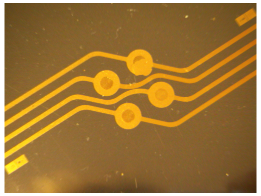

Linear Control of Turbulent Channel Flow and the Role of Pressure

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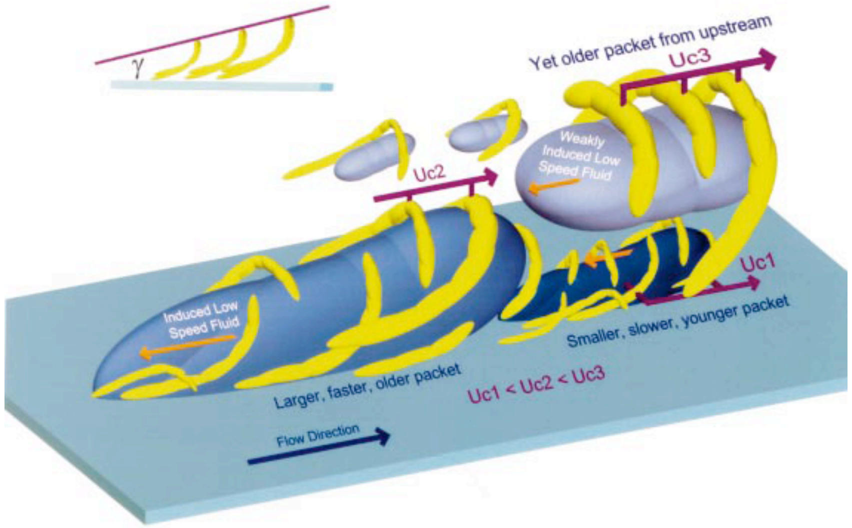
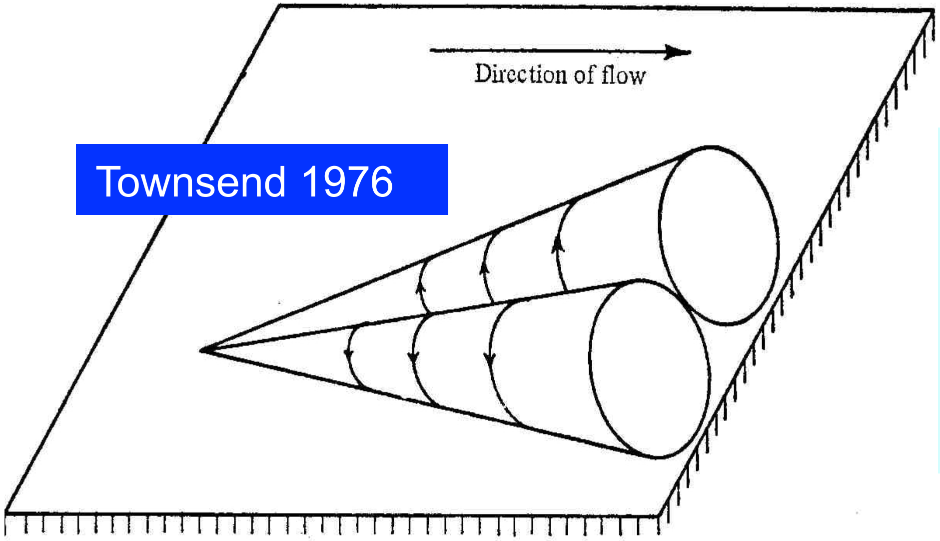
FLOW-NORDITA Spring School on Turbulent Boundary Layers

Inner and Outer: attached wall-eddies

“Bottom-up”: Adrian *et al.* 2000



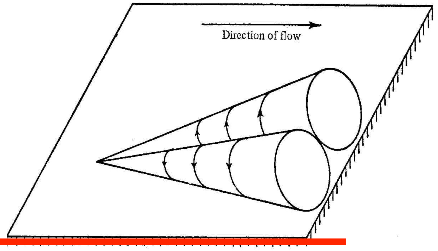


Townsend 1976

“Top-down”: Hunt & Morrison 2000

- packets carry roughly half the turbulence kinetic energy and shear stress
- fill most of the boundary layer
- reach to the wall
- at least 20δ in length – “meandering”

Attached wall eddies

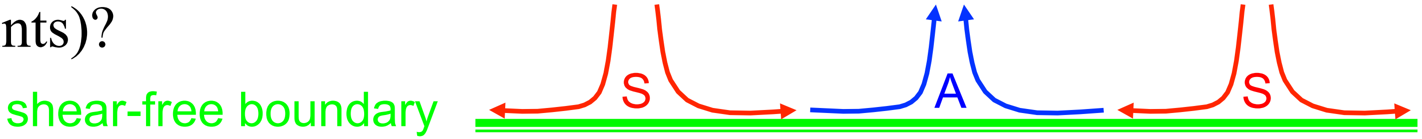


- Does top-down effect lead to:

  "modulation" of near-wall motion (Hutchins & Marusic 2007)?
  or streamwise vortices (Hunt & Morrison 2000) and hence –
  plane (oblique) waves (Sirovich 1990, Carpenter 2007)?

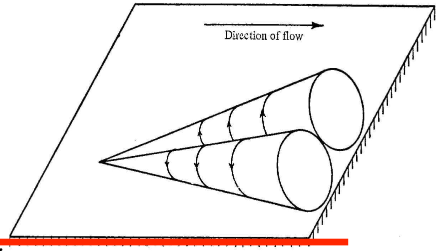
- Are viscous waves dynamically significant at high Re? – far too long a timescale?

- What is the role of wall-normal velocity and pressure fluctuations – "Anti-splats" as well as Splats (local surface stagnation points)?



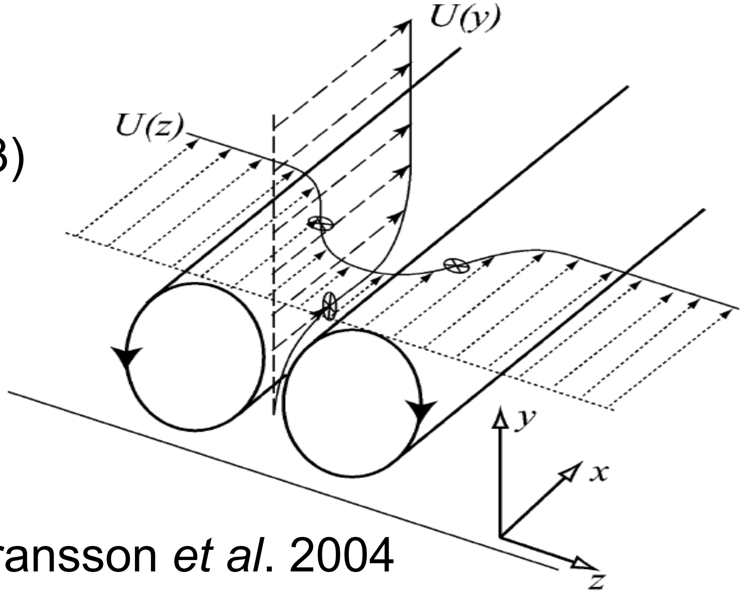
- Viscosity alters the balance between A and S: pressure-strain effects transfer of energy from v – component to u and w (Perot & Moin 1995)

Non-normality

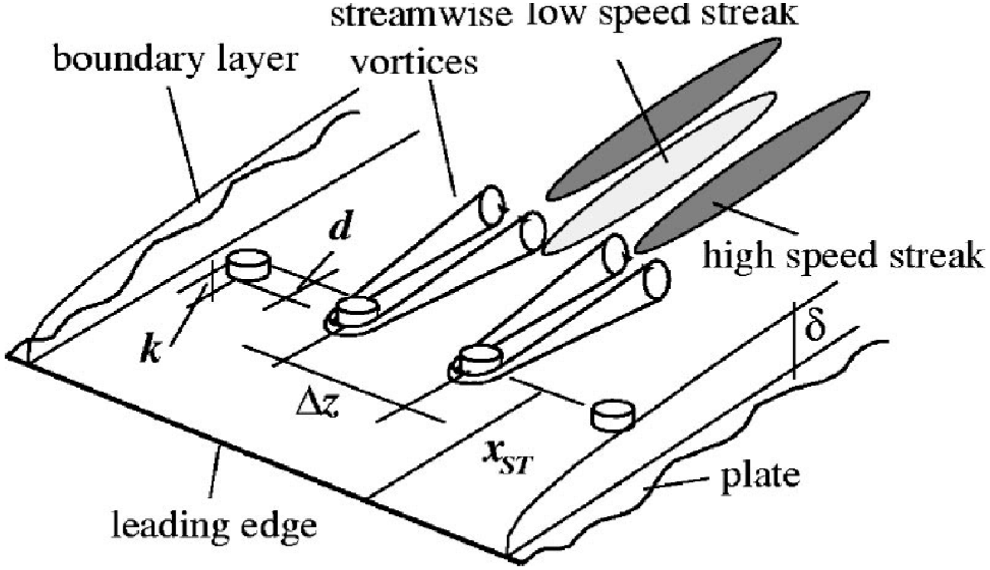


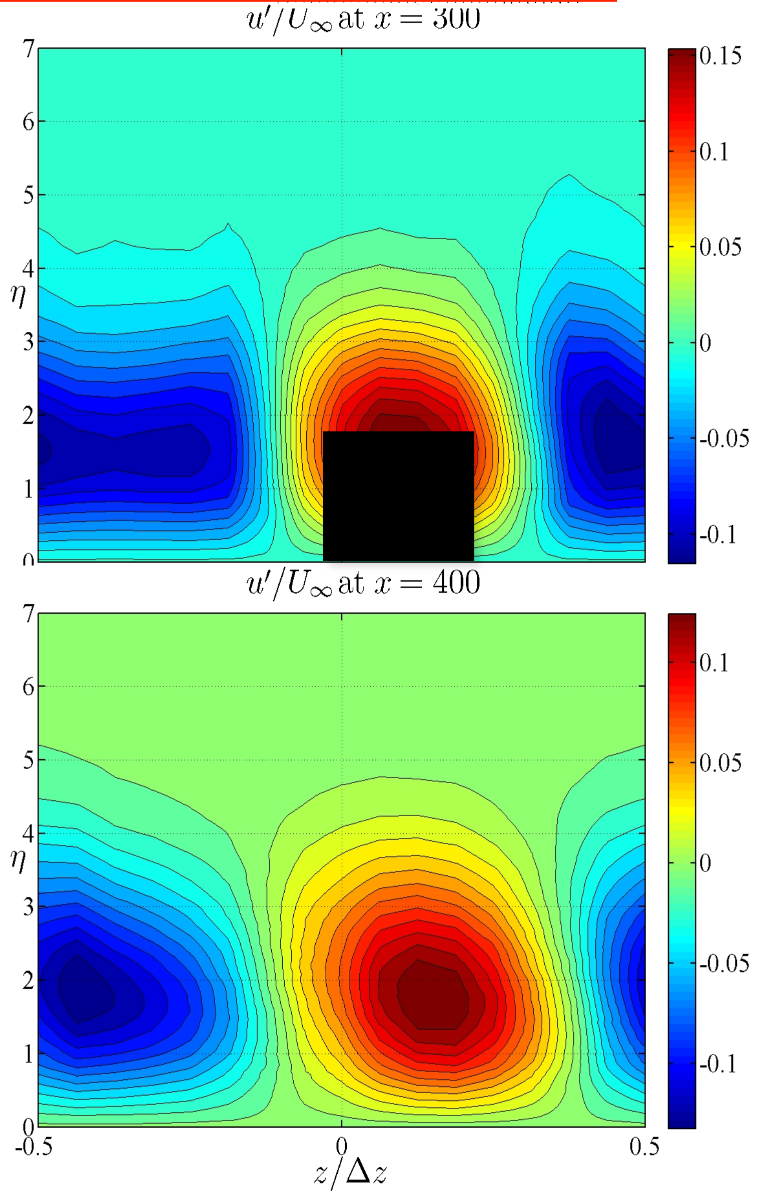
Sketch from

Rempfer (2003)



Sketch from Fransson *et al*. 2004





Linear theory

- Linear disturbance equations (*p*=*q*= 0):

  wall-normal velocity and vorticity

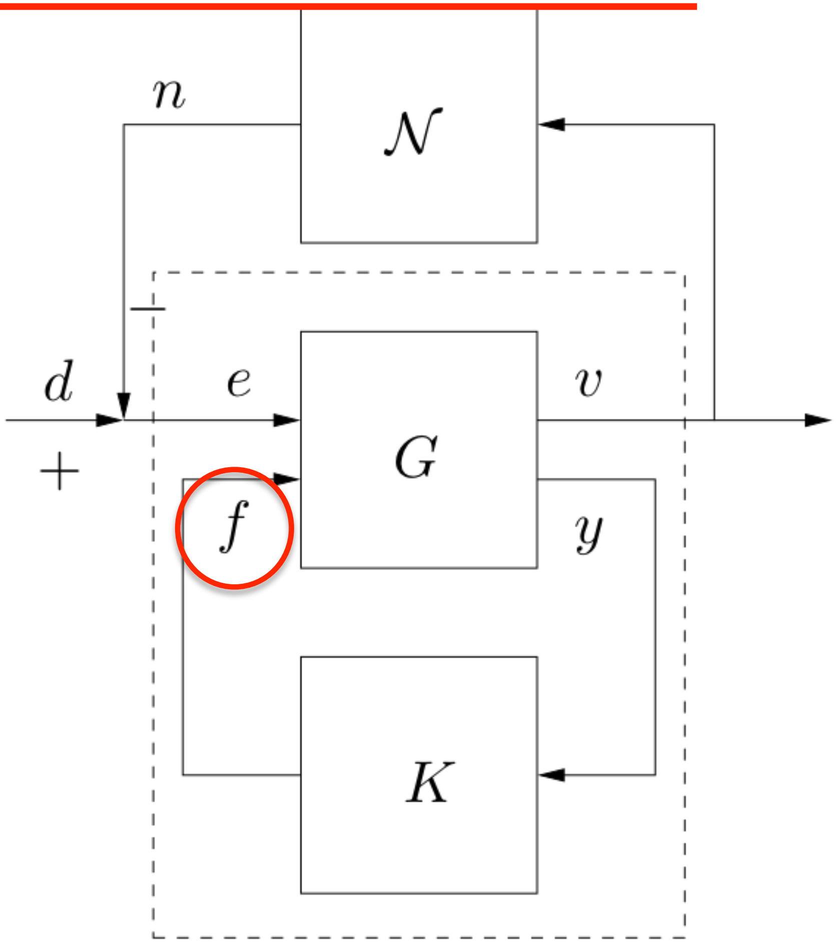
$$\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)\nabla^2 v - U''\frac{\partial v}{\partial x}-\frac{\nabla^4 v}{\mathrm{Re}}=0$$

$$\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)\eta - \frac{\nabla^2 \eta}{\mathrm{Re}}=-U'\frac{\partial v}{\partial z}$$

- v evolves via OS operator, η evolves via Squire operator. Coupling, $\boldsymbol{L_c}$, appears through v-forcing of η.
- Streaks and vortices decay if $\boldsymbol{L_c}$ suppressed (Kim & Lim '00), but nonlinearity required to form structures of correct scales (Waleffe & Kim '97)
- Several suggest structure formation requires linear mechanisms only:
  1. Jang, Benney & Gran ('86): "direct resonance" between eigenvalues of η and v leads to streak formation with spacing $\approx 100\nu/u_\tau$
  2. RDT (Lee *et al.* '90) and linearised NS + stochastic forcing (Farrell & Ioannou '93) produce vortices & streaks
  3. stability analysis using turbulent velocity profile with variable eddy viscosity – two peaks of maximum amplification (one inner, one outer) del Álamo & Jiménez (2006), Cossu et al. (2009).

Linear Globally Stabilising Controller

- Navier-Stokes equations written as a linear system *G* with control *K* and nonlinear forcing, *f*
- Nonlinear term *N* is conservative w.r.t. disturbance energy
- The linear terms always dissipate disturbance energy
- Linear controller works in presence of nonlinearity by characterising it as positive real i.e. passive.
- Stability: choose *K* such that linear part of closed loop is passive

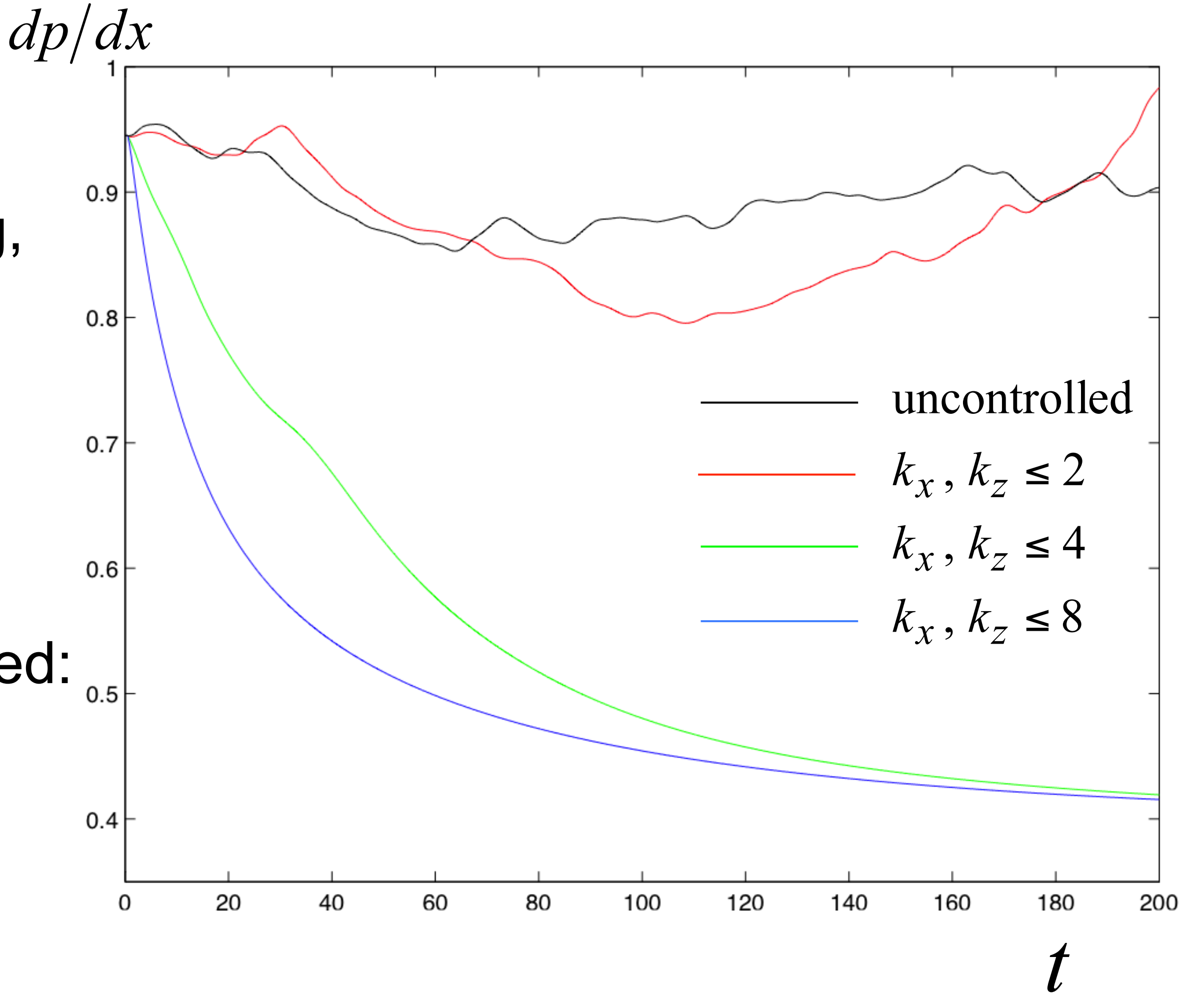


$$\frac{dE}{dt} = \oint_{x\in\Omega}\left(U_L'(y)uv + \frac{\varepsilon}{\mathrm{Re}}\right) \leq 0$$

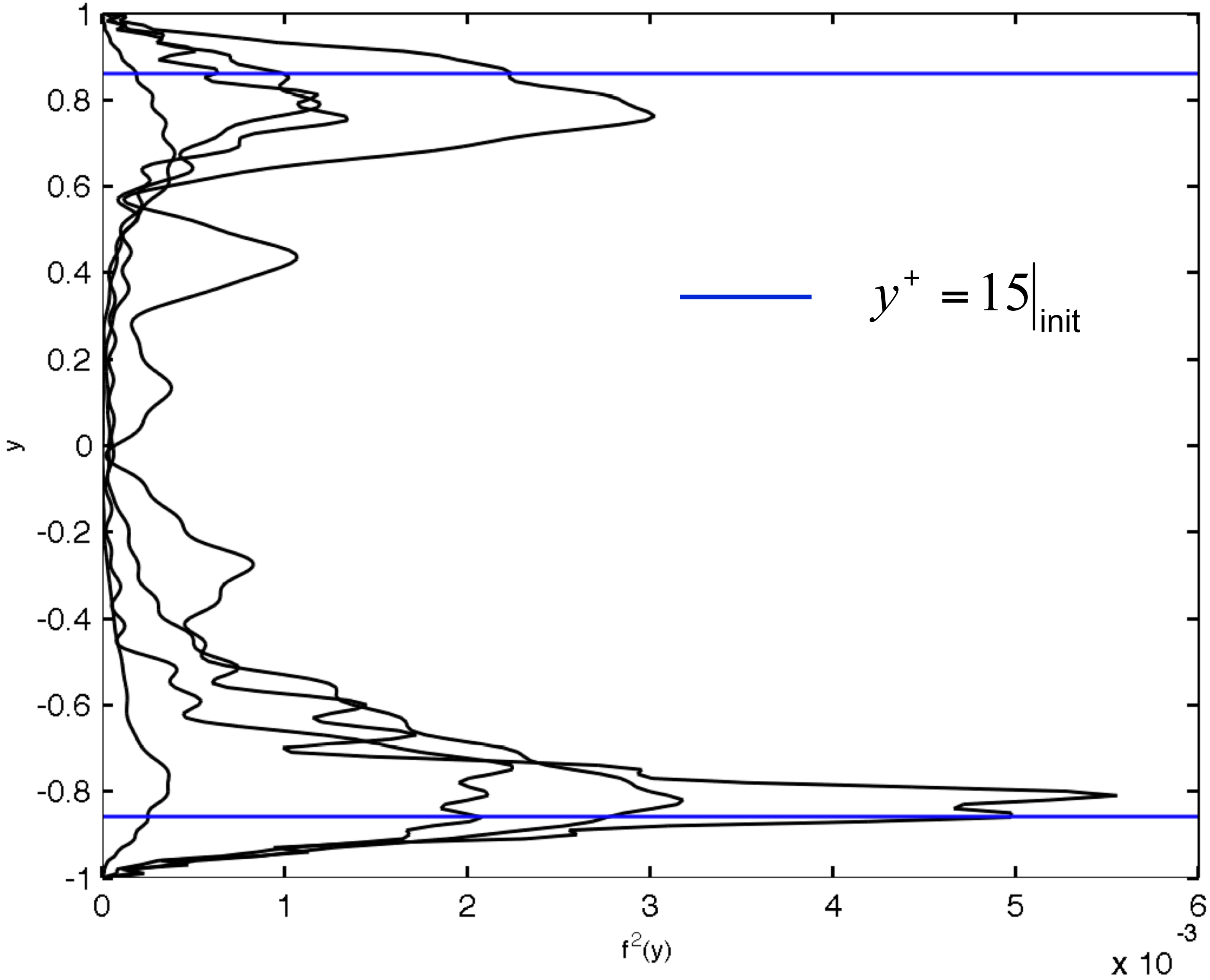
Turbulent channel flow, $Re_\tau = 100$

- Domain $4\pi \times 2 \times \frac{4}{3}\pi$
- Full-domain sensing, actuation on v
- Control focuses on vU'
- Forcing bandwidth progressively reduced: $k_x, k_z \leq 8$
- Details for $k_x, k_z \leq 4$,

  $y^+ = 15|_{\text{init}}$

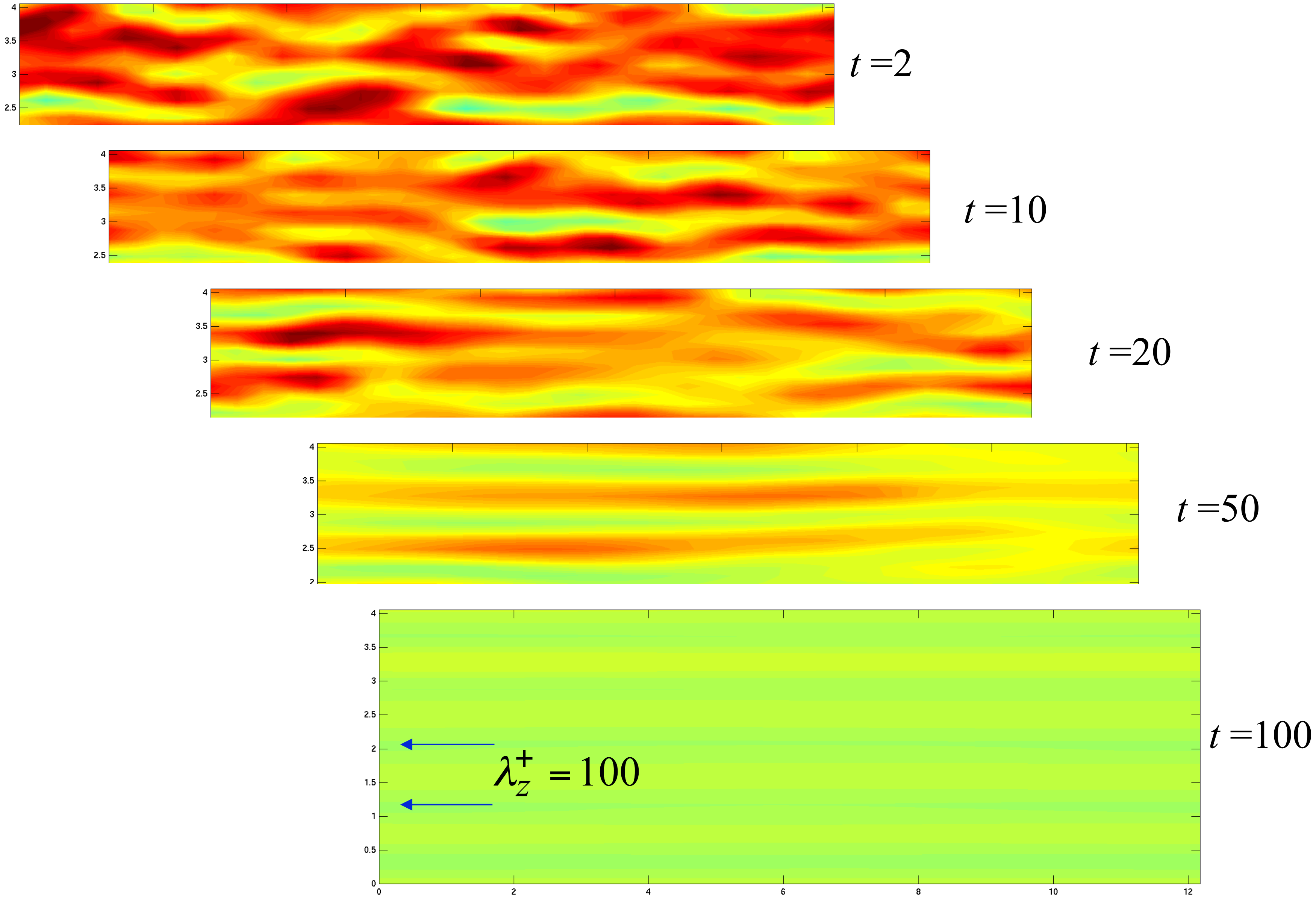


Turbulent channel flow: Forcing

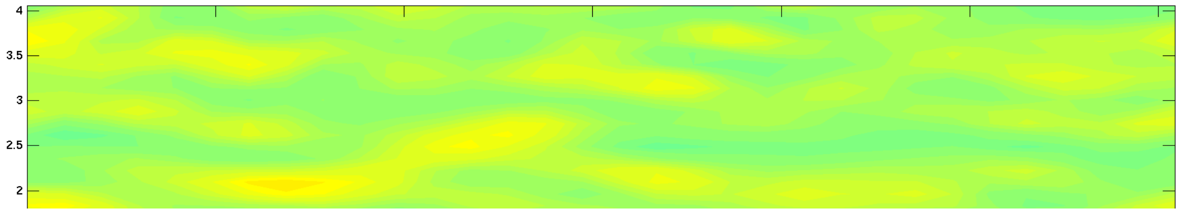


$$\overline{f^2}(y)$$

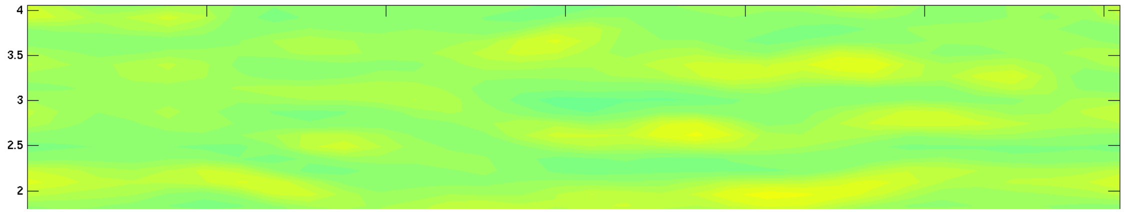
Controlled u: $y^+ = 15|_{\text{init}}$



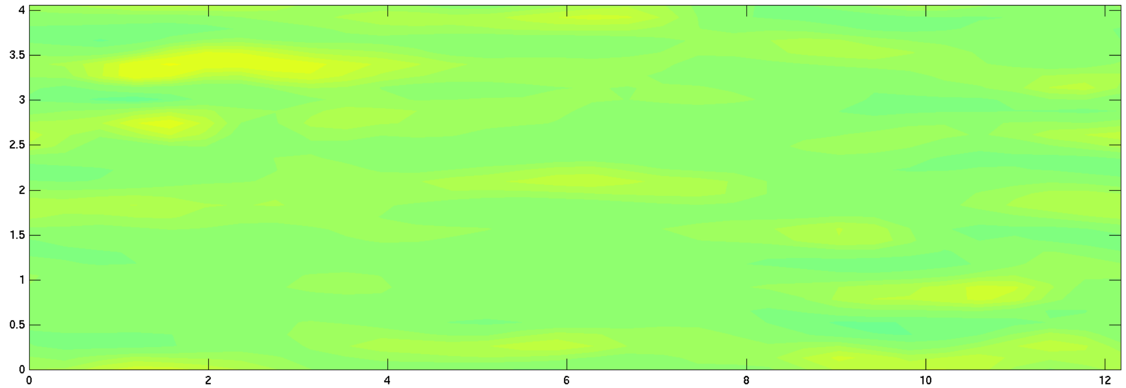
Controlled p: $y^+ = 15\big|_{\text{init}}$



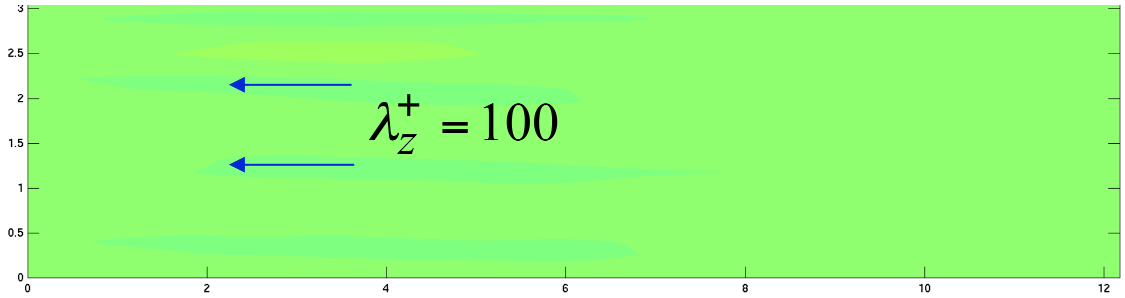
$t = 2$



$t = 10$



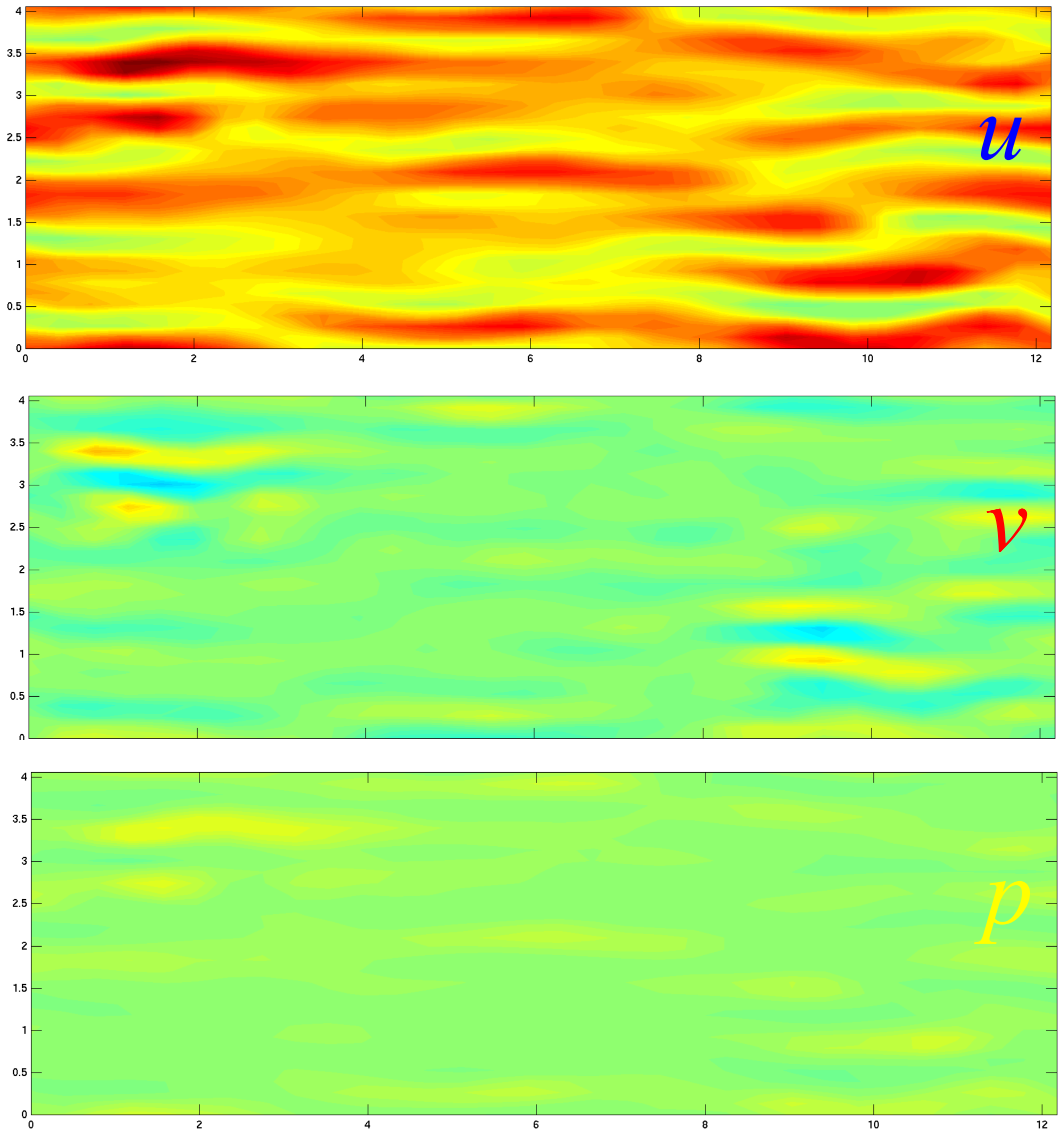
$t = 20$



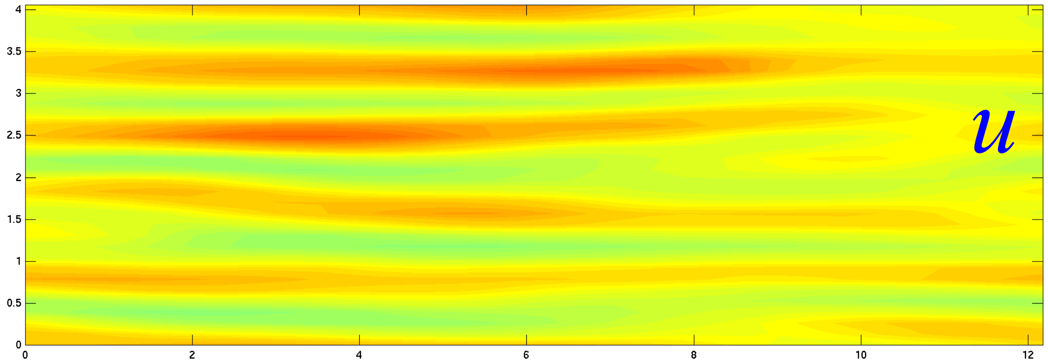
$\lambda_z^+ = 100$

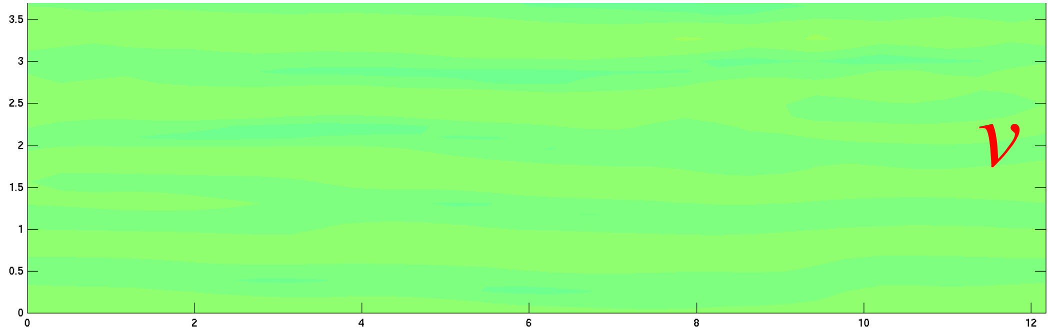
$t = 50$

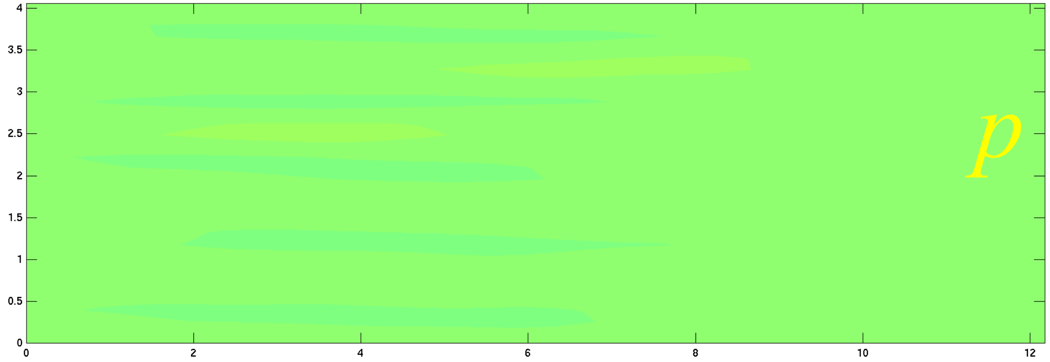
Controlled u,v and p: $y^{+} = 15\big|_{\text{init}}$ $t = 20$



Controlled u,v and p: $y^{+}=15\big|_{\text{init}}$ t = 50





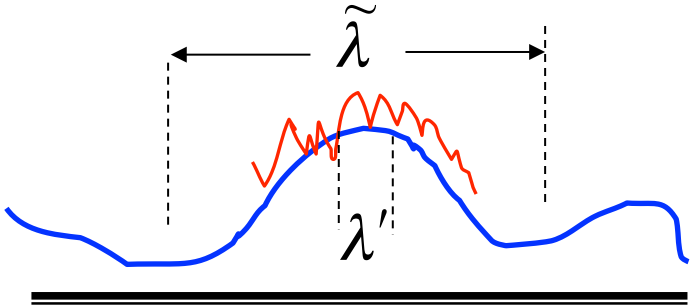


A useful theory for Inner-Outer Interaction?

- Landahl ('93, '90, '75): initial disturbance scales $L,\ u_0$ with timescales: shear interaction $\{U'_w\}^{-1}$ << viscous $\{L^2/(\nu U'^2)\}^{1/3}$ << nonlinear L/u_0 .
- Large and small-scale decomposition: $u_i = \tilde{u}_i + u'_i$
- Small scale, λ' , large scale $\tilde{\lambda}$, where $\lambda'/\tilde{\lambda} = \varepsilon << 1$
- To first order in ε, large-scale and small-scale fields may be represented separately by the same equations:

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\nabla^2 v - U''\frac{\partial v}{\partial x} - \frac{\nabla^4 v}{\mathrm{Re}} = q$$

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\eta + U'\frac{\partial v}{\partial z} - \frac{\nabla^2 \eta}{\mathrm{Re}} = p$$



$$U_i = U(y)\delta_{1i} + u_i(x_j, t)$$

- *p, q* nonlinear source terms (turbulent stresses) significant only in local regions: "intense small-scale turbulence of an intermittent nature" interspersed with "laminar-like unsteady motion of a larger scale".

Pressure fluctuations

- High Reynolds numbers: local isotropy and negligible viscous diffusion
- Mean-square acceleration becomes

$$\overline{\left(\frac{Du_i}{Dt}\right)^2} \approx \overline{\left(\frac{\partial p}{\partial x_i}\right)^2} + \nu^2 \overline{\left(\frac{\partial^2 u_i}{\partial x_j^2}\right)^2}$$

- where $\overline{\left(\frac{\partial p}{\partial x_i}\right)^2} \approx 20\nu^2 \overline{\left(\frac{\partial^2 u_i}{\partial x_j^2}\right)^2}$

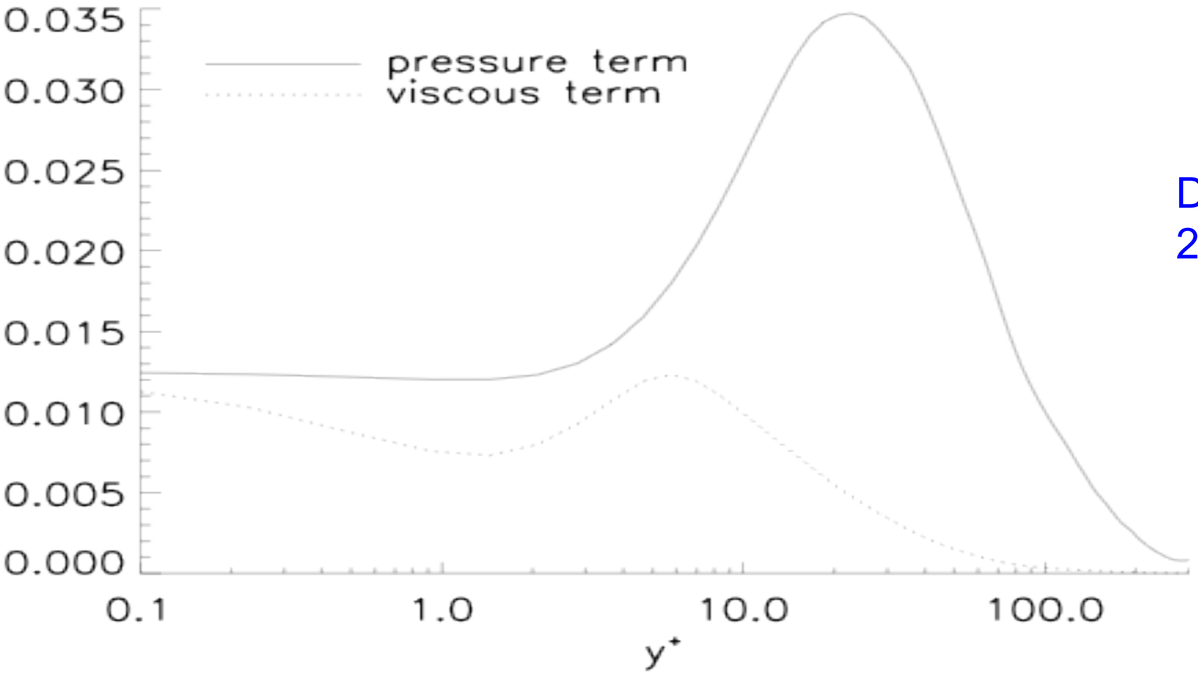
- Therefore, even the smallest scale motion is driven by pressure gradients and not by viscous forces.

Batchelor & Townsend 1956

Batchelor & Townsend: near-wall channel flow

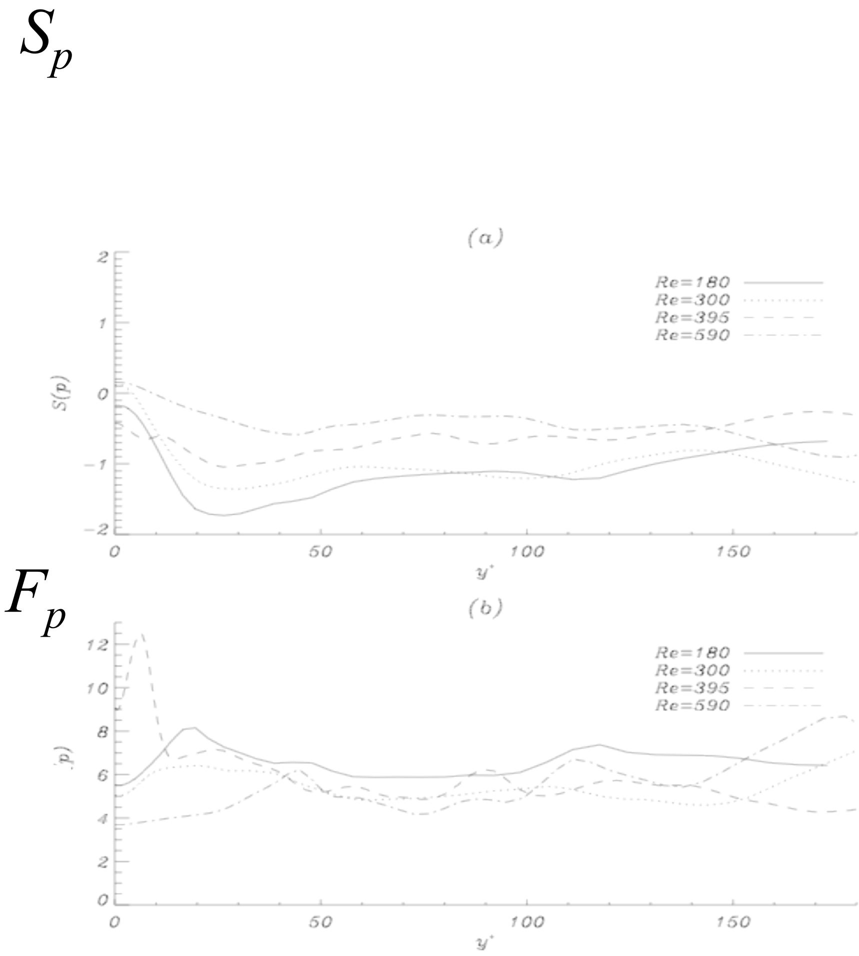
$$\mathrm{Re}_\tau = 300$$

$$\overline{\left(\frac{\partial p}{\partial x_i}\right)^2} \approx 20\nu^2 \overline{\left(\frac{\partial^2 u_i}{\partial x_j^2}\right)^2}$$



Dunn & Morrison
2003

Near-wall pressure statistics



- **Spatial intermittency: pressure has small skewness, large flatness (> 6)**
- **Green's function integral shows that contribution to wall pressure comes mostly from near-wall velocity field**

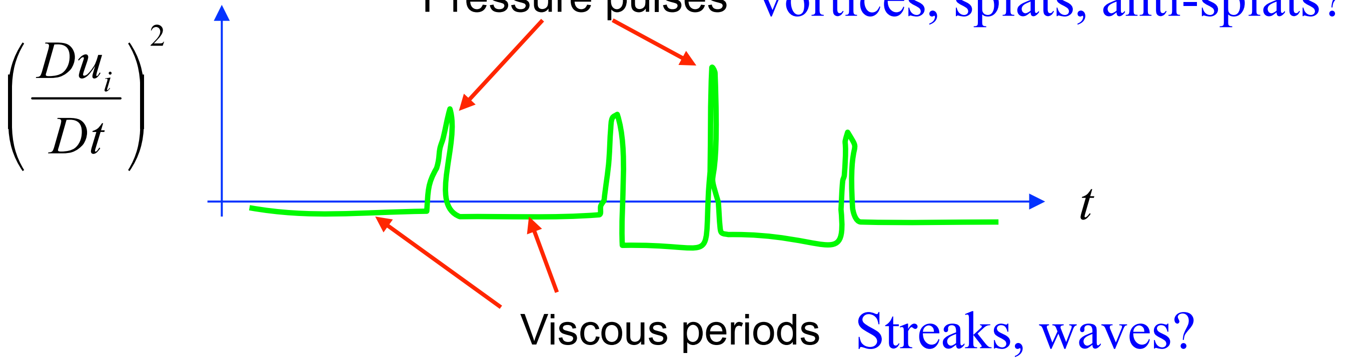
$$-\nabla^2 p = 2U' \frac{\partial v}{\partial x} - \frac{\partial^2}{\partial x \partial y}\left[uv - \overline{uv}\right]$$

How relevant is this picture to near-wall turbulence?

- DNS channel flow – (Kim 1989):

$$\overline{\left(\frac{\partial p}{\partial x}\right)^2} \approx 0.5\overline{\left(\frac{\partial p}{\partial y}\right)^2} \approx 0.5\overline{\left(\frac{\partial p}{\partial z}\right)^2}$$

- $\partial p/\partial y,\ \partial p/\partial z$ are good indicators of quasi-streamwise vortices
- spatial intermittency caused by the mean-square pressure gradient – effect is predominant in the nonlinear source term

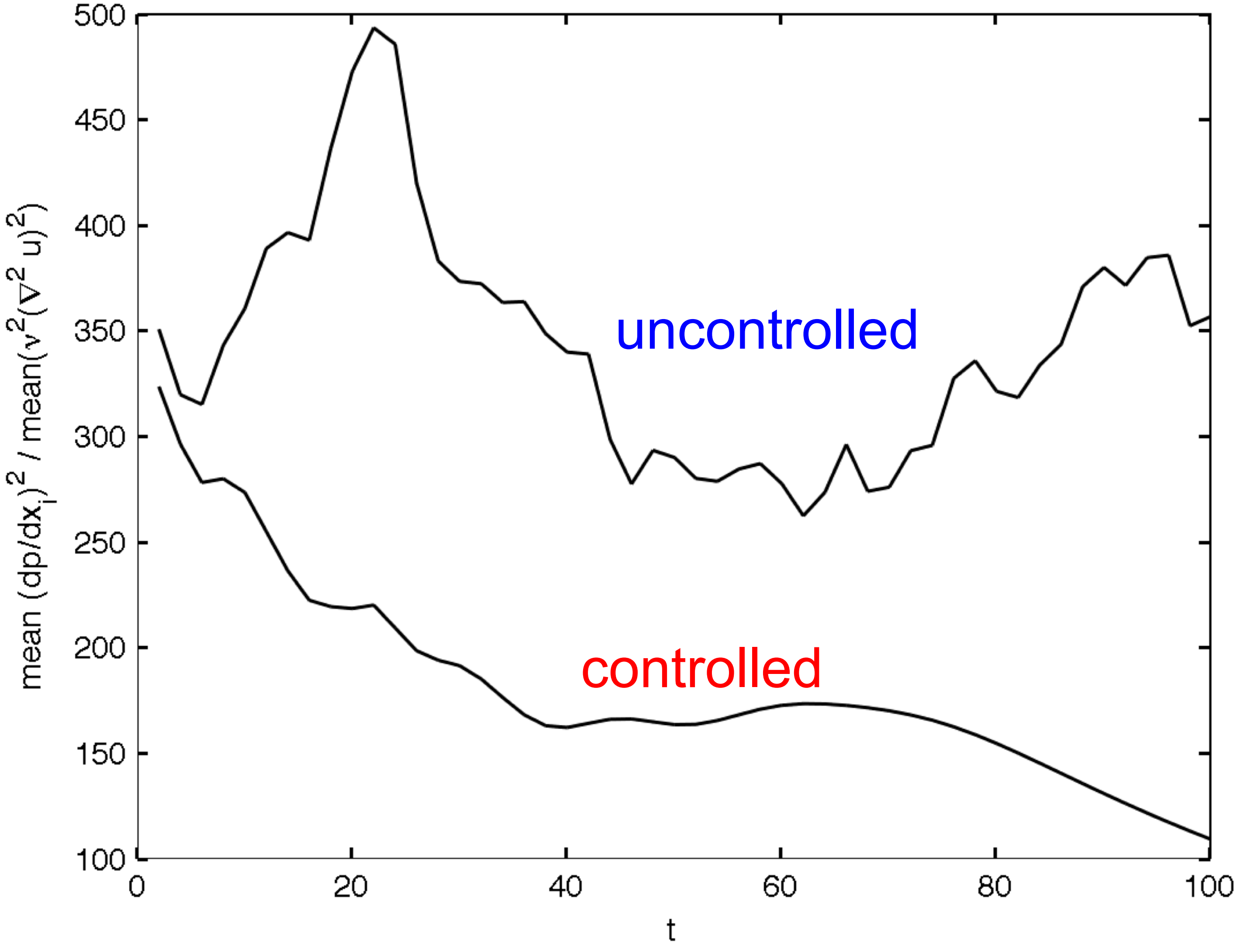


Pressure pulses vortices, splats, anti-splats?

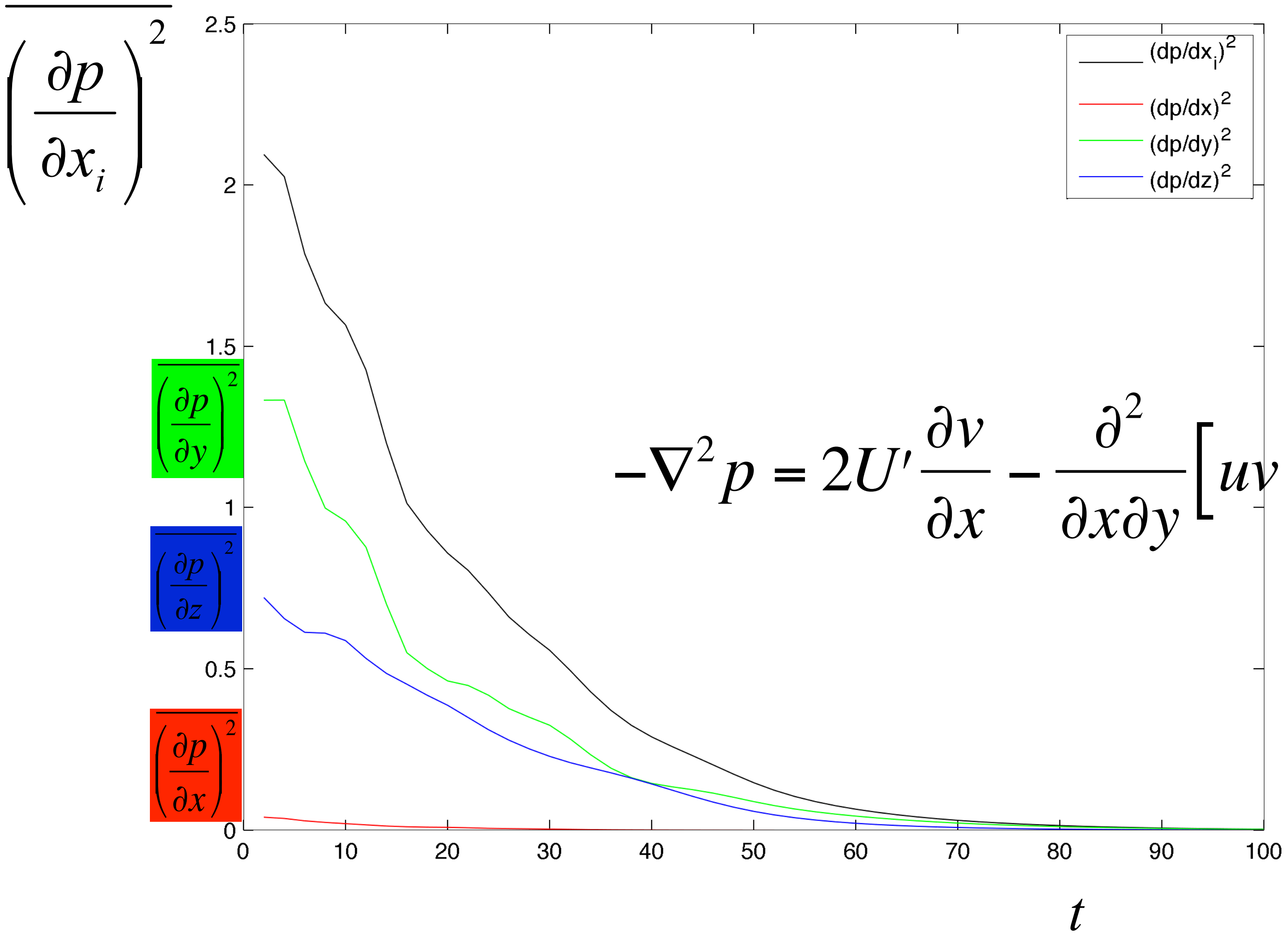
$$\left(\frac{Du_i}{Dt}\right)^2$$

Viscous periods Streaks, waves?

$$\overline{\left(\frac{\partial p}{\partial x_i}\right)^2} \Big/ \nu^2 \overline{\left(\frac{\partial^2 u_i}{\partial x_j^2}\right)^2}$$



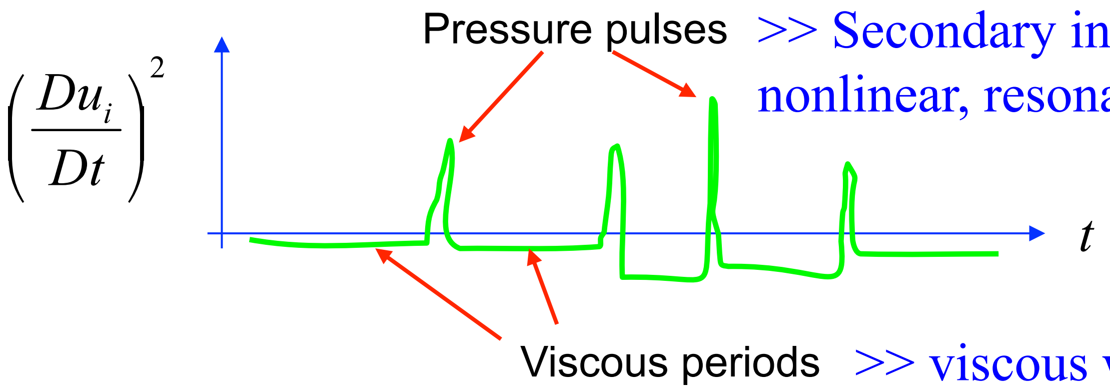
Controlled $\overline{\left(\frac{\partial p}{\partial x_i}\right)^2}$



$$-\nabla^2 p = 2U'\frac{\partial v}{\partial x} - \frac{\partial^2}{\partial x \partial y}\left[uv - \overline{uv}\right]$$

BLT theory

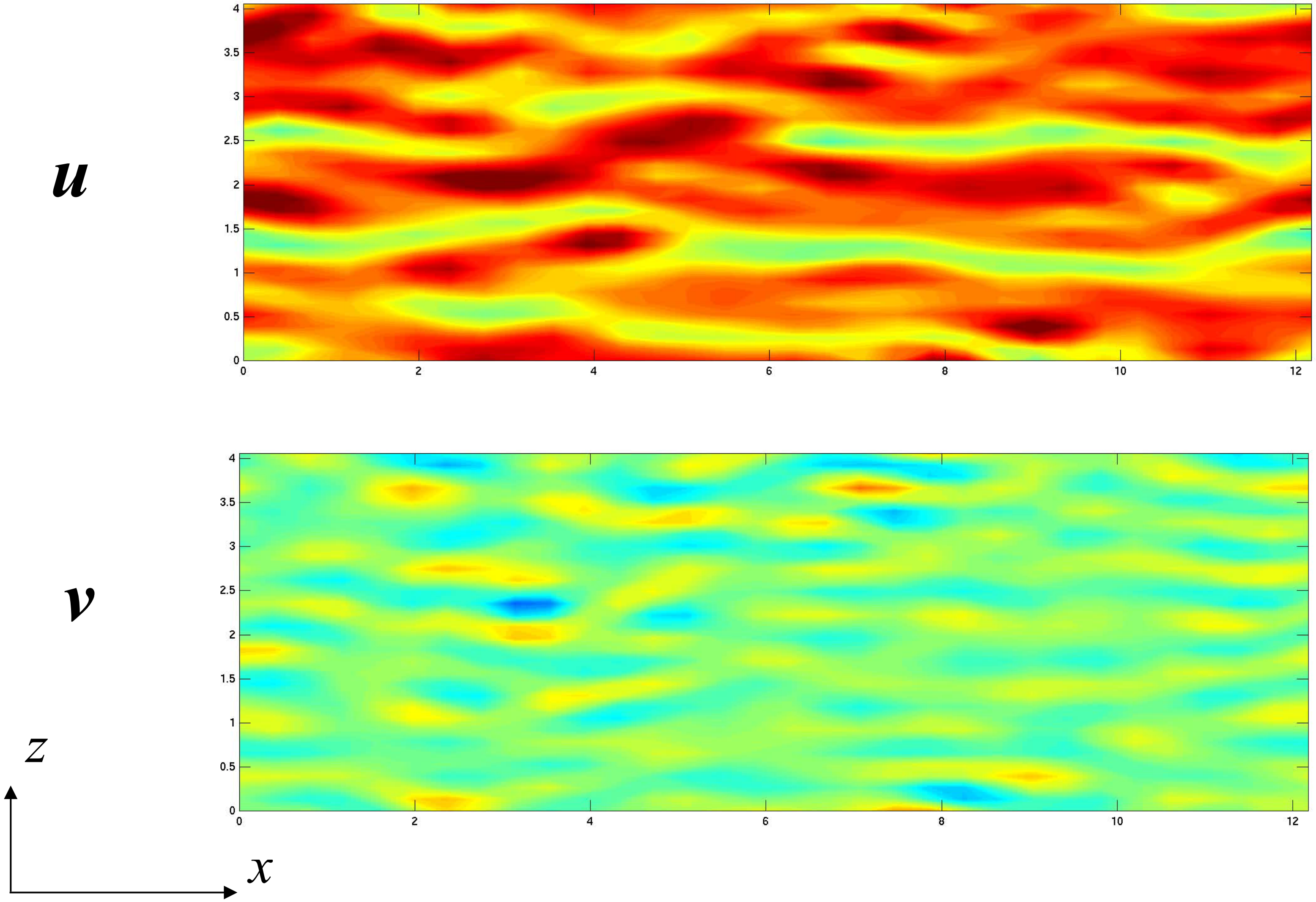
- Sublayer as a waveguide: primarily for p and v
- u and w also wave-like but including convected eddy behaviour
- Description of both large & small scales by OSS permits resonance – Inner-outer interaction
- Pressure sources can 'trigger' bursts near wall = short shear: interaction timescale



>> Secondary instability, nonlinear, resonance?

>> viscous waves, primarily in p and v?

Controlled u & v: $y^+ = 15$



Conclusions

- Linear full-domain forcing via vU' at low wavenumbers effective in attenuating turbulent channel flow.
- Control-theoretic approach (“passivity”) explained by conservative nature of nonlinear terms contributing to Reynolds-Orr equation
- Control acts on v–component field and hence pressure field via rapid source term of Poisson equation.
- Qualitative support for Landahl’s theory: Inner-outer interaction significantly enhanced by linear shear-interaction which proceeds on short timescales.
- Resonance of OSS eigenvalues explained physically by near wall spikes in mean-square acceleration due to pressure-gradient field (discrete and continuous modes not distinguished).