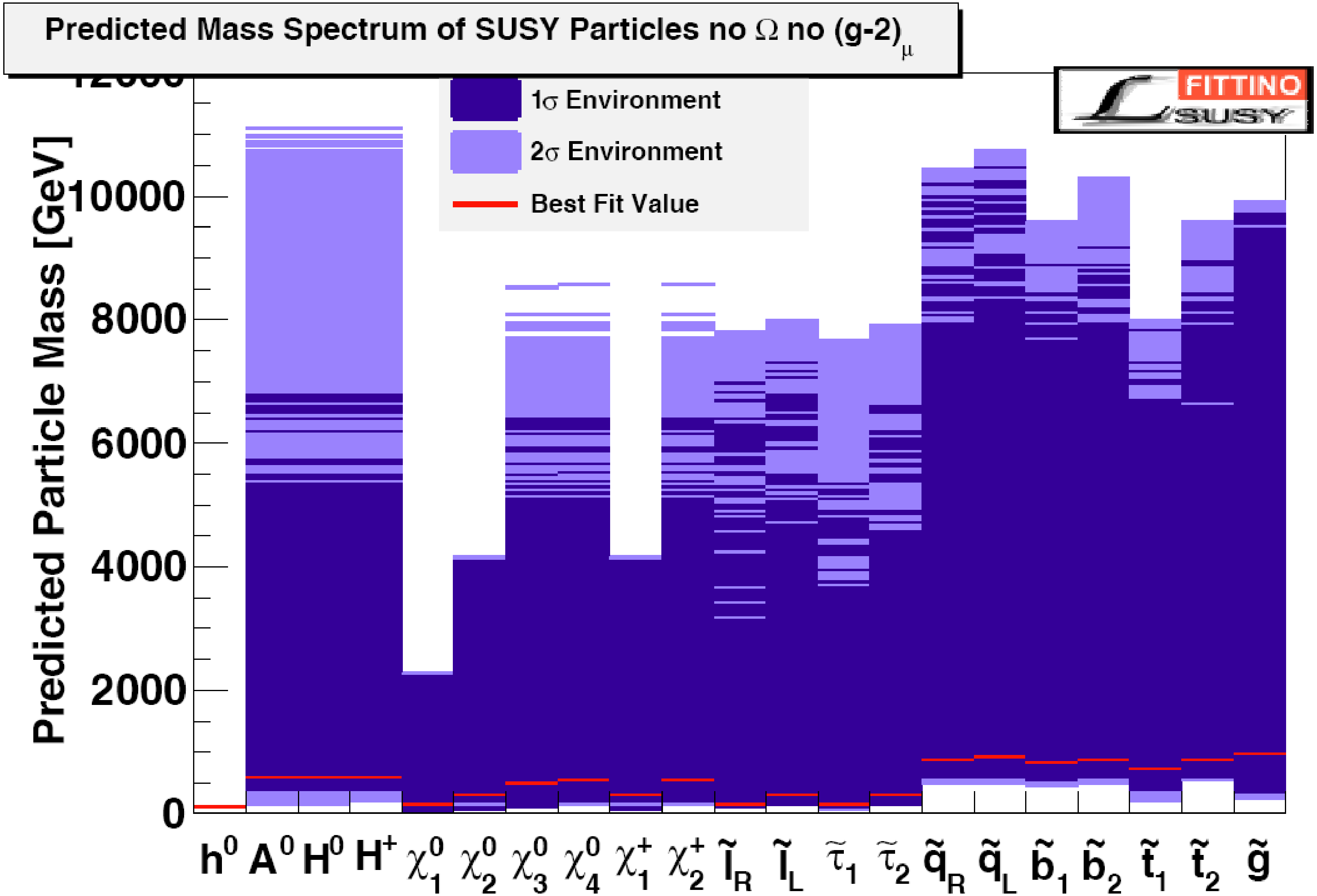
With Ωh^2 but with $(g-2)_\mu^{tau}$



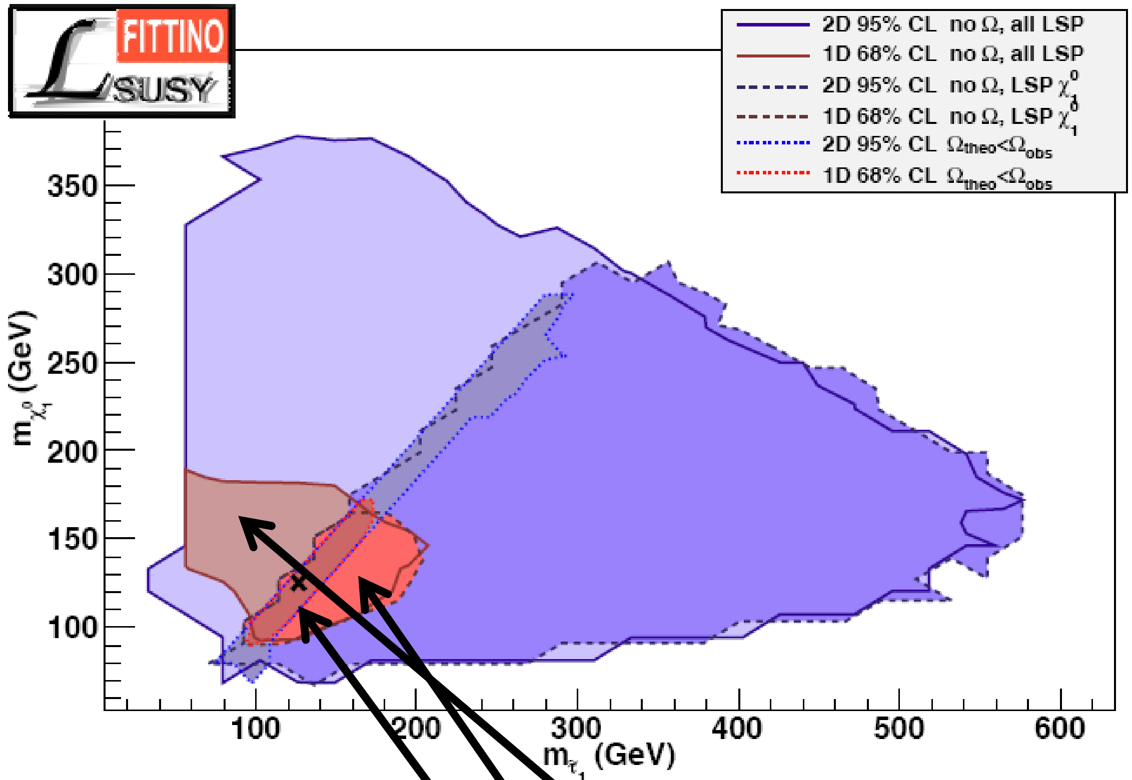
With Ωh^2 but with $(g-2)_\mu^{SM}$



Without both Ωh^2 and $(g-2)_\mu$



Mass difference of LSP and NLSP (stau1)



stau co-annihilation region clearly preferred if
LE data + DM constraint applied

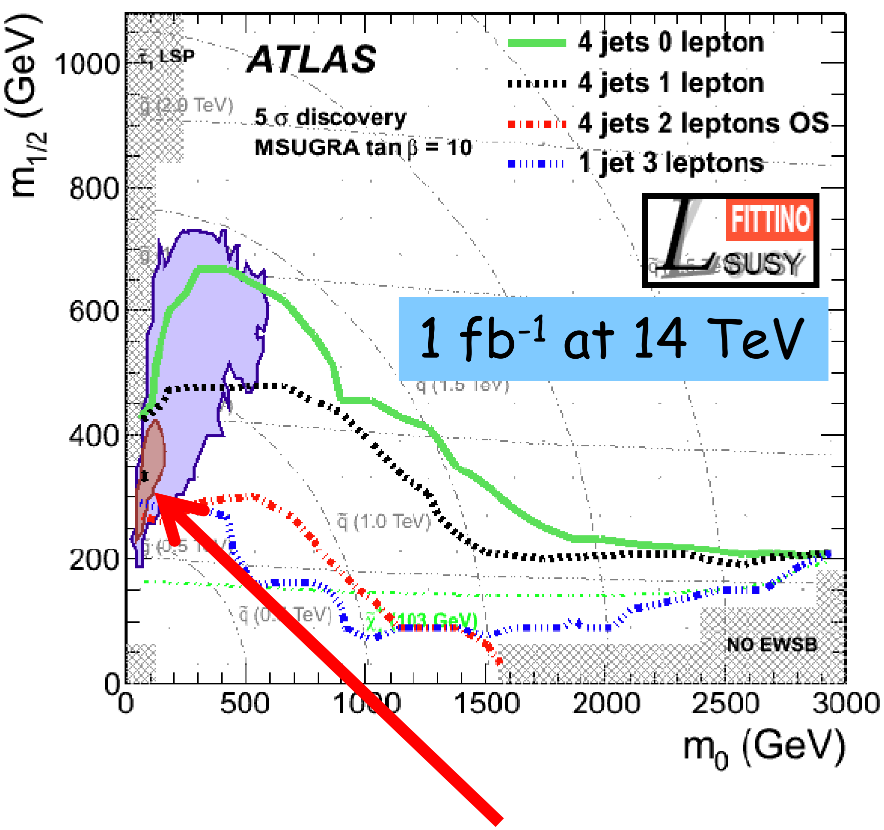
→ low NLSP-LSP mass difference

no Ω constraint, all LSPs allowed

no Ω constraint, neutralino LSP

with $\Omega_{SUSY} \leq \Omega_{obs}$

LHC discovery?



68% preferred region by data

Existing data are very consistent with mSUGRA (and GMSB)

General MSSM18 fit in preparation…

IF

- sth. like mSUGRA is realized
- deviation of $(g-2)_\mu$ from SM is real

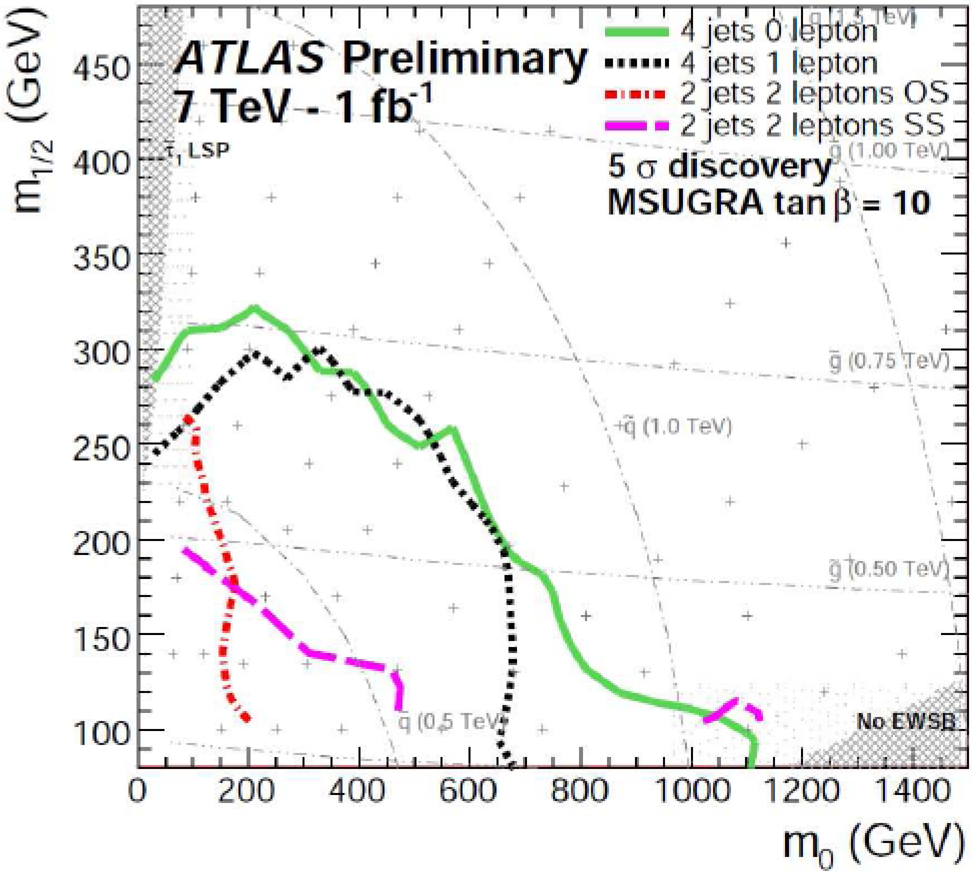
THEN

SUSY particles are rather light and should be seen by the ATLAS+CMS rather easily

If LSP is stable, and Ωh^2 is taken at face value, then

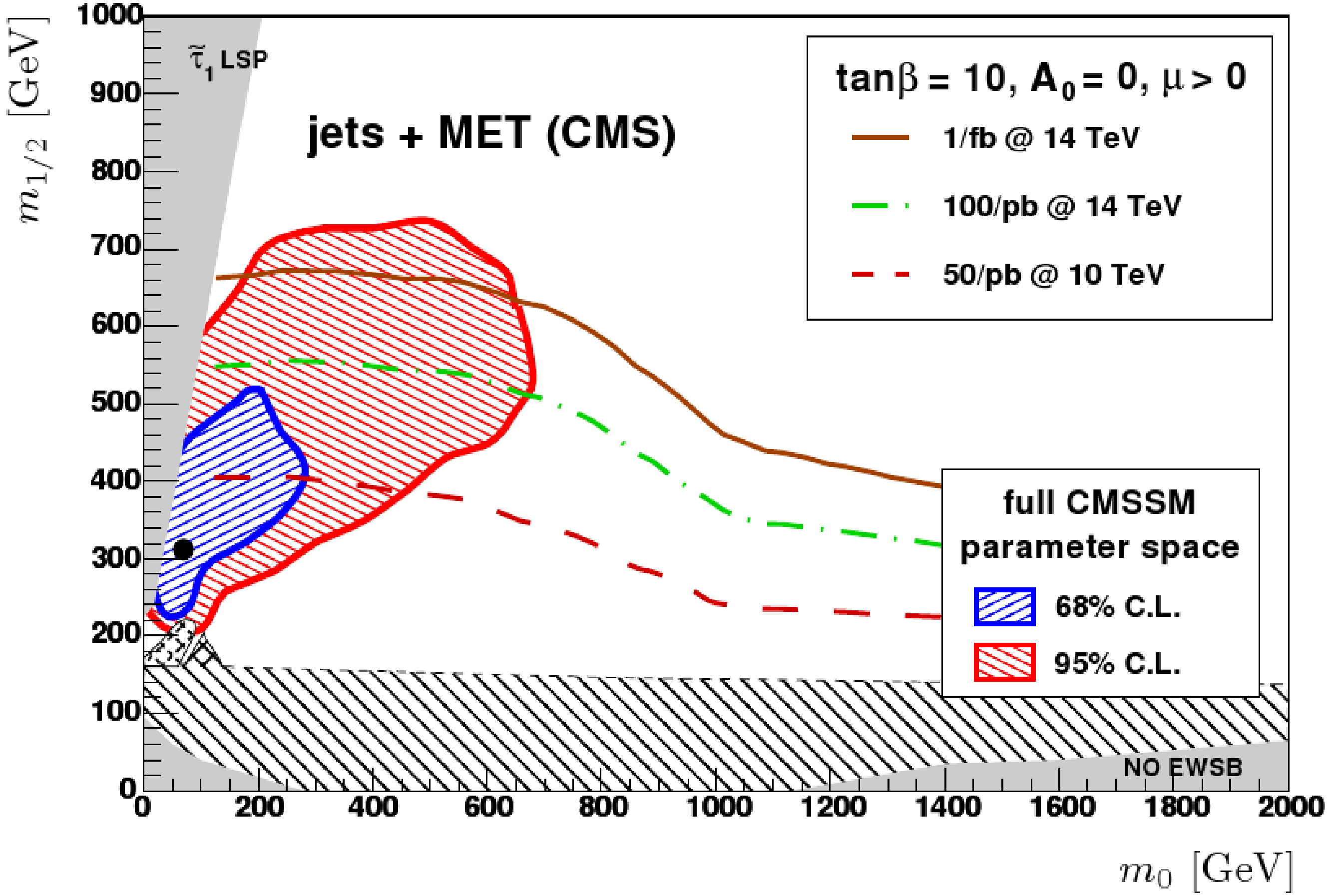
NLSP-LSP mass difference is rather small (~9 GeV for best fit point)

LHC discovery?



$$1\,\mathrm{fb}^{-1}\,@\,7\,\mathrm{TeV}$$

LHC discovery?



Result fully consistent with Mastercode crew
[Buchmüller et al, arXiv:0808.4128]

Prospects for LHC

LHC prospects for measurements of SUSY pars

Ideally one would investigate LHC prospects for the current „best fit" point – but would need to do all LHC prospect studies for this point (too expensive…, use real data soon)

	„best fit"	1σ error	SPS1a
$M_{1/2}$	331.5	± 86.6	250
M_0	76.2	+79.2 -29.1	100
$\tan\beta$	13.2	± 7.2	10
A_0	383.8	± 647	-100

„SPS1a" is just in the middle of the single-parameter errors

Mass scales and hierarchies in the right ballpark

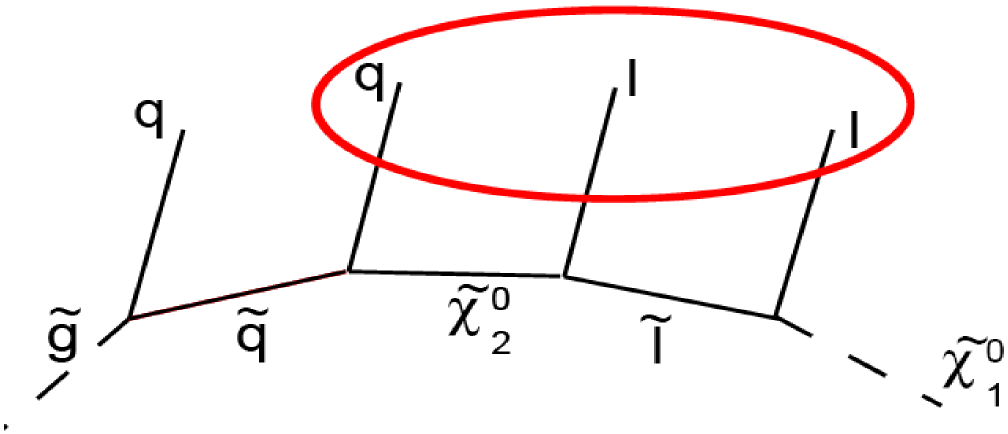
Main deficit: not compatible with Ωh^2

→ SPS1a has somewhat too large $\Delta m_{NLSP\text{-}LSP}$

(we use it anyhow because of largest number of exp. inputs)

Expected LHC inputs

Most information comes from the reconstruction of the famous χ^0_2 chain



Various kinematic endpoints with l = e,μ,τ and q = (udsc), b

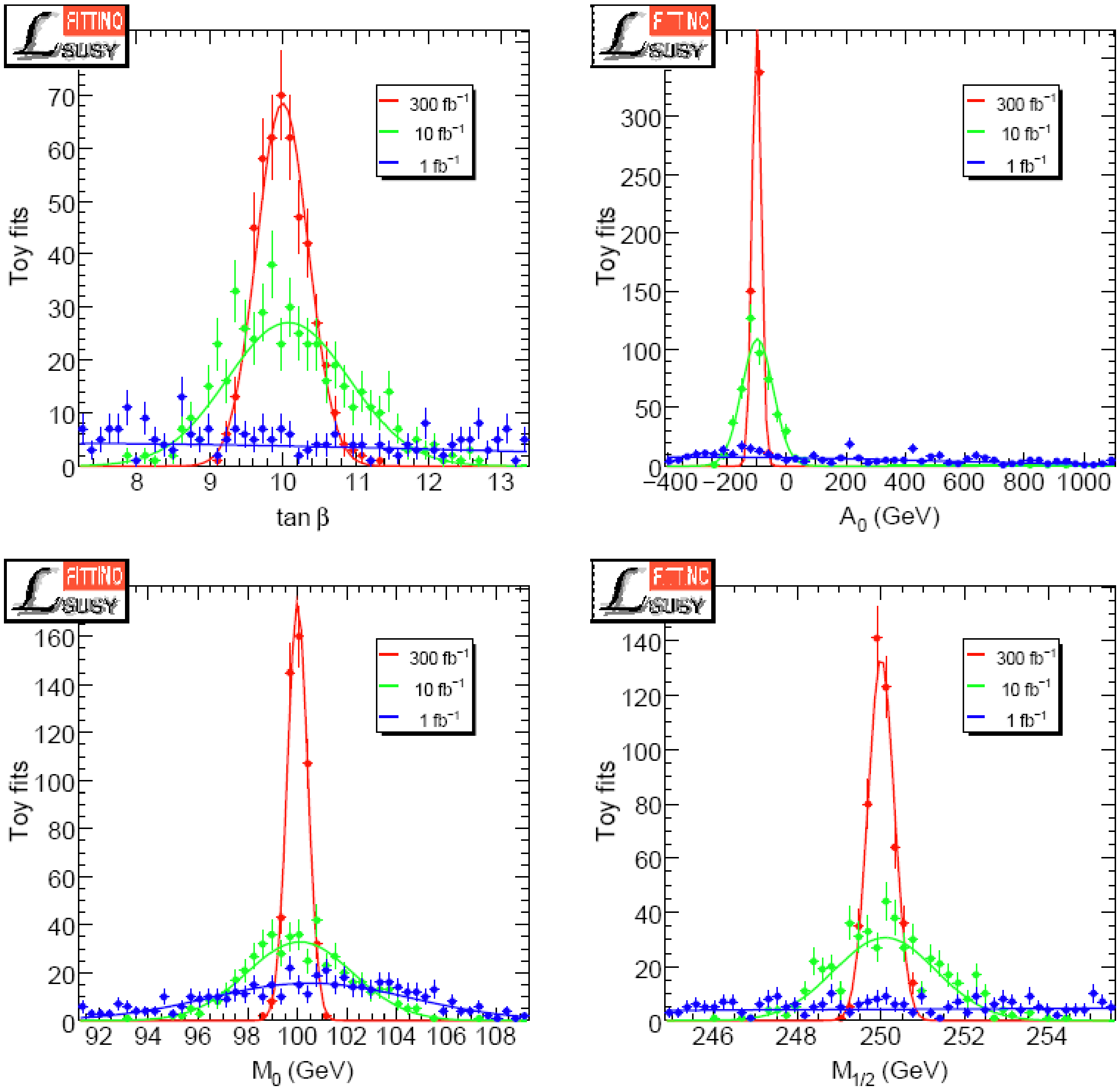
Precision and number of accessible observables increases with L
e.g. m_h only after >~10 fb^{-1}, sbottom only after >~100 fb^{-1}

1. Fit mSUGRA for 1,10,300 fb^{-1}
2. Distinguish/handle <u>ambiguities</u>
3. Fit the data within a (much) larger model class (MSSM-18)

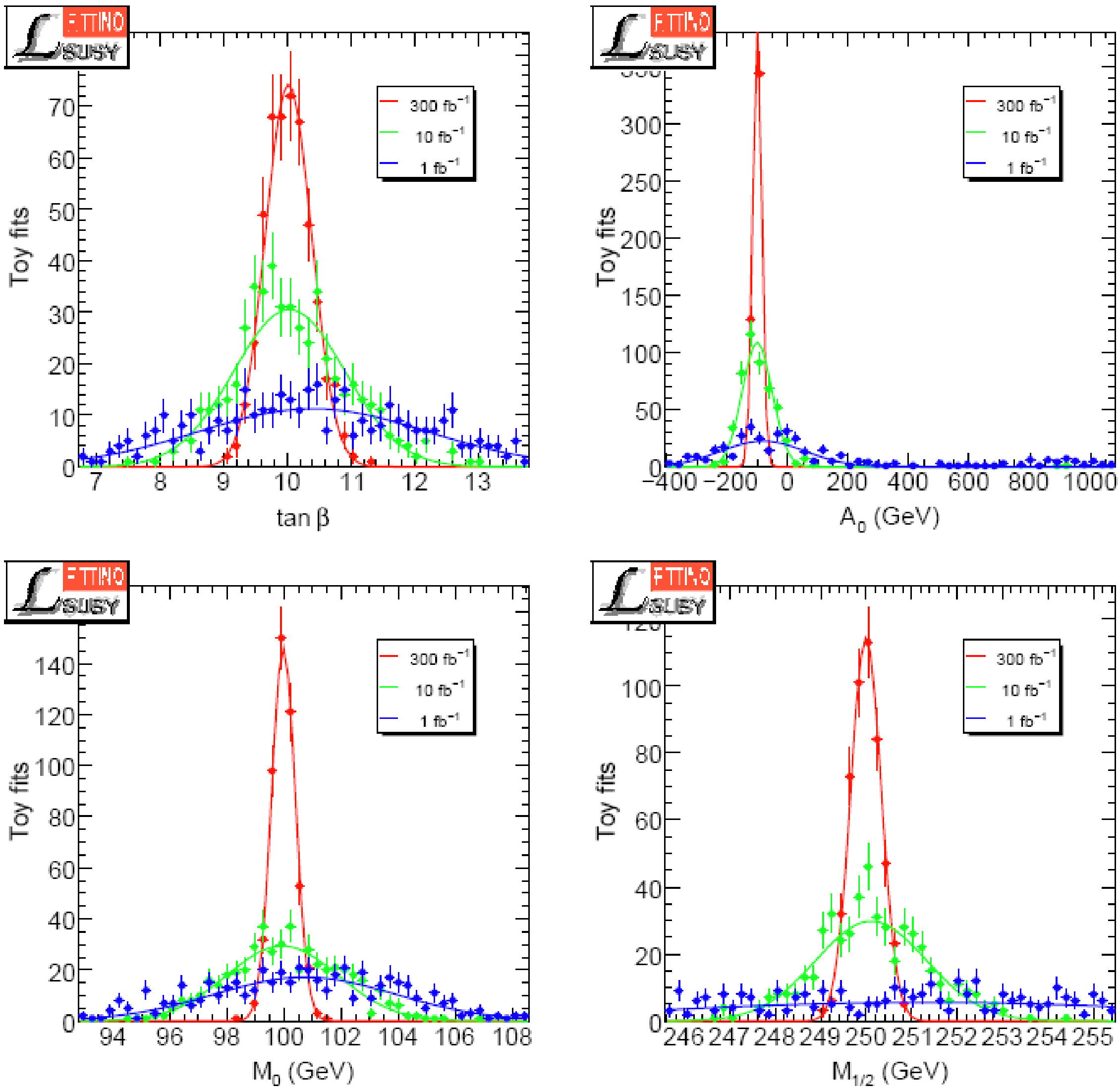
List of assumed LHC inputs

Observable	Nominal Value	Uncertainty							
		1 fb^{-1}	10 fb^{-1}	300 fb^{-1}	LES_1	$\mathrm{LES}_{10,300}$	JES_1	$\mathrm{JES}_{10,300}$	syst.
m_h	109.6		1.4	0.1		0.1			
m_t	172.4	1.1	0.05	0.01			1.5	1.0	
$m_{\tilde\chi_1^\pm}$	180.2			11.4				1.8	
$\sqrt{m_{\tilde\ell_L}^2 - 2m_{\tilde\chi_1^0}^2}$	148.8			1.7		0.1			6.0
$m_{\tilde g} - m_{\tilde\chi_1^0}$	507.7		13.7	2.5				5.1	10.0
$\sqrt{m_{\tilde q_R}^2 - 2m_{\tilde\chi_1^0}^2}$	531.0	19.6	6.2	1.1			22.7	4.5	10.0
$m_{\tilde g} - m_{\tilde b_1}$	88.7			1.5				0.9	
$m_{\tilde g} - m_{\tilde b_2}$	56.8			2.5				0.6	
$m_{\ell\ell}^{\max}(m_{\tilde\chi_1^0}, m_{\tilde\chi_2^0}, m_{\tilde\ell_R})$	80.4	1.7	0.5	0.03	0.16	0.08			
$m_{\ell\ell}^{\max}(m_{\tilde\chi_1^0}, m_{\tilde\chi_4^0}, m_{\tilde\ell_L})$	280.6		12.6	2.3		0.28			
$m_{\tau\tau}^{\max}(m_{\tilde\chi_1^0}, m_{\tilde\chi_2^0}, m_{\tilde\tau_1})$	83.4	12.6	4.0	0.73			4.2	0.8	5.7
$m_{\ell\ell q}^{\max}(m_{\tilde\chi_1^0}, m_{\tilde q_L}, m_{\tilde\chi_2^0})$	452.1	13.9	4.2	1.4			22.7	4.5	
$m_{\ell q}^{\mathrm{low}}(m_{\tilde\ell_R}, m_{\tilde q_L}, m_{\tilde\chi_2^0})$	318.6	7.6	3.5	0.9			16.2	3.2	
$m_{\ell q}^{\mathrm{high}}(m_{\tilde\chi_1^0}, m_{\tilde\chi_2^0}, m_{\tilde\ell_R}, m_{\tilde q_L})$	396.0	5.2	4.5	1.0			19.9	4.0	
$m_{\ell\ell q}^{\mathrm{thres}}(m_{\tilde\chi_1^0}, m_{\tilde\chi_2^0}, m_{\tilde\ell_R}, m_{\tilde q_L})$	215.6	26.5	4.8	1.6			10.8	2.2	
$m_{\ell\ell b}^{\mathrm{thres}}(m_{\tilde\chi_1^0}, m_{\tilde\chi_2^0}, m_{\tilde\ell_R}, m_{\tilde b_1})$	195.9		19.7	3.6				2.0	
$m_{tb}^{\mathrm{w}}(m_t, m_{\tilde t_1}, m_{\tilde\chi_1^\pm}, m_{\tilde g}, m_{\tilde b_1})$	359.5	43.0	13.6	2.5			18.0	3.6	
$\frac{\mathcal{B}(\tilde\chi_2^0\to\tilde\ell_R\ell)\times\mathcal{B}(\tilde\ell_R\to\tilde\chi_1^0\ell)}{\mathcal{B}(\tilde\chi_2^0\to\tilde\tau_1\tau)\times\mathcal{B}(\tilde\tau_1\to\tilde\chi_1^0\tau)}$	0.076	0.009	0.003	0.001					0.008
$\frac{\mathcal{B}(\tilde g\to\tilde b_2 b)\times\mathcal{B}(\tilde b_2\to\tilde\chi_2^0 b)}{\mathcal{B}(\tilde g\to\tilde b_1 b)\times\mathcal{B}(\tilde b_1\to\tilde\chi_2^0 b)}$	0.168			0.078					

Evolution of precision (mSUGRA) with Lumi

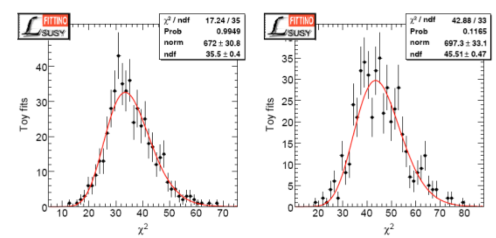


Combination with „LE“ data helps for low L



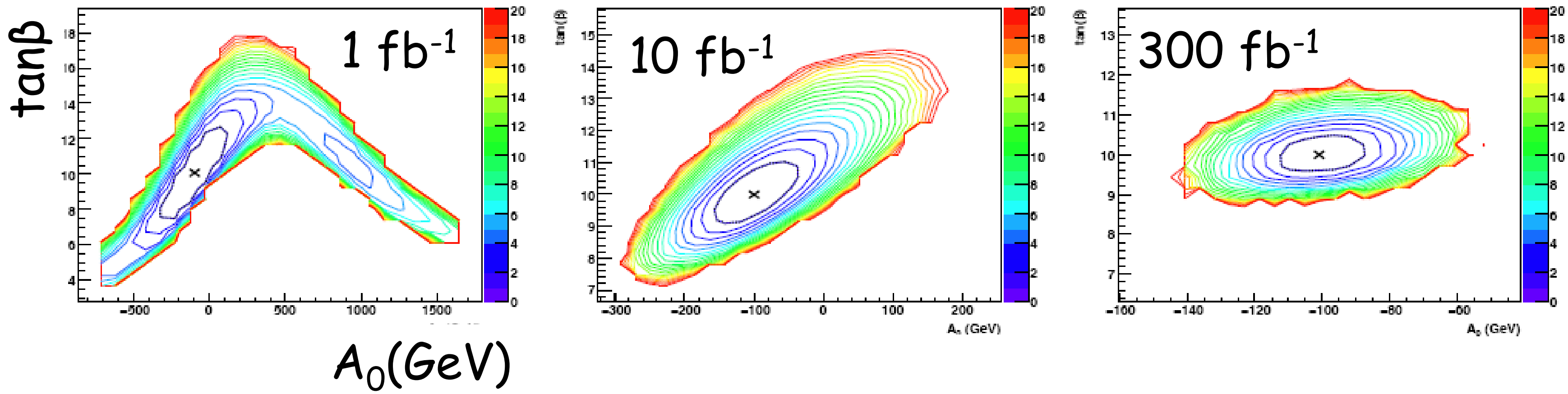
Note: χ^2 distribution nice at all luminosities

„quality control of fit“

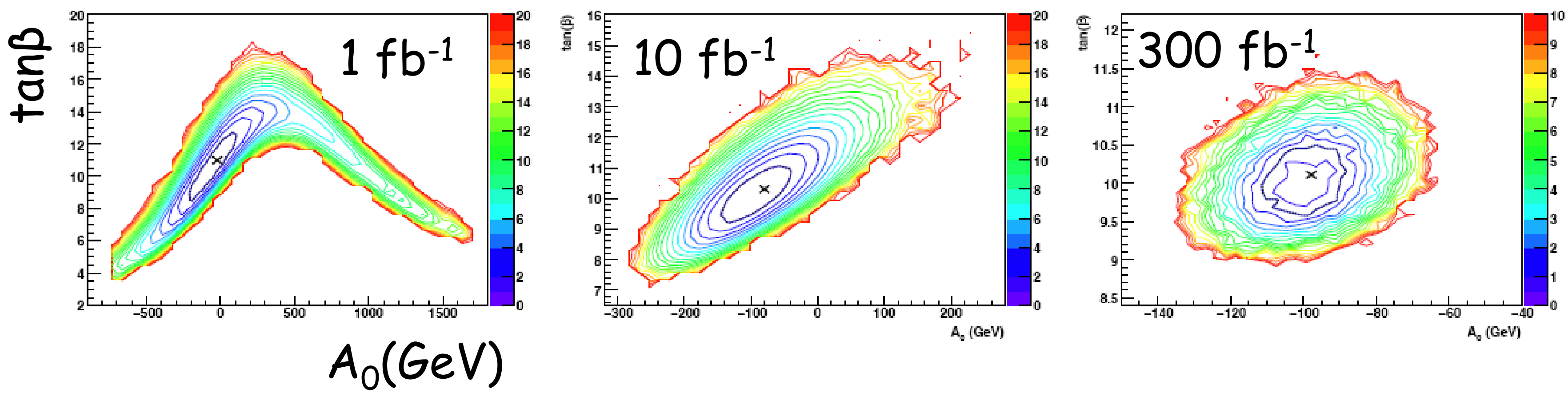


Frequentist vs Bayes

Frequentist (take (minimal) local χ^2 as probability/likelihood measure)



Bayesian (take marginalized sampling density as probability/likelihood measure)
(with flat priors)

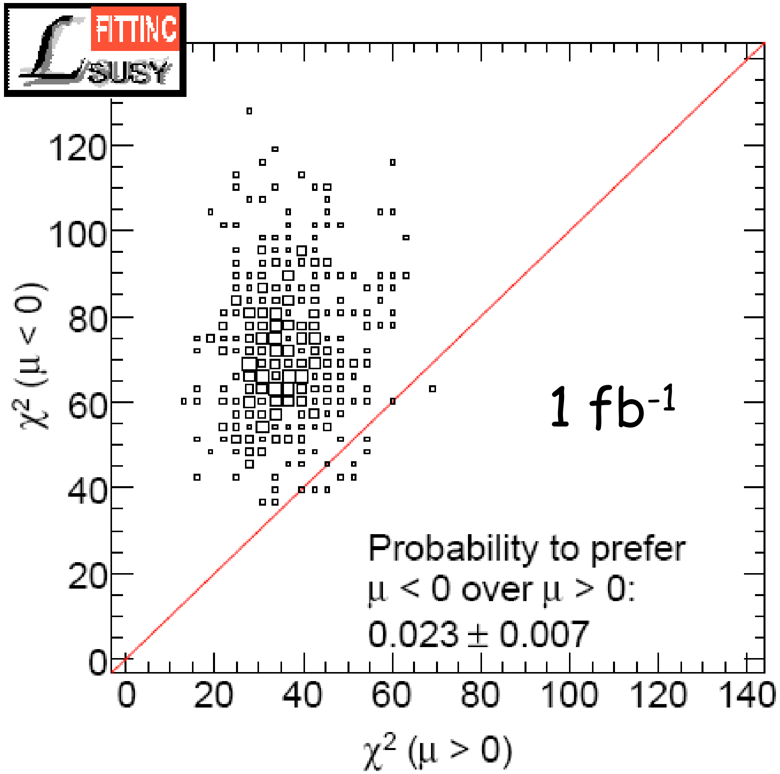


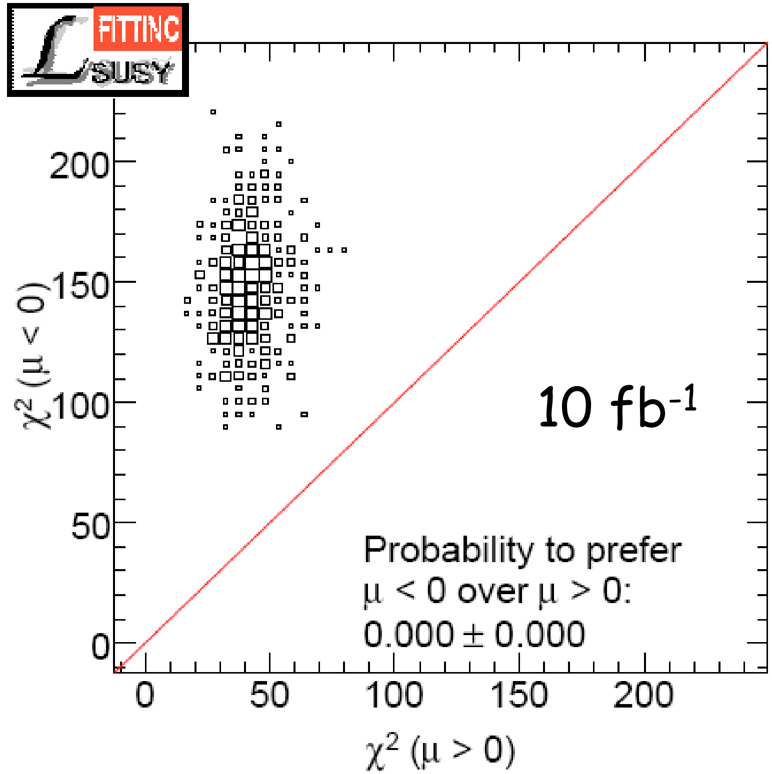
No significant difference (but Bayesian produces ~10x more CO_2)

Ambiguities 1: digital model parameters

In the fit „digital" model parameters, e.g. sign(μ), have to chose beforehand

Can they be distinguished with data?

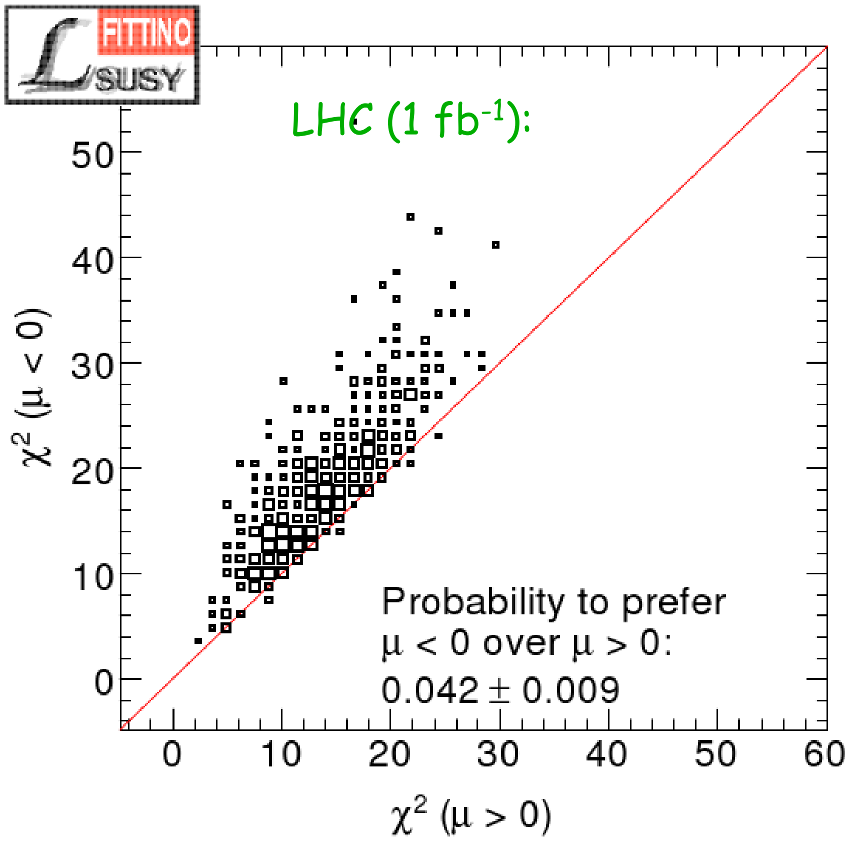


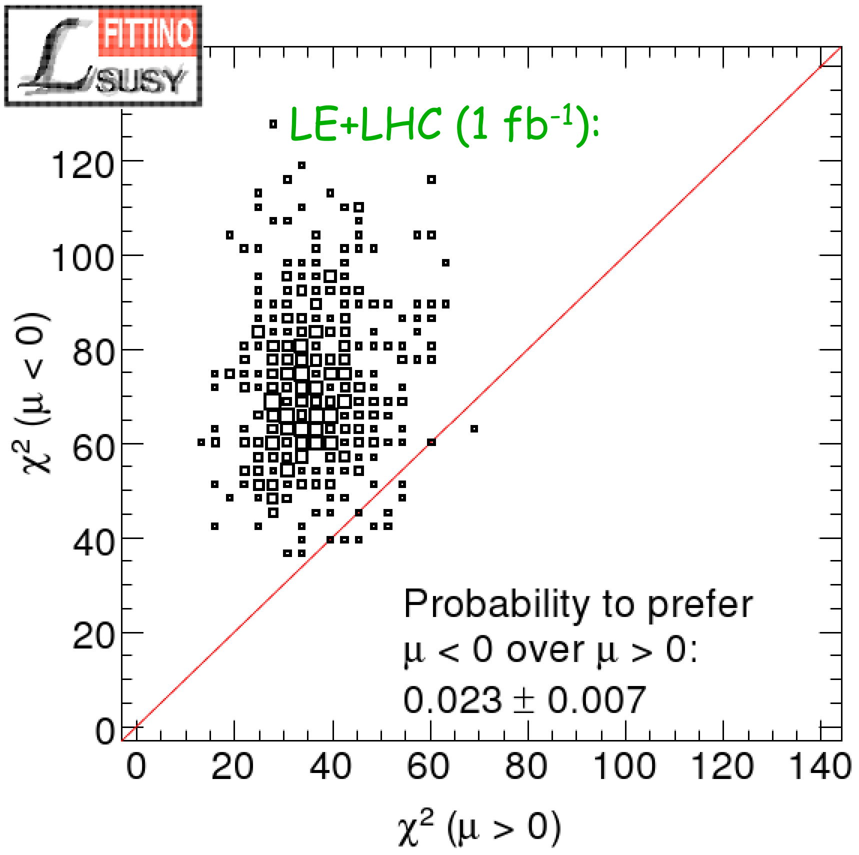


YES!

LHC only vs. LE+LHC

Even after SUSY discovery, low-energy data will remain important for a while…



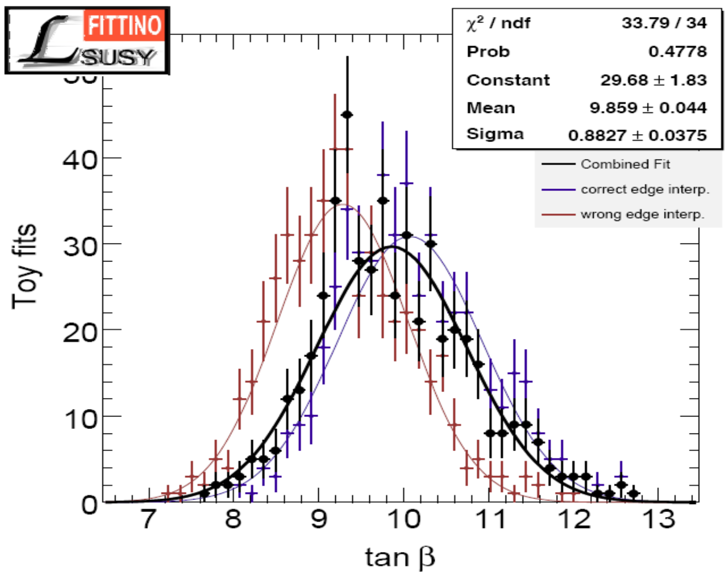


Ambiguities 2: chain assignments (a first example)

When using certain LHC measurements (e.g. endpoints) they have to be assigned ad hoc to originate from a certain decay chain.

What if this assignment is wrong?

New approach: if we don´t know what´s right or wrong, let χ^2 decide! (chose the hypothesis with the better χ^2)



Ambiguity automatically gets folded into the

error estimate

for the parameters

→could in principle be done for an arbitrary number of ambiguities

correct interpretation: $\chi^0_4 \rightarrow \tilde{\ell}_L\, \ell \rightarrow \ell\ell\chi^0_1$

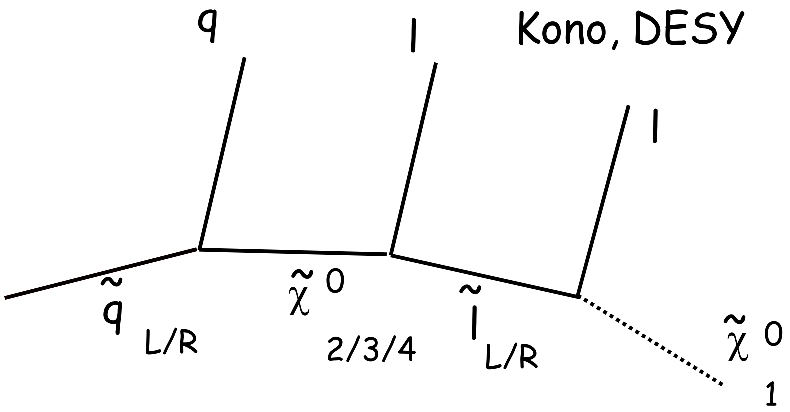
wrong interpretation: $\chi^0_4 \rightarrow \tilde{\ell}_R\, \ell \rightarrow \ell\ell\chi^0_1$

Ambiguities 2: chain assignments (recent progress)

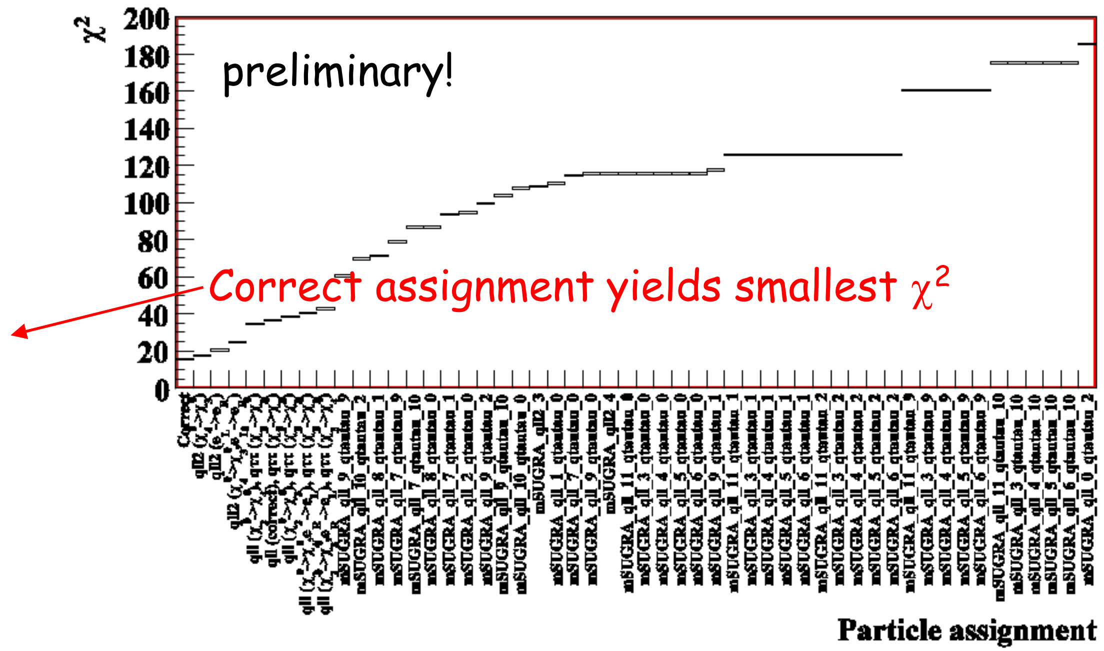
Kono, DESY

Consider all possible decay chain ambiguities in parameter analysis of LHC data

First steps started:



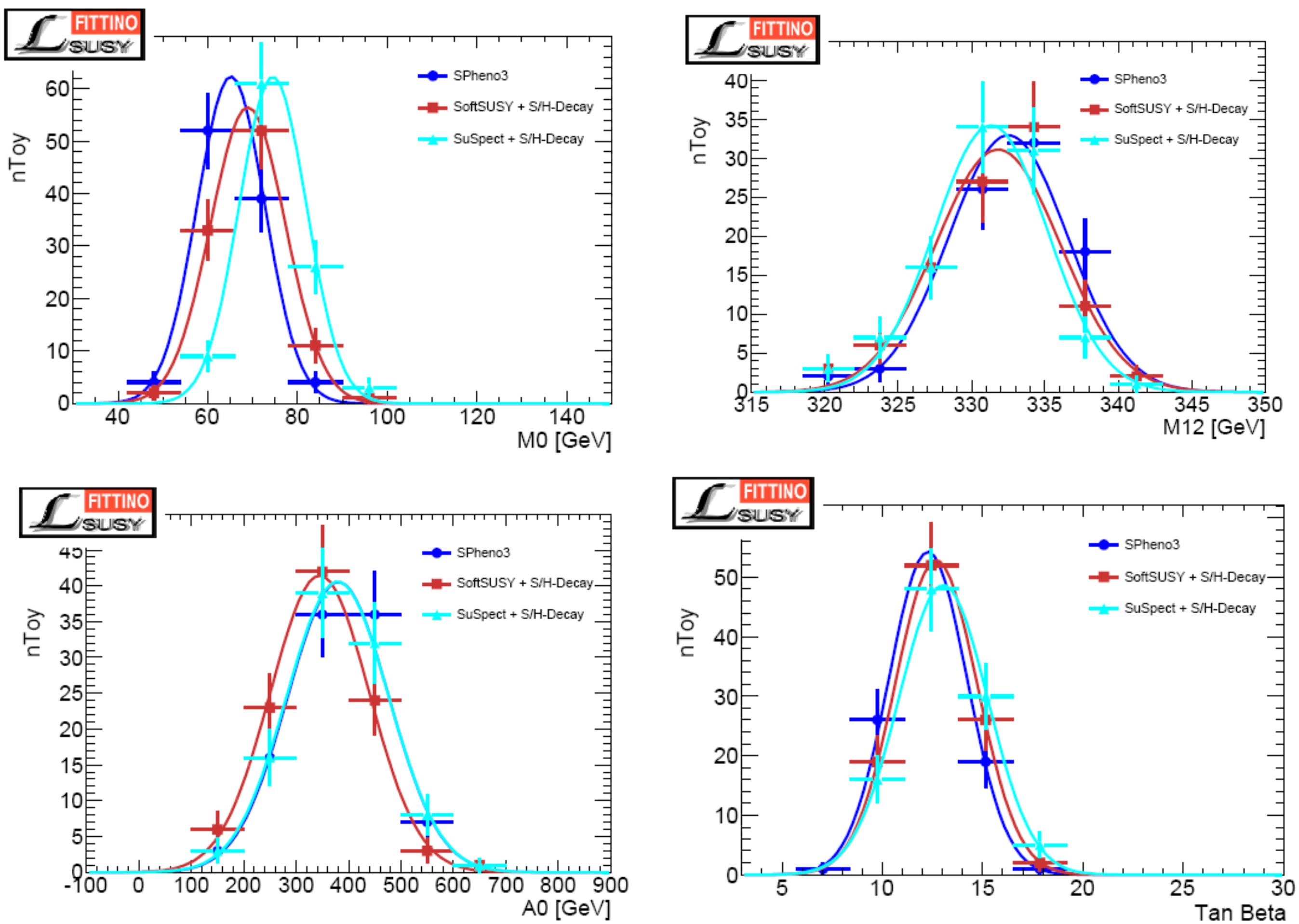
Perform separate fit for each option



Different SUSY Calculators

for CMSSM-Fit - LHC-only

Matthias Hamer, Dipl. Thesis



not too bad – probably ok for LHC, ILC needs more study

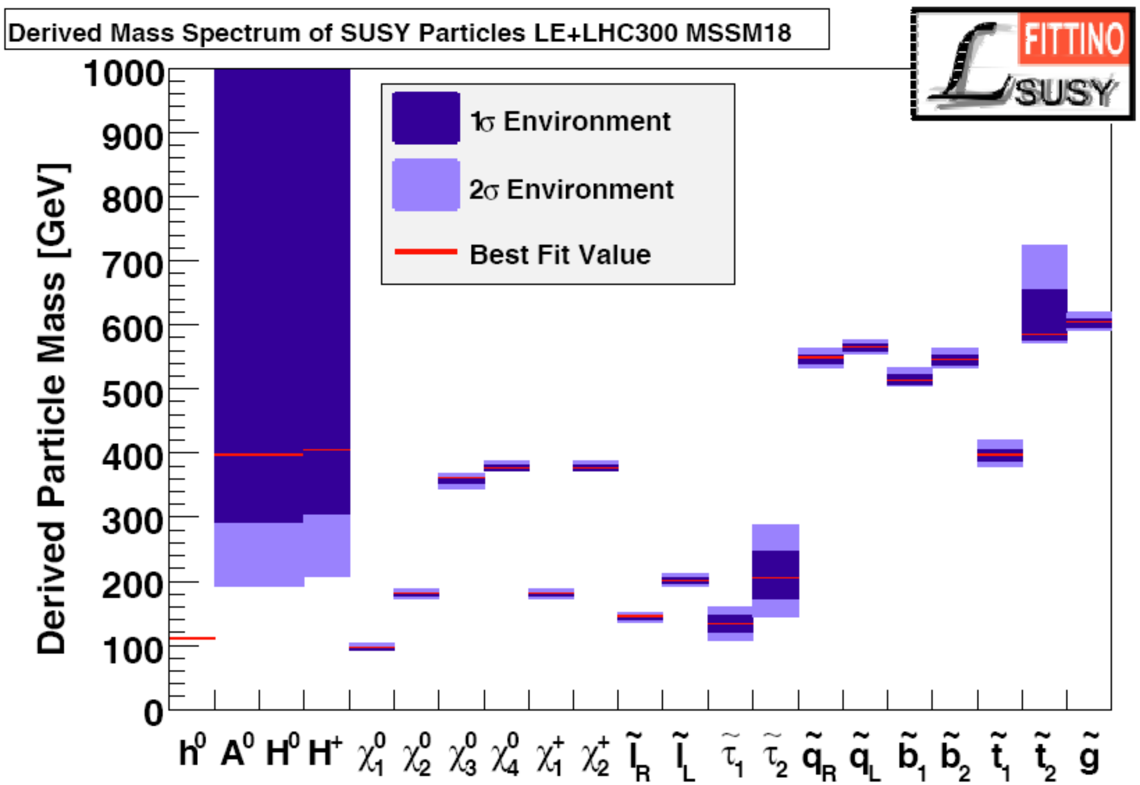
study for LE observables (Mastercode vs. SPHENO) ongoing

Extending the scope: MSSM-18 with LHC?

J.Ellis: „the CMSSM probably is complete bullshit“

No assumption on high-scale model – vital for „bottom-up“ reco of ~~SUSY~~

„Mild“ assumptions on EW-scale Lagrangian:
1. flavour-diagonality 2. no CPV 3. unification of 1st and 2nd generation



reasonably stable fit with 300 fb^{-1} and combination with LE observables

Most sparticle masses can be predicted with 10-20% precision

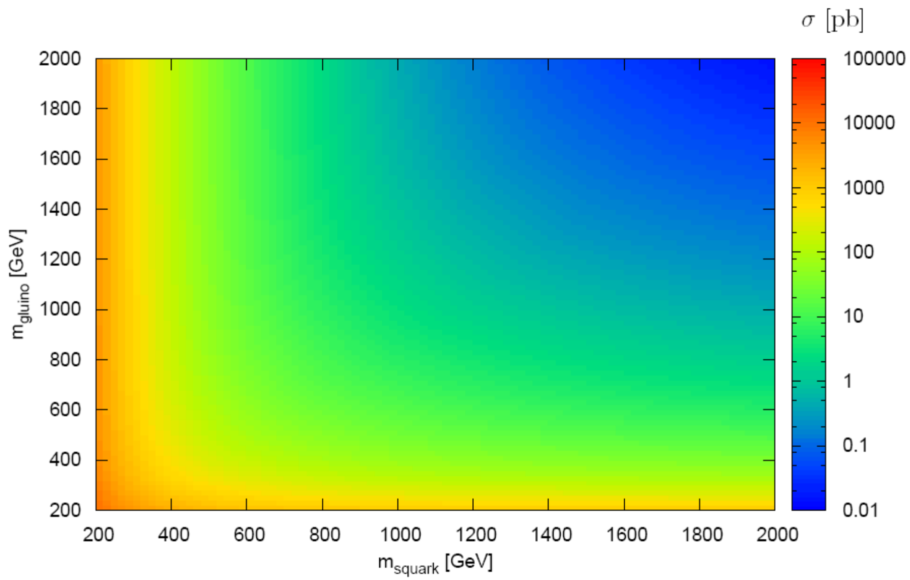
Good news for LC energy planning!

Not all tricks played yet… (1)

Conventional calculation of production rates at LHC too slow.

Possible way out:

- Prospino look-up table (only 2 parameters: squark and gluino mass)
- Calculation of BRs by SPheno/SDecay
- “Generic” acceptance cuts

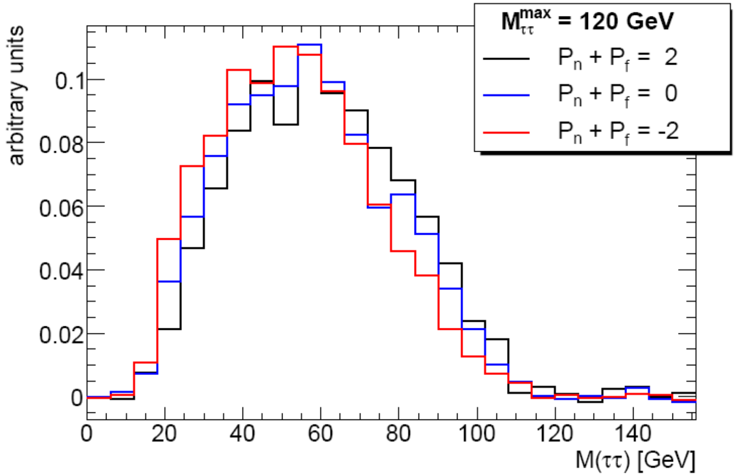


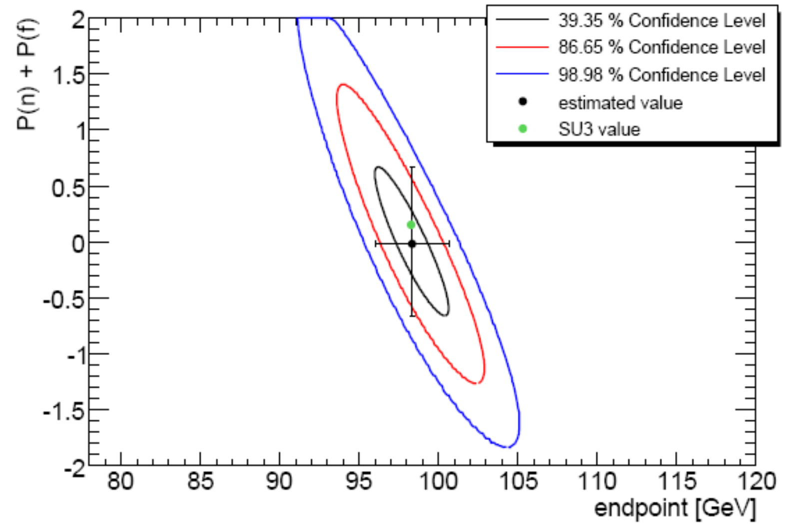
→ Ben´s talk yesterday

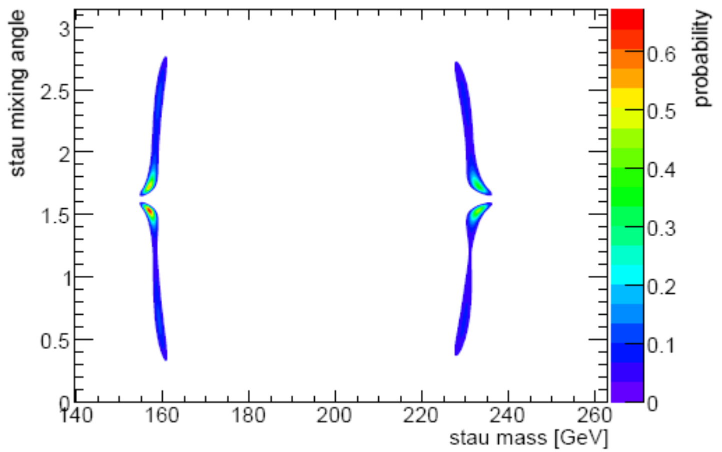
Krämer, Lindert, O'Leary, Dreiner 1003.2648 [hep-ph]

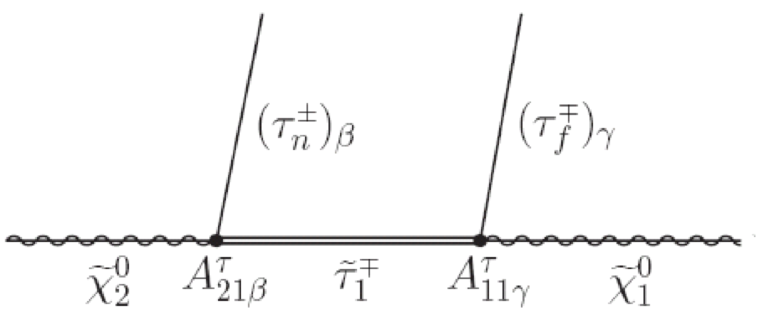
Not all tricks played yet…   (2)

Tau polarisation in Neutralino-2 decay chain:





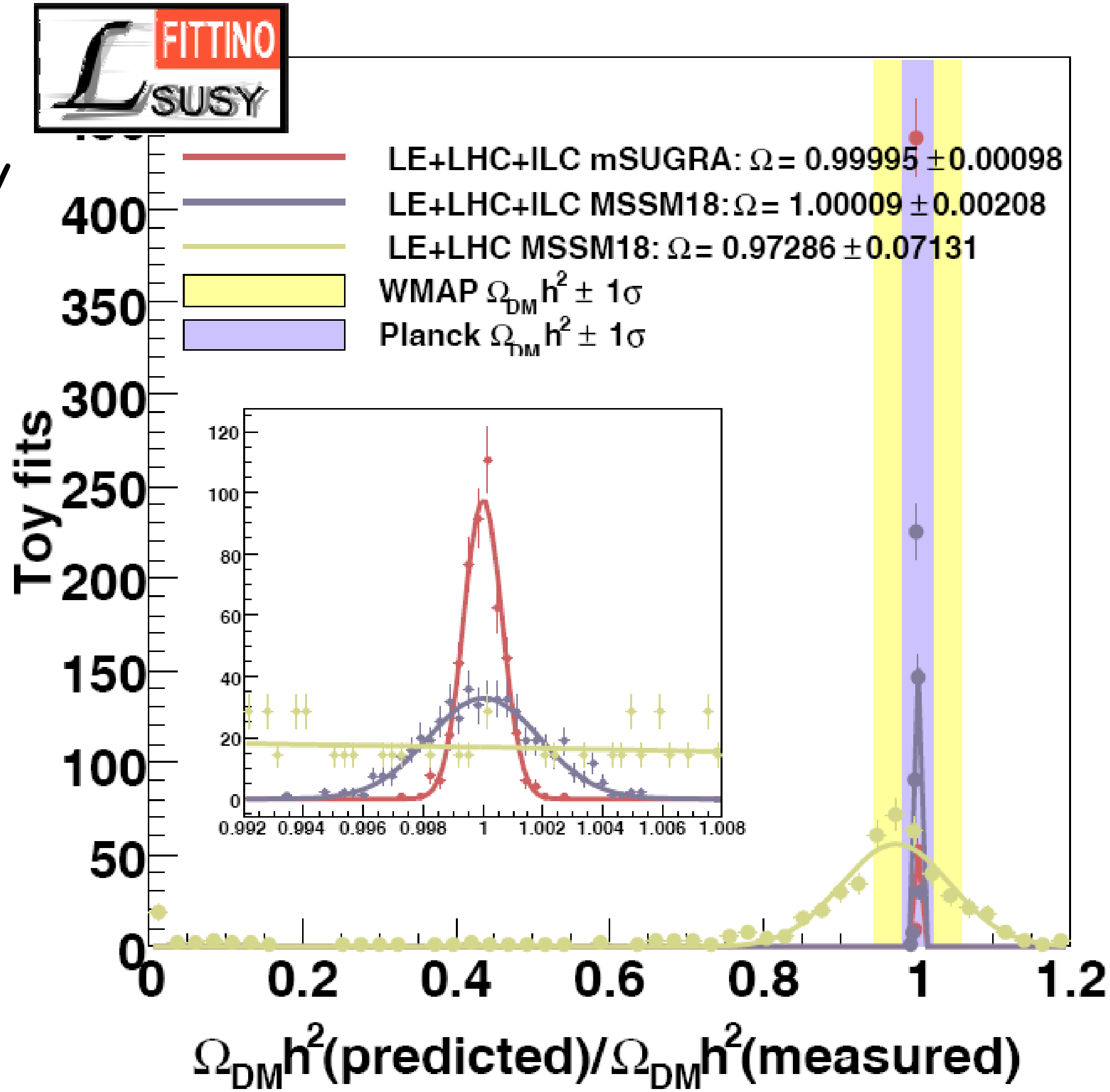




Nattermann, Wienemann, Zendler, KD (´09)

Ceterum censeo… (why still need a linear collider)

Only one example: pre(post)dict DM density to compare with future Planck measurement



Summary & Conclusions
(while „waiting“ for SUSY discovery)

- Parameter determination in complex parameter spaces needs dedicated fitting/estimation techniques
- Existing data point towards light SUSY (at least within mSUGRA-like models)
- then LHC can constrain simple high-scale models well
- even in LHC era, LE data help, in particular for MSSM18
- new methods to incorporate ambigiuities into error analysis → inverse problem solving
- Linear collider vital to achieve precision necessary for bottom-up reconstruction of the model + comparison to cosmology

BACKUP

Virtual but real?

All collider searches for SUSY particles so far only yielded exclusions (otherwise I would have chosen a different topic…)
But what about virtual effects from SUSY particles?

1 loop

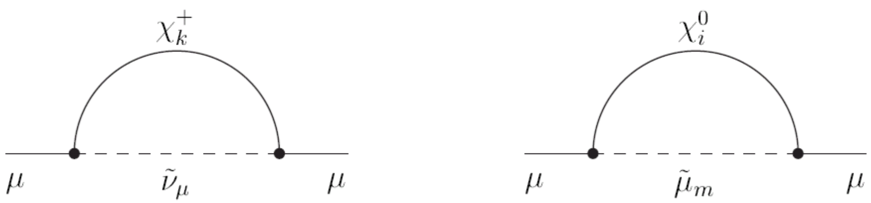


Figure 3. The two SUSY one-loop diagrams, written in terms of mass eigenstates. The external photon line has to be attached to the charged internal lines.

(add an external photon line to the loop to get g-2)

2 loop (1 loop SUSY)

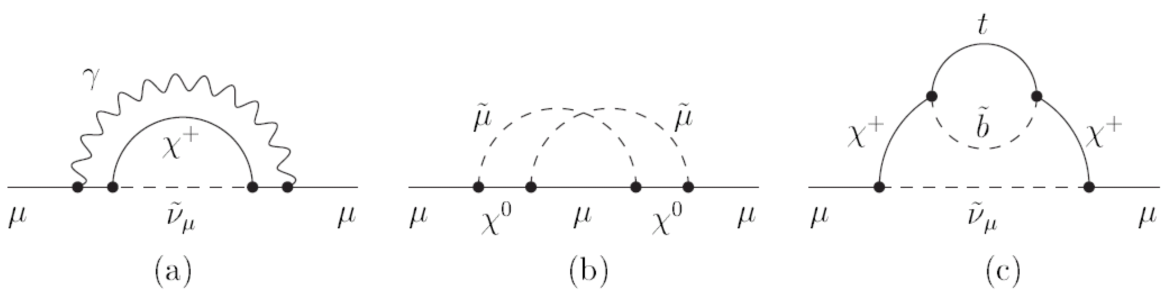
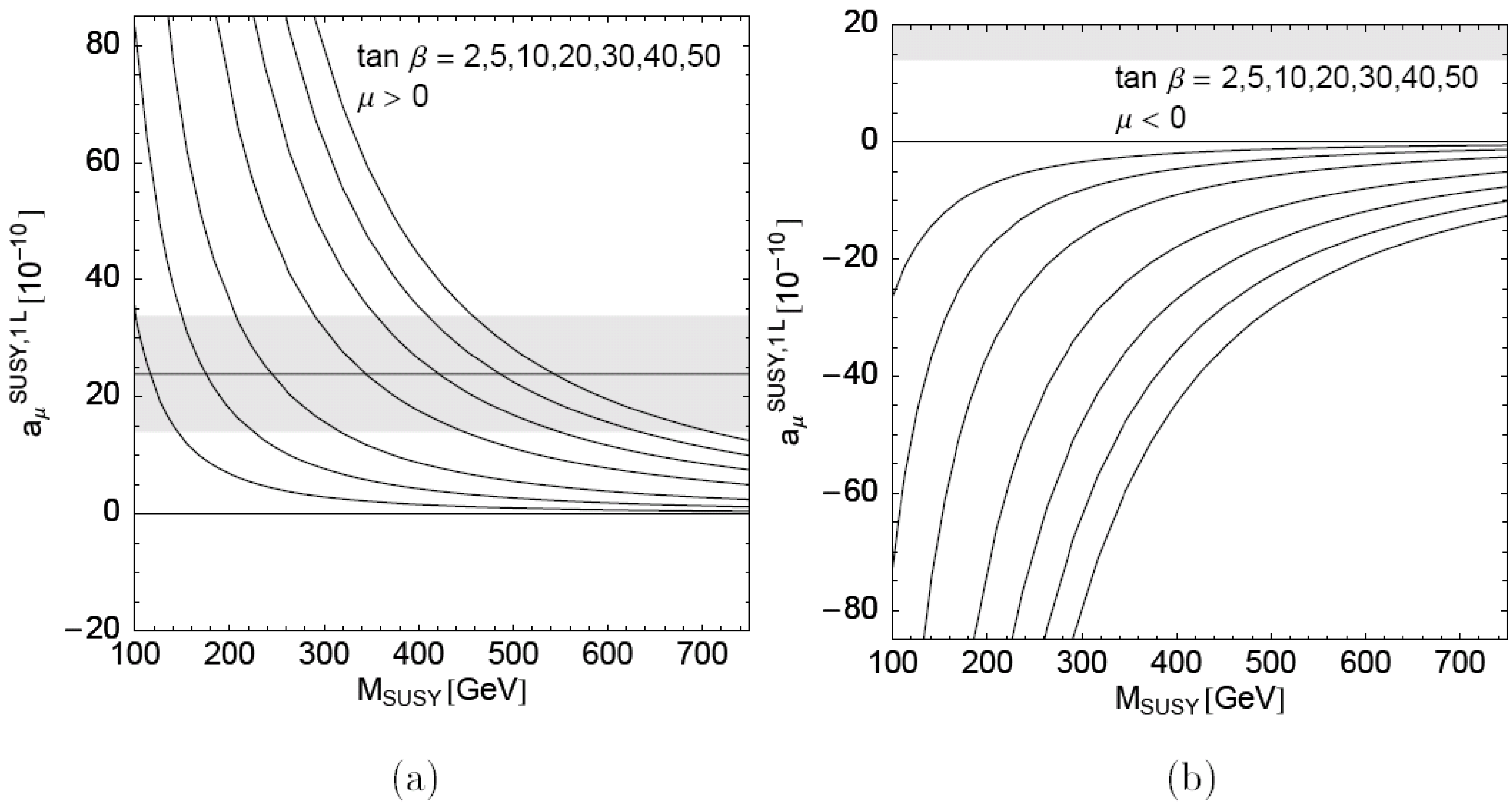


Figure 6. Sample two-loop diagrams contributing to $a_\mu^{\mathrm{SUSY,2L(b)}}$, i.e. involving a SUSY one-loop diagram. The external photon can be attached to all charged internal lines. (a) shows a diagram with additional photon loop, giving rise to large QED-logarithms. (b) shows a diagram of the class computed in [63]. (c) shows a diagram with an additional fermion/sfermion loop.

comprehensive review by D.Stöckinger, hep-ph/0609168

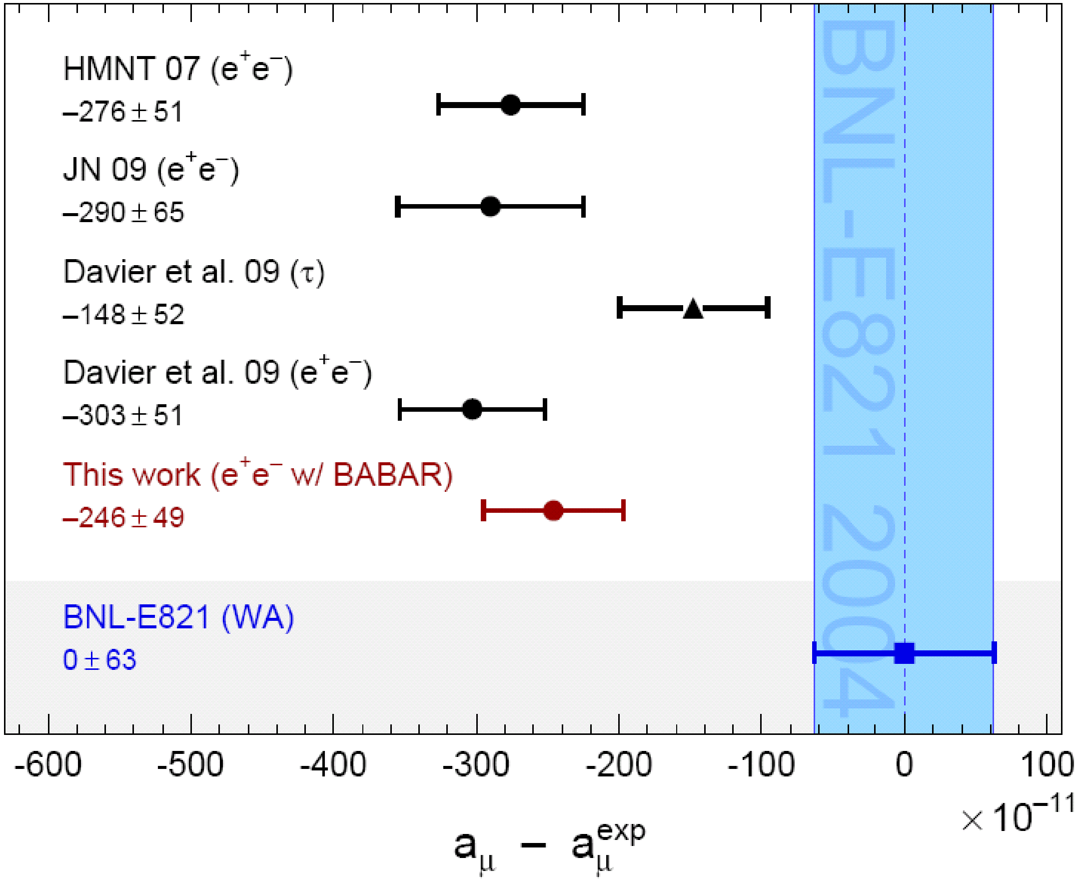
Virtual but real?

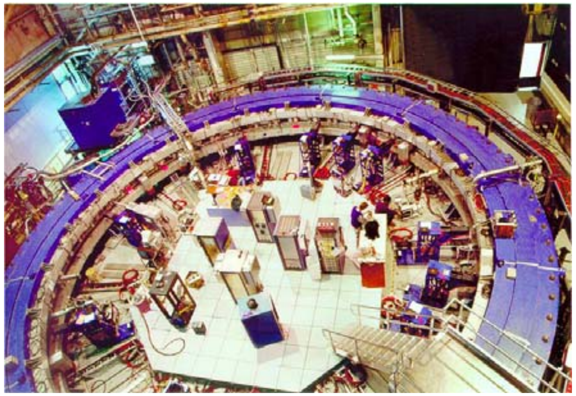


correction large for small SUSY masses and large tanβ
sign (correction) = sign(μ)

D.Stöckinger, hep-ph/0609168

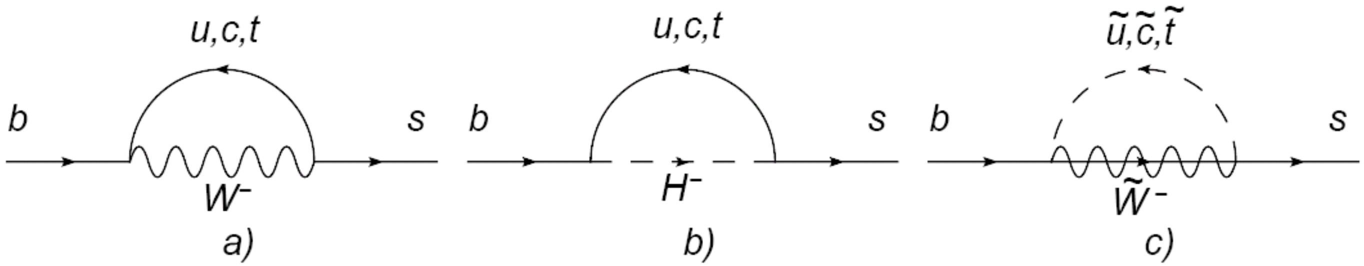
Data?



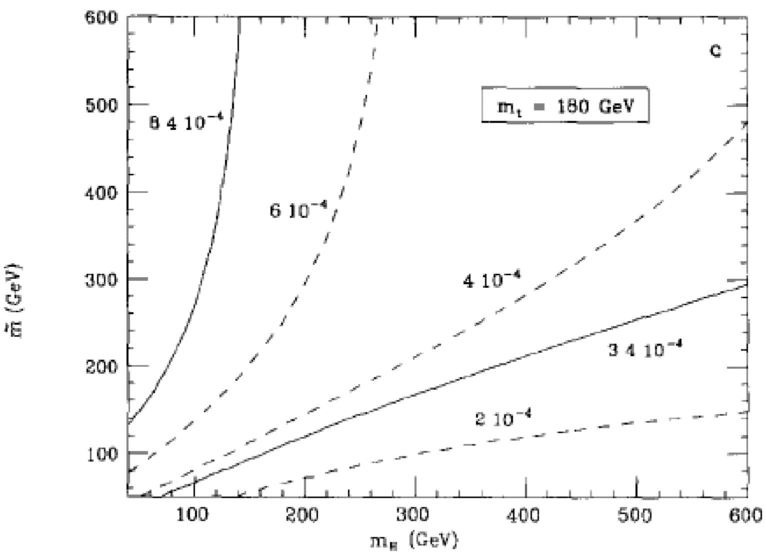


[Davier et al arXiv:0908:4300]

Further virtualities: b → s γ



SM and SUSY may cancel (for certain SUSY pars)



Barbieri,Guidice hep-ph/9303270 (1993!)

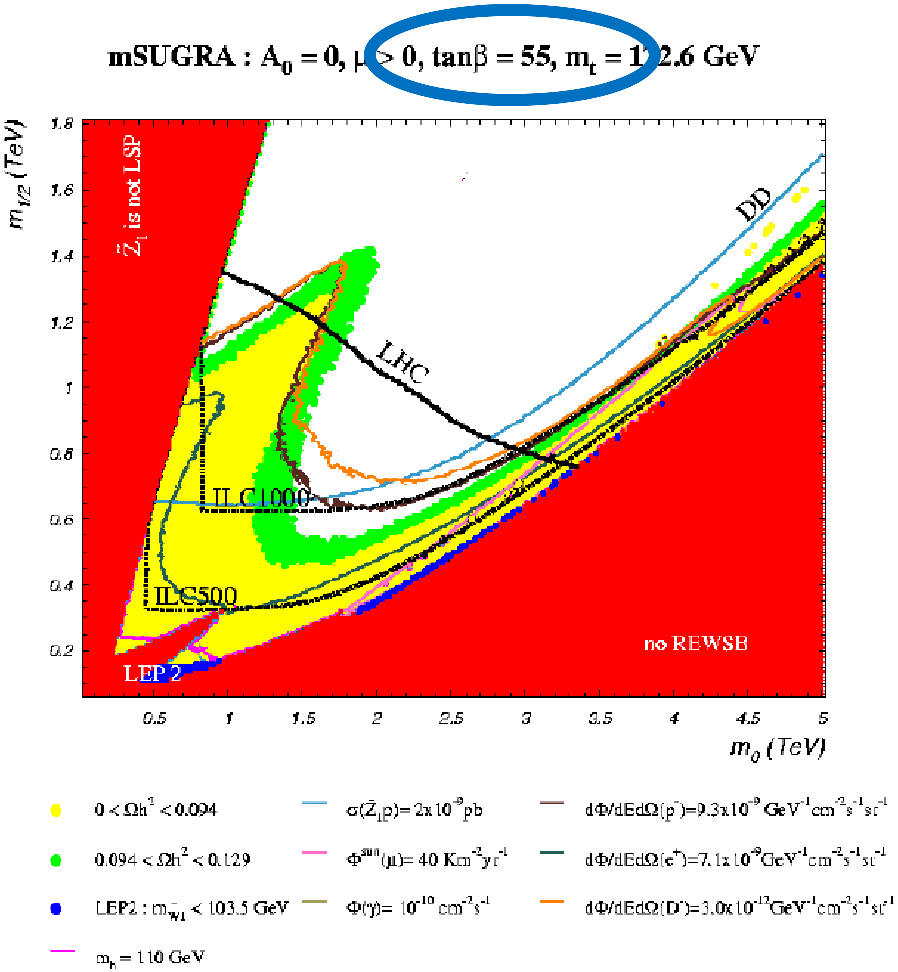
Fig 1 Contour plots of BR$(b \rightarrow s\gamma)$ in the plane $(m_H, \tilde{m})$ of the charged Higgs mass m_H and the common squark mass $\tilde{m}$, for $m_t = 120$ (a), 150 (b), and 180 GeV (c), under the assumptions stated in the text ($M = \mu = A = 0$, $\tan\beta = 1$, see appendix) The present 90% CL bound, BR$(b \rightarrow s\gamma)$ $< 8\ 4 \times 10^{-4}$ [3], and the standard model prediction, for any given m_t, are shown by solid lines

Measurement: b → s γ = $(3.56 \pm 0.25) \times 10^{-4}$

Does not necessarily constrain SUSY to be light – but limits viable parameter space

Real but invisible

Requirement that LSP is/contributes to CDM constrains SUSY parameters heavily!
(usually, i.e. in „most" of the parameter space SUSY-LSP would predict significantly more DM than observed!)



H.Baer arXiv:0912:0883

Backup - LHC observables

Observable	Nominal Value	Uncertainty 1 fb^{-1}	10 fb^{-1}	300 fb^{-1}	LES_1	$\mathrm{LES}_{10,300}$	JES_1	$\mathrm{JES}_{10,300}$	syst.
m_h	109.6		1.4	0.1		0.1			
m_t	172.4	1.1	0.05	0.01			1.5	1.0	
$m_{\tilde{\chi}_1^\pm}$	180.2			11.4				1.8	
$\sqrt{m_{\tilde{\ell}_L}^2 - 2m_{\tilde{\chi}_1^0}^2}$	148.8			1.7		0.1			6.0
$m_{\tilde{g}} - m_{\tilde{\chi}_1^0}$	507.7		13.7	2.5				5.1	10.0
$\sqrt{m_{\tilde{q}_R}^2 - 2m_{\tilde{\chi}_1^0}^2}$	531.0	19.6	6.2	1.1			22.7	4.5	10.0
$m_{\tilde{g}} - m_{\tilde{b}_1}$	88.7			1.5				0.9	
$m_{\tilde{g}} - m_{\tilde{b}_2}$	56.8			2.5				0.6	
$m_{\ell\ell}^{\max}(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\ell}_R})$	80.4	1.7	0.5	0.03	0.16	0.08			
$m_{\ell\ell}^{\max}(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_4^0}, m_{\tilde{\ell}_L})$	280.6		12.6	2.3		0.28			
$m_{\tau\tau}^{\max}(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\tau}_1})$	83.4	12.6	4.0	0.73			4.2	0.8	5.7
$m_{\ell\ell q}^{\max}(m_{\tilde{\chi}_1^0}, m_{\tilde{q}_L}, m_{\tilde{\chi}_2^0})$	452.1	13.9	4.2	1.4			22.7	4.5	
$m_{\ell q}^{\mathrm{low}}(m_{\tilde{\ell}_R}, m_{\tilde{q}_L}, m_{\tilde{\chi}_2^0})$	318.6	7.6	3.5	0.9			16.2	3.2	
$m_{\ell q}^{\mathrm{high}}(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\ell}_R}, m_{\tilde{q}_L})$	396.0	5.2	4.5	1.0			19.9	4.0	
$m_{\ell\ell q}^{\mathrm{thres}}(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\ell}_R}, m_{\tilde{q}_L})$	215.6	26.5	4.8	1.6			10.8	2.2	
$m_{\ell\ell b}^{\mathrm{thres}}(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\ell}_R}, m_{\tilde{b}_1})$	195.9		19.7	3.6				2.0	
$m_{tb}^{\mathrm{w}}(m_t, m_{\tilde{t}_1}, m_{\tilde{\chi}_1^\pm}, m_{\tilde{g}}, m_{\tilde{b}_1})$	359.5	43.0	13.6	2.5			18.0	3.6	
$\frac{\mathcal{B}(\tilde{\chi}_2^0 \to \tilde{\ell}_R \ell) \times \mathcal{B}(\tilde{\ell}_R \to \tilde{\chi}_1^0 \ell)}{\mathcal{B}(\tilde{\chi}_2^0 \to \tilde{\tau}_1 \tau) \times \mathcal{B}(\tilde{\tau}_1 \to \tilde{\chi}_1^0 \tau)}$	0.076	0.009	0.003	0.001					0.008
$\frac{\mathcal{B}(\tilde{g} \to \tilde{b}_2 b) \times \mathcal{B}(\tilde{b}_2 \to \tilde{\chi}_2^0 b)}{\mathcal{B}(\tilde{g} \to \tilde{b}_1 b) \times \mathcal{B}(\tilde{b}_1 \to \tilde{\chi}_2^0 b)}$	0.168			0.078					

Backup - predictions for sign mu) +1 / -1

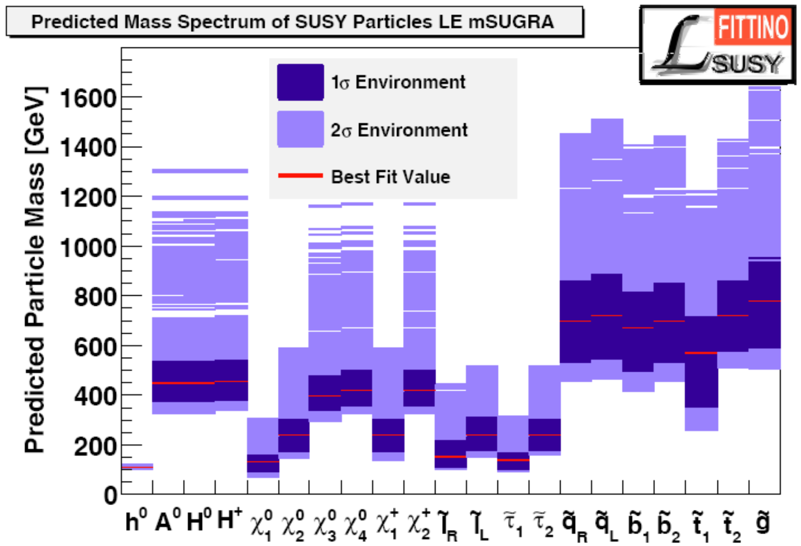


Fig. 6: SUSY mass spectrum as predicted by mSUGRA parameter fit to low energy measurements with sign(μ) fixed to $+1$.

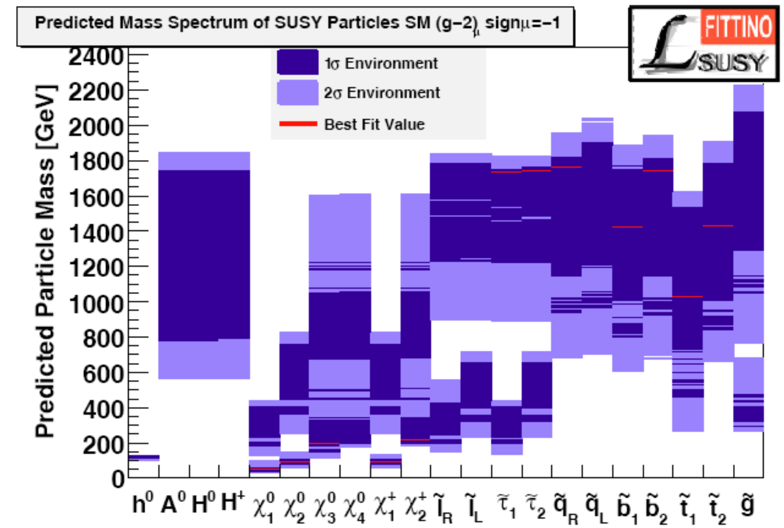
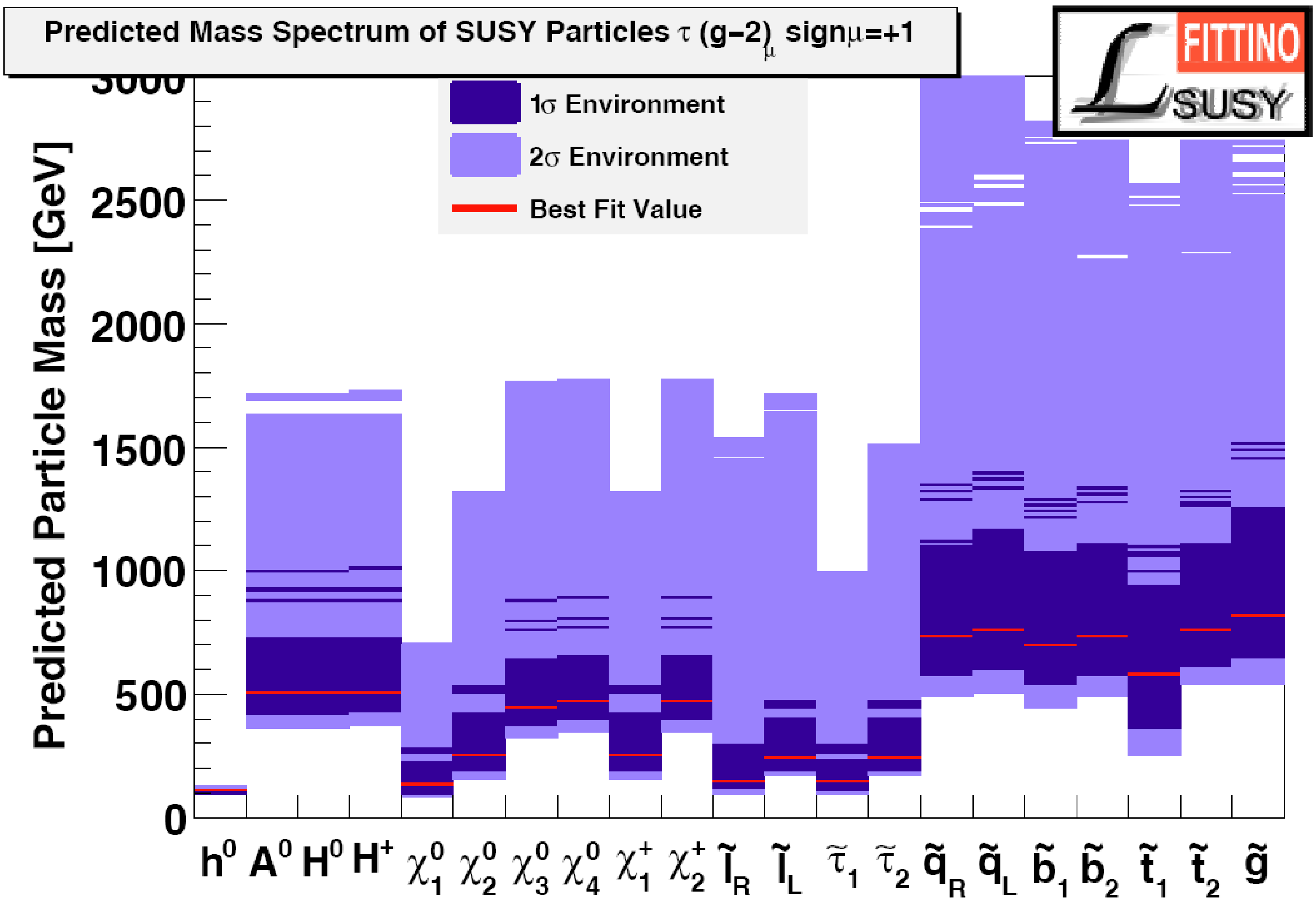


Fig. 15: SUSY mass spectrum as predicted by mSUGRA parameter fit to low energy measurements with $a_\mu^{\exp} = a_\mu^{\mathrm{SM}}$ and with sign(μ) fixed to -1.

Backup - prdediction for (g-2) from tau data



Backup - BR predictions

Decay Mode	Expected Branching Fraction	Uncertainty
$\tilde{\chi}_2^0 \to \tilde{\tau}_1 \tau$	0.46	${}^{+0.38}_{-0.44}$
$\tilde{\chi}_2^0 \to \tilde{\nu}_{\tau_1} \nu_\tau$	0.076	${}^{+0.039}_{-0.067}$
$\tilde{\chi}_2^0 \to \tilde{e}_R e$	0.040	${}^{+0.044}_{-0.038}$
$\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 h$	0.036	${}^{+0.13}_{-0.035}$
$\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z$	0.018	${}^{+0.0098}_{-0.018}$
$\tilde{\chi}_2^0 \to \tilde{e}_L e$	0.00018	${}^{+0.14}_{-0.00018}$
$\tilde{\chi}_2^0 \to \tilde{\tau}_2 \tau$	0.	${}^{+0.014}_{-0}$
$\tilde{\chi}_1^\pm \to \tilde{\tau}_1 \nu_\tau$	0.40	${}^{+0.42}_{-0.39}$
$\tilde{\chi}_1^\pm \to \tilde{\nu}_{e_L} e$	0.15	${}^{+0.10}_{-0.15}$
$\tilde{\chi}_1^\pm \to \tilde{\nu}_{\tau_1} \tau$	0.15	${}^{+0.10}_{-0.15}$
$\tilde{\chi}_1^\pm \to \tilde{\chi}_1^0 W^\pm$	0.12	${}^{+0.079}_{-0.12}$
$\tilde{\chi}_1^\pm \to \tilde{\tau}_2 \nu_\tau$	0	${}^{+0.14}_{-0}$
$\tilde{g} \to \tilde{u}_L u$	0.052	${}^{+0.020}_{-0.023}$
$\tilde{g} \to \tilde{u}_R u$	0.094	${}^{+0.030}_{-0.054}$
$\tilde{g} \to \tilde{d}_L d$	0.051	${}^{+0.018}_{-0.021}$
$\tilde{g} \to \tilde{d}_R d$	0.093	${}^{+0.031}_{-0.055}$
$\tilde{g} \to \tilde{t}_1 t$	0.090	${}^{+0.166}_{-0.066}$
$\tilde{g} \to \tilde{t}_2 t$	0	${}^{+0.056}_{-0}$
$\tilde{g} \to \tilde{b}_1 b$	0.179	${}^{+0.022}_{-0.056}$
$\tilde{g} \to \tilde{b}_2 b$	0.11	${}^{+0.009}_{-0.038}$
$\tilde{e}_R \to \tilde{\chi}_1^0 e$	1	${}^{+0}_{-0}$
$\tilde{e}_L \to \tilde{\chi}_1^0 e$	1	${}^{+0}_{-0.23}$
$\tilde{e}_L \to \tilde{\chi}_2^0 e$	0	${}^{+0.085}_{-0}$
$\tilde{\tau}_1 \to \tilde{\chi}_1^0 \tau$	1	${}^{+0}_{-0}$
$\tilde{\tau}_2 \to \tilde{\chi}_1^0 \tau$	0.99	${}^{+0.011}_{-0.42}$
$\tilde{\tau}_2 \to \tilde{\chi}_2^0 \tau$	0.0007	${}^{+0.040}_{-0.0007}$
$\tilde{\tau}_2 \to \tilde{\chi}_1^\pm \nu_\tau$	0.0044	${}^{+0.078}_{-0.0043}$
$\tilde{u}_L \to \tilde{\chi}_1^0 u$	0.008	${}^{+0.0049}_{-0.0026}$
$\tilde{u}_L \to \tilde{\chi}_2^0 u$	0.31	${}^{+0.013}_{-0.0061}$
$\tilde{u}_L \to \tilde{\chi}_1^\pm d$	0.64	${}^{+0.019}_{-0.0094}$
$\tilde{u}_L \to \tilde{\chi}_2^\pm d$	0.025	${}^{+0.0092}_{-0.018}$
$\tilde{u}_R \to \tilde{\chi}_1^0 u$	0.99	${}^{+0.012}_{-0.0080}$
$\tilde{u}_R \to \tilde{\chi}_2^0 u$	0.0078	${}^{+0.0064}_{-0.0072}$
$\tilde{t}_1 \to \tilde{\chi}_1^0 t$	0.18	${}^{+0.073}_{-0.037}$
$\tilde{t}_1 \to \tilde{\chi}_2^0 t$	0.15	${}^{+0.006}_{-0.069}$
$\tilde{t}_1 \to \tilde{\chi}_1^\pm b$	0.51	${}^{+0.21}_{-0.28}$
$\tilde{t}_1 \to \tilde{\chi}_2^\pm b$	0.15	${}^{+0.11}_{-0.15}$
$\tilde{b}_1 \to \tilde{\chi}_1^\pm t$	0.37	${}^{+0.13}_{-0.039}$
$\tilde{b}_1 \to \tilde{\chi}_2^0 b$	0.25	${}^{+0.12}_{-0.068}$
$\tilde{b}_1 \to \tilde{t}_1 W^\pm$	0.043	${}^{+0.29}_{-0.043}$

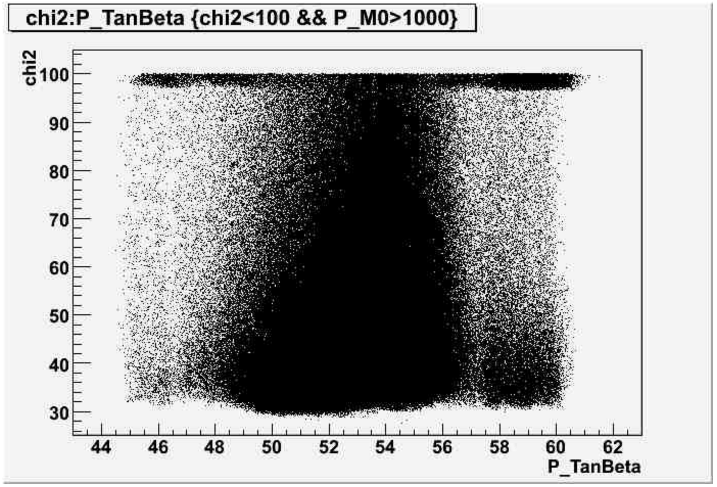
MSSM-18 and the Linear Collider (.5 + 1 TeV)

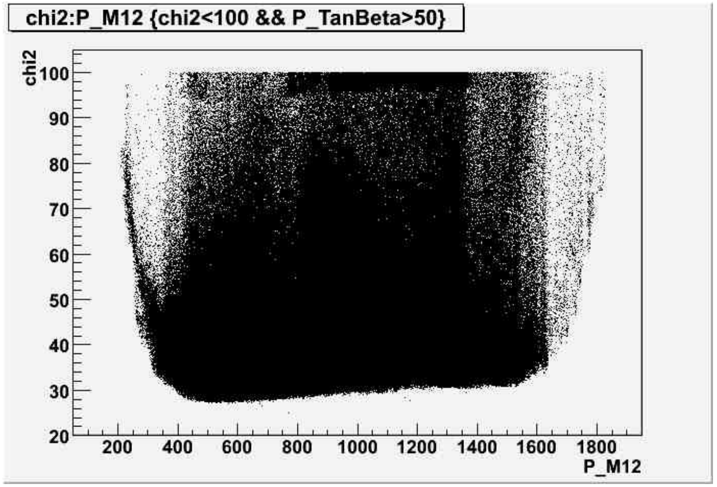
Parameter	Nominal value	ILC Fit		$\sigma_{\mathrm{LE+LHC\,300}}$	$\sigma_{\mathrm{LE+LHC300+ILC}}$
$M_{\tilde{\ell}_L}$ (GeV)	194.31	194.315	$\pm$	6.4	0.068
$M_{\tilde{\ell}_R}$ (GeV)	135.76	135.758	$\pm$	10.5	0.071
$M_{\tilde{\tau}_L}$ (GeV)	193.52	193.46	$\pm$	43.0	0.33
$M_{\tilde{\tau}_R}$ (GeV)	133.43	133.45	$\pm$	38.2	0.35
$M_{\tilde{q}_L}$ (GeV)	527.57	527.61	$\pm$	3.4	0.64
$M_{\tilde{q}_R}$ (GeV)	509.14	509.3	$\pm$	9.0	9.0
$M_{\tilde{b}_R}$ (GeV)	504.01	504.2	$\pm$	33.3	2.4
$M_{\tilde{t}_L}$ (GeV)	481.69	481.6	$\pm$	15.5	1.5
$M_{\tilde{t}_R}$ (GeV)	409.12	409.2	$\pm$	103.8	1.6
$\tan\beta$	10	10.01	$\pm$	3.3	0.29
μ (GeV)	355.05	355.02	$\pm$	6.2	0.88
X_τ (GeV)	-3799.88	-3795.1	$\pm$	3053.5	46.6
X_t (GeV)	-526.62	-526.8	$\pm$	299.2	4.7
X_b (GeV)	-4314.33	-4252.1	$\pm$	5393.6	728.7
M_1 (GeV)	103.15	103.154	$\pm$	3.5	0.046
M_2 (GeV)	192.95	192.95	$\pm$	5.5	0.11
M_3 (GeV)	568.87	568.66	$\pm$	6.9	1.65
m_A (GeV)	359.63	360.07	$\pm$	${}^{+1181}_{-99.3}$	1.83

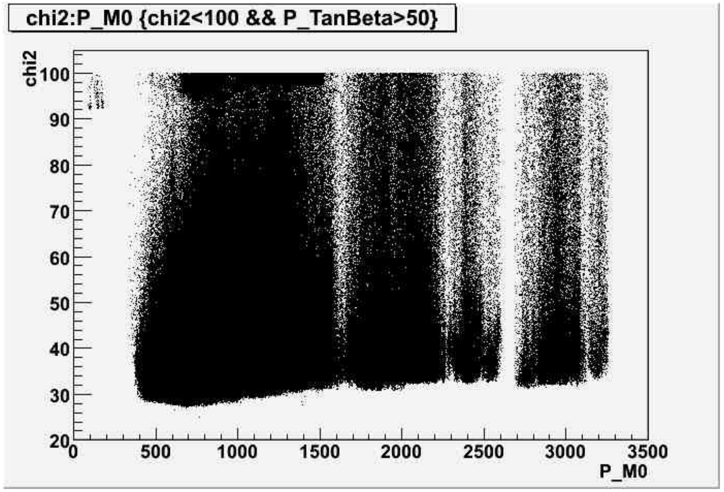
1-2 orders of magnitude improvement possible with ILC

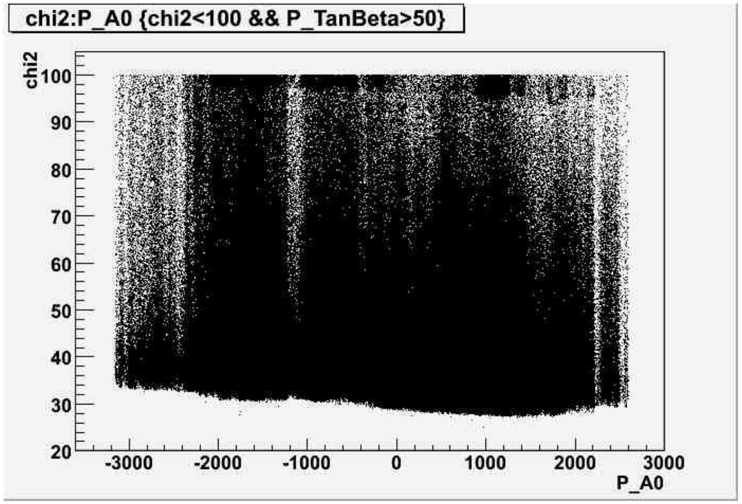
Precise bottom-up reconstruction possible!

LE-Fit at large m_0









compare with χ^2_{min} = 20.6 (for ndf = 22)