Digesting DNS-LES data: statistical analysis of structural dynamics of turbulent boundary layers

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Outline

1.Methods

2.Objectives

3.Open discussions: Interactions and collaborations

1. Methods:

Traditional approach:

1. Mean velocity profile
2. Kinetic energy profile
3. Correlation coefficients
4. Energy spectra
5. Energy budget terms
6. Higher order moments (skewness, flatness, etc.)
7. etc.

→ Empirical fitting – asymptotic expansion and matching

→ For qualitative understanding

→ Scaling,

→ For qualitative understanding

→ Examining intermittency

We need to use these measures in a more productive way!
We need to define other more sensitive measures!

SED approach:

1. Existence of an ensemble
2. **Balance equations**
   - **mass, momentum, energy, etc.**
3. State functions
   - main variables
4. **Order functions**
   - **a combination of correlation functions**

→ Sub-ensemble decomposition, if necessary!

→ **Scaling analysis step-by-step on equations**

→ Solution of the closure problem!

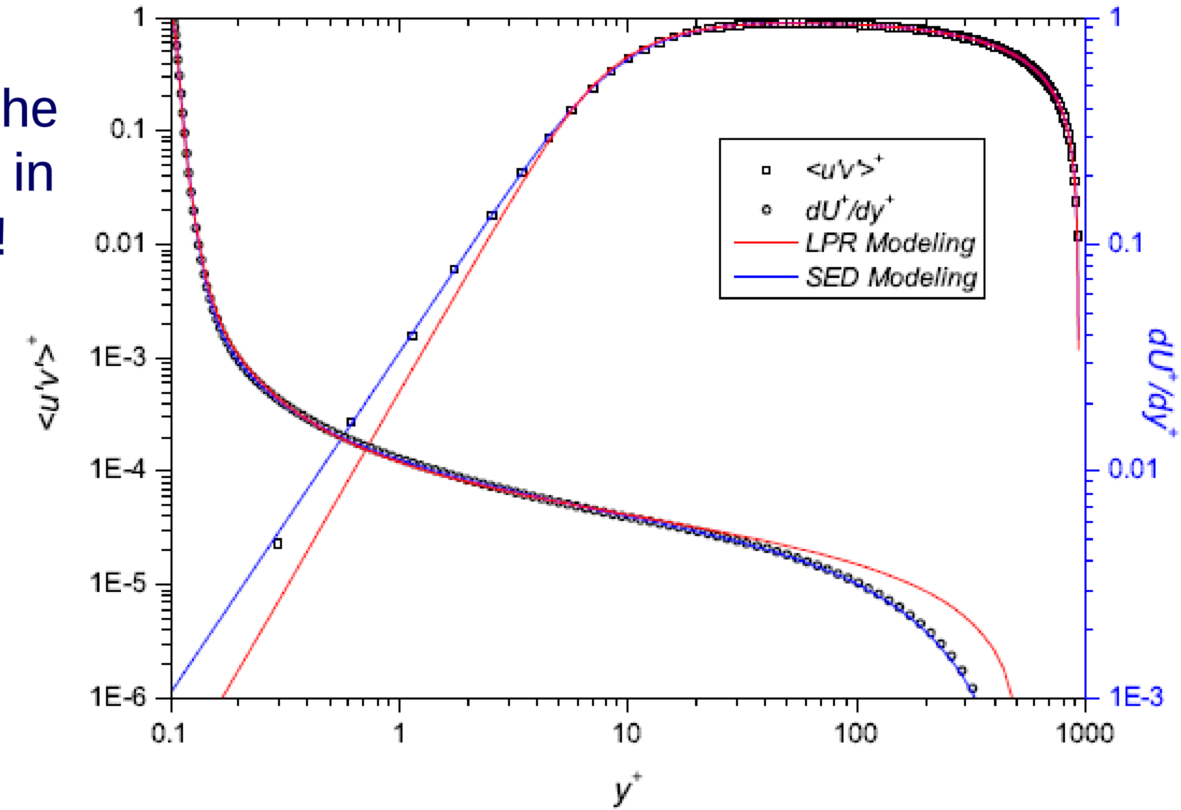
→ **Interpretation in terms of structural dynamics**

We will illustrate how the method is applied to turbulent boundary layer DNS data!

1. Methods:

- Well-known measure: Reynolds stress and mean shear in channel flow!
- Discrepancy of W near the wall and S at the center in LPR model are revealed!

$$S^+ \equiv dU^+/dy^+$$
$$W^+ \equiv -\overline{u'v'}^+$$



LPR: L'vov, Procaccia & Rodenko, PRL, 2008

SED: current theory

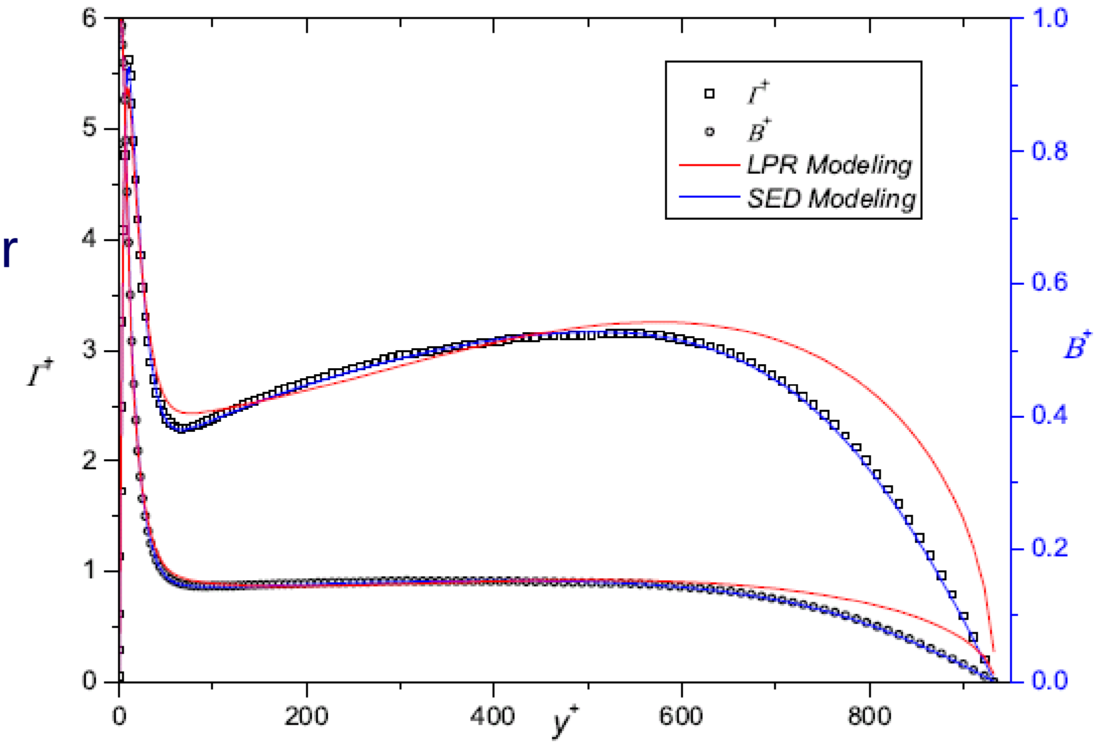
Dots: DNS data

1. Methods:

$$\Gamma^+ = y^+ dU^+/dy^+$$

$$\mathcal{B}^+ = y^+/U^+ dU^+/dy^+$$

- Two sensitive indicators:
- Discrepancy near the center in LPR model is revealed
- Wider constancy range for B is observed, indicating power-law leading-order term may have higher accuracy!

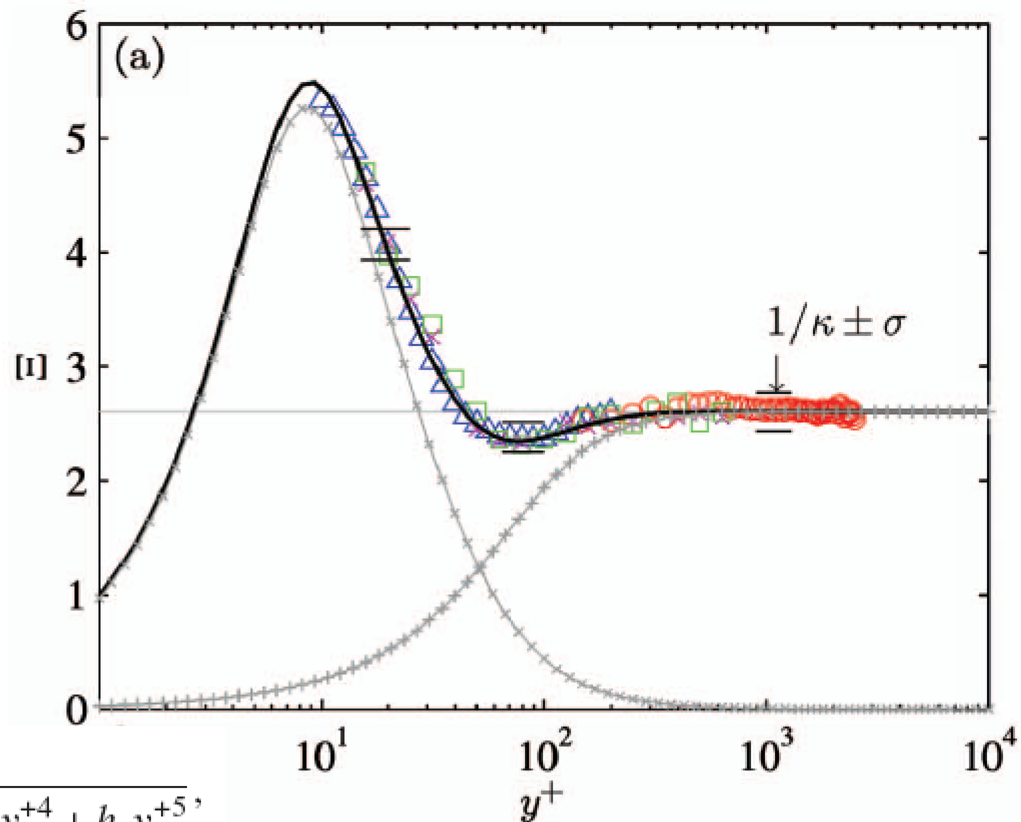


LPR: L’vov, Procaccia & Rodenko, PRL, 2008
SED: current theory
Dots: DNS data

1. Methods:

- Boundary layer data (KTH) fitting from Monkewitz et al (2007) with two Pade approximants

$$\Gamma^+ = y^+ dU^+/dy^+$$



$$\frac{dU^+_{\text{inner}}}{dy^+} = P_{23} + P_{25},$$

$$P_{23} = b_0 \frac{1 + b_1 y^+ + b_2 y^{+2}}{1 + b_1 y^+ + b_2 y^{+2} + \kappa b_0 b_2 y^{+3}},$$

$$b_0 = 1.000 \times 10^{-2} \kappa^{-1} \quad (\kappa = 0.384),$$

$$b_1 = 1.100 \times 10^{-2},$$

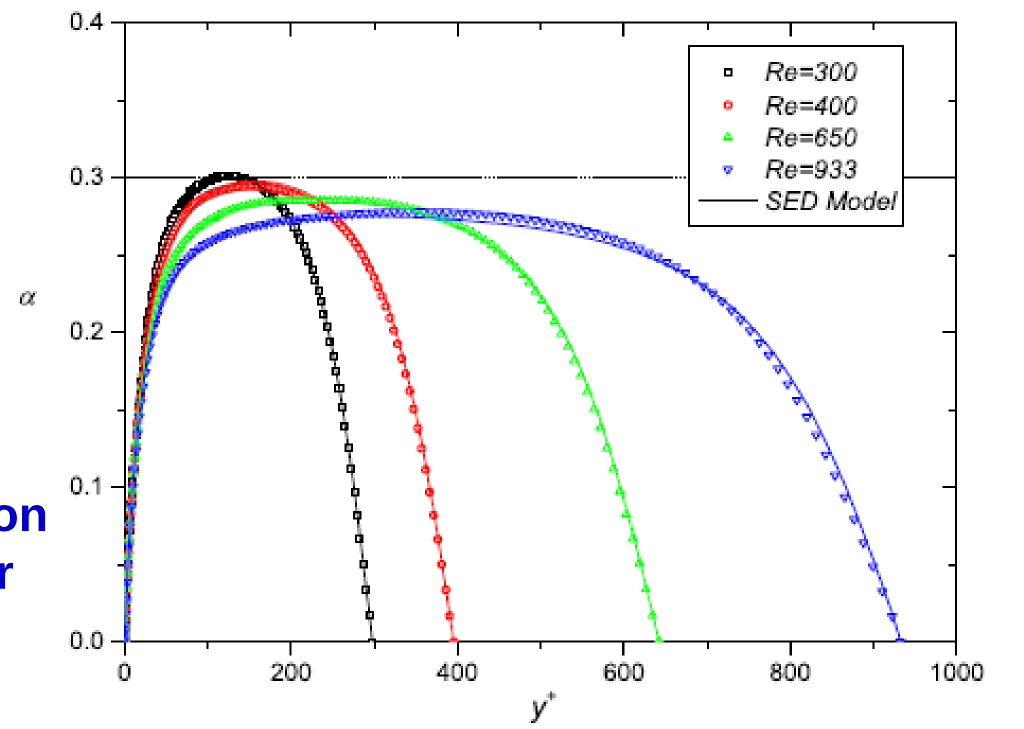
$$b_2 = 1.100 \times 10^{-4},$$

$$P_{25} = (1 - b_0) \frac{1 + h_1 y^+ + h_2 y^{+2}}{1 + h_1 y^+ + h_2 y^{+2} + h_3 y^{+3} + h_4 y^{+4} + h_5 y^{+5}},$$

1. Methods:

- Bradshaw function is anothe
  example of the order functi

$$\alpha_k = W^+/K^+$$

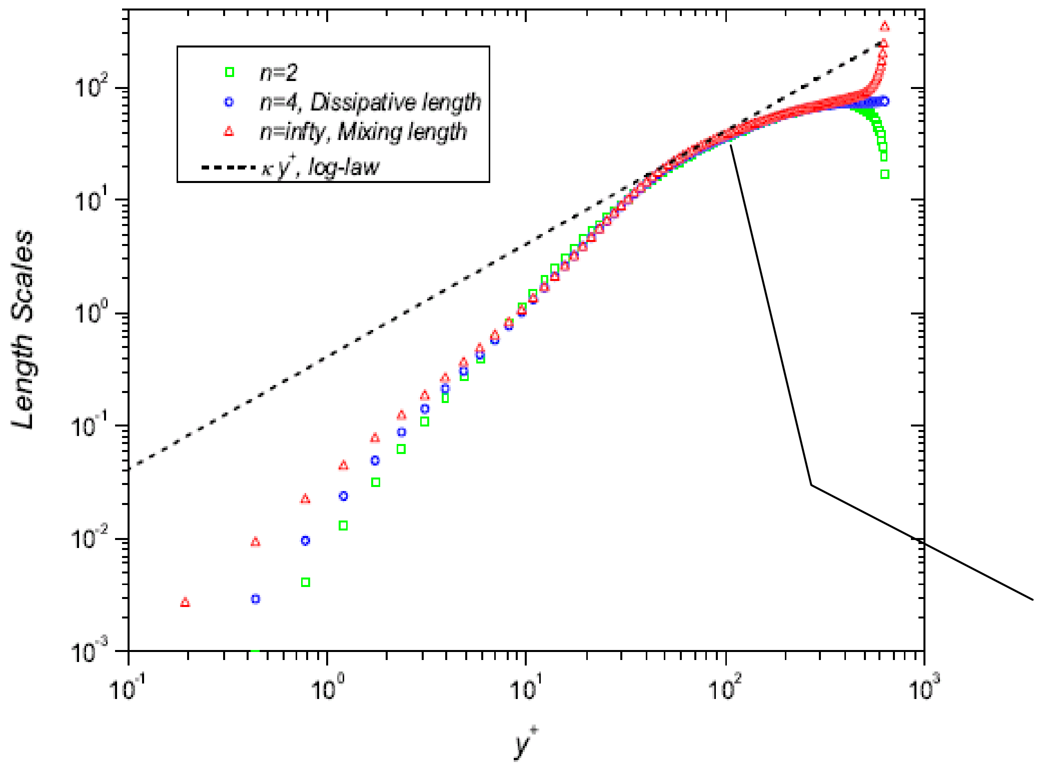


We look for simplest expression for its description! It is simpler than Pade approximants.

$$\alpha_k = B_\alpha \left(1 + \frac{64}{y^+}^{1.26} Re^{-0.2}\right)^{-1/1.2} \left(1 + \frac{0.78}{z}^{2} Re^{-0.4}\right)^{-1/2}$$

1. Methods:

- We define a length function for the energy dynamics involving dissipation and mean shear production, and identify multiple physical domains.



Dissipative Length (n=4)

$$\ell_\nu^+ = [(\frac{W^{+3}}{S^{+3}})/\varepsilon^+]^{1/4}.$$

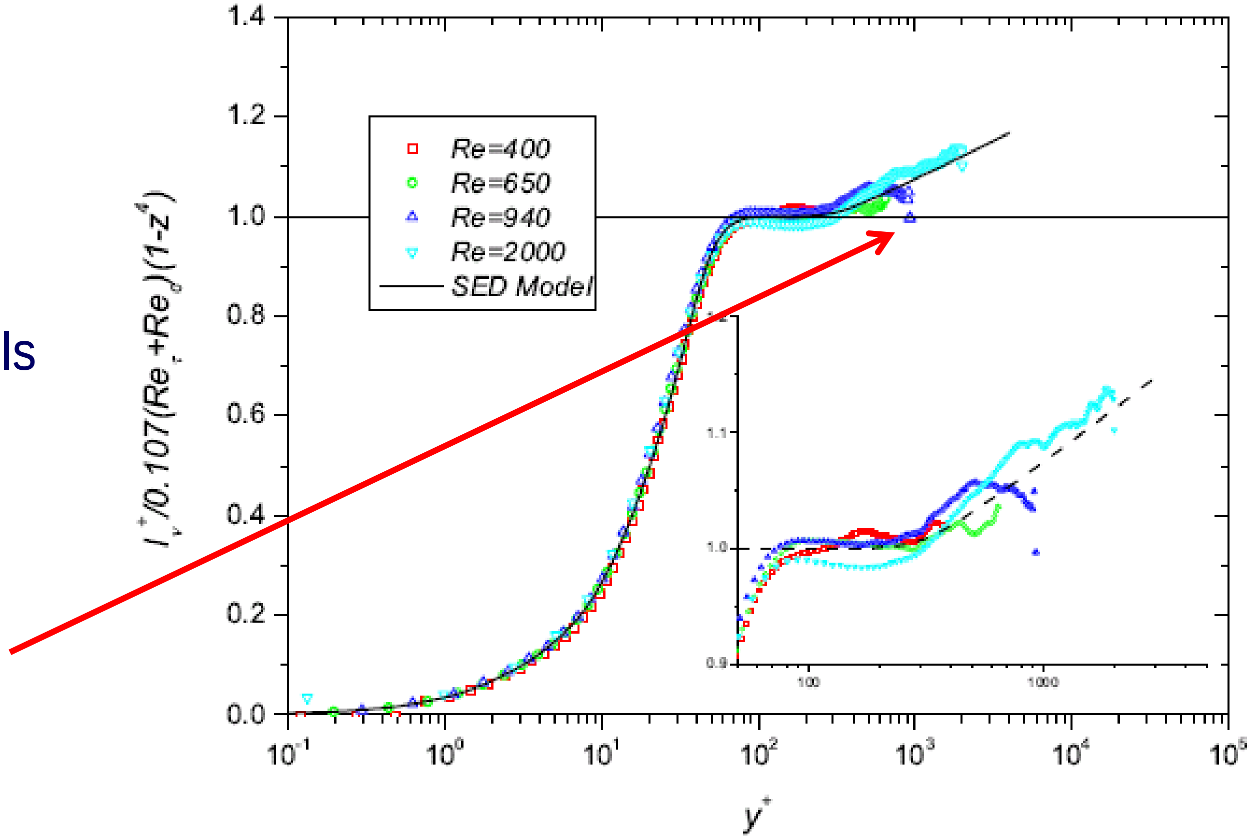
Mixing Length (n=∞)

$$l_M = W^{+1/2}/S^+.$$

Collapse in logarithm region, an evidence for decoupling of mean-field property from fluctuation structures.

1. Methods:

- Now, we try to model the length function!
- Near-wall behavior:
  - a transition function
  - Its modeling calls for an order function!
- Discover a new Reynolds number effect!



1. Methods:

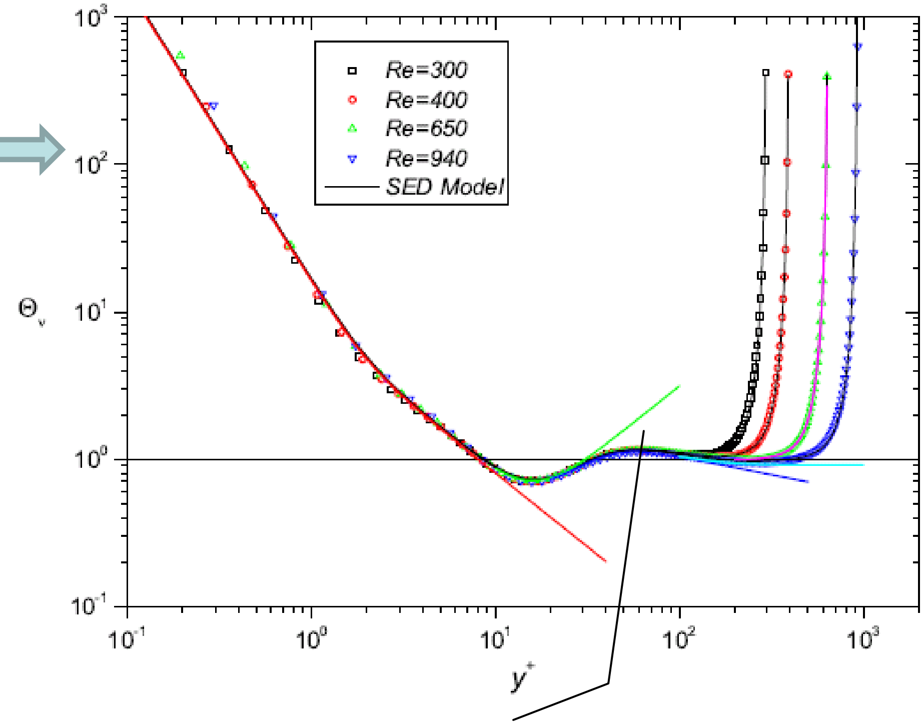
- Define another order function specifying details of other energy processes, e.g. turbulent and pressure transport!

$$\frac{\ell^+_{n_1}}{\ell^+_{n_2}} = \left(\frac{\varepsilon^+}{S^+W^+}\right)^{\frac{1}{n_2}-\frac{1}{n_1}}.$$

- It reveals a few transitions from the wall to the center – rich physics captured!

Dissipation-shear ratio

$$\Theta_\nu = \frac{\varepsilon^+}{S^+W^+}.$$



Almost equal to 1 in log-layer.

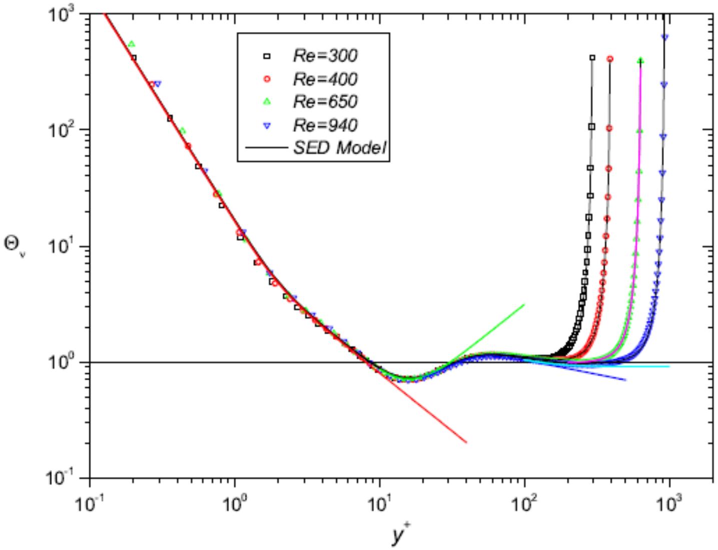
1. Methods:

- The series of transitions can be modeled by the product of a series of rational functions.

$$\frac{\ell^+_{n_1}}{\ell^+_{n_2}} = \left(\frac{\varepsilon^+}{S^+W^+}\right)^{\frac{1}{n_2}-\frac{1}{n_1}}.$$

Dissipation-shear ratio

$$\Theta_\nu = \frac{\varepsilon^+}{S^+W^+}.$$



$$\Theta_\nu = 16(y^+)^{-2}\left(1+\left(\frac{y^+}{2}\right)^4\right)^{\frac{1}{4}}\left(1+\left(\frac{y^+}{16}\right)^4\right)^{\frac{2}{4}}\left(1+\left(\frac{y^+}{41}\right)^4\right)^{-\frac{1.25}{4}}\left(1+\left(\frac{y^+}{170}\right)^4\right)^{\frac{0.25}{4}}\left(1+\left(\frac{z}{0.26}\right)^{-2.6}\right)^{\frac{2}{2.6}},$$

a refined description for all Reynolds number!

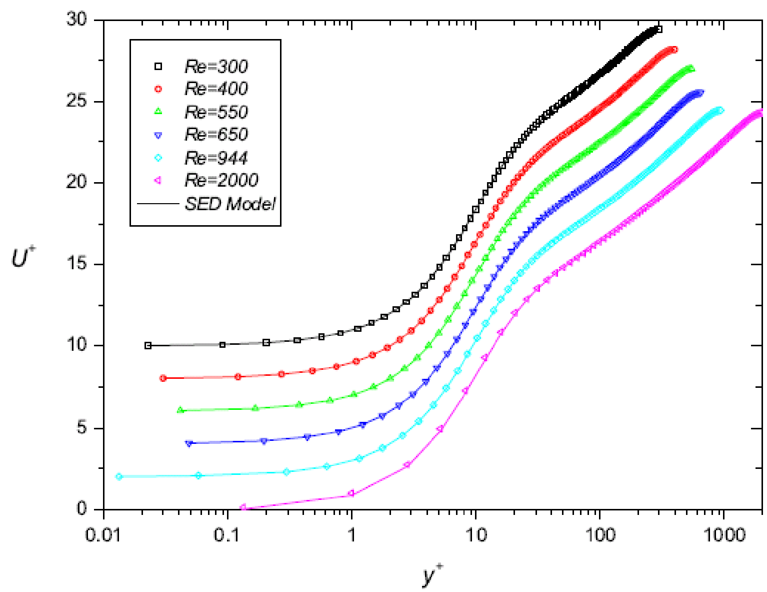
Why?

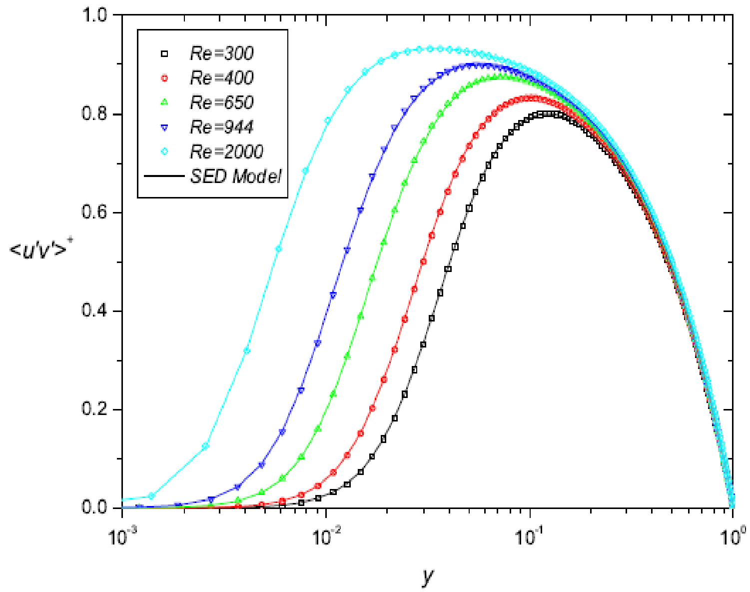
1. Methods:

- A complete 3/2 closure from SED, with the specification of two order functions.
- Maximum relative error: 0.5%.
- **These are solutions of the equations.**

$$
\begin{aligned}
S^+ + W^+ &= 1 - y \\
\Theta_\nu \mathcal{P}^+ &= \varepsilon^+ \\
\mathcal{P}^+ &= S^+ W^+ \\
\varepsilon^+ &= (W^{+3}/S^{+3})/\ell_\nu^{+4}.
\end{aligned}
$$

$$l_M^{SED} = \ell_\nu^+ \Theta_\nu^{1/4}.$$

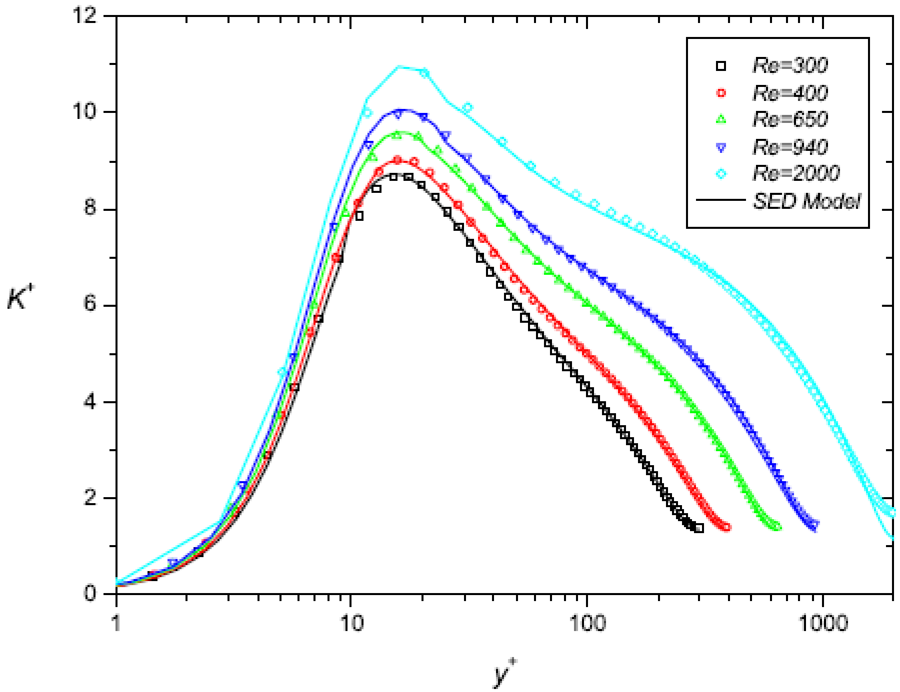




1. Methods:

- Prediction of the kinetic ene
(or rms velocity).

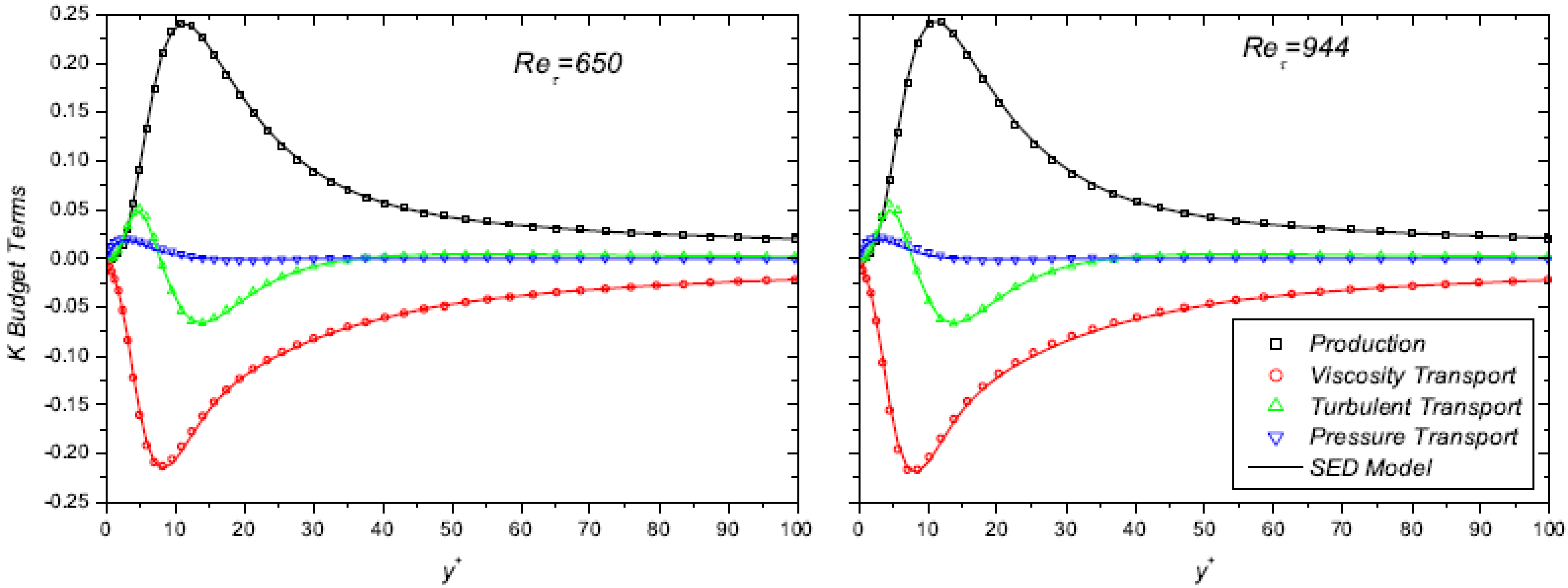
$$k^+ = W^+/\alpha_k$$



1. Methods:

- A complete second-order closure can be formed with four order-functions in total.
- Then, all budget terms can be predicted for all Reynolds numbers.

$$
\begin{aligned}
k^+ &= W^+/\alpha_k \\
c^+ &= \Theta_C S^+ W^+ \\
\Pi^+ &= (\Theta_\nu - \Theta_C - 1) S^+ W^+.
\end{aligned}
$$



1. Methods:

Summary:

- We suggest to examine the behavior (Reynolds number dependences) of a series state functions and order functions.
- We suggest to define the order functions from the momentum/energy equations.

$$\Gamma^+ = y^+ dU^+/dy^+$$

$$\mathcal{B}^+ = y^+/U^+ dU^+/dy^+$$

$$\ell_\nu^+ = [(\frac{W^{+3}}{S^{+3}})/\varepsilon^+]^{1/4}.$$

$$\Theta_\nu = \frac{\varepsilon^+}{S^+W^+}.$$

$$\alpha_k = W^+/K^+$$

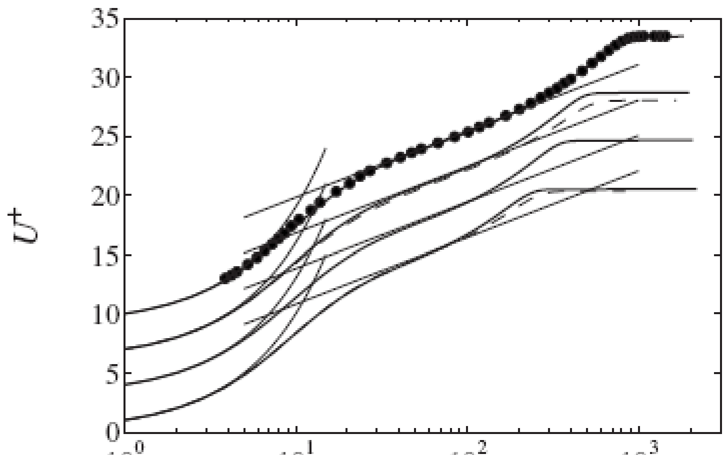
$$\begin{aligned} S^+ + W^+ &= 1 - y \\ \Theta_\nu \mathcal{P}^+ &= \varepsilon^+ \\ \mathcal{P}^+ &= S^+W^+ \\ \varepsilon^+ &= (W^{+3}/S^{+3})/\ell_\nu^{+4}. \end{aligned}$$

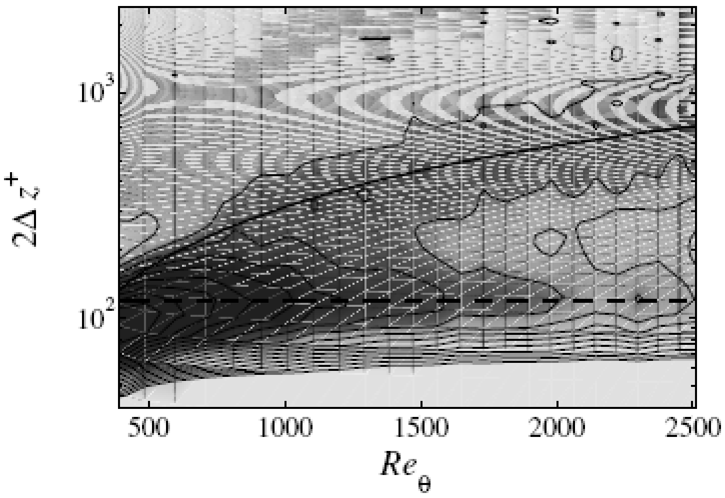
$$\begin{aligned} k^+ &= W^+/\alpha_k \\ \mathcal{C}^+ &= \Theta_C S^+W^+ \\ \Pi^+ &= (\Theta_\nu - \Theta_C - 1)S^+W^+. \end{aligned}$$

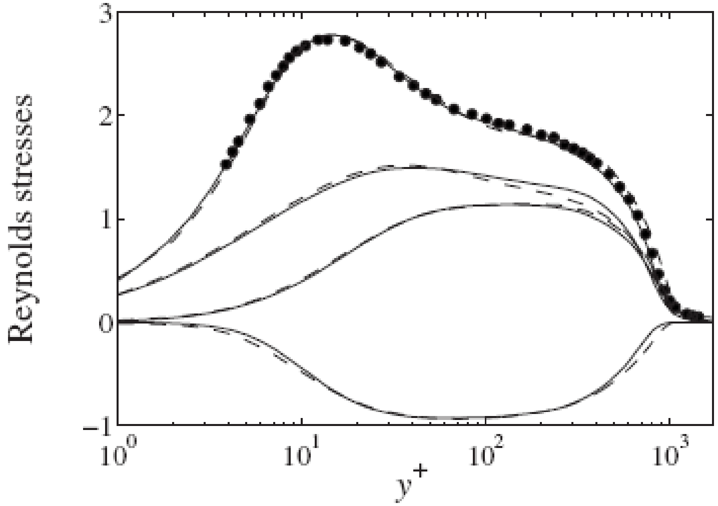
Applying SED analysis to boundary layer data (Schlatter et al.)

Schlatter et al., Phy Fluids, 2009

- **mean velocity profiles**
- **Reynolds stresses**
- **spanwise correlation coefficients**



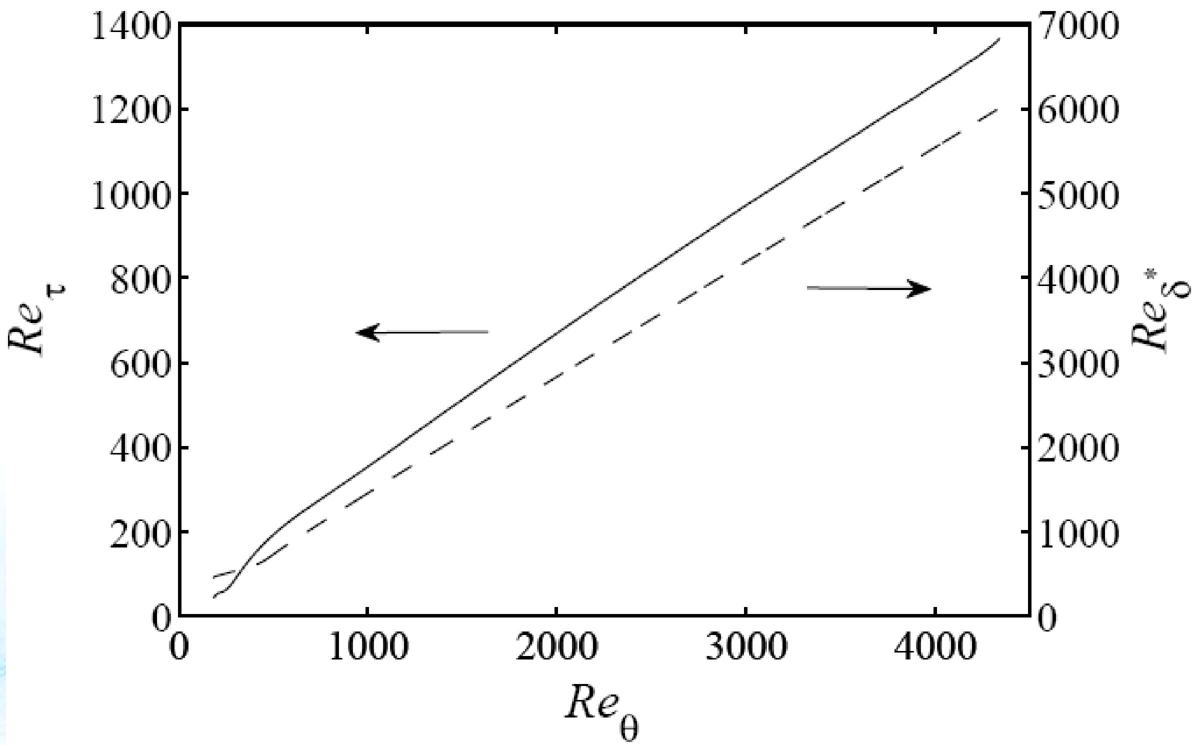




2. Objectives

Applying SED analysis to boundary layer data (Schlatter et al.)

Examine the scaling for global properties:

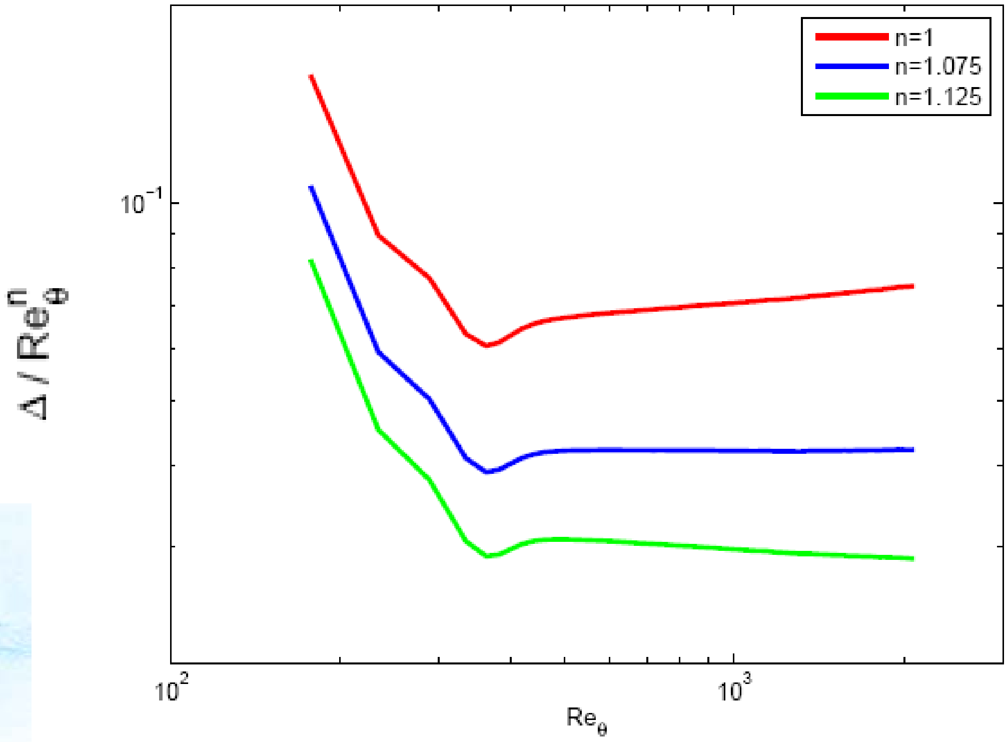


2. Objectives

Applying SED analysis to boundary layer data (Schlatter et al.)

Examine the scaling for global properties:

$$\Delta = \frac{\delta^* U_\infty}{u_\tau} = \frac{y^+}{Re_\delta^*}$$



n=1
n=1.075
n=1.125

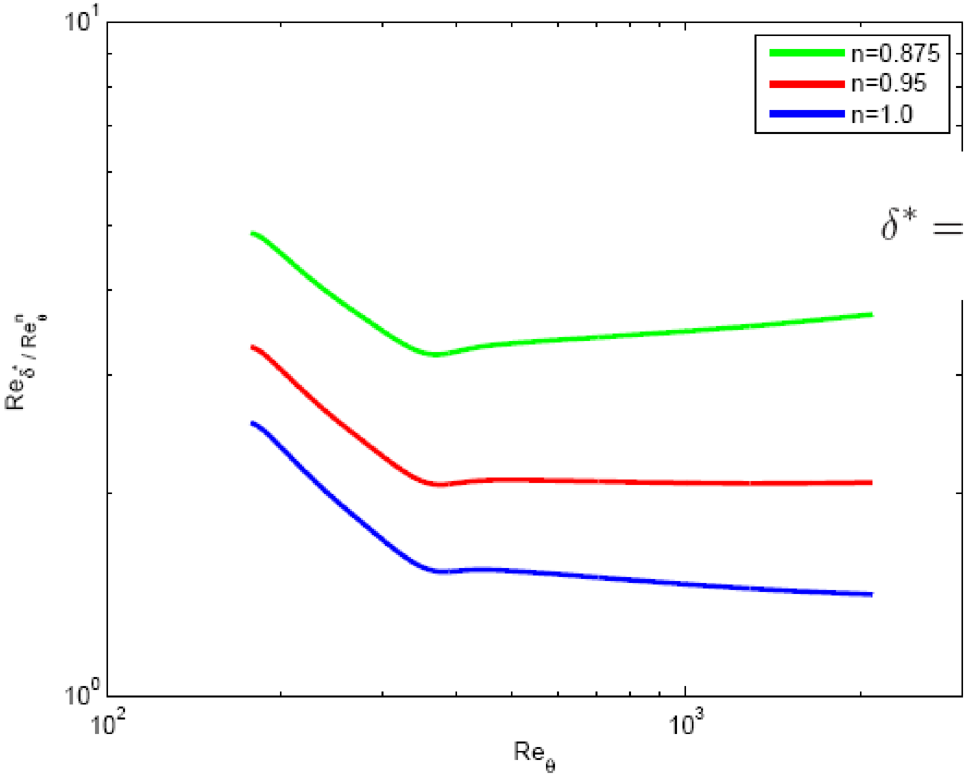
2. Objectives

Applying SED analysis to boundary layer data (Schlatter et al.)

Examine the scaling for global properties:

$$\theta = \int_0^\infty \left(\frac{\langle u \rangle}{U_\infty} (1 - \frac{\langle u \rangle}{U_\infty}) \right) \mathrm{d}y \ .$$

$$\Delta = \frac{\delta^* U_\infty}{u_\tau} = \frac{y^+}{Re_\delta^*}$$

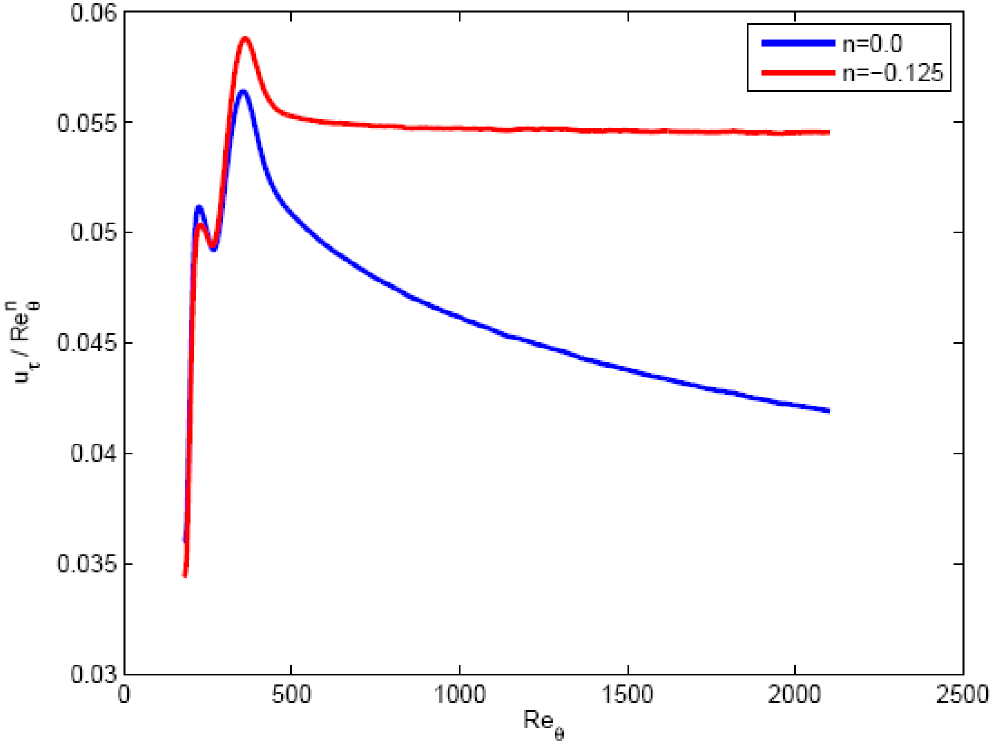


$$\delta^* = \int_0^\infty \left(1 - \frac{\langle u \rangle}{U_\infty} \right) \mathrm{d}y \ .$$

$$(Re_\theta)$$

Applying SED analysis to boundary layer data (Schlatter et al.)

Examine the scaling for global properties:

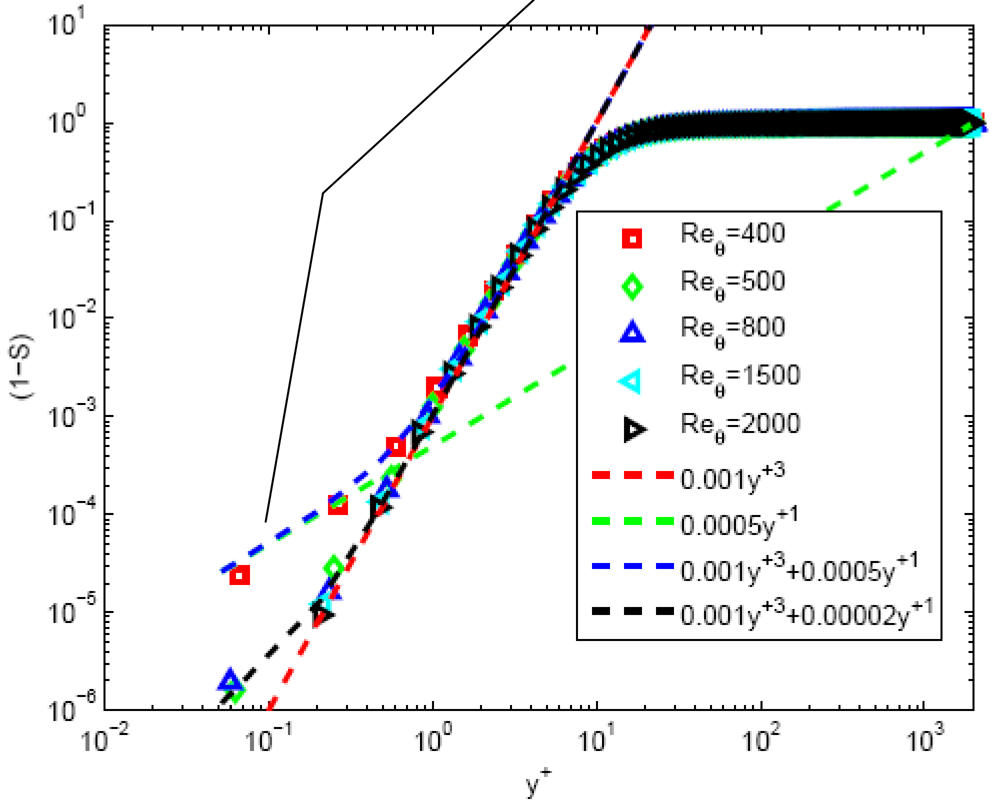


2. Objectives

Applying SED analysis to boundary layer data (Schlatter et al.)

Examine an indicator function, 1-S:

Near wall behavior to understand, and test for sufficient resolution.

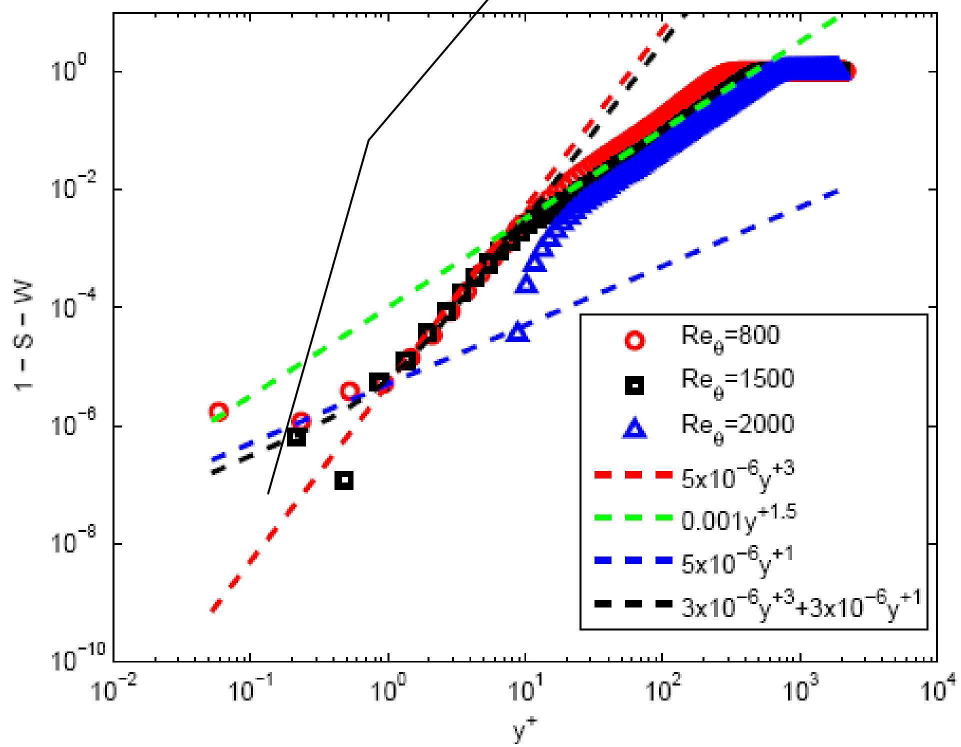


2. Objectives

Applying SED analysis to boundary layer data (Schlatter et al.)

Near wall behavior to understand, and test for sufficient resolution.

Examine an indicator function, 1-S-W:

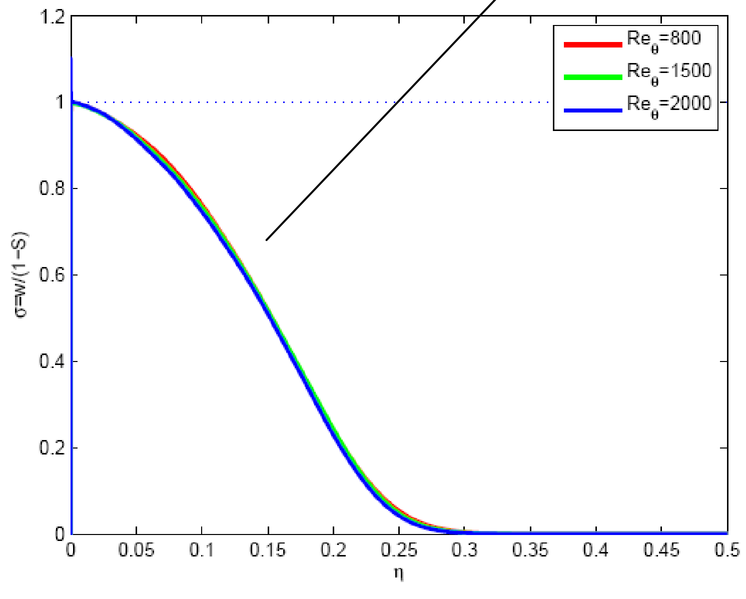


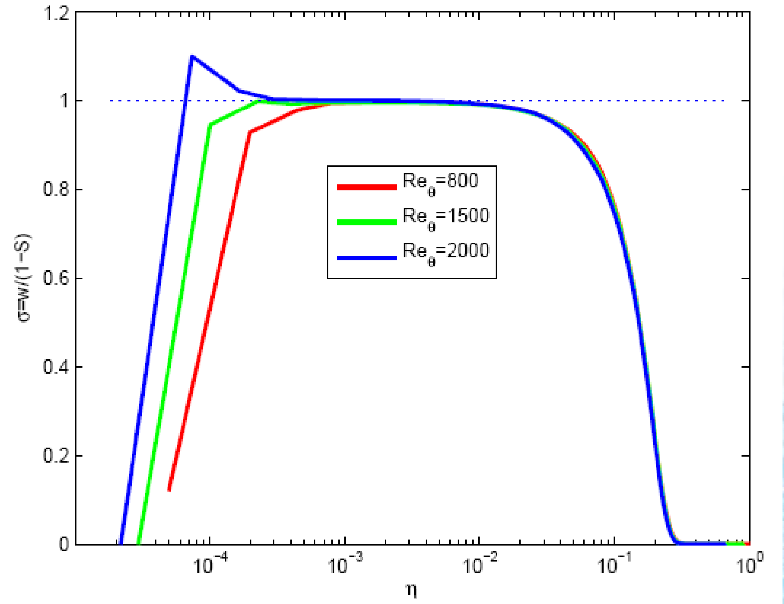
2. Objectives

Applying SED analysis to boundary layer data (Schlatter et al.)

Complete collapse in eta variable.

Examine a new order function:



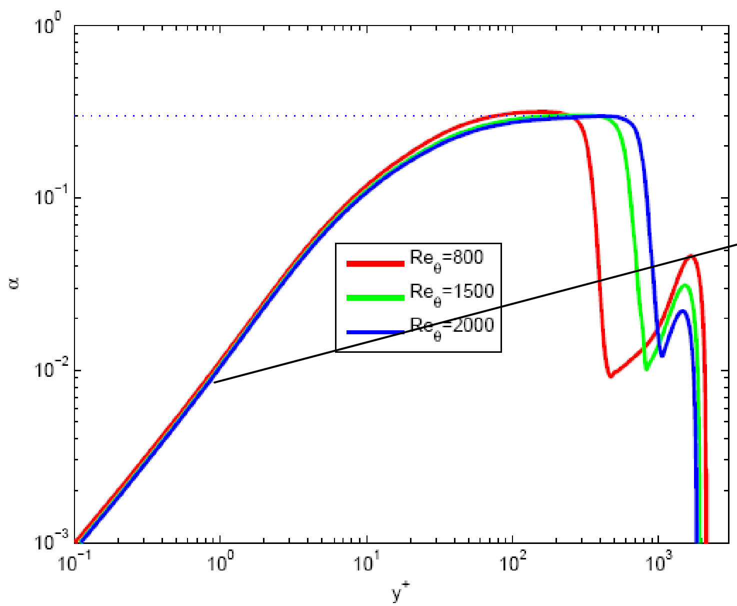


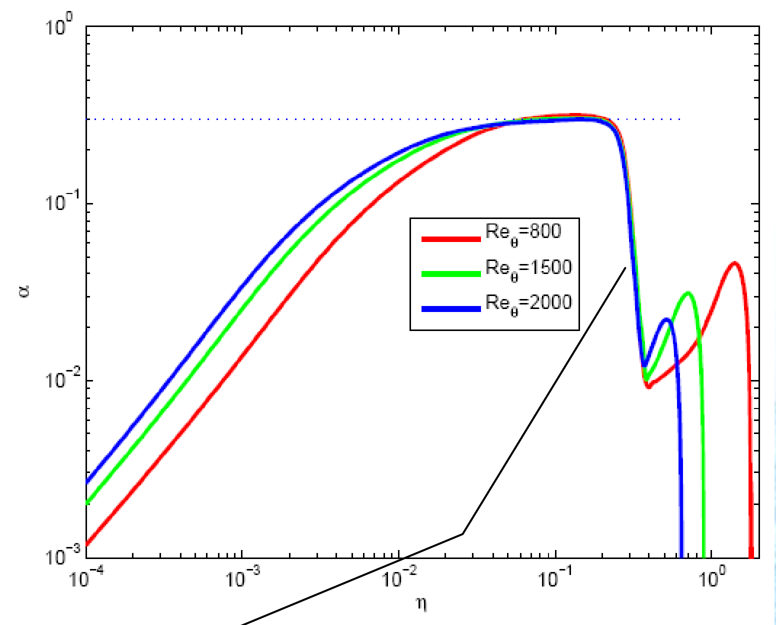
$$\sigma = \frac{W}{1 - S}$$

Applying SED analysis to boundary layer data (Schlatter et al.)

Examine the Bradshaw function:

Collapse near the wall and in eta=0.3.





$$\alpha_k = W^+/K^+$$

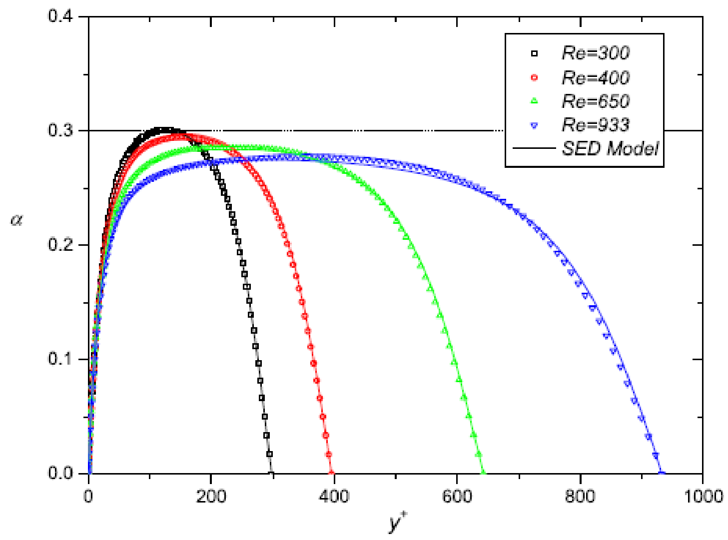
Collapse near the wall and in eta=0.3.

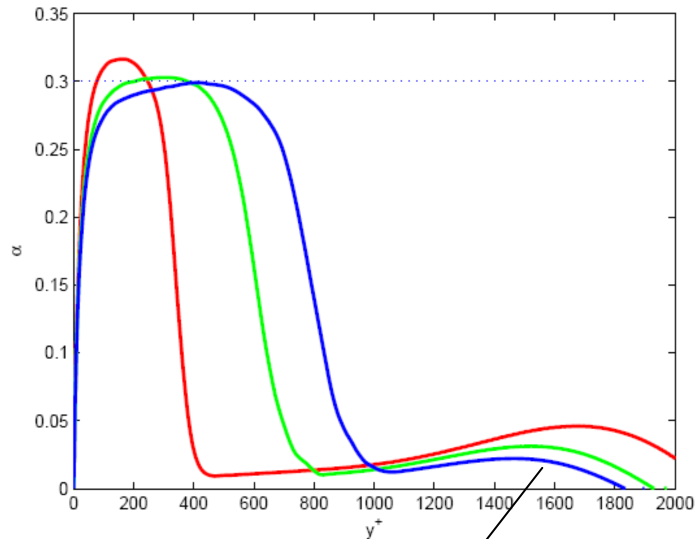
2. Objectives

Applying SED analysis to boundary layer data (Schlatter et al.)

Compare with the channel flow:

$$\alpha_k = W^+/K^+$$

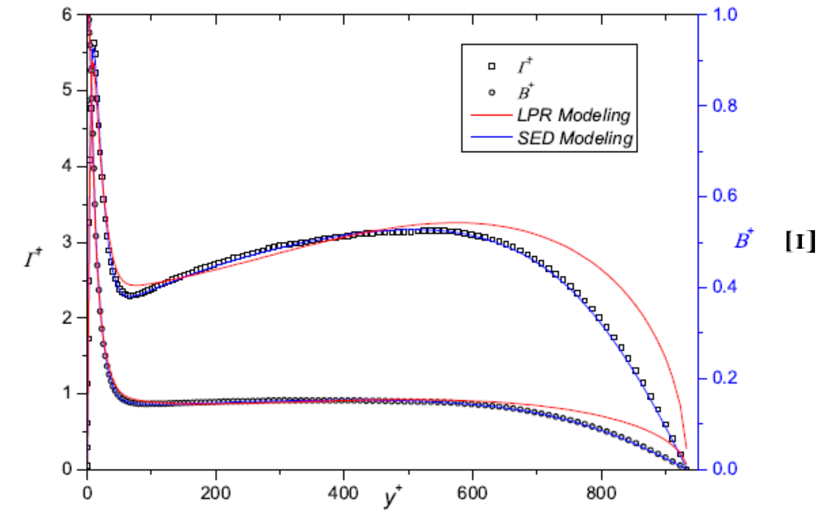


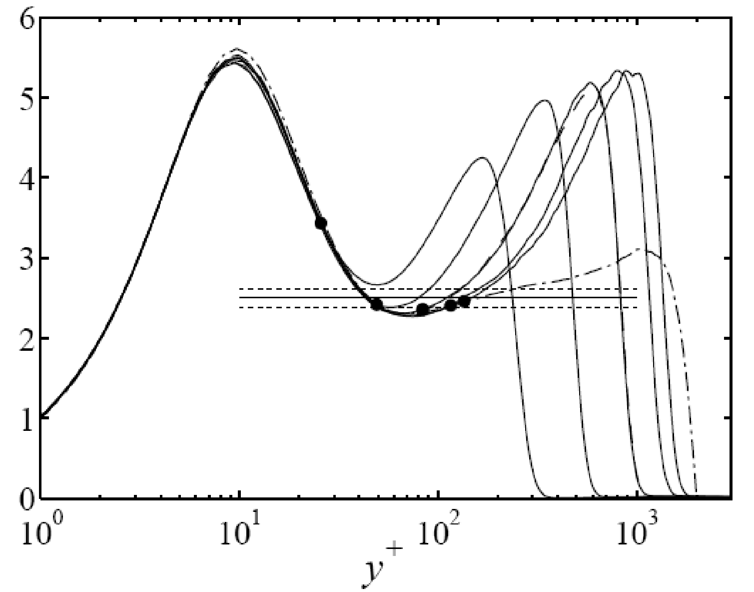


A specific outer zone behavior needs further attentions

Applying SED analysis to boundary layer data (Schlatter et al.)

Compare with the channel flow:





2. Objectives

- SED is a new theoretical framework, designed to readdress the old closure problem of turbulence
- It is a platform to analyze DNS (and experimental) data.
- What I have shown is only the first part of SED, namely, a systematic revelation of spatial and parametric variations. The second part will come to interpret the variations, which will emphasize physical mechanisms and role of structures.
- Then, a connected study of structure-profile would become possible!
- **More exciting things would come, when many complex flows are analyzed, and laws behind the variations are revealed!**

3. Open discussion

- A list of issues that Philipp has raised this morning!
- Issues of numerical calculations: we need sensitive indicators for determining the validity of the simulations, DNS or LES.
- Issues of physical mechanisms: we need to address Reynolds number effects, Prandtl number effects, Rossby number effects, etc. in a quantitative way.
- Issues of turbulence models: we need to transfer the knowledge into accomplishing the accuracy of the computation with reduced (RANS) models.
- How do we collaborate, to analyze data?