Evolution and lifetimes of flow topology in a turbulent boundary layer

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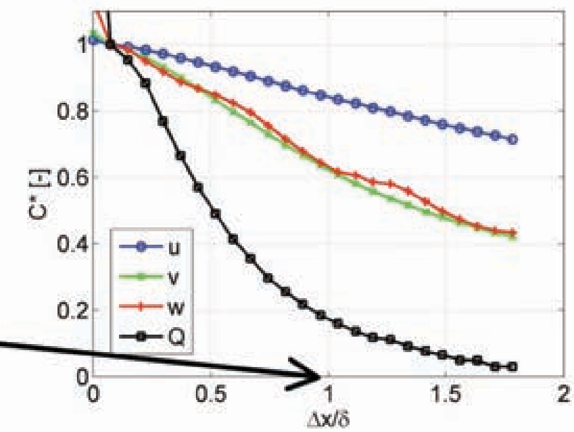
Temporal behavior coherent structures in the outer layer

Convection and lifetimes

- space-time correlations suggest
  short time-scale for local flow topology
  ($\sim\delta/U_e$)
- characteristic eddy lifetime in visualizations
  ($>\delta/U_e$, beyond measurement domain)
  => need a measure

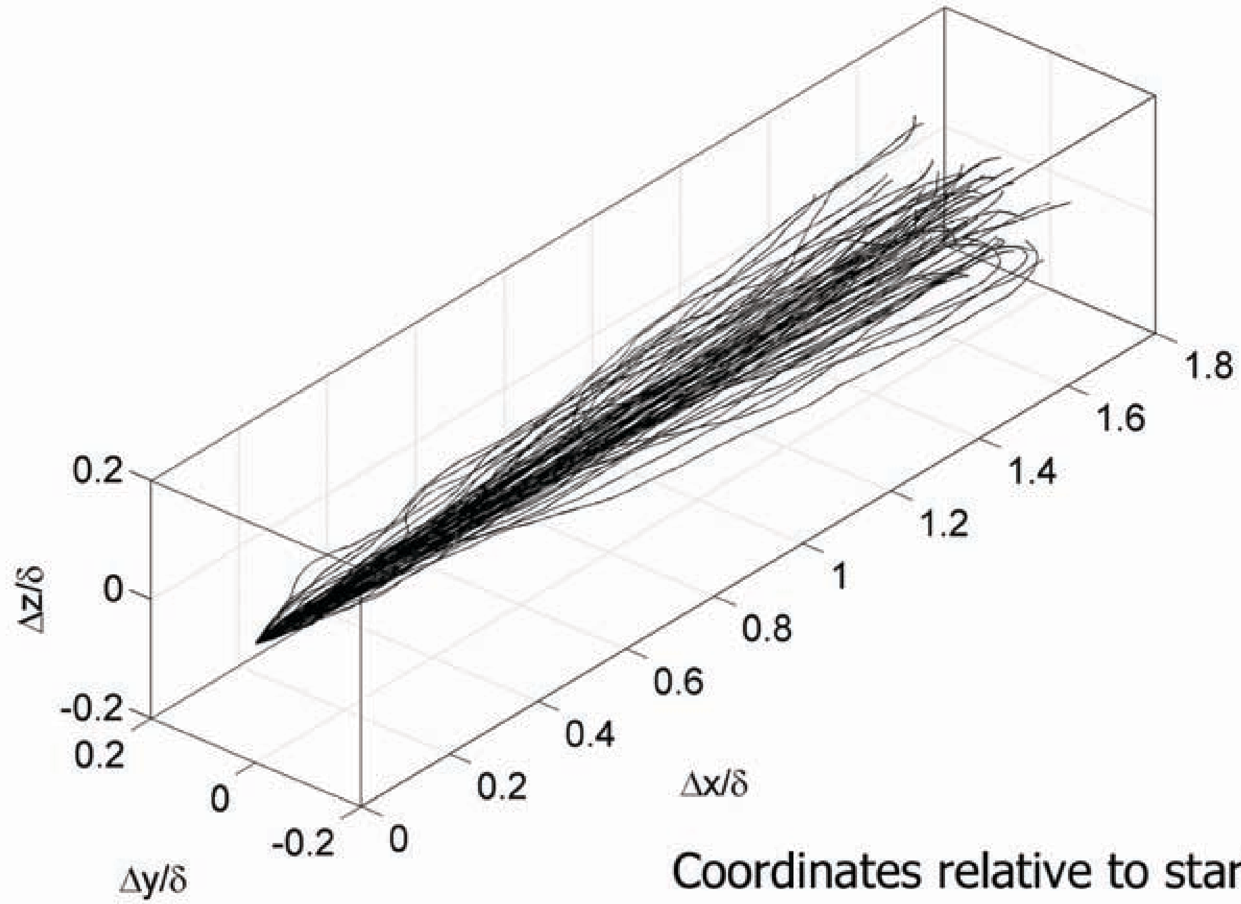


Dynamics

- Temporal evolution (outer layer) flow topologies

Movement flow structure in time

38 trajectories



Coordinates relative to starting position

Movement flow structure in time

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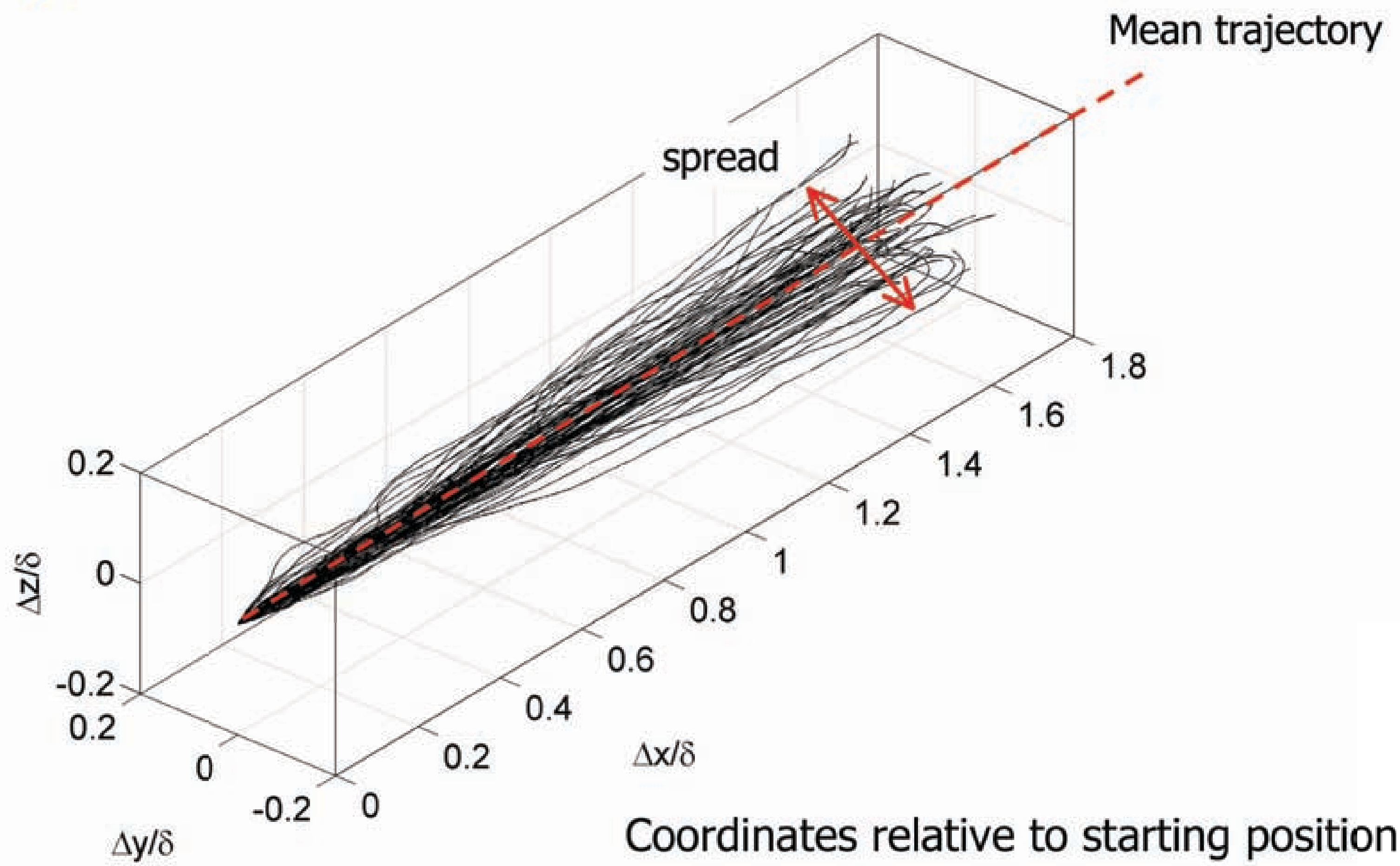

Coordinates relative to starting position

Movement flow structure in time

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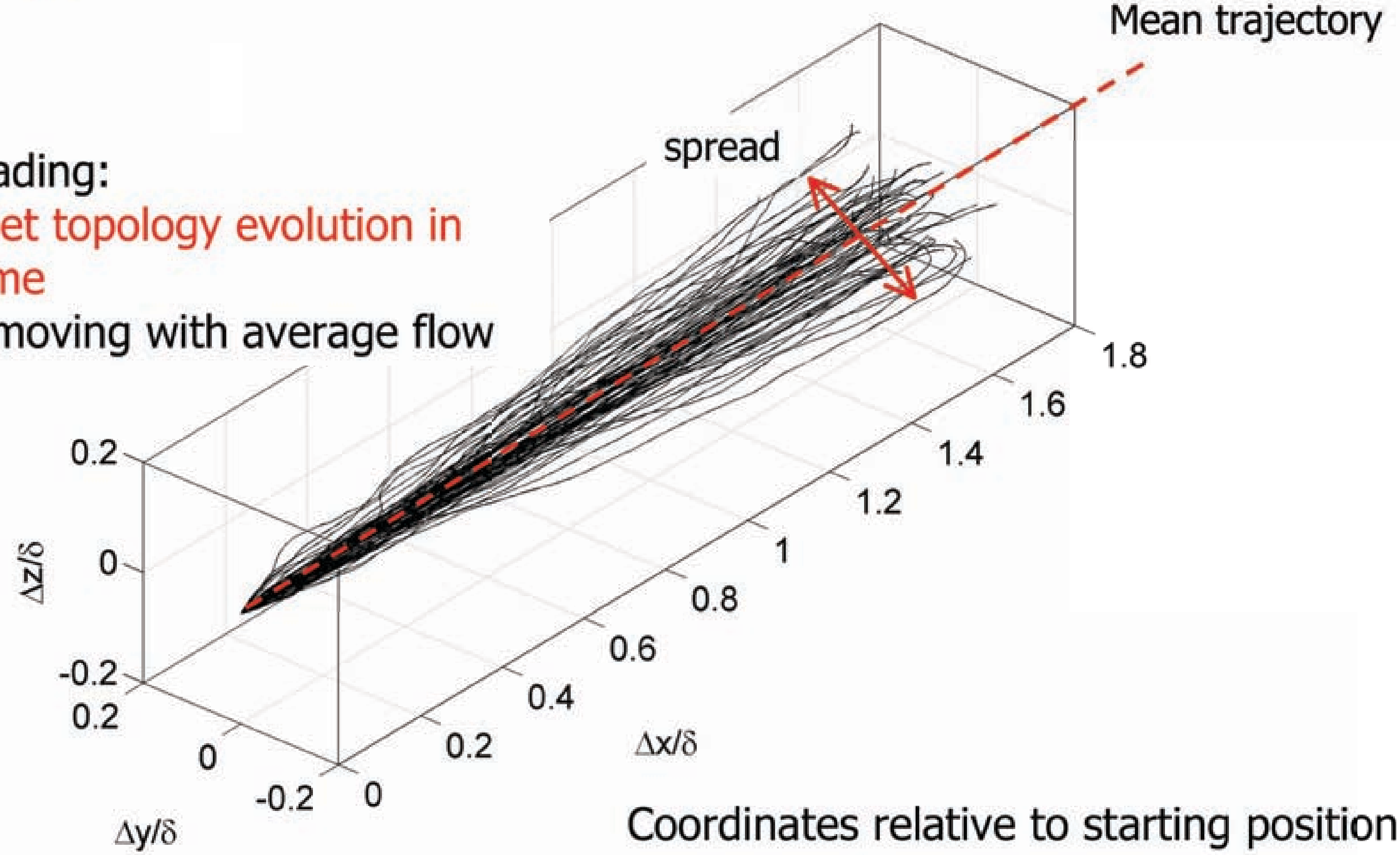
Due to spreading:
Difficult to get topology evolution in Eulerian frame
Even when moving with average flow



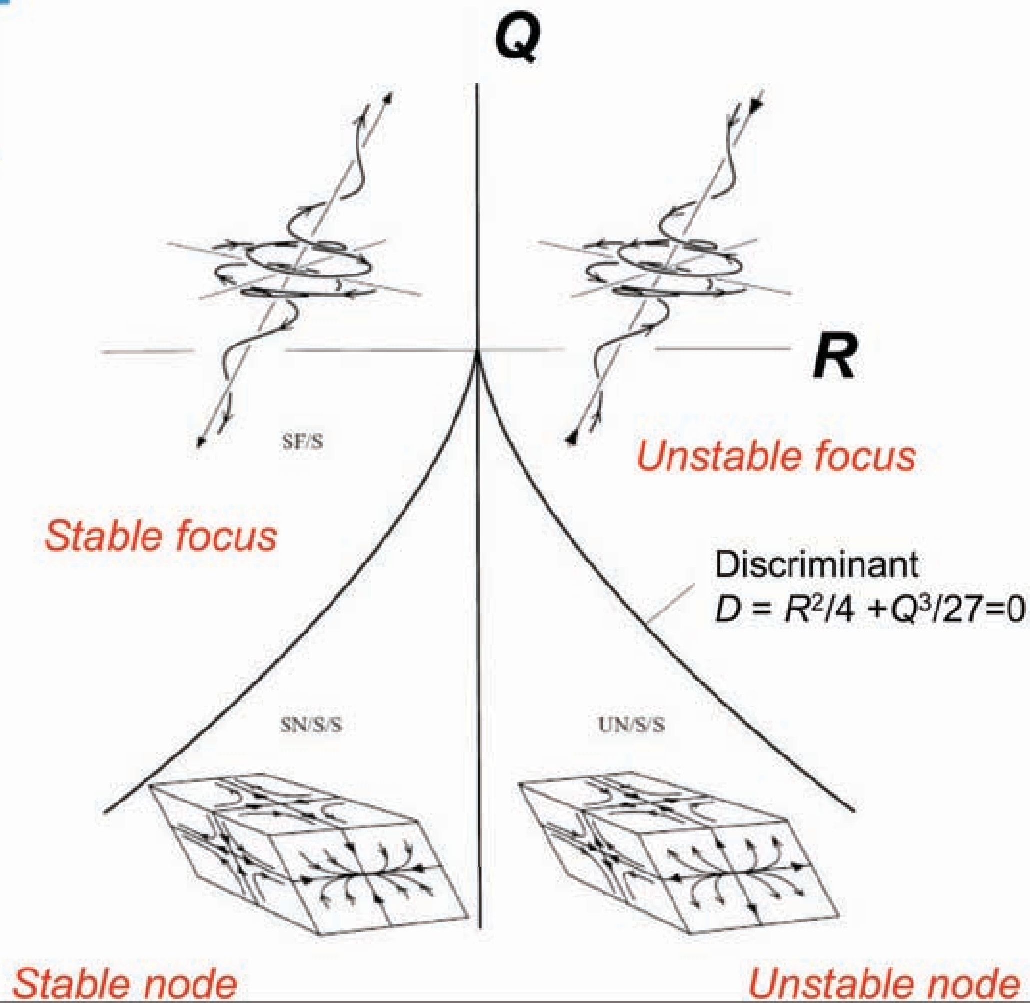
Coordinates relative to starting position

Aim

Obtain average dynamic evolution of local flow topology in a Lagrangian frame of reference

Estimate a characteristic topology lifetime *(which cannot be inferred from Eulerian correlation functions due to important variations in convection velocities)*

Quantifying local flow topology



Invariants of the velocity gradient tensor *A*

[Chong et al. 1990 Phys Fluids]

For incompressible flow:

$$P = -A_{ii} = 0$$

$$Q = -\frac{1}{2} A_{ij} A_{ji}$$

$$R = -\frac{1}{3} A_{ij} A_{jk} A_{ki}$$

$$A = \nabla \vec{V}$$

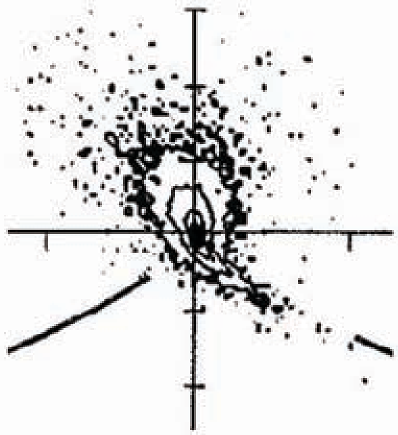
[Ooi et al 1999 JFM]

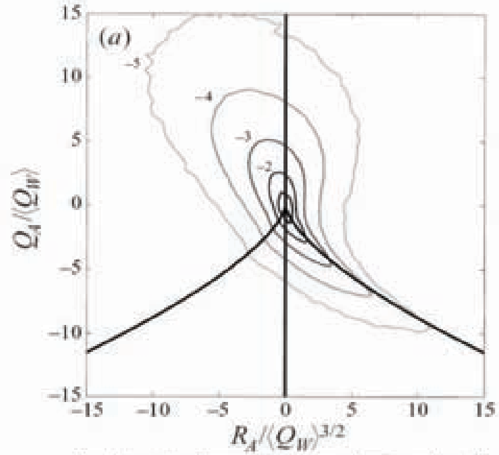
Universality in instantaneous QR distribution (joint-PDF)

Mixing layer

DNS

[Soria et al 1994 PoF]

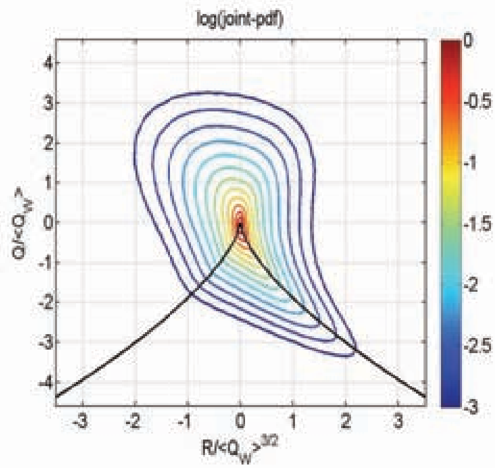


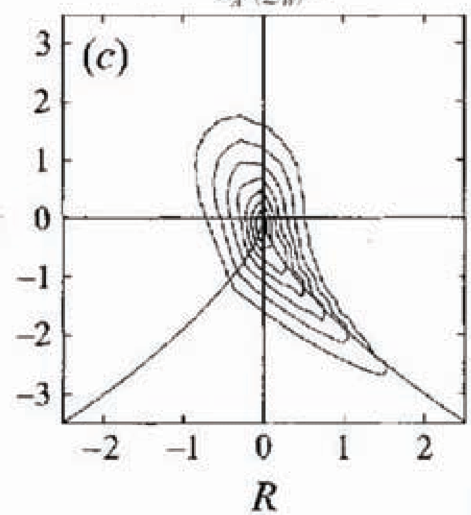


Isotropic Turbulence

DNS

[Ooi et al 1999 JFM]





Boundary layer

3D-PIV, present data

Channel

DNS

[Blackburn et al 1996 JFM]

Topology dynamics

Topology dynamics

A method based on invariants *QR*

Previously used in DNS of isotropic turbulence [Ooi et al 1999 JFM]

Conditional averaging of

the invariants rates of change following a fluid particle:

Topology dynamics

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Conditional averaging of

the invariants rates of change following a fluid particle:

$$\left\langle \frac{DQ}{Dt} \right\rangle (Q_0, R_0) = \left\langle \frac{DQ}{Dt} \middle| -\Delta Q/2 \le (Q - Q_0) < \Delta Q/2; -\Delta R/2 \le (R - R_0) < \Delta R/2 \right\rangle$$

$$\left\langle \frac{DR}{Dt} \right\rangle (Q_0, R_0) = \left\langle \frac{DR}{Dt} \middle| -\Delta Q/2 \le (Q - Q_0) < \Delta Q/2; -\Delta R/2 \le (R - R_0) < \Delta R/2 \right\rangle$$

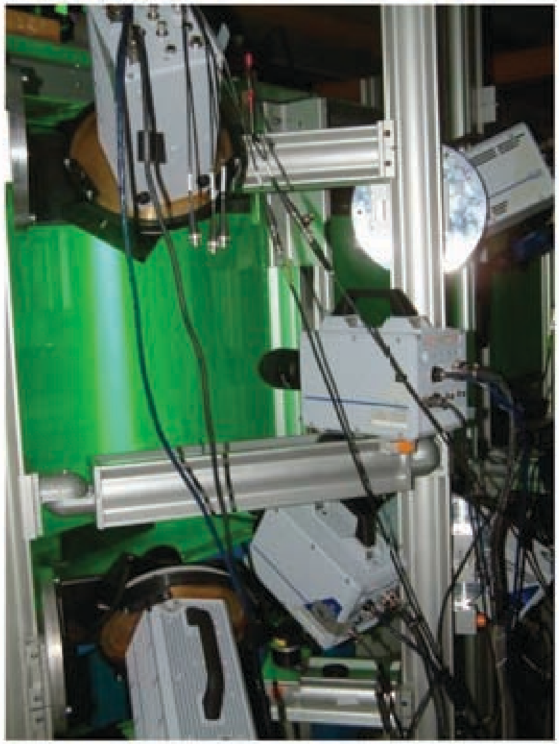
(Q_0, R_0) bin center ΔQ, ΔR bin width

Experimental dataset

Time-resolved 3-D velocity (tomographic-PIV)

- $Re_\theta = 2460$, $Re_\tau = 800$
- $0.1 < z/\delta < 0.3$ (z - wall normal coordinate)
- medium: water
- free stream velocity 0.53 m/s
- boundary layer thickness 37 mm
- spatial resolution 55 wall units
- measurement frequency 1 kHz

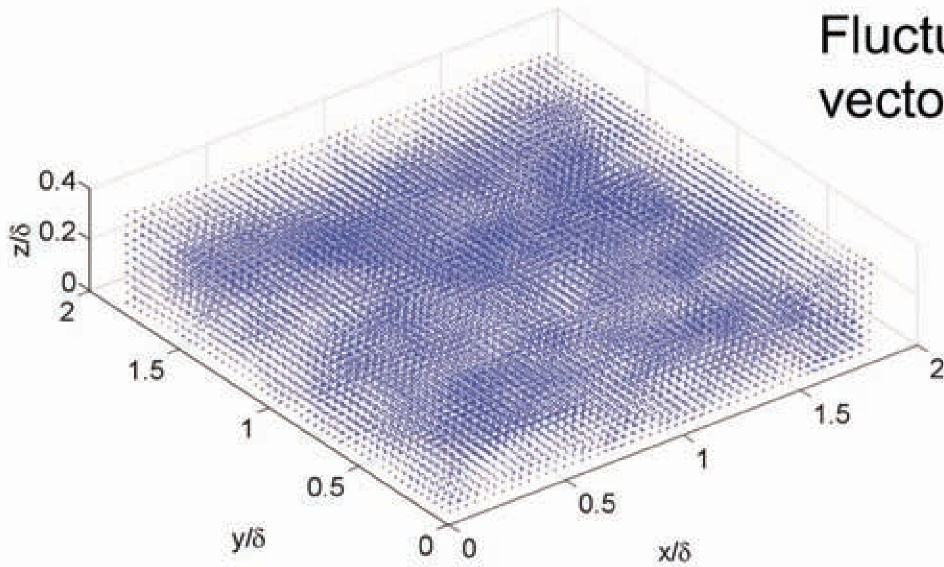
  Corresponding to 10 wall units convection distance



[Schröder et al., Exp fluids, in prep]

TU Delft DLR

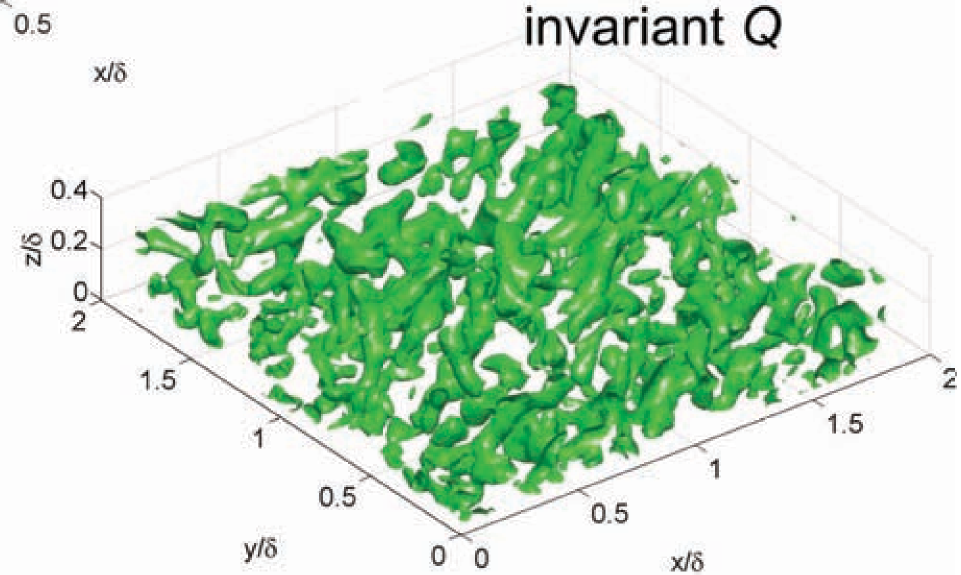
Typical result



Fluctuating velocity vectors

U_e

Iso-surface second invariant Q



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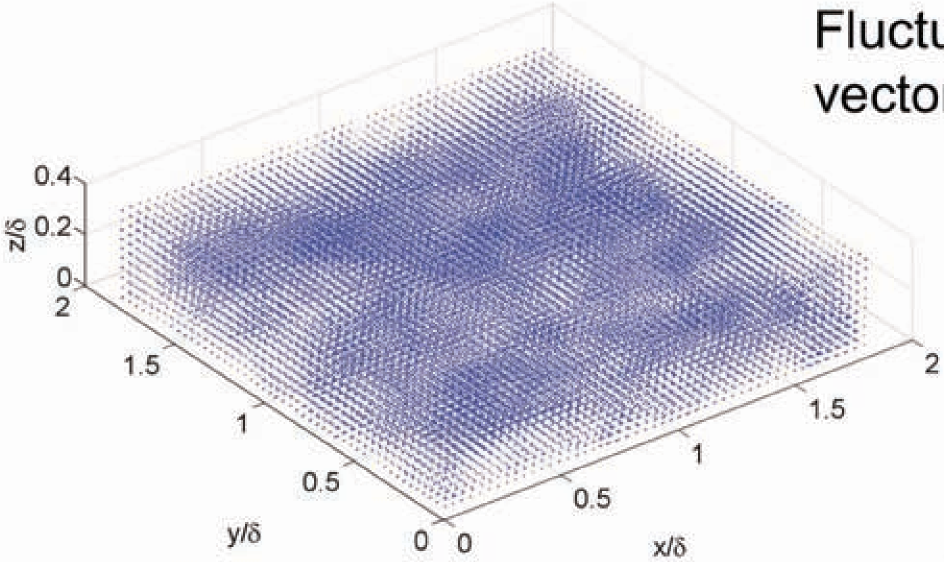
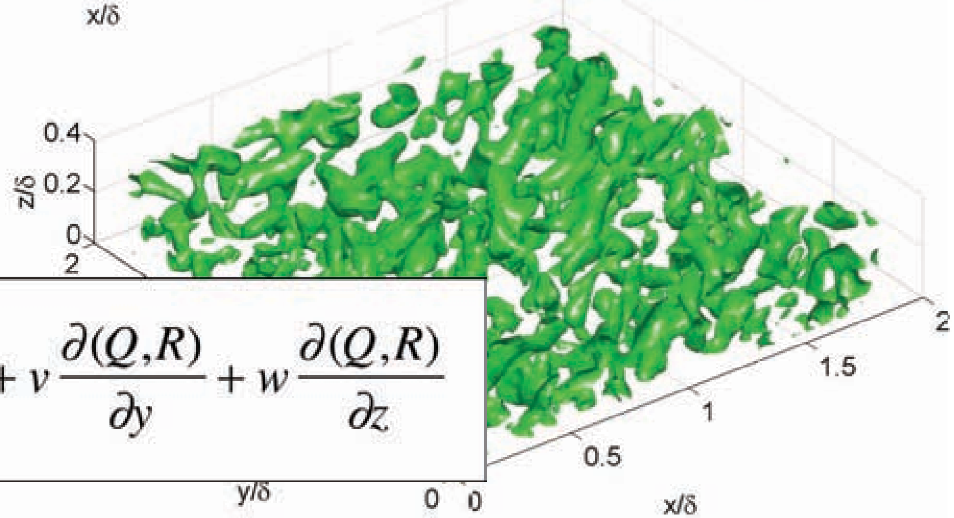
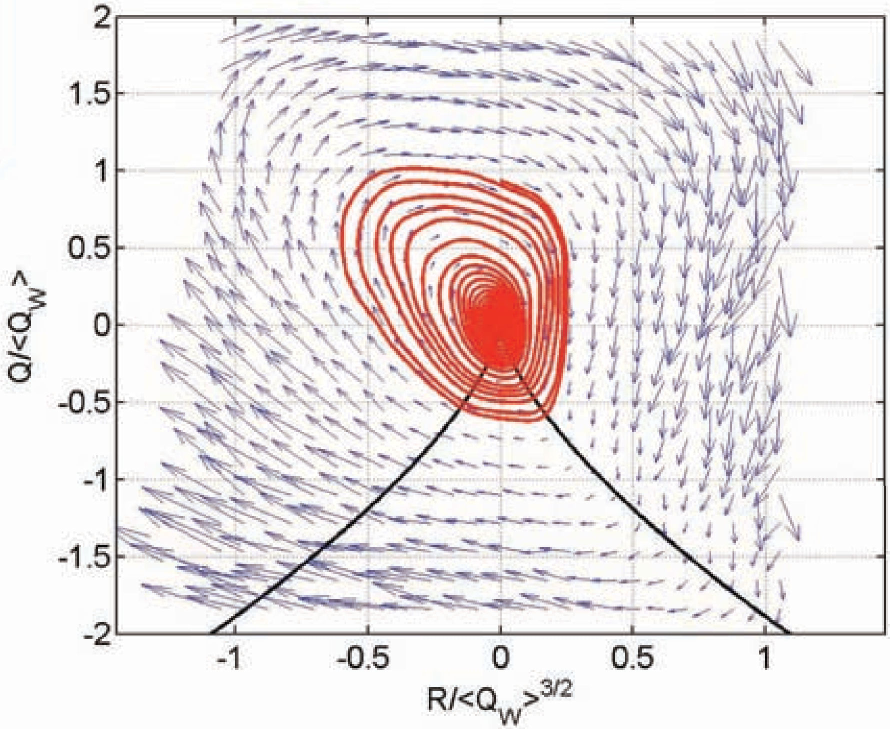


Iso-surface second invariant *Q*



$$\frac{D(Q,R)}{Dt} = \frac{\partial(Q,R)}{\partial t} + u\frac{\partial(Q,R)}{\partial x} + v\frac{\partial(Q,R)}{\partial y} + w\frac{\partial(Q,R)}{\partial z}$$

QR average dynamics



Vectors represent:

$$\left\langle \frac{DQ}{Dt} \right\rangle (Q,R)$$

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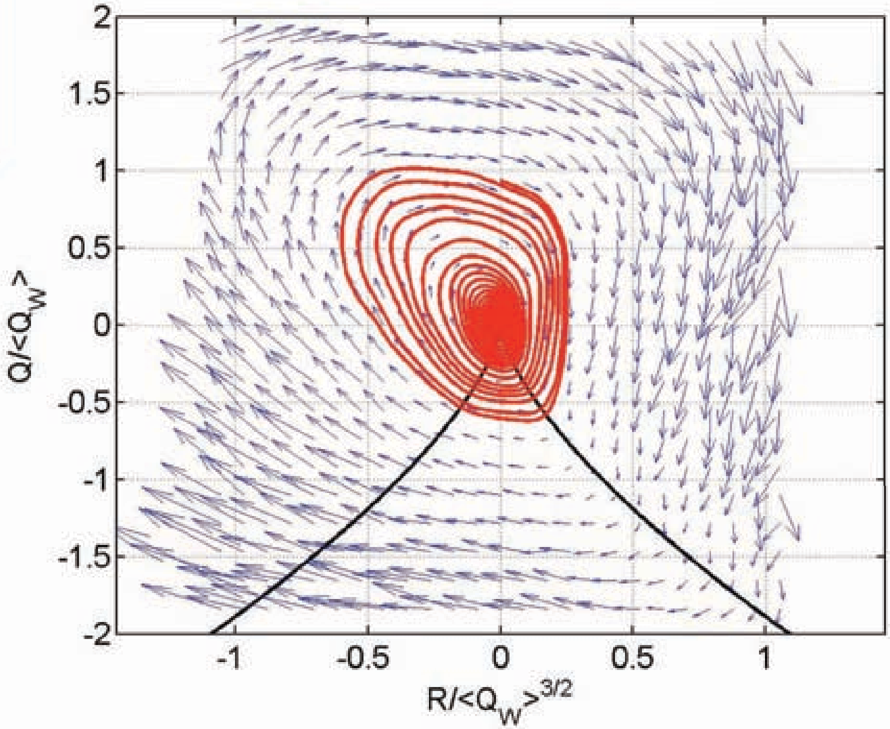
QR average dynamics



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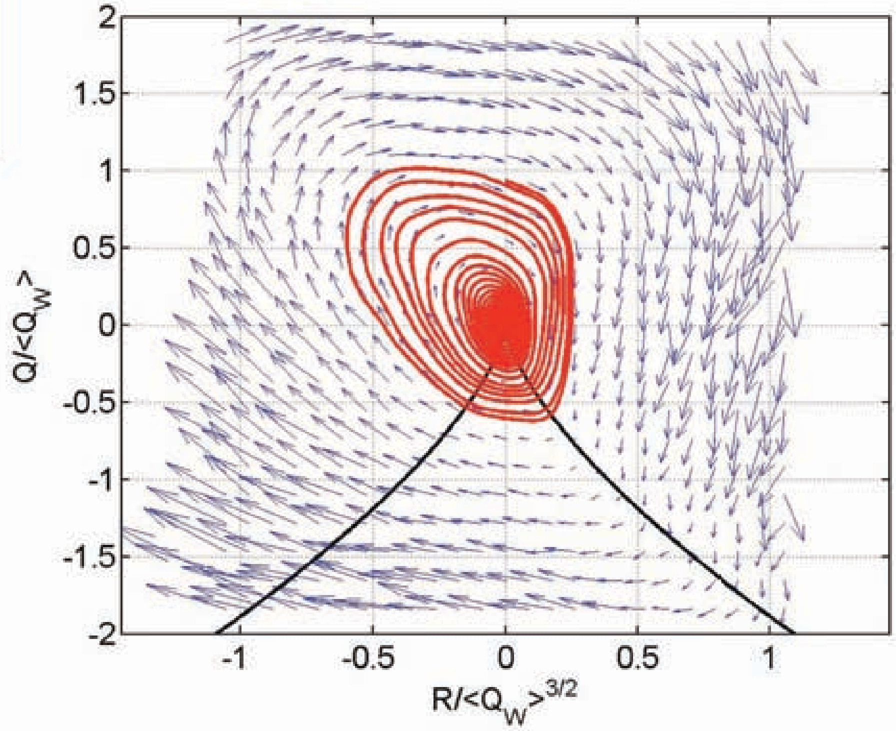
Spiraling orbits:

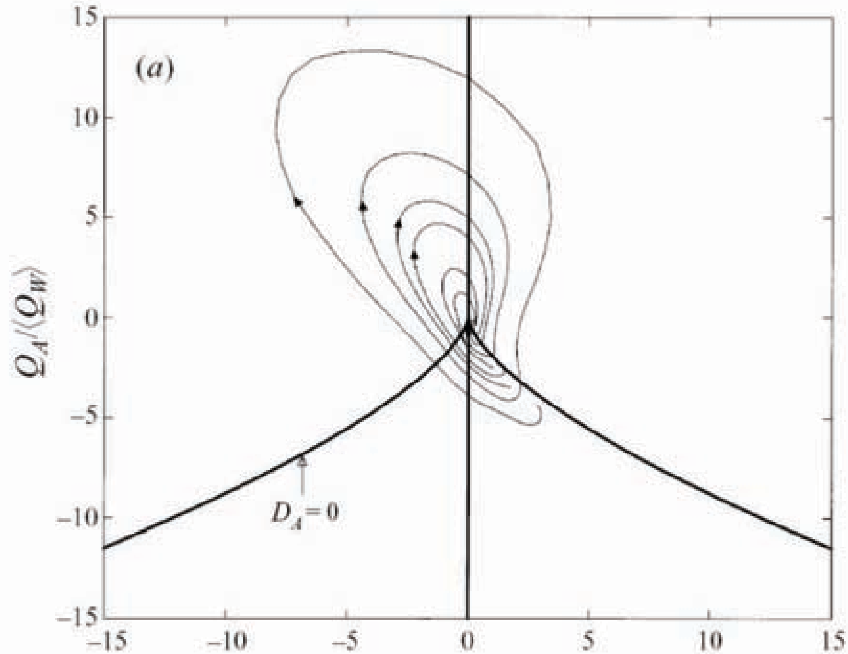
Stable focus → *Unstable focus*

↑ ↓

Stable node ← *Unstable node*

QR average dynamics





DNS of isotropic turbulence
(Note difference in scale)
[Martin et al 1998 PoF
Ooi et al 1999 JFM]

Vectors represent: $\left\langle \frac{DQ}{Dt} \right\rangle (Q,R)$ $\left\langle \frac{DR}{Dt} \right\rangle (Q,R)$

Spiraling orbits:

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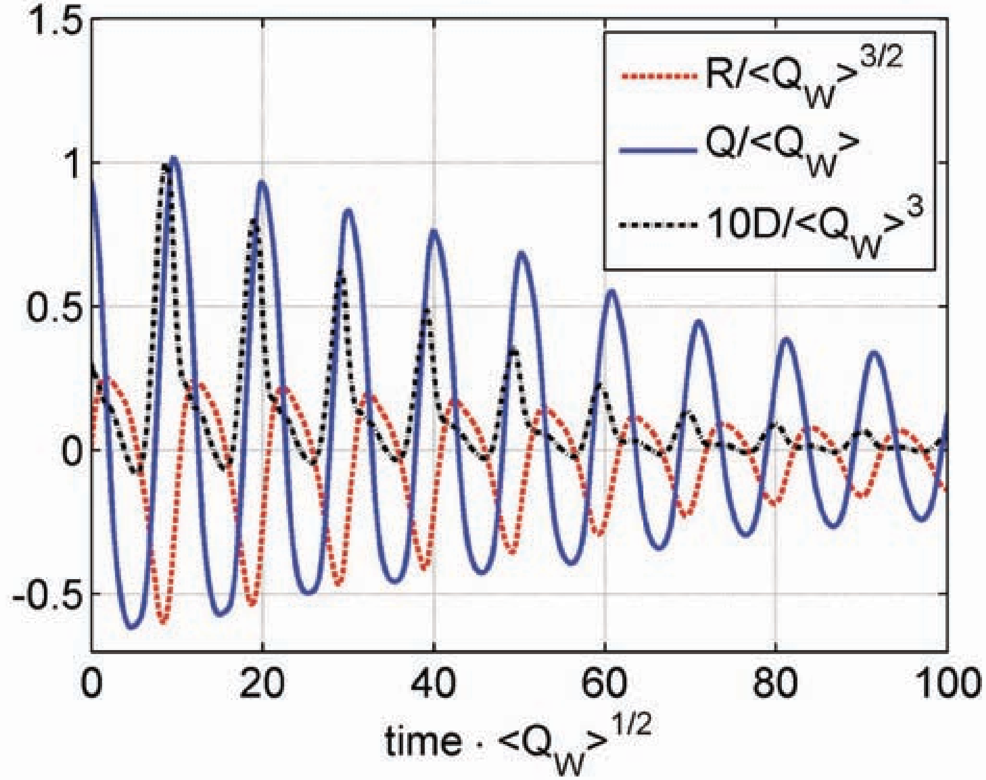
Characteristic lifetime: the *QR* orbit's period



Orbit's period nearly constant

Characteristic lifetime of the eddies

14 δ/U_e

10 $\langle Q_W \rangle^{-1/2}$

compare with 15 $\langle Q_W \rangle^{-1/2}$

reported for isotropic DNS

[Ooi et al 1999 JFM]

Viscous decay rate >300 δ/U_e

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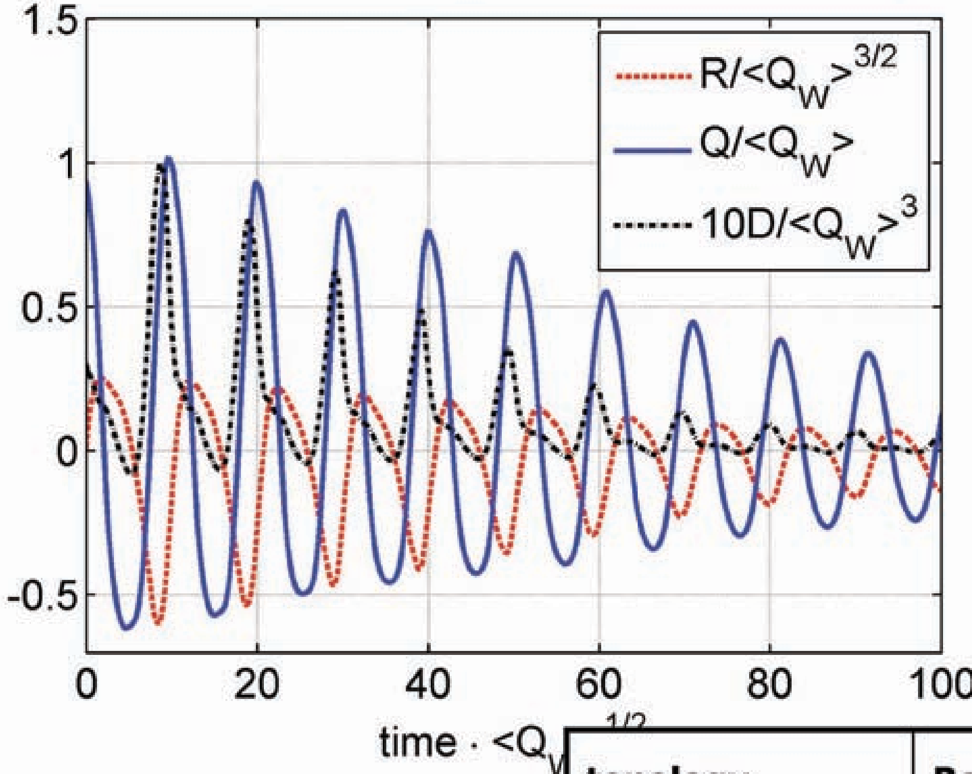

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Time fraction of the orbit:

topology	Boundary layer	Isotropic turb.
Stable focus	39%	22%
Unstable focus	38%	20%
Unstable node	15%	53%
Stable node	8%	6%

Dynamic Model for *QR*

Navier-Stokes rewritten (Cantwell 1992):

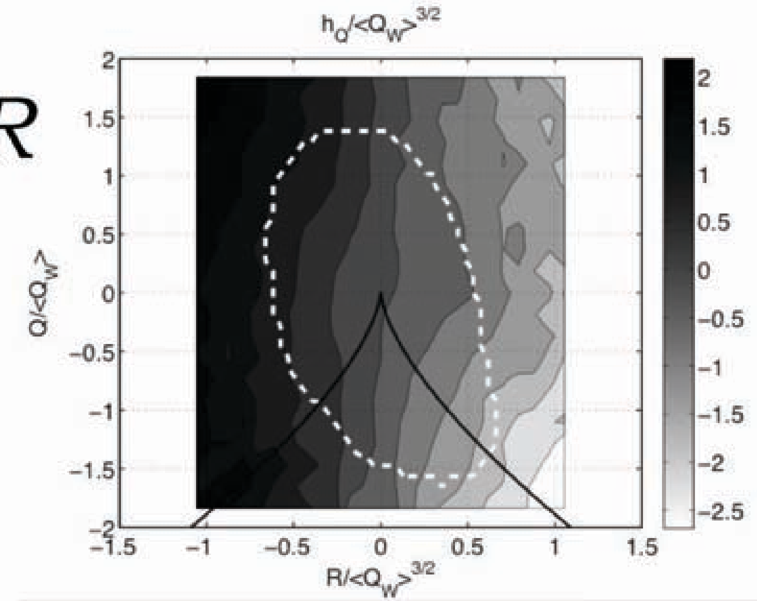
$$\left\langle \frac{DQ}{Dt} \right\rangle (Q,R) = -3R - h_Q(Q,R)$$

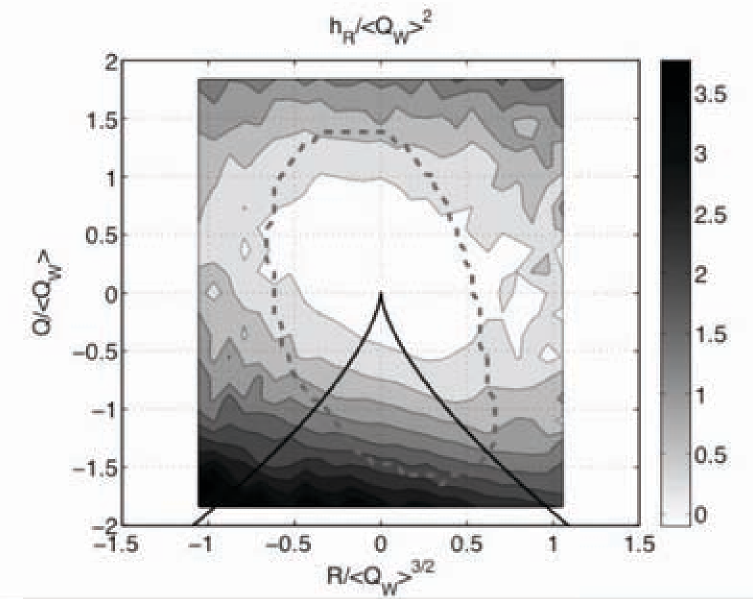
$$\left\langle \frac{DR}{Dt} \right\rangle (Q,R) = \tfrac{2}{3}Q^2 - h_R(Q,R)$$

↑ *Reduced Euler*

↑ *Eddy interactions trough pressure and viscous forces*

(are countering the reduced Euler behavior)





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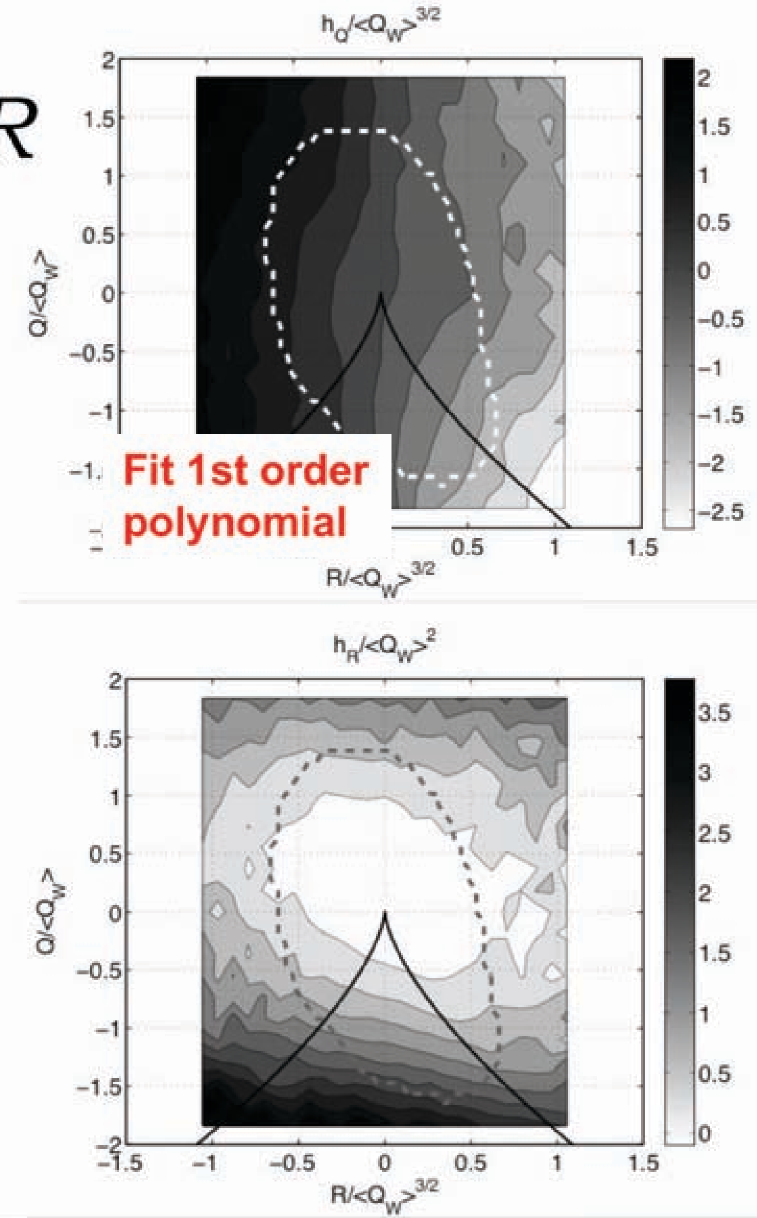

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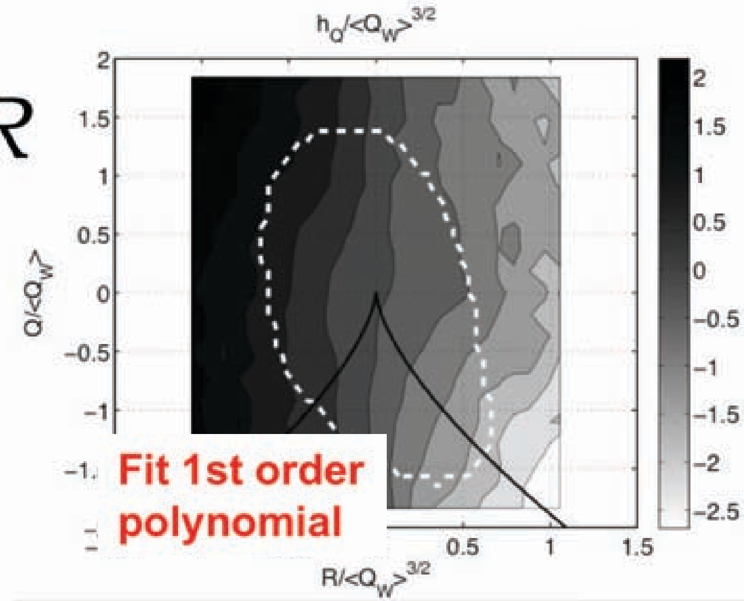

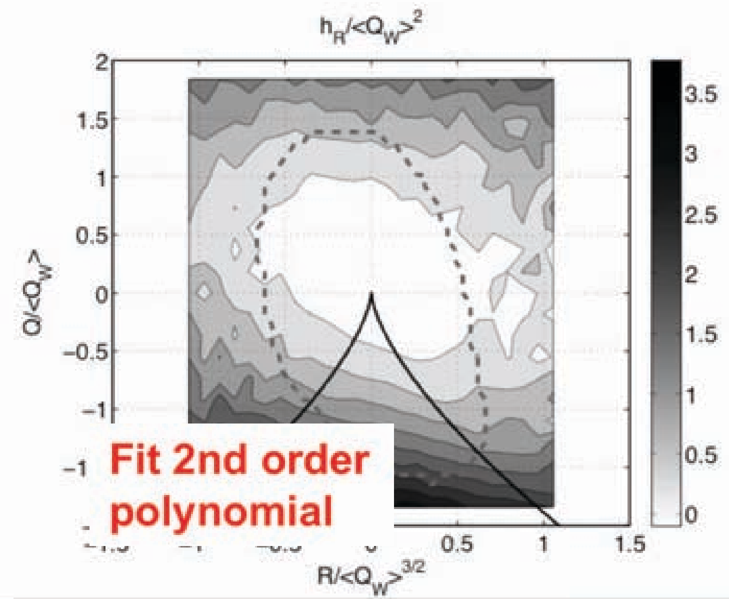

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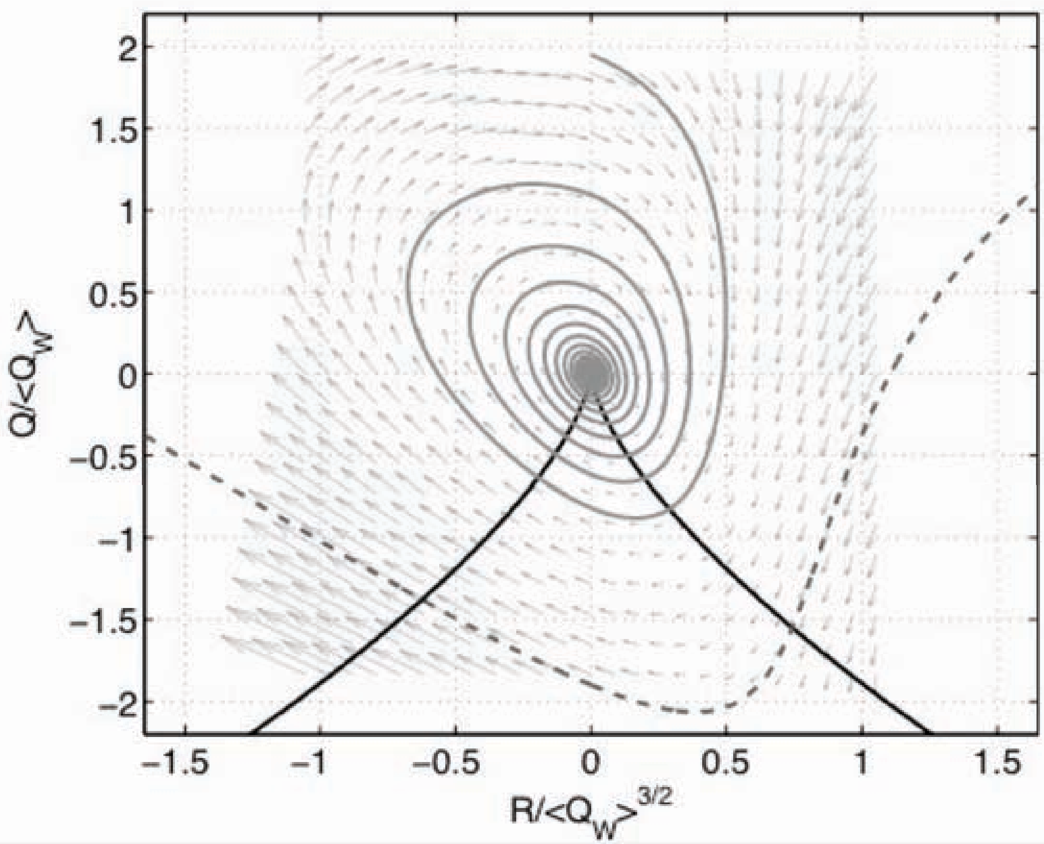
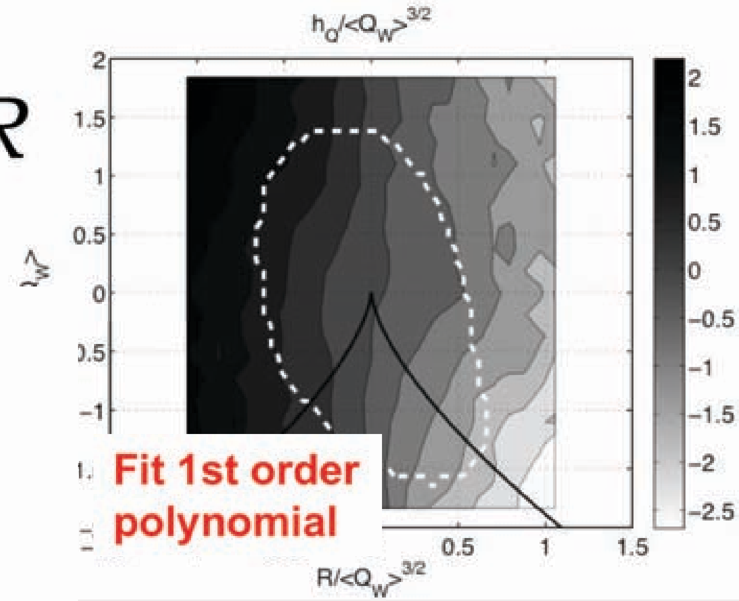
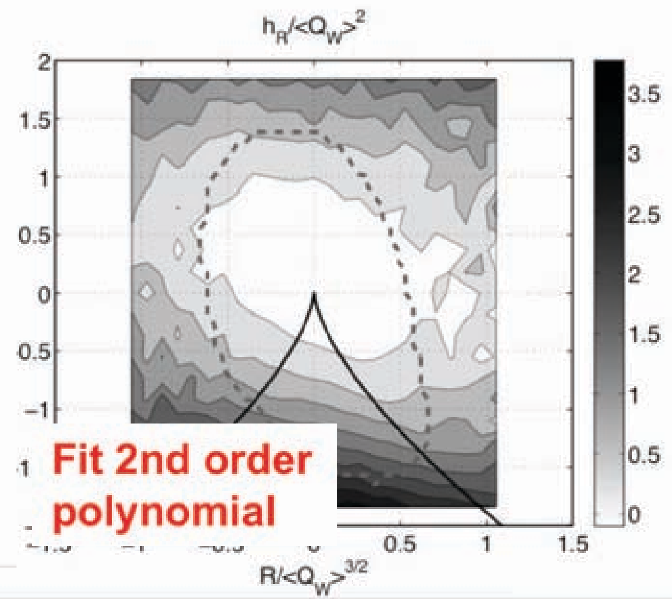
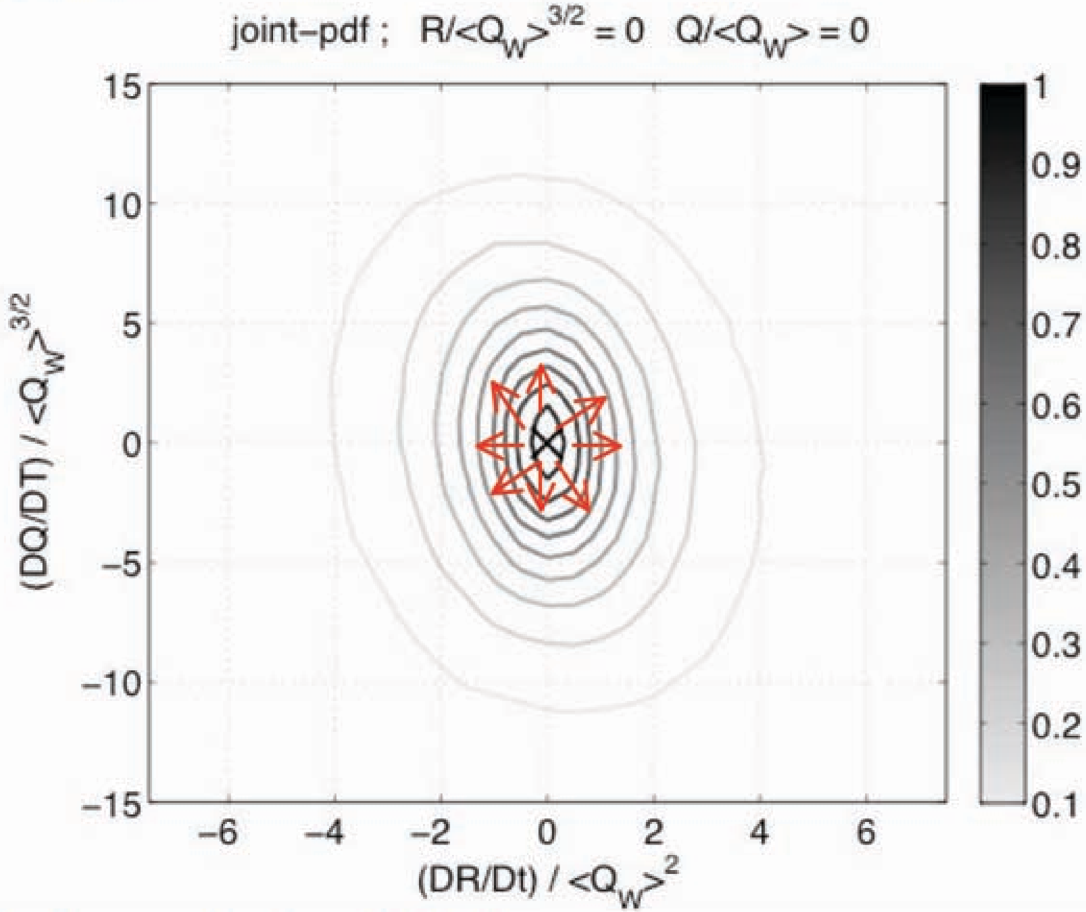






What happens when Q=R=0 is reached?



Possible scenario:

Q=R=0 represents degenerate topologies like

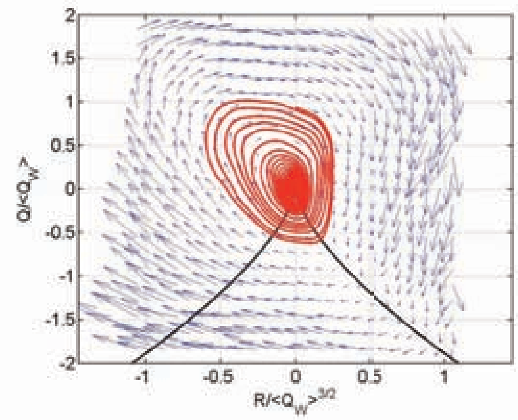
shear layer (du/dy≠0 => Q=R=0) roll-up into vortex with saddle point

TU Delft

Conclusions

Experimentally determined conditional average rate of change of the velocity gradient tensor invariants *QR*

in the outer region of a turbulent boundary layer



Integration reveals inward, clockwise spiraling orbits

The orbit's period 14 δ/U_e

is considered to be characteristic eddy lifetime

corresponding spatial wavelength 11δ *(constant convection velocity)*

Results have been summarized in an empirical model

[Elsinga & Marusic, 2010 Phys Fluids]