Bound states and E_8 symmetry effects in perturbed quantum Ising chains

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First experimental evidence of E_8 - symmetry

Quantum criticality in an Ising chain:

Experimental evidence for emergent E_8 Symmetry

[Coldea 10] R. Coldea, D. A. Tennant, E. M. Wheeler, E. Wawrzynska, D. Prabhakaran, M. Telling, K. Habicht, P. Smeibidl and K. Kiefer, **Science** 326, 177 (2010).

- Theoretically derived in a 2D classical Ising model

Integrals of motion and S-matrix of the (scaled) $T=T_c$ Ising model with magnetic field

[Zamolodchikov 89] A. B. Zamolodchikov**, Int. J. Mod. Phys. A** 4, 4235 (1989).

Time Evolving Block Decimation (TEBD)

- Numerical simulation of 1D quantum system
- Based on a matrix product state (MPS) representation
- Descendant of the DMRG algorithm
- Ground state by time evolution in imaginary time

$$|\psi_0\rangle = \lim_{\tau\to\infty} \frac{e^{-H\tau}|\psi_i\rangle}{||e^{-H\tau}|\psi_i\rangle||}$$

Real time evolution

$$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$$

- Dynamical structure function – neutron scattering data

$$S(q,\omega) = \sum_x \int_{-\infty}^{\infty} dt e^{-iqx} e^{i\omega t} C(x,t)$$

$$C(x,t) = \langle \psi_0 | S_x^-(t) S_0^+(0) | \psi_0 \rangle$$

- Computationally hard to simulate long enough times
  - Calculate C(x,t) for every 10th time steep and interpolate its value between
  - ”Light-cone” like spread of the entanglement
  - Linear prediction

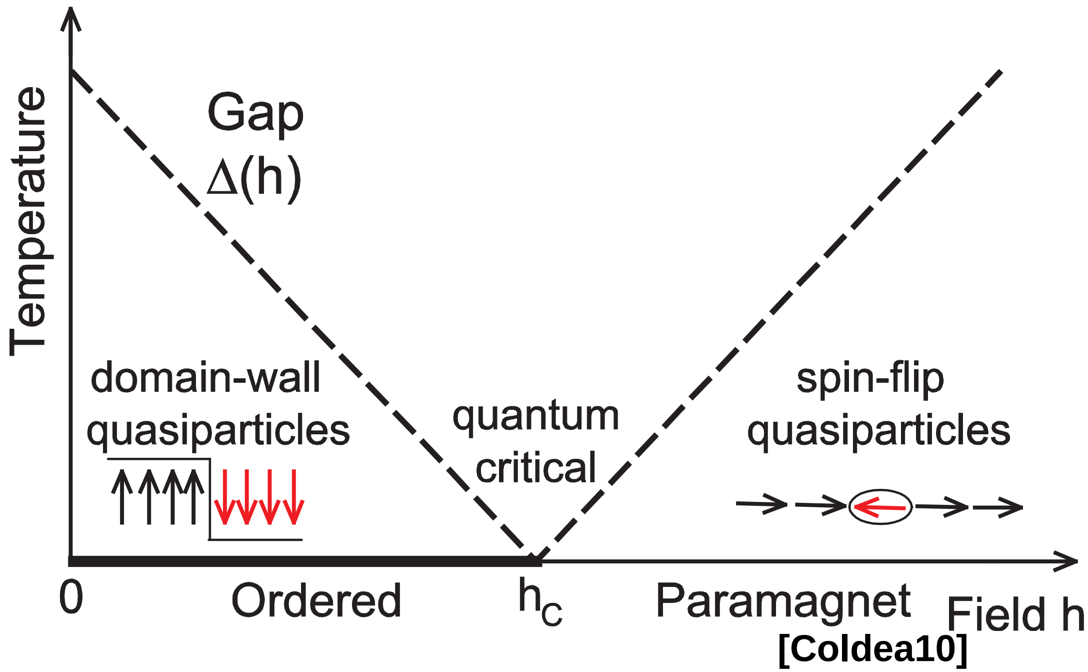
Quantum Ising chain

$$H = -J \sum_n S_n^z S_{n+1}^z - h^x \sum_n S_n^x$$

- J > 0 favors a ferromagnetic state

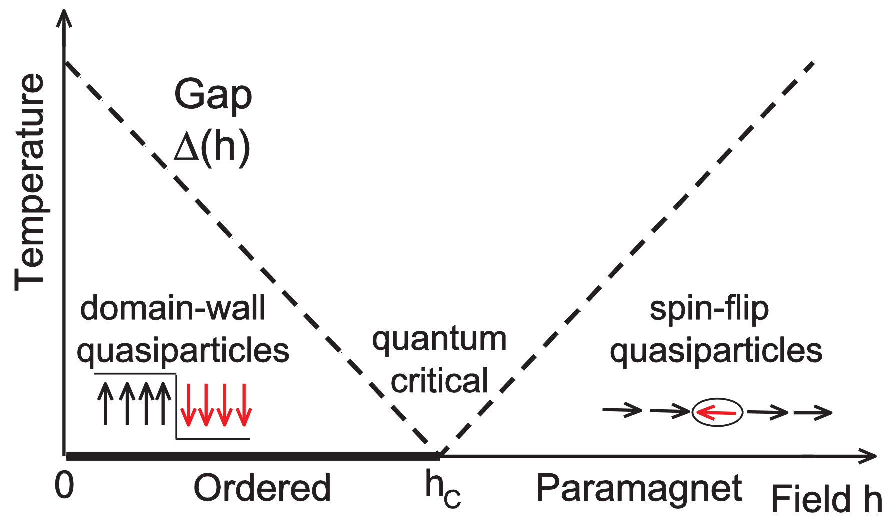
| ↑↑ ... ↑> or | ↓↓ ... ↓>

- Phase transition to a paramagnetic state | →→ ... → > at QCP $|h_{xc}| = J/2$,



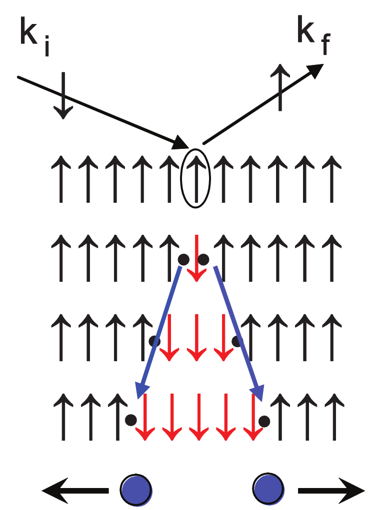
Excitations

- Elementary excitation
  - domain wall or kink (ferromagnetic phase)
  - spin flip (paramagnetic phase)



- Excitation gap closes at h_c

- Experimentally not possible to create one kink
  Spin flip ➡ two freely moving kinks



Longitudinal field

- Breaks the two-fold degeneracy of the ferromagnetic state

- Opens up a gap

$$H_L = -h^z \sum_n S_n^z$$

- Moves the minimum gap to higher longitudinal field

- Confines the kinks into bound states

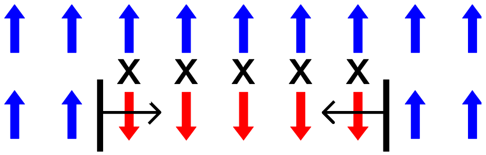
Confinement into bound states

- Ferromagnetic interchain coupling

- A longitudinal field

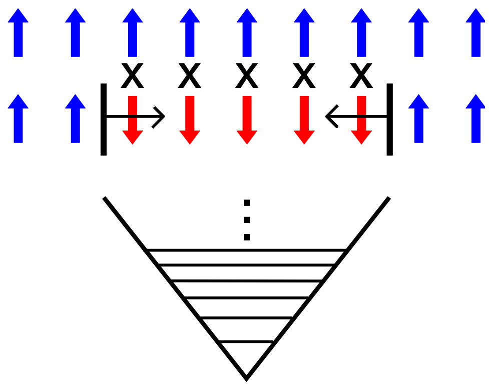
➡ confines the kinks into bound states

➡ splits up the excitation continuum



Analytical solution

- At low transverse field and small bound state momentum



➡ 1 D Schrödinger equation with a linear confining potential

➡ Energy levels are the zeros of the Airy function

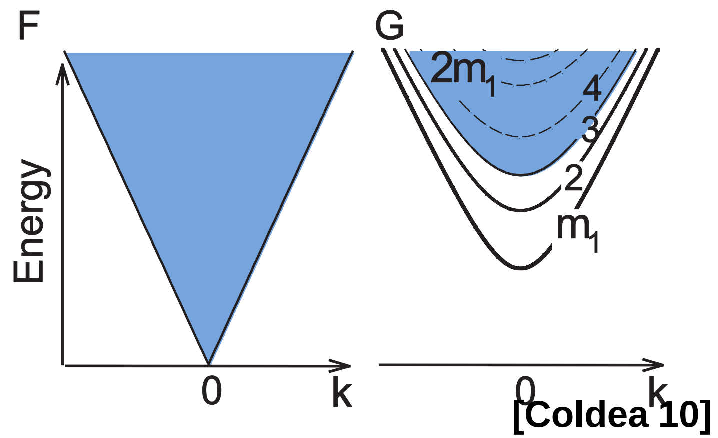
Analytical solution - Hidden E_8 symmetry

- Close to the QCP

  - solvable by CFT

  8 bound state masses

  (and sums of them)



- m_i / m_1

1.000	1.618 ← golden ratio
1.989	2.405
2.956	3.218
3.891	4.783

“Spinon jets”

- Additional bound state

- High energy bound state – kinks far apart

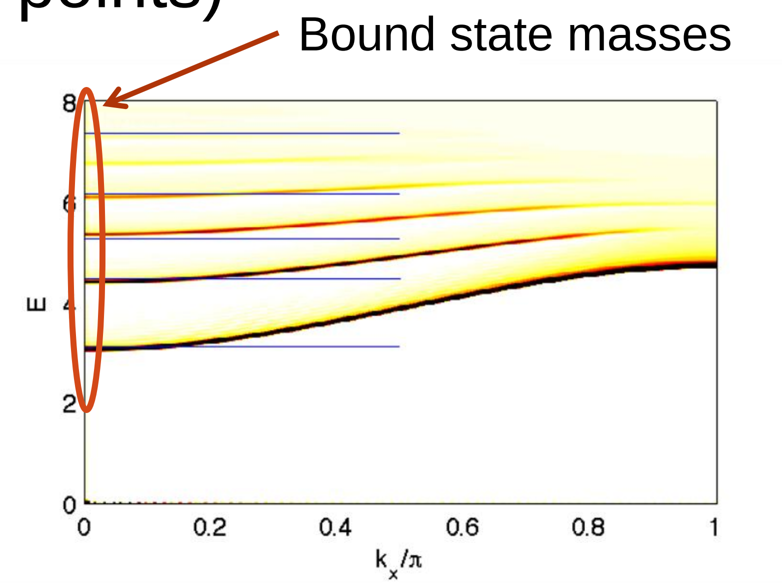
⇨ Energetically favorable to flip intermediate spin

⇨ Two new kinks

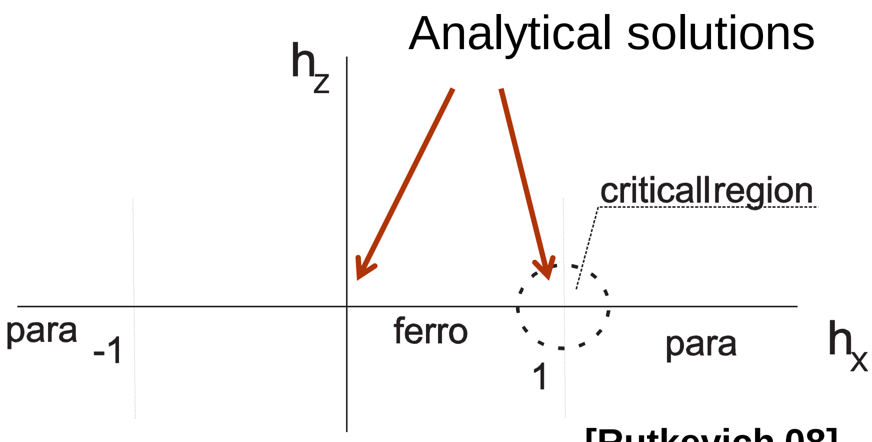
⇨ Each one forms a bound state with one original

⇨ Continuum in the excitation spectra

Excitations-Bound state masses
(between analytical solvable points)



- Minimum gap at zero bound state momenta
- Many bound states close to E=J for h_x<<J, h_z<<J
- 8 bound states close to E=0 for h_x≈J/2, h_z<<J



[Rutkevich 08]

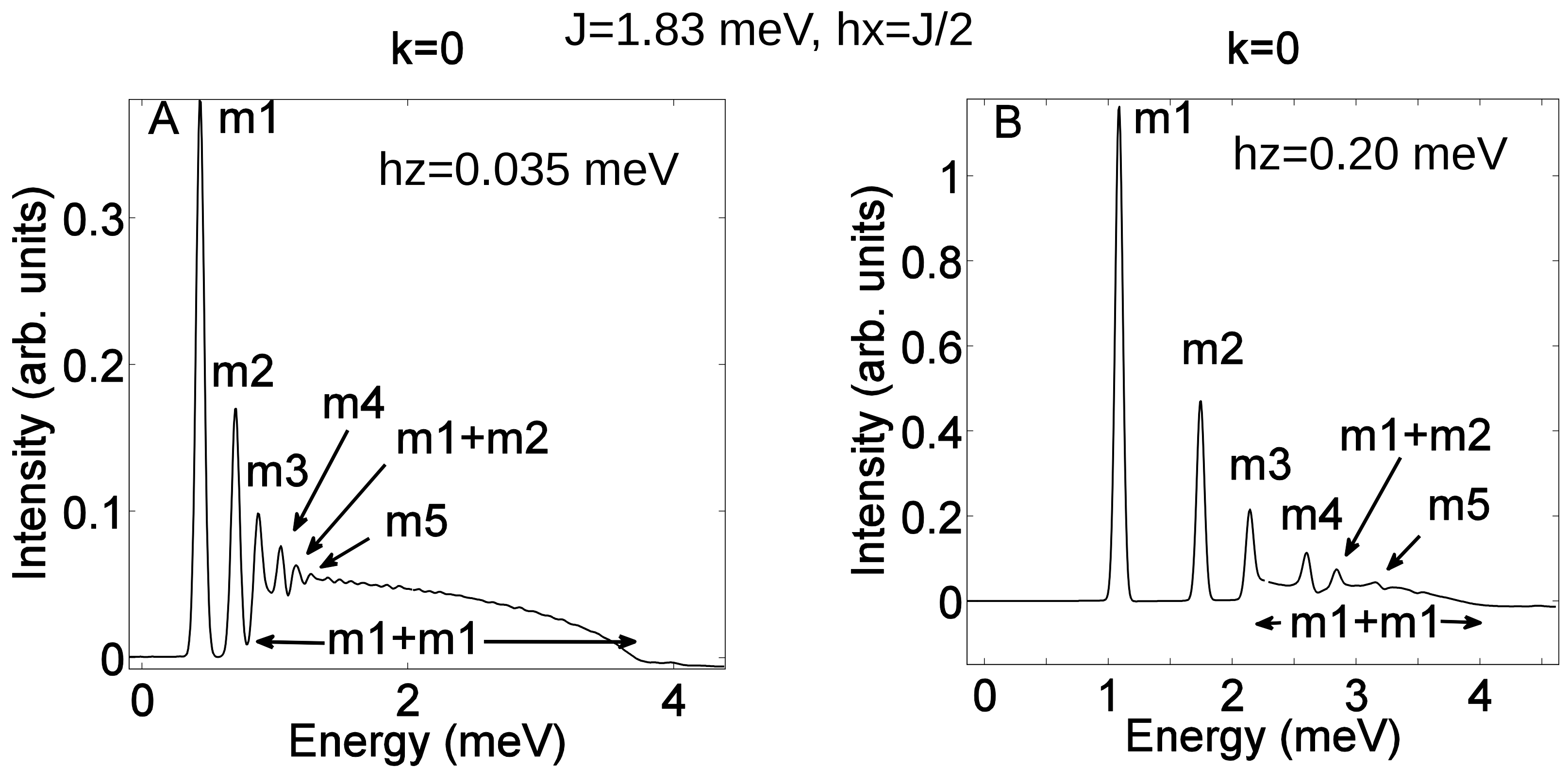
The goal with our work

1. Simulate the excitation spectra for the transverse Ising chain in a longitudinal field around h_x=J/2

2. Derive an accurate microscopic model of $CoNi_2O_6$

3. Simulate the excitation spectra for the model of $CoNi_2O_6$ around h_x=J/2

Longitudinal field

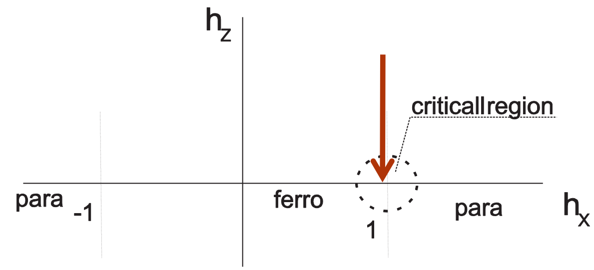


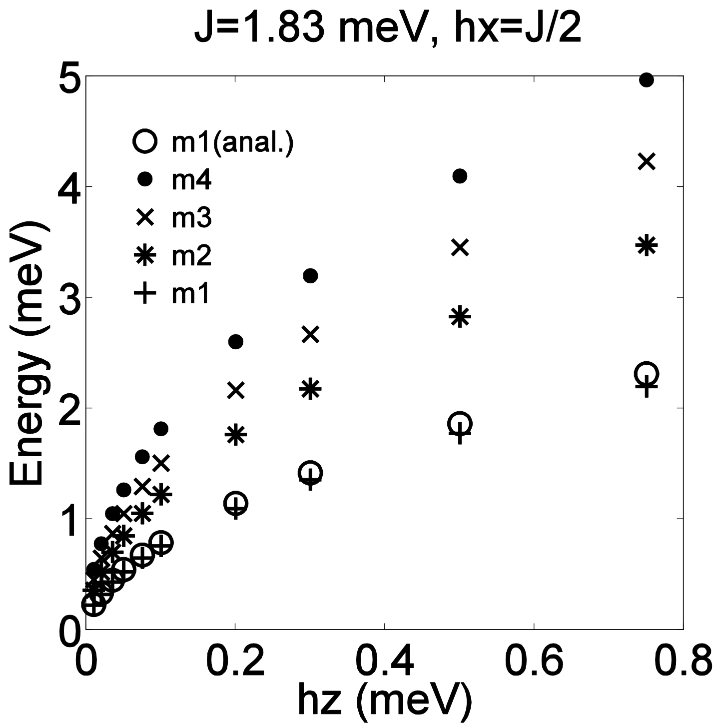
- Increasing longitudinal field

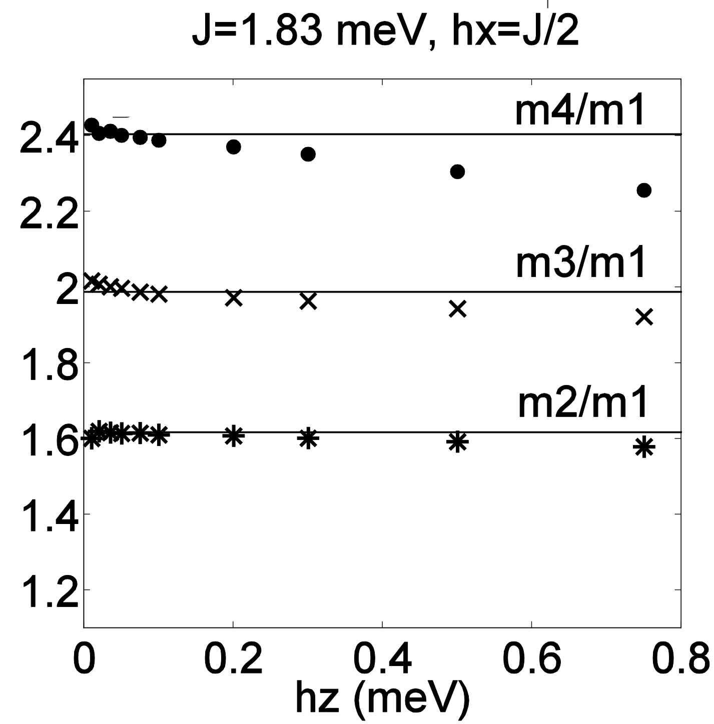
➡ Bound state spaced further apart

➡ Weight of continuum decreases

E_8 bound states





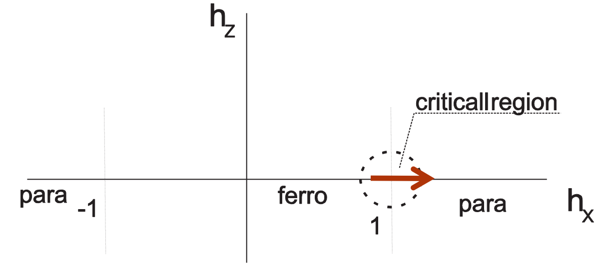


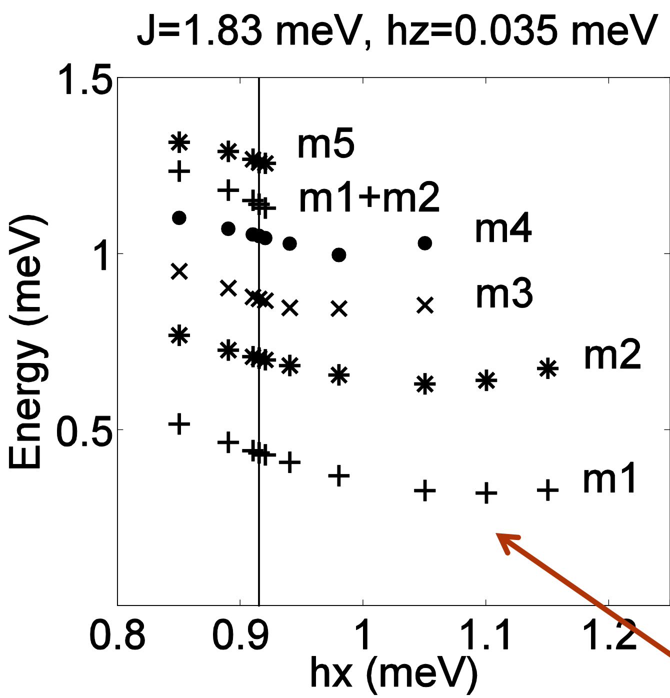
$$m_1 \approx JC(2h/J)^{8/15}, \text{ with } C = 4.40490858/0.7833$$

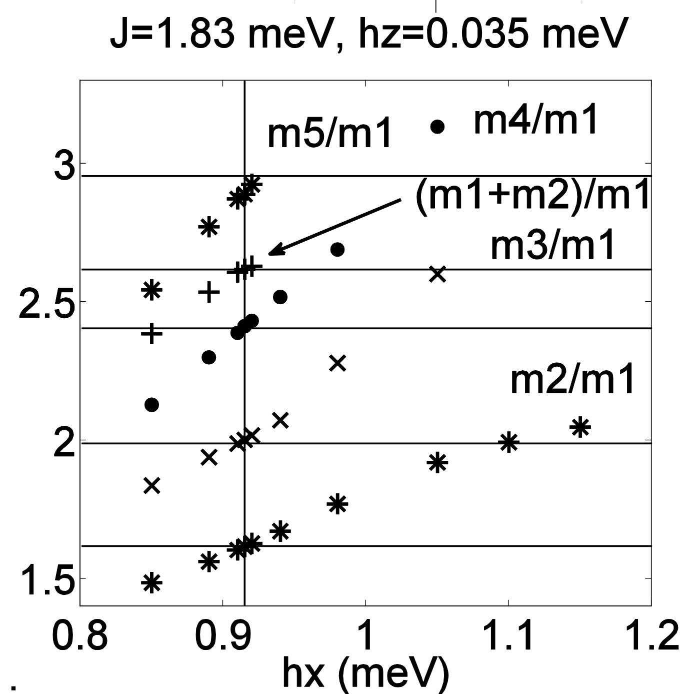
[Fateev 94]

- Surprisingly good agreement to strong long. fields

Gap minimum







Gap minimum

- Gap minimum indicates 3D phase transition

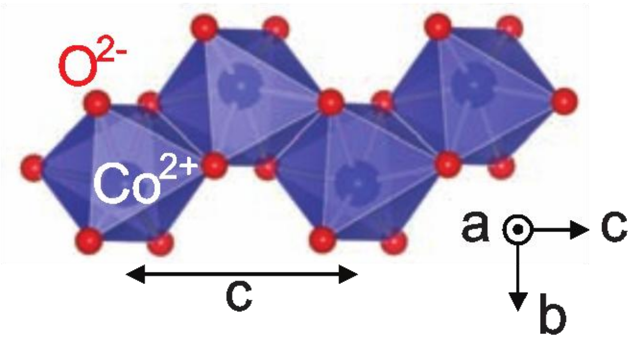
- Gap minimum moved far away

[Carr , Tsvelik 03]

- Good agreement with RPA calculations $h^x \approx h^x_c + 1.42meV(2h^z/J)^{4/7} \approx 1.12\ \mathrm{meV}$

Cobalt niobate $CoNi_2O_6$

- Ising spins on each Co(2+) ion
- Weakly coupled zigzag ferromagnetic chains



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3D model of $CoNi_2O_6$

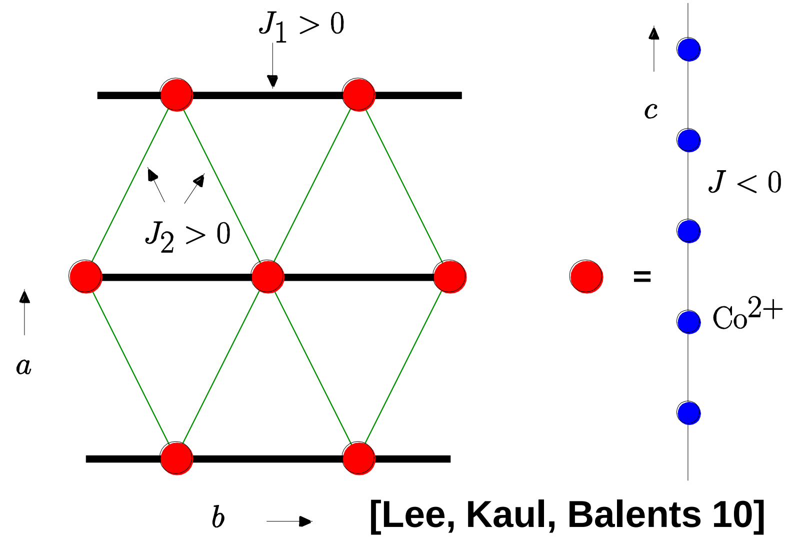
- Perpendicular plane has triangular structure

- Ferrimagnetic structure

- Magnetic ordered at low temperature

⇨ $$h^z = \sum_\delta J_\delta \langle S^z \rangle$$

[Carr, Tsvelik 03]



[Lee, Kaul, Balents 10]

Microscopic model of $CoNi_2O_6$

- Strong easy axis

➡ weak XX-term still present

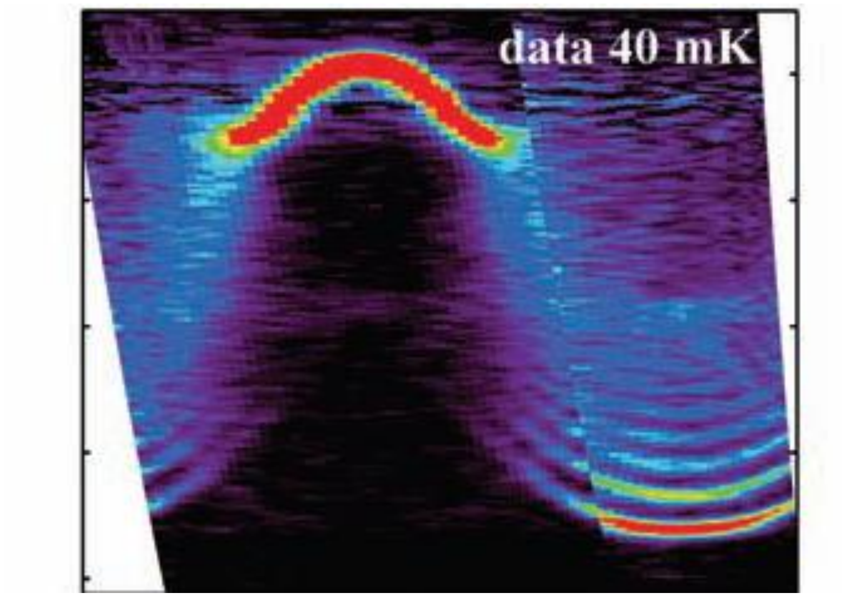
- Zigzag chain

➡ next nearest neighbor interaction

$$
\begin{aligned}
H &= -J' \sum_n S_n^z S_{n+1}^z - h^x \sum_n S_n^x - h^z \sum_n S_n^z \\
&- J_p \sum_n \left(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y \right) + J_B \sum_n S_n^z S_{n+2}^z .
\end{aligned}
$$

Fitting of parameters (low transverse field)

Neutron scattering data



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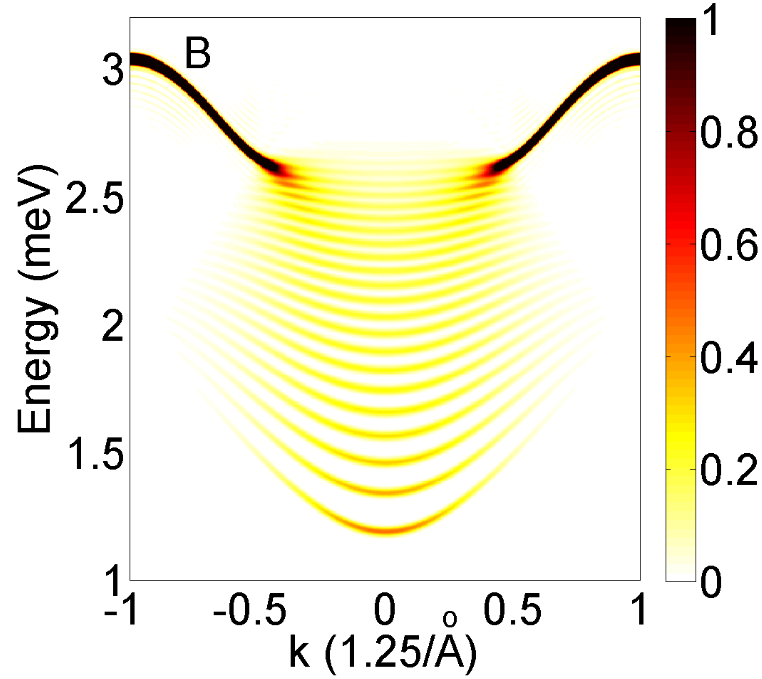
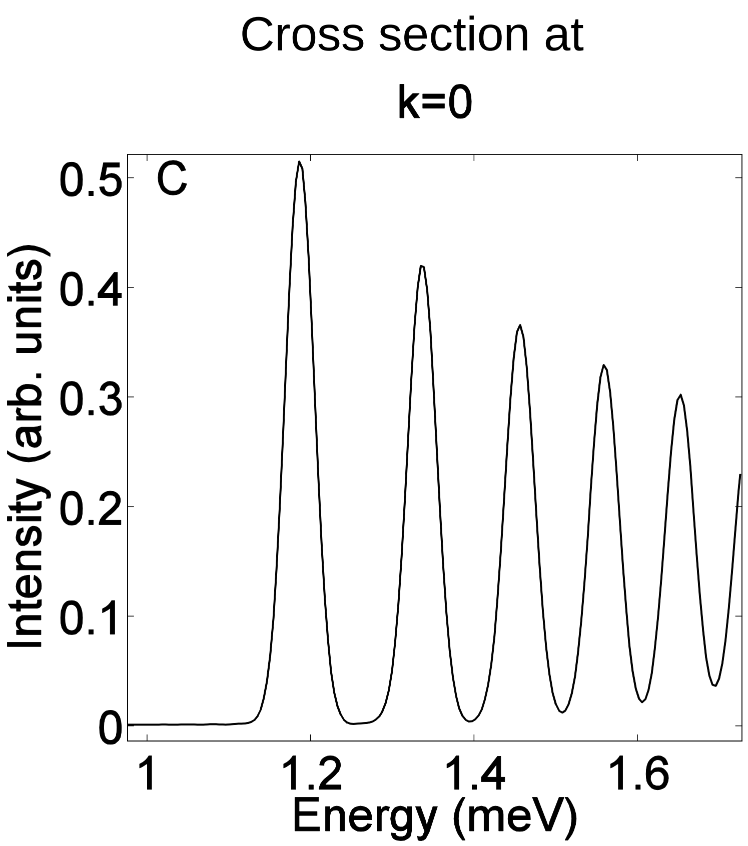
Dynamical Structure Function

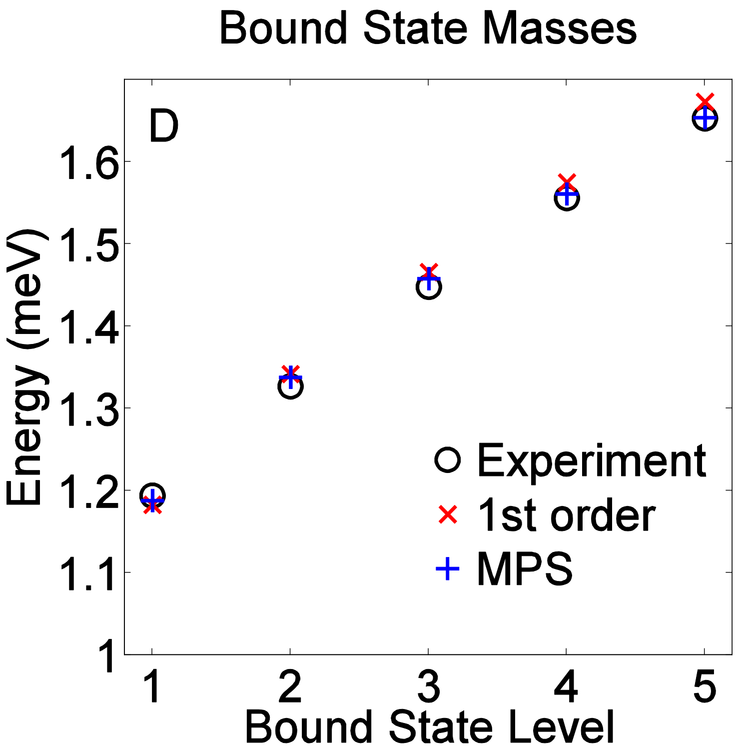


$J' = J + J_B = 2.43$ meV, $h^x = 0.354$ meV, $h^z = 0.035$ meV, $J_p = 0.52$ meV, $J_B = 0.60$ meV

Varied

Fitting of parameters (low transverse field)





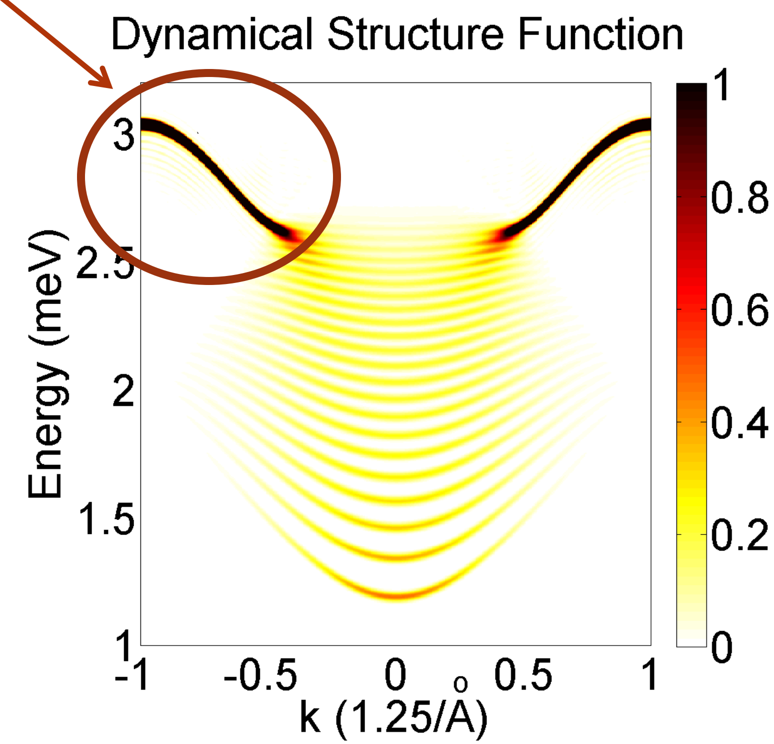
- The microscopic model can explain the experimental data

Kinetic bound state

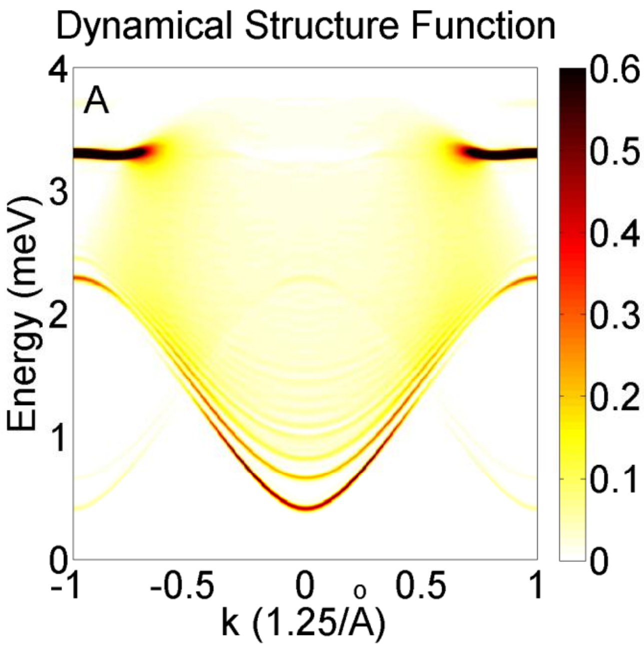
- Bound state stabilized by kinks moving together
- Short range interaction
- Energy gain in nn kink hopping

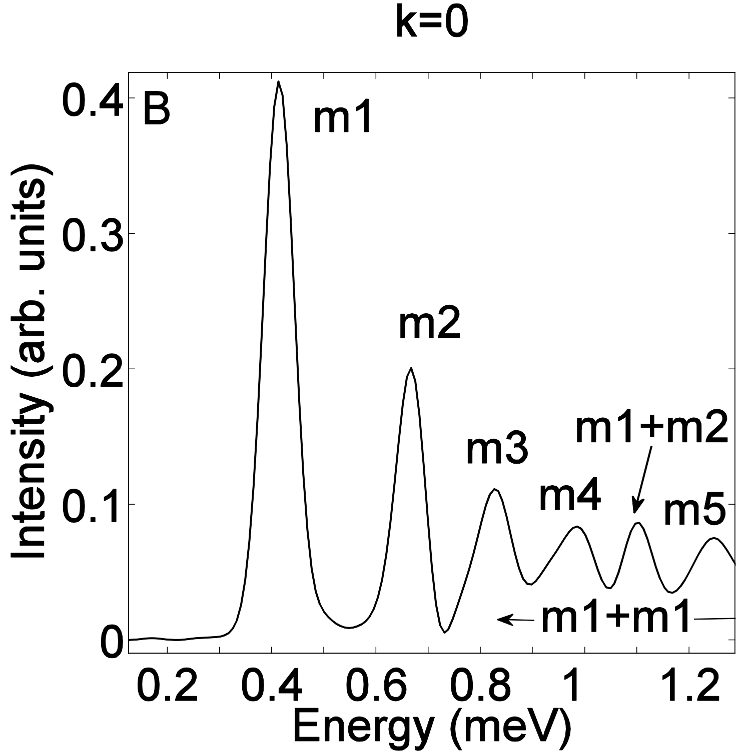
$$S_n^+ S_{n+1}^- + S_n^- S_{n+1}^+$$

Kinetic bound state



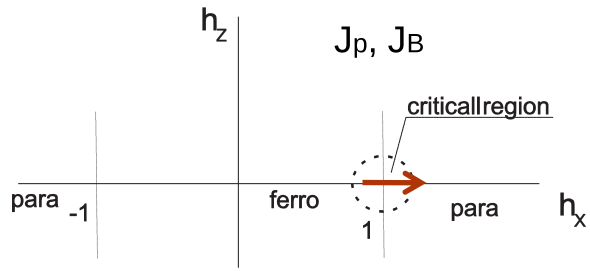
$CoNi_2O_6$ close to the QCP

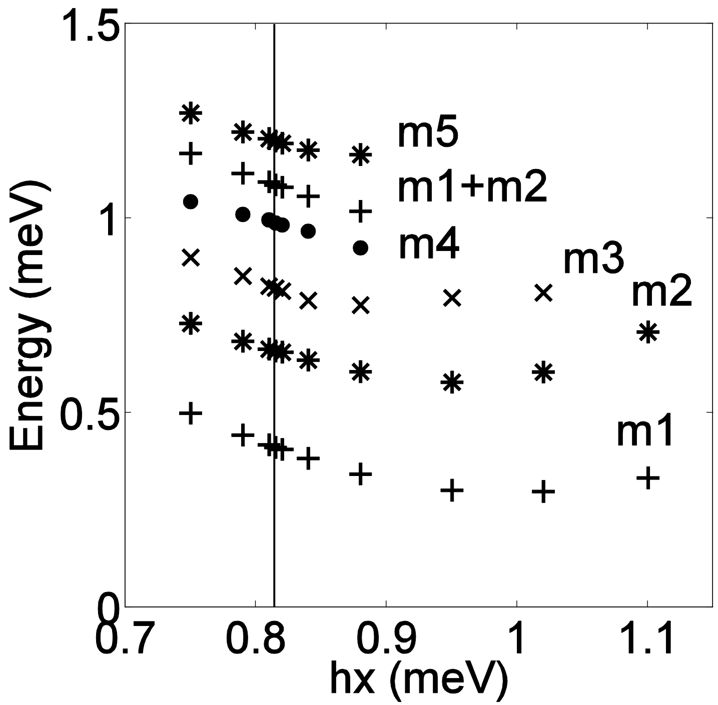


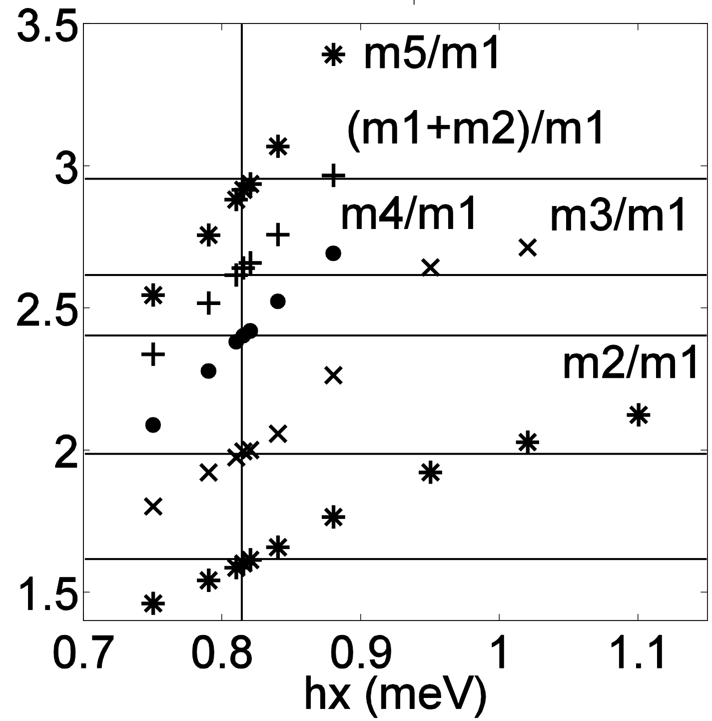


- Flattening of the kinetic bound state
- The relative intensity of the bound state masses are unaltered

$CoNi_2O_6$ close to the QCP

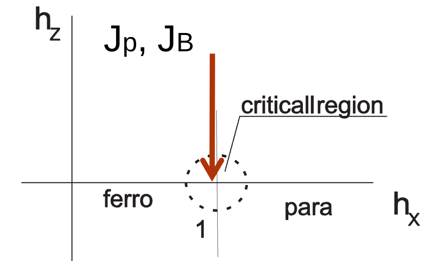




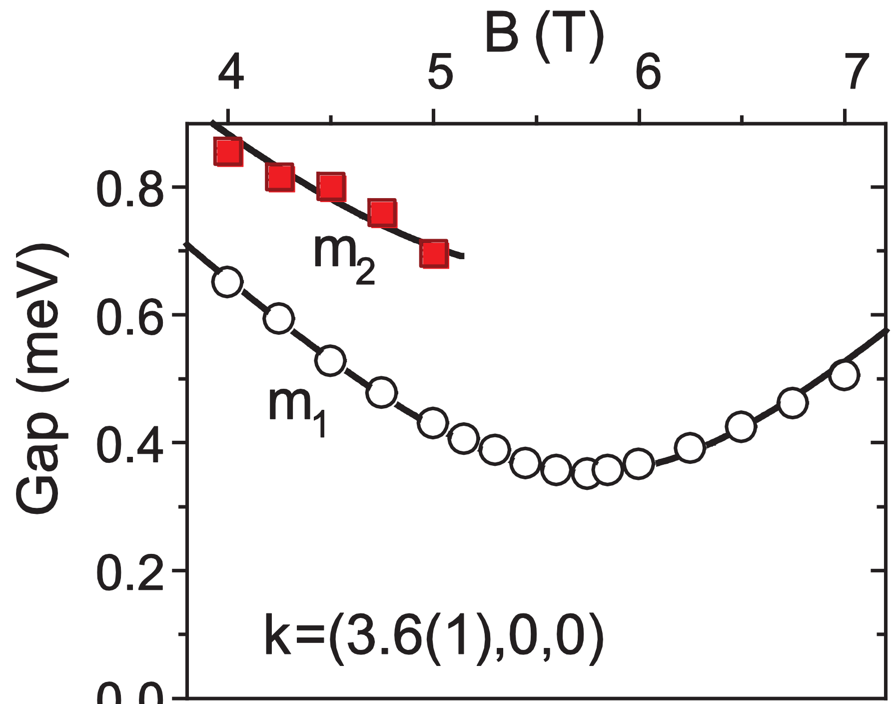


- Both axis rescaled by ~ 10%
  - QCP moved to hx ≈ 0.814 meV, gap minimum moved to hx ≈ 0.99 meV
  - Gap minimum decreased to E ≈ 0.295 meV
- Mass ratios unaltered

Comparison with experimental data

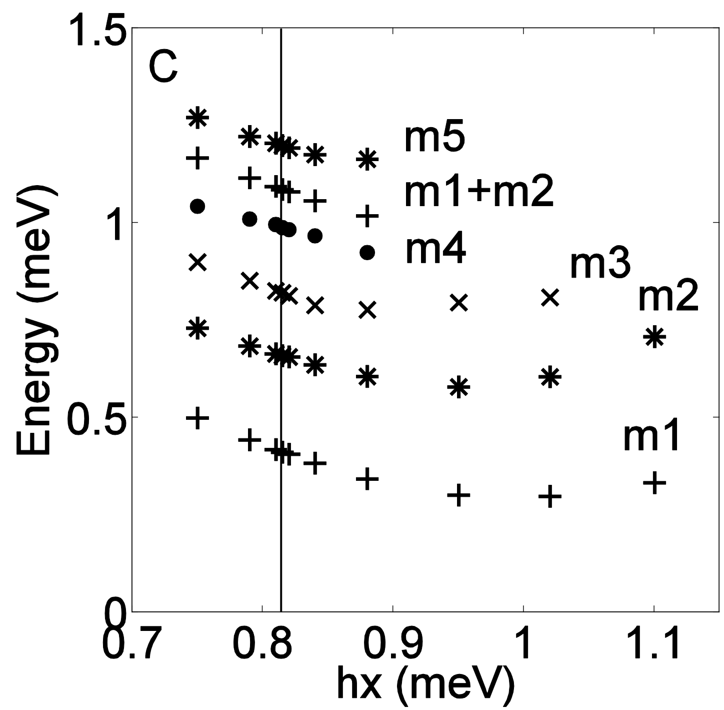


Experimental data



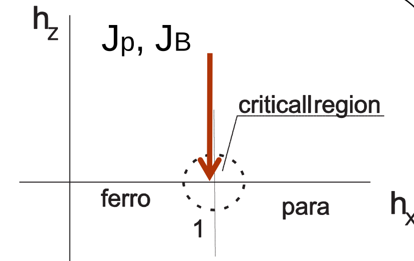
[Coldea 10]

Simulation

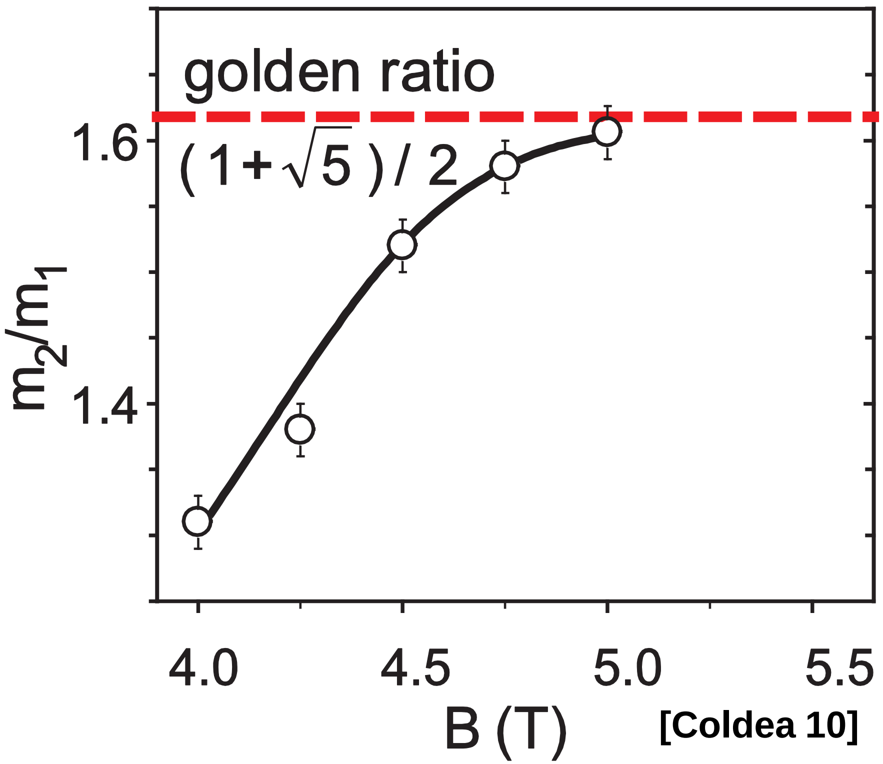


- Same general behaviour
  - Experimental gap minimum E ≈ 0.36 meV

Comparison with experimental data

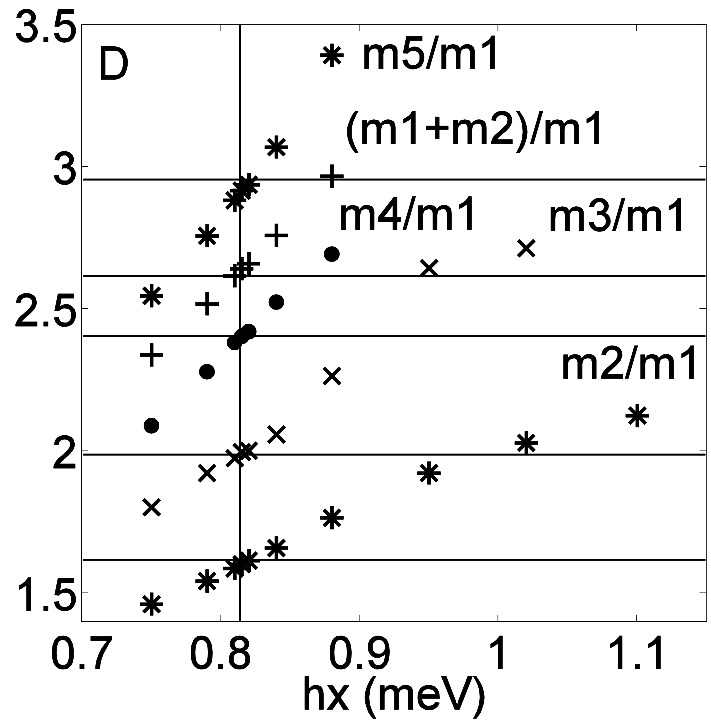


Experimental data



[Coldea 10]

Simulation



- The mass ratios passes the analytical values at the critical field, not approach it at the gap minimum

Conclusions

- The microscopic model of $CoNi_2O_6$ reproduce the experimental data well

- Mass ratios follow straight lines through the analytical values at the critical field strength

- Improved experiments should be as likely to detect higher bound states as in a pure QIC.

Thank You!

References

[Coldea 10] R. Coldea, D. A. Tennant, E. M.Wheeler, E. Wawrzynska,

D. Prabhakaran, M. Telling, K. Habicht, P. Smeibidl and K. Kiefer, **Science** 326, 177 (2010).

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[Fateev 94]]V. A. Fateev, Phys. Lett. B 324, 45 (1994).

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