# Energy cascade and spatial fluxes in filtered wall-turbulent flows

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# Subgrid energy transfer

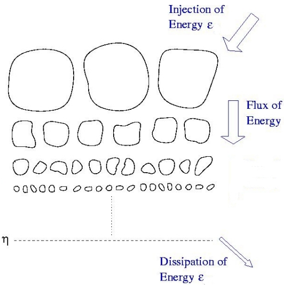
The energy transfer is one of the basic physical processes influencing the evolution of a turbulent field.

$\Longrightarrow$ The most important feature of the LES models should be their ability to reproduce accurately the energy transfer between resolved and subgrid scales.

Kolmogorov phenomenology: the energy flux through the inertial range is constant, from large to small scales and equal to the energy dissipation.

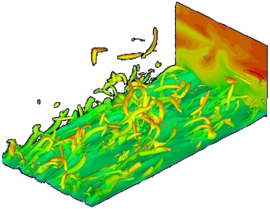
$$\frac{\langle(\delta u_{||}(x,r))^3\rangle}{r} = -\frac{4}{5}\epsilon$$

$\epsilon = \nu S_{ij} S_{ij}$: rate of energy dissipation



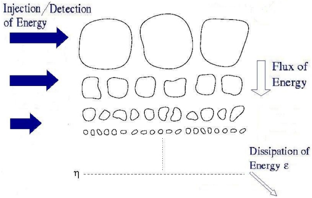

Universality: small-scale isotropy recover of any flow at sufficiently large Reynolds number

# Subgrid energy transfer in wall-turbulent flows



The picture of the Richardson energy cascade is modified by

- Anisotropy $\Longrightarrow$ turbulent energy production
- Inhomogeneity $\Longrightarrow$ spatial energy flux



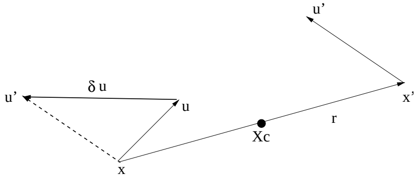
# The second-order structure function

The turbulent phenomena occurring simultaneously in physical and scale space can be studied with the second order-structure function.

$$\langle \delta u^2 \rangle (r_i, X_{ci})$$

where $\delta u^2 = \delta u_i \delta u_i$ and $\delta u_i = u_i(x_s + r_s) - u_i(x_s)$ is the fluctuating velocity increment.

- scale-energy at $r = \sqrt{r_s r_s}$
- function of the separation vector $r_i$
- function of the mid-point $X_{ci} = 1/2(x_i' + x_i)$



# The generalized Kolmogorov equation I

The balance equation of $\langle \delta u^2 \rangle$ for anisotropic inhomogeneous flows reads

$$\frac{\partial \langle \delta u^2 \delta u_i \rangle}{\partial r_i} + \frac{\partial \langle \delta u^2 \delta U \rangle}{\partial r_x} + 2 \langle \delta u \delta v \rangle \left( \frac{dU}{dy} \right)^* + \frac{\partial \langle v^* \delta u^2 \rangle}{\partial Y_c} =$$

$$-4 \langle \epsilon^* \rangle + 2\nu \frac{\partial^2 \langle \delta u^2 \rangle}{\partial r_i \partial r_i} - \frac{2}{\rho} \frac{\partial \langle \delta p \delta v \rangle}{\partial Y_c} + \frac{\nu}{2} \frac{\partial^2 \langle \delta u^2 \rangle}{\partial {Y_c}^2}$$

which can be written

$$\nabla_r \cdot \mathbf{\Phi}_r(\mathbf{r}, Y_c) + \frac{d\Phi_c(\mathbf{r}, Y_c)}{dY_c} = s(\mathbf{r}, Y_c)$$

Energy flux across scales + Spatial energy flux = Sink or source of energy

Hill JFM 468 (2002), Marati et al. JFM 521 (2004), Casciola et al. JFM 476 (2003)

# The generalized Kolmogorov equation II

The r-averaged form over a two-dimensional square domain

- highlight the energy processes through the spectrum of scales
- scale-by-scale budget as function of a single scale parameter $r$ and of the wall-distance $Y_c$.

$$\Pi_e(r, Y_c) + T_r(r, Y_c) = E_e(r, Y_c)$$

where

$$\Pi_e = (\Pi + T_c - P) \Rightarrow \left( 2\langle \delta u \delta v \rangle \left( \frac{dU}{dy} \right)^* + \frac{\partial \langle v^* \delta u^2 \rangle}{\partial Y_c} + \frac{2}{\rho} \frac{\partial \langle \delta p \delta v \rangle}{\partial Y_c} \right)$$

$$T_r \Rightarrow \frac{\partial \langle \delta u^2 \delta u_i \rangle}{\partial r_i}$$

$$E_e = (D_r + D_c + E) \Rightarrow \left( 2\nu \frac{\partial^2 \langle \delta u^2 \rangle}{\partial r_i \partial r_i} + \frac{\nu}{2} \frac{\partial^2 \langle \delta u^2 \rangle}{\partial {Y_c}^2} - 4\langle \epsilon^* \rangle \right)$$

# Homogeneity and isotropy recover in the small scales

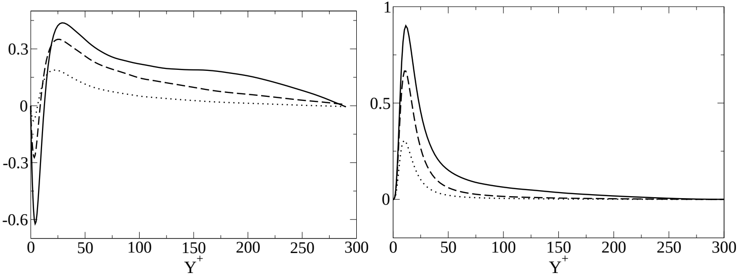
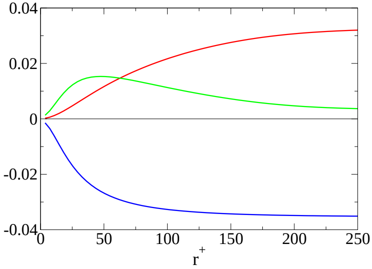



Figure: Left: spatial flux $\phi_c$. Right: production of turbulent fluctuations $\Pi$. $r^+ = 20$ (dotted line), $r^+ = 40$ (dashed line), $r^+ = 300$ (solid line).

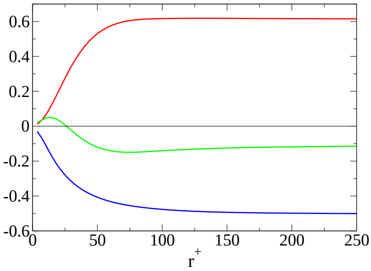
- The Kolmogorov phenomenology can be recovered only in the bulk region of the flow
- Near the wall production and spatial energy flux cannot be neglected also in the small scales

# Energy cascade and spatial fluxes in wall-turbulent flows

- In the bulk and logarithmic regions the large-scale production range is followed by an inertial transfer range which is closed by dissipation at local dissipative scales.
- In the buffer the inertial transfer reveals a classical energy cascade at small scales and an inverse cascade at large scales.




(a) $y^+ = 160$

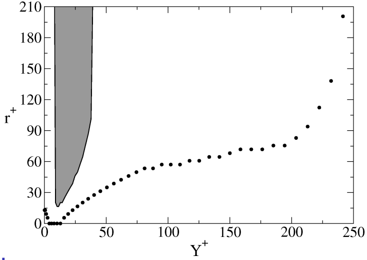



(b) $y^+ = 20$

Figure: $\Pi_e + T_r = E_e$ (see Marati et al. JFM 521 (2004))

# Cross-over scale $l_c$ and reverse energy cascade region $\Omega_B$

- $l_c(Y)$: splits the range of scales where the classical energy cascade is recovered from a production dominated range at large scales $(\Pi(l_c, Y) = T_r(l_c, Y))$.
- $\Omega_B(r, Y)$, $l_B(Y)$: at the wall production and spatial fluxes strongly modify the energy cascade up to lead a reverse energy cascade region $(T_r(r > l_B, Y) < 0)$.



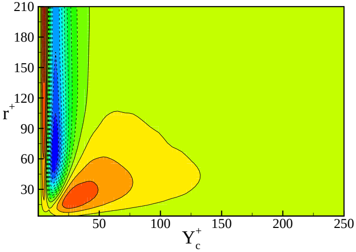




Figure: Left: $l_c(Y)$ (circle) and the reverse cascade region $\Omega_B(r, Y)$ (grey region). Right: $-T_r(r, Y)$.

# The Kolmogorov equation for filtered velocity field

$$\frac{\partial \langle \delta \bar{u}^2 \delta u_i \rangle}{\partial r_i} + \frac{\partial \langle \delta \bar{u}^2 \delta U \rangle}{\partial r_x} + 2 \langle \delta \bar{u} \delta \bar{v} \rangle \left( \frac{dU}{dy} \right)^* + \frac{\partial \langle \bar{v}^* \delta \bar{u}^2 \rangle}{\partial Y_c} = -4 \langle \bar{\epsilon}^* \rangle$$

$$+2\nu \frac{\partial \langle \delta \bar{u}^2 \rangle}{\partial r_i \partial r_i} - \frac{2}{\rho} \frac{\partial \langle \delta \bar{p} \delta \bar{v} \rangle}{\partial Y_c} + \frac{\nu}{2} \frac{\partial^2 \langle \delta \bar{u}^2 \rangle}{\partial {Y_c}^2} - 4 \langle \epsilon^*_{sgs} \rangle - 4 \frac{\partial \langle \tau^*_{ij} \delta \bar{u}_i \rangle}{\partial r_j} - 4 \frac{\partial \langle \delta \tau_{i2} \delta \bar{u}_i \rangle}{\partial Y_c}$$

r-averaged form

$$\left( \bar{\Pi}_e + T_c^{sgs} \right) + \left( \bar{T}_r + T_r^{sgs} \right) = \left( \bar{E}_e + E_{sgs} \right)$$

Subgrid stresses effects

$$T_c^{sgs} \Rightarrow 4 \frac{\partial \langle \delta \tau_{i2} \delta \bar{u}_i \rangle}{\partial Y_c} \; ; \; T_r^{sgs} \Rightarrow 4 \frac{\partial \langle \tau^*_{ij} \delta \bar{u}_i \rangle}{\partial r_j} \; ; \; E_{sgs} \Rightarrow -4 \langle \epsilon^*_{sgs} \rangle$$

# DNS data set

DNS data of a turbulent channel flow.

| $Re$ | $Re_\tau$ | $L_x$ | $L_y$ | $L_z$ | $\Delta x^+$ | $\Delta z^+$ |
|---|---|---|---|---|---|---|
| 10000 | 297 | $2\pi$ | 2 | $\pi$ | 3.64 | 3.64 |

- Pseudo-spectral code;
- High resolution;

Sharp spectral cutoff filter

- in the homogeneous directions;
- for filter scales equal in the homogeneous directions ($l^+_{F0} = 20$, $l^+_{F1} = 30$ and $l^+_{F2} = 60$);

$\Rightarrow$ Analysis of the scale-energy processes of a filtered field as function of the single filter parameter $l_F$ compared with the single scale parameter $l_c$ and $l_B$ for different wall-distances $Y_c$.

# Filtered turbulent kinetic energy budget

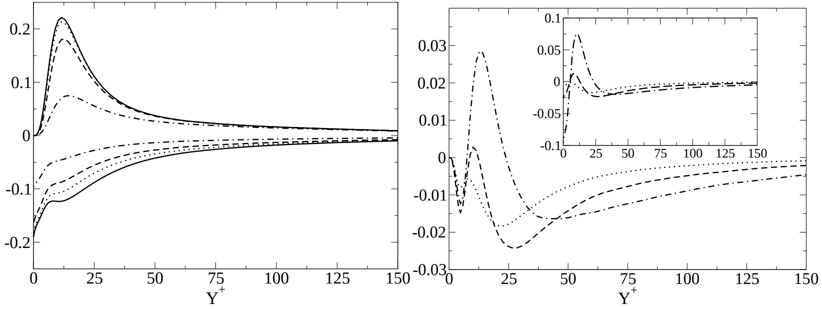
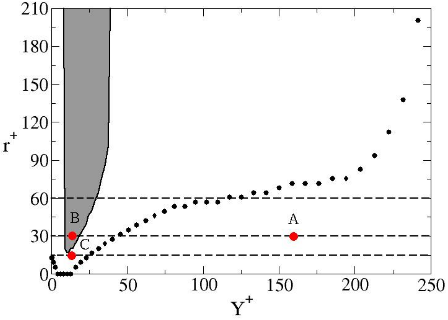



Figure: Left: $\bar{P} = \langle \bar{u}\bar{v} \rangle dU/dy$ vs $\bar{\epsilon}$. Right: $E_{sgs}$. Inset: $\left(P - \bar{P}\right) - (\epsilon - \bar{\epsilon})$. DNS (solid line), $I_{F0}$ (dotted line) $I_{F1}$ (dashed line) and $I_{F2}$ (dashed-dotted line).

- Subgrid dissipation $\Rightarrow$ Viscous dissipation in the subgrid scales;
- Backward energy transfer $\Rightarrow$ Excess of scale-energy in the subgrid scales;

# Analysis of the filtered scale-by-scale budget



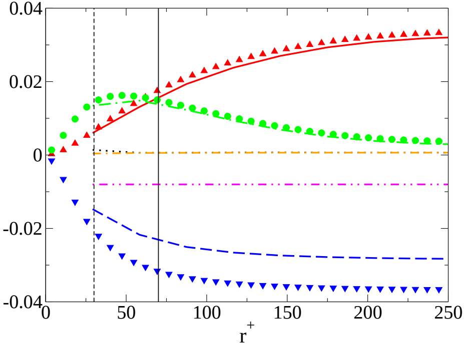

- A: log-layer, $l_F < l_c$
- B: buffer layer, $l_F > l_B$
- C: buffer layer, $l_F < l_B$

# Scale-by-scale budget in the log-layer

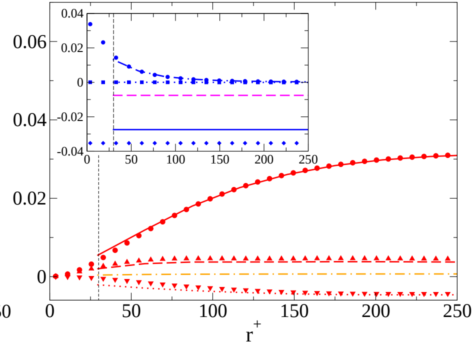
A: Logarthmic region, $l_F < l_c$ (see Gualtieri et al JFM 592 (2007))

$\Longrightarrow$ The main energy processes are resolved: $\bar{\Pi} \approx \Pi$, $\bar{E} + E_{sgs} \approx E$

$\Longrightarrow$ $T_r^{sgs}$ and $T_c^{sgs}$ are negligbles




$\left(\bar{\Pi}_e + T_c^{sgs}\right) + \left(\bar{T}_r + T_r^{sgs}\right) = \left(\bar{E}_e + E_{sgs}\right)$




$\left(\bar{\Pi}_e + T_c^{sgs}\right)$ and $\left(\bar{E}_e + E_{sgs}\right)$ (inset)

# Scale-by-scale budget in the buffer layer I

In the wall region:

- $l_c \to \eta$ and $l_F > l_c$;
- the resolved field is largely affected by the position of $l_F$ with respect $\Omega_B$;

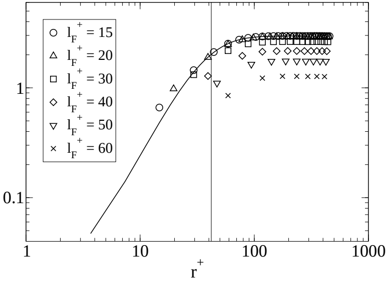



Figure: Mixed structure function: $\bar{S}_{12} = \langle \delta\bar{u}\delta\bar{v} \rangle$ for different filter scale at $Y^+ = 25$.

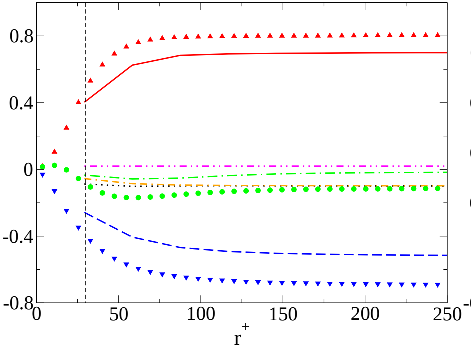
# Scale-by-scale budget in the buffer layer II

B: Buffer layer, $l_F > l_B$

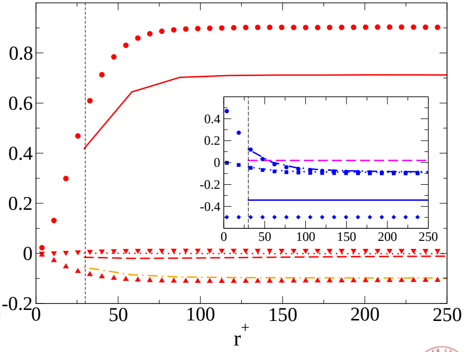
$\Longrightarrow$ The main energy processes act in the subgrid scales, $\bar{\Pi} \ll \Pi$, $\bar{E} + E_{sgs} \ll E$

$\Longrightarrow$ $T_r^{sgs}$ and $T_c^{sgs}$ cannot be neglected

$\Longrightarrow$ The subgrid scales feeds the resolved ones




$\left(\bar{\Pi}_e + T_c^{sgs}\right) + \left(\bar{T}_r + T_r^{sgs}\right) = \left(\bar{E}_e + E_{sgs}\right)$




$\left(\bar{\Pi}_e + T_c^{sgs}\right)$ and $\left(\bar{E}_e + E_{sgs}\right)$ (inset)

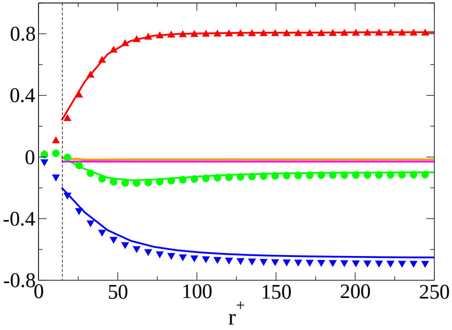
# Scale-by-scale budget in the buffer layer III

C: Buffer layer, $l_F < l_B$

$\Longrightarrow$ The main energy processes are resolved, $\bar{\Pi} \approx \Pi$

$\Longrightarrow$ $T_r^{sgs}$ and $T_c^{sgs}$ are negligeble

$\Longrightarrow$ The subgrid scales drain resolved energy




$$\left(\bar{\Pi}_e + T_c^{sgs}\right) + \left(\bar{T}_r + T_r^{sgs}\right) = \left(\bar{E}_e + E_{sgs}\right)$$

# Resolved spatial flux I

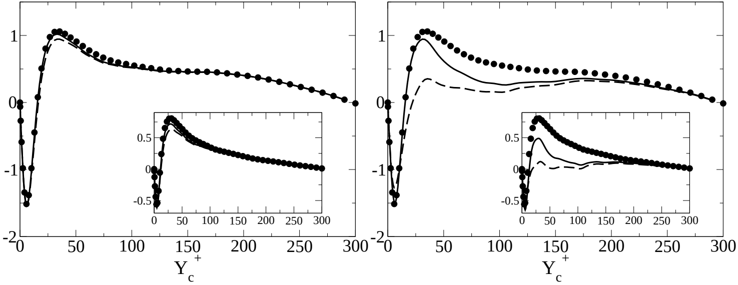



Figure: Y-behaviour of the spatial flux for large scales $r^+ = 310$ and small scales $r^+ = 30$ (inset plots). The unfiltered spatial flux $\Phi_c$ (circle), the resolved spatial flux $\bar{\Phi}_c$ (dashed line) and the resolved and subgrid spatial flux $\bar{\Phi}_{c+sgs}$ (solid line). (a) $l_F^+ = 15$ and (b) $l_F^+ = 30$.

$\Rightarrow$ The amount of resolved energy which leaves the buffer layer to feed the viscous and the core flow recover the unfiltered behaviour only for $l_F \notin \Omega_B$.

# Resolved spatial flux II

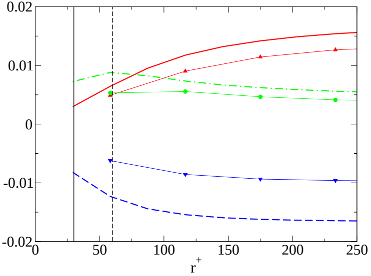



Figure: $(\bar{\Pi}_e + T_c^{sgs}) + (\bar{T}_r + T_r^{sgs}) = (\bar{E}_e + E_{sgs})$. $l_F = 30$ (line) and $l_F = 60$ (symbols).

$\Rightarrow$ Even if the local main processes are resolved, $l_F < l_c$, a depletion of the resolved physics occurs if $l_F \in \Omega_B$ in the buffer layer.

# The scale $l_B$

In the buffer layer the scale $l_B$ marks an intermediate range of scales which loses energy to feed both larger (reverse cascade) and smaller (forward cascade) scales in the same physical location and other range of scales of the adjacent regions of the flow via spatial flux.

If $l_F > l_B$:

- the subgrid stresses $T_r^{sgs}$ and $T_c^{sgs}$ account for a significant part of the energy cascade and spatial fluxes in the resolved scales;
- a large energy-excess in the subgrid scales due to turbulent production feeds the resolved motion, $E_{sgs} > 0$;

# Conclusions

A scale by scale budget of a filtered wall turbulent flow has been performed singling out the prominent role of the filter scale compaired to $l_c$ and $\Omega_B$

- In the bulk the condition $l_F < l_c$ allows to resolve the main physical processes. Recover of the Kolmogorov framework.
- In the buffer layer anisotropy and inhomogeneity strongly modify the energy cascade leading a reverse energy cascade region $\Omega_B$ which is correlated with turbulent coherent structures and then with production and spatial energy flux.

Perspectives

- The present tool appears to be appropriate to *a priori/a posteriori* test of the abilty of LES models to capture the relevant processes occurring both in physical and scale space.