Negative effective magnetic pressure instability

- Les Houches 1987
- Cargese 1988
- Cambridge 1992 …
- But now for something completely non-helical…

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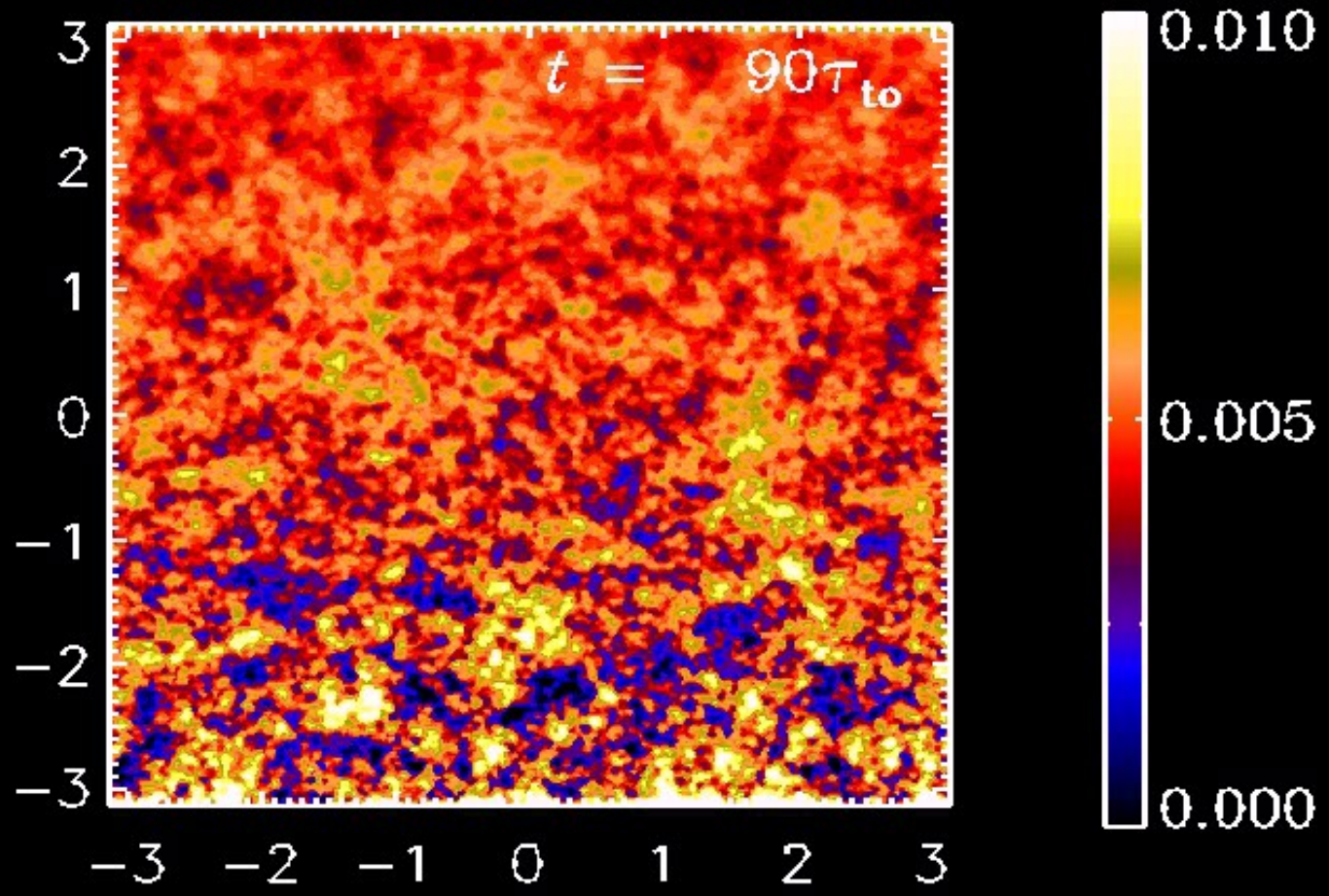
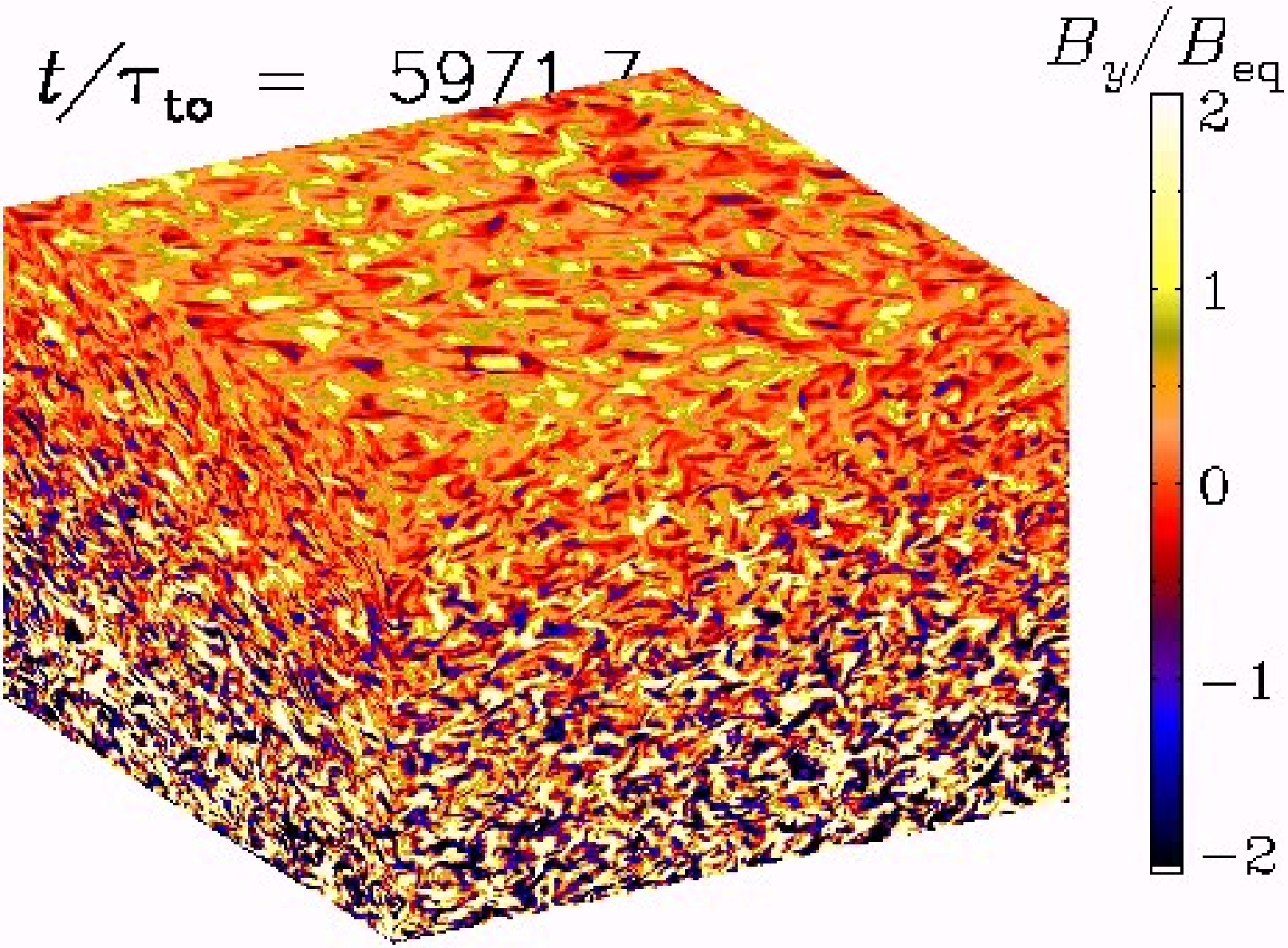
Setup

- 3-D box, size $(2\pi)^3$, isothermal MHD
- Random, nonhelical forcing at $k_f/k_1=15$
- Stratified in z, $\rho \sim \exp(z/H)$, $H=1$, $\Delta\rho=535$
- Periodic in x and y
- stress-free, perfect conductor in z
- Weak imposed field B_0 in y
- Run for *long* times: what happens?
- Turnover time $\tau_{to}=(u_{rms}\, k_f)^{-1}$, turb diff $\tau_{td}=(\eta_t\, k_1^2)^{-1}$
- Is longer by factor $3(k_f/k_1)^2 = 3\ 15^2 = 675$
- Average B_y over y and $\Delta t=80\tau_{to}$

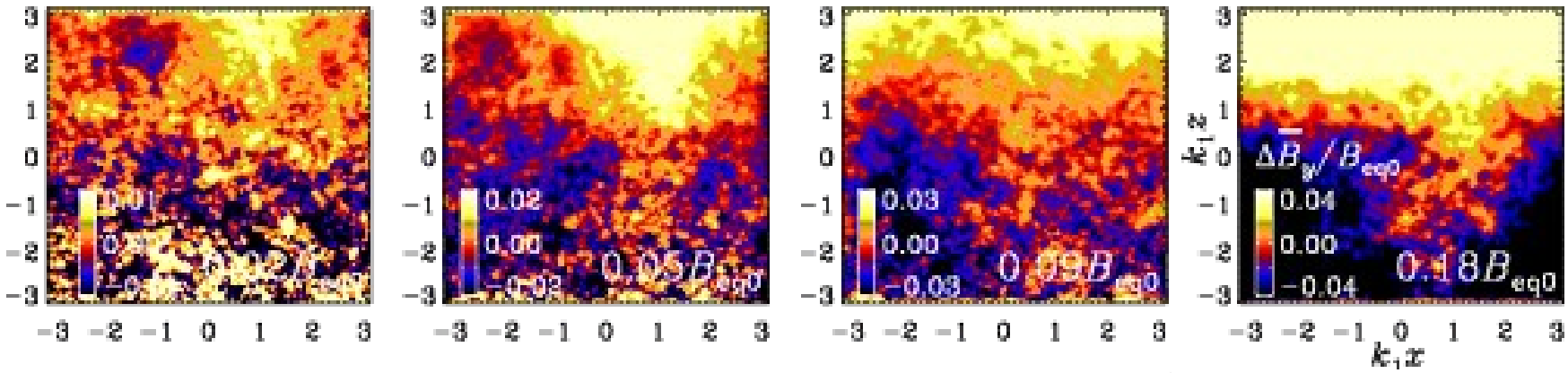
Negative effective magnetic pressure instability in action (Re_M=70)

Brandenburg et al. (ApJL 740, 50L)

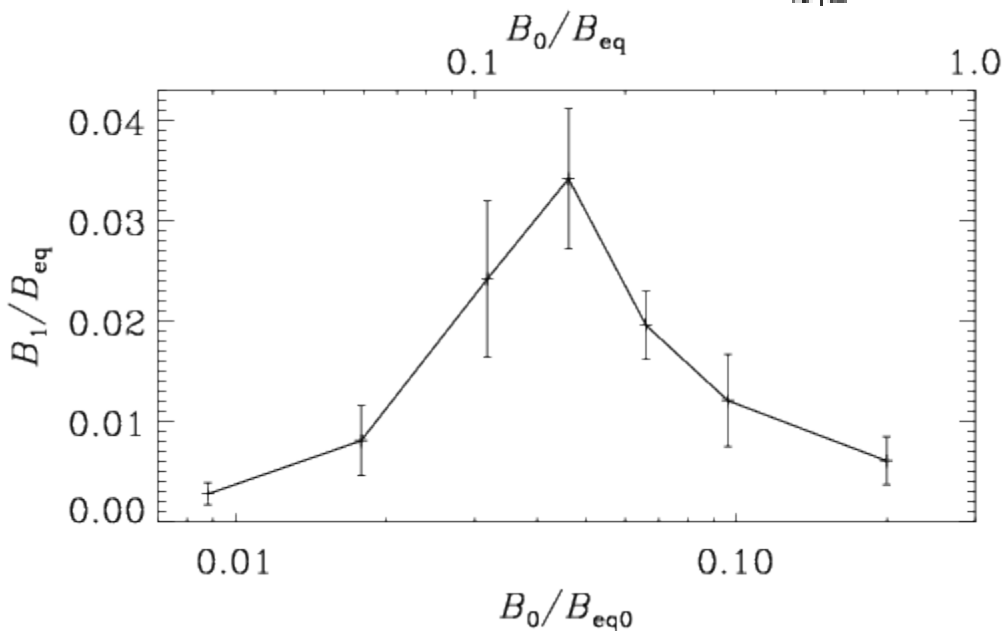



$t/\tau_{to} = 5971.7$
B_y/B_{eq}
2
1
0
−1
−2

Works only in a certain *B* range

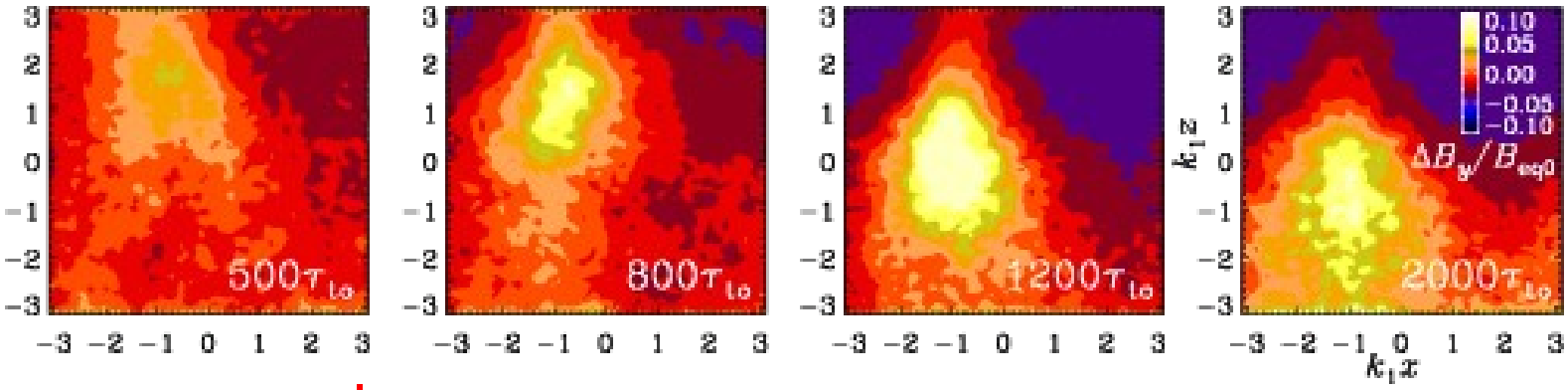


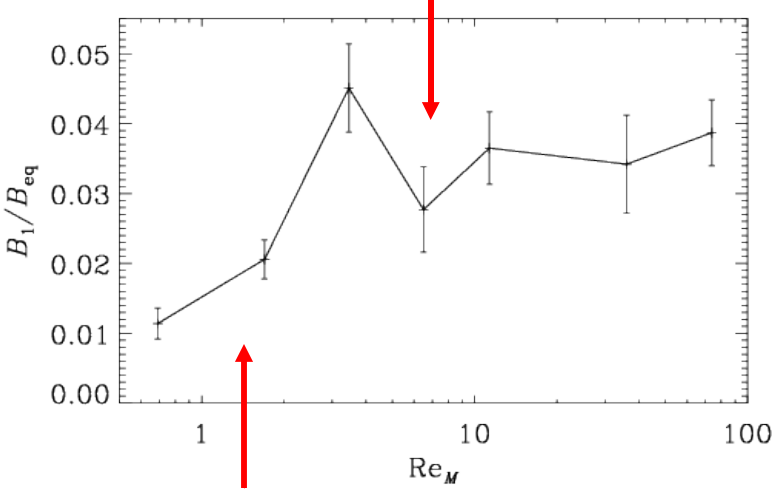
- is expected
- B_0/B_{eq}~0.15
- Vertical inhomo. for stronger field



Rm dependence

$$U_iU_j - B_iB_j + \tfrac{1}{2}\delta_{ij}\mathbf{B}^2 = \tfrac{1}{3}\delta_{ij}\left(\mathbf{U}^2 + \tfrac{1}{2}\mathbf{B}^2\right) \approx \tfrac{1}{3}\delta_{ij}\left(\underbrace{\mathbf{U}^2 + \mathbf{B}^2}_{\approx const} - \tfrac{1}{2}\mathbf{B}^2\right)$$





Breakdown of quasi-linear theory

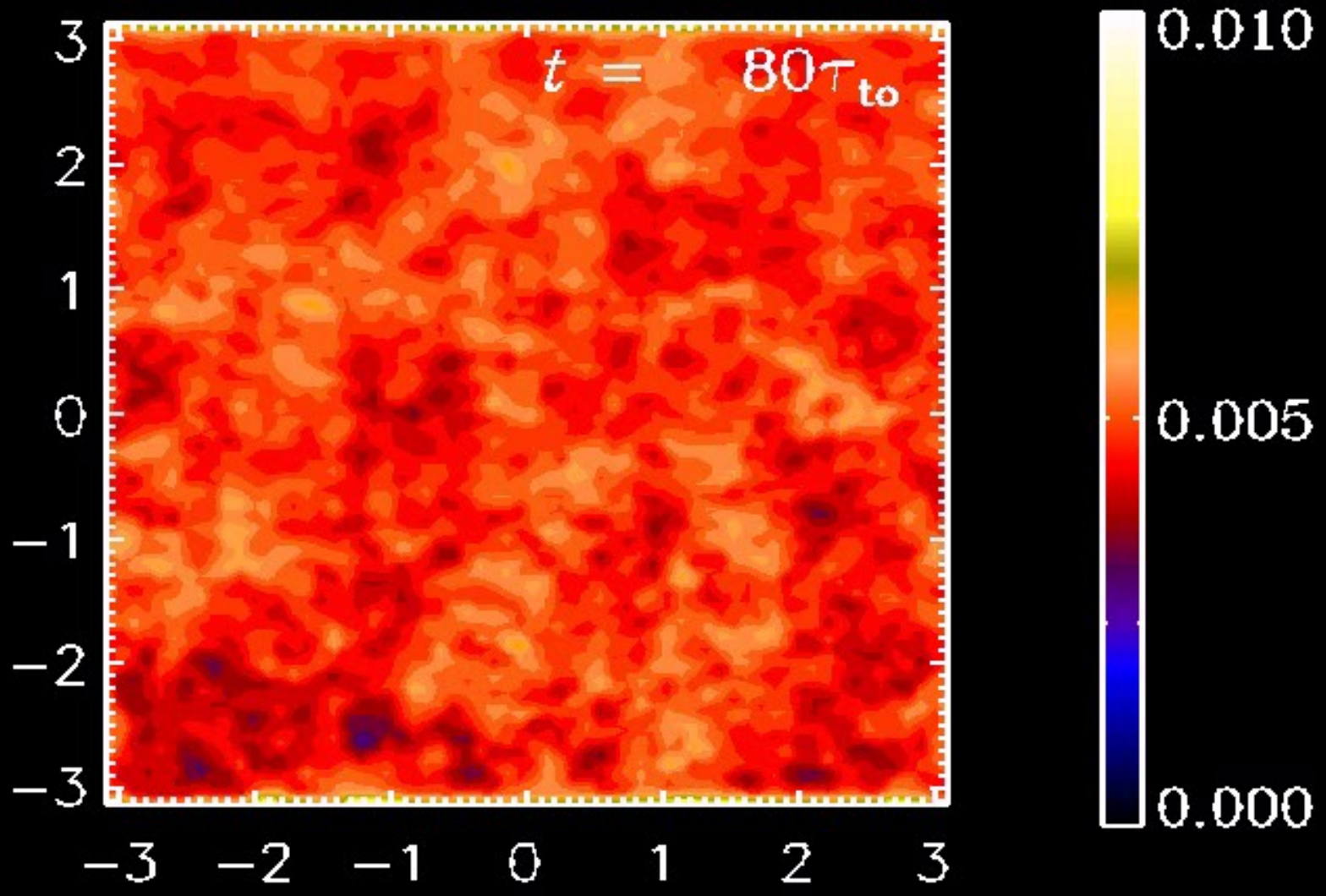
Re_M here based on forcing $\boldsymbol{k}$

Here 15 eddies per box scale

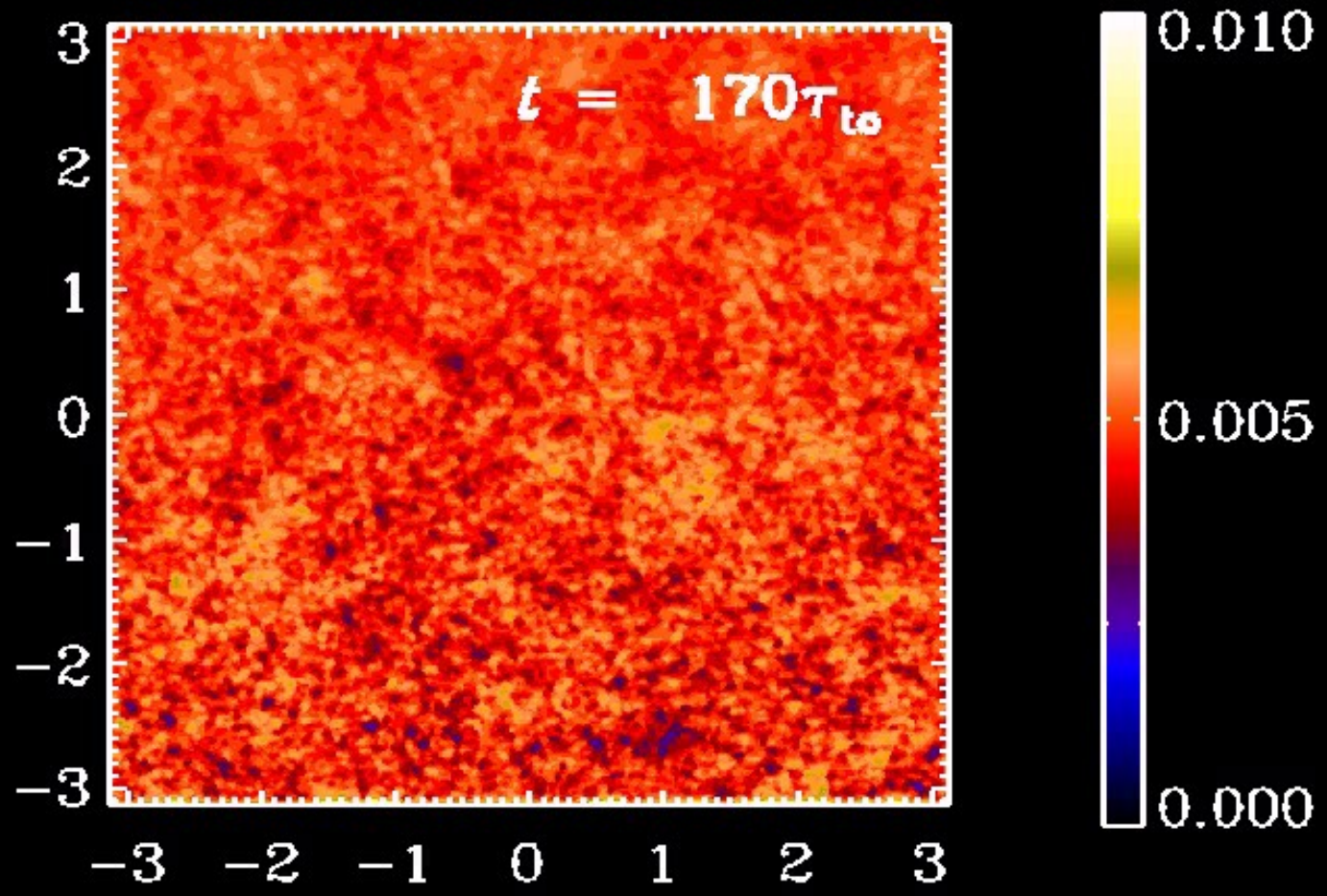
Re_M=70 means 70x15x2π =7000

based on box scale

Rm=6 potato sack

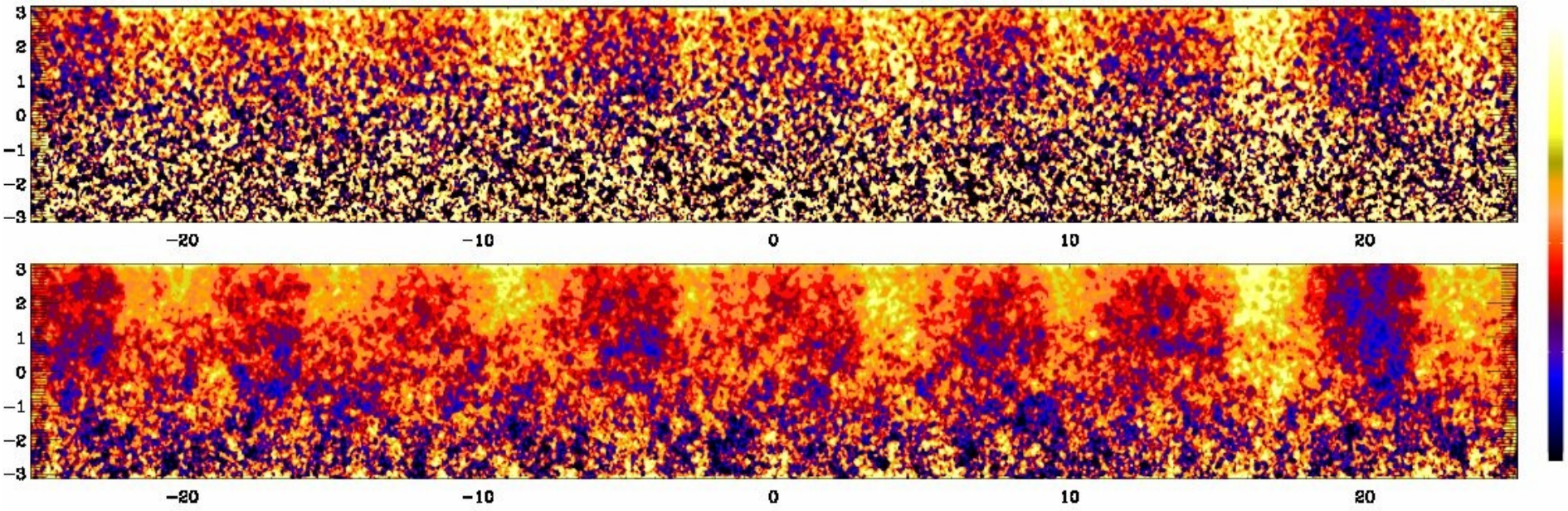


Larger scale separation: 30 instead of 15



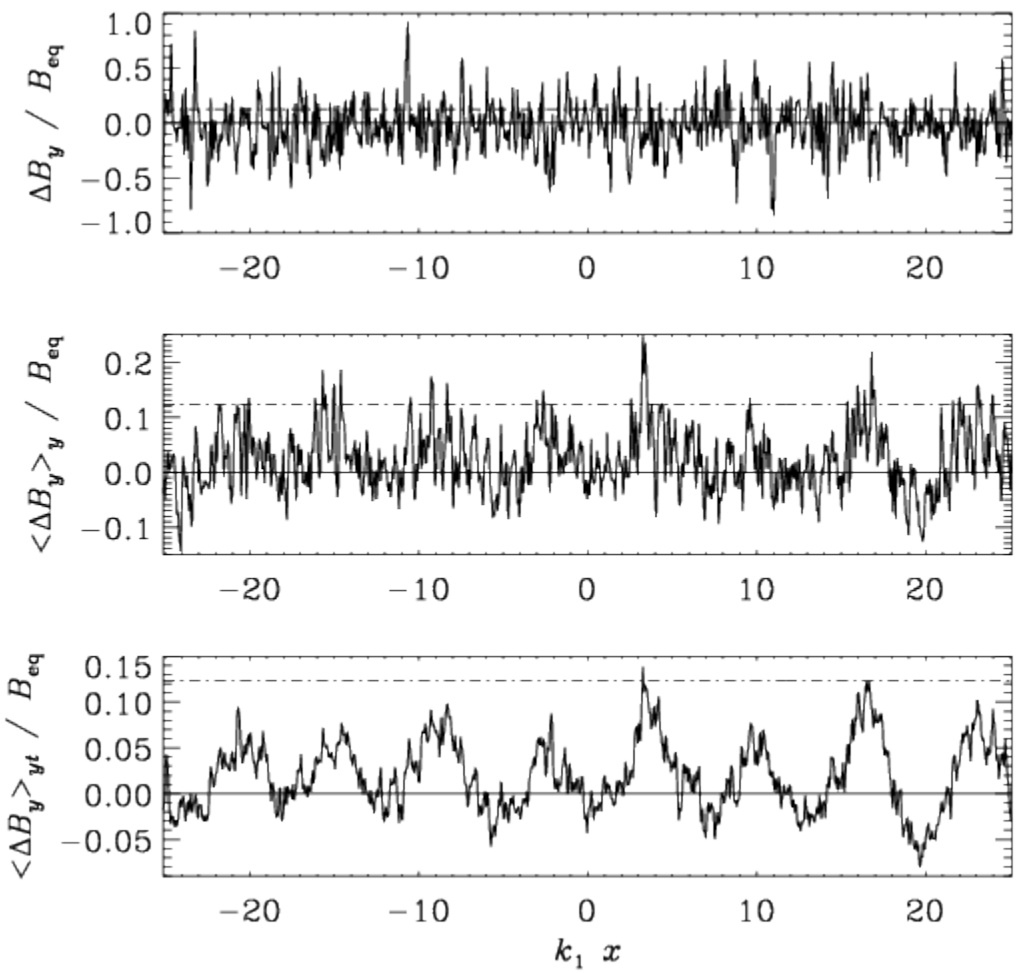
Large aspect ratio: more cells

- not all cells equally strong
- reasonably well seen in just y-averages



Large aspect ratio runs

- But not in *y*-slice
- $B \sim B_{eq}$, while
- $B_1 \sim 0.1\ B_{eq}$
- Not centered around B_0



New aspects in mean-field concept

Ohm's law

$$\eta\overline{\mathbf{J}} = \overline{\mathbf{E}} + \overline{\mathbf{U}} \times \overline{\mathbf{B}} + \overline{\mathbf{u} \times \mathbf{b}}$$

Theory and simulations: x effect and turbulent diffusivity

$$\overline{\mathbf{u} \times \mathbf{b}} = \alpha\overline{\mathbf{B}} - \eta_{\mathrm{t}}\overline{\mathbf{J}} + \ldots$$

Turbulent viscosity and other effects in momentum equation

$$\overline{u_i u_j} = \ldots - \nu_{\mathrm{t}}\left(\overline{U}_{i,j} + \overline{U}_{j,i}\right) + q_{\mathrm{s}}\overline{B}_i\overline{B}_j - \tfrac{1}{2}\delta_{ij}q_{\mathrm{p}}\overline{\mathbf{B}}^2 + \ldots$$

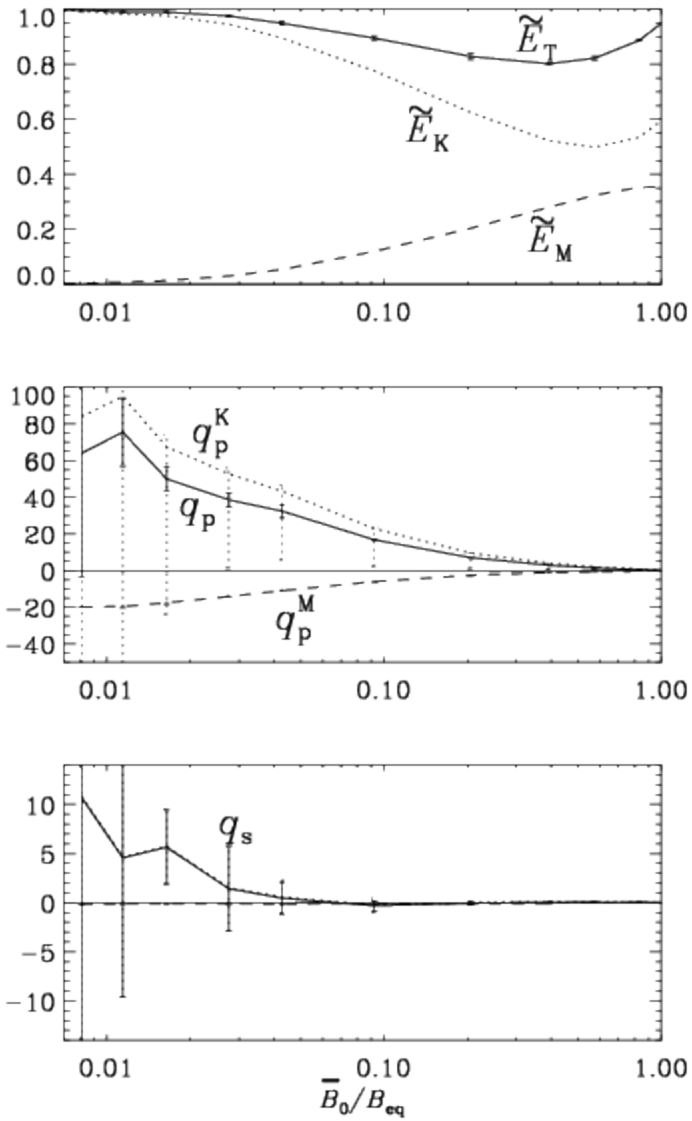
Magnetic contribution to pressure & energy different!

$$U_i U_j - B_i B_j + \tfrac{1}{2}\delta_{ij}\mathbf{B}^2$$

$$\approx \tfrac{1}{3}\delta_{ij}\left(\mathbf{U}^2 + \tfrac{1}{2}\mathbf{B}^2\right)$$

$$\approx \tfrac{1}{3}\delta_{ij}\left(\underbrace{\mathbf{U}^2 + \mathbf{B}^2}_{\approx const} - \tfrac{1}{2}\mathbf{B}^2\right)$$

Can lead to reversed mean-field buoyancy in *stratified* system



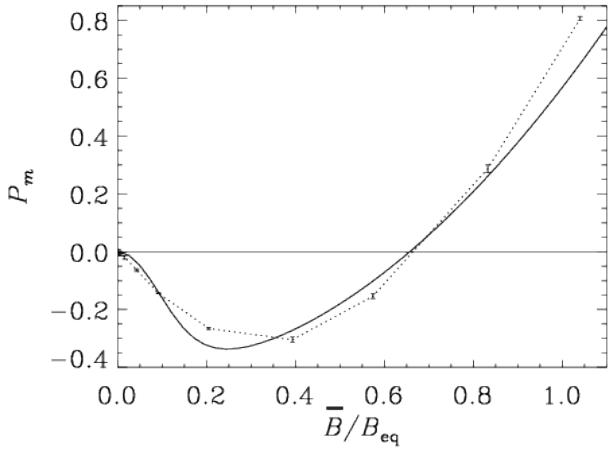
Earlier results for low Rm

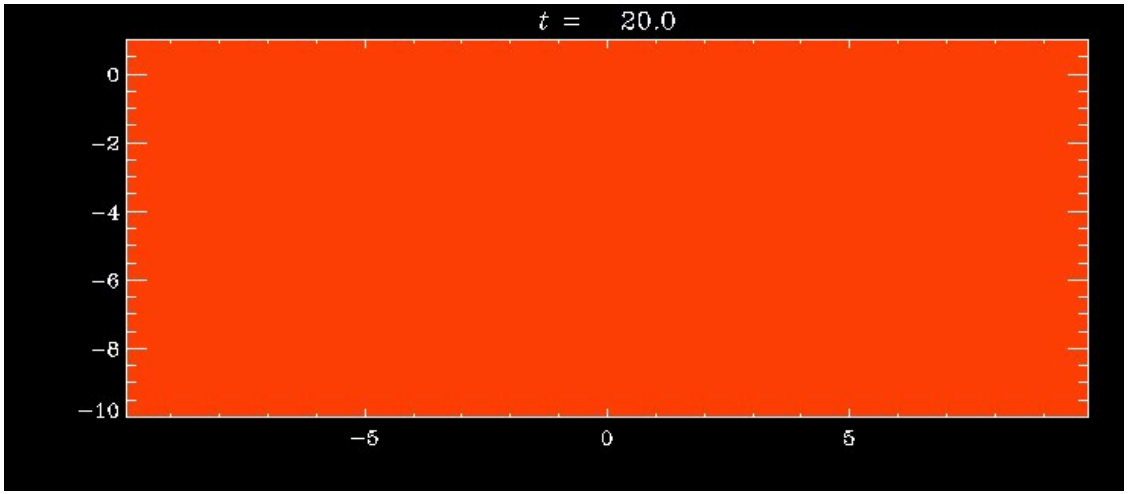
- Rädler (1974) computed magnetic suppression (for other reasons)
- Rüdiger (1974) → works only for Pm < 8
- Rüdiger et al. (1986) Maxwell tension formally negative for Rm > 1, but invalid
- Rüdiger et al. (2011, arXiv), no negative effective magnetic pressure for Rm < 1.
- Kleeorin et a. (1989, 1990, 1996), Kleeorin & Rogachevskii (1994, 2007)

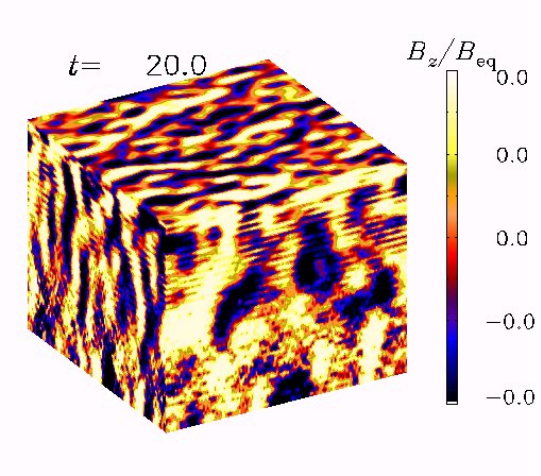
Formation of flux concentrations

Recent work with Kleeorin & Rogachevskii (2010, AN 331, 5)

$$\overline{u_i u_j} + \ldots = q_s\left(\overline{B}\right)\overline{B}_i\overline{B}_j - \tfrac{1}{2}\delta_{ij} q_p\left(\overline{B}\right)\overline{\mathbf{B}}^2 + \ldots$$

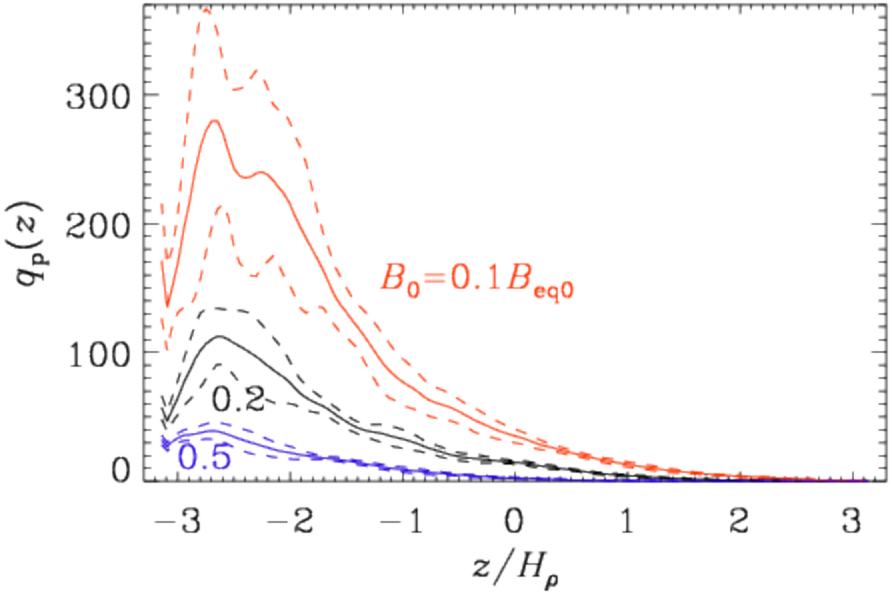


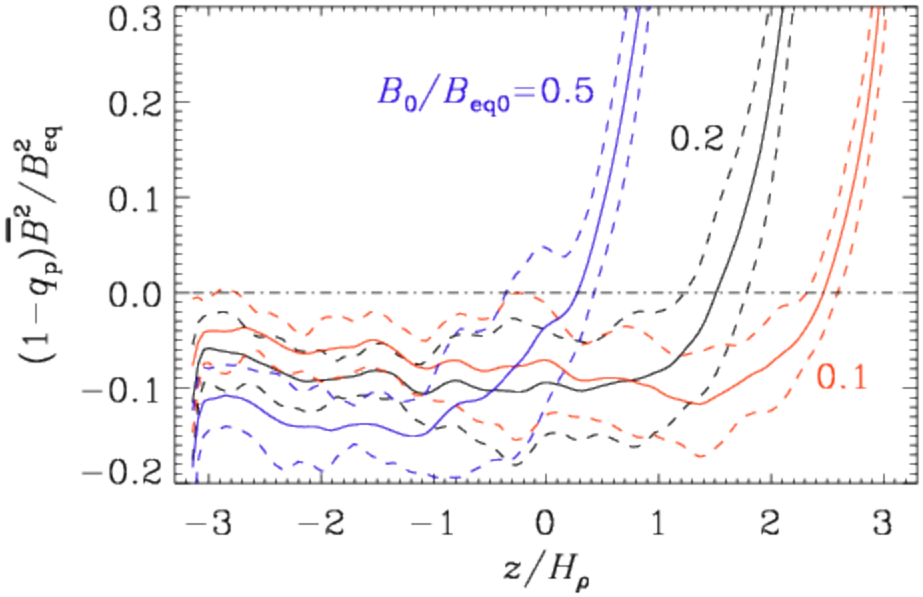


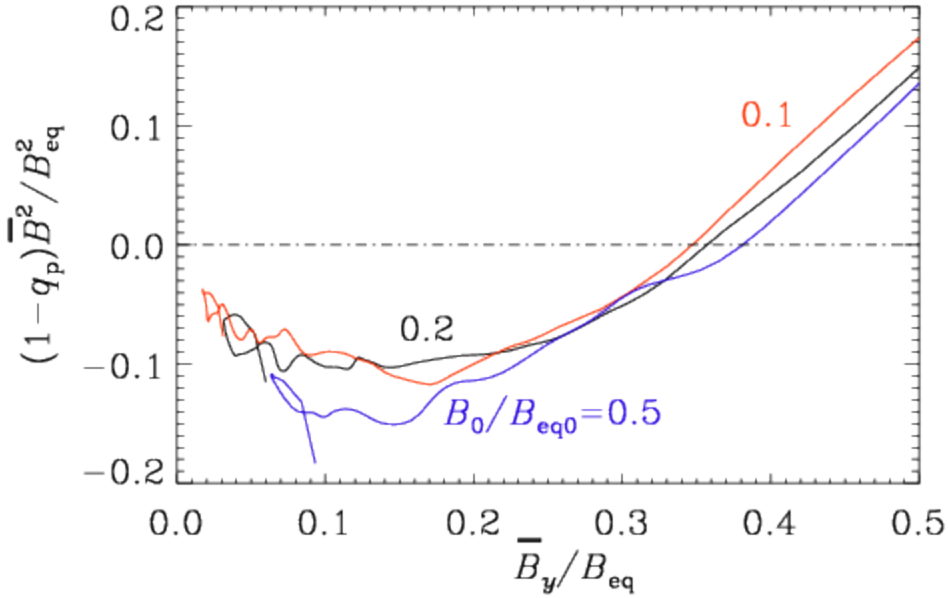


Stratified runs

isothermal → constant scale height,
density contrast, exp(2π)=535,
B_{eq} varies, so B/B_{eq} varies,
get whole curve in one run

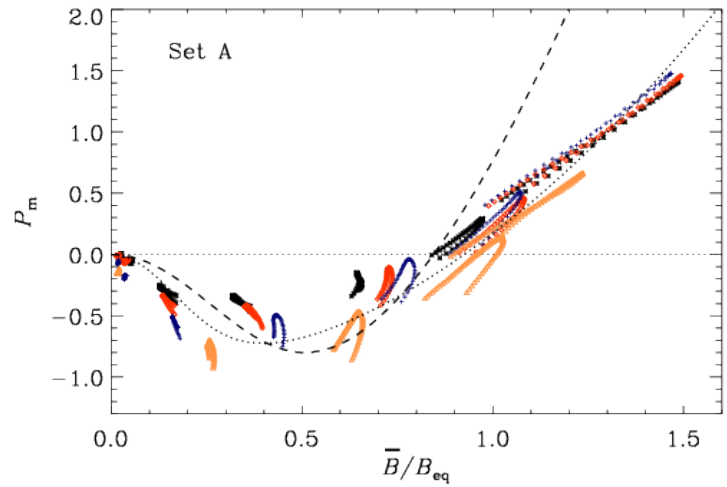


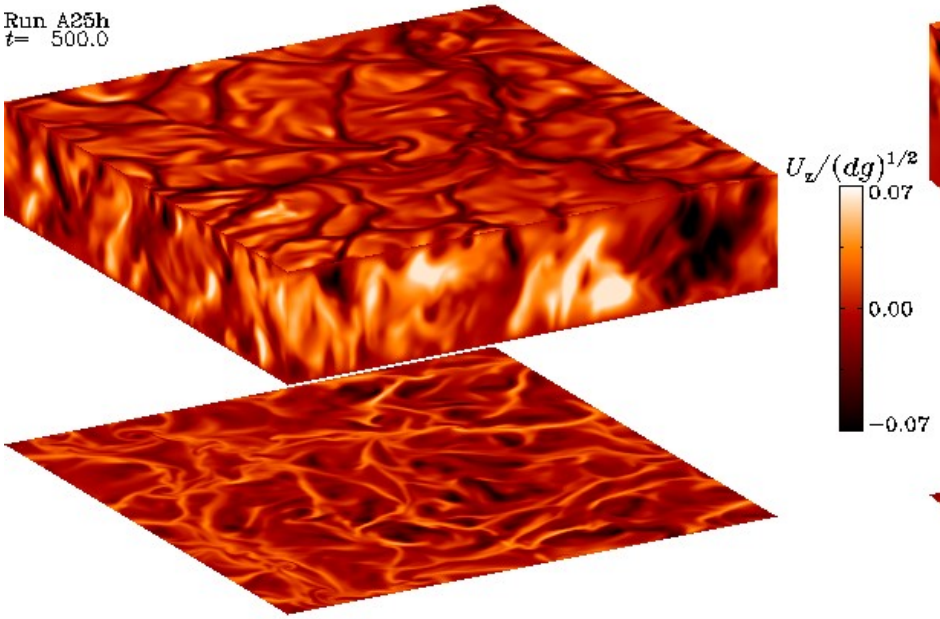


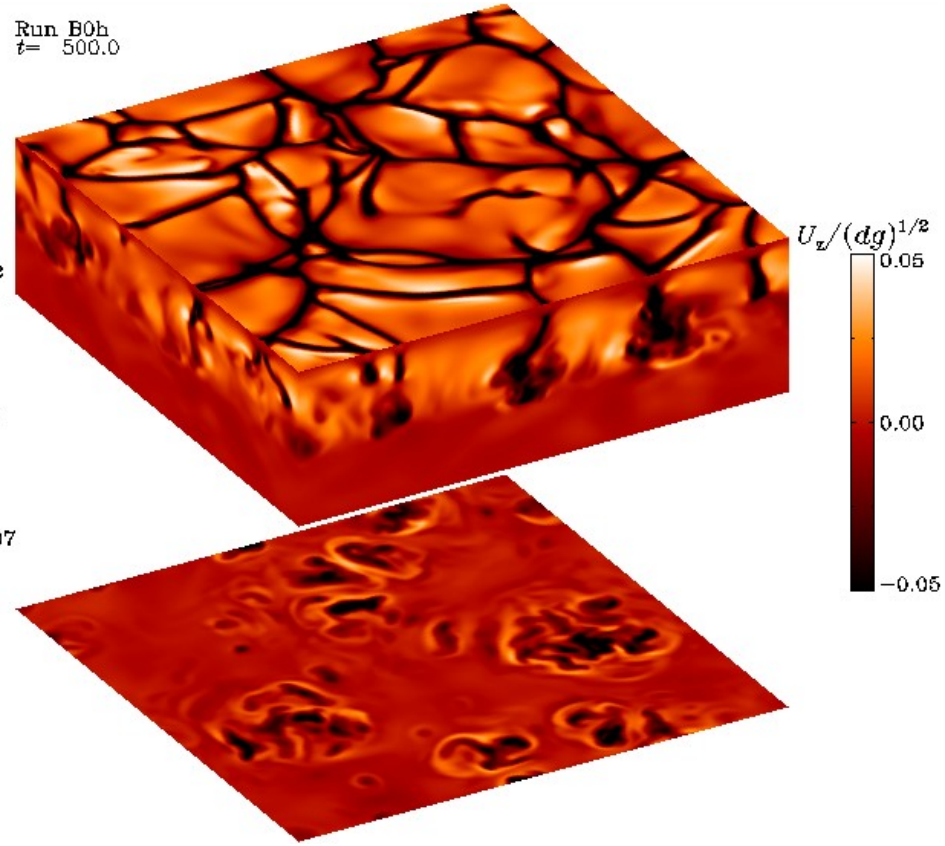


Convection

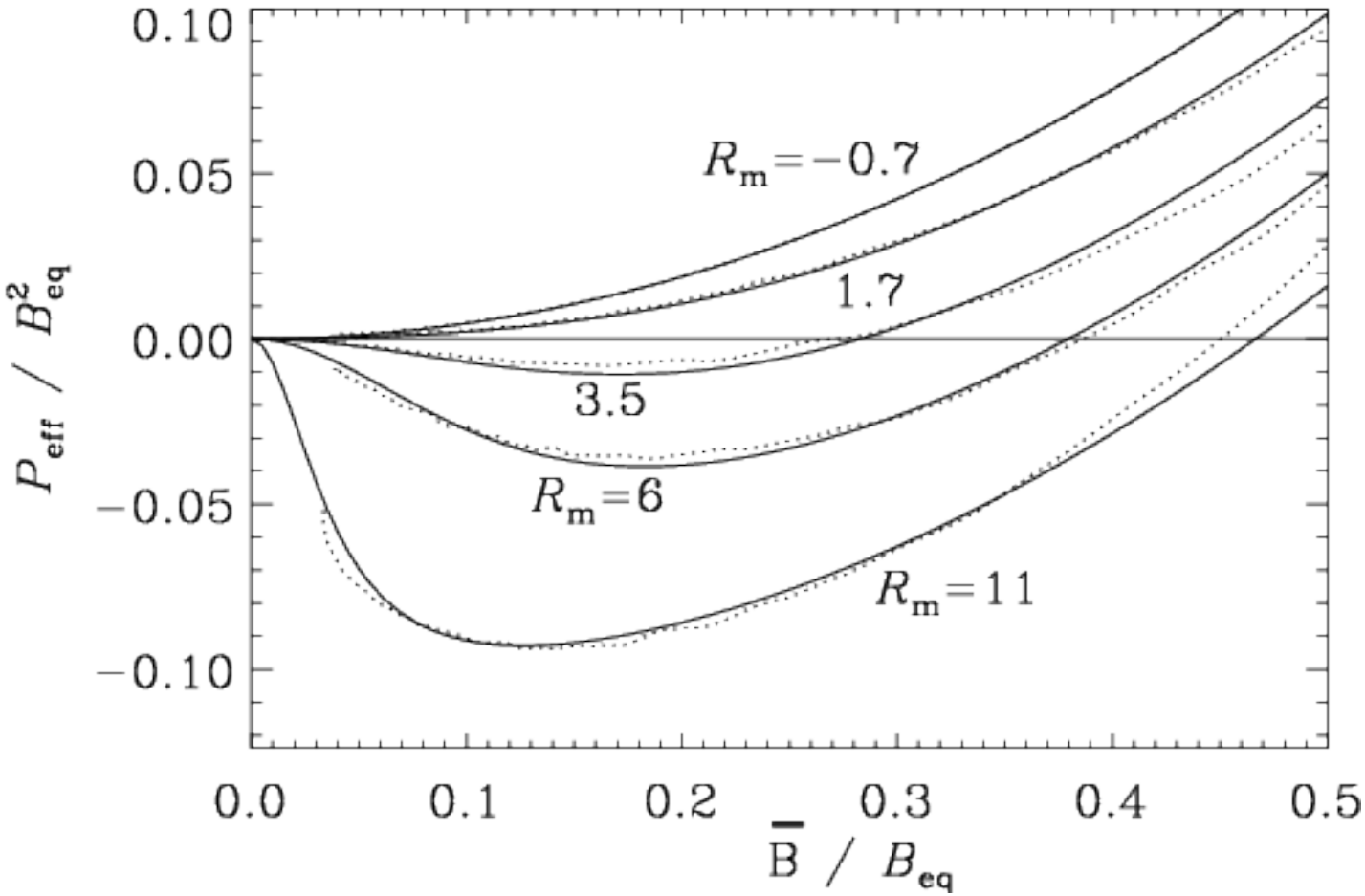
not just for forced turbulence,
→ result is robust!







Effective magnetic pressure

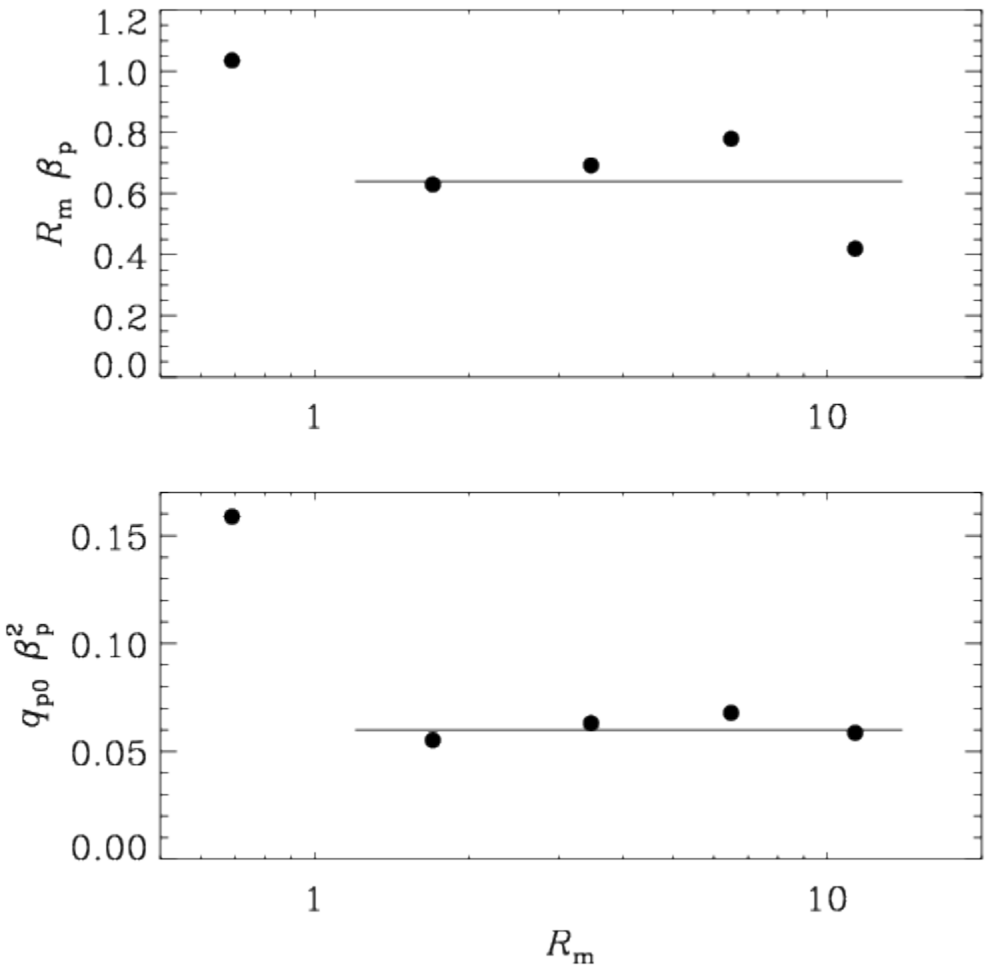


Fit formula and Rm dependence

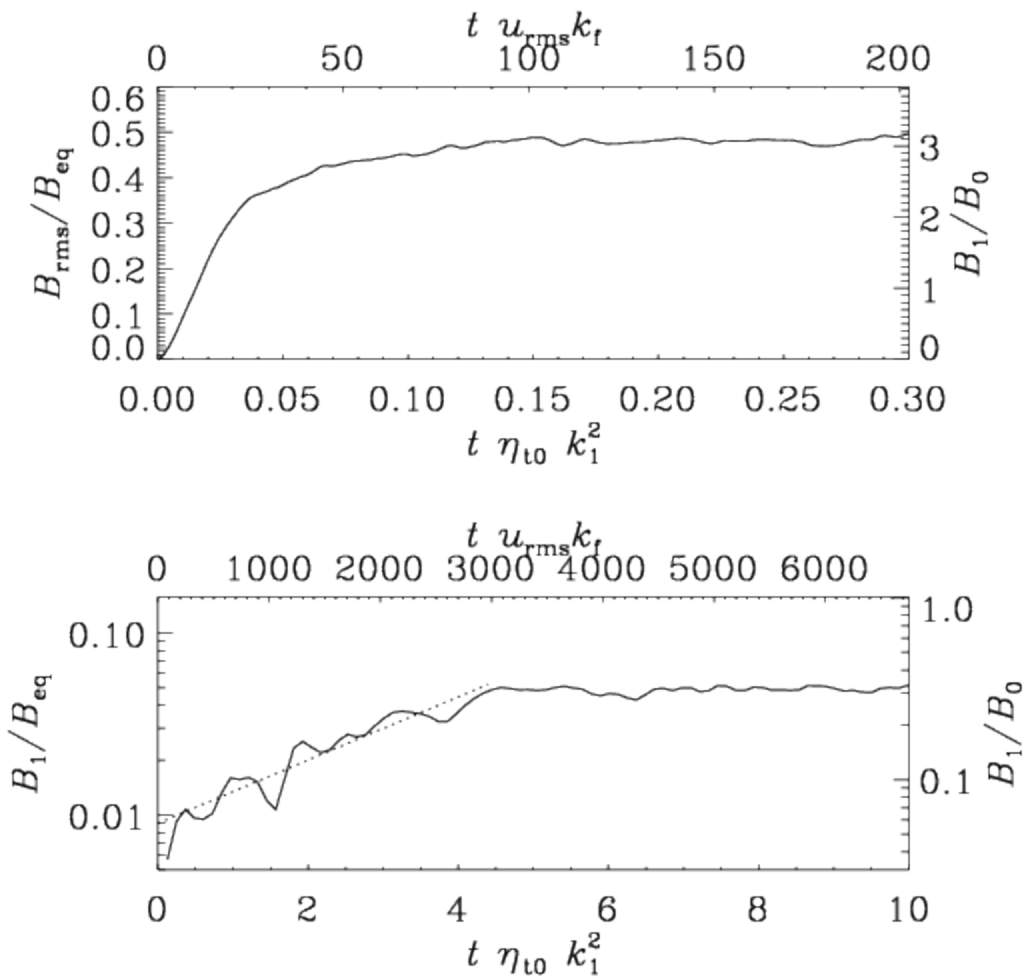
$$\boldsymbol{\beta} = \overline{\mathbf{B}} / B_{\mathrm{eq}}$$

$$P_{\mathrm{eff}} = \left[1 - q_{\mathrm{p}}(\beta)\right] \beta^2$$

$$q_{\mathrm{p}}(\beta) = \frac{q_{\mathrm{p0}}}{1 + \beta^2 / \beta_p^2}$$

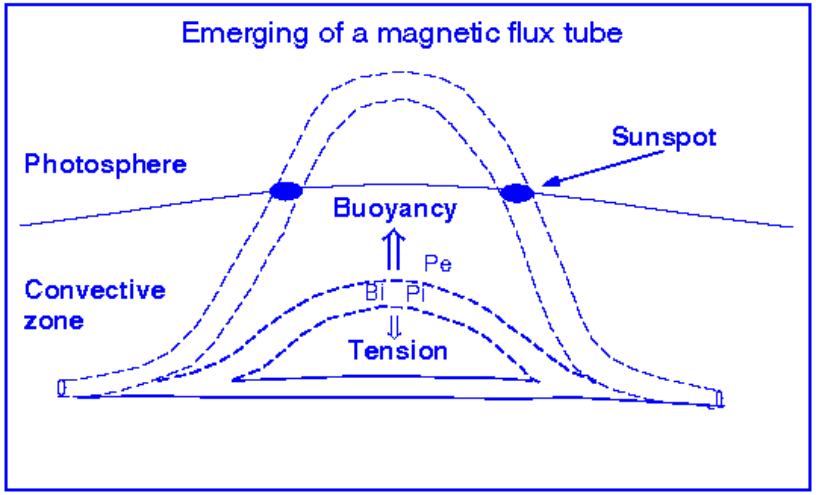


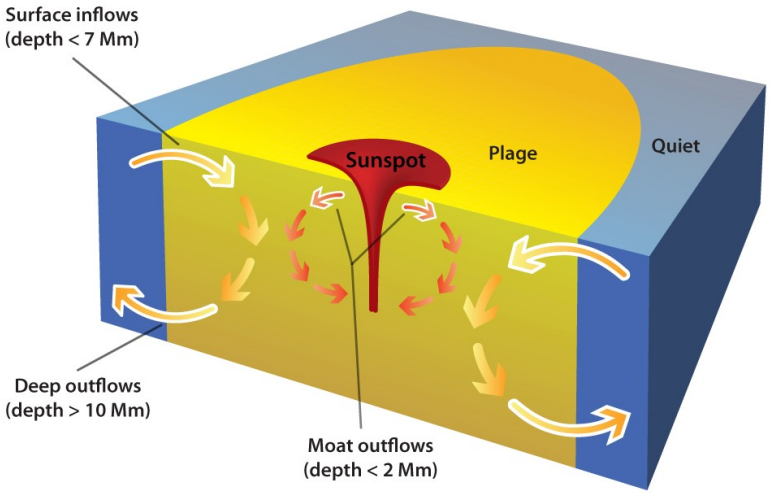
Slow growth



- Several thousand turnover times
- Or ½ a turbulent diffusive time
- Exponential growth → linear instability of an already turbulent state

How deep are sunspots rooted?





Hindman et al. (2009, ApJ)

- Solar activity may not be so deeply rooted
- The dynamo may be a distributed one
- Near-surface shear important

Conclusions: new food for thought

- Essential ingredient for shallow sunspot acticity identified (ApJL 740, L50)
- Effect does not exist below Rm=1
- Becomes stronger with better scale separation (e.g. 30 instead of 15)

Thanks to the Astrophysics group at Nordita