Linear non-normal energy amplification of harmonic and stochastic forcing in the turbulent channel flow

Carlo Cossu

Institut de Mécanique des Fluides de Toulouse
(IMFT - CNRS) & Département de Mécanique
École polytechnique, France

Yongyun Hwang

Laboratoire d'Hydrodynamique (LadHyX)
École polytechnique, France

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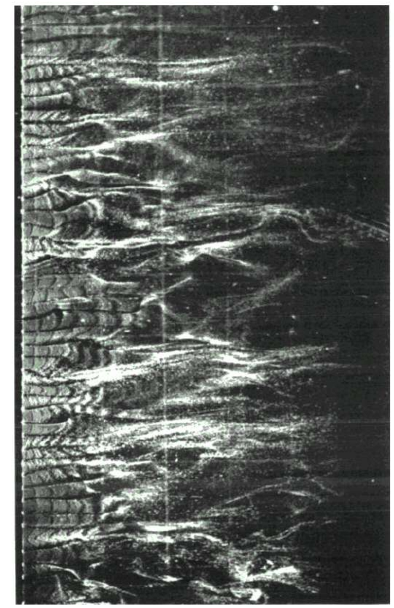
Streaky structures in turbulent flows

Near wall streaks in a turbulent boundary layer

mean spanwise spacing $\lambda^+ \approx 100$

most probable spacing $\lambda^+ \approx 80$

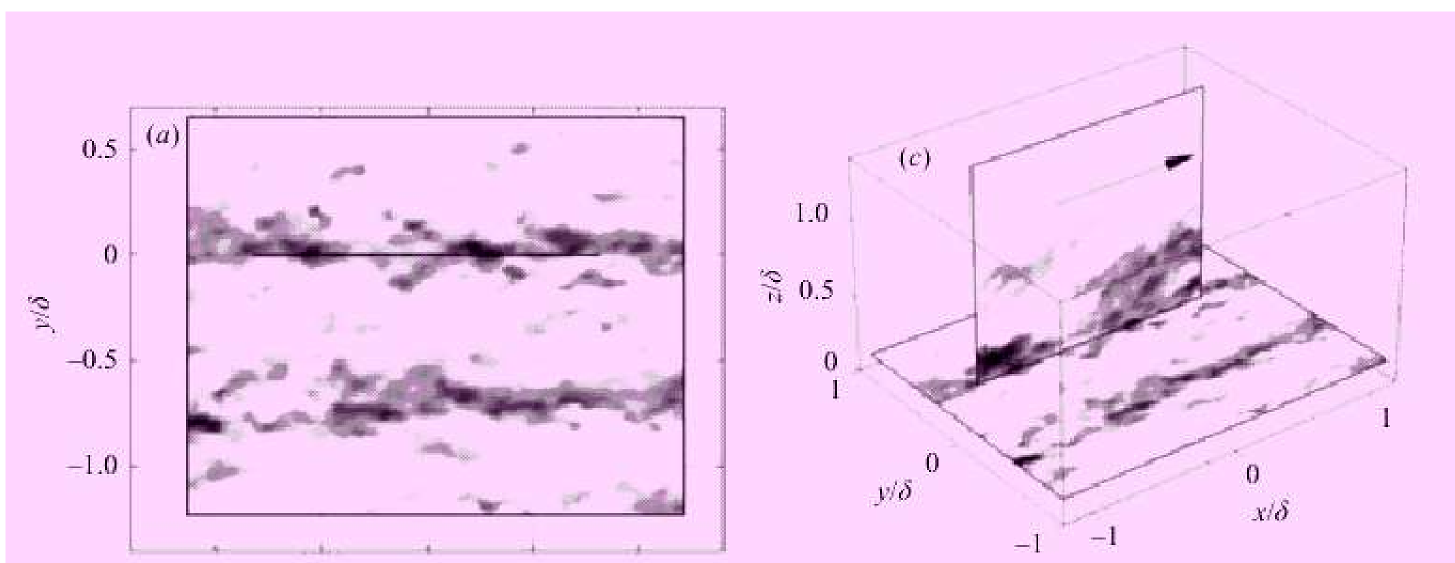


Kline et al. JFM 1967

Large scale streaks in a turbulent boundary layer

Streks spacing

$\lambda \approx \delta$



Hutchins & Marusic JFM 2007

Some questions

Is a 'coherent' lift-up effect at work in fully developed turbulent flows?

Does it select the spanwise scales of the 'natural' streaks?

Are the optimal amplifications large?

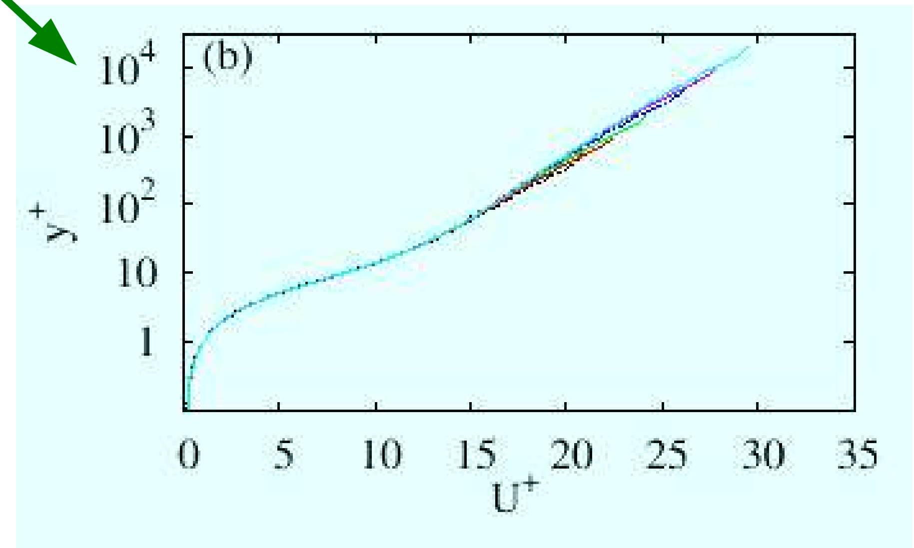
Can we force these streaks for control purposes?

Linearization near the mean flow

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \mathbf{u} \cdot \mathbf{U} + \nabla \mathbf{U} \cdot \mathbf{u} = -\frac{1}{\rho}\nabla p + \nabla \cdot \left[\nu_T \left(\nabla \mathbf{u} + \nabla \mathbf{u}^T\right)\right] + \mathbf{f},$$

U=(U(y),0,0)**:** base mean flow (no coherent perturbations)

Cess-Reynolds-Tiedermann U(y) profile



ν_T(y): eddy viscosity in equilibrium with U(y) (Cess fit used)

u: small coherent perturbations to **U** (obtained with coherent forcing or IC)

Reynolds & Hussain *JFM* 1972
del Alamo & Jiménez *JFM* 2006
Pujals, Garcia-Villalba, Depardon & Cossu, *Phys. Fluids* 2009
Cossu, Pujals & Depardon, *JFM* 2009
Hwang & Cossu *JFM* 2010

Linearized system for parallel basic flows

$$\widehat{\mathbf{u}}(\alpha, y, \beta, t)\, \mathrm{e}^{i(\alpha x + \beta z)}$$

Fourier modes

streamwise wavenumber   spanwise wavenumber

$$\hat{\mathbf{q}} = \mathbf{D}\hat{\mathbf{u}};$$

$$\hat{\mathbf{u}} = \mathbf{C}\hat{\mathbf{q}};$$

$$\hat{\mathbf{q}} = [\hat{v}, \hat{\omega}_y]^T.$$

State vector

Dynamical system formulation

$$\frac{\partial \hat{\mathbf{q}}}{\partial t} = \mathbf{A}\hat{\mathbf{q}} + \mathbf{B}\hat{\mathbf{f}}.$$

generalized Orr-Sommerfeld operator

$$\mathbf{A} = \begin{bmatrix} \Delta^{-1}\mathcal{L}_{OS} & 0 \\ -i\beta U' & \mathcal{L}_{SQ} \end{bmatrix}$$

Coupling term

generalized Squire operator

Optimals for the initial value problem

$$\hat{\mathbf{f}} = 0$$

IVP with no forcing

$$\hat{\mathbf{q}} = e^{t\mathbf{A}}\hat{\mathbf{q}}_0$$

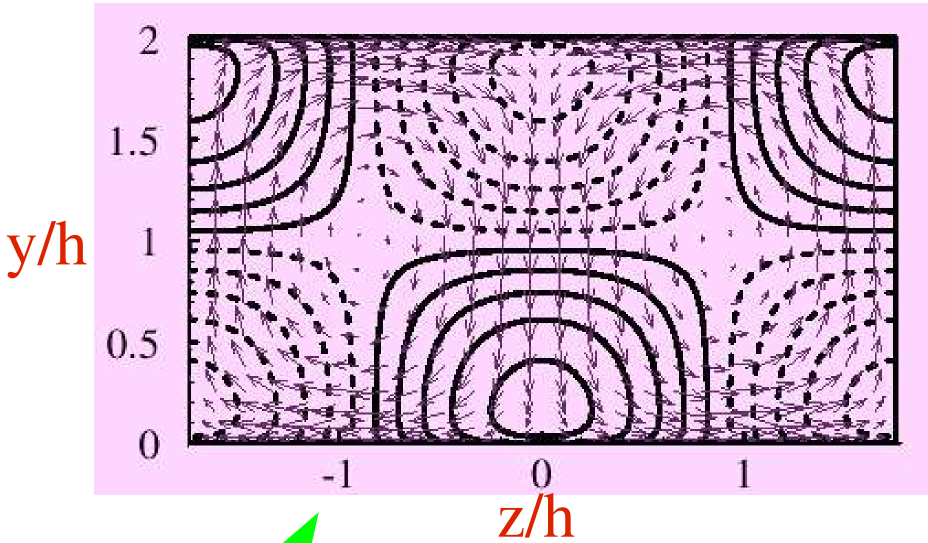
$$\hat{\mathbf{u}} = \mathbf{C}e^{t\mathbf{A}}\mathbf{D}\hat{\mathbf{u}}_0$$

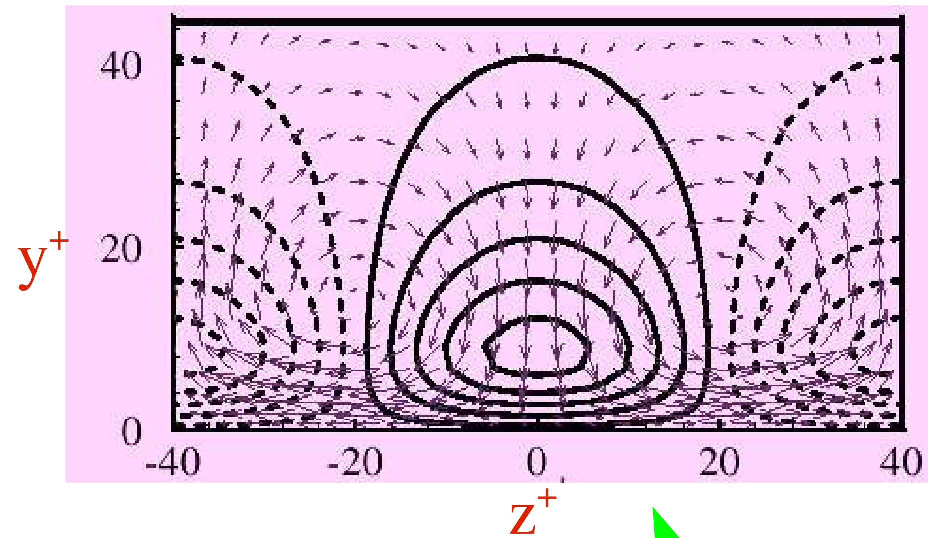
Optimal amplification of initial conditions

$$G(t,\alpha,\beta) = \max_{\hat{\mathbf{u}}_0 \neq 0} \frac{\|\hat{\mathbf{u}}\|^2}{\|\hat{\mathbf{u}}_0\|^2} = \|\mathbf{C}e^{t\mathbf{A}}\mathbf{D}\|^2$$

$$G_{max}(\alpha,\beta) = \max_t G(t,\alpha,\beta)$$

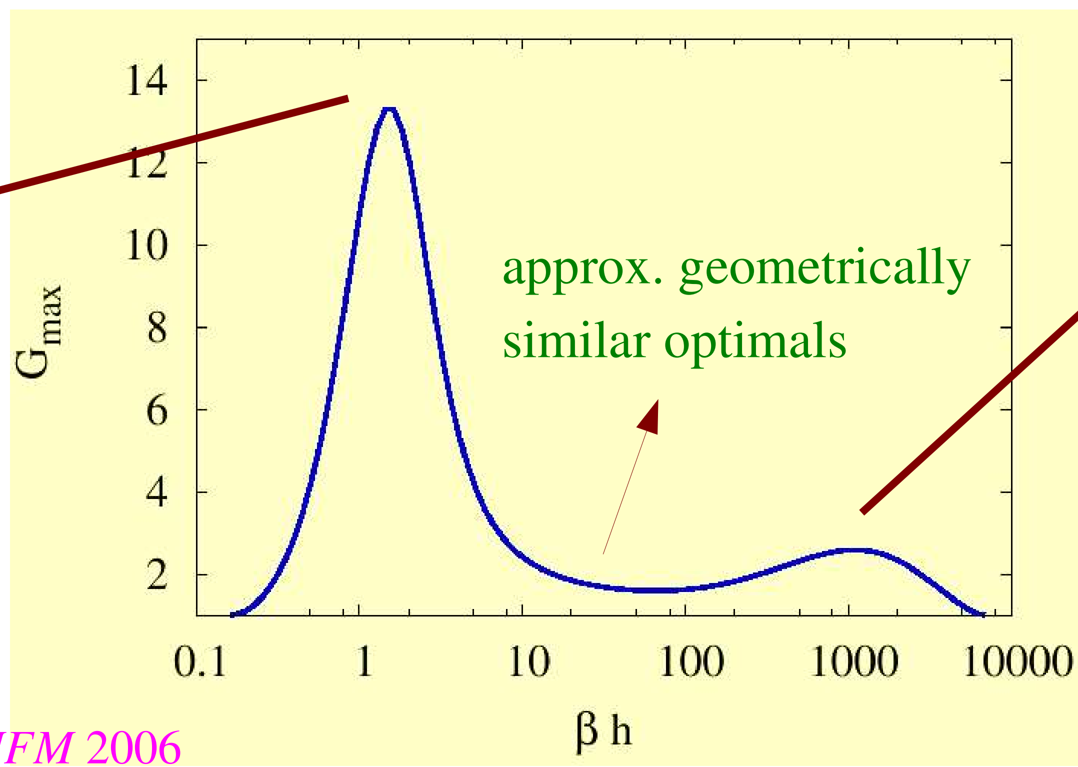
Gmax for $\alpha=0$ and $Re_\tau=16000$





$\lambda_z \approx 4\ h$

$\lambda_z^+ \approx 92$



del Alamo & Jiménez *JFM* 2006

Pujals, Garcia-Villalba, Depardon & Cossu, *Phys. Fluids* 2009

Active & passive control ~ deterministic forcing

Influence of free-stream turbulence (in TBL)

& nonlinear interactions with other scales

~ stochastic forcing

Same outer-inner peak structure for the amplifications of forcing?

In previous analysis of turbulent Couette flow, $Re_\tau=52$ was too low for inner-outer scale separation

Hwang & Cossu *JFM* 2010

Response to harmonic forcing (scale by scale)

forcing frequency

$$\hat{\mathbf{f}}(y,t) = \tilde{\mathbf{f}}(y)e^{i\omega_f t}$$

$$\hat{\mathbf{u}}(y,t) = \tilde{\mathbf{u}}(y)e^{i\omega_f t}$$

$$\tilde{\mathbf{u}} = \mathbf{H}\tilde{\mathbf{f}}$$

$$\mathbf{H} = \mathbf{C}(i\omega_f \mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$$

transfer function (aka resolvent operator evaluated on the imaginary axis)

Optimal amplification of harmonic forcing

$$R(\omega_f;\alpha,\beta) = \max_{\tilde{\mathbf{f}}\neq \mathbf{0}} \frac{\|\tilde{\mathbf{u}}\|^2}{\|\tilde{\mathbf{f}}\|^2} = \|\mathbf{H}\|^2$$

$$R_{\max}(\alpha,\beta) = \max_{\omega_f} R(\omega_f;\alpha,\beta)$$

Response to stochastic forcing (scale by scale)

$$\langle \hat{\mathbf{f}} \rangle = \mathbf{0} \qquad \langle \hat{\mathbf{f}}(t)\hat{\mathbf{f}}^H(t') \rangle = \mathbf{I}\delta(t - t').$$

stochastic forcing: zero-mean, isotropic, flat spectrum, Gaussian distribution

$$V = \langle \|\hat{\mathbf{u}}\|^2 \rangle$$ variance of the response to stochastic forcing

variance of the response = trace of < ûû >

$$V(\alpha, \beta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} trace\left(\mathbf{H}\mathbf{H}^{\dagger}\right) d\omega = trace(\mathbf{C}\mathbf{X}_{\infty}\mathbf{C}^{\dagger})$$

KL (POD) modes from spectral analysis of < ûû >

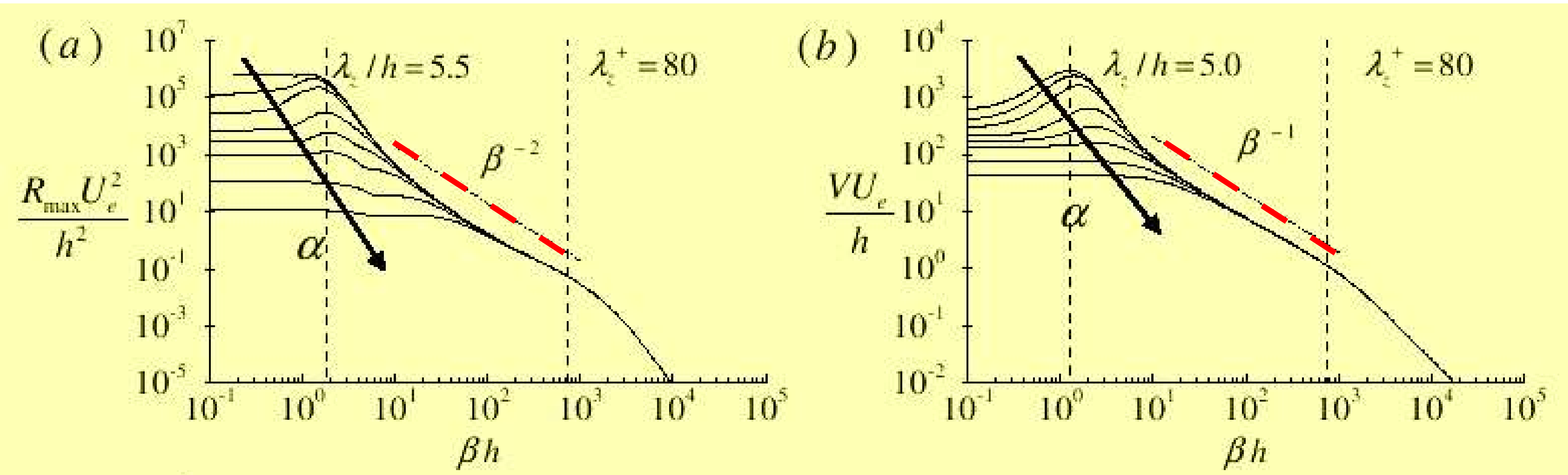
$$\mathbf{A}\mathbf{X}_{\infty} + \mathbf{X}_{\infty}\mathbf{A}^{\dagger} + \mathbf{B}\mathbf{B}^{\dagger} = 0.$$

algebraic Lyapunov eqn.

see e.g. Farrell & Ioannou *Phys. Fluids* 1993

Rmax & V for α=0 and Re_τ=10000

Harmonic forcing (Rmax) **Stochastic forcing (V)**



(approximate) β^{-2} intermediate régime

(approximate) β^{-1} intermediate régime

approximate power law régime

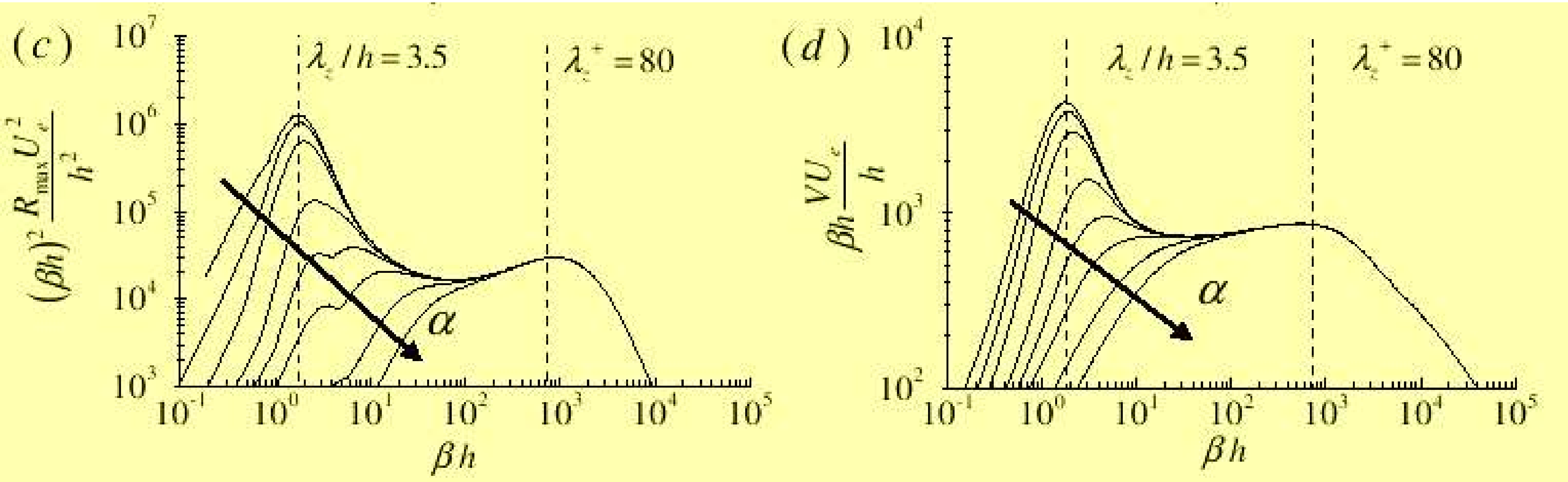
extreme limits: $\lambda_z^+ \approx 80$ and $\lambda_z \approx 5$ h

Hwang & Cossu , submitted to *JFM* (2010)

Premultiplied Rmax & V (same data)

deviations from (approximate) power law régime revealed on ***premultiplied* amplifications**

$\beta^2 R_{max}$ βV



outer units scaling peak: $\lambda_z = 3.5$ h

inner units scaling peak: $\lambda_z^+ \approx 80$

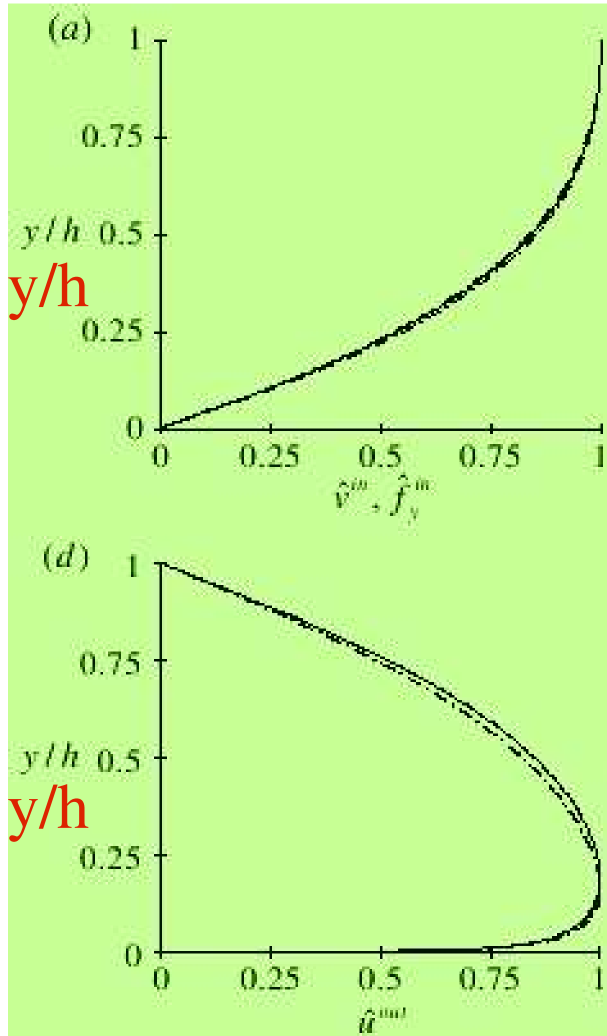
Hwang & Cossu , submitted to *JFM* (2010)

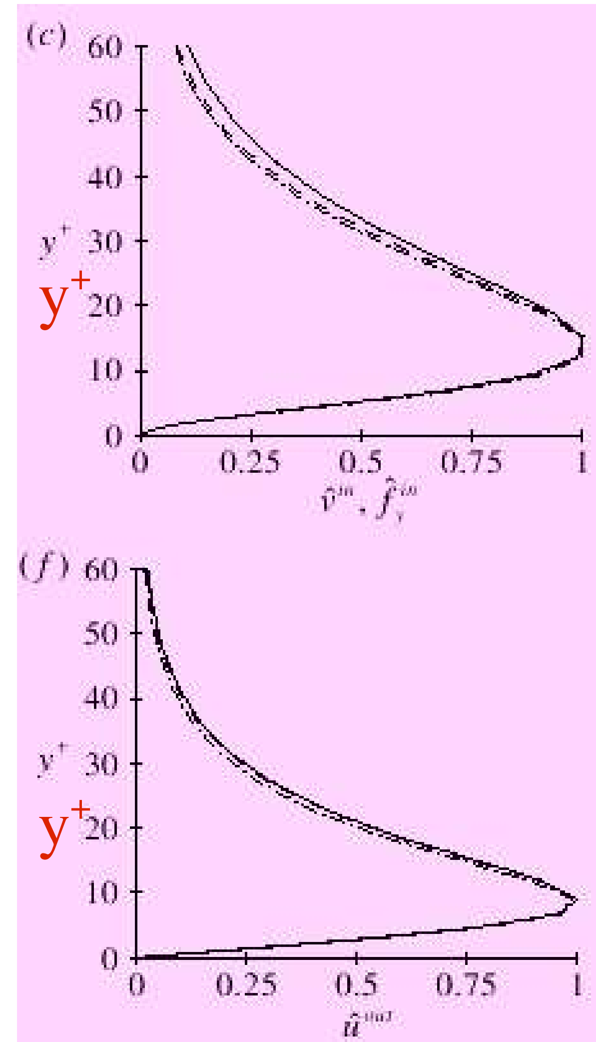
Wall-normal Fourier-mode profiles of optimals

outer peak λ_z=3.5-4 h | **inner peak λ_z^+=80**

v^ profile of the optimal input (forcing or IC): ~ vortices

û profile of the optimal output ~ streaks





Same shapes for Gmax, Rmax and V-KL1!

A question

Can we explain the intermediate régime (approximate) power scalings?

$$\mathbf{G_{max} \approx \beta^{0}\ ;\ V \approx \beta^{-1}\ ;\ R_{max} \approx \beta^{-2}}$$

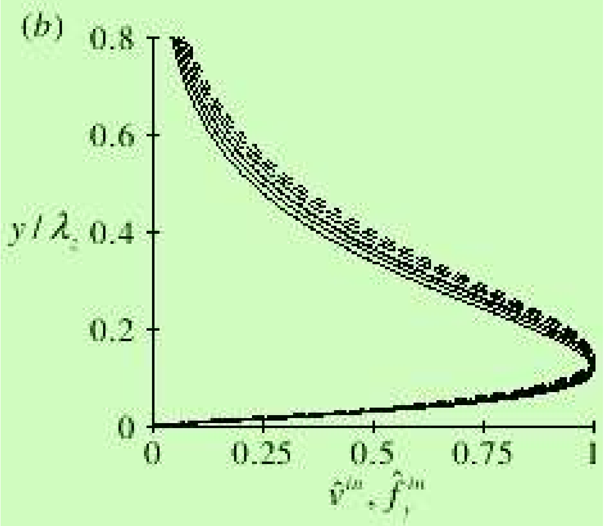
Geometrical similarity in the intermediate β region

In the intermediate region the relevant scale is λ_z ->

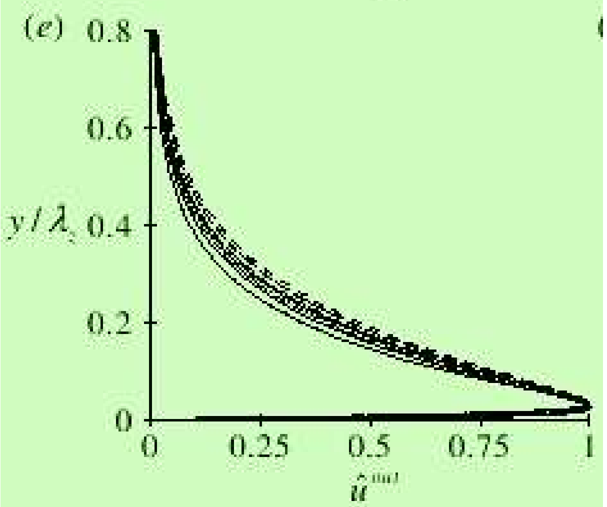
plot versus y/λ_z

(approximate) geometrical similarity observed

v^ of input for several β



û of output for several β



Rescaled system

$$\check{y} = \bar{\beta} y \qquad \check{\mathbf{q}} = \left[\hat{v}, \hat{\omega}_y / \bar{\beta}\right]^T (\check{y}, t) \qquad \bar{\beta} = (\beta / \check{\beta})$$

geometrical similarity assumption

$$\nu_T = \bar{\beta}^{-1} \check{\nu}_T$$

from log-layer assumption:

$\nu_T = u_\tau \kappa y$

=> $$\partial \check{\mathbf{q}} / \partial t = \bar{\beta} \check{\mathbf{A}} \check{\mathbf{q}} + \check{\mathbf{B}} \hat{\mathbf{f}}$$

$$\bar{t} = \bar{\beta} t$$

rescaled time

Only β dependence in the system

Evolution equation in rescaled variables

$$\partial \check{\mathbf{q}} / \partial \bar{t} = \check{\mathbf{A}} \check{\mathbf{q}} + \bar{\beta}^{-1} \check{\mathbf{B}} \hat{\mathbf{f}}.$$

Rescaling of initial value problem

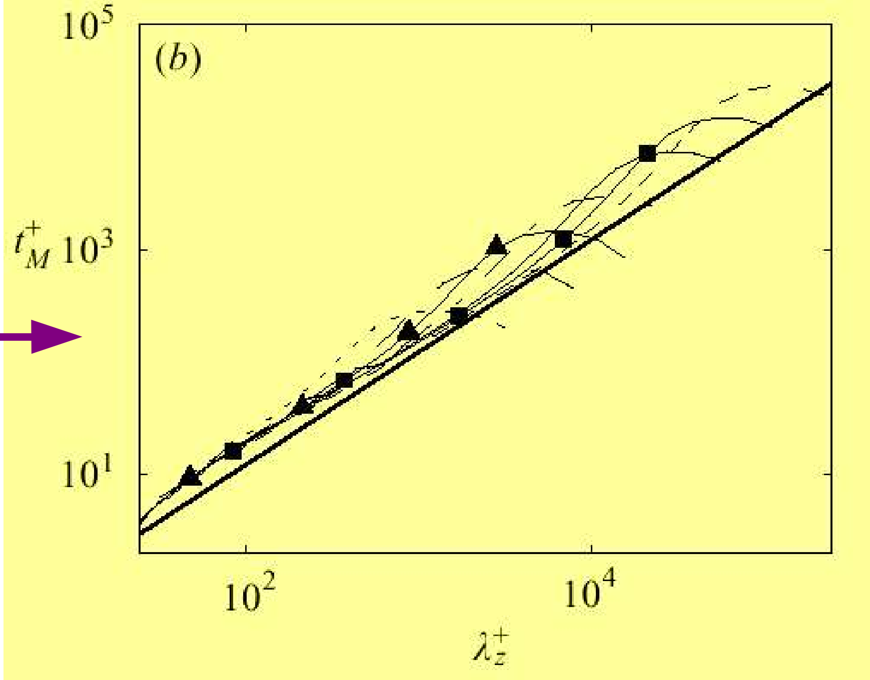
$$G = \max_{\hat{\mathbf{u}}_0} \|\hat{\mathbf{u}}\|^2/\|\hat{\mathbf{u}}_0\|^2 = \|\check{\mathbf{C}}e^{\bar{t}\check{\mathbf{A}}}\check{\mathbf{D}}\|^2.$$

$$G(\beta, t) = G(\check{\beta}, \bar{t}\check{\beta}/\beta).$$

<=> $G_{max} \approx \beta^0$

$$t_{max} = \bar{t}_{max}\check{\beta}/\beta$$

max G time proportional to λ_z



del Alamo & Jiménez *JFM* 2006

same in Pujals et al. PoF 2009 &

Cossu et al. JFM 2009,

Rescaling of response to harmonic forcing

$$\bar{\omega}_f = \omega_f / \bar{\beta}$$

$$\hat{\mathbf{f}} = \tilde{\mathbf{f}} e^{i\bar{\omega}_f \bar{t}}$$

$$\tilde{\mathbf{u}} = \bar{\beta}^{-1} \check{\mathbf{H}} \tilde{\mathbf{f}}$$

$$\check{\mathbf{H}} = \check{\mathbf{C}}(i\bar{\omega}_f \mathbf{I} - \check{\mathbf{A}})^{-1} \check{\mathbf{B}}$$

$$R = \bar{\beta}^{-2} \|\check{\mathbf{H}}\|^2$$

<=> $R_{max} \approx \beta^{-2}$

$$R(\beta, \omega_f) = (\check{\beta}/\beta)^2 R(\check{\beta}, \bar{\omega}_f \beta / \check{\beta}).$$

Rescaling of response to stochastic forcing

$$V = (1/2\pi) \int_{-\infty}^{\infty} trace(\mathbf{H}\mathbf{H}^{\dagger}) d\omega.$$

β^{-2} β^{1}

$$V = (\bar{\beta}^{-1}/2\pi) \int_{-\infty}^{\infty} trace[(\check{\mathbf{H}}\check{\mathbf{H}}^{\dagger}) d\bar{\omega}$$

$$V(\beta) = (\check{\beta}/\beta) V(\check{\beta}).$$

<=> $V \approx \beta^{-1}$

Summary

* **Approximate power law régime for log-layer geometrically similar structures**
* **Two peaks in the deviation from this régime**
* **Forced response can be very large if at large scale**
* **Same optimal perturbations for all frameworks (new?)**
* **Large-scale forced peaks -> large wall influence**

* **Current work: extension to low α/β régime -> (approximate) k^{-1} dependence in the log régime?**

  (relation with Perry & Chong, Perry & Marusic, ...?)

THANK YOU FOR YOUR ATTENTION

http://yakari.polytechnique.fr/people/carlo/

http://www.imft.fr/page_perso/ccossu/