Unitarity in lattices and Cooper pair insulators

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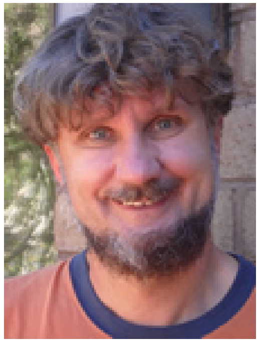
Acknowledgments

- Collaborators



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Overview

- Introduction and motivation
- Unitarity in lattice potentials (renormalization group)
- Cooper pair insulators (“pseudogap” at T=0)
- Applications: re-entrant SC, PDW, topological insulators, cuprates...
- Conclusions

Introduction and motivation

- The quest for universality
  - Common phenomena independent of complicated microscopic details
  - The fundamental principle behind understanding things

- Universality in fermionic systems
  - In metals: Fermi surface instabilities (not a subject of this talk)
  - In band insulators: scattering resonances and pairing … unitarity
  - In correlated states with emergent degrees of freedom

- Can unitarity teach us about:
  - The birth of some correlated states in atomic and electronic materials?
  - “Unconventional” superconducting transitions?

Universality from band insulators

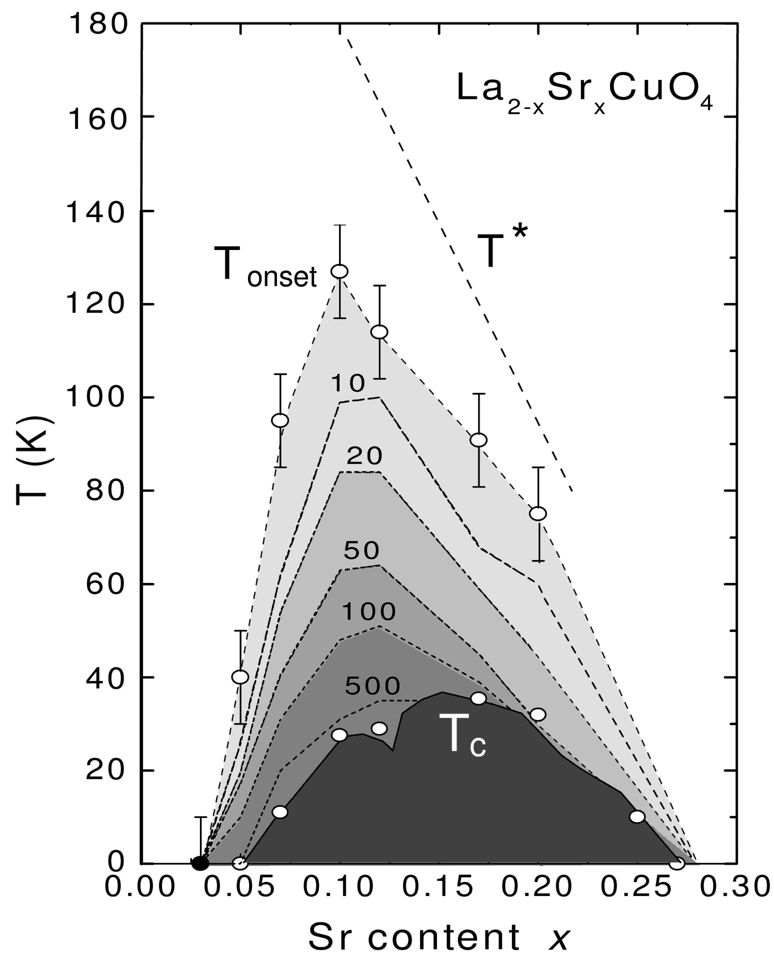
- Gapped interacting fermionic excitations
  - A direct path to correlated states in which...
  - … low-energy bosons dominate dynamics

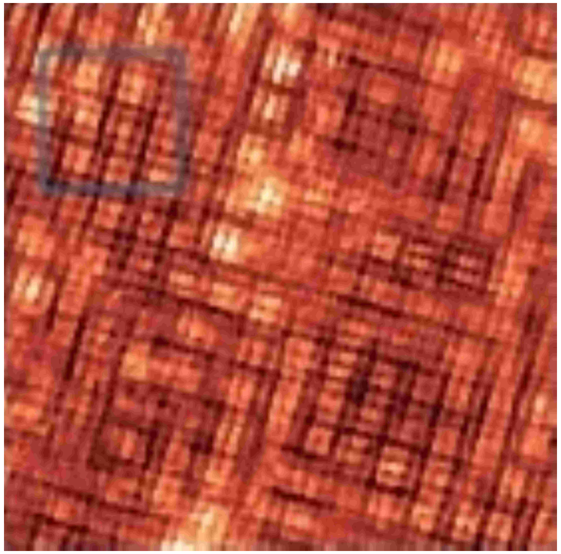
- The simplest setup
  - A band insulator with short-range interactions
  - Weak: BCS pairing transitions
  - Strong: “unconventional” transitions in bosonic universality classes
  - Fixed points describe scattering resonances

- Applicability
  - Cold atoms in lattice potentials
  - PDW and topological insulators
  - High-temperature superconductors?

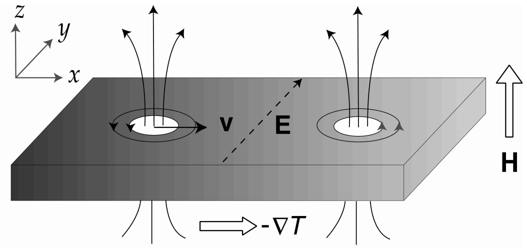
Quantum vortex liquid in cuprates?

- Vortices in the normal phase of cuprates, even at T=0





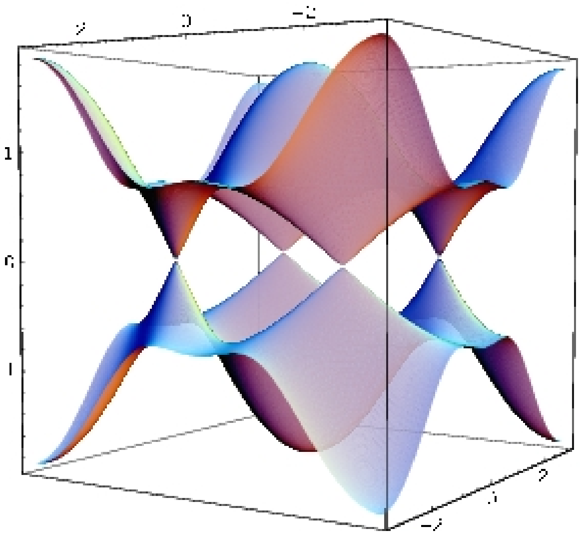
T.Hanaguri, et.al.; Nature **430**, 1001 (2004)

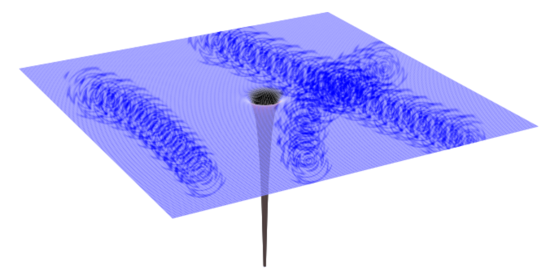


Y.Wang, et.al.; Phys.Rev.B **73**, 024510 (2006)

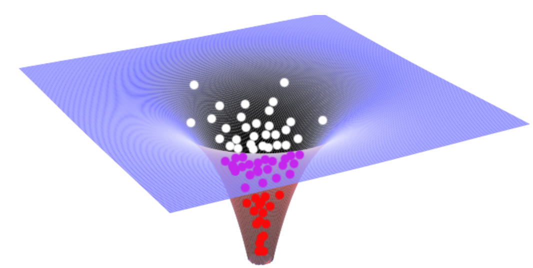
Vortices in superconductors

- “Fluctuating” *d*-wave superconductivity
  - Massless Dirac fermions
  - *d*-wave → no vortex core states
  - Small cores
  - Light and friction-free vortices
  - Quantum vortex dynamics





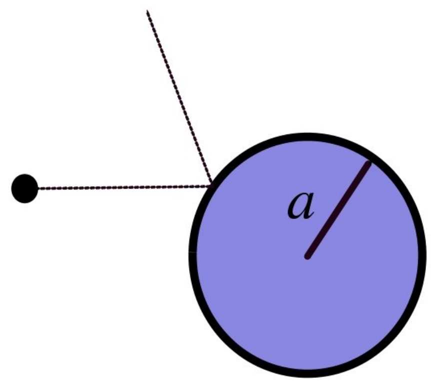
- Conventional BCS superconductivity
  - *s*-wave → vortex core states
  - Large cores
  - Heavy vortices, large friction
  - Semi-classical vortex dynamics



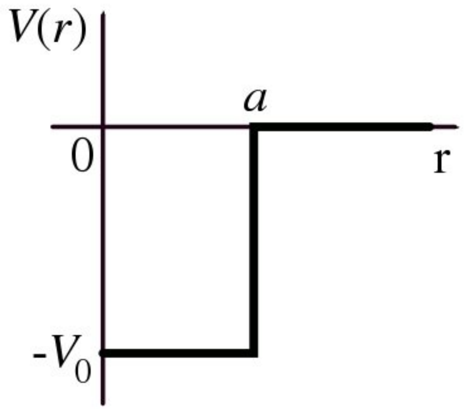
P.N., S.Sachdev; Phys.Rev.B **73**, 134511 (2006)

Unitarity: two-body picture

- Universality: irrelevant microscopic details
  - Two-body resonant scattering
  - Bound state at zero energy



$$\sigma \approx 4\pi a^2 \quad , \quad ka \ll 1$$

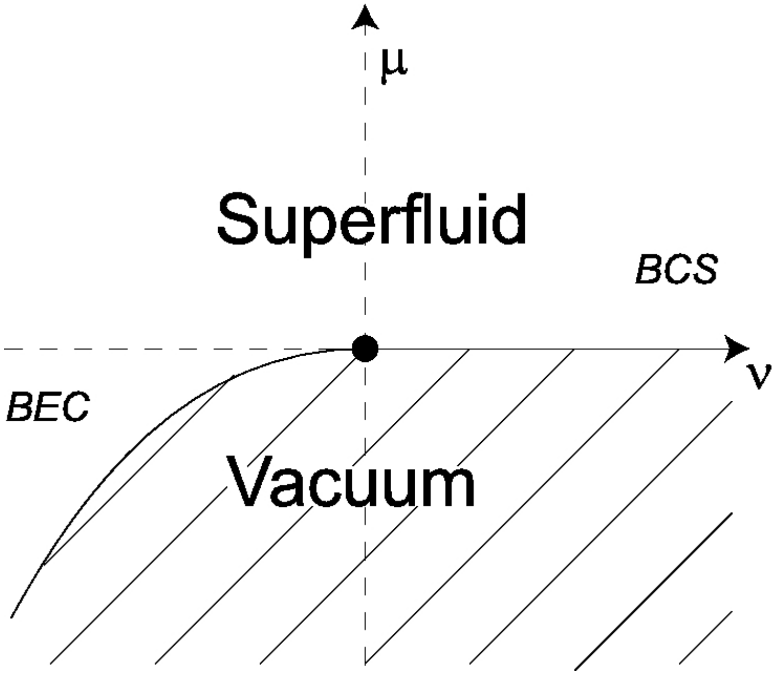


$$\sigma \approx 4\pi a^2 \left(1 - \frac{\tan(\alpha_0 a)}{\alpha_0 a}\right) \quad , \quad ka \ll 1$$

$$\sigma \approx \frac{4\pi}{k^2 + \alpha_0^2 \cot(\alpha_0 a)} \approx \frac{4\pi}{k^2} \quad , \quad \alpha_0 = \sqrt{2mV_0}$$

Unitarity: many-body picture

$$S = \int \mathrm{d}\tau \mathrm{d}^d x \Big[\psi^\dagger_{i\alpha} \Big(\frac{\partial}{\partial\tau} + \frac{(-i\boldsymbol{\nabla} - \boldsymbol{A})^2}{2m} - \mu + V(\boldsymbol{x}) \Big) \psi_{i\alpha}$$
$$+ h\big(\psi^\dagger_{i\uparrow}\psi_{i\uparrow} - \psi^\dagger_{i\downarrow}\psi_{i\downarrow}\big) + N\frac{m\nu}{4\pi}\Phi^\dagger\Phi + \Phi^\dagger\psi_{i\downarrow}\psi_{i\uparrow} + \Phi\psi^\dagger_{i\uparrow}\psi^\dagger_{i\downarrow} \Big]$$



- Universality
  - Quantum critical point
  - Zero density, T=0

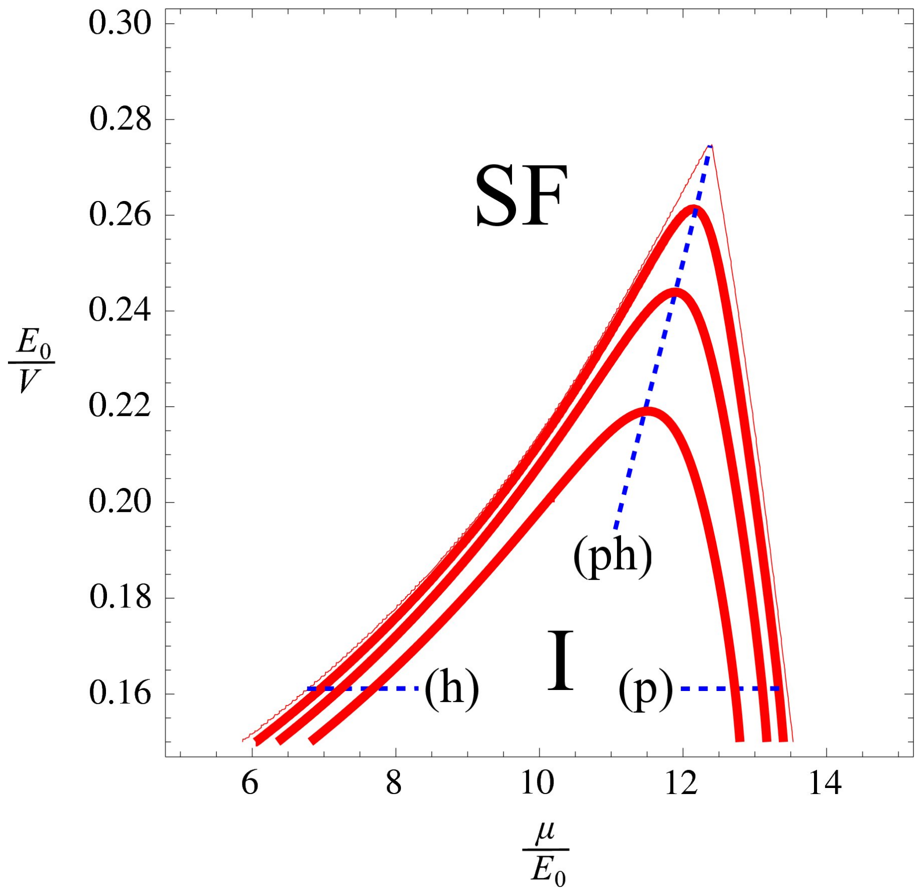
M.Y.Veillette, D.E.Sheehy, L.Radzihovsky; Phys.Rev. A 75, 043614 (2007)

Y.Nishida, D.T.Son; PRL **97**, 050403 (2006)

P.N., S.Sachdev; Phys.Rev. A **75**, 033608 (2007)

New fixed points in lattice potentials

- Unitarity at finite densities
  - Every band insulator is a vacuum of particles and holes



- SF-I pairing transitions
  - (p) … particle dominated
  - (h) … hole dominated
  - (ph) .. relativistic

- Tuning parameters
  - Chemical potential μ
  - Interaction strength ν
  - Lattice depth V

Transitions involving only particles (holes)

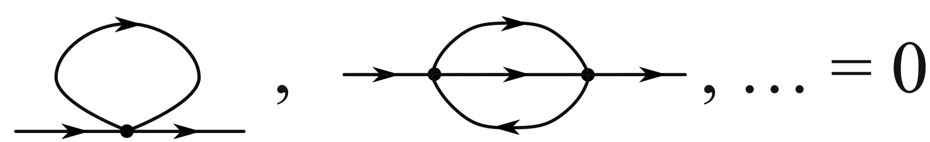
- Effective action

$$S = \int \mathcal{D}k \; f^\dagger_{k,\alpha} \left(-i\omega + E(k)\right) f_{k,\alpha} + U \int \mathcal{D}k_1 \mathcal{D}k_2 \mathcal{D}q \; f^\dagger_{k_1,\alpha} f_{k_1+q,\alpha} f^\dagger_{k_2,\beta} f_{k_2-q,\beta}$$

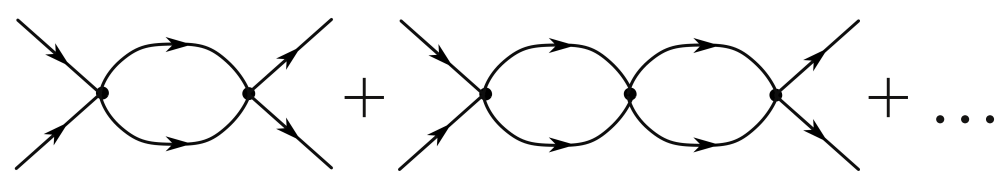
$$E(k) = E_0 + \frac{k^2}{2m} \qquad\qquad E_g = E_0 + U$$

- Exact renormalization group

$$\frac{dE_g}{dl} = 2E_g$$

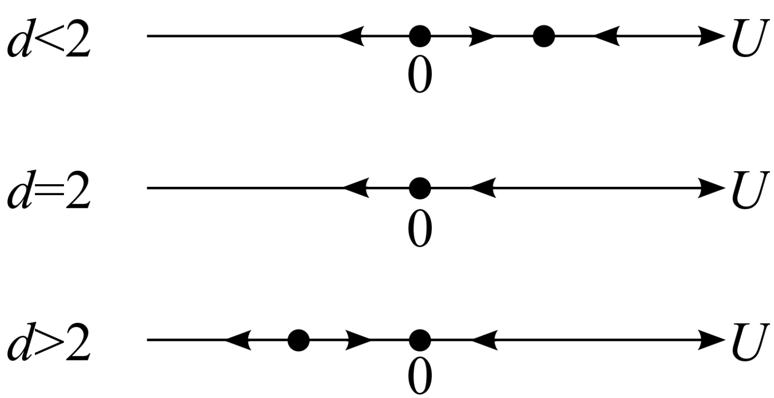


$$\frac{dU}{dl} = (2-d)U - \Pi U^2$$



Transitions involving only particles (holes)

- Fixed points & RG flow



  - Gaussian & Tonks-Girardeau
  - "Gaussian"
  - Gaussian & Unitarity

- Run-away flow for $U<U^*<0$
  - Asymptote at a finite l
  - High-energy pairing
    - ⇒ bound-state pairs
    - ⇒ BEC regime

$$U(l) = \left\{ \begin{array}{ll} \frac{U(0)}{1+\Pi U(0) l} & , \quad d=2 \\ \frac{U(0)}{[1+\Pi U(0)]e^{l} - \Pi U(0)} & , \quad d=3 \end{array} \right\}$$

- BCS: $U^*<U<0$

Transitions involving both particles and holes

- Effective action

$$S = \sum_n \int \mathcal{D}k \; f^\dagger_{n,k,\alpha} \left(-i\omega + E_n(k)\right) f_{n,k,\alpha}$$

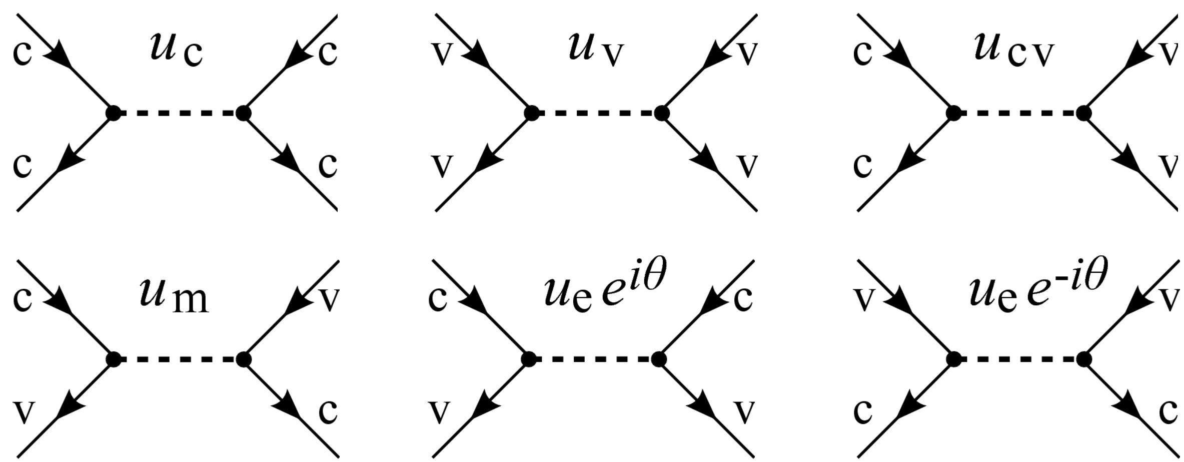
$$+ \sum_{n_1 m_1} \sum_{n_2 m_2} U^{m_1 m_2}_{n_1 n_2} \int \mathcal{D}k_1 \mathcal{D}k_2 \mathcal{D}q f^\dagger_{m_1,k_1+q,\alpha} f_{n_1,k_1,\alpha} f^\dagger_{m_2,k_2-q,\beta} f_{n_2,k_2,\beta}$$

$$E_c(k) = E_{c0} + \frac{k^2}{2m_c}$$

$n \equiv c$ … conduction band

$$E_v(k) = -E_{v0} - \frac{k^2}{2m_v}$$

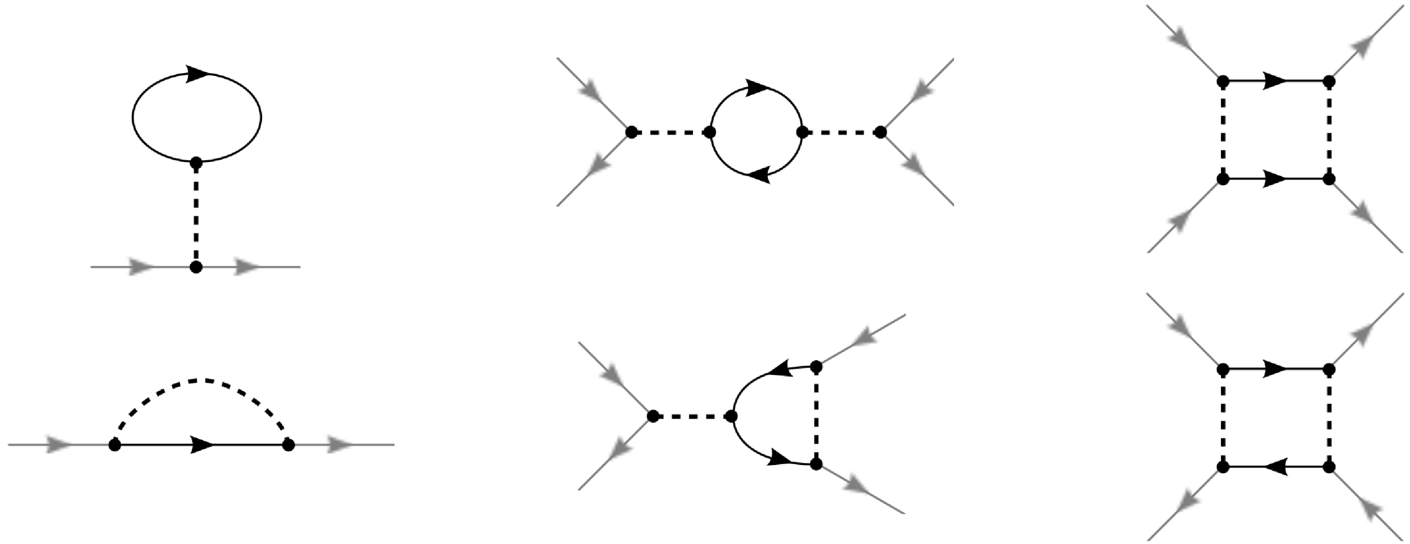
$n \equiv v$ … valence band



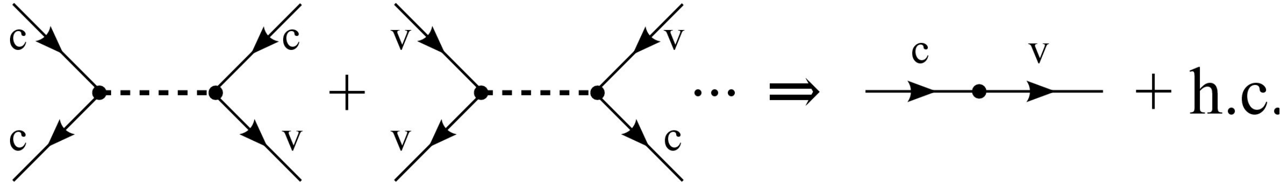
+ scattering with one band-conversion ...

Transitions involving both particles and holes

- Renormalization group: one loop, $\varepsilon = d - 2$ expansion



- Band mixing
  - Generated by scattering with one band-conversion



Transitions involving both particles and holes

- RG equations with no band-mixing: admit analytical solution

$$\frac{du_c}{dl} = \epsilon\Big[-u_c - 4u_c^2 - 4u_e^2\Big] \qquad \beta = \frac{m_c - m_v}{m_c + m_v}$$

$$\frac{du_v}{dl} = \epsilon\Big[-u_v - 4u_v^2 - 4u_e^2\Big]$$

$$\frac{du_{cv}}{dl} = \epsilon\Big[-u_{cv} + 2u_{cv}^2 + 8(1-\beta^2)u_e^2\Big]$$

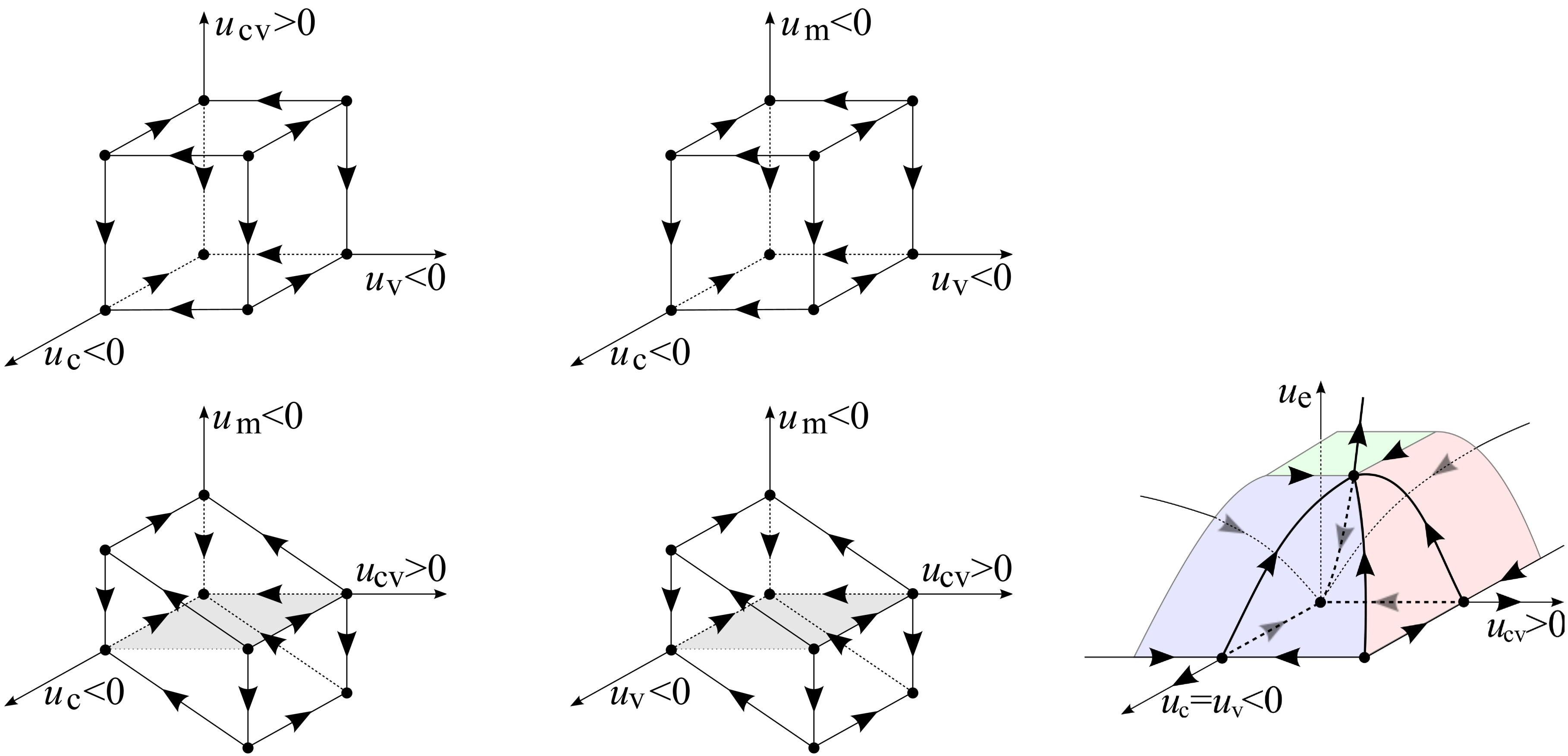
$$\frac{du_m}{dl} = \epsilon\Big[-u_m - 4u_m^2 + 4u_{cv}u_m\Big]$$

$$\frac{du_e}{dl} = \epsilon\Big[-u_e + u_e\Big(-4u_c - 4u_v + 8u_{cv} - 4u_m\Big)\Big]$$

$$\frac{de_g}{dl} = 2e_g - \frac{2u_v}{1-\beta} + \frac{2u_{cv}}{1-\beta^2} - \frac{u_m}{1-\beta^2}$$

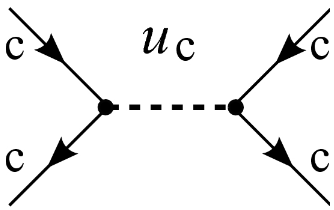
Transitions involving both particles and holes

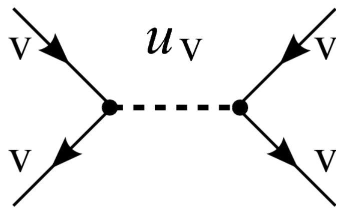
- 17 fixed points with no band-mixing
  - Gaussian + 15 resonant scattering fixed points in various channels
  - A "pair-scattering" fixed point



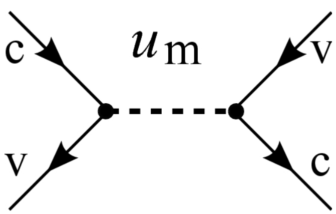
Transitions involving both particles and holes

- Scattering resonances
  - BEC-BCS crossovers in various channels

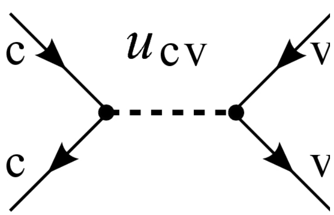




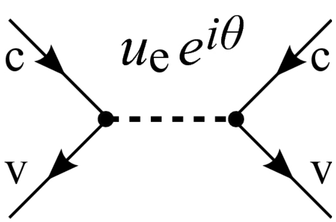
Bound singlet of two conduction band or two valence band fermion

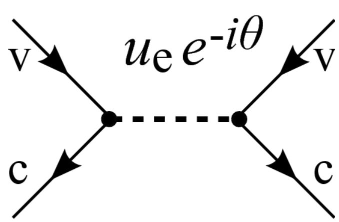


Bound singlet of a conduction and a valence band fermion



Exciton: bound singlet of a particle and hole

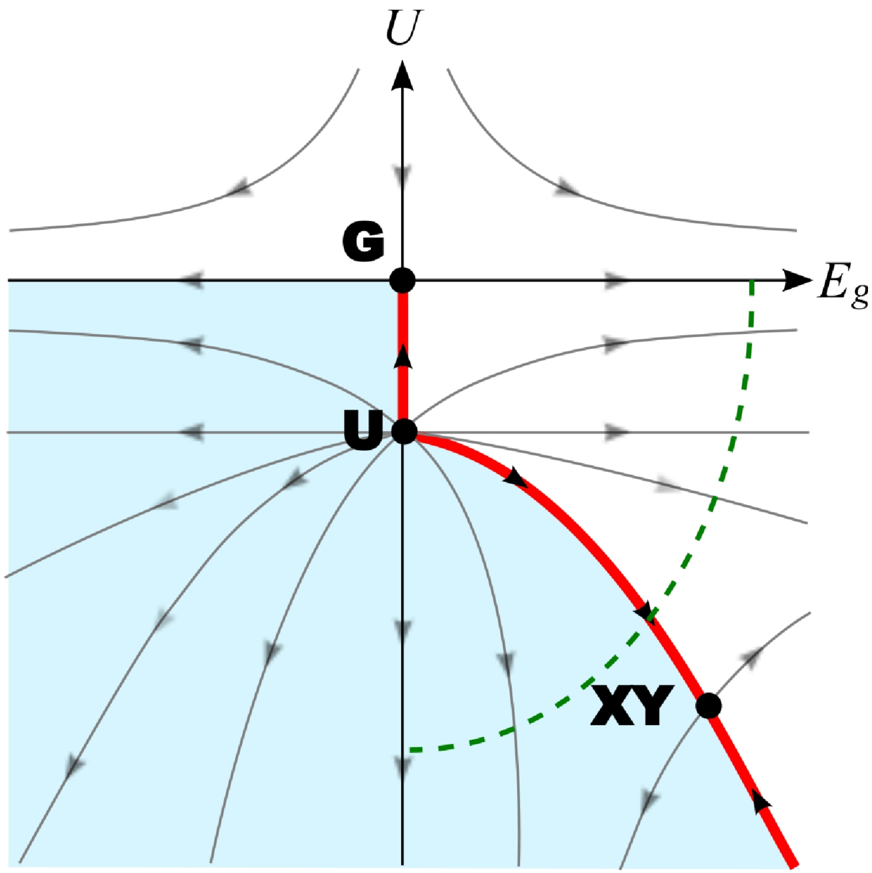




Assisted resonance?
Extended s-wave resonating singlet

RG summary

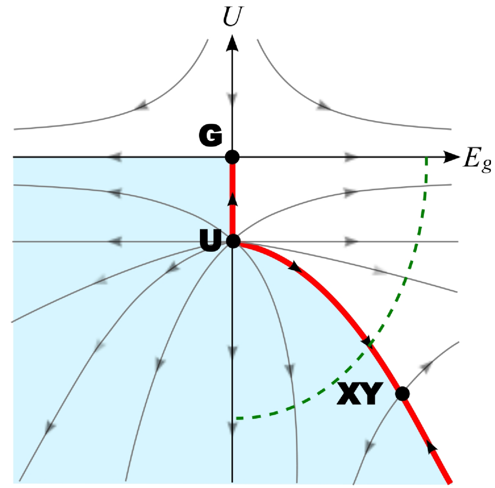
- Most general cases
  - Scattering resonances with multiple particle/hole species
  - Additional fixed points with band-mixing
- Unitarity universality classes
  - Universal ratios of observables (μ, E_f, T_c, P...)
  - Slight modifications of the vacuum universality
- Global RG
  - Run-away flows in BEC limits
  - Strong-coupling fixed points

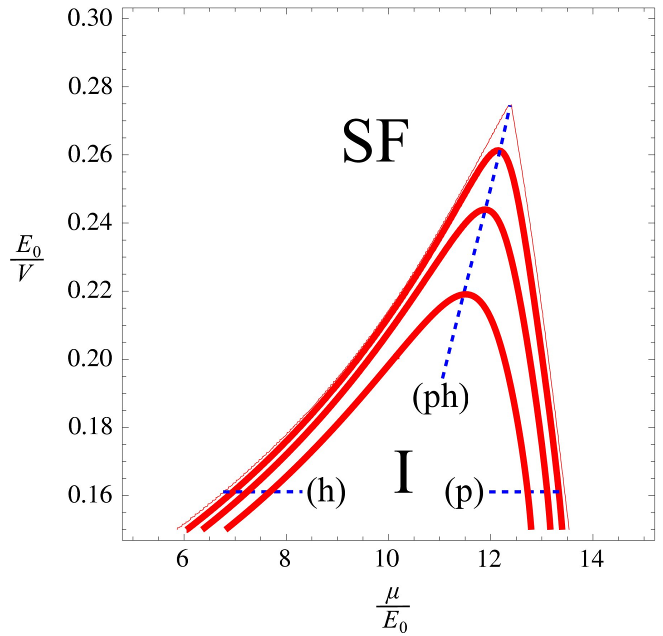


P.N., Arxiv:1006.2378 (2010)

Effective BEC regimes

- RG run-away flow (bound states)
  - Any interaction strength in d=2
  - Requires strong interaction in d>2
- BEC regime: bound states
  - Quasiparticles are gapped
  - Bosonic universality class for the SF transition mean-field or XY
- BCS regime: no bound states
  - BCS (pairing) transition
  - Must close the fermion gap in order to induce SF





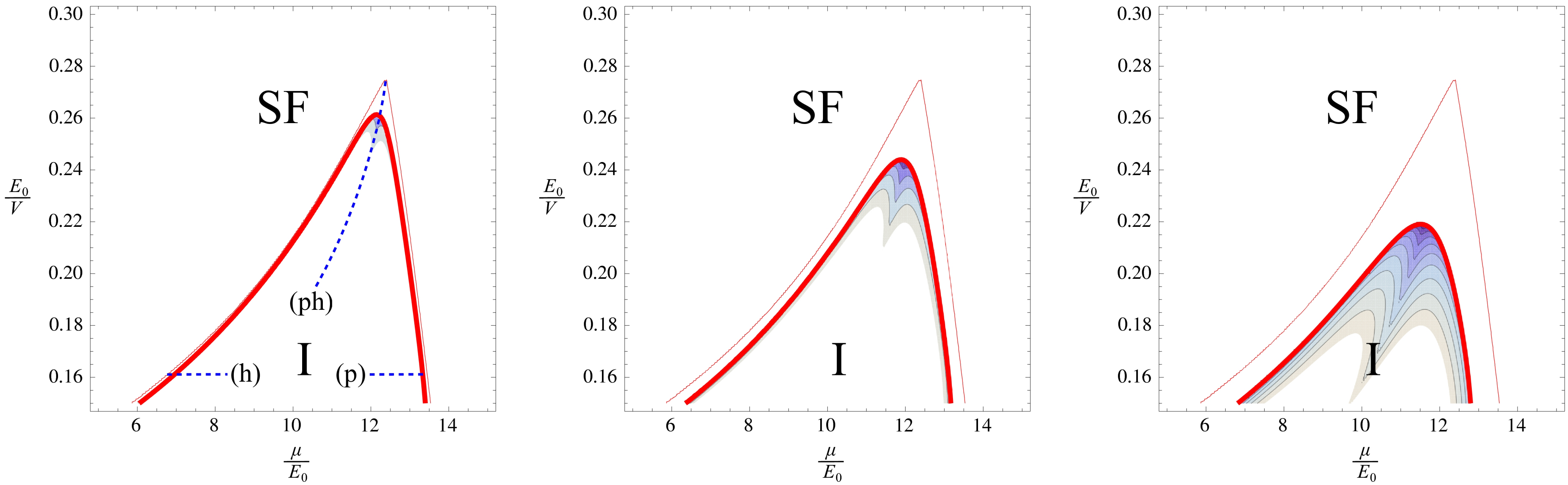
Bosonic Mott Insulator

- Insulator adjacent to SF
  - BEC: Bosonic Mott
  - BCS: Band insulator

D.M.Eagles; Phys.Rev. **186**, 456 (1969)

P.N., Zlatko Tešanović (unpublished)

- Bosonic Mott insulator
  - No symmetry breaking (but not excluded either)
  - Bosonic lowest energy excitations
  - Large fermion gap



Mott/band insulator distinction

- Quantum phase transition
  - Non-analytic change of the ground state manifold as a function of tuning parameters

- A generalization to the entire spectrum
  - Non-analytic change of the Hamiltonian (density matrix) as a function of tuning parameters

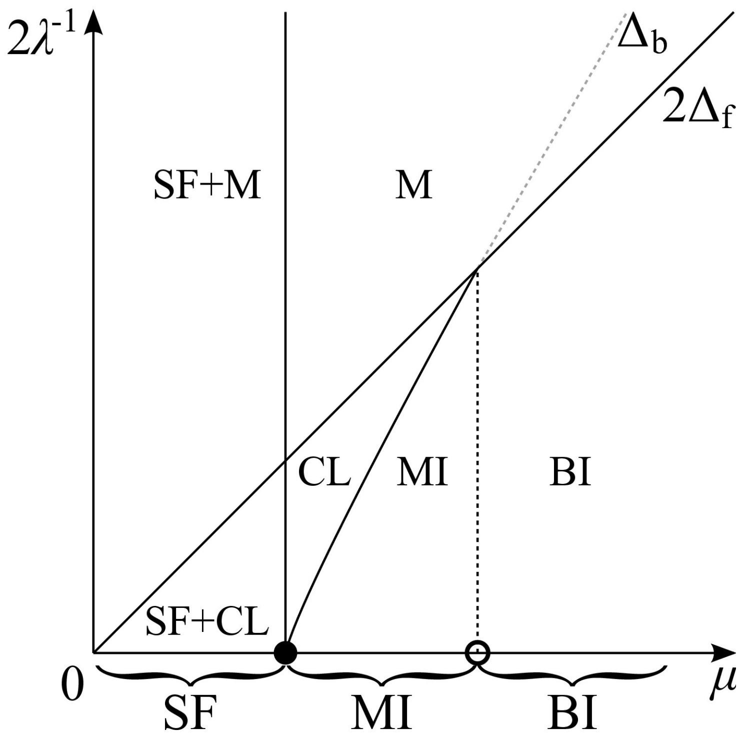
- Mott insulator "order parameter"

$$\rho(E,\mathbf{P}) = \frac{1}{\mathcal{V}} \sum_n \delta(E - E_n)\delta(\mathbf{P} - \mathbf{P}_n)$$

$$\rho'(\mathbf{k}) = \lim_{\Delta_\varepsilon \to 0} \lim_{\Delta_p^d \to 0} \int_{\Delta_\varepsilon} d\delta\varepsilon \int_{\Delta_p^d} d^d\delta p \frac{1}{\mathcal{V}} \sum_N \rho(N\varepsilon_{\mathbf{k}} + \delta\varepsilon, N\mathbf{p}_{\mathbf{k}} + \delta\mathbf{p})$$

Non-equilibrium pairing transitions

- Cooper pair laser
  - Sharp non-equilibrium distinction between Mott and band insulators



- Numerics?
  - Quantum Monte Carlo
  - Negative-U Hubbard model
- Experiments?
  - SC / narrow bandgap material heterostructures
  - Superlattices
  - Cold atoms

P.N., Zlatko Tešanović (unpublished)

Non-trivial Mott insulators

- Broken symmetries
  - Pair density waves (particle-particle BEC)
  - Magnetic, nematic... (*p*-wave pairs?)
  - Density waves (particle-hole BEC)
  - Valence bond crystals (inter-valley particle-particle BEC)

- Topological orders
  - Fractional quantum Hall states with even-denominator filling factor
  - Fractional spin quantum Hall states?
  - Spin liquids

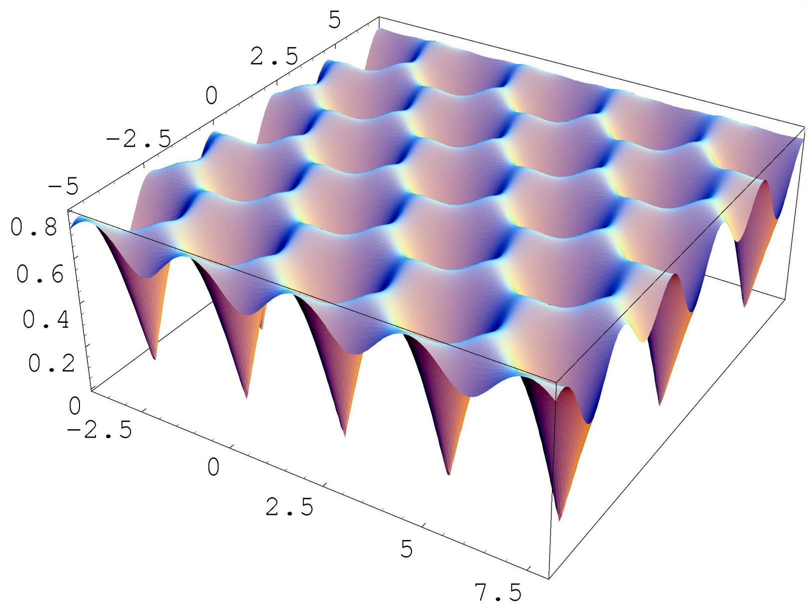
Superfluids in the quantum Hall regime

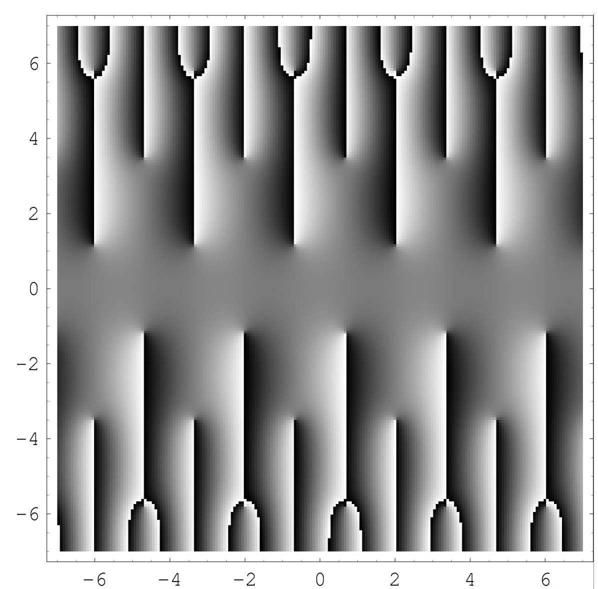
- Normal state → quantum Hall insulator
  - Localized particles (cyclotron orbitals)
  - Discrete Landau levels
  - Macroscopic degeneracy: two particles per flux quantum

$$\epsilon_n = \omega_c \left(n + \frac{1}{2}\right)$$

- Superfluid

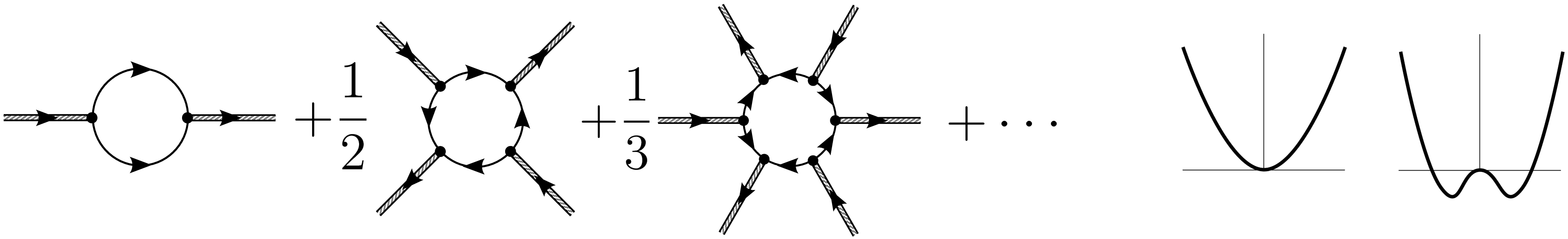
$$\Phi((r)) = \Delta_0 e^{-2m\omega y^2} \theta_3 \left(\left(\pi\sqrt{3}m\omega\right)^{\frac{1}{2}} (x+iy) \middle| e^{i\pi/3} \right)$$





Pairing instability

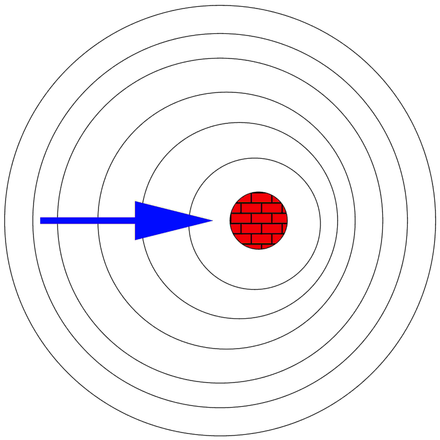
$$\Pi_{n,n'}(p_x, p_z, i\Omega) \propto \sum_{m_1,m_2} \int \frac{dk_z}{2\pi}\frac{dk_x}{2\pi}\frac{f(\varepsilon_{m_1,k_z+\frac{p_z}{2}}) - f(-\varepsilon_{m_2,-k_z+\frac{p_z}{2}})}{-i\Omega + \varepsilon_{m_1,k_z+\frac{p_z}{2}} + \varepsilon_{m_2,-k_z+\frac{p_z}{2}}} \times$$
$$\times \Gamma^{n}_{m_1,m_2}\left(\frac{k_x}{\sqrt{B}}\right)\Gamma^{n'*}_{m_1,m_2}\left(\frac{k_x}{\sqrt{B}}\right) + \mathcal{O}\left(\frac{1}{N}\right)$$



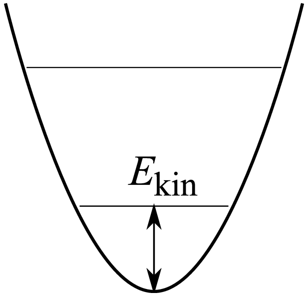
- No p_x dependence to all orders of $1/N$
  - "charged" bosonic excitations live on degenerate Landau levels
  - Macroscopically many modes turn soft simultaneously
  - The nature of "condensate" is determined by interactions

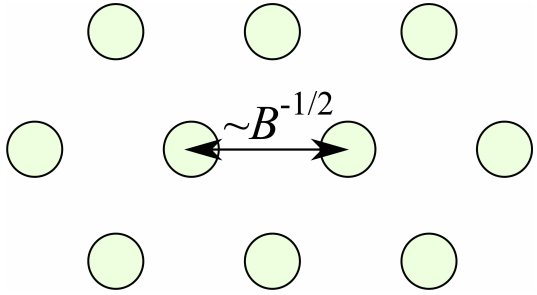
Quantum vortex lattice melting

- Vortex mass
  - Compression of the stiff superfluid
  - Neutral: $$m_v \approx \frac{\rho_s}{s^2} \log\left(\frac{R}{\xi}\right)$$
    $$\rho_s, s^2 \propto |\Phi|^2$$



- Vortex localization energy
  - $E_{kin} \sim p^2/2m_v$ ... $p^2 \sim$ B



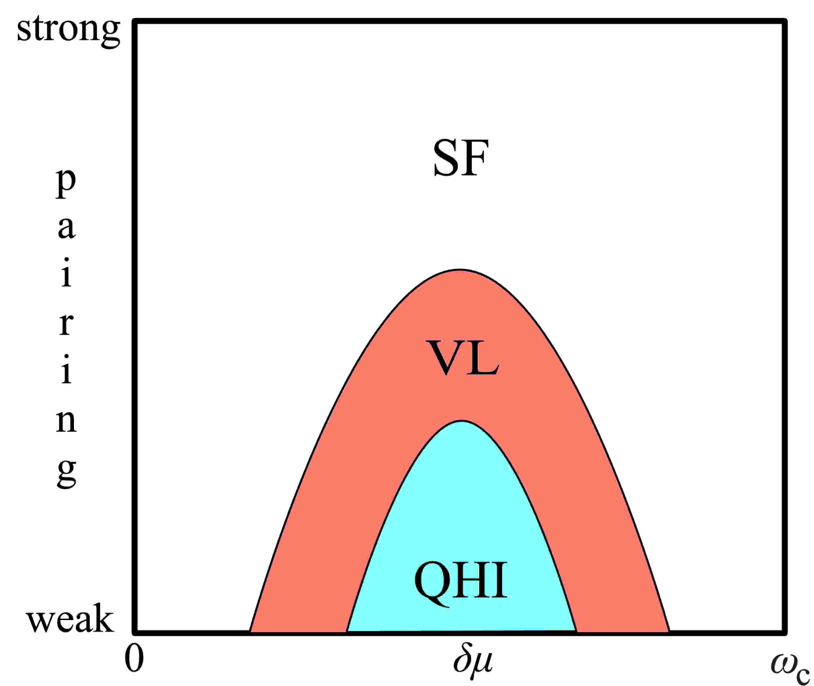


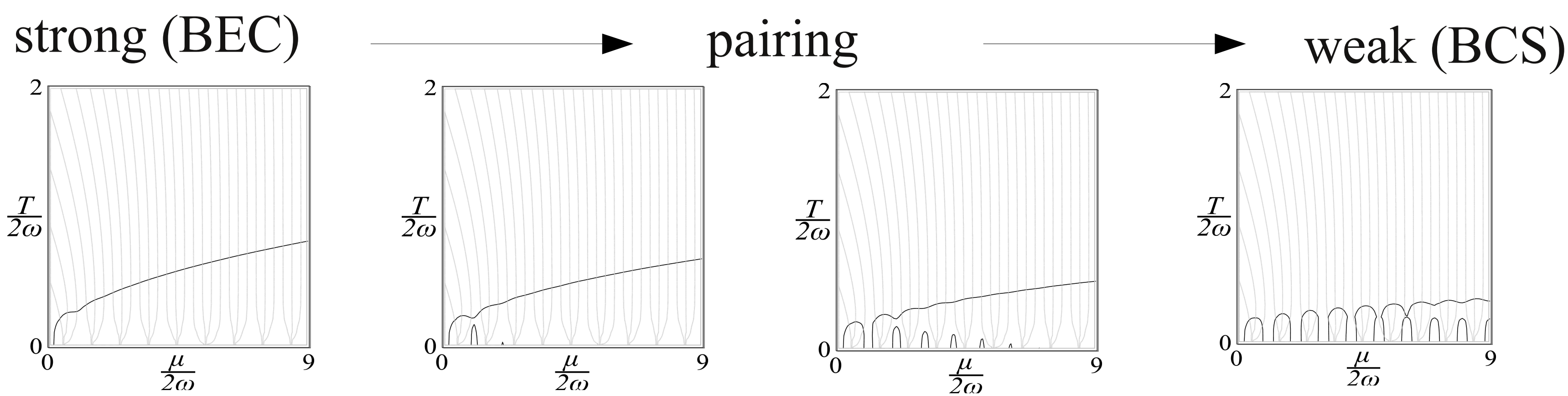
- Vortex lattice potential energy
  - Π is degenerate $\rightarrow E_{pot} \sim \Phi_0^4$

$$\frac{\mathcal{F}(\Phi_0)}{N} = \frac{\mathcal{F}_0}{N} + \hat{\Pi}_{ij}\Phi_0^{i*}\Phi_0^j + \hat{U}_{ijkl}\Phi_0^{i*}\Phi_0^{j*}\Phi_0^k\Phi_0^l + \mathcal{O}(\Phi^6)$$

Vortex liquid

- Genuine phases at T=0
  - Vortex lattice potential energy: $\Delta_0^{\,4}$
  - Melting kinetic energy gain: $\log^{-1}(\Delta_0)$
  - 1^{st} order vortex lattice melting as $\Delta_0 \to 0$
  - Low energy spectrum inconsistent with fermionic quantum Hall states
  - Non-universal properties (by RG)





P.N, Phys.Rev.B **79**, 144507 (2009)

The nature of vortex liquids

- Non-universal properties
  - At Gaussian and unitarity fixed points of RG

$$S = \int d\tau d^{d-2} r_\perp \Biggl\{ \sum_n \int \frac{dk_x}{2\pi} \psi^\dagger_{n,k_x} \left(\frac{\partial}{\partial\tau} + n\omega_c - \frac{\boldsymbol{\nabla}^2_\perp}{2m} - \mu' \right) \psi_{n,k_x} \quad + N \sum_{n_1 n_2} \int \frac{dp_x}{2\pi} \Phi^\dagger_{n_1,p_x} \hat{\Pi}^{(0)}_{n_1,n_2} \Phi_{n_2,p_x}$$

$$+ g \sum_{n m_1 m_2} \int \frac{dk_x}{2\pi} \frac{dp_x}{2\pi} \Gamma^n_{m_1 m_2} \left(\frac{k_x}{\sqrt{B}} \right) \left[\Phi^\dagger_{n,p_x} \psi_{m_1, k_x + \frac{p_x}{2}} \psi_{m_2, -k_x + \frac{p_x}{2}} + \text{h.c.} \right]$$

$$+ u_2 \sum_{m_1 \dots m_4} \int \frac{dk_{x1}}{2\pi} \frac{dk_{x2}}{2\pi} \frac{dq_x}{2\pi} \Gamma'_{m_1 \dots m_4} (k_{x1}, k_{x2}, q_x) \psi^\dagger_{m_1,k_{x1}} \psi^\dagger_{m_2,k_{x2}} \psi_{m_3,k_{x2}+q_x} \psi_{m_4,k_{x1}-q_x} \Biggr\} + \cdots$$

- All interactions are relevant in *d*=2
  - Dimensional reduction
  - Many stable interacting fixed points?

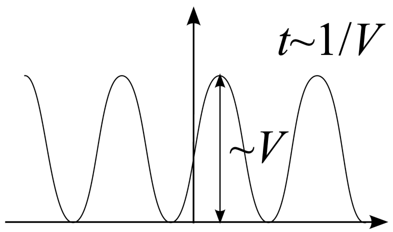
$$\frac{dg}{dl} = \left(3 - \frac{d}{2}\right) g - bNg^3$$

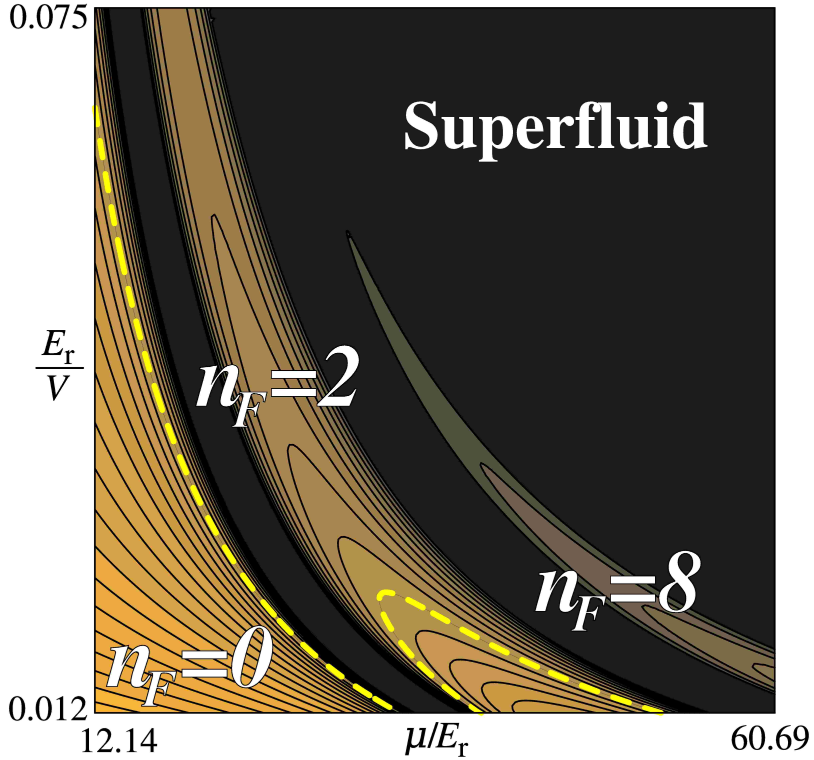
$$\frac{du_n}{dl} = \left[d + (2-d)n\right] u_n + \mathcal{O}(u^2)$$

P.N, Phys.Rev.B **79**, 144507 (2009)

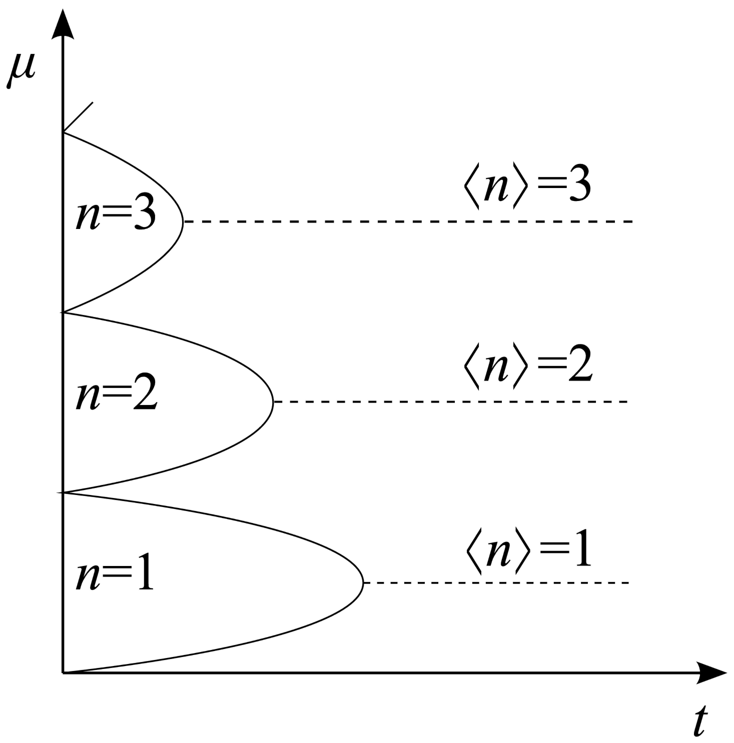
BCS-BEC crossover in lattice potentials

- 2[nd] order superfluid-insulator phase transition at T=0, h=0
  - Band-Mott insulator crossover at unitarity (s-wave)





E.G.Moon, P.Nikolić, S.Sachdev;
Phys.Rev.Lett. **99**, 230403 (2007)

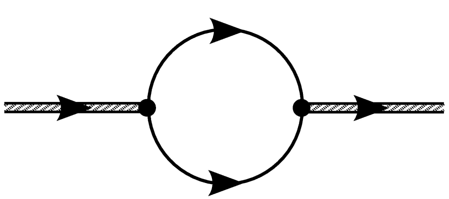


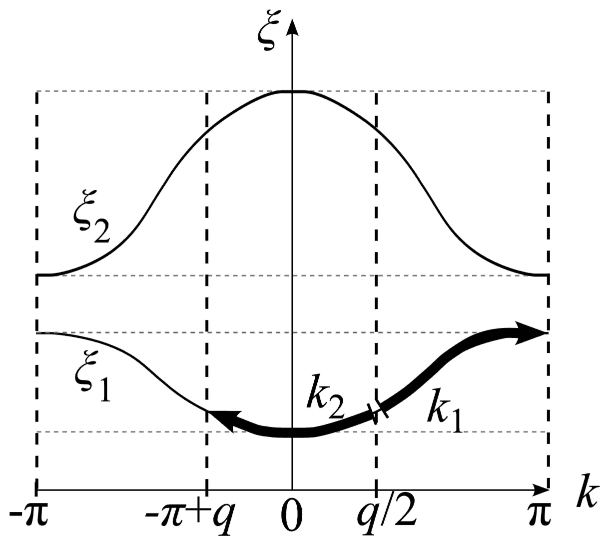
M.P.A.Fisher, P.B.Weichman, G.Grinstein, D.S.Fisher;
Phys.Rev.B **40**, 546 (1989)

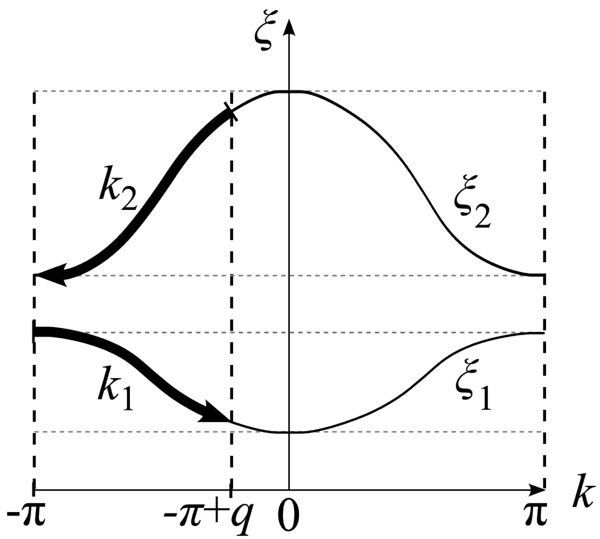
Pair density wave

- Pair density wave
  - Supersolid without the uniform component
  - Pairing instability in a band-insulator **generally** occurs at a finite crystal momentum

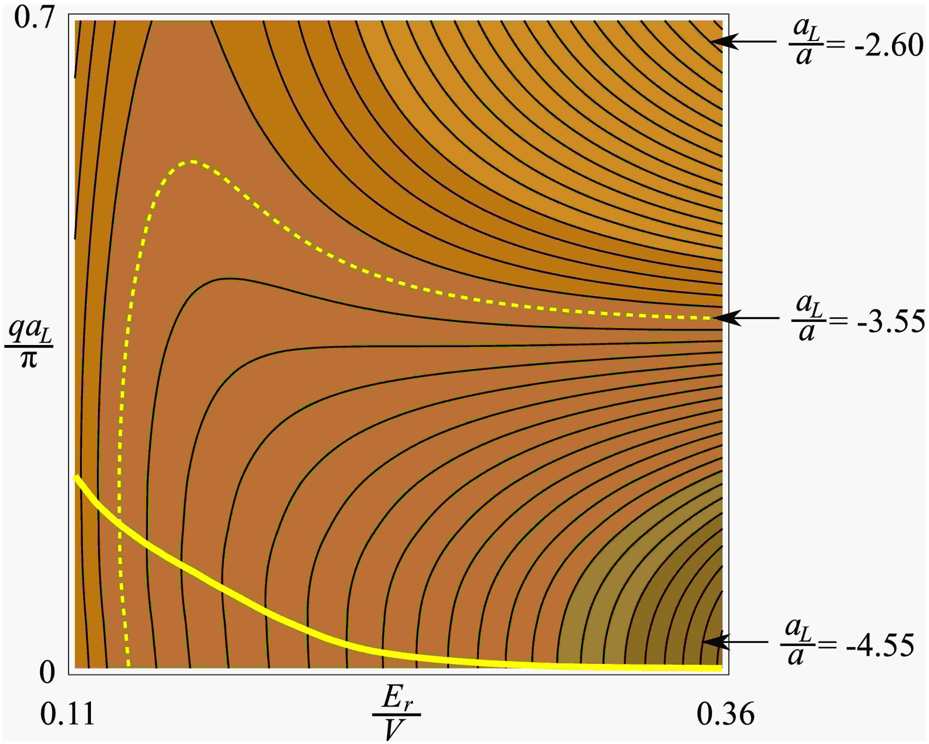
$$\Pi_{\mathbf{Gq};\mathbf{G'q'}} = \sum_{\mathbf{n}_1\mathbf{n}_2} \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \frac{f\left(\xi_{\mathbf{n}_1\mathbf{k}_1}\right) - f\left(-\xi_{\mathbf{n}_2\mathbf{k}_2}\right)}{\xi_{\mathbf{n}_1\mathbf{k}_1} + \xi_{\mathbf{n}_2\mathbf{k}_2}} \Gamma^{\mathbf{Gq}*}_{\mathbf{n}_1\mathbf{k}_1;\mathbf{n}_2\mathbf{k}_2} \Gamma^{\mathbf{G'q'}}_{\mathbf{n}_1\mathbf{k}_1;\mathbf{n}_2\mathbf{k}_2}$$

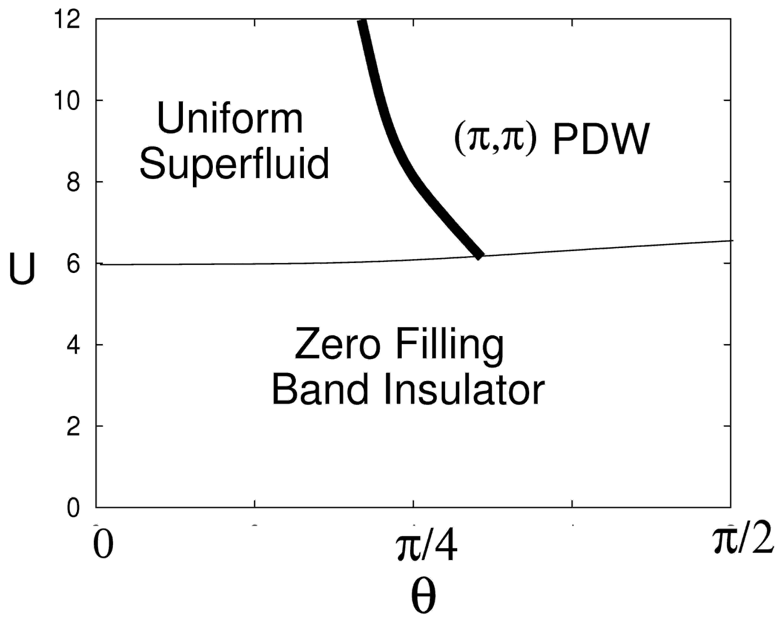






PDW evolution



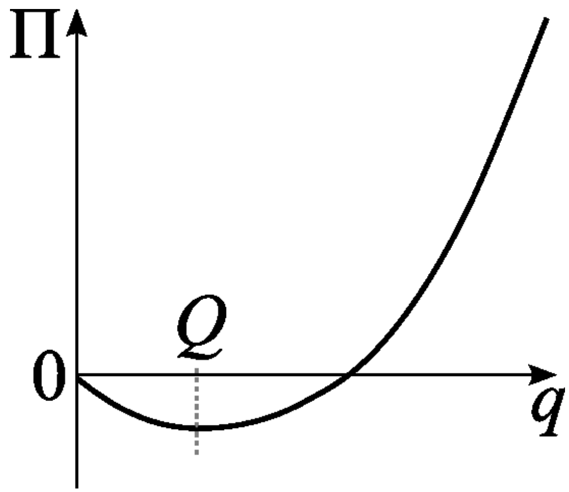


- Incommensurate PDW
  - Vertex *q*-dependence
  - Weak coupling (BCS limit)
- Commensurate PDW
  - Energy *q*-dependence
  - Strong inter-band coupling
  - Halperin-Rice in p-p

P.N., A. Burkov, A.Paramekanti, Phys.Rev.B **81**, 012504 (2010)

Fluctuation effects

- Incommensurate supersolid?
  - Pairing bubble has non-analytic linear q-dependence at small q
  - Inconsistent with q=0 pairing ($\omega \sim \sqrt{|\mathbf{q}|}$ Goldstone modes)
  - Robust finite-q pairing against fluctuations
  - But, frustrated on the lattice!



- Fluctuation effects
  - Stabilize a commensurate supersolid order?
  - Looks like Mott physics!
  - Are there non-trivial paired insulators?

- Near the superfluid-insulator transition
  - Fermions have a large (band) gap
  - Collective bosonic modes are low energy excitations
  - Charge conservation => infinite lifetime for gapped bosons

Cuprates, d-wave pairing and Mottness

- Microscopic mechanism in underdoped cuprates?
  - Short-range AF correlations ⇒ gap (antinodal)
  - Hole pair hopping doesn't frustrate spins
    ⇒ effective weak attractive interaction (antinodal)
  - Two-dimensional dynamics
  - Effective BEC regime for antinodal quasiparticles
- Consequences
  - Mottness adjacent to SC, quantum vortex dynamics...
- Complications due to d-wave
  - Nodal pairbreaking occurs, but anomalously slow
  - Low-energy bosons exist (superohmic decay) with large DOS

Conclusions

- Effective scattering resonances
  - “Weak-coupling” universality in generic band insulators
  - Particle-particle and particle-hole channels
  - Path to strongly correlated states of fermions
- Bosonic Mott insulators from fermions
  - Adjacent to superfluid phases in effective BEC regimes (especially 2D)
  - Susceptible to symmetry breaking or topological order
- Systems of interest
  - PDW in cold atom gases
  - Re-entrant superconductivity, topological insulators
  - Cuprates