NORDITA-FLOW Spring School on Turbulent Boundary Layers
27 April 2010

– DNS of the turbulent Ekman layer –

“Lessons Past, Present & (perhaps) Future”

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With thanks to Philipp Schlatter

Outline of talk

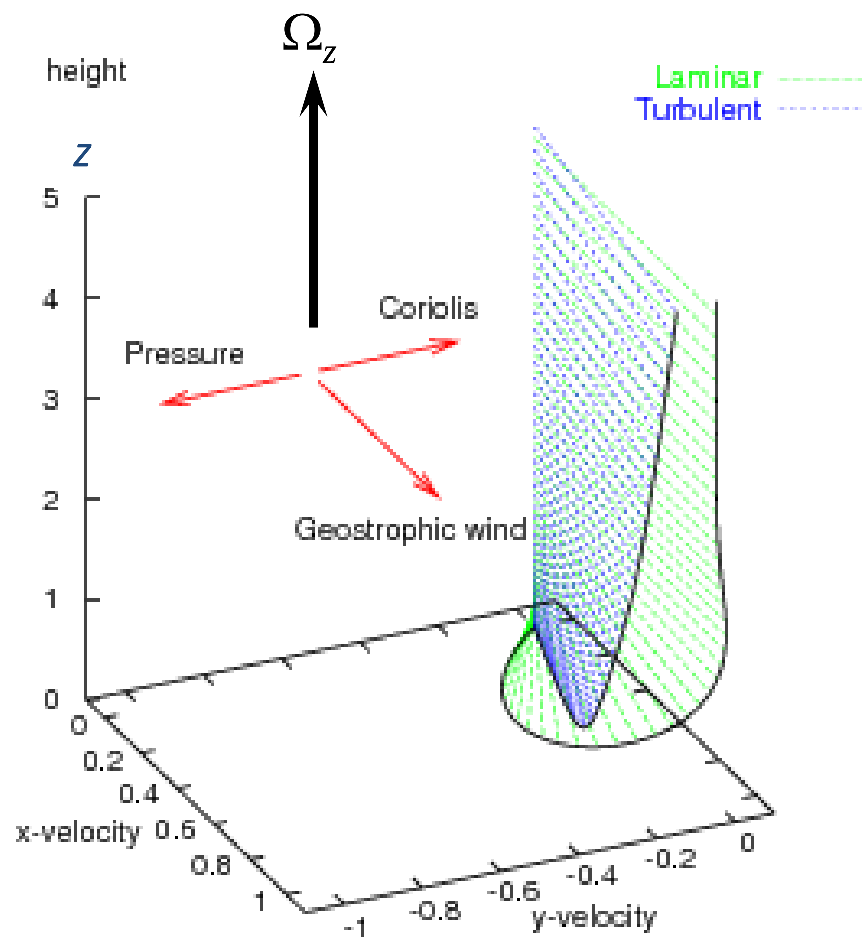
- Introduction: rotating, pressure-driven TBL
- Results (past, present):
  - The **idealised ABL**
    - Role of stratification, latitude dependence
  - The **canonical TBL**
    - Near-wall similarity: κ and the Law of the Wall
- Future directions
  - Wind-farm arrays
  - Entrainment dynamics
  - Turbulence structures?

Introduction: Turbulent (pressure-driven) Ekman layer:

Balance between:

(1) horizontal pressure gradient
(2) Coriolis acceleration and
(3) stress divergence …

…Leads to **3D equilibrium boundary layer** (*cf nonequilibrium* 3DBLs over swept-wings, in curved ducts, upstream of obstacles, etc)

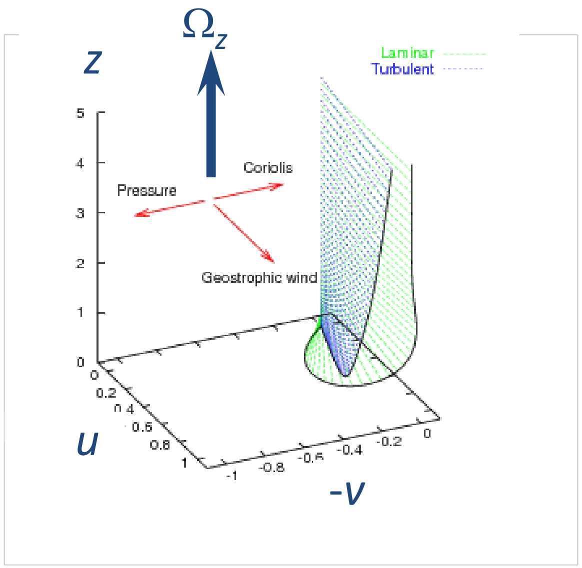


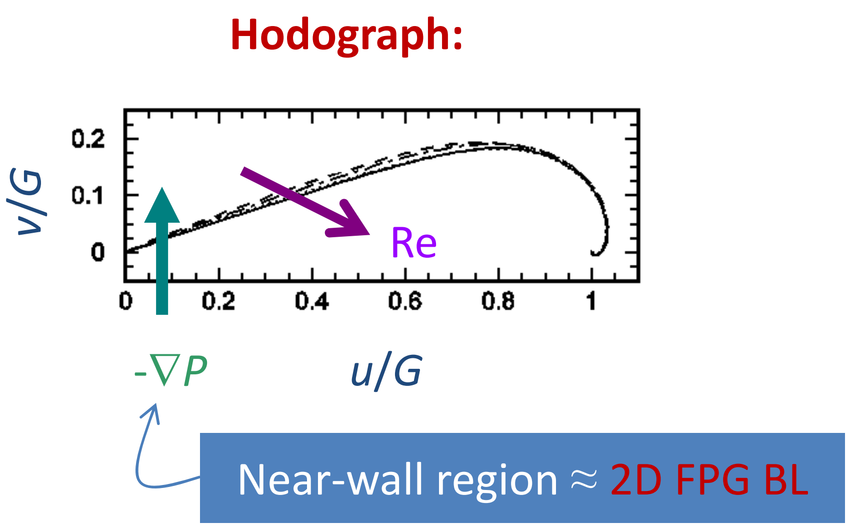
⟶ Coriolis term allows BL **homogeneous** in *x, y* **and *t***

(Allows time averaging. Includes *no mean entrainment.*)

Turbulent (pressure-driven) Ekman layer:

- Balance between pressure gradient, Coriolis and "friction"
  ⇒ ***3D equilbrium boundary layer***…





- Defining parameter: **Reynolds number** $Re = GD/\nu$, where
  - G ← freestream/geostrophic wind speed
  - $D = (2\nu/f)^{1/2}$ ← viscous boundary-layer depth
  - $f = 2\Omega_z$ ← Coriolis/rotation parameter
  - $\nu = \mu/\rho$ ← kinematic viscosity

Lessons **past**…

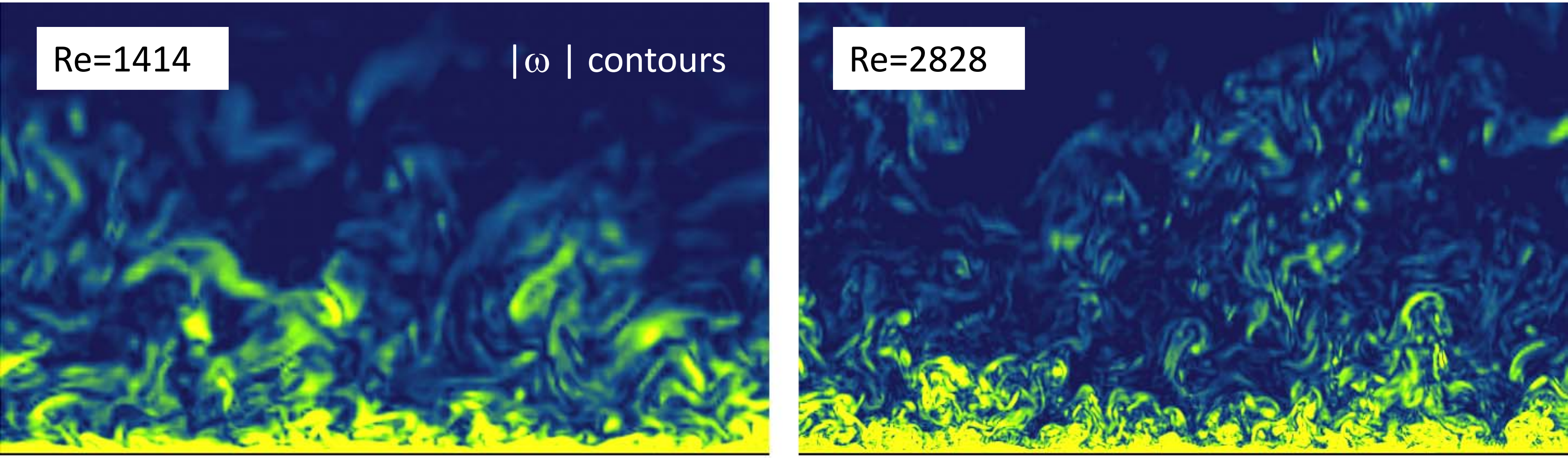
- Viewed as idealised *Atmospheric Boundary Layer*:
  - Unstable stratification required for roll cells
  - Horizontal rotation (and therefore wind direction) matters
- Viewed as *canonical TBL*:
  - The quest for κ (a cautionary tale)…

The quest for κ…

Ekman DNS at Re=1000, 1414, 2000, 2828

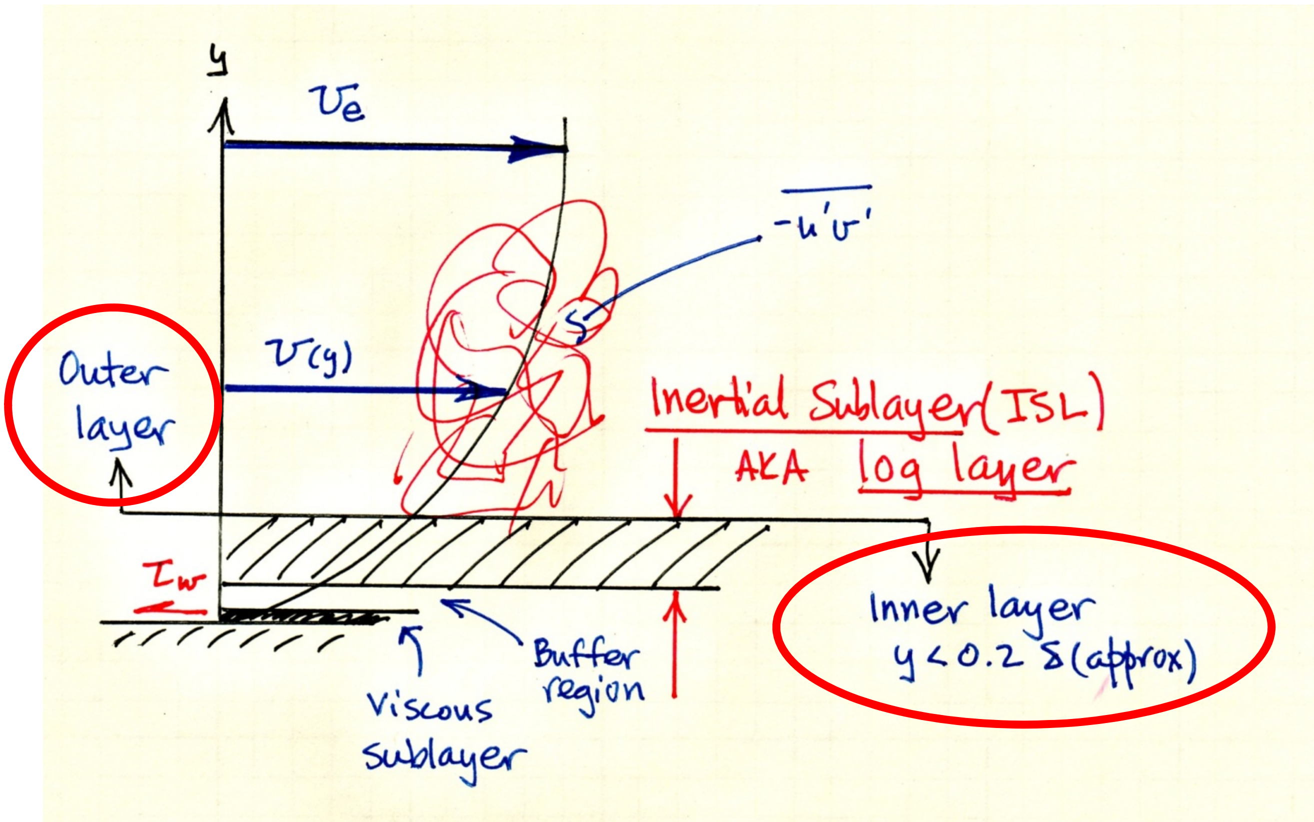
(Spalart, Coleman & Johnstone 2008)

[Using $\Delta x^+=20$, $\Delta y^+=7$, $n=10$ at $z^+=10$]



Behaviour of Inertial Sublayer?...

<u>Recall</u>: *Near-wall similarity* of mean velocity in low-speed turbulent boundary layers – the *non-overlap perspective:*



ISL Assumptions: $U = U(y, \tau_w, \nu) \overset{DA}{\Rightarrow} \frac{dU}{dy} = \frac{u_\tau}{y} f(y^+)$

where $u_\tau^2 = \tau_w/\rho$

and $y^+ = y u_\tau/\nu$

As $y^+ \rightarrow$ 'large enough', effect of ν becomes negligible, and $f(y^+) \rightarrow \frac{1}{\kappa}$, where κ is (presumably) a universal constant

$$\therefore \frac{dU}{dy} = \frac{u_\tau}{\kappa y} \Rightarrow U = \frac{u_\tau}{\kappa} \ln y^+ + C$$

Validation:

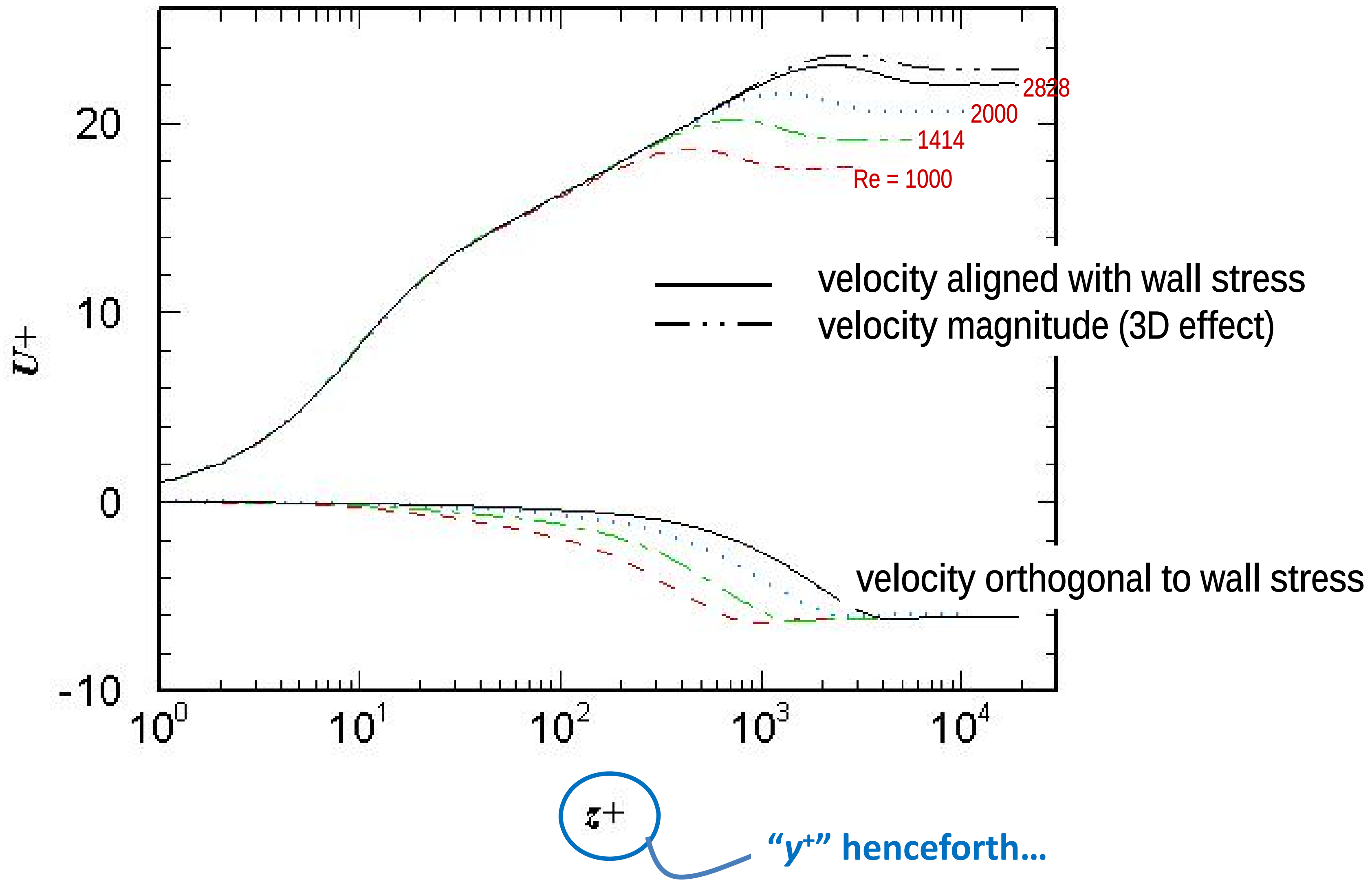
(1) κ = constant?

(2) κ = universal?

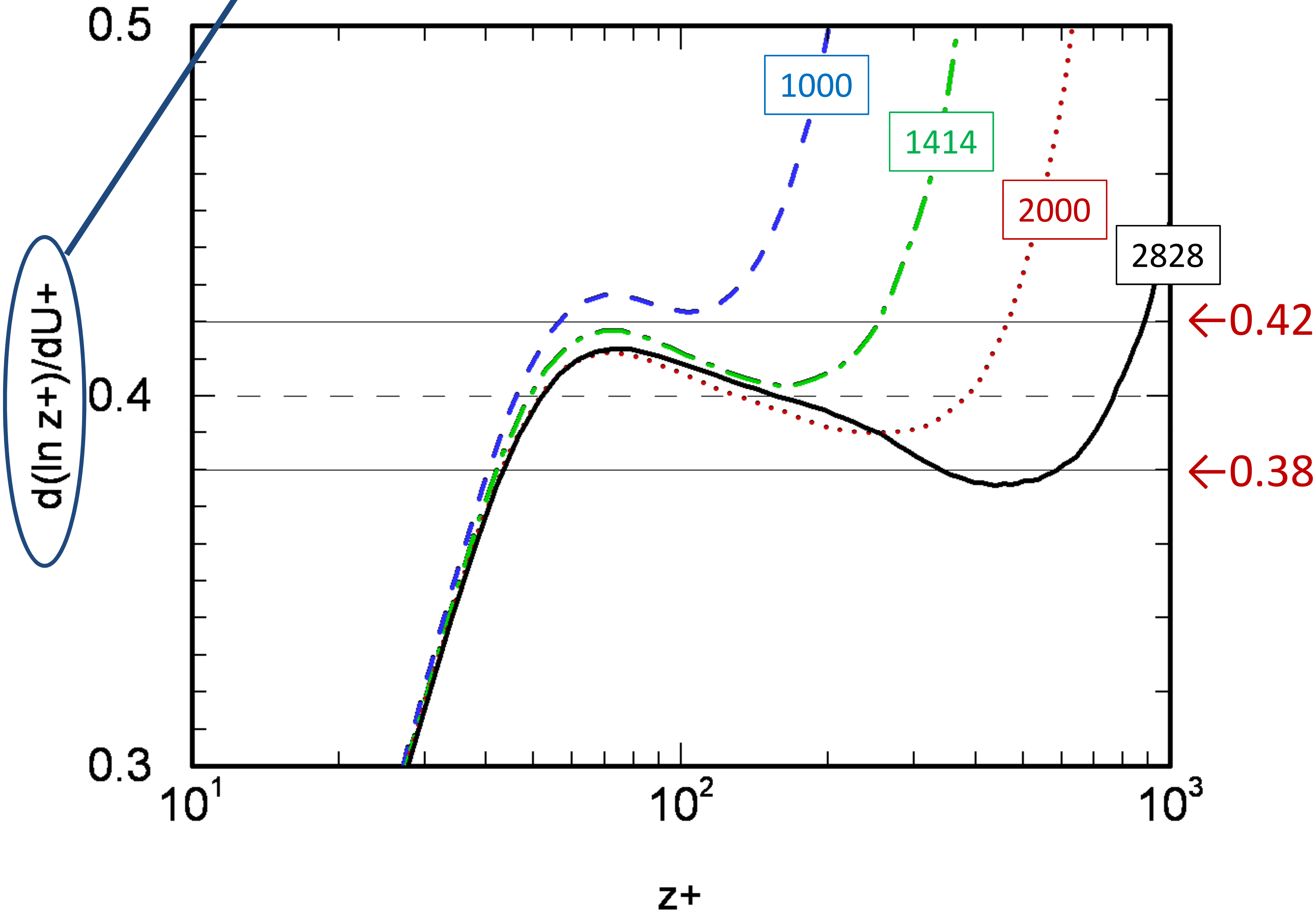
Check via *Karman Measure*

KM = $(y^+ dU^+/dy^+)^{-1} = d(\ln y^+)/dU^+$…

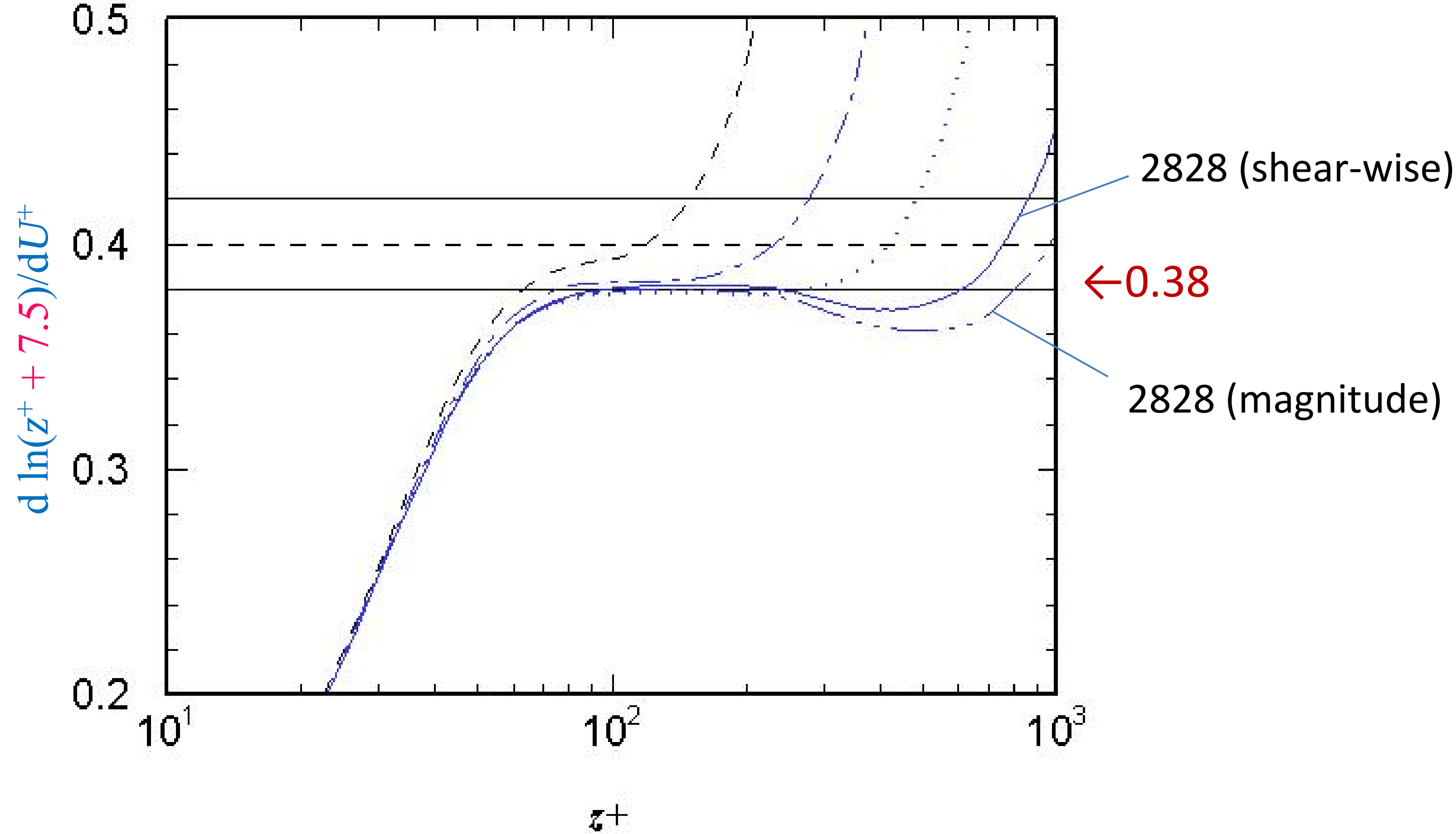
Log Law in Ekman-Layer DNS?



Karman Measure in Ekman-Layer DNS



Karman Measure in Ekman-Layer DNS **with Shift**



- Shifting to ln(z^++7.5) gives plateau at 0.38 (see *Phys. Fluids* 2008), *BUT*…

…But…

- 2008 Ekman DNS used Rules of Thumb for resolution *that affect Karman Measure to same degree as current uncertainty in* κ…

  - 1988 Resolution (R_θ=1410 benchmark):

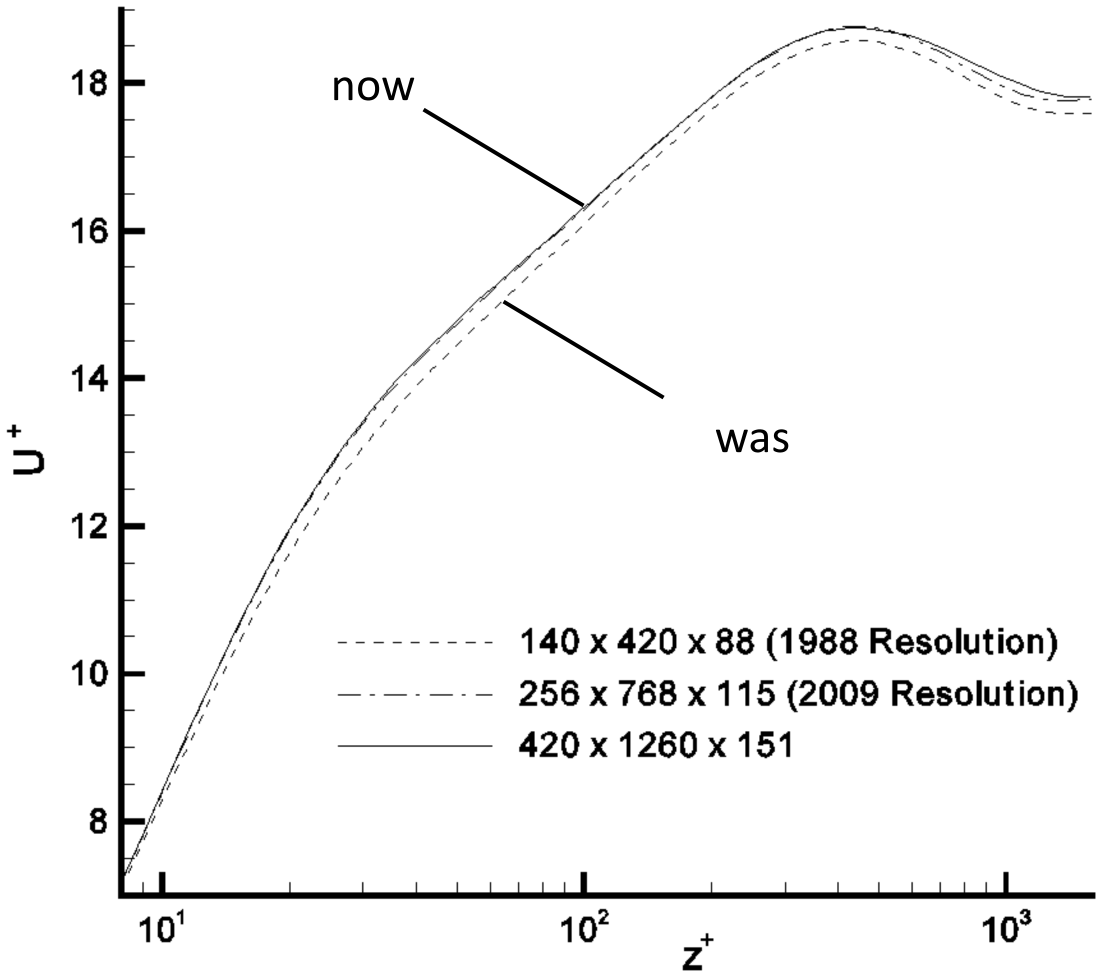
    $\Delta x^+ = 20$, $\Delta y^+ = 7$, n=10 at $z^+ = 10$

    (*cf* fig 2 of Spalart 1988)

  - 2009 Resolution:

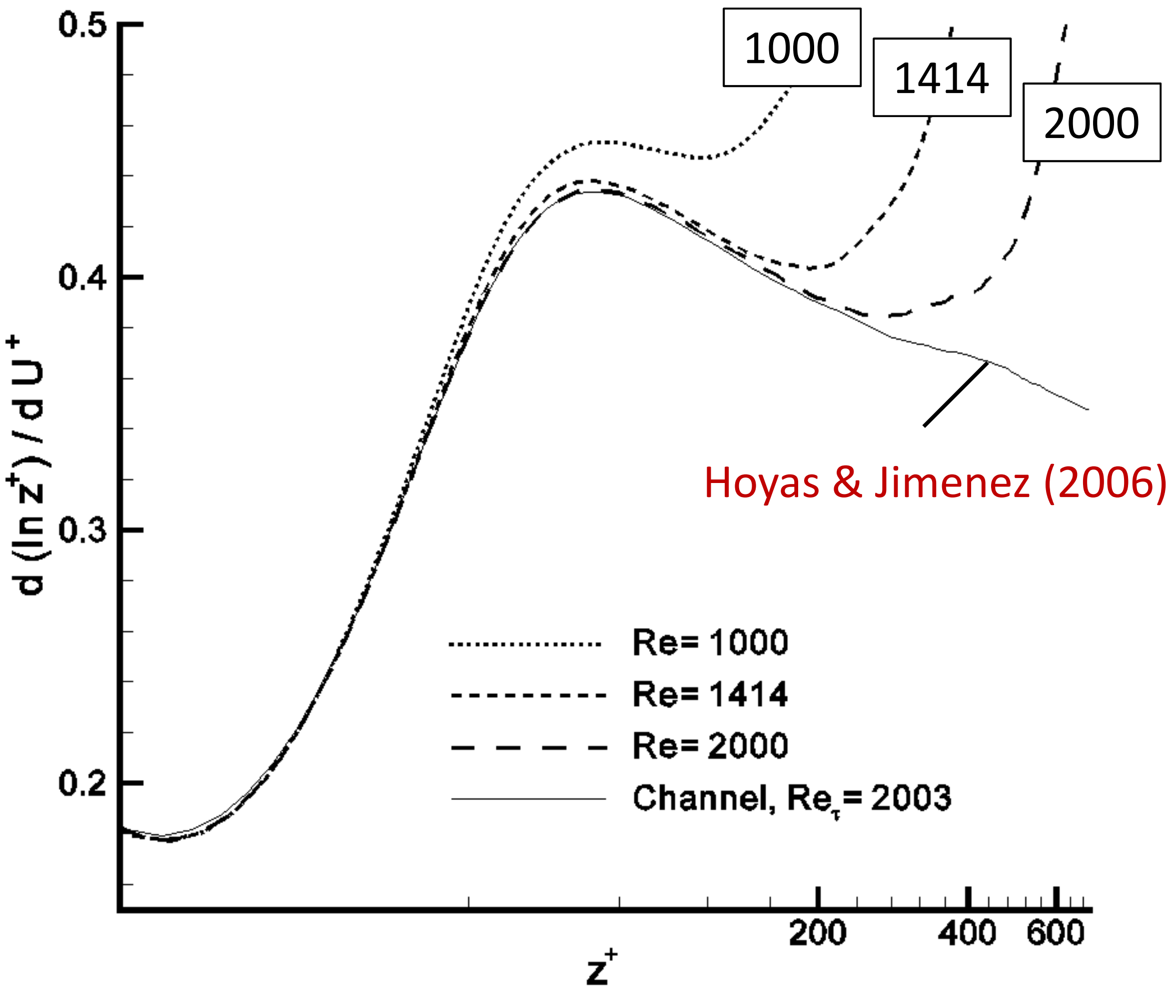
    $\Delta x^+ = 12$, $\Delta y^+ = 4$, n=10 at $z^+ = 6$

Resolution check: Ekman DNS at Re=1000



⇒ still a role for grid-resolution checks in DNS!

Karman Measure for high-res (2009) Ekman DNS



Slope now too steep to justify an offset ☹

However, *note agreement with channel DNS*

Implications

(*cf* Phys. Fluids **21**, 109901, 2009)

- Using Standard Rules of Thumb regarding DNS resolution requires caution, and does not in-and-of itself guarantee perfection *at every level*
- No sign of plateau in Karman Measure (without an implausibly large offset ($a^+ \approx 25$), with $\kappa \approx 0.35$)
- The Log Law is established for $z^+ > 200$ at best (conventional ideas about Viscous-, Buffer- and beginning of Inertial-Sublayer need revision)
- Determination of universal/constant κ (assuming it exists) via DNS many years away
- **But** unique Law of the Wall $dU^+/dz^+=f(z^+)$ apparent for z^+ up to 200, for flat-walls with FPGs (with and without rotation). *True for other cases too?...*

Ekman-layer DNS

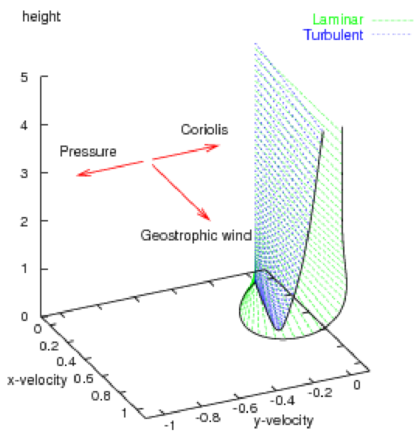
…Lessons *present*

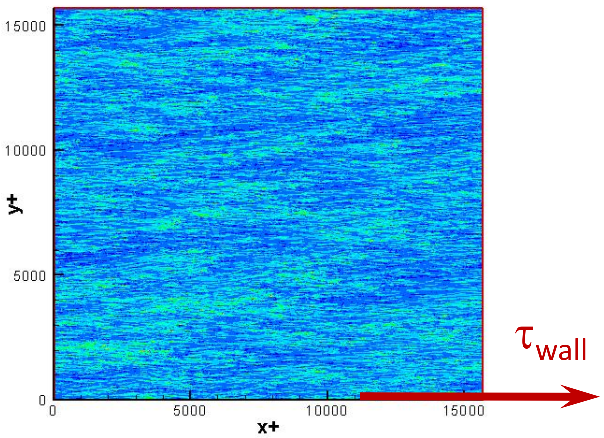
Given that the κ=constant question is currently beyond our reach, what ***can*** we say about near-wall similarity?

Q. How does the Ekman layer compare to other wall-bounded flows, in terms of satisfying a *general Law of the Wall*?

→ To what extent is dU^+/dz^+ *solely a function of* z^+*?*...

…Revisiting recent(2009)/new(2010) Ekman DNS



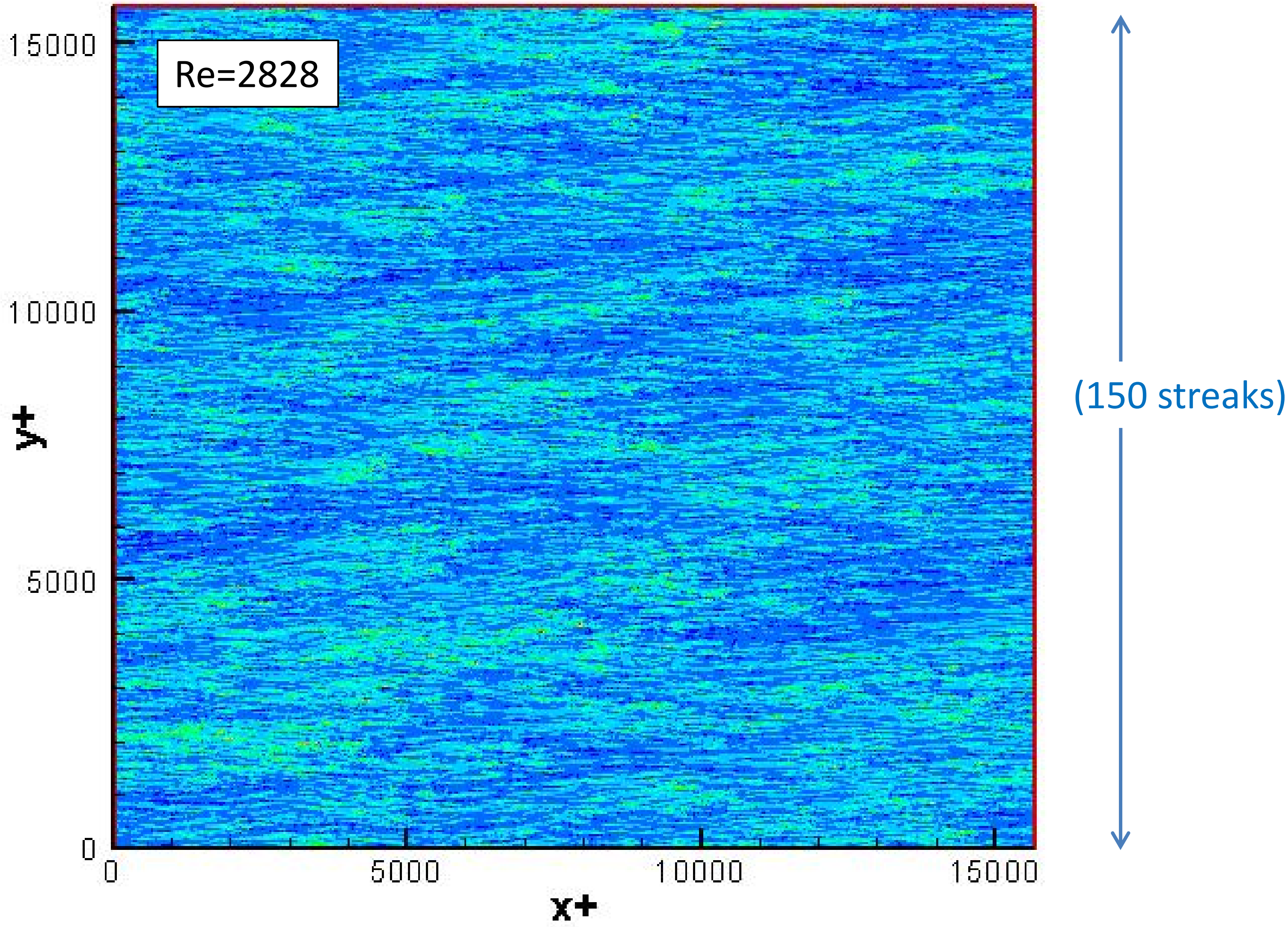


- Recall: Fourier²/Jacobi code (Spalart 1988; Ashworth & Johnstone 2004)
- *x*-axis aligned to direction of surface shear stress (estimated using 3DBL similarity theory) – *greatly increases max attainable* Re

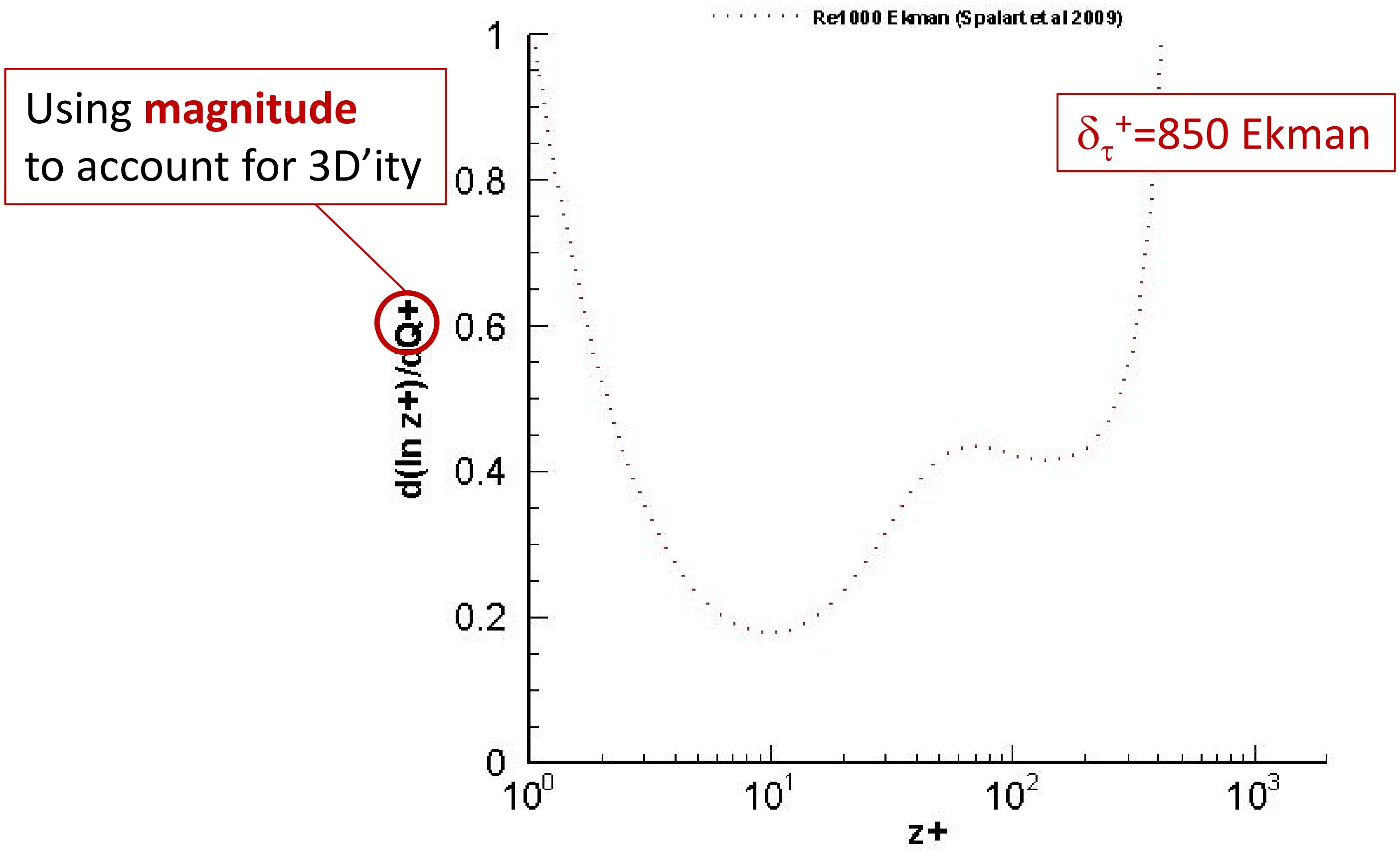
date	Re	*Nx*	*Ny*	*Nz*	*Nx* x *Ny* x *Nz*	δ_τ^+
2009	1000	256	768	115	22.6M	850
2009	1414	420	1260	151	79.9M	1400
2009	2000	768	2304	204	360.9M	2500
2010	**2828**	**1344**	**4032**	**273**	**1,479M**	**4000**

- All cases use high-resolution grid: $\Delta x^+=12$, $\Delta y^+=4$, $n=10$ at $z^+=6$

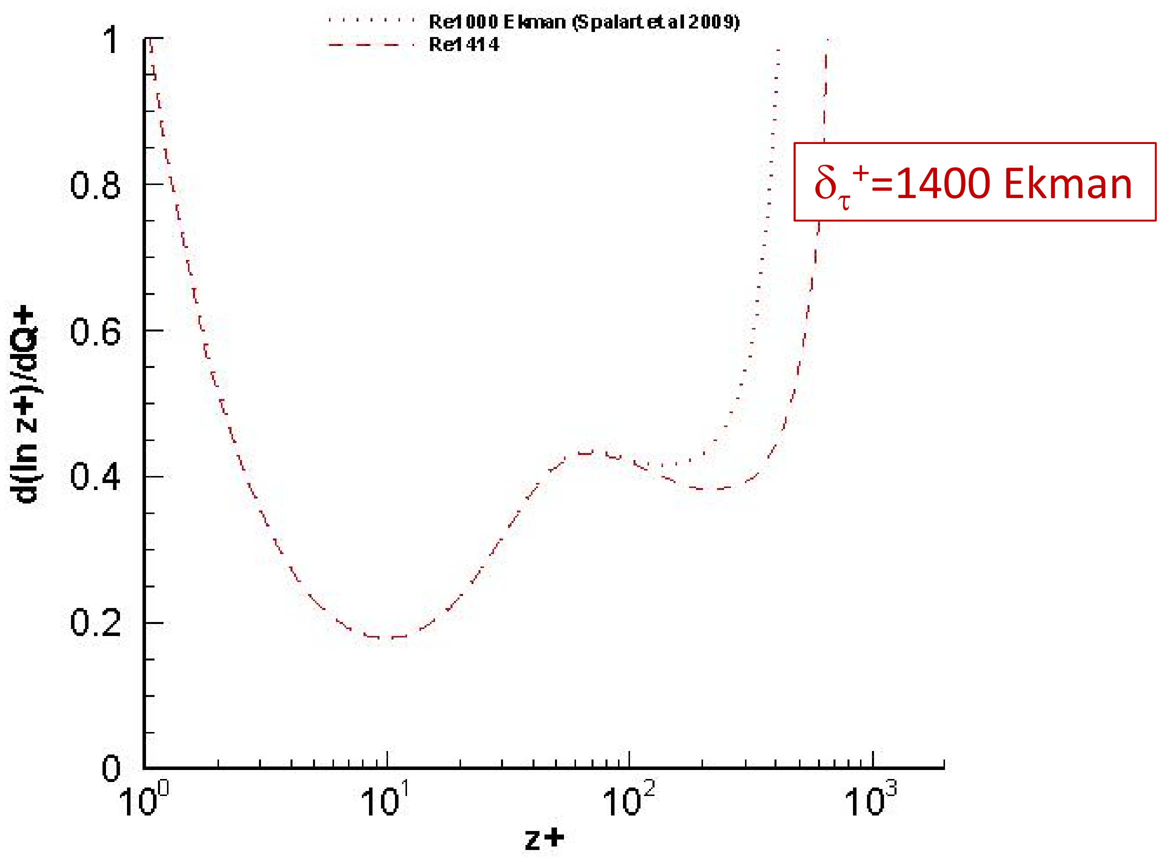
...Revisiting recent/new Ekman DNS (2009,2010)



Unique Law of the Wall?



Unique Law of the Wall?



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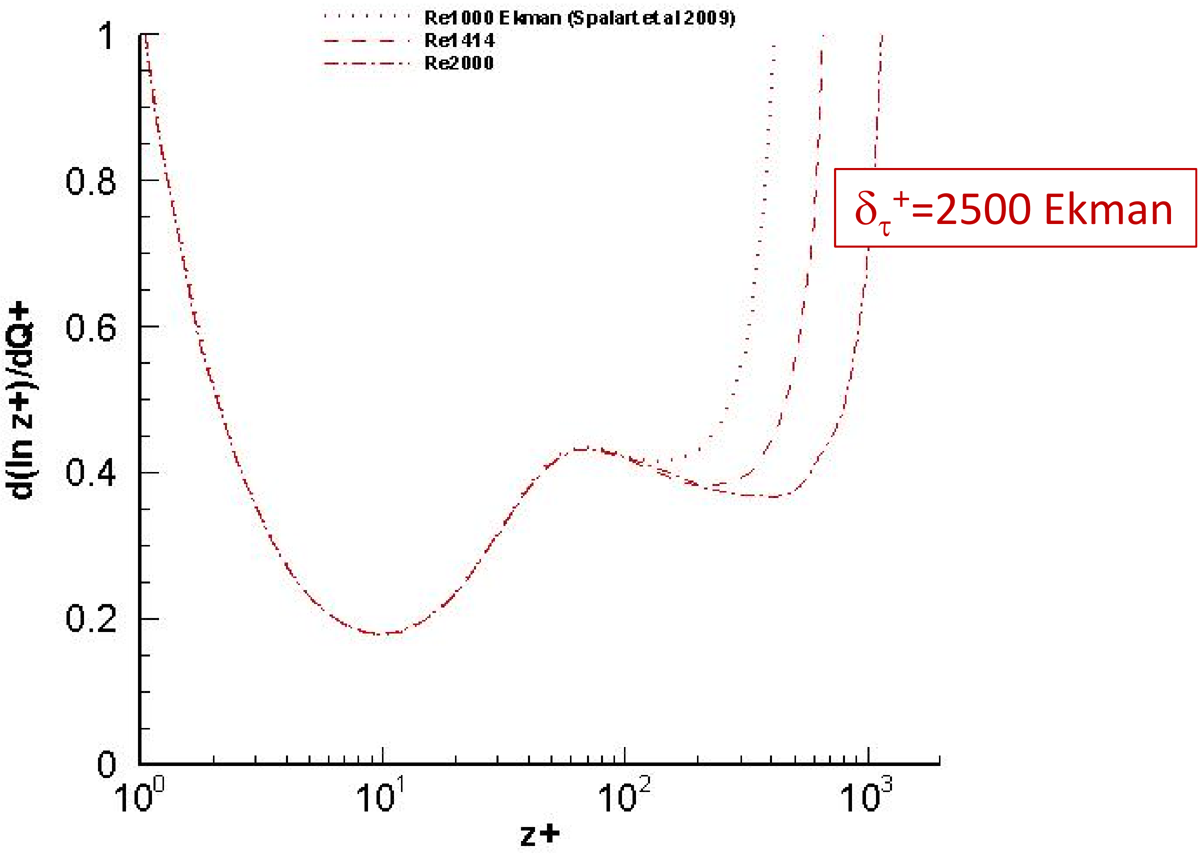
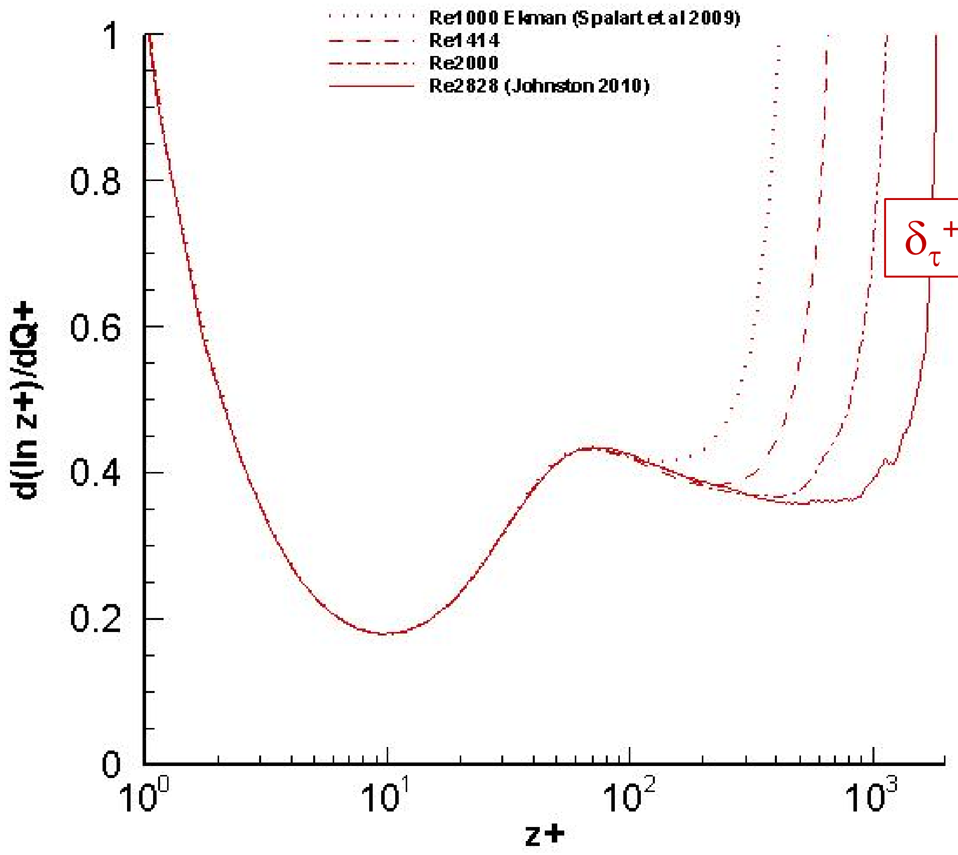
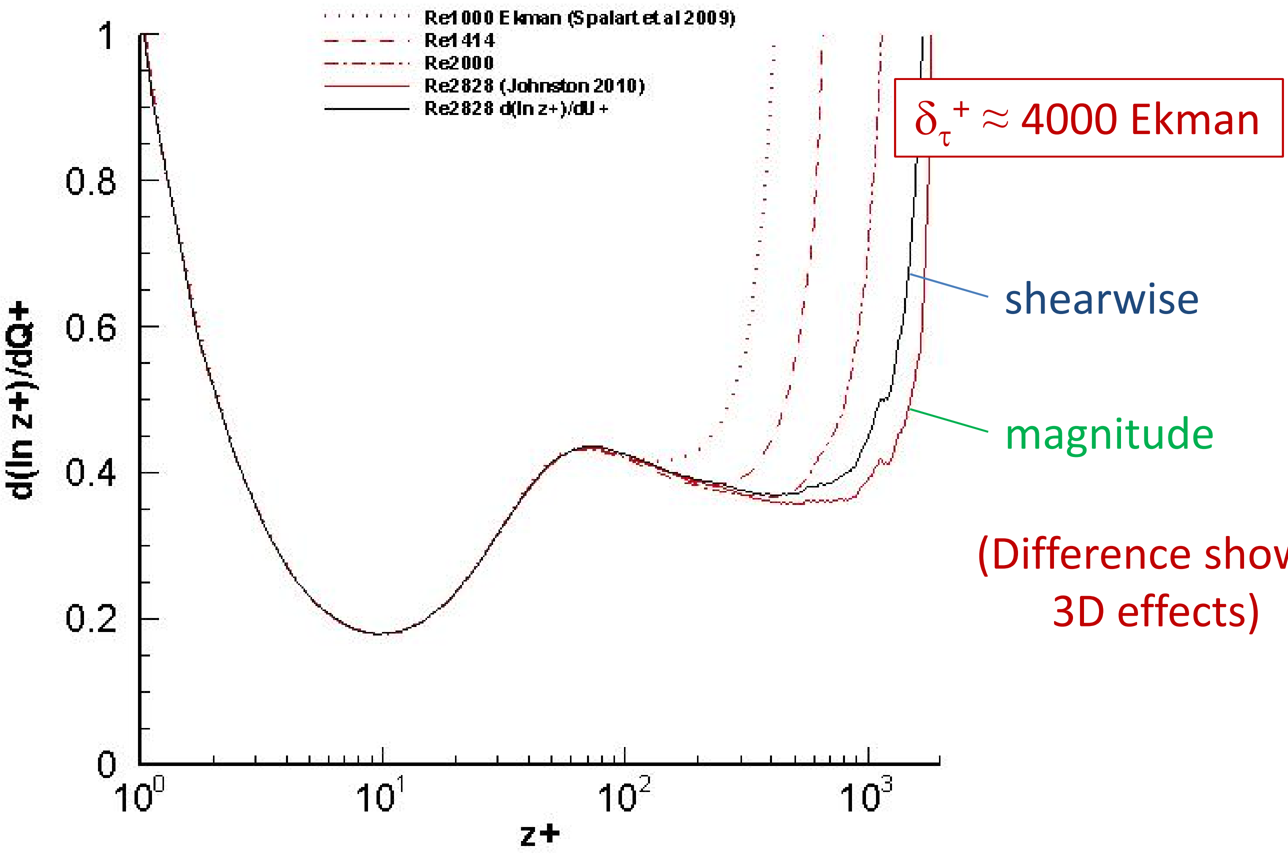


Unique Law of the Wall?



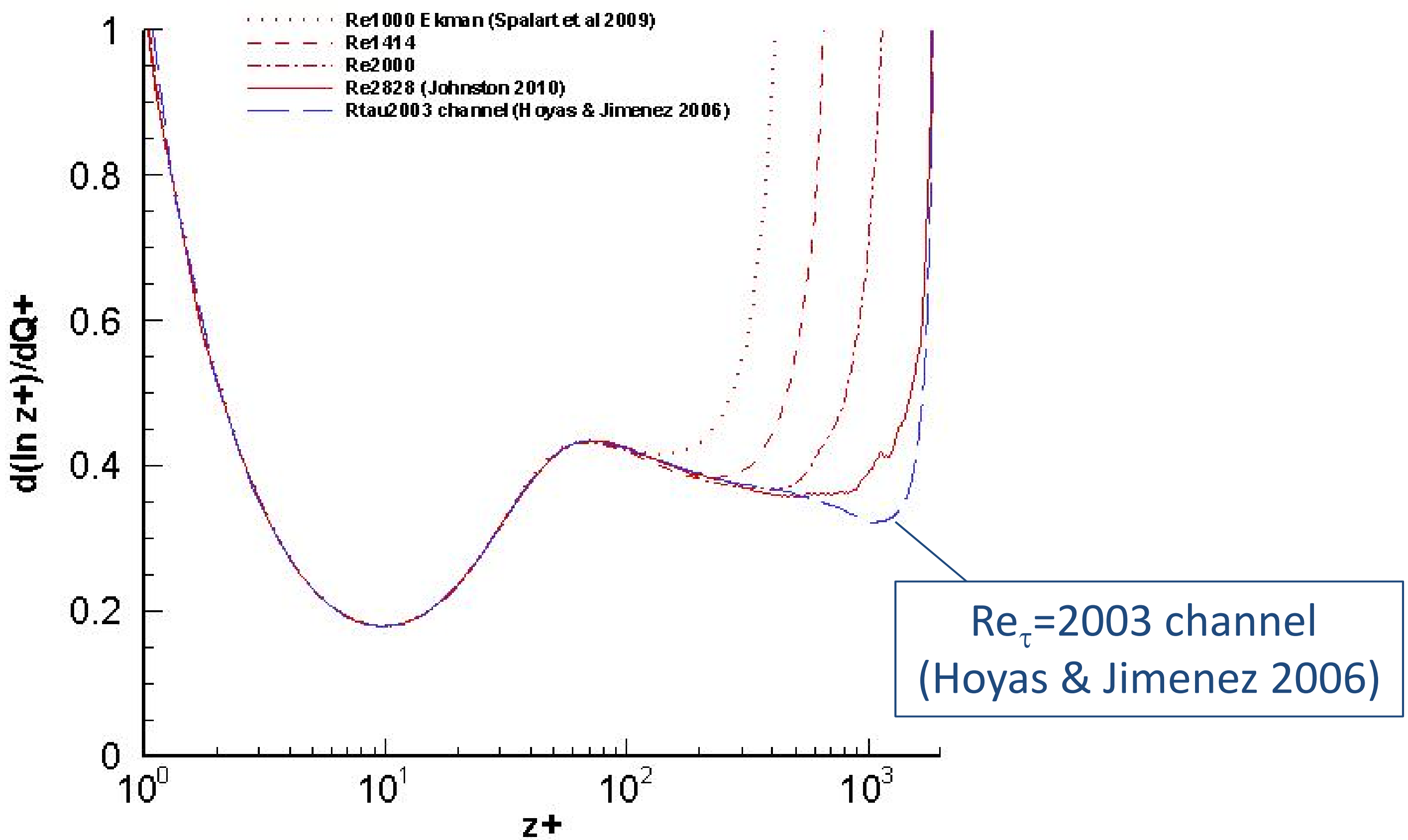
$\delta_\tau^+ \approx$4000 Ekman (new)

Unique Law of the Wall?



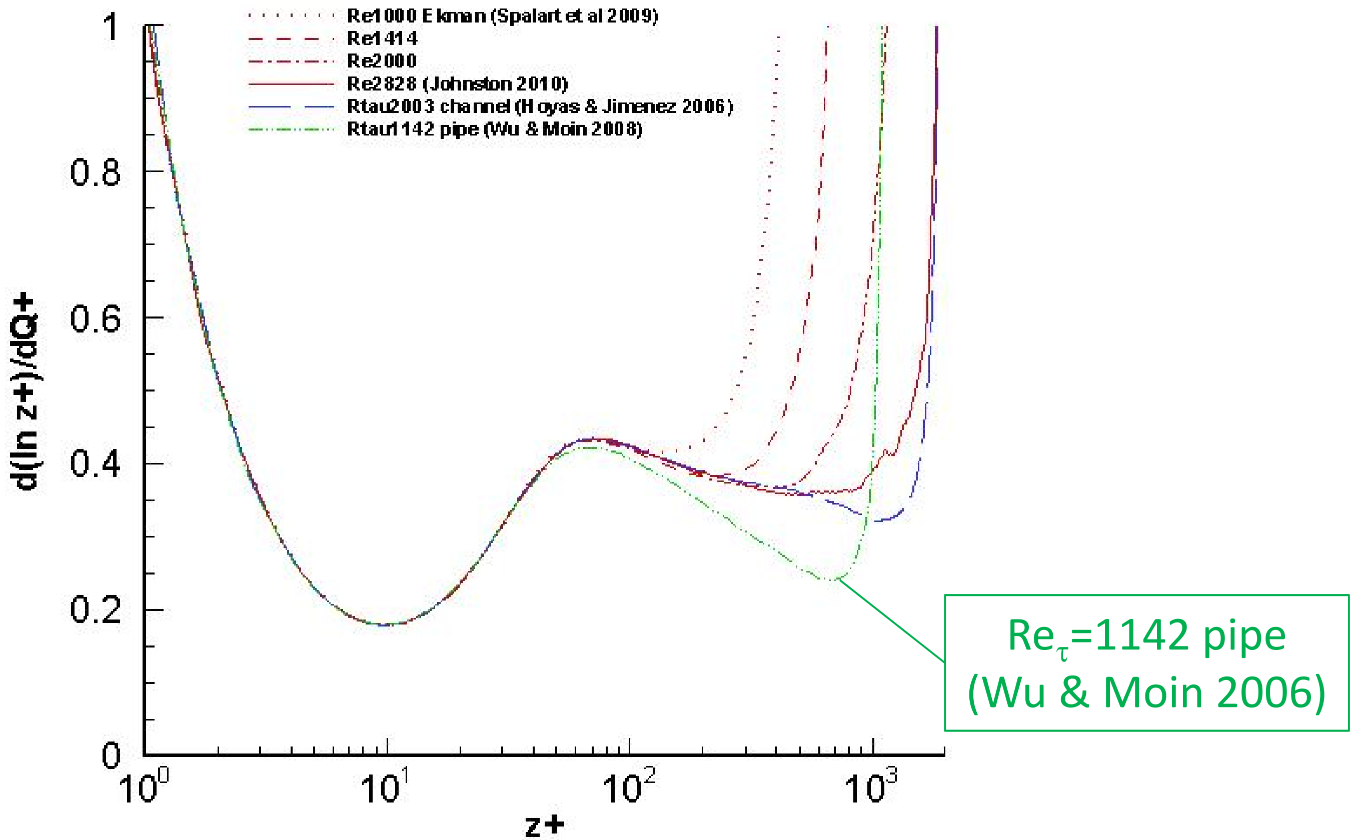
(Difference shows 3D effects)

Unique Law of the Wall?

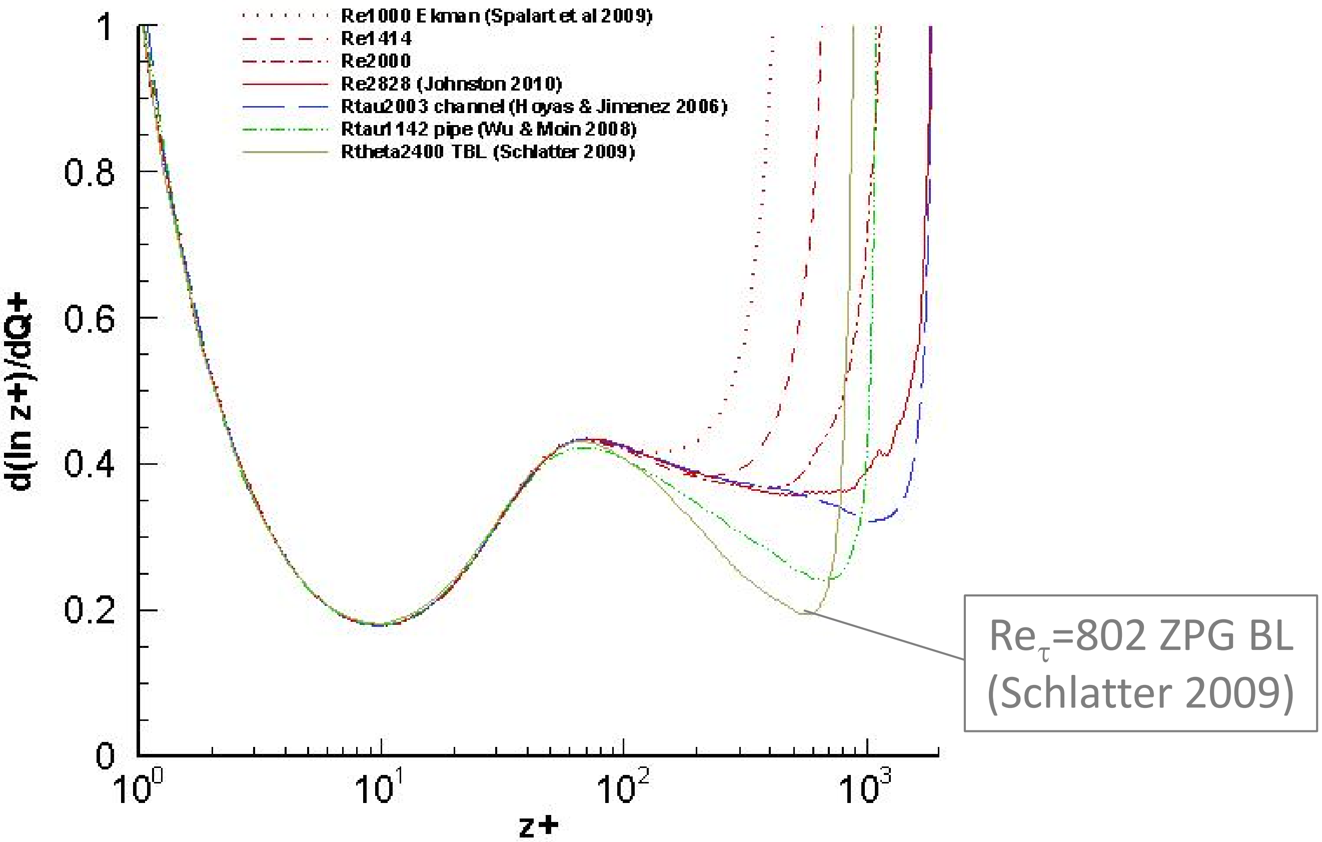


<u>Summary</u>: “flat-surface FPG flows” (w/ and w/o rotation) agree below $z^+ \approx 300$

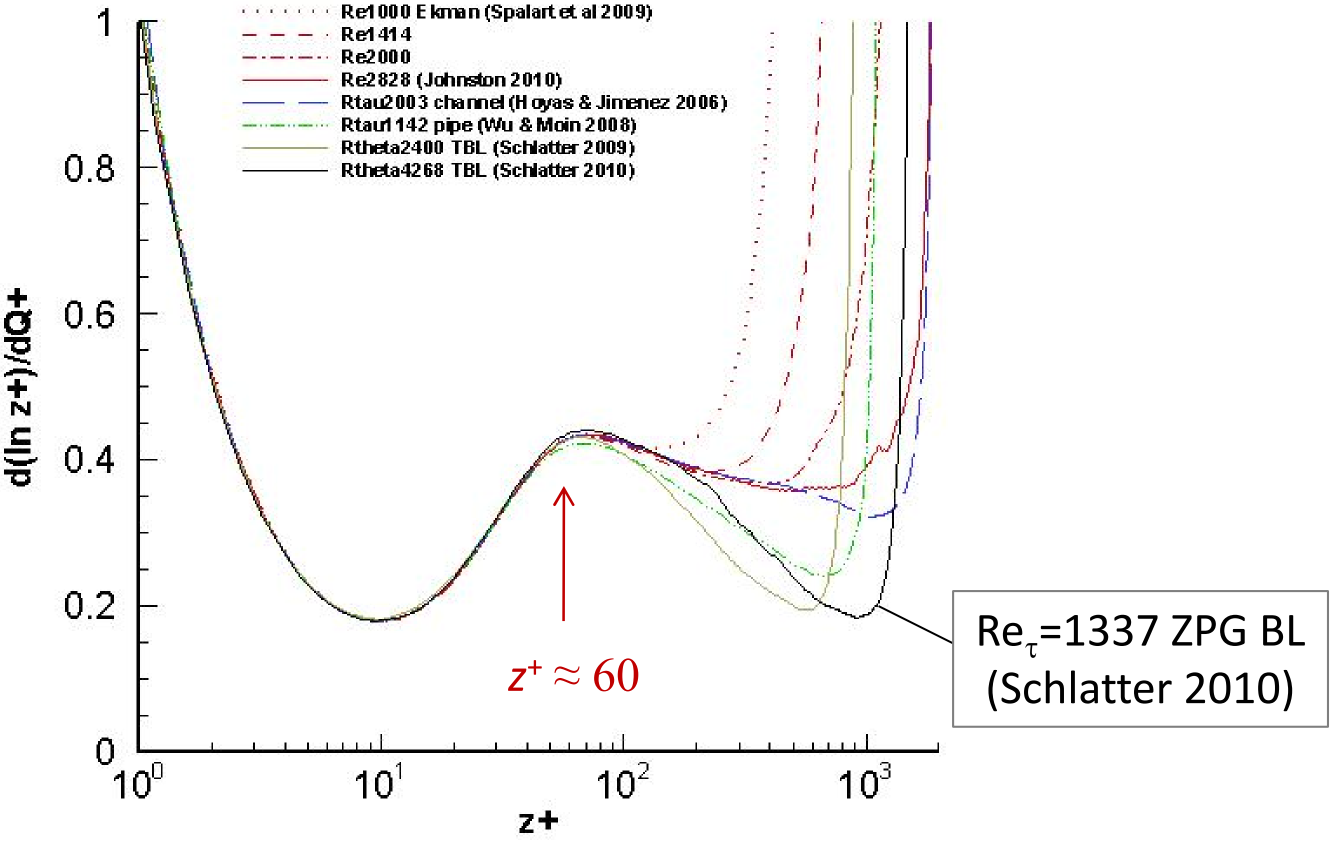
Unique Law of the Wall?



Unique Law of the Wall?



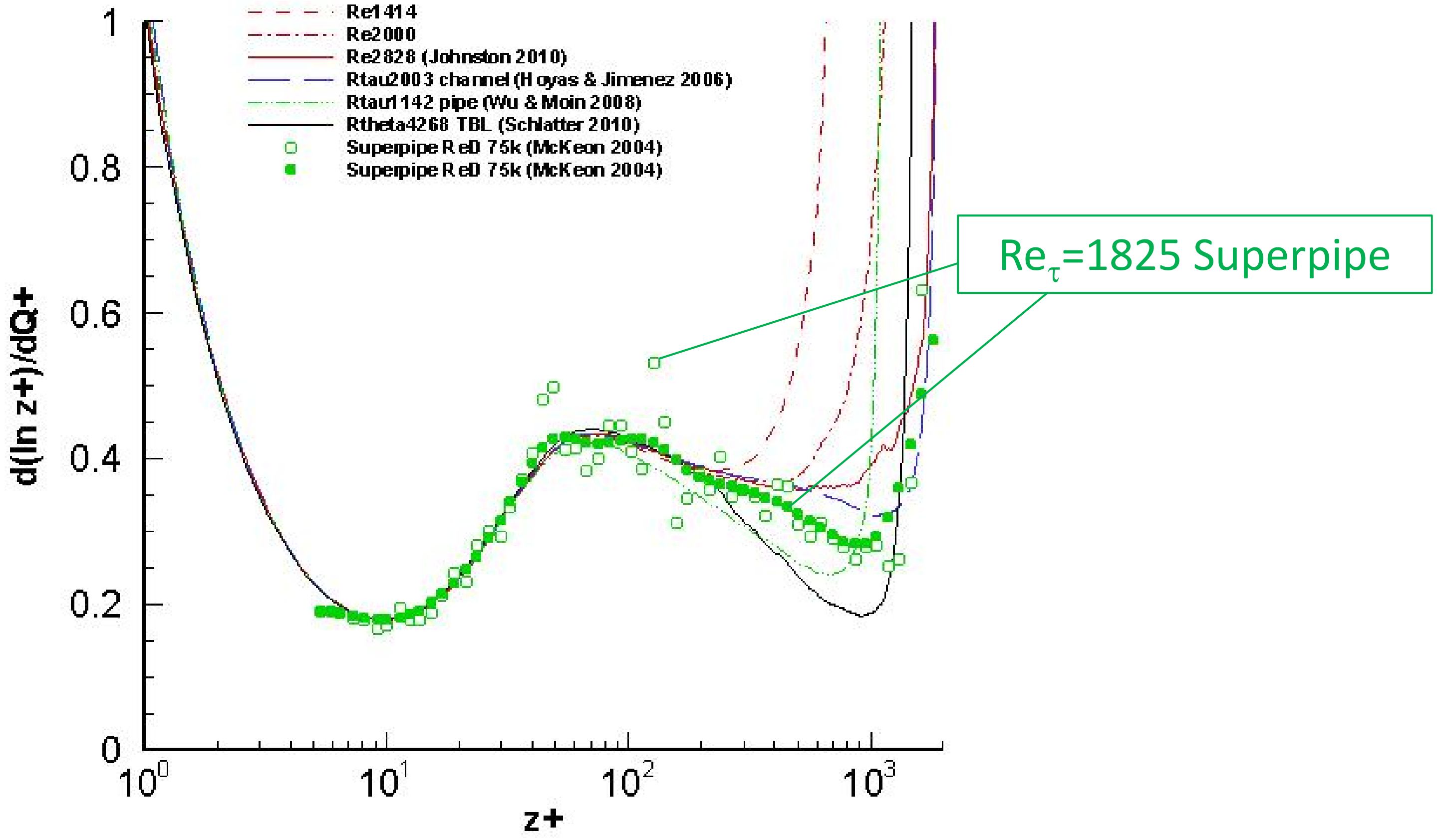
Unique Law of the Wall?



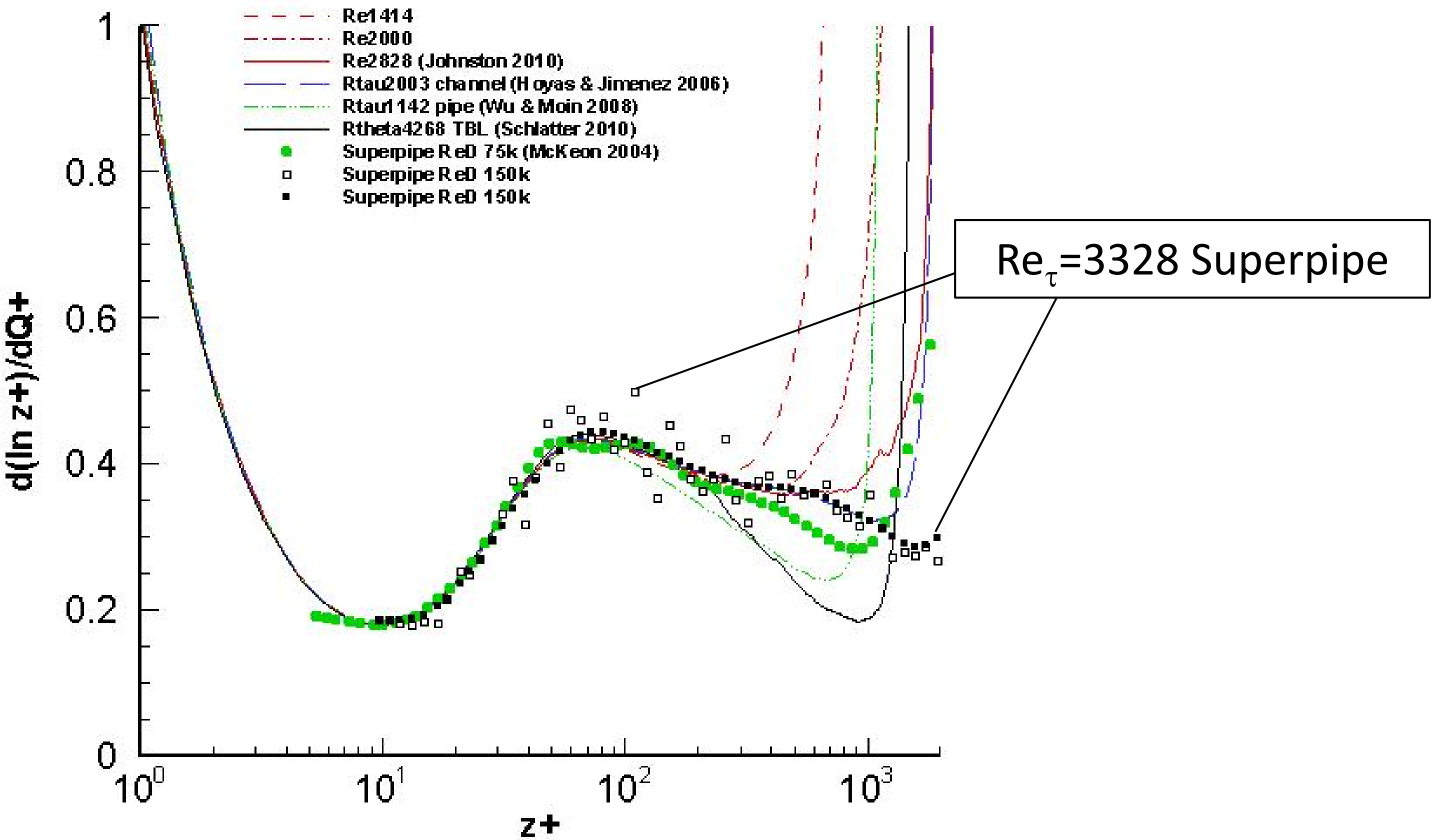
All DNS flows (pipe, channel, Ekman, ZPG BL) agree below $z^+ \approx 60$

Unique Law of the Wall?

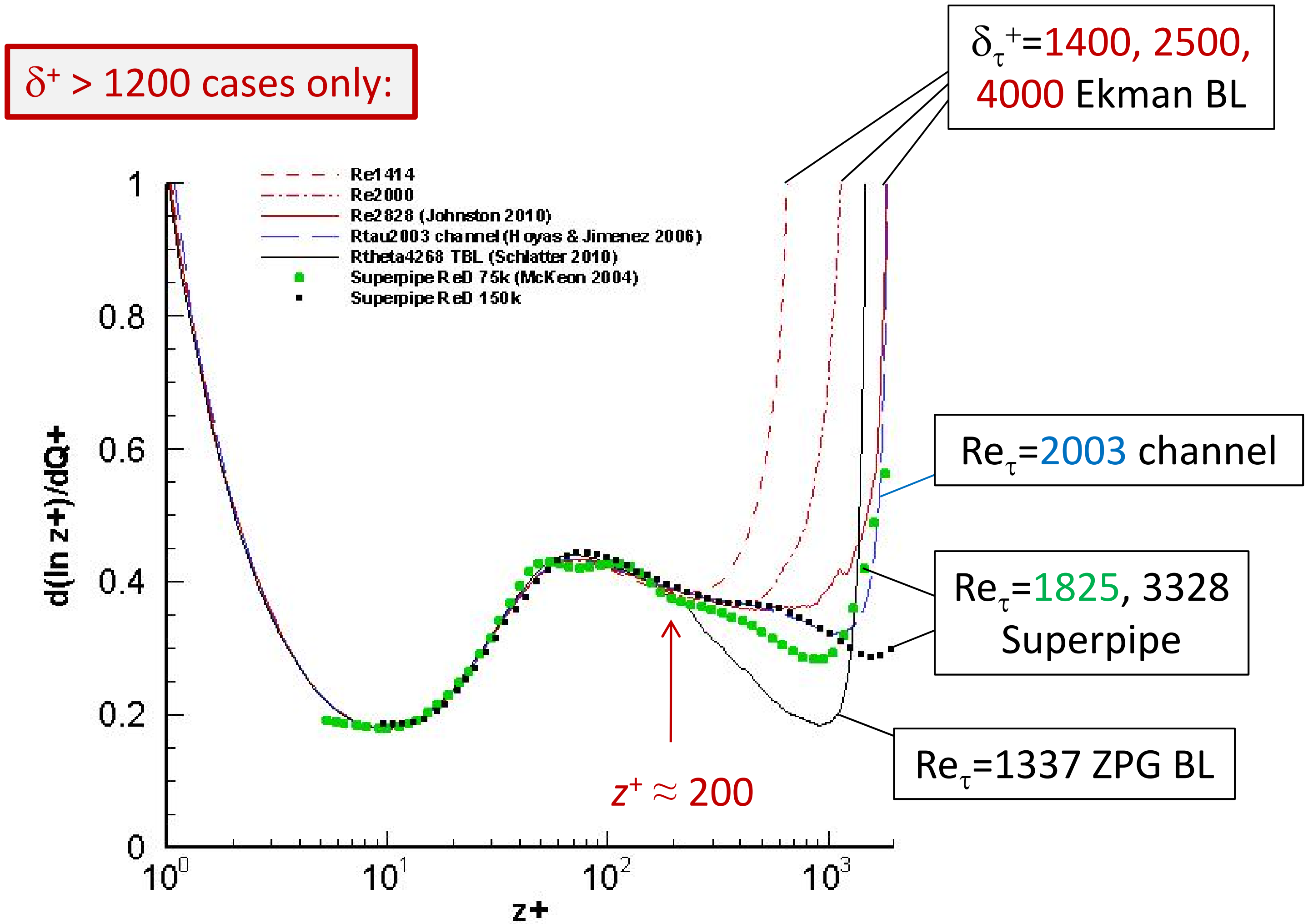
Consider higher Re pipe flow (exp):



Unique Law of the Wall?



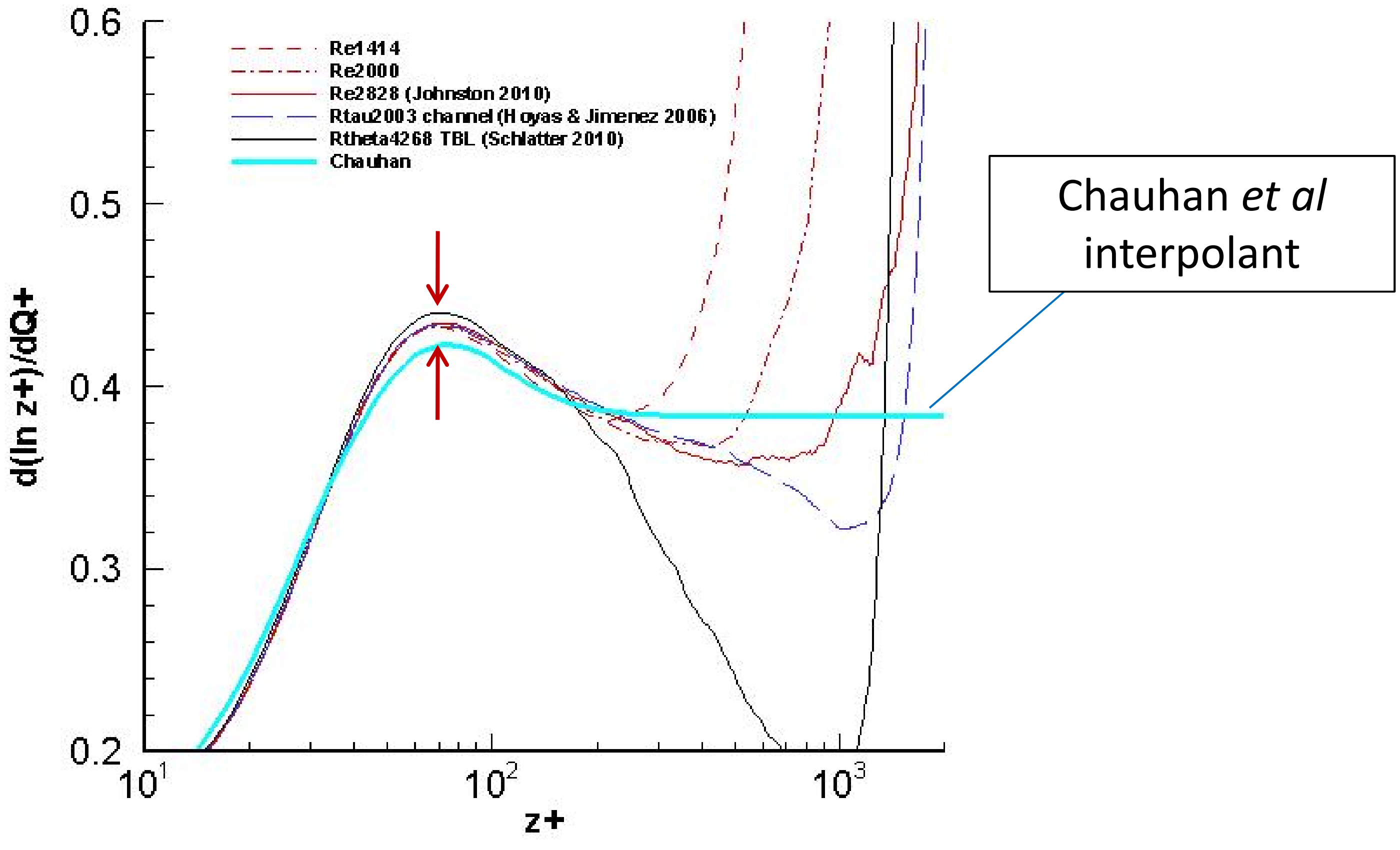
Unique Law of the Wall?



All <u>"higher-Re" flows</u> (DNS and exp, for pipe, channel, Ekman, ZPG BL) <u>agree below $z^+ \approx 200$</u>

cf Chauhan *et al* (2007)

DNS only:



KM peak a bit too low in Chauhan's composite, constant KM not observed in DNS

Two (parenthetical) points regarding LotW

1. In <u>*very*</u> *near-wall region* (VSL, $z \rightarrow 0$),

$$\text{RANS eqn} \Rightarrow U^+ \equiv z^+ + (½)p^+(z^+)^2,$$

$$\text{where } p^+ = (\mathrm{d}P/\mathrm{d}x)/(u_\tau^3/\nu).$$

Therefore $\mathrm{d}U^+/\mathrm{d}z^+ = f(z^+)$ only <u>*not possible in VSL*</u>

[***Inertial* SL**: integrating $\mathrm{d}U^+/\mathrm{d}z^+ = 1/\kappa z^+ \Rightarrow U^+ = (1/\kappa)\ln z^+ + C$, where $C = U_0^+ - (1/k)\ln z_0^+$. That is, additive constant C defined by thickness of VSL z_0^+. <u>Suggests $C=C(p^+)$</u>? (*cf* Kays & Crawford 1993).]

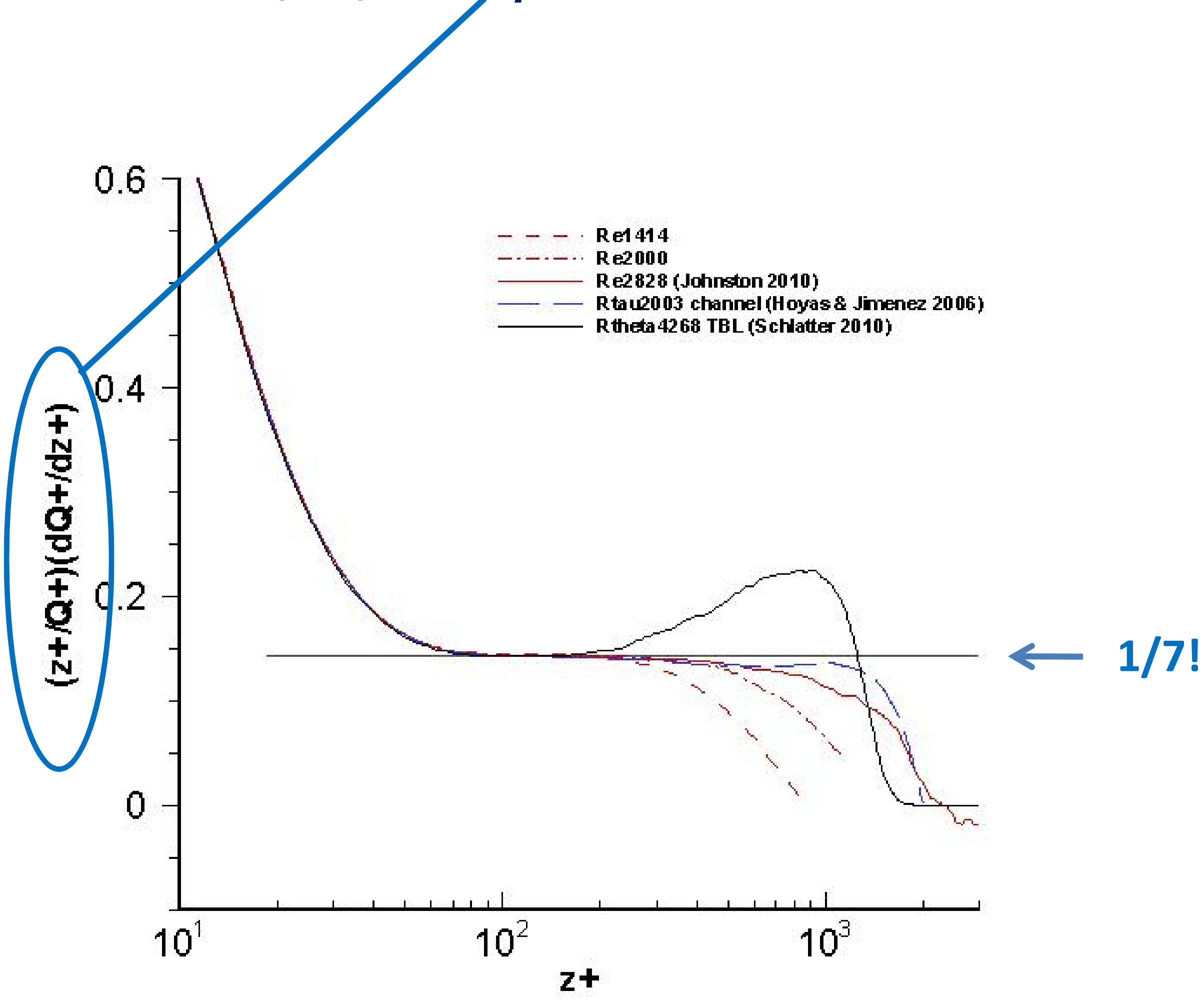
…Two (parenthetical) points regarding LotW

2. For $70 < z^+ < 200$ (and probably higher), $U^+ = f(z^+)$ profile well represented by <u>power law</u>:

$$U^+ = A^+(z^+)^\beta, \text{ with } \beta = 1/7\ldots$$

Power-law behaviour:

$$U^+ = A^+(z^+)^\beta \Rightarrow \beta = (z^+/Q^+)(dQ^+/dz^+)$$



Summary:

- Unique Law of the Wall (i.e. $dU^+/dz^+=f(z^+)$ only) evident for $z^+ < 200$, for a variety of flows – *unaffected by rotation, lateral curvature and pressure gradient*
- Conceivable (likely?) that range of validity will increase with Re
- Is κ constant and universal? *Time will tell…*

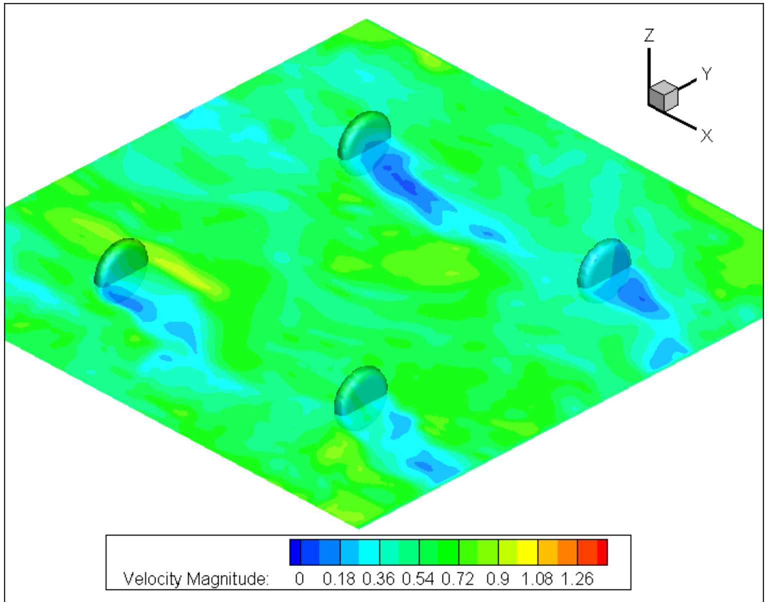
Ekman-layer DNS

Future lessons?

- Wind-farm/turbine-array DNS
- Near-wall structures: hairpins, QSVs, etc?
- Interfacial dynamics for “non-entraining” TBLs?

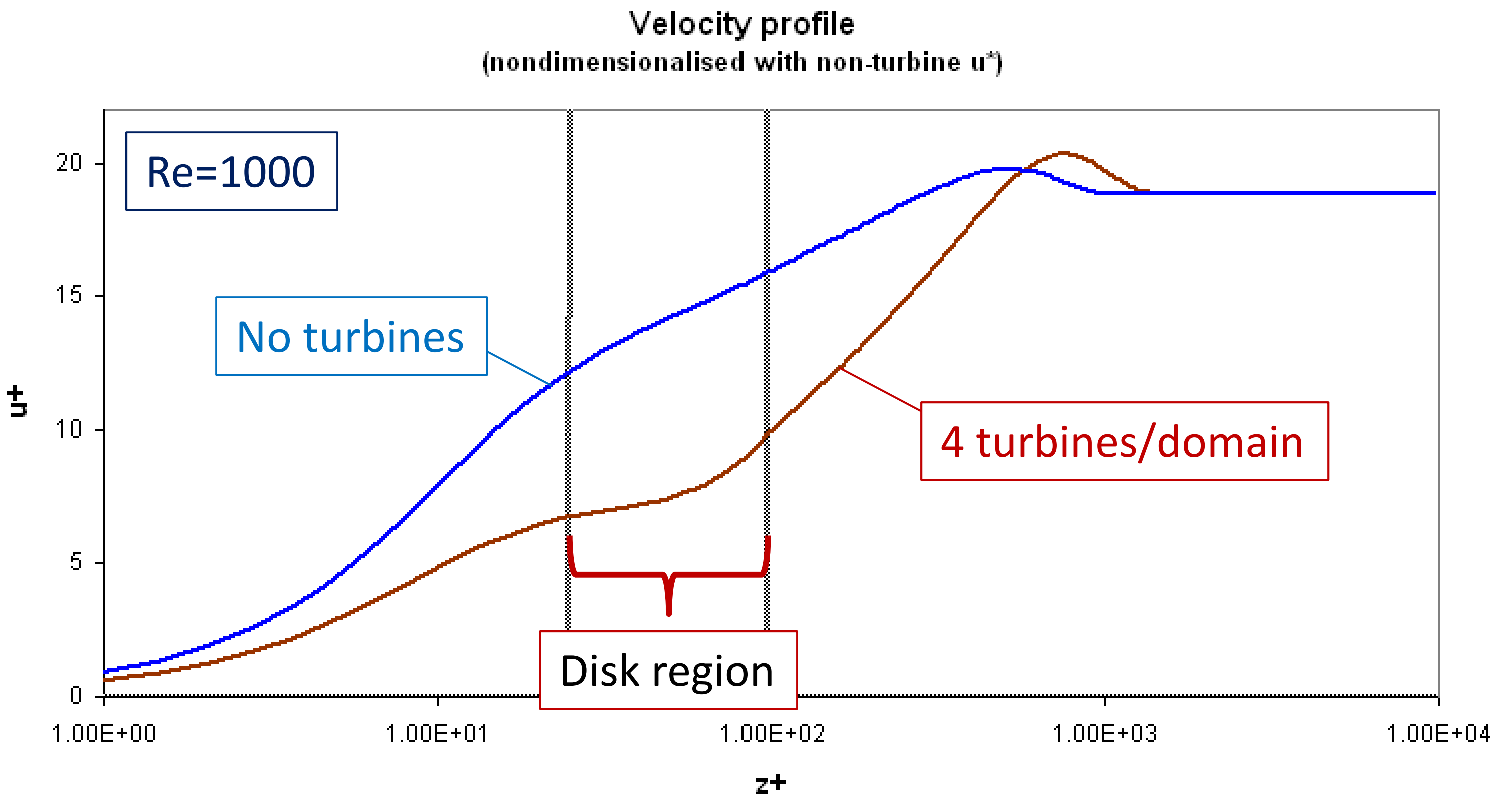
Ekman-layer wind-farm DNS

- **Approach**
  - Emulate turbine arrays as dynamic actuator-disks (*cf* Calaf *et al* 's 2010 LES)
  - Mimic roughness via viscosity
  - ABL depth scale not quite right, in terms of fraction of actual u_τ/f, but mean skewing and more-realistic (floating) $\tau(z)$ included
- **Issues**
  - ABL/turbine interactions
  - Maximum/optimal size of single turbine
  - Optimal array configurations
  - Actual-vs-Betz power

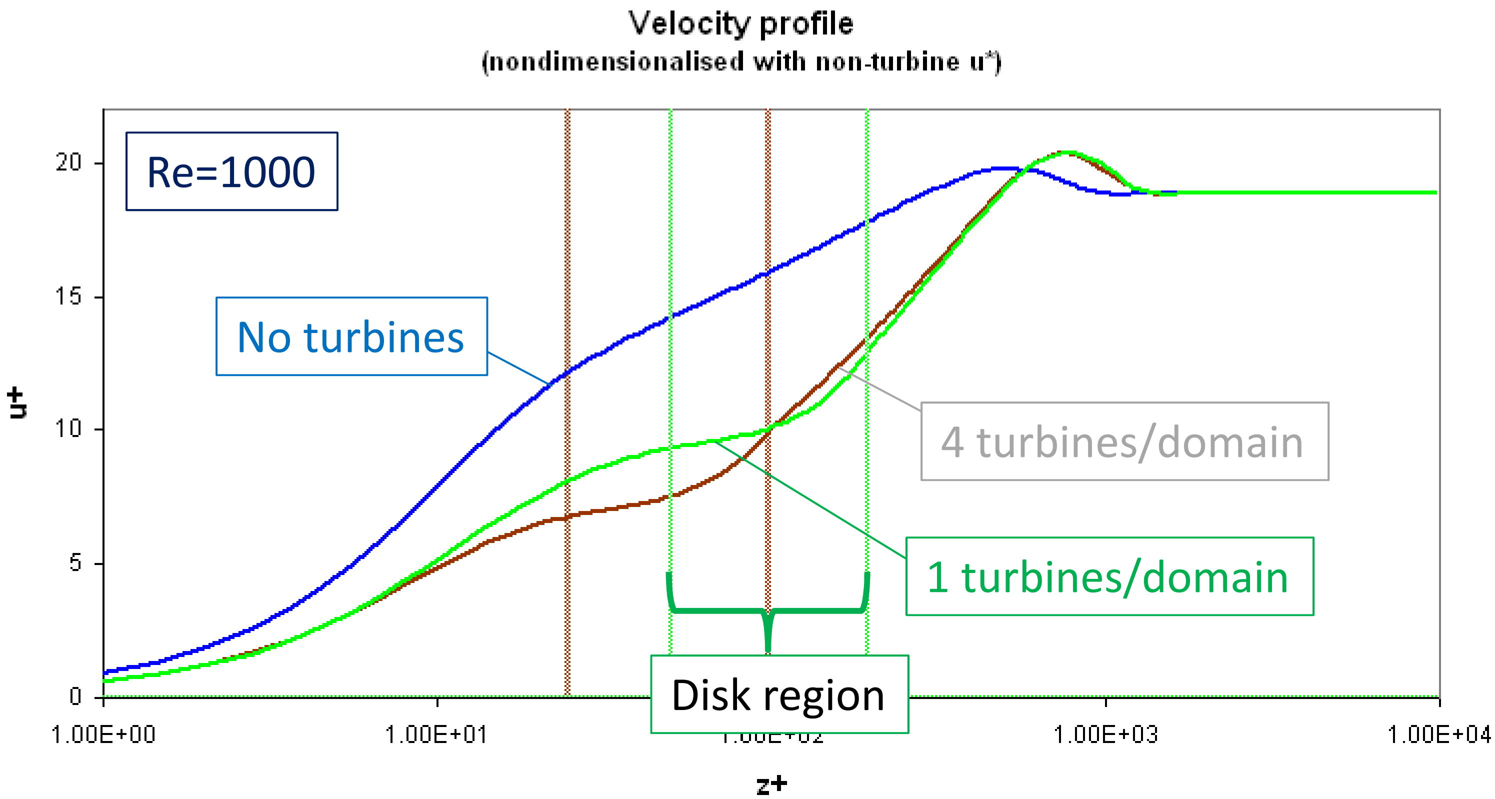


Horizontal slice at about hub height, and actuator disk isosurfaces

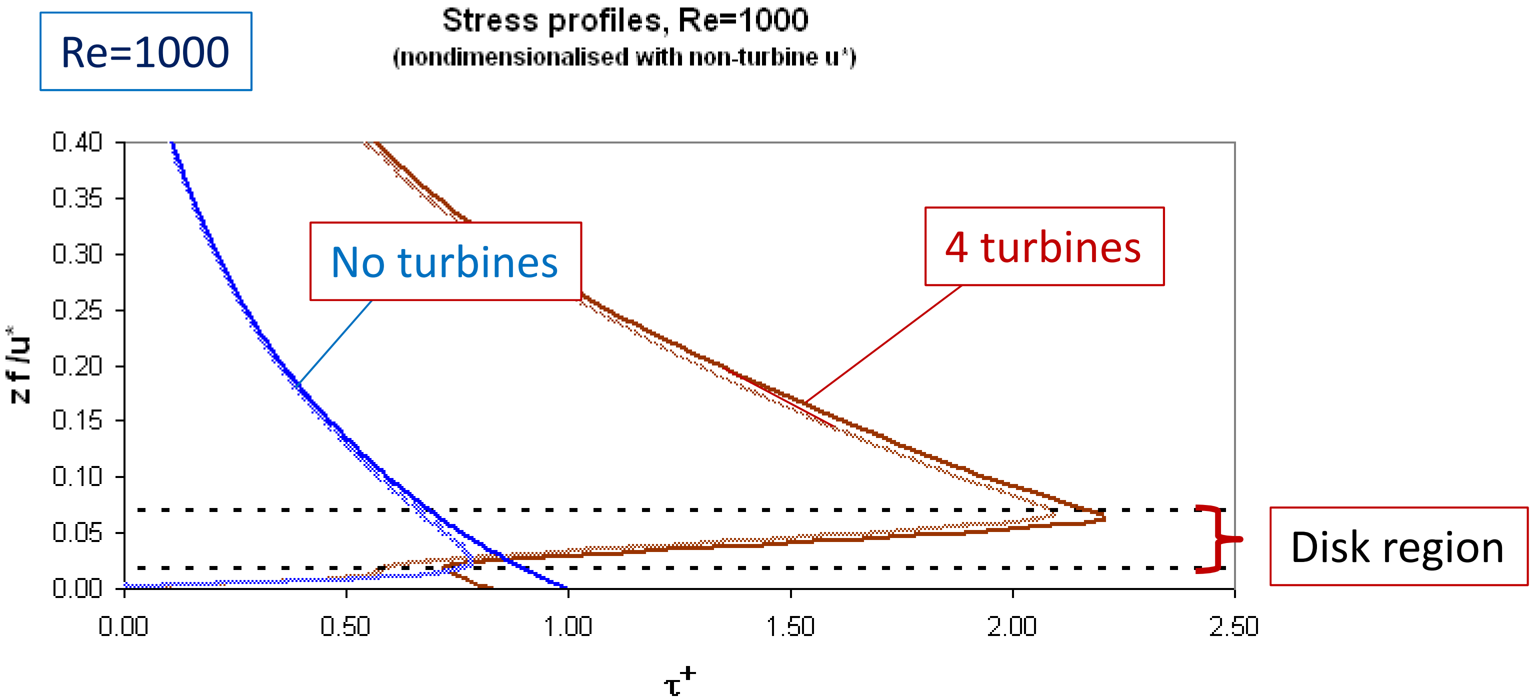
Ekman-layer wind-farm DNS



Ekman-layer wind-farm DNS

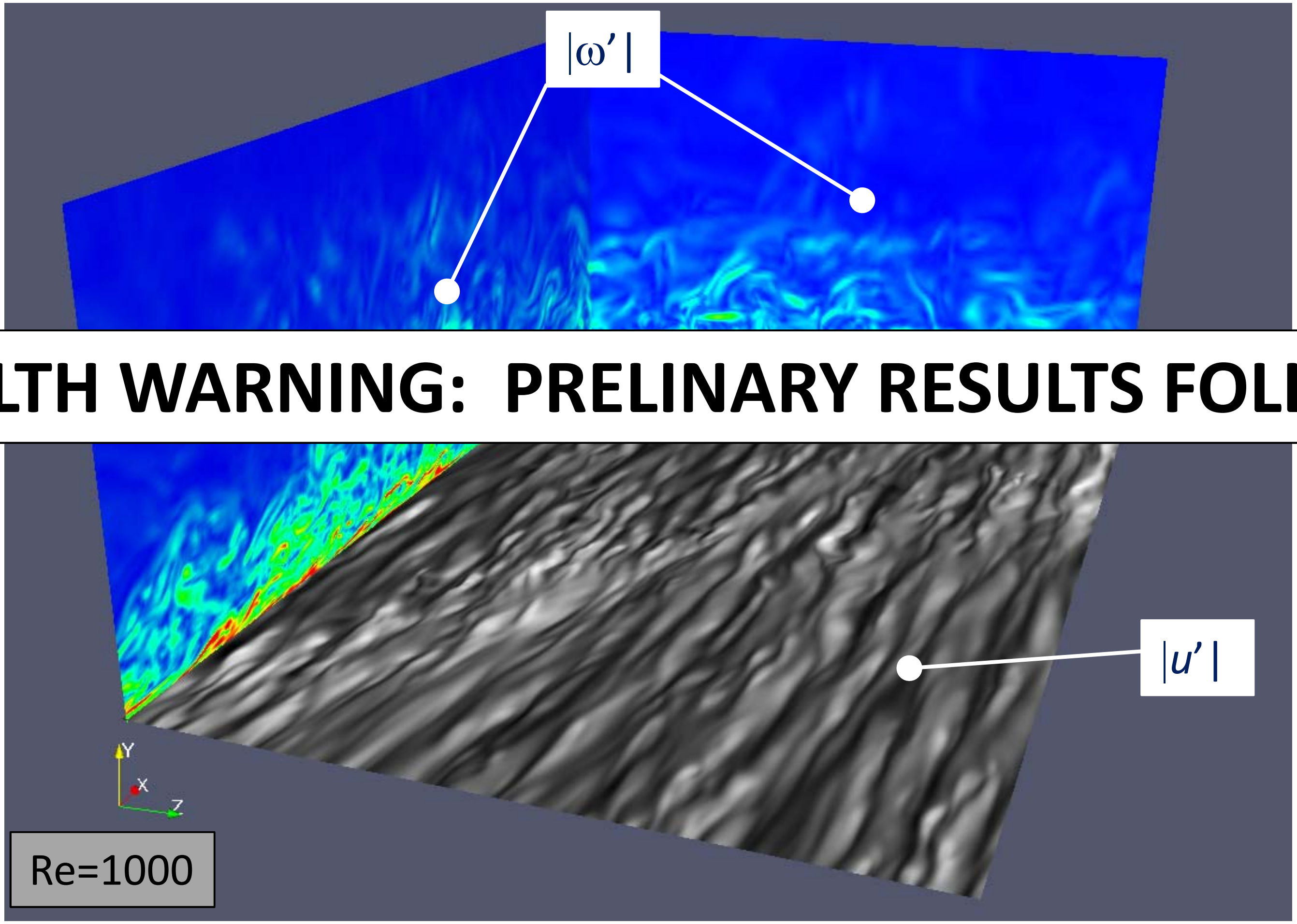


Ekman-layer wind-farm DNS

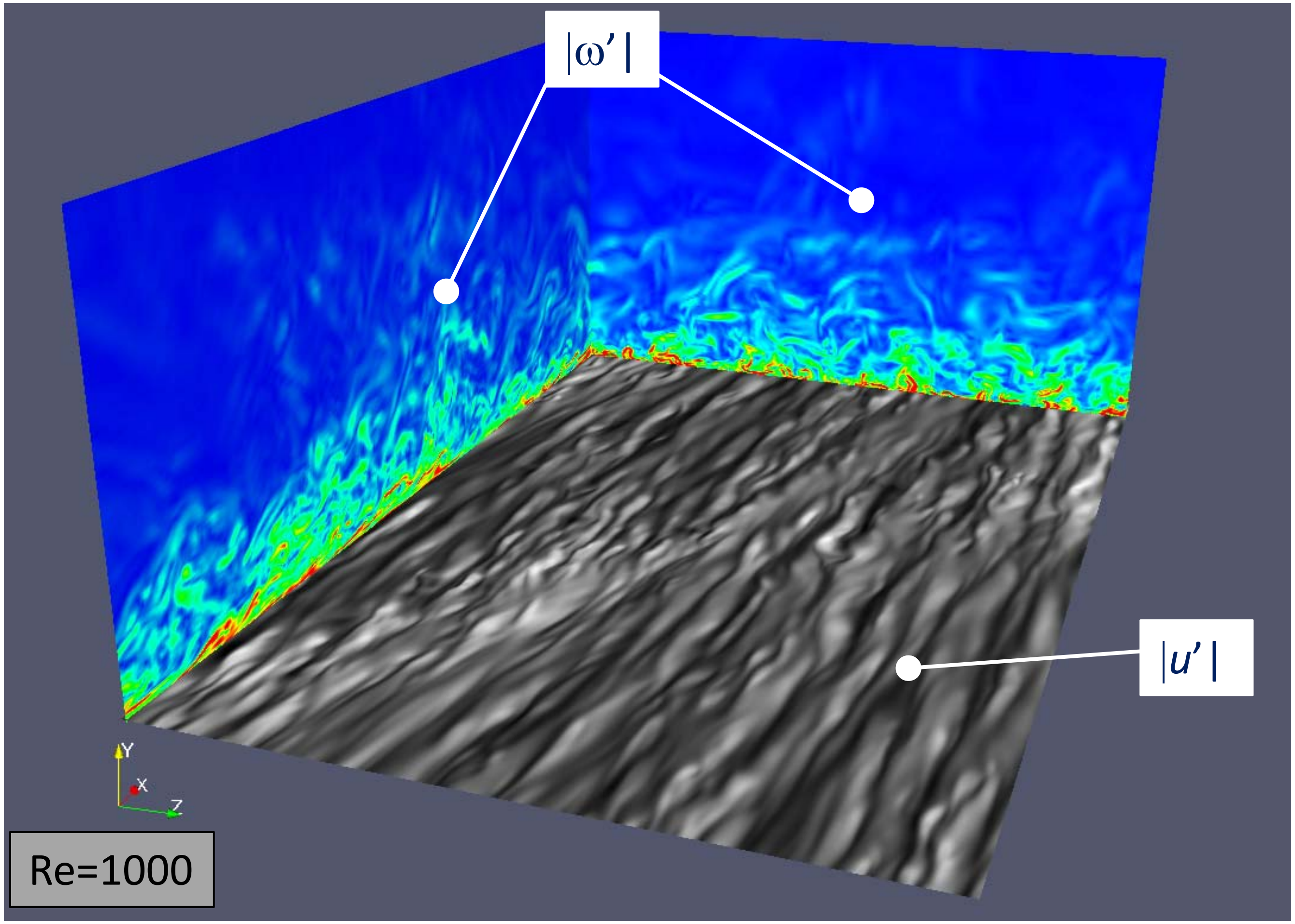


Turbulence structures?

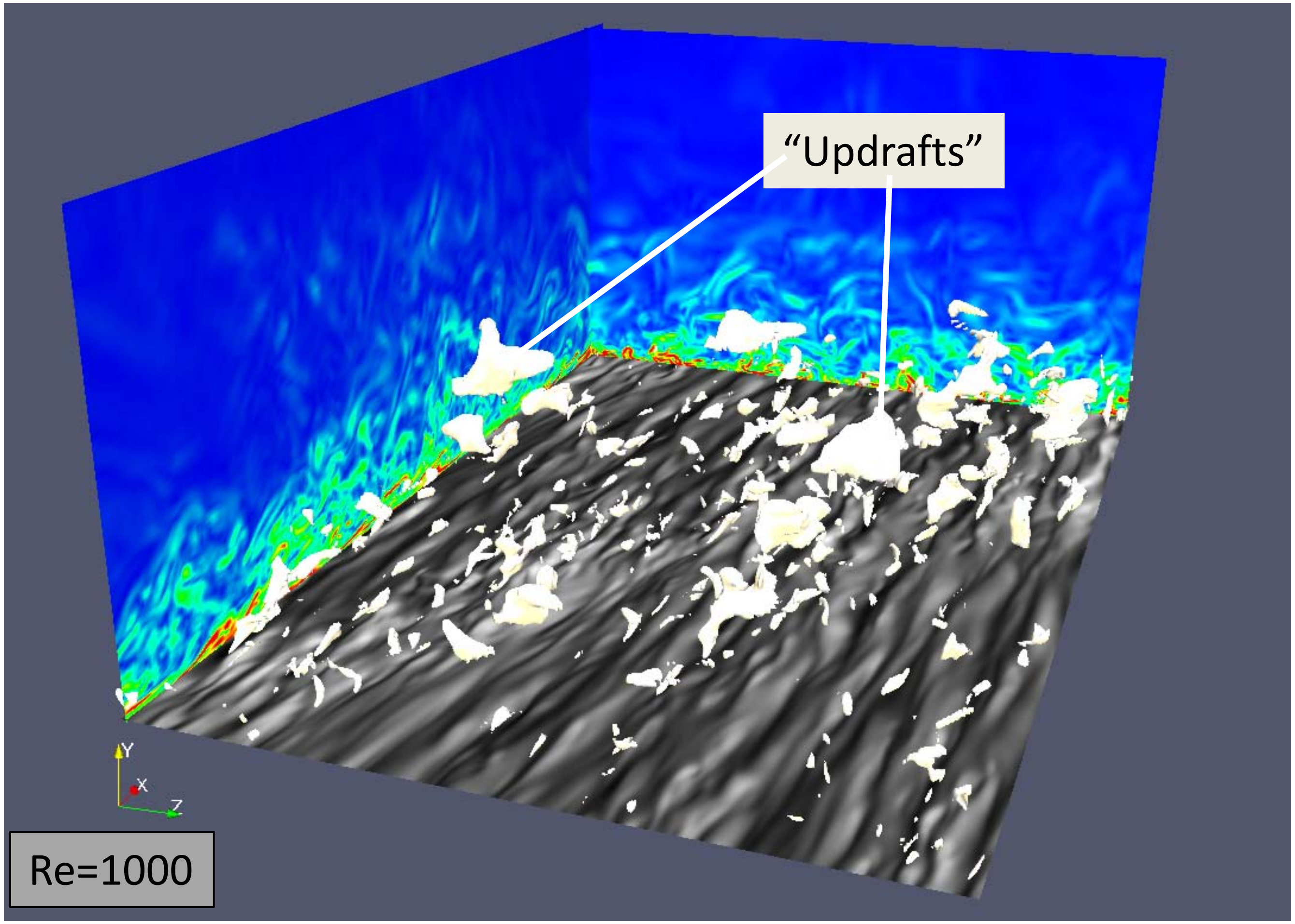
HEALTH WARNING: PRELINARY RESULTS FOLLOW!



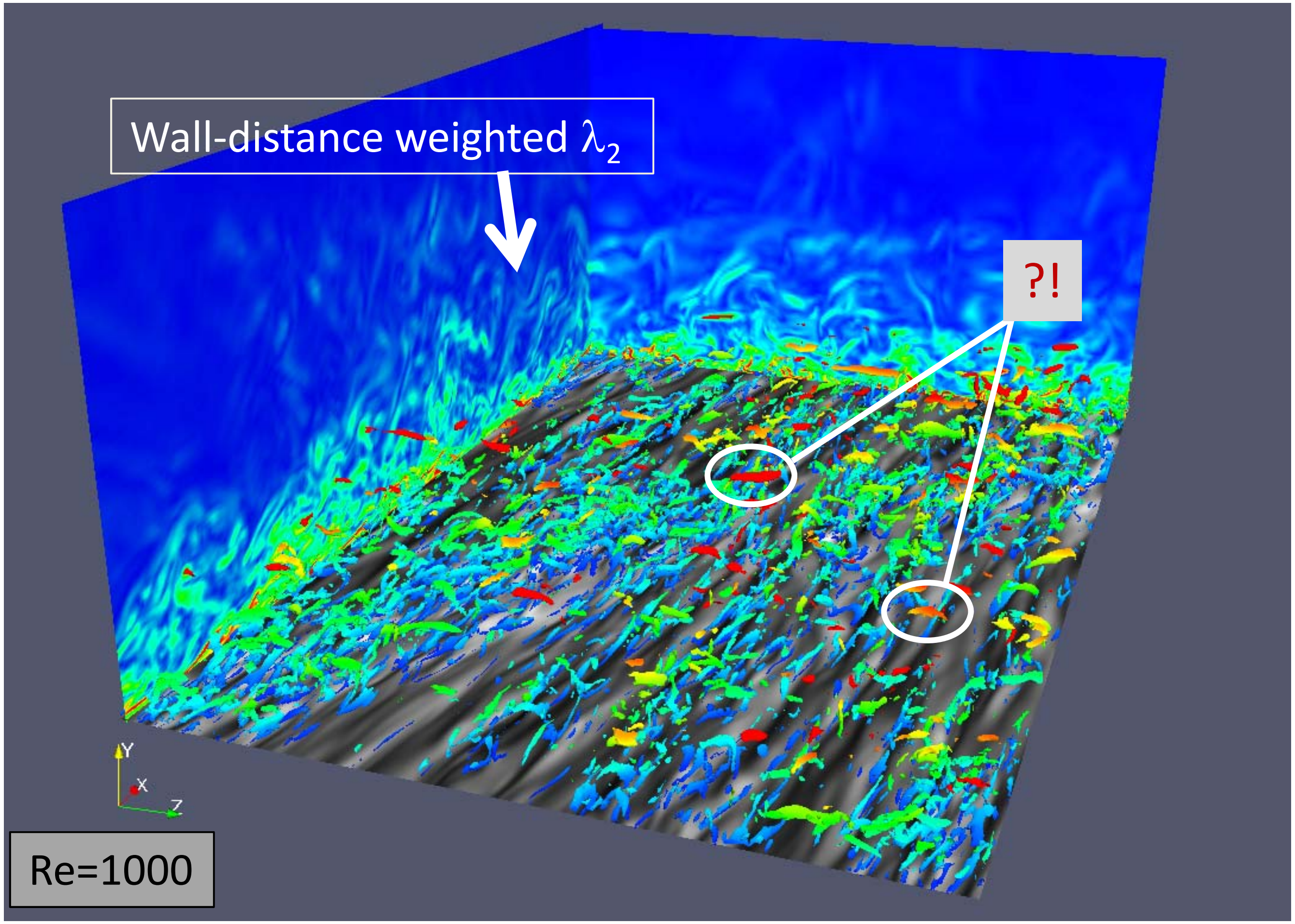
Turbulence structures?



Turbulence structures?



Turbulence structures?

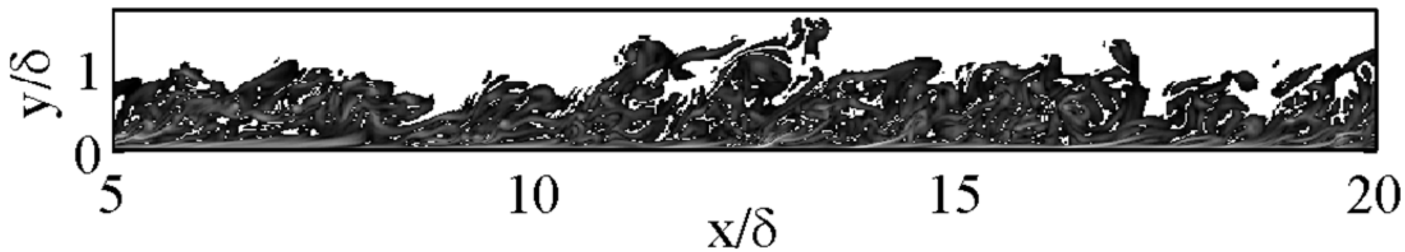


Contrast with channel, ZPG BL, etc? **TBC**…

Interfacial dynamics/intermittency…

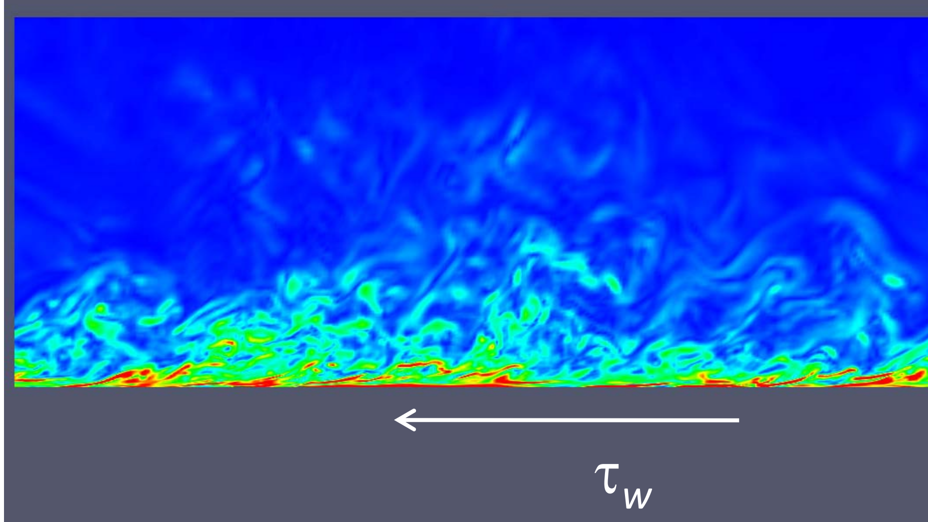
cf Jimenez et al (2010) ZPG TBL:

$|\omega'|$ contours:



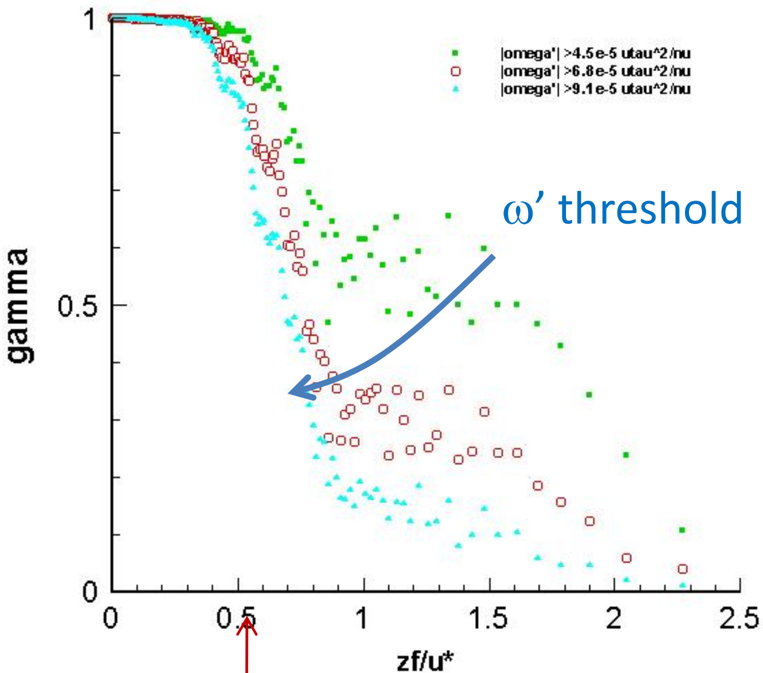
Re=1000 EBL:

$|\omega'|$ contours:



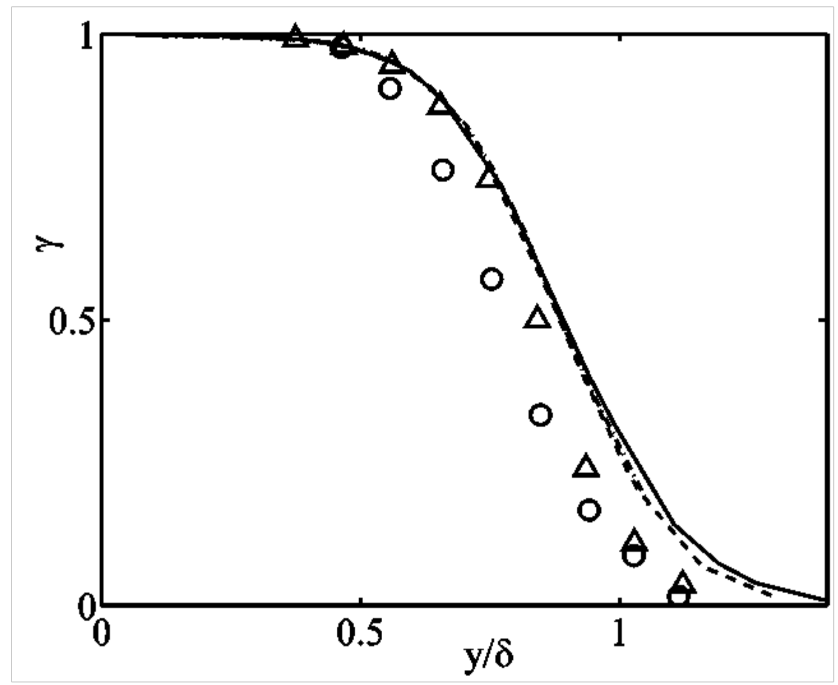
(Preliminary) Intermittency statistics

Re=2828

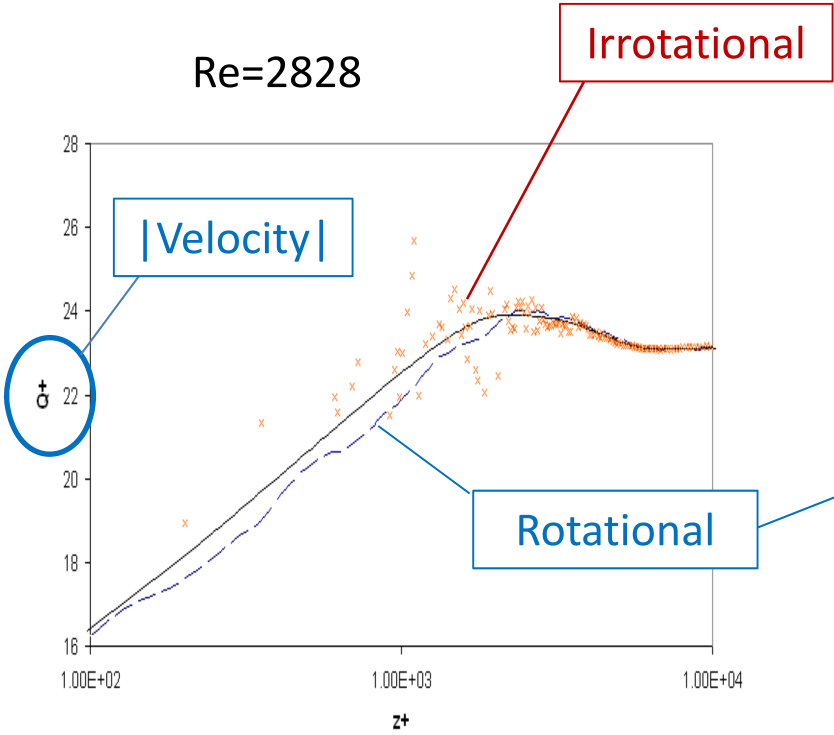


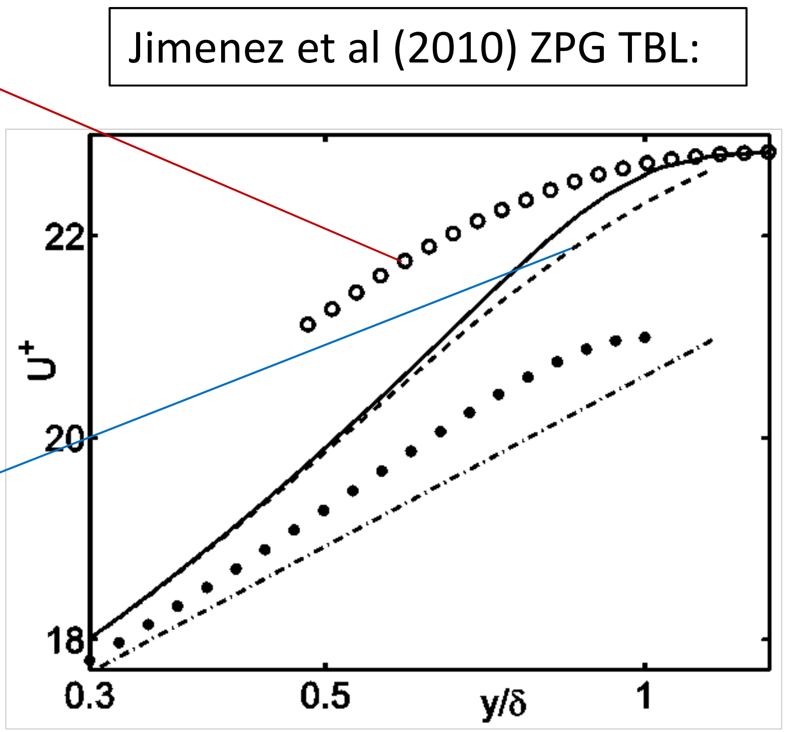
"$\delta_{\tau,95}$"

Jimenez et al (2010) ZPG TBL:



Streamwise velocity conditioned on |ω'|





(*TBC, with p' stats,* etc)…

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