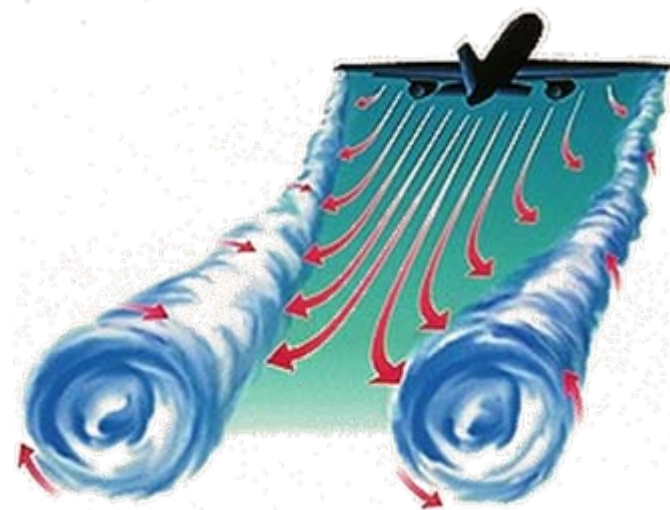


CS-Turbulence Interaction

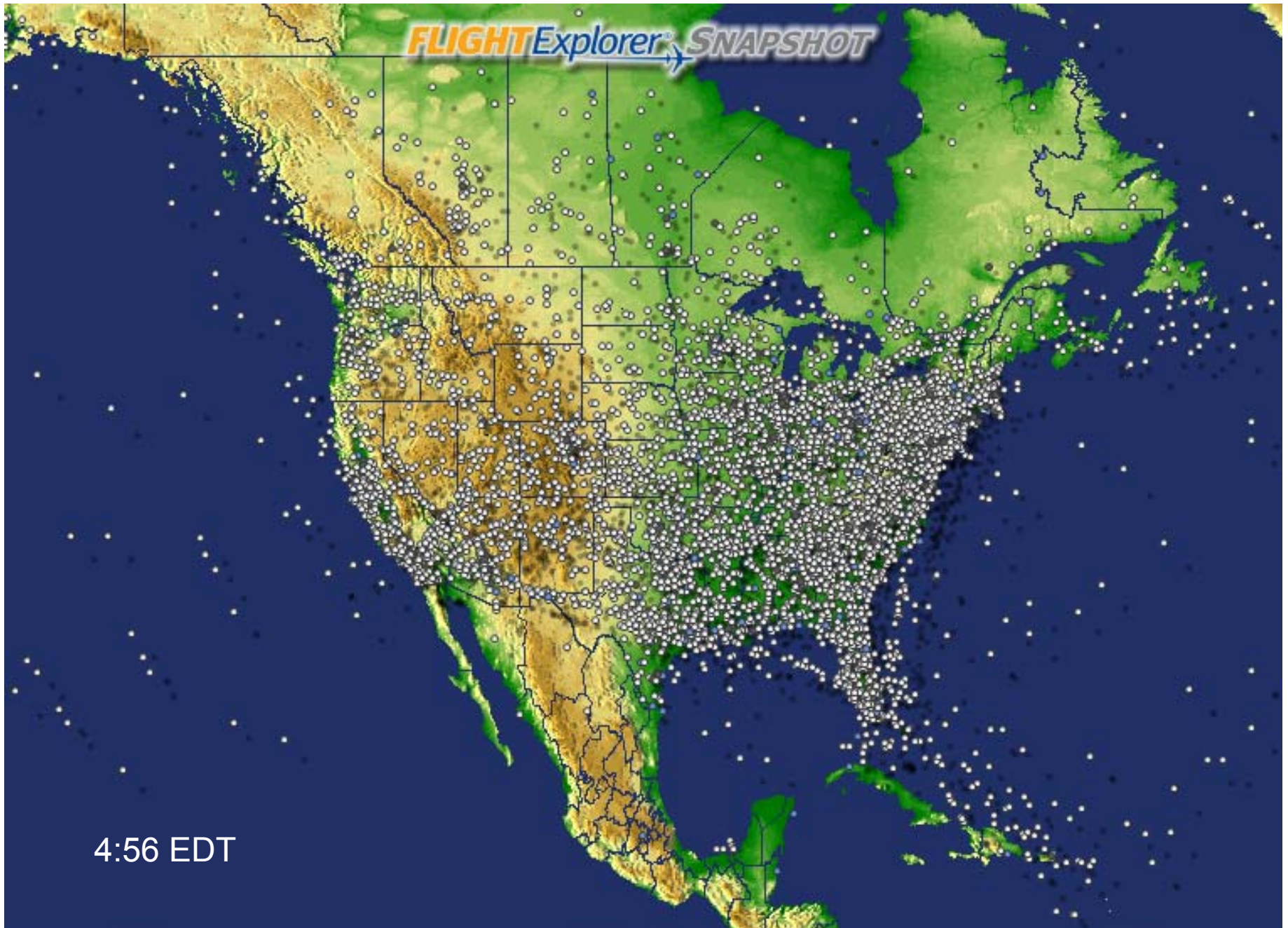
Fazle Hussain, D. S. Pradeep & Eric Stout

University of Houston





FLIGHT Explorer *SNAPSHOT*



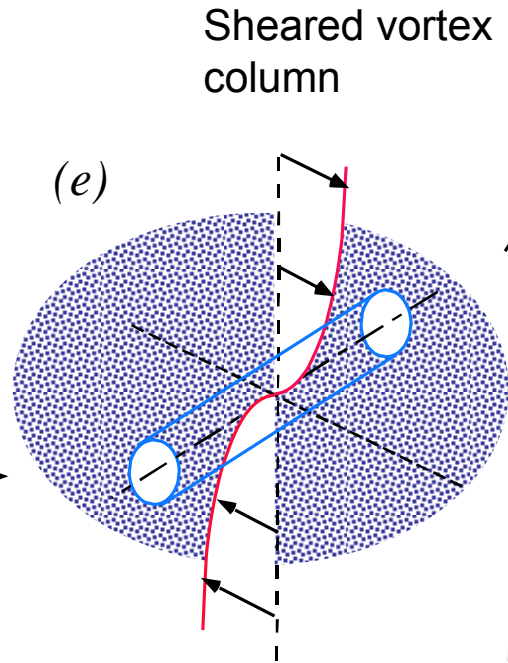
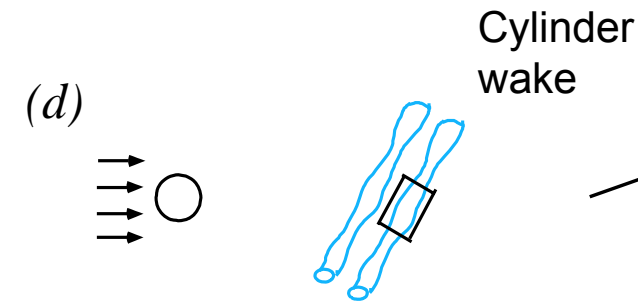
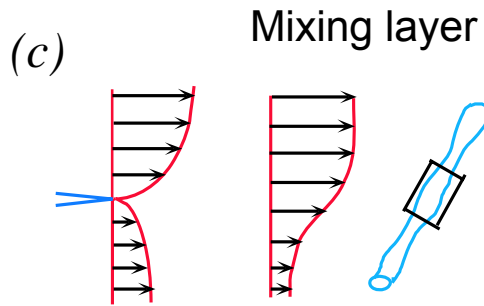
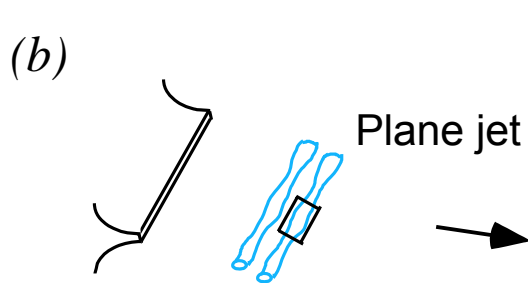
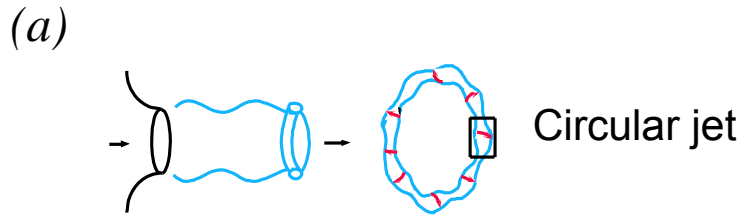
4:56 EDT



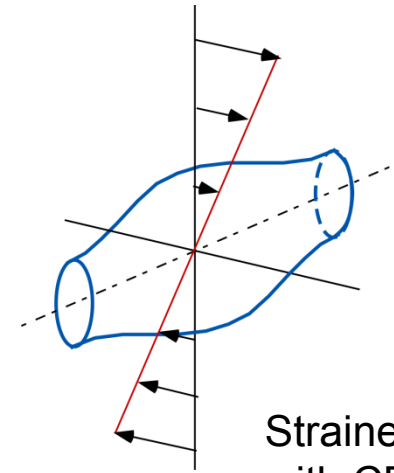
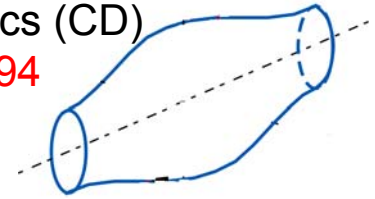
Trailing vortex:

A simple example of **coherent structure**

Motivation:

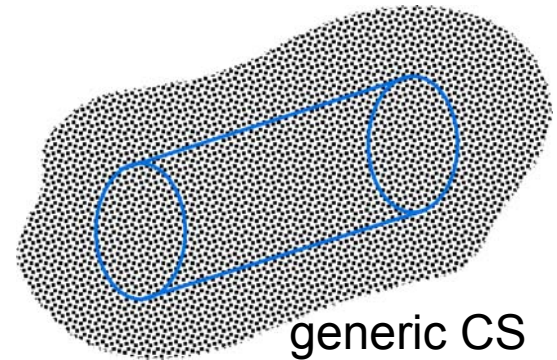


Core Dynamics (CD)
Fl.Dyn.Res. '94



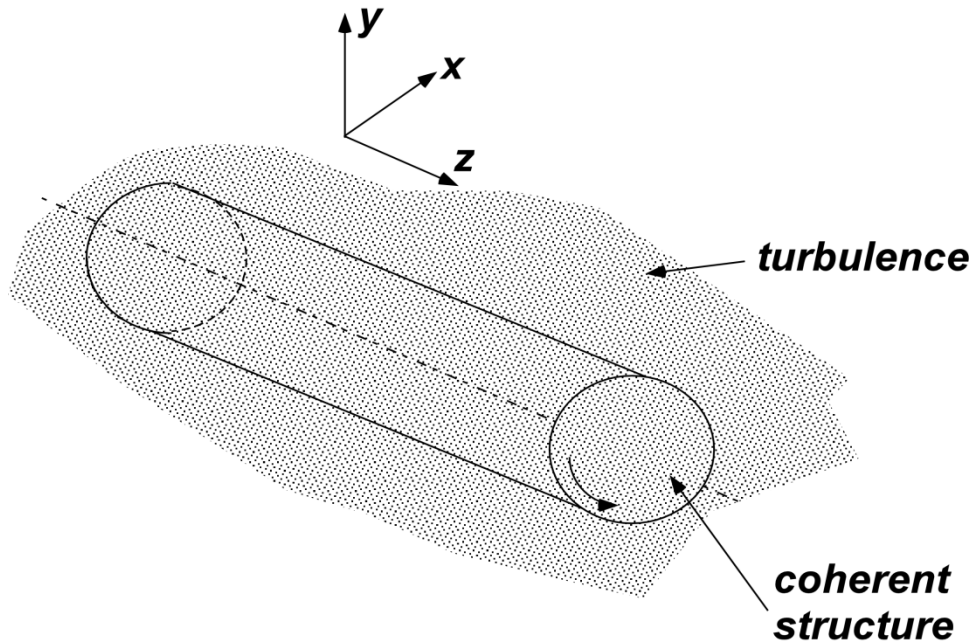
Strained vortex
with CD
JFM '01

vortex column in
turbulence
PRE '93
JFM '06



generic CS

CS-turbulence interaction: Idealized flow



DNS

$Re \equiv \Gamma/\nu$

1k – 20k

Oseen-Lamb
vortex

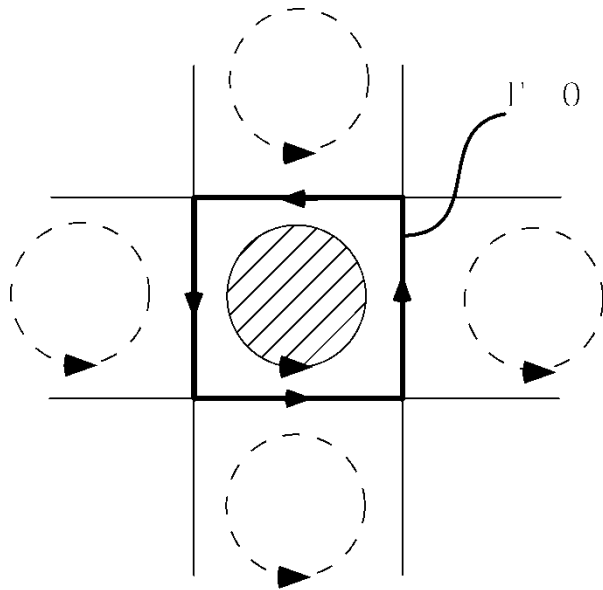
Idealizations:

- No interaction with other CS
- No background shear
- Rectilinear, cylindrical CS
- Random, fine-scale fluctuations

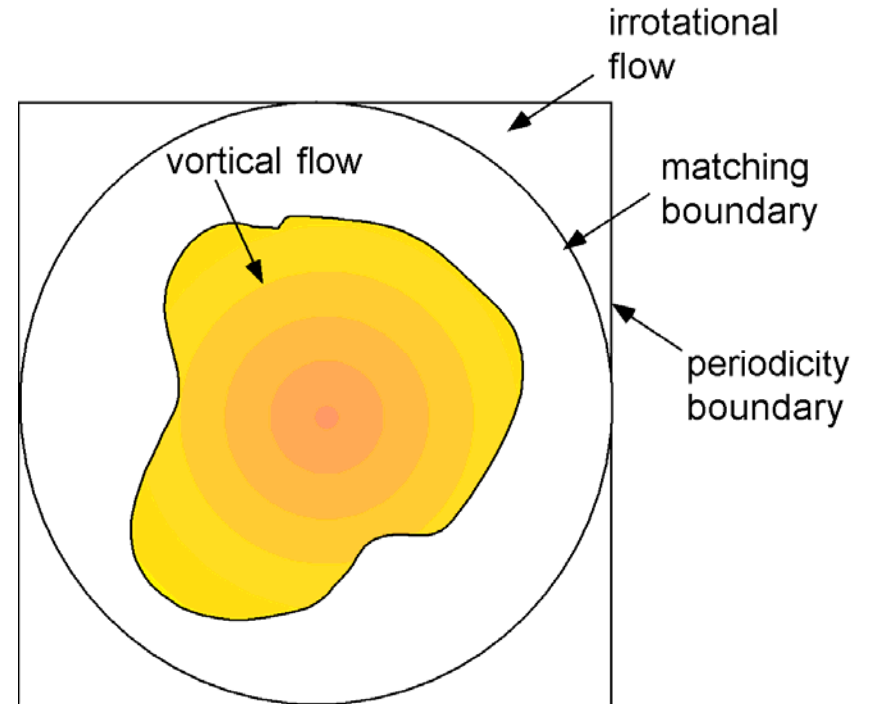
no pairing or reconnection
no elliptic instability
no self-induced motion
homog., isotrop. k – sep.

Flow evolution using DNS initialized with 3-D vort. from lin. analysis
Pseudo-spectral method (Rennich & Lele '97; Pradeep & Hussain '04)
periodic in z , pot. flow @ $r \rightarrow \infty$

Numerical simulation method:



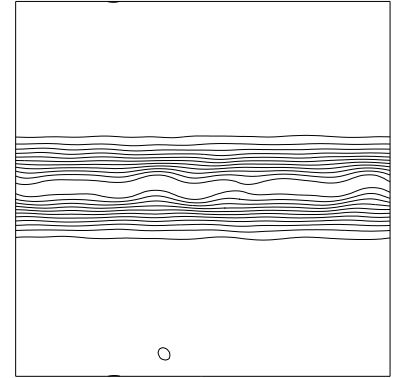
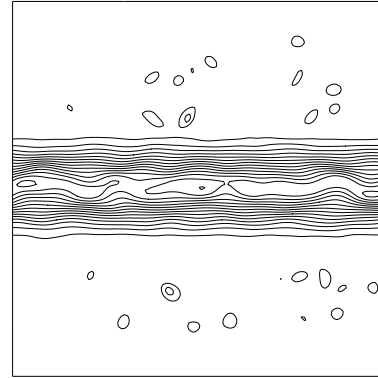
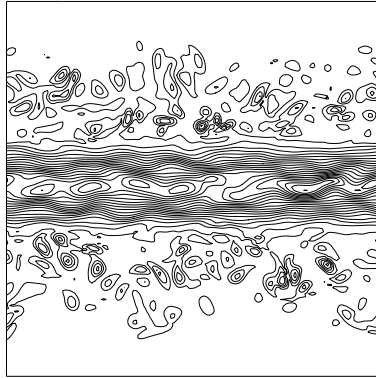
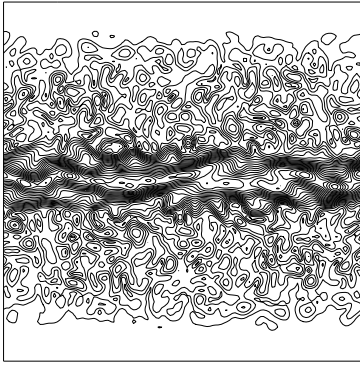
Triply-periodic



“unbounded” flow
(Rennich-Lele '97)
Pradeep & H. '04

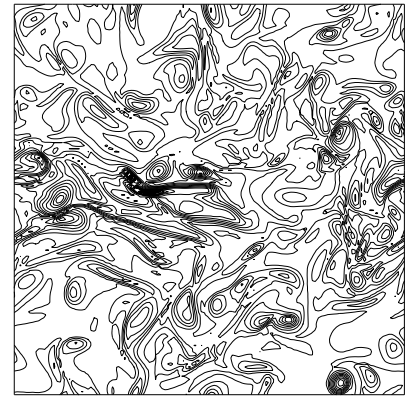
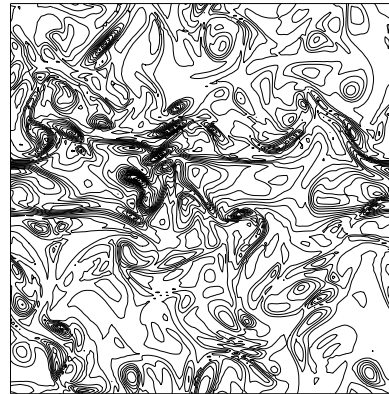
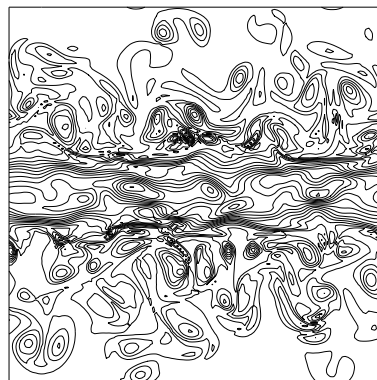
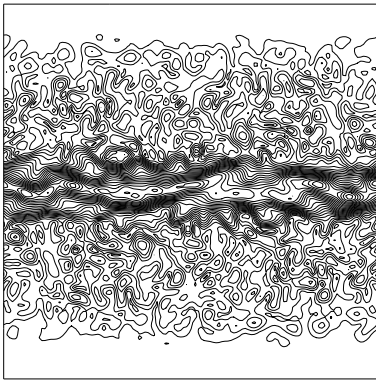
Comparison:

Unbounded BC



→ time

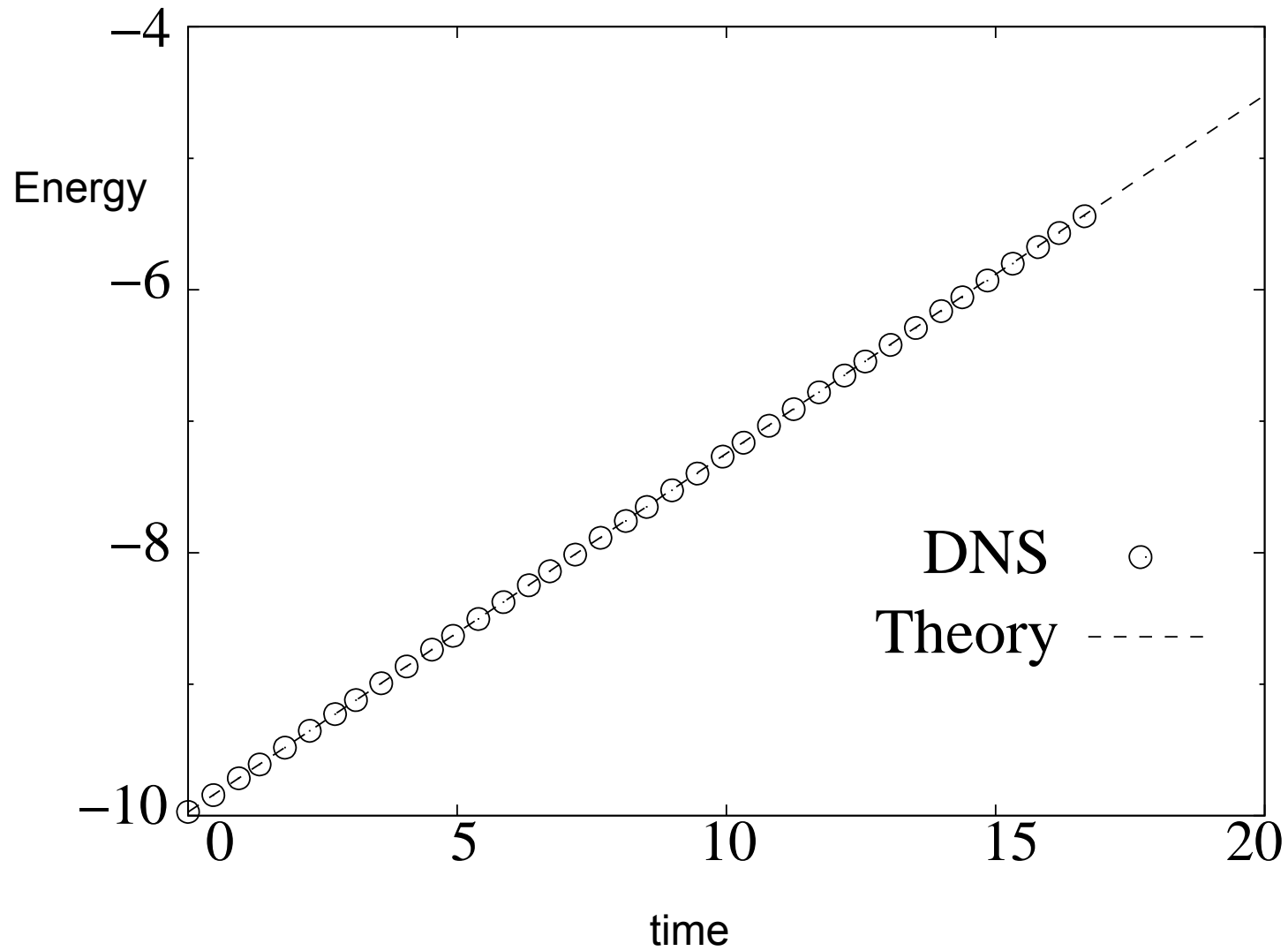
Periodic BC



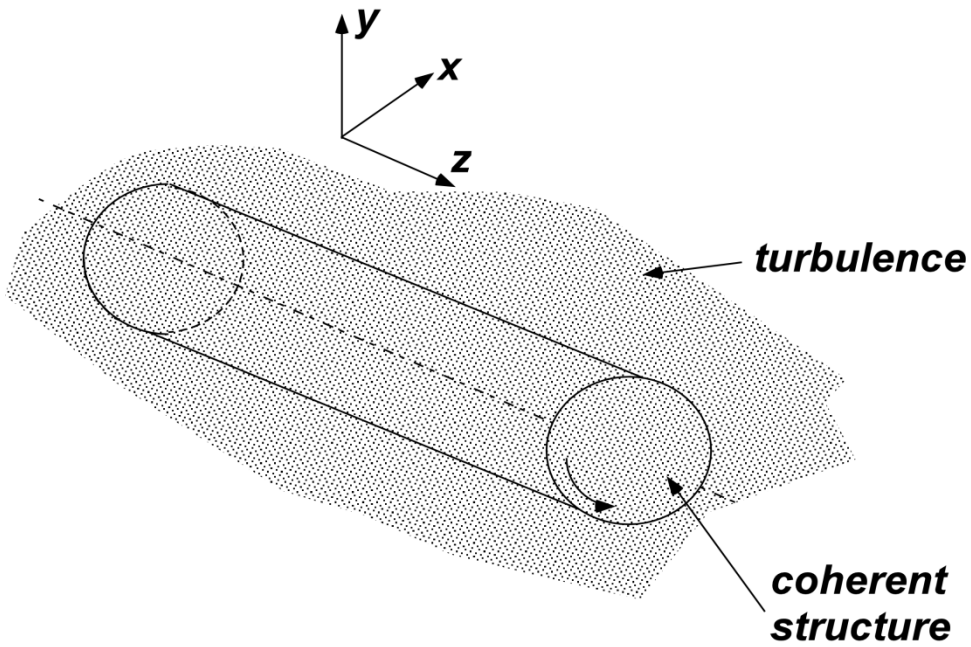
Growth of q-vortex instability mode:

Bending wave

Re = 10^5 (DNS)



Vortex-Turbulence Interaction

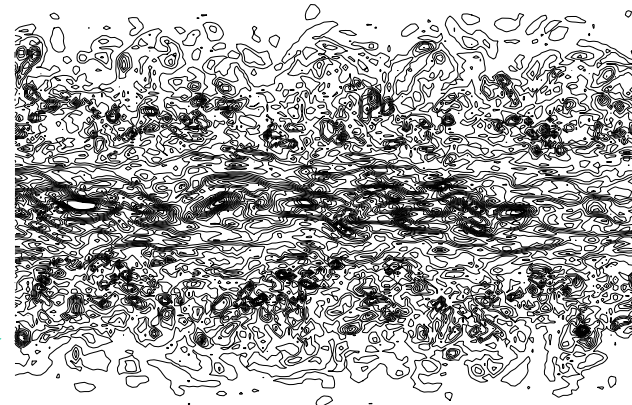
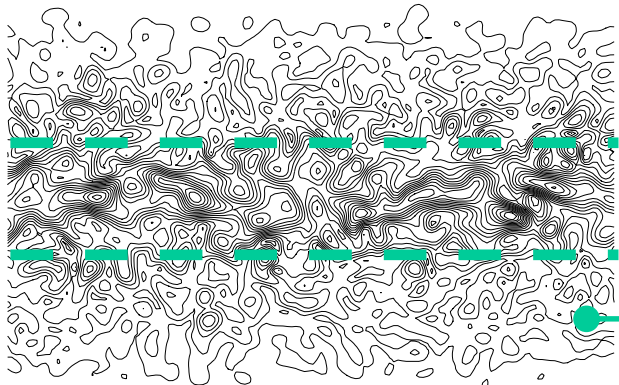


Organization of turbulence:

meridional plane $|\omega|$ contours

$\Gamma/\nu = 12.5k$

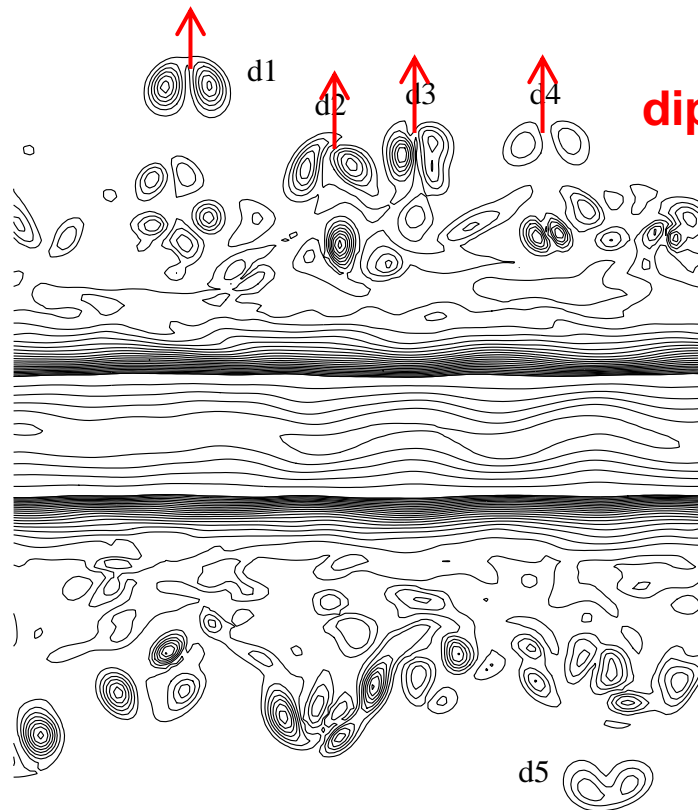
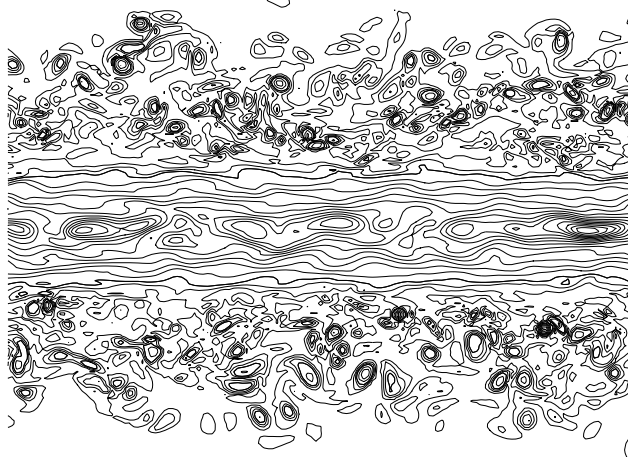
$T = 0$



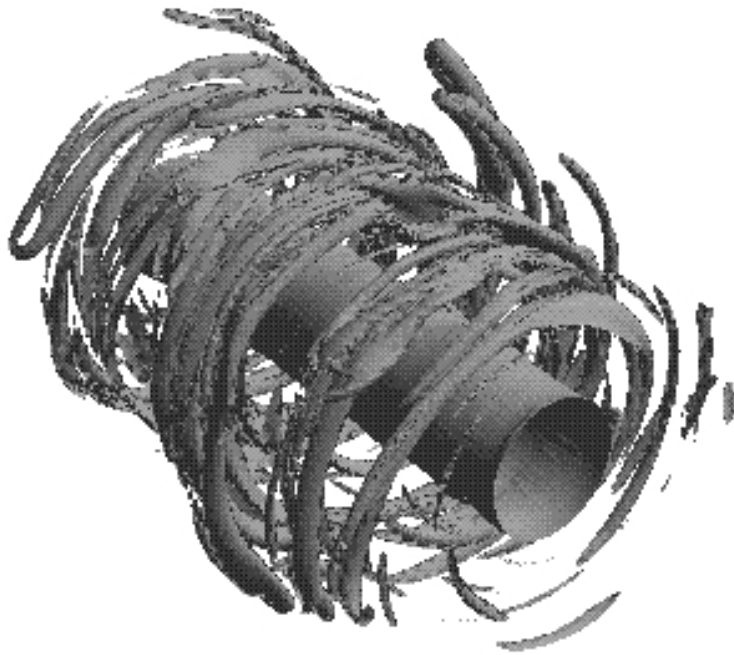
$T = 10$

LS

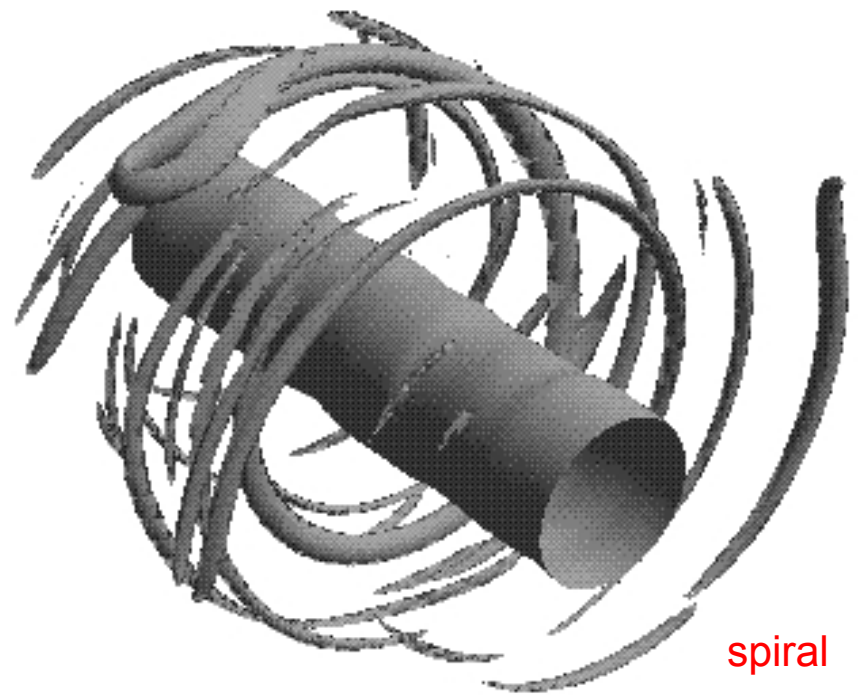
$T = 30$



$T = 120$

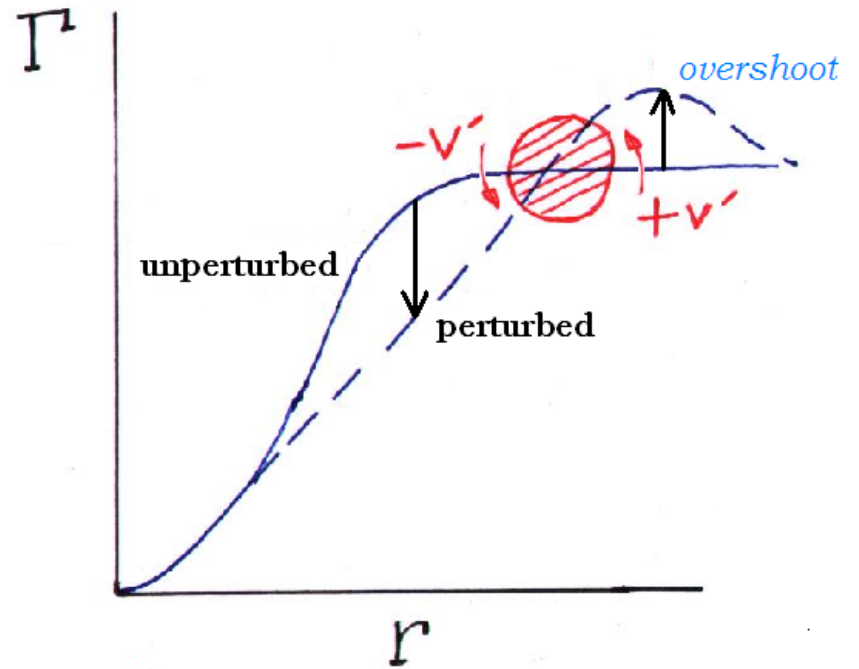
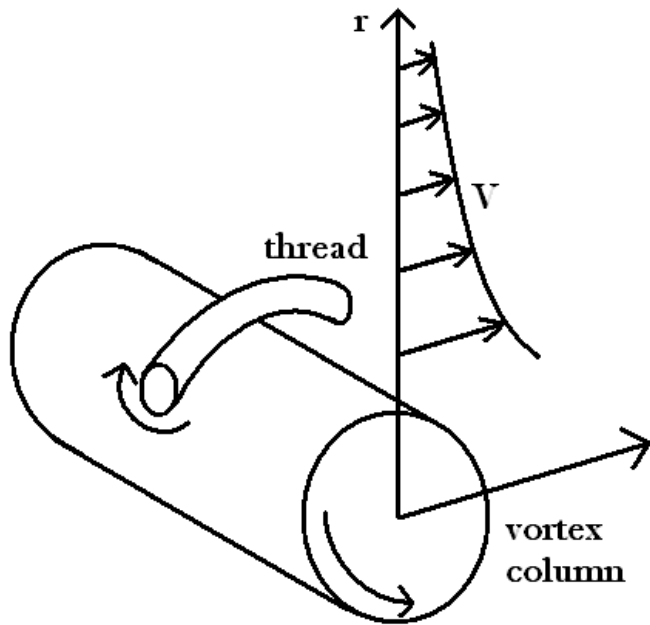


dipoles



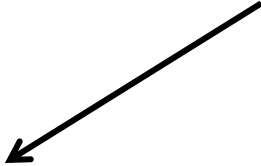
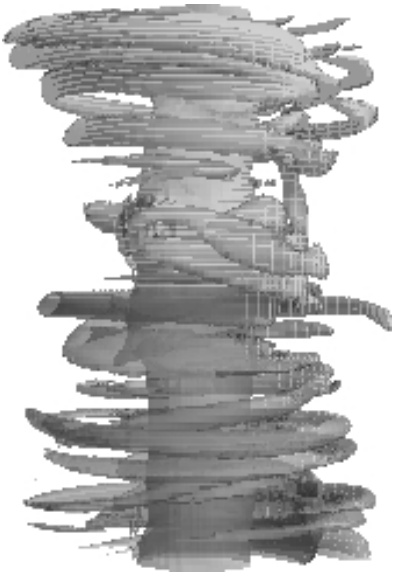
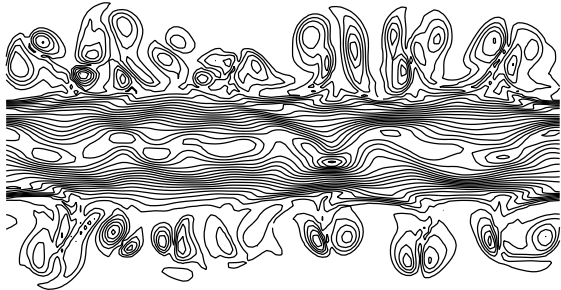
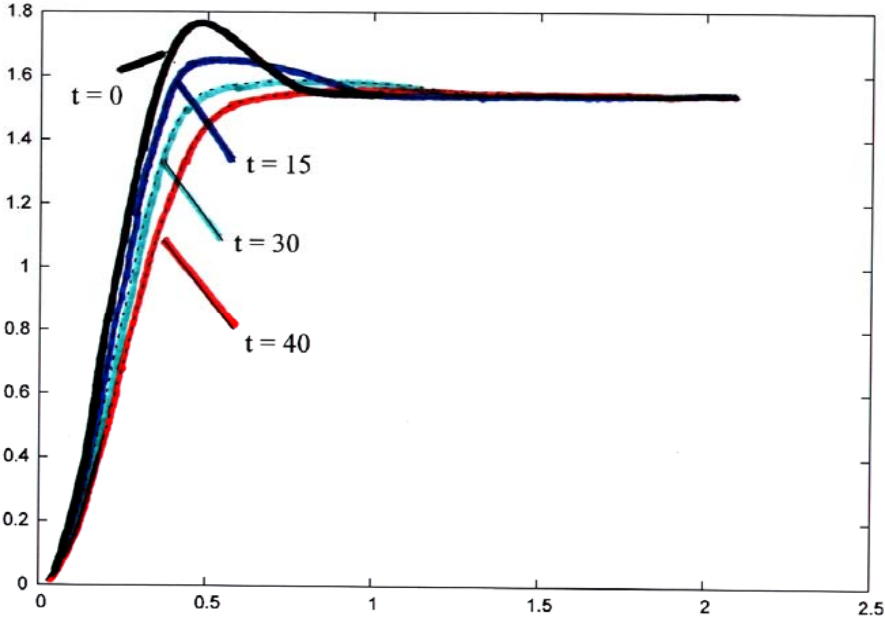
spiral
threads

Transport effect of *threads*:

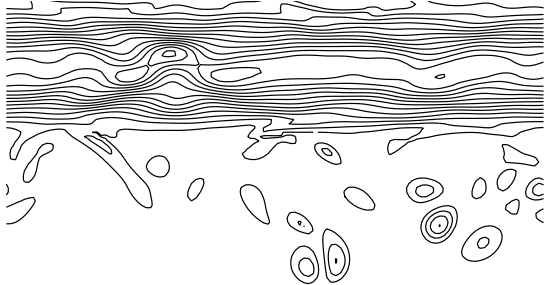


centrifugal instability

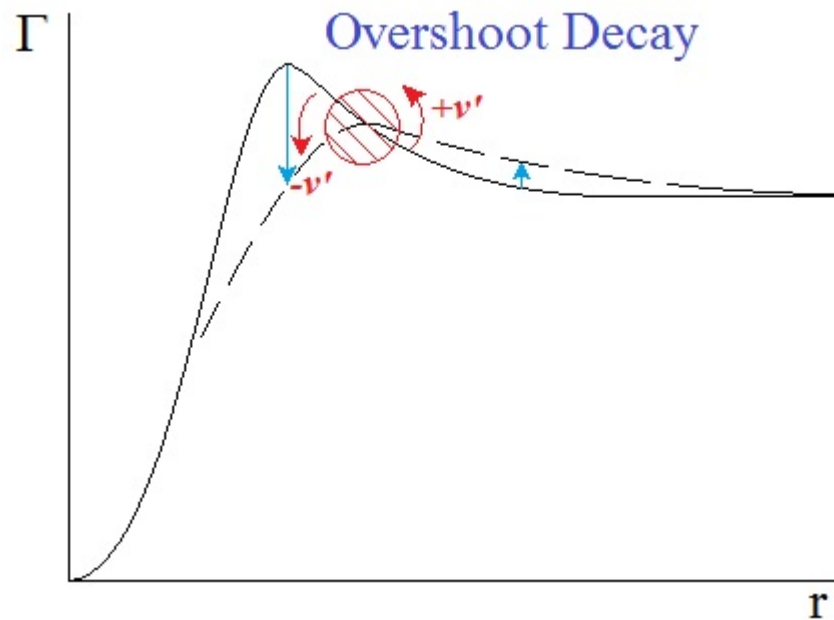
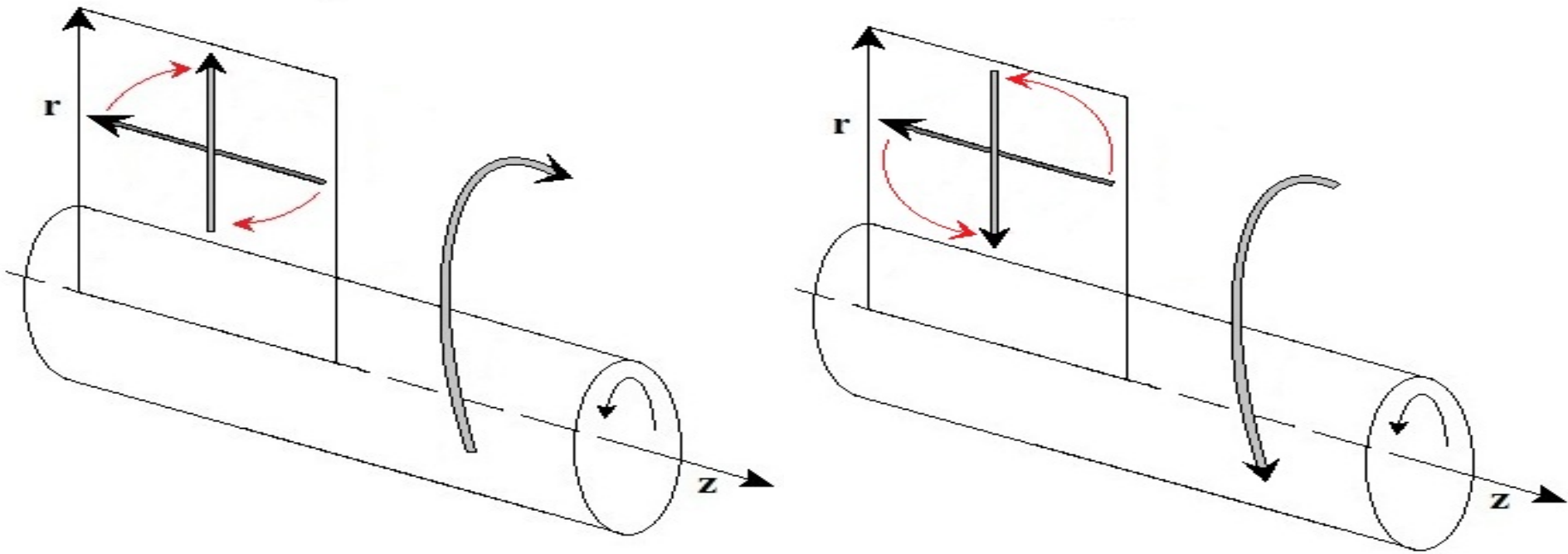
$Re = 5000$

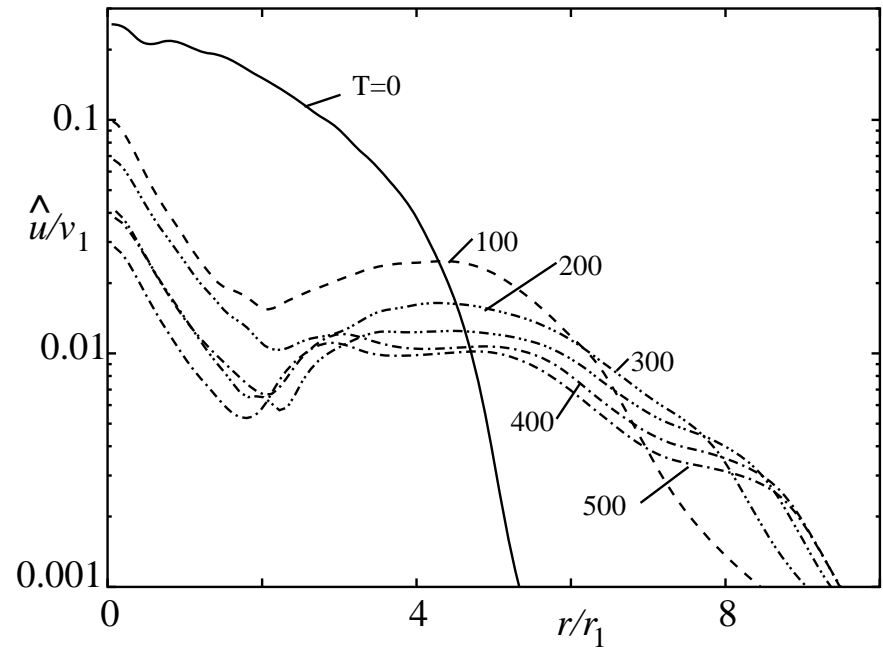
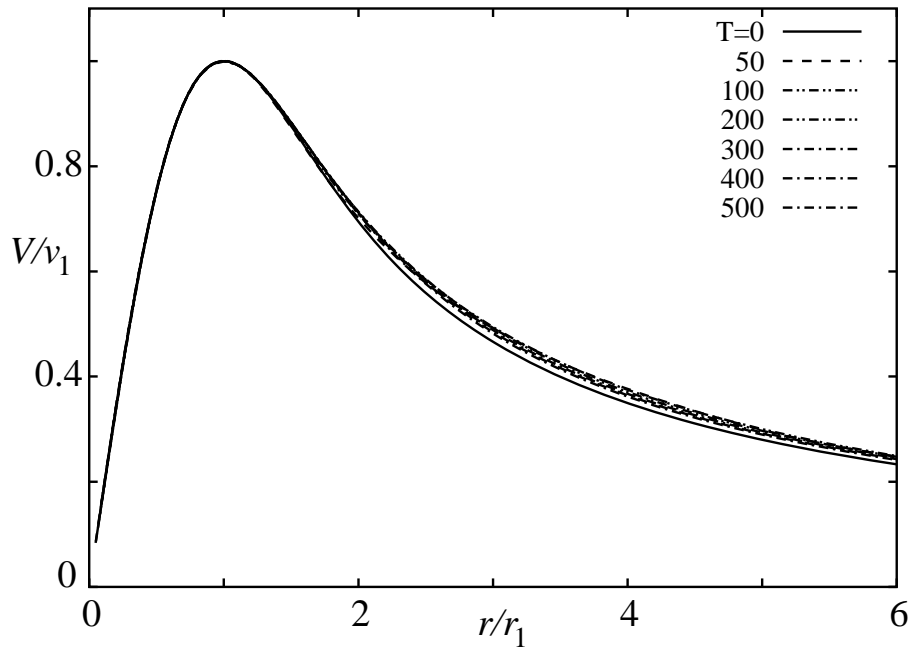


Self-limiting



Instability Driven Intensification:



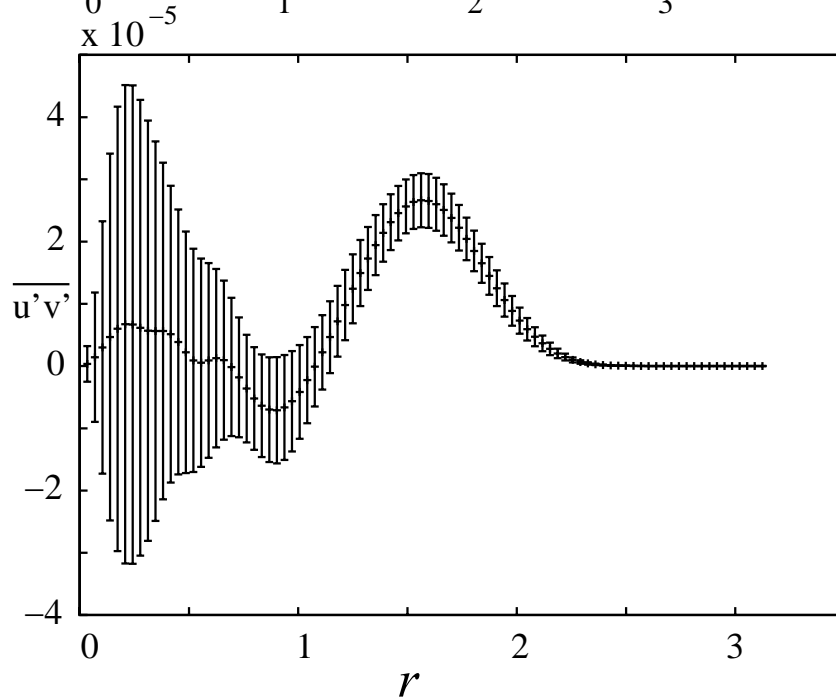
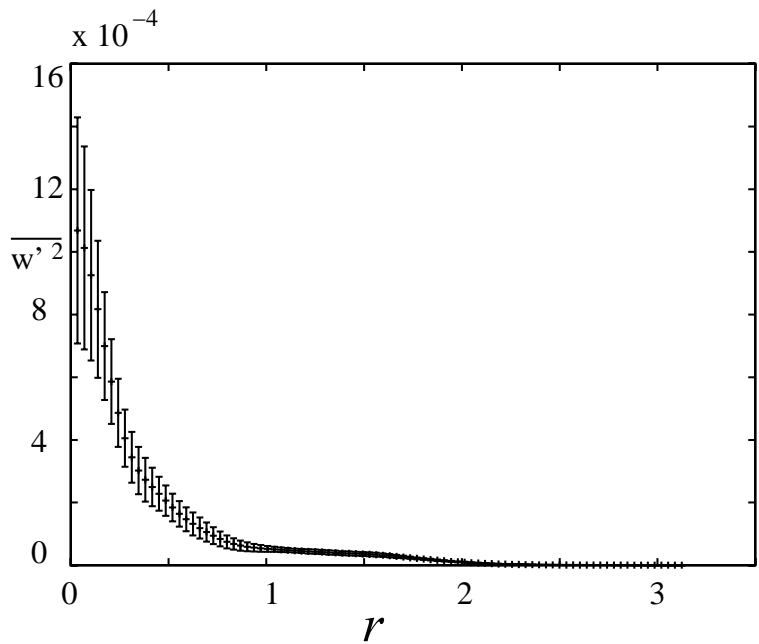
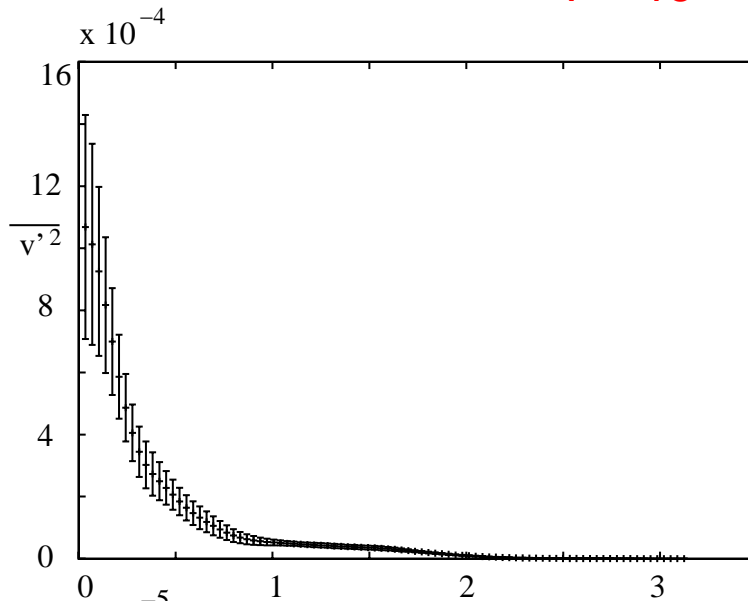
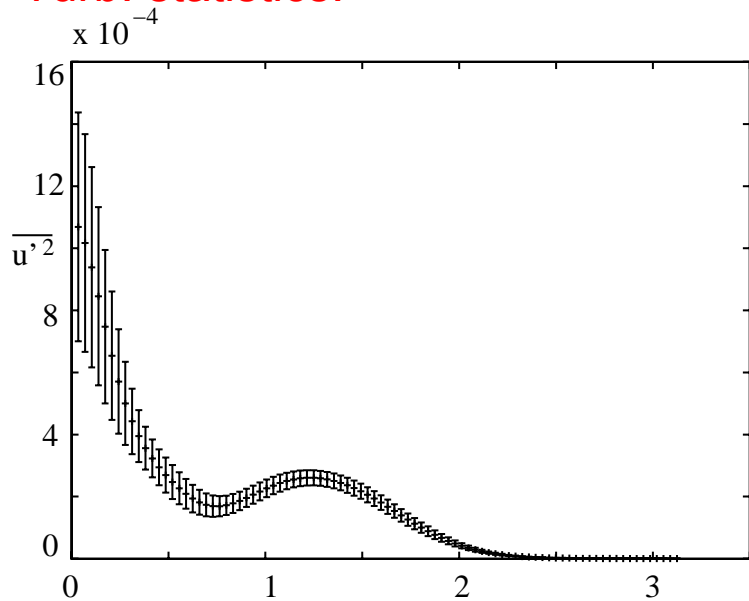
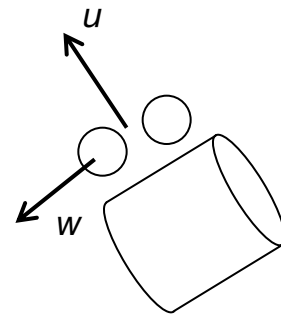


- surprisingly little effect of turbulence on vortex decay

More rapid decay possible?

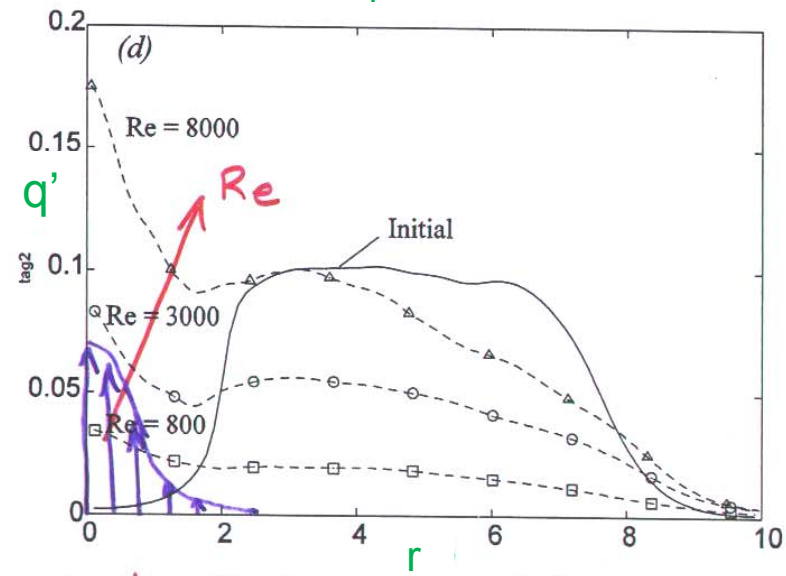
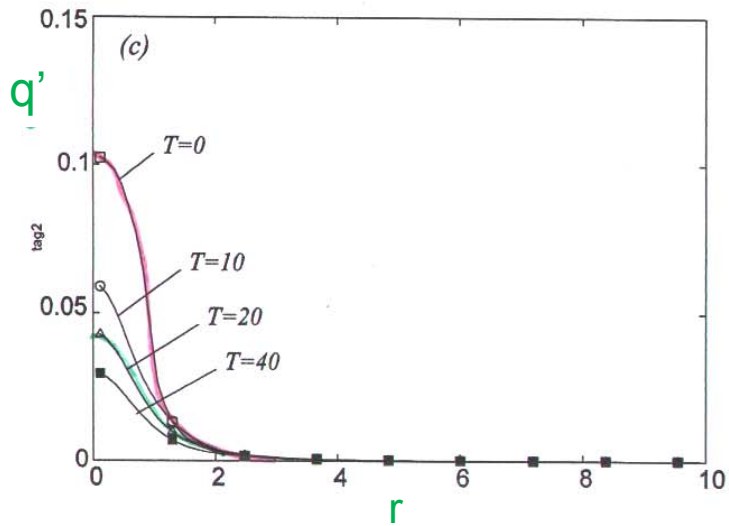
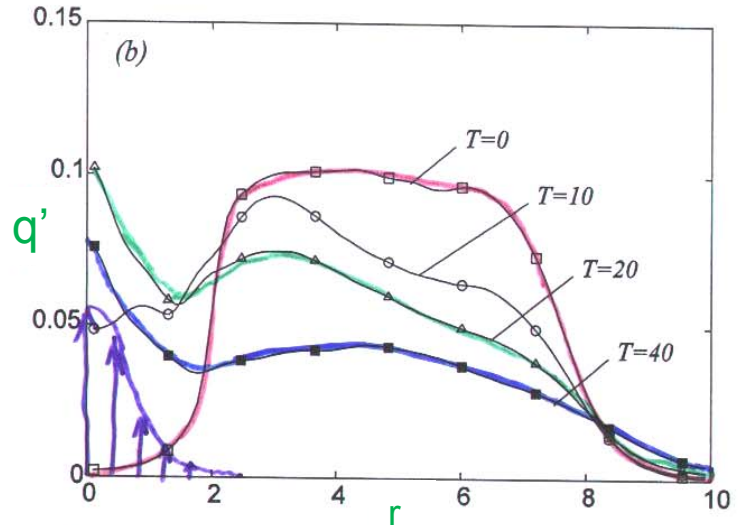
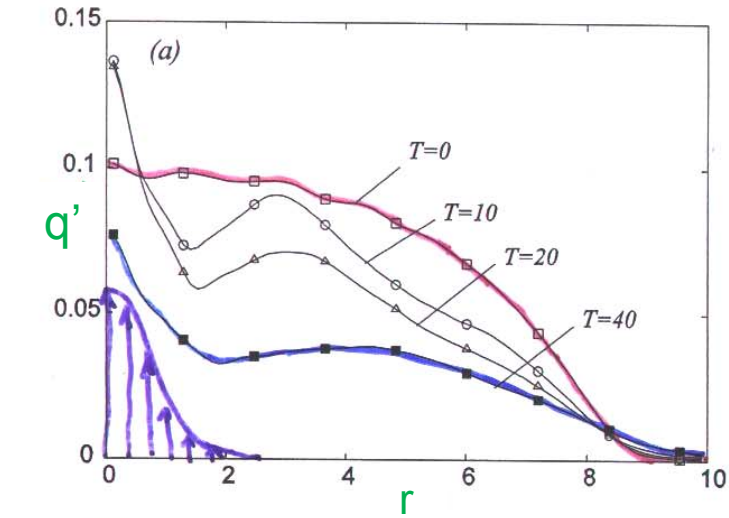
Turb. statistics:

T = 15



Re=3000

Growth of core fluctuations:



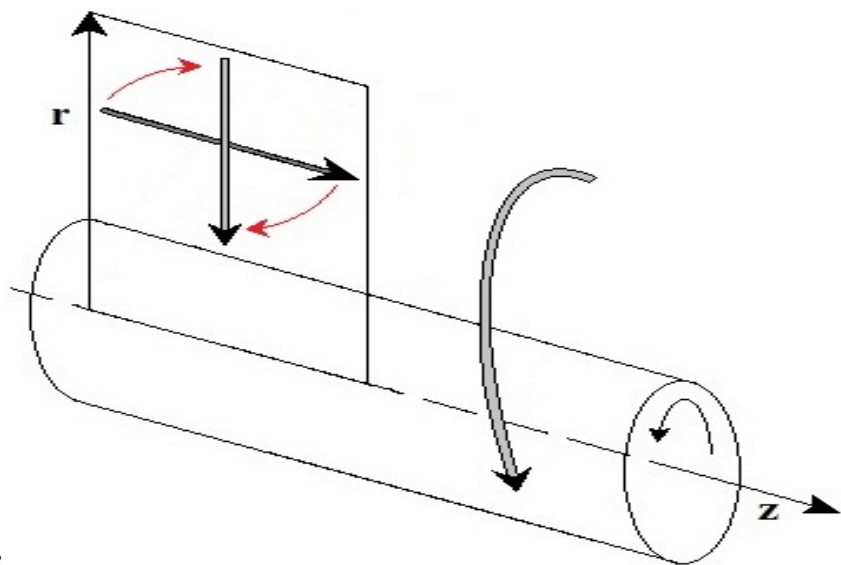
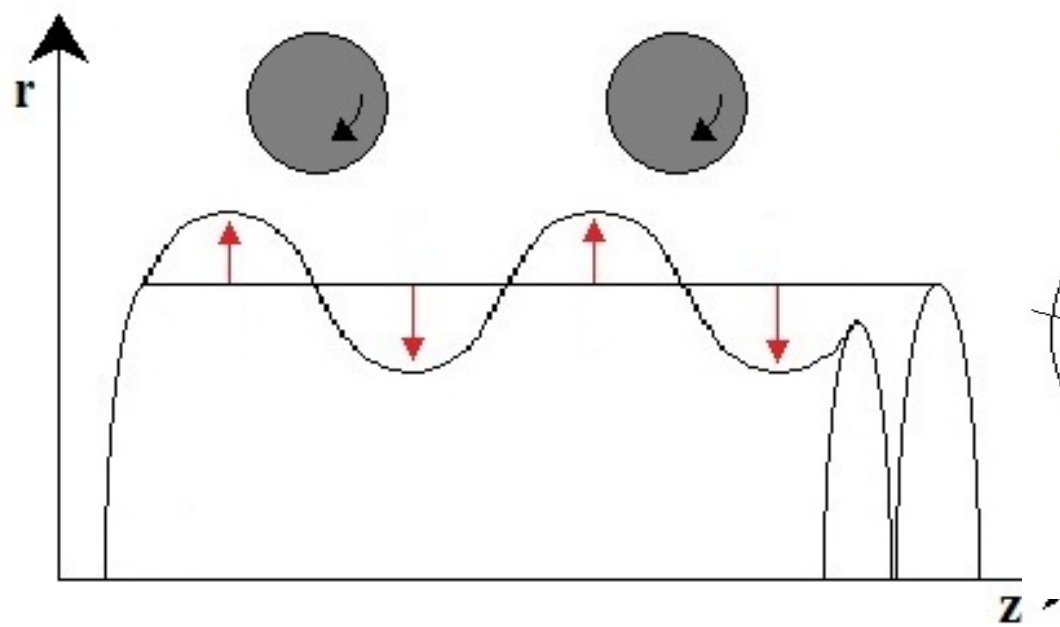
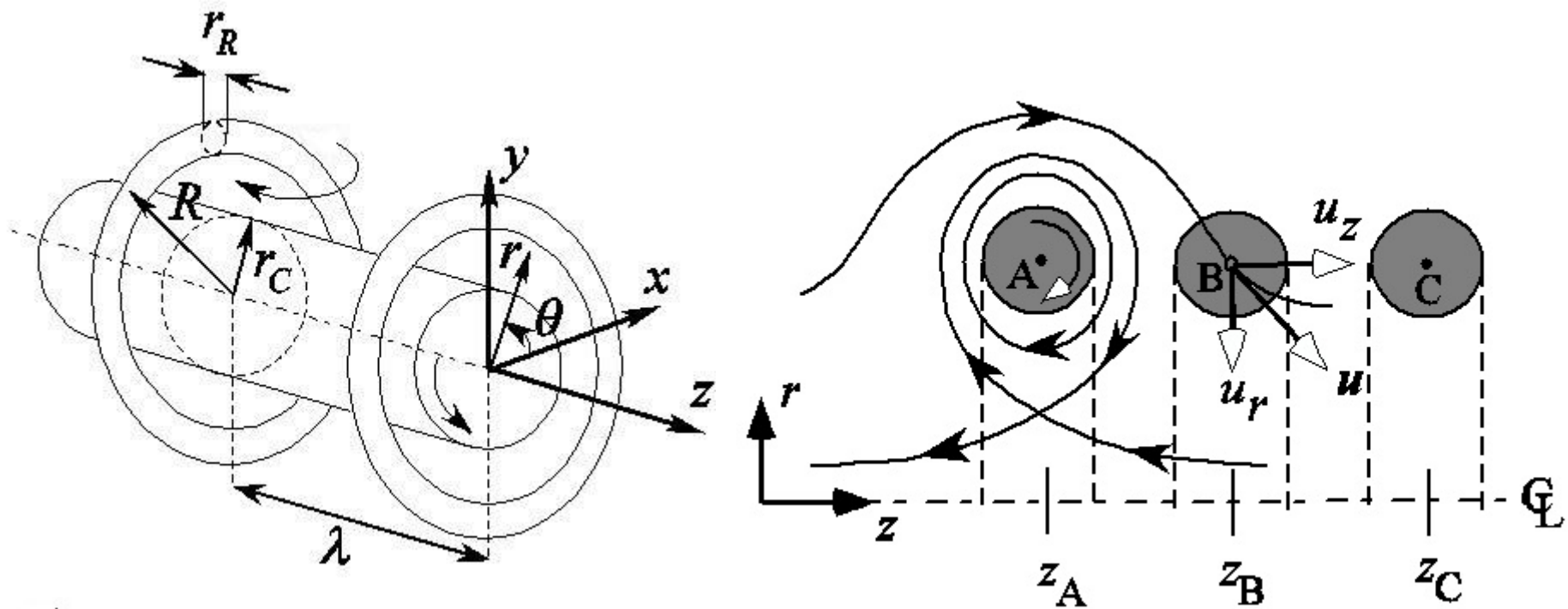
Oseen vortex stable; Γ overshoot effect damped?
Why $E \uparrow$ Transient Growth?

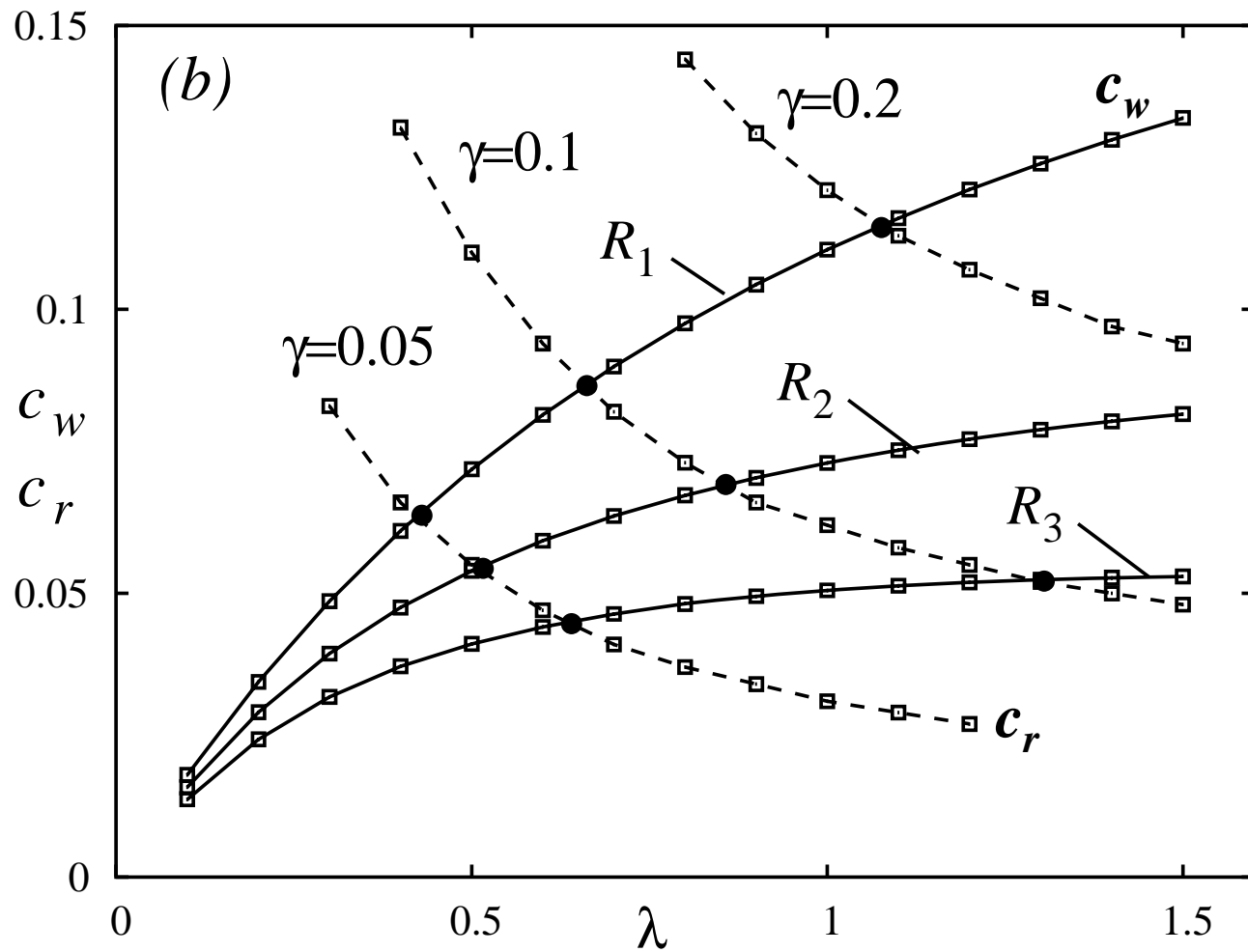
Mechanisms of core perturbation growth:

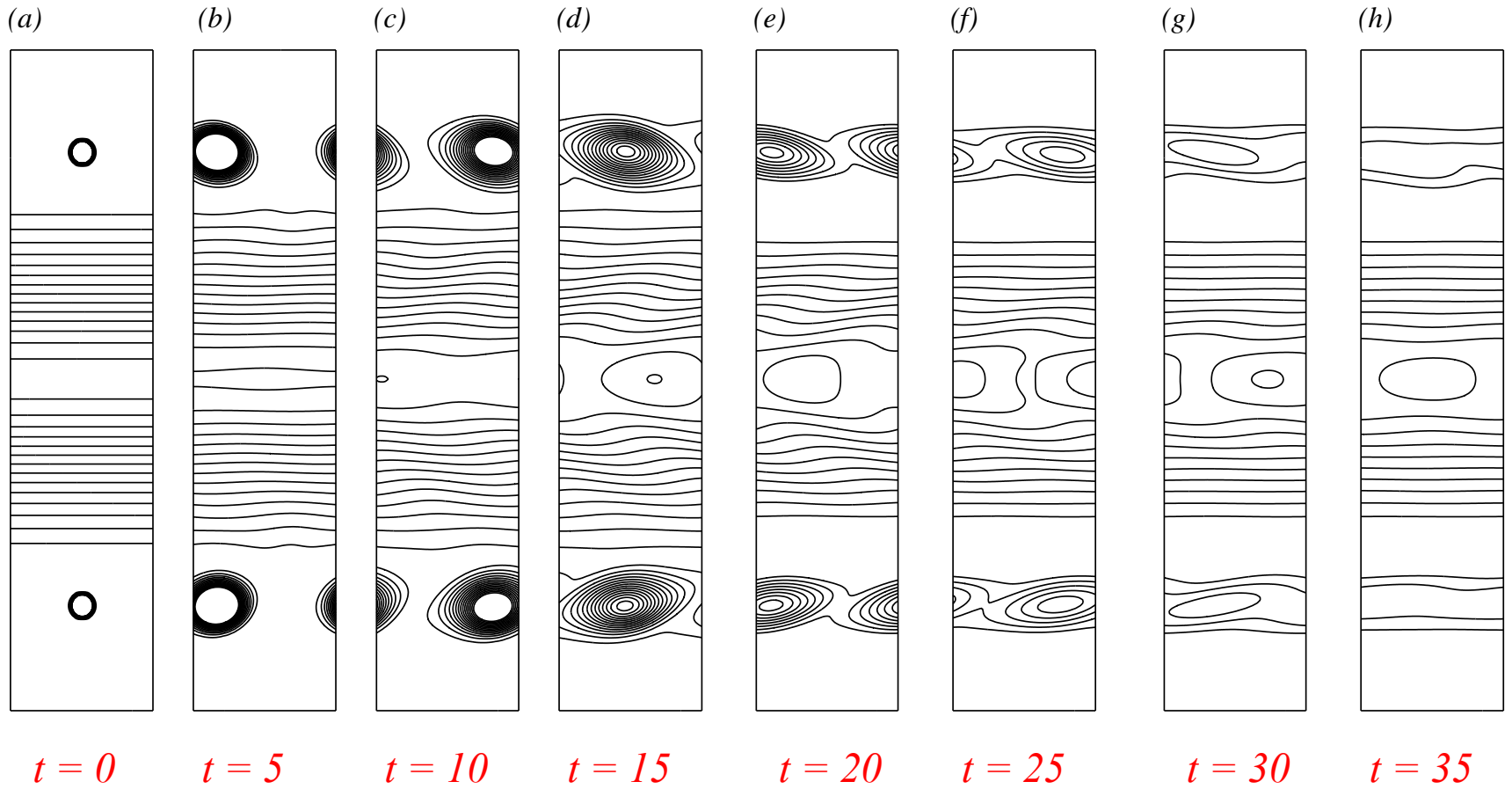
Centrifugal instability is self-limiting

Other mechanisms?

- Thread/Vortex wave resonance
- Transient growth

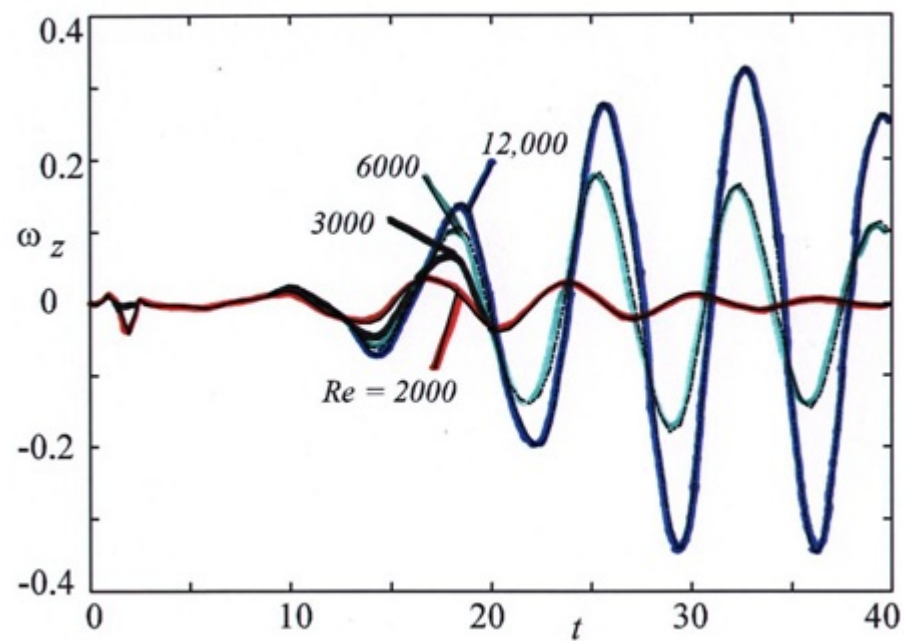
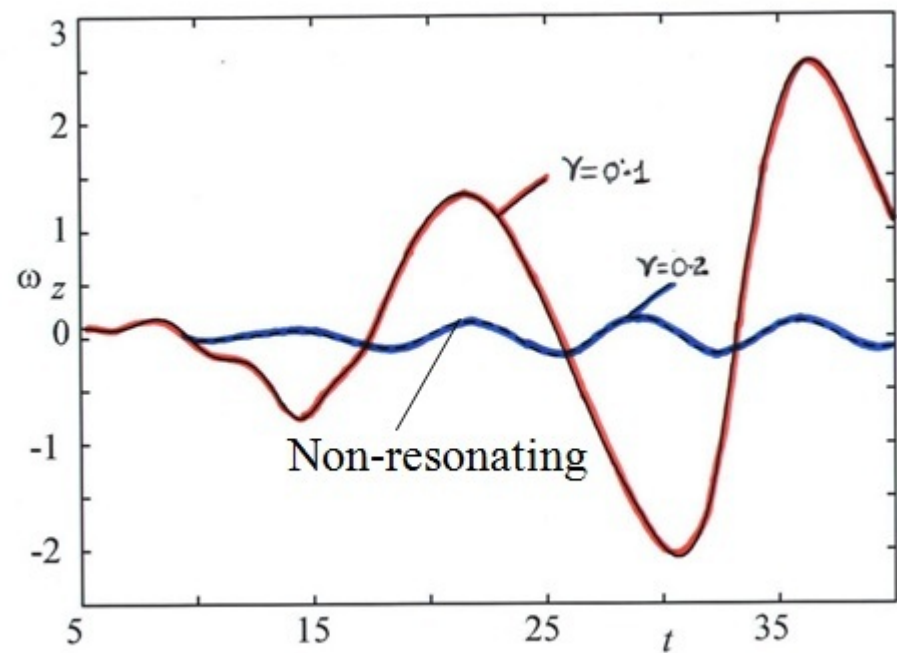
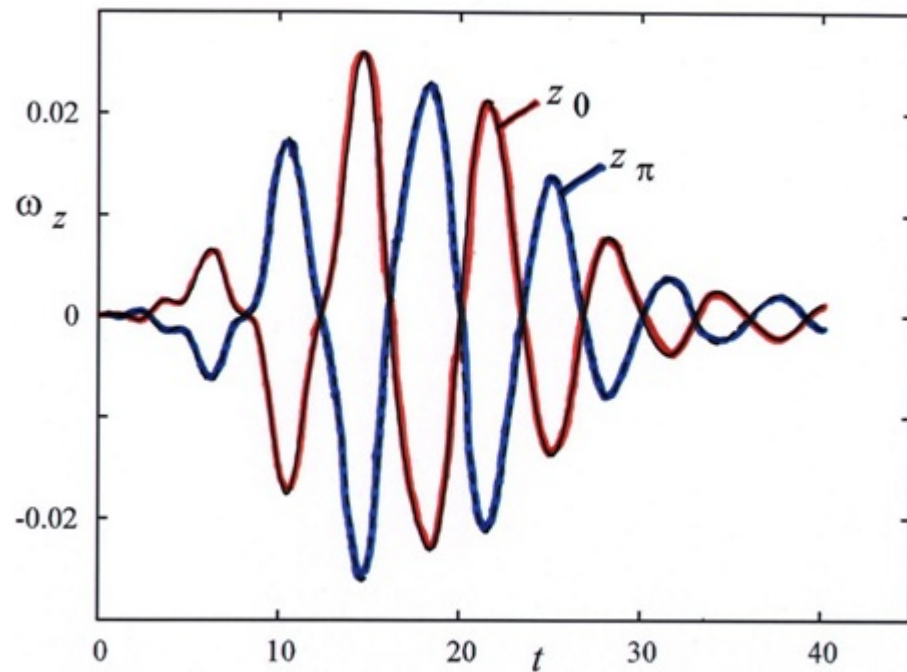






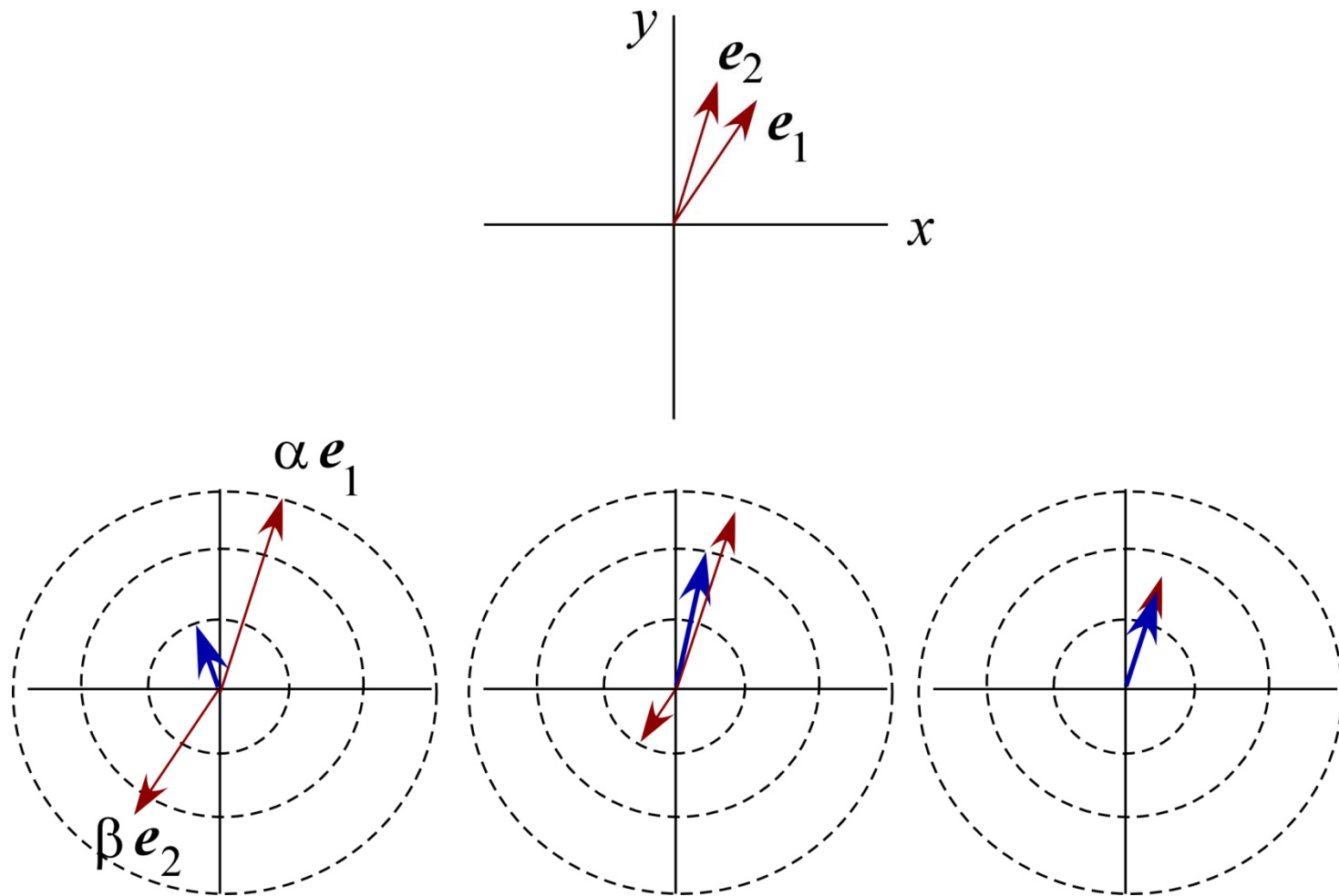
$$\gamma = 0.1$$

$$\text{Re} = 2000$$



TRANSIENT GROWTH

A rudimentary example:

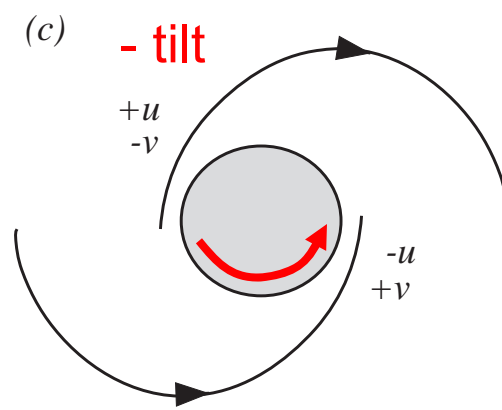
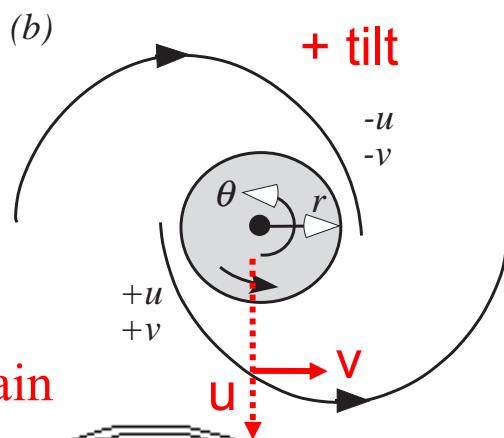
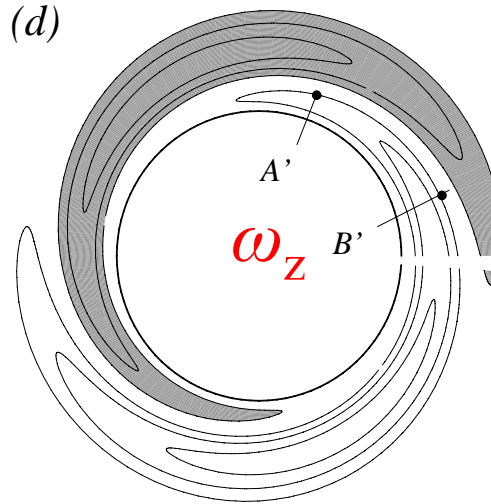
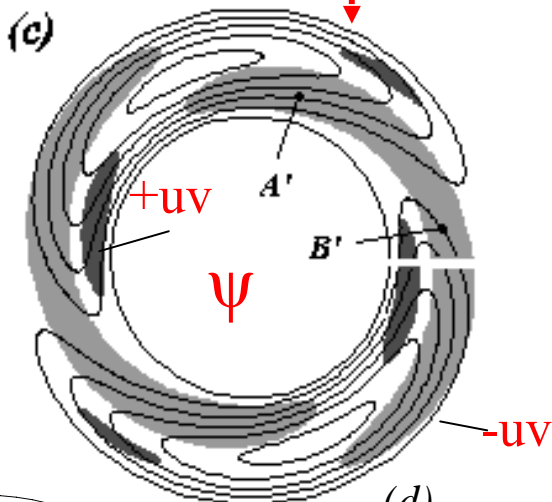
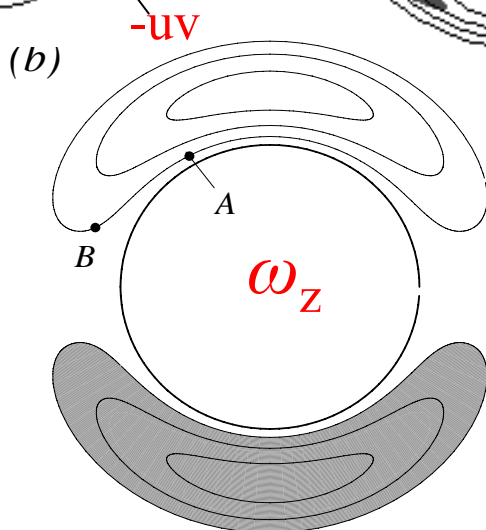
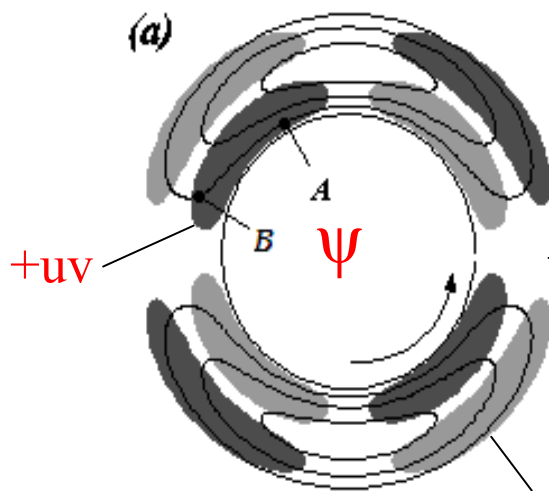


TG: Temporary growth followed by decay

2-D mechanism:

Rigid Rotating Rod

$$\frac{dE}{dt} \sim -\int uv \, \underline{r(V/r)'} \, dV$$



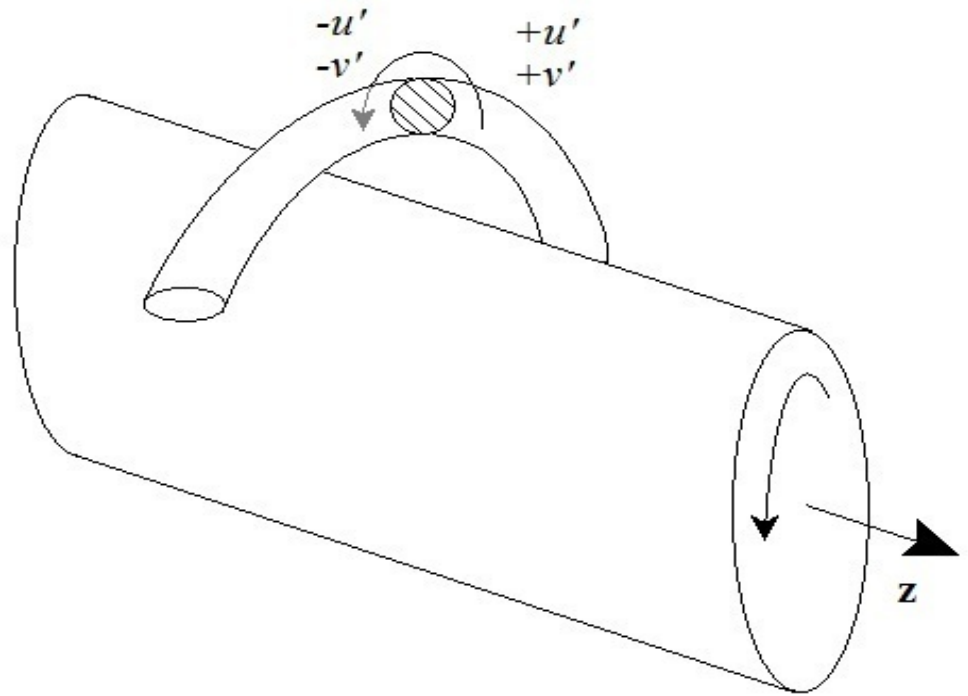
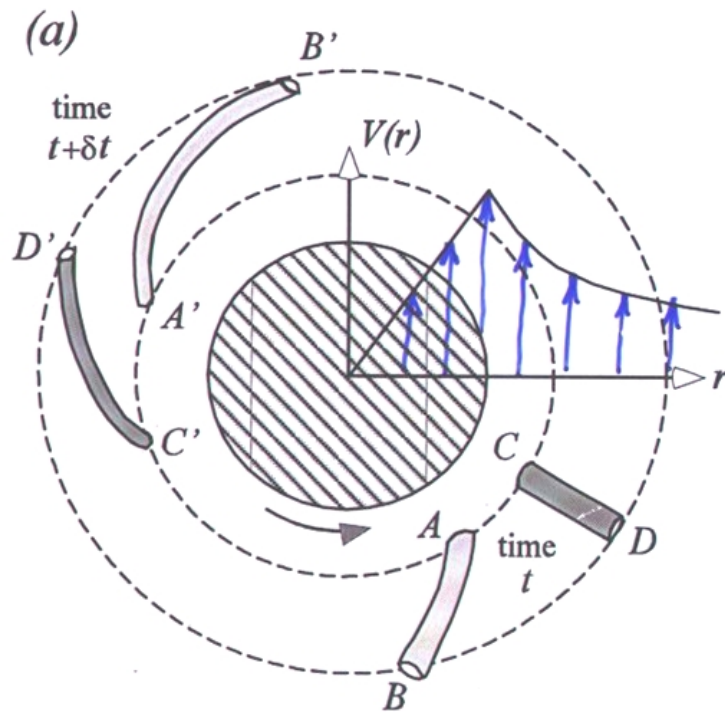
**only shearing,
no stretching**

Three-dimensional mechanism:

$$\frac{\partial \omega_r}{\partial t} = 0; \quad \frac{\partial \omega_z}{\partial t} = 0; \quad \frac{\partial \omega_\theta}{\partial t} = \omega_r S$$

mean vorticity & strain: opposite effects
 limits growth (mag. & period) ← core
 growth ← outside

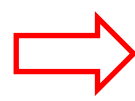
Pure straining flow



Rotation only

@ vort. Surface

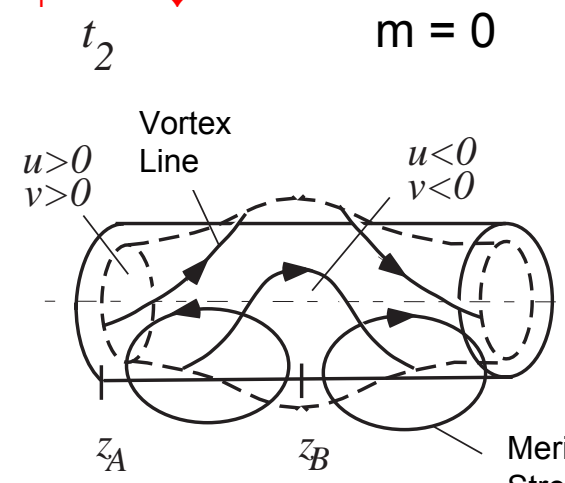
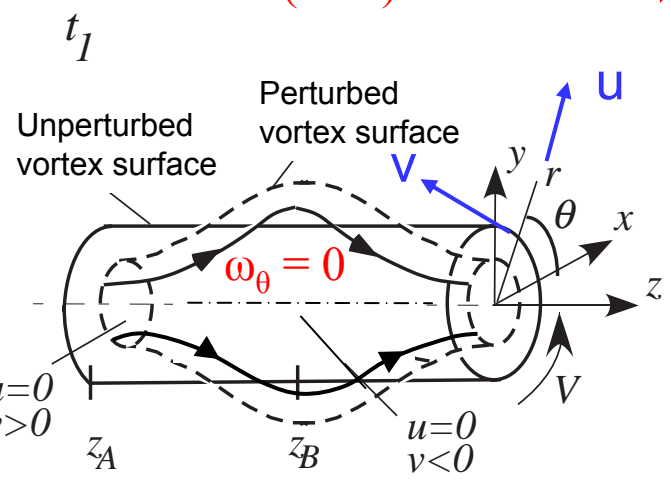
$$\Gamma = 2\pi r(V+v) = \text{const.}$$



$$\begin{aligned} r \downarrow &\Rightarrow v \uparrow \\ r \uparrow &\Rightarrow v \downarrow \end{aligned}$$

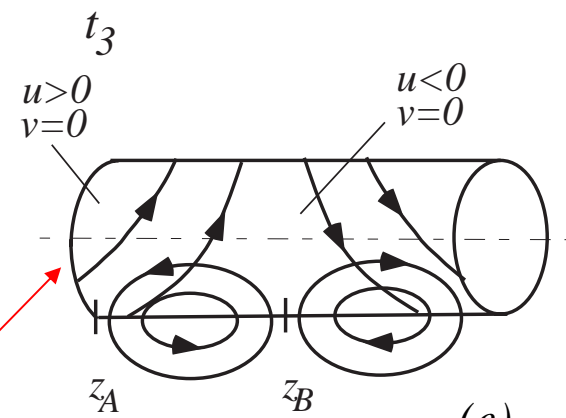
Core dynamics

$m = 0$

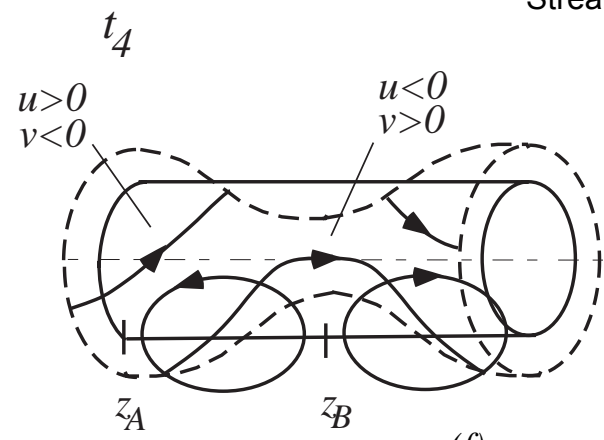


$$uv > 0 \Rightarrow E \uparrow$$

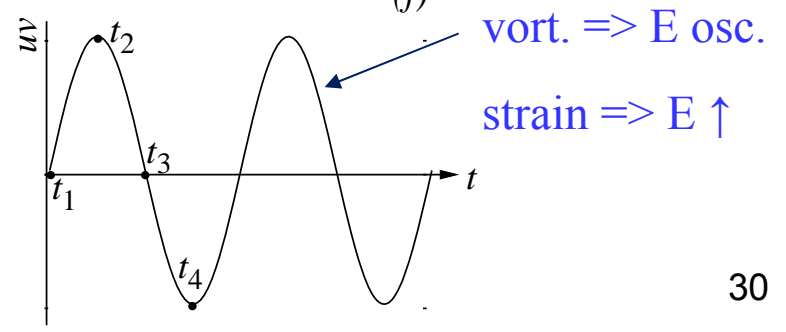
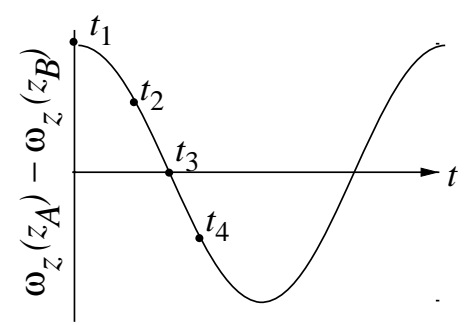
$$uv = 0$$



$$\omega_\theta = \text{max.}$$



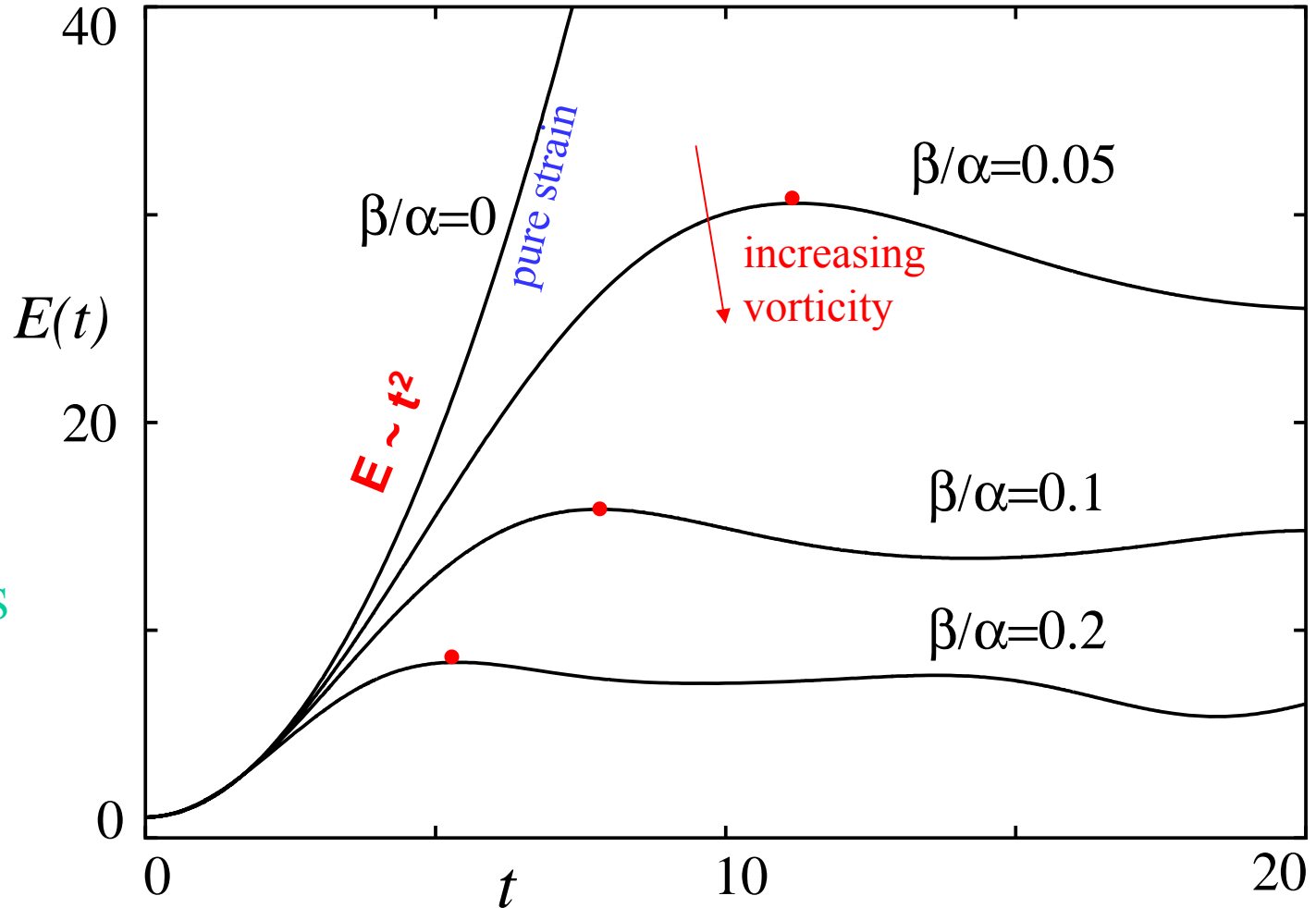
$$uv < 0 \Rightarrow E \downarrow$$



$$V = \frac{\alpha}{r} + \frac{\beta r}{2}$$

model flow

$\beta/\alpha = \text{rotation/strain}$



$$\frac{\partial \omega_\theta}{\partial t} = \omega_r S$$

Higher strain S

i.e. $\alpha \uparrow$

$\Rightarrow \omega_\theta$ gen.

$\Rightarrow uv \uparrow$

$\Rightarrow E \uparrow$

inc. vort(β)

\Rightarrow arrest $E \uparrow$

sooner

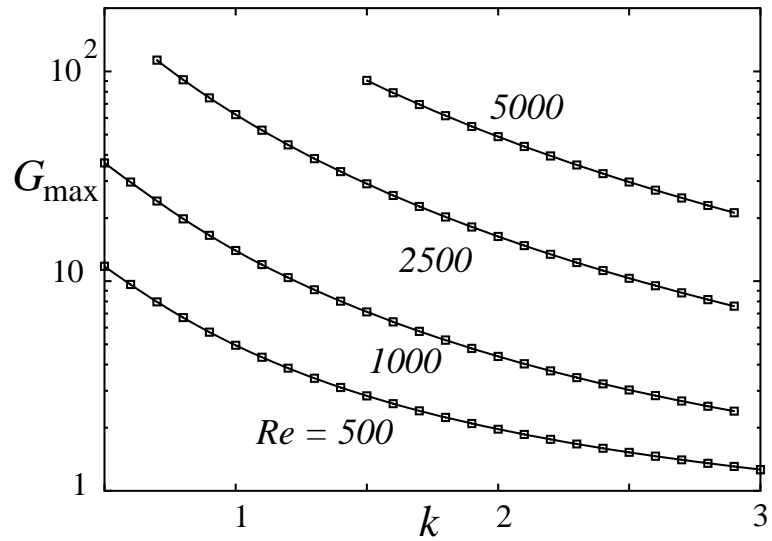
LIN. INVISCID TG

Strain: unbounded growth (lin. sense)
eventually saturate at NL level

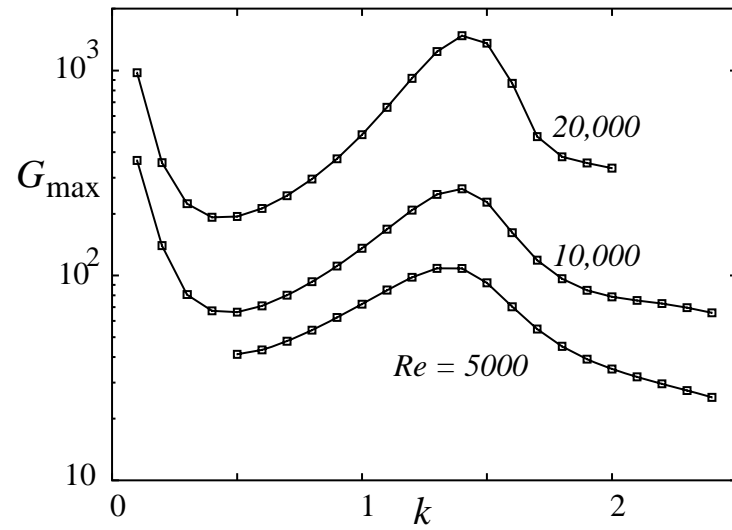
core vorticity: arrest growth & period of growth
→ core oscillation

VISCOSITY damps both

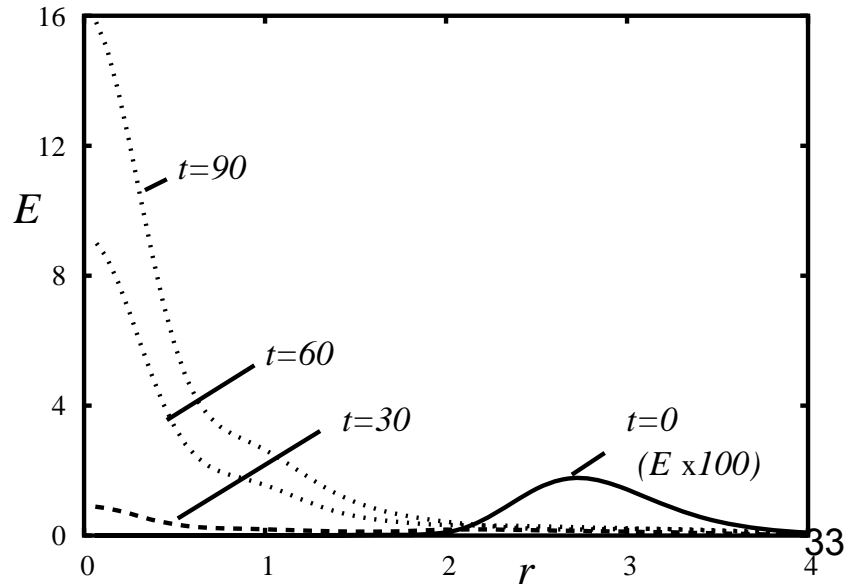
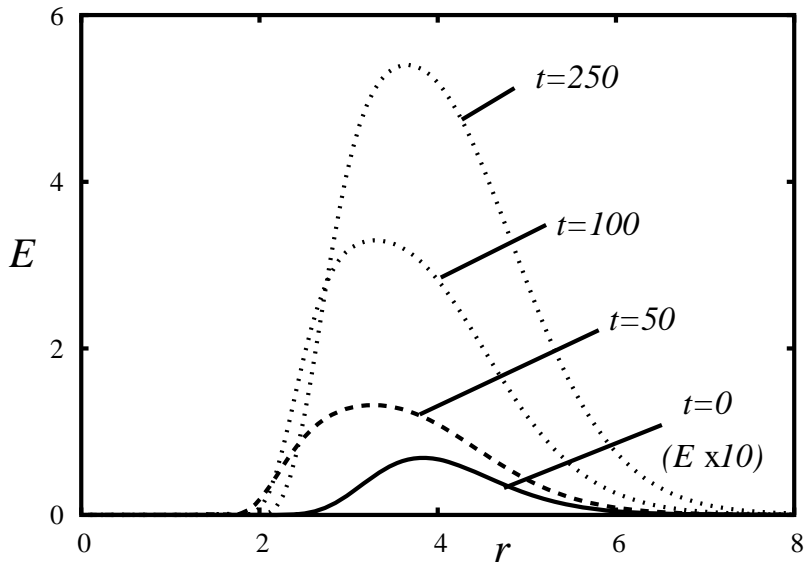
Optimal gains: *axi-sym* $m = 0$



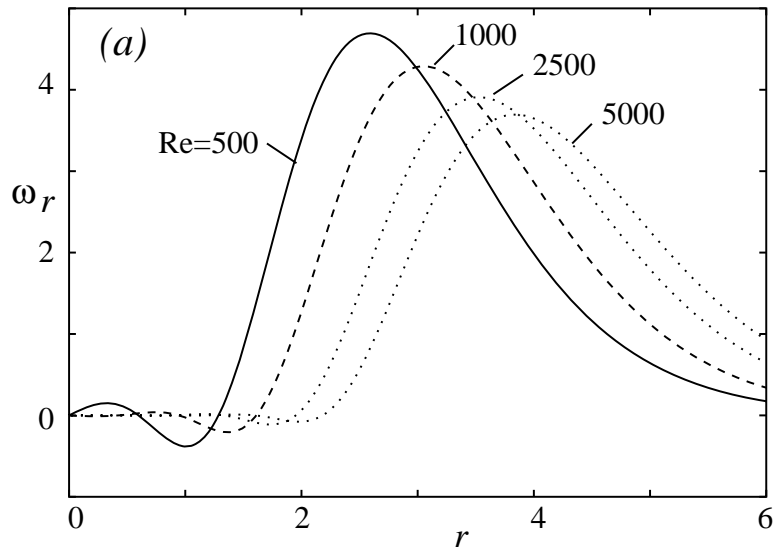
Bending $m = 1$



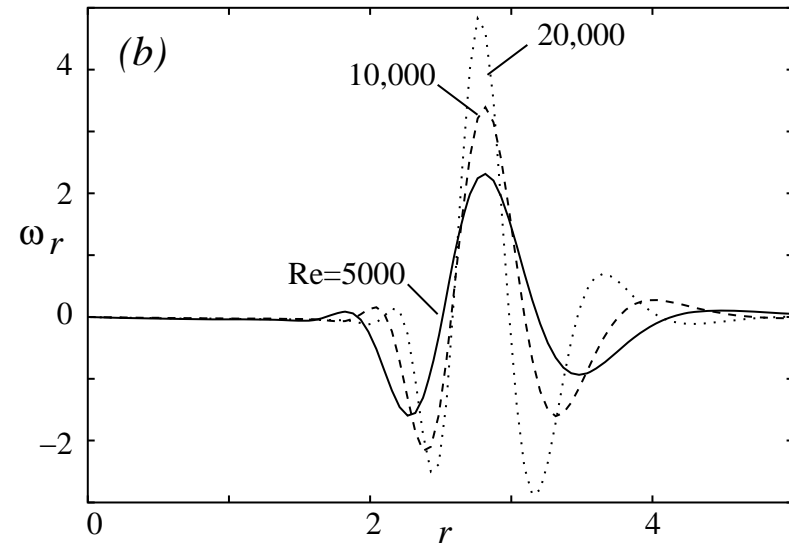
Energy evolution:



Re effect on tilting/stretching



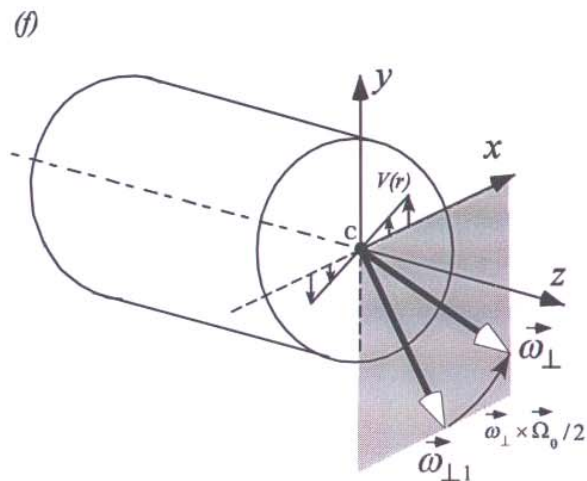
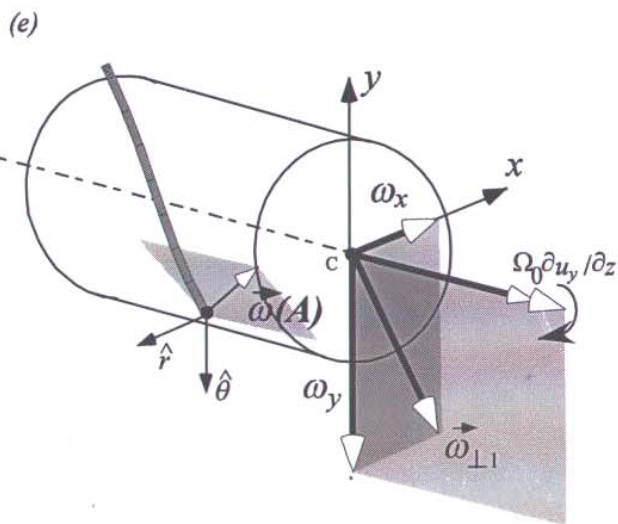
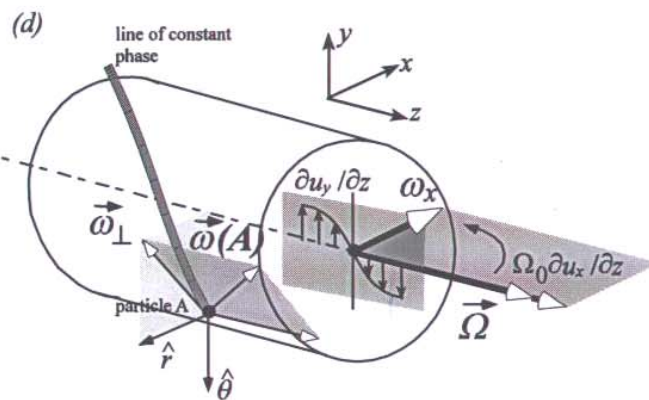
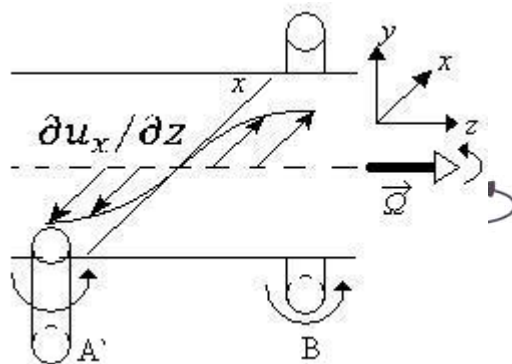
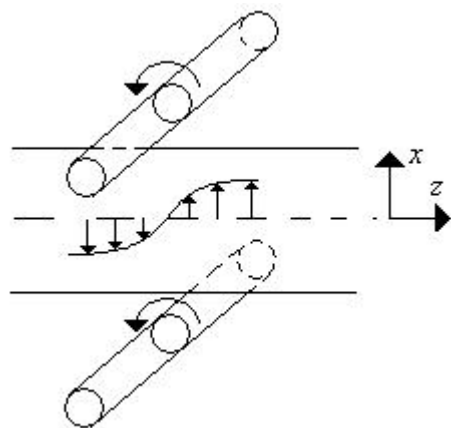
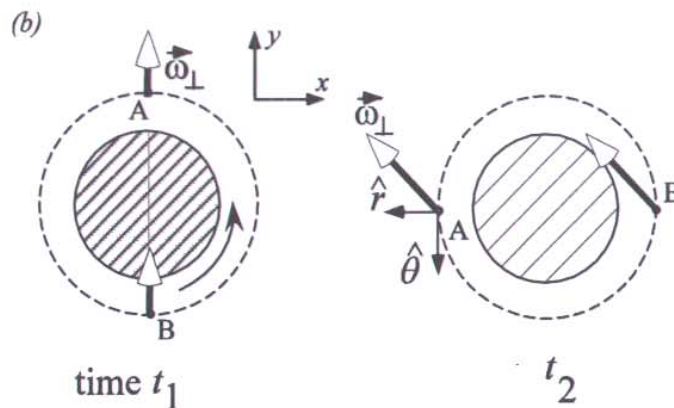
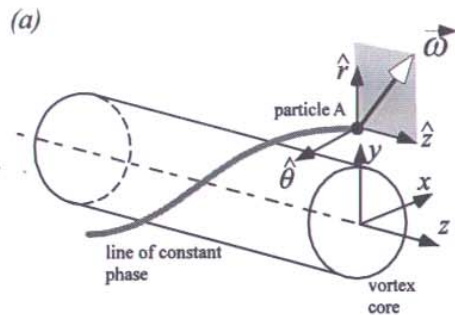
$m = 0$



$m = 1$

Mechanism of core fluctuation growth:

$m = 1$

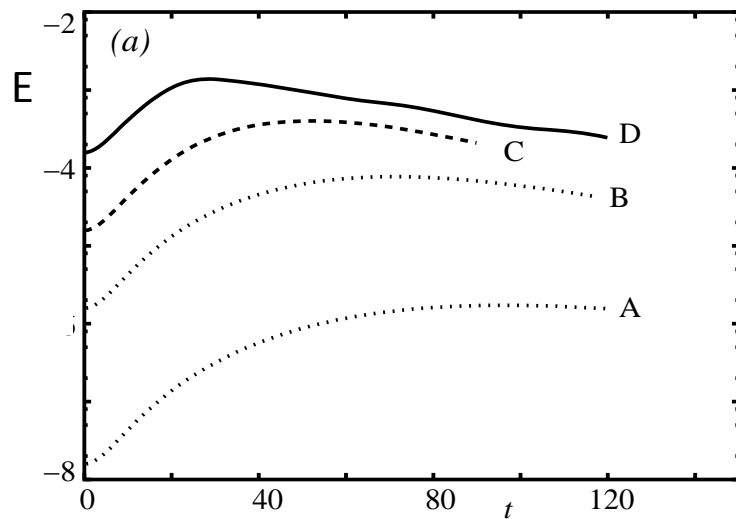


NONLINEAR TRANSIENT GROWTH

of optimal modes

initial perturbation amplitude: $m = 1$

$Re = 5000$



Case **A**

Linear

B

0.6%

C

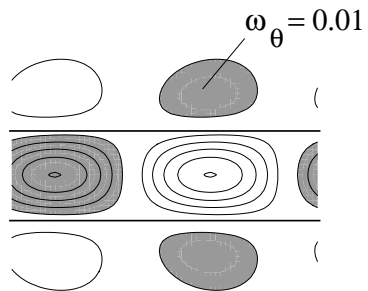
2%

D

6%

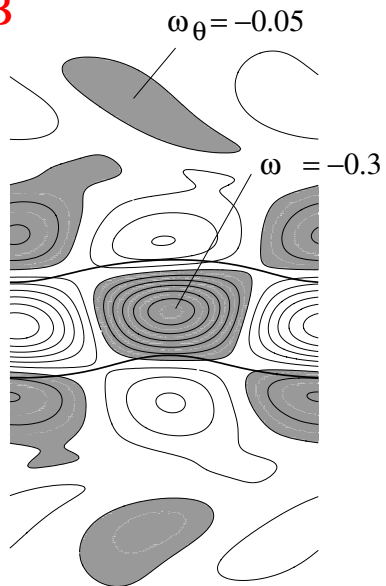
Structure at time of max. energy:

A



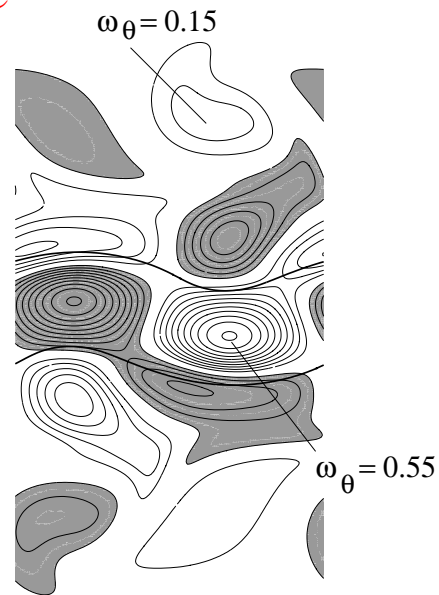
$t = 90$

B



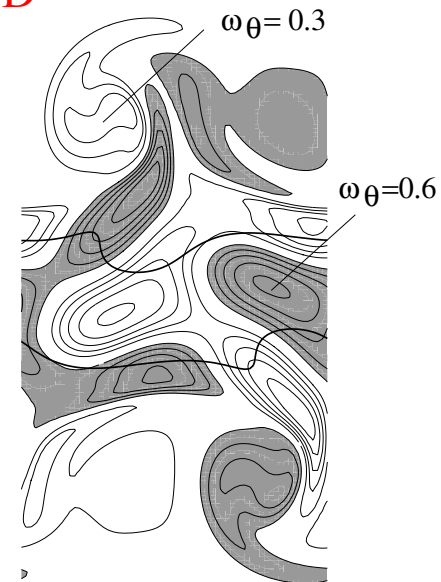
$t = 70$

C

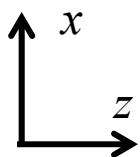


$t = 50$

D



$t = 30$



High-amplitude perturbation evolution:

Case D, $m = 1$

$Re = 5000$

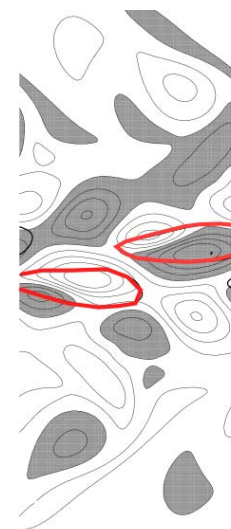
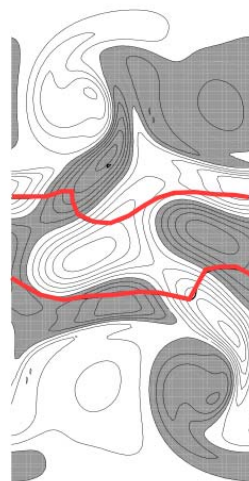
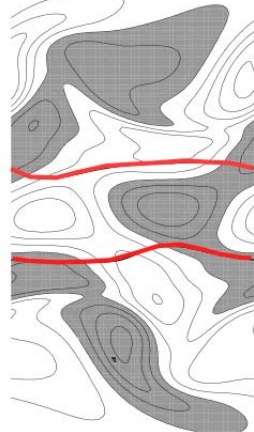
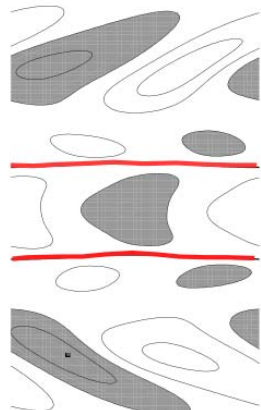
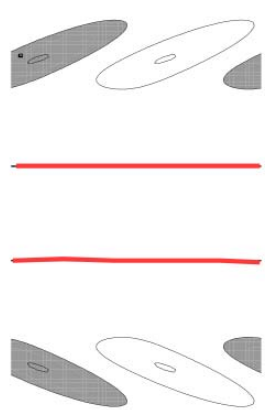
$t = 0$

$t = 10$

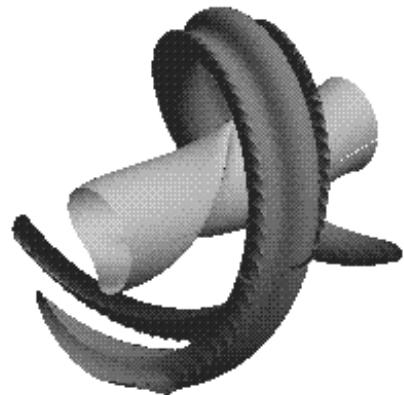
$t = 20$

$t = 30$

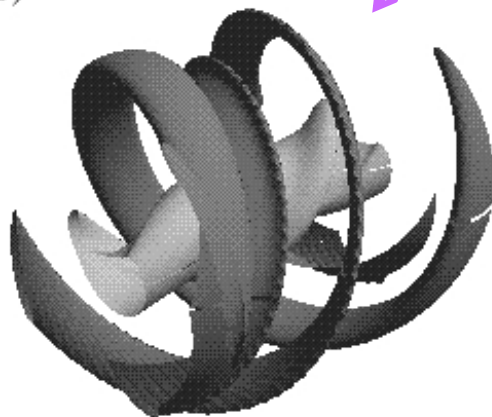
$t = 100$



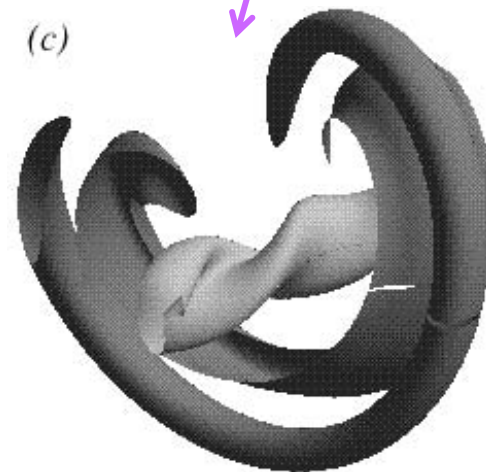
(a)



(b)

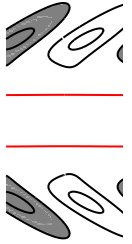


(c)

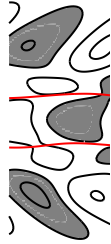


Re effect:

t=0



t=10



t=30



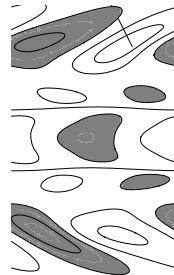
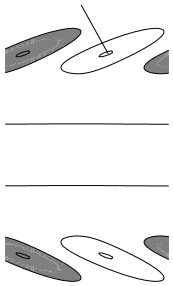
t=75



t=100

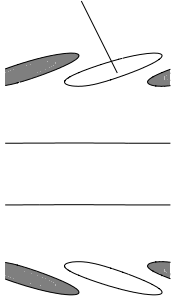


Re = 2000

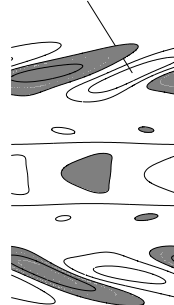


Re = 5000

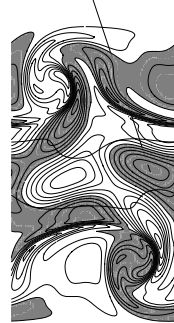
$\omega = 0.05$



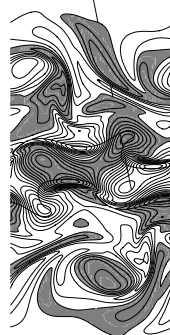
$\omega = 0.25$



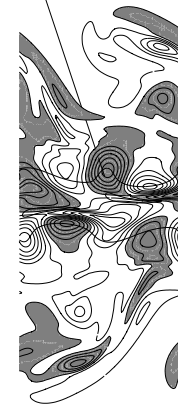
$\omega = 0.65$



$\omega = 0.95$

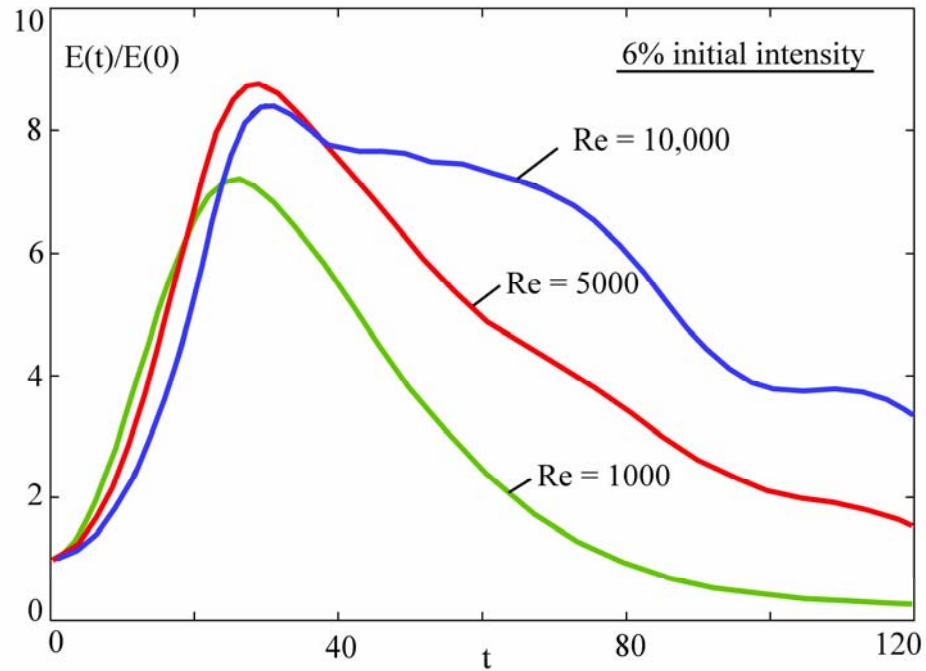


$\omega = 0.65$



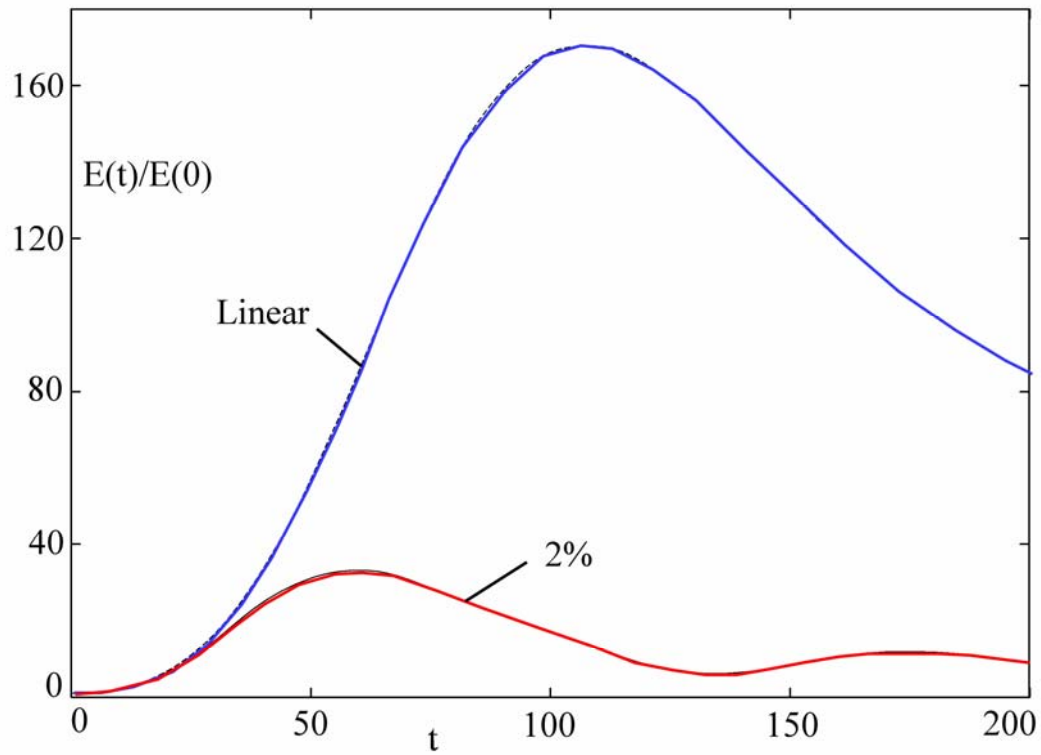
Re = 10,000

Re-effect in time



Regenerative Transient Growth?

Re = 5000



Regenerative Transient Growth?

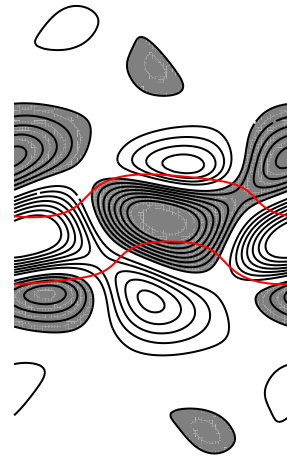
t = 100

t = 120

t = 140

t = 160

t = 180



Some conclusions

- Turbulence induces and amplifies core fluctuations – amplitudes exceeding those of external perturbations.
- Several potential mechanisms of core transition / accelerated vortex decay studied.
- Circulation overshoot => **centrifugal instability**: amplifies perturbations, but inherently self-limiting.
- Weak “threads” can **resonate** with vortex core dynamics waves, but not strong perturbations.
- **Transient growth**: orders-of-magnitude amplification
- Strongest transient growth for bending waves.

Further study

Nonlinear transient growth, regenerative transient growth, vortex breakup and turbulence self-sustenance