Inverse Energy Cascade in 3d Isotropic Turbulence

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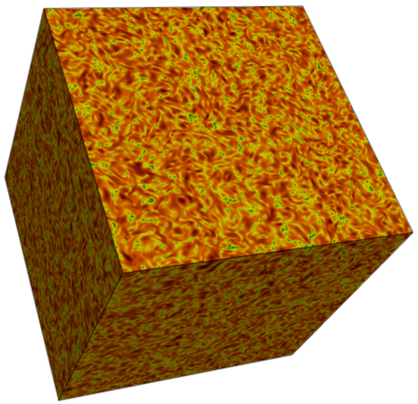

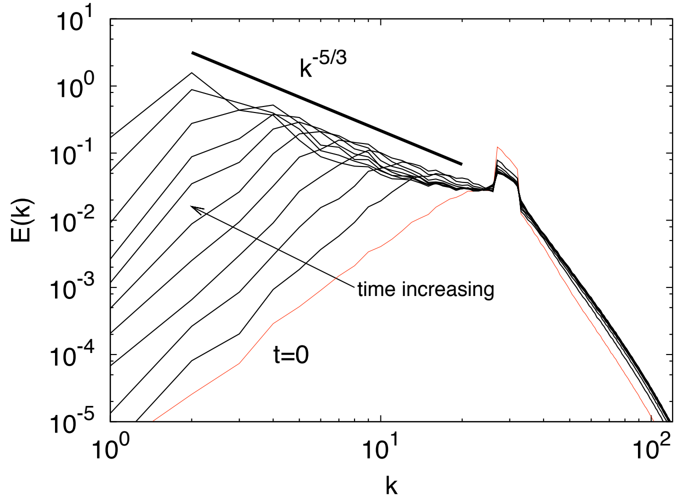

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Credits: S. Musacchio (CNRS-France)

F. Toschi (University of Eindhoven, The Netherlands) breaking parity invariance: helicity input

right handed

left handed

breaking isotropy   3D - > 2D

THE PHYSICS OF FLUIDS VOLUME 16, NUMBER 8 AUGUST 1973

Research Notes

Research Notes published in this Section include important research results of a preliminary nature which are of special interest to the physics of fluids and new research contributions modifying results already published in the scientific literature. Research Notes cannot exceed five printed columns in length including space allowed for title, abstract, figures, tables, and references. The abstract should have three printed lines. Authors must shorten galley proofs of Research Notes longer than five printed columns before publication.

Helicity cascades in fully developed isotropic turbulence

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(Received 12 June 1972; final manuscript 7 March 1973)

Based on total helicity conservation in inviscid incompressible flows, the existence of simultaneous energy and helicity cascades is envisaged.

both Energy & Helicity forward cascades

$$E(k) \propto \epsilon^{\frac{2}{3}} k^{-\frac{5}{3}}$$

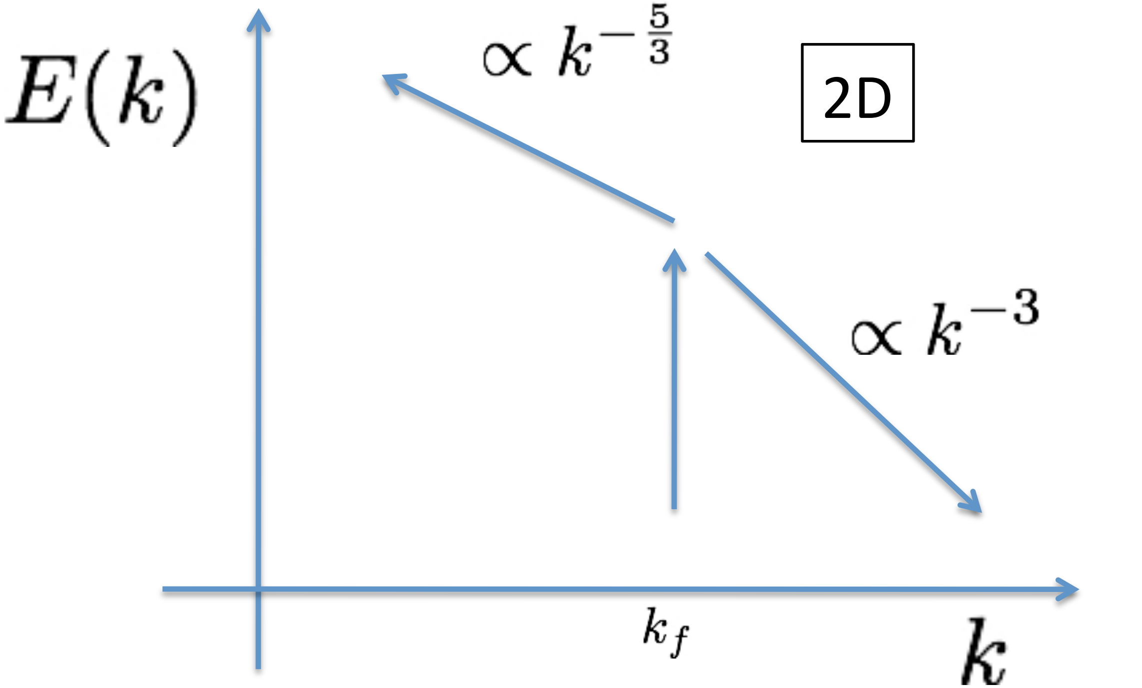
$$H(k) \propto \eta \epsilon^{-\frac{1}{3}} k^{-\frac{5}{3}}$$

only Helicity forward cascades

$$H(k) \propto \eta k^{-\frac{4}{3}}$$

$$E(k) \propto \eta k^{-\frac{7}{3}}$$

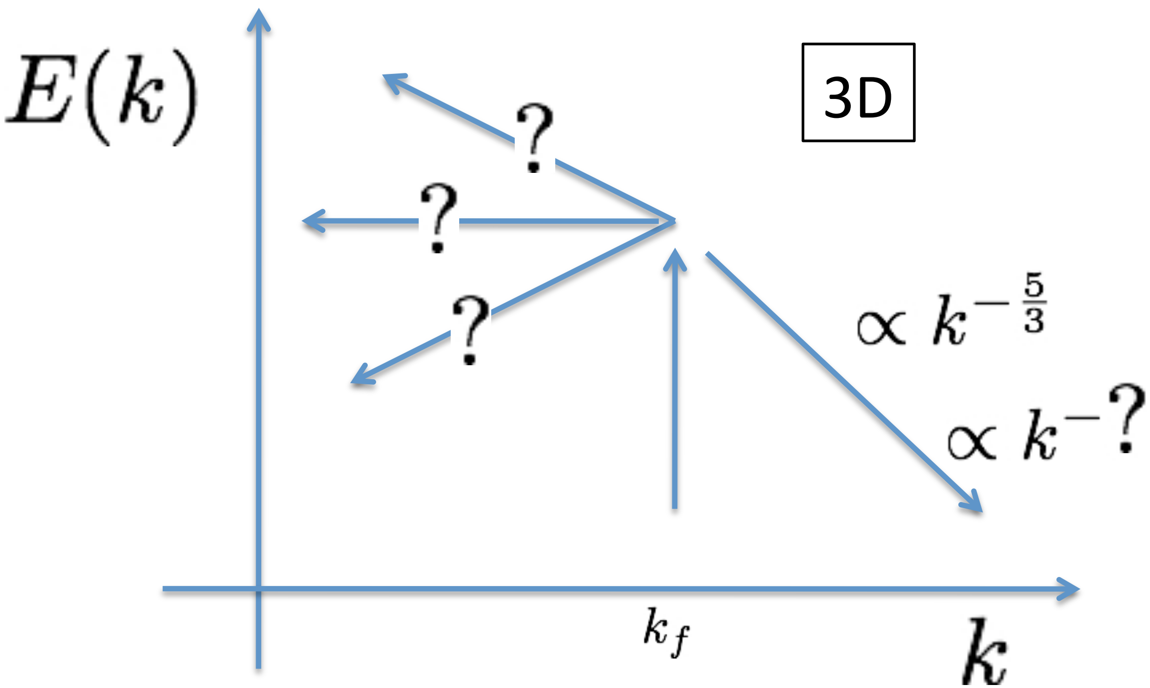
However, a problem arises with a pure helicity cascade: it appears difficult to inject helicity into the fluid without at the same time injecting some energy. Possibly this difficulty can be overcome, as for two-dimensional turbulence, by assuming that energy and helicity are fed into the fluid at a certain wavenumber k_i; helicity then cascades toward large wavenumbers according to (8) while energy cascades toward small wavenumbers (inverse cascade) according to the usual Kolmogoroff law. In the energy inverse cascade range,



2 Invariants (positive defined)

$$\Omega = \int d^3x\, \omega \cdot \omega$$

$$E = \int d^3x\, \mathbf{v} \cdot \mathbf{v}$$



2 invariants

$$E = \int d^3x\, \mathbf{v} \cdot \mathbf{v}$$

$$H = \int d^3x\, \omega \cdot \mathbf{v}$$

The joint cascade of energy and helicity in three-dimensional turbulence

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$$H(k) \propto \eta \epsilon^{-\frac{1}{3}} k^{-\frac{5}{3}}$$
$$E(k) \propto \epsilon^{\frac{2}{3}} k^{-\frac{5}{3}}$$

The role of helicity in three-dimensional turbulence is, in our opinion, still somewhat mysterious. In particular, it is still unclear how energy and helicity dynamics interact in detail. The role of helicity in geophysical flows has been considered[3]—without being fully resolved—while its appearance and influence in engineering applications is still largely unexplored. We hope that this work will be a helpful step in the direction of better understanding the subtle manifestations of helicity in three-dimensional turbulence.

parity invariance restoration

PRL 104, 184506 (2010) PHYSICAL REVIEW LETTERS week ending 7 MAY 2010

Turbulence in More than Two and Less than Three Dimensions

Antonio Celani,[1] Stefano Musacchio,[2,3] and Dario Vincenzi[3]

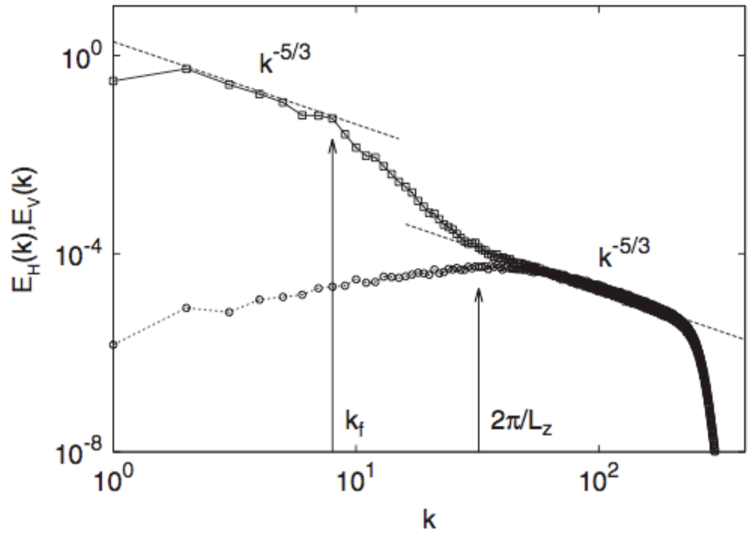


FIG. 3. Kinetic energy spectrum of horizontal (squares) and vertical (circles) velocities. Dashed lines represents Kolmogorov scaling. Parameters of the simulation: $L_x = 2\pi$, $\ell_f/L_x = 1/8$,

CONFINEMENT 3D -> 2D

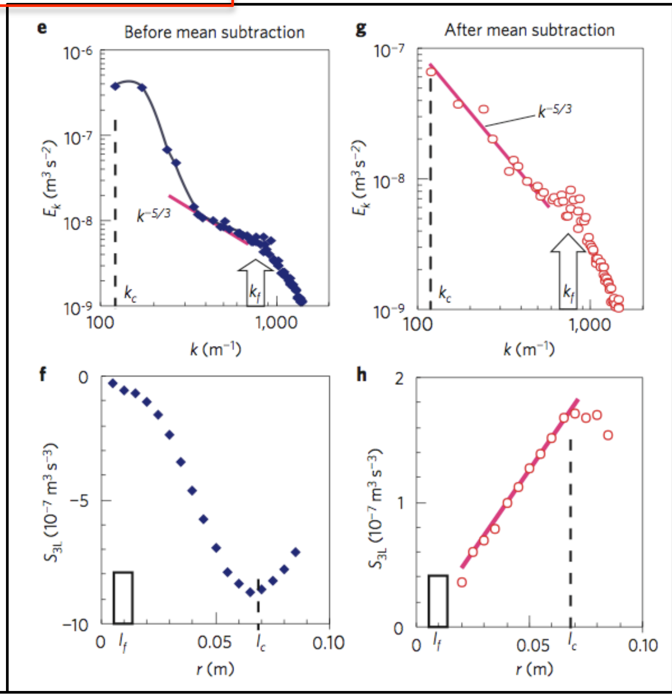
ANISOTROPY; HELICITY == 0

$$H = \int d^3x \, \omega \cdot \mathbf{v}$$

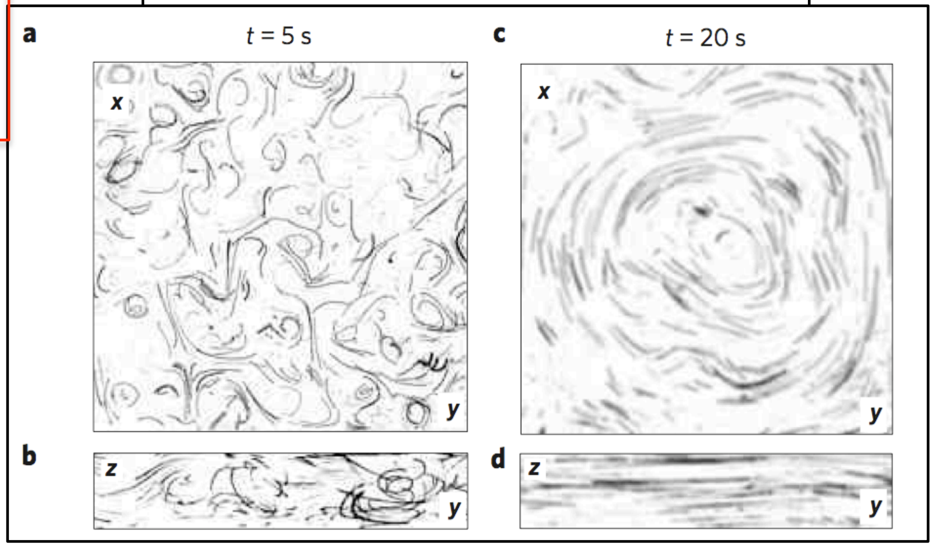
Nat Phys. 2011

Upscale energy transfer in thick turbulent fluid layers

H. Xia[1], D. Byrne[1], G. Falkovich[2] and M. Shats[1]*



Flows in natural fluid layers are often forced simultaneously at scales smaller and much larger than the depth. For example, the Earth's atmospheric flows are powered by gradients of solar heating: vertical gradients cause three-dimensional (3D) convection whereas horizontal gradients drive planetary scale flows. Nonlinear interactions spread energy over scales[1,2]. The question is whether intermediate scales obtain their energy from a large-scale 2D flow or from a small-scale 3D turbulence. The paradox is that 2D flows do not transfer energy downscale whereas 3D turbulence does not support an upscale transfer. Here we demonstrate experimentally how a large-scale vortex and small-scale turbulence conspire to provide for an upscale energy cascade in thick layers. We show that a strong planar vortex suppresses vertical motions, thus facilitating an upscale energy cascade. In a bounded system, spectral condensation into a box-size vortex provides for a self-organized planar flow that secures an upscale energy transfer.



CONFINEMENT 3D -> 2D

ANISOTROPY; HELICITY == 0

Crossover from Two- to Three-Dimensional Turbulence

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(Received 22 March 1996)

Forced rotating turbulence is simulated within a periodic box of small aspect ratio. Critical parameter values are found for the stability of a 2D inverse cascade of energy in the presence of 3D motions at small scales. There is a critical rotation rate below which 2D forcing leads to an equilibrated 3D state, while for a slightly larger rotation rate, 3D forcing drives a 2D inverse cascade. It is shown that inverse and forward cascades of energy can coexist. This study is relevant to geophysical flows, and contains physics beyond the scope of quasigeostrophic models. [S0031-9007(96)01175-1]

ROTATION 3D -> 2D

HELICITY ?

PHYSICAL REVIEW E **79**, 026304 (2009)

Helicity cascades in rotating turbulence

P. D. Mininni[1,2] and A. Pouquet[2]

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(Received 4 September 2008; published 5 February 2009)

ROTATION 3D -> 2D

The effect of helicity (velocity-vorticity correlations) is studied in direct numerical simulations of rotating turbulence down to Rossby numbers of 0.02. The results suggest that the presence of net helicity plays an important role in the dynamics of the flow. In particular, at small Rossby number, the energy cascades to large scales, as expected, but helicity then can dominate the cascade to small scales. A phenomenological interpretation in terms of a direct cascade of helicity slowed down by wave-eddy interactions leads to the prediction of non-Kolmogorovian inertial indices for the small-scale energy and helicity spectra.

DOI: 10.1103/PhysRevE.79.026304 PACS number(s): 47.32.Ef, 47.27.Gs, 47.27.Jv

HELICITY ?
forward cascade

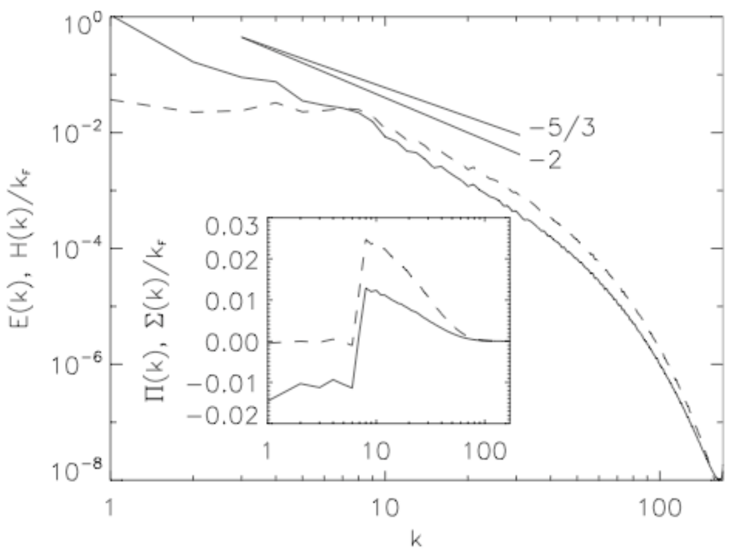

FIG. 2. Energy (solid) and helicity (dash) spectra in run A3 with the same forcing as run A2 but lower Rossby number. Different

Generation of large-scale structures in three-dimensional flow lacking parity-invariance

By P. L. SULEM[1,2], Z. S. SHE[1,3], H. SCHOLL[1,4]
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AKA EFFECT

ANISOTROPY; HELICITY

BIDIMENZIONALIZATION VIA GEOMETRY-> INVERSE ENERGY CASCADE; NO HELICITY INPUT
ROTATION -> BIDIMENZIONALIZATION -> INVERSE ENERGY + HELICITY INPUT
AKA -> BREAKING OF ISOTROPY AND PARITY BY SMALL SCALE FORCING

ALL CASES ARE STRONGLY ANISOTROPIC

ROLE OF HELICITY FOR 3D ISOTROPIC (INVERSE) ENERGY CASCADE?

The nature of triad interactions in homogeneous turbulence

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(Received 24 July 1991; accepted 22 October 1991)

$$\boxed{\boldsymbol{u}(\boldsymbol{k}) = u^+(\boldsymbol{k})\boldsymbol{h}^+(\boldsymbol{k}) + u^-(\boldsymbol{k})\boldsymbol{h}^-(\boldsymbol{k})}$$

$$\boldsymbol{h}^\pm = \hat{\boldsymbol{\nu}} \times \hat{\boldsymbol{k}} \pm i\hat{\boldsymbol{\nu}}$$

$$\hat{\boldsymbol{\nu}} = \boldsymbol{z} \times \boldsymbol{k}/||\boldsymbol{z} \times \boldsymbol{k}||$$

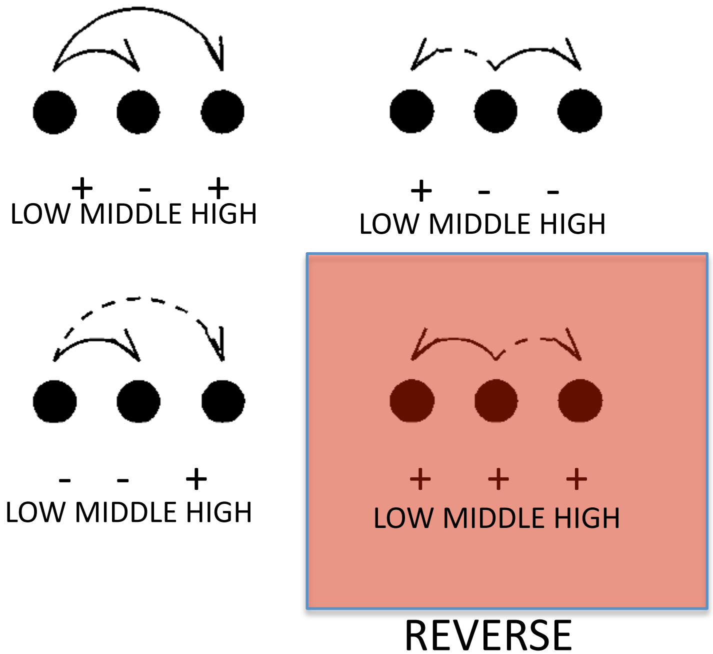
$$\boxed{i\boldsymbol{k} \times \boldsymbol{h}^\pm = \pm k\boldsymbol{h}^\pm}$$

$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + |u^-(\boldsymbol{k})|^2; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - |u^-(\boldsymbol{k})|^2). \end{cases}$$

$$u^{s_k}(\mathbf{k},t) \quad (s_k = \pm 1)$$

$$\frac{d}{dt}u^{s_k}(\mathbf{k}) + \nu k^2 u^{s_k}(\mathbf{k}) = \sum_{\mathbf{k}+\mathbf{p}+\mathbf{q}=0} \sum_{s_p, s_q} g_{\mathbf{k},\mathbf{p},\mathbf{q}}(s_p p - s_q q) \times [u^{s_p}(\mathbf{p}) u^{s_q}(\mathbf{q})]^* . \tag{15}$$

Eight different types of interaction between three modes $u^{s_k}(\mathbf{k})$, $u^{s_p}(\mathbf{p})$, and $u^{s_q}(\mathbf{q})$ with $|\mathbf{k}|<|\mathbf{p}|<|\mathbf{q}|$ are allowed according to the value of the triplet (s_k, s_p, s_q)



$$\dot{u}^{s_k} = r(s_p p - s_q q)\frac{s_k k + s_p p + s_q q}{p}(u^{s_p} u^{s_q})^* ,$$

$$\dot{u}^{s_p} = r(s_q q - s_k k)\frac{s_k k + s_p p + s_q q}{p}(u^{s_q} u^{s_k})^* ,$$

$$\dot{u}^{s_q} = r(s_k k - s_p p)\frac{s_k k + s_p p + s_q q}{p}(u^{s_k} u^{s_p})^* .$$

$$\mathcal{P}^{\pm} \equiv \frac{\boldsymbol{h}^{\pm} \otimes \overline{\boldsymbol{h}^{\pm}}}{\overline{\boldsymbol{h}^{\pm}} \cdot \boldsymbol{h}^{\pm}}. \qquad \boldsymbol{v}^{\pm}(\boldsymbol{x}) \equiv \sum_{\boldsymbol{k}} \mathcal{P}^{\pm} \boldsymbol{u}(\boldsymbol{k});$$

$$\boxed{\boldsymbol{u}(\boldsymbol{k}) = u^{+}(\boldsymbol{k})\boldsymbol{h}^{+}(\boldsymbol{k}) + \cancel{u^{-}(\boldsymbol{k})\boldsymbol{h}^{-}(\boldsymbol{k})}}$$

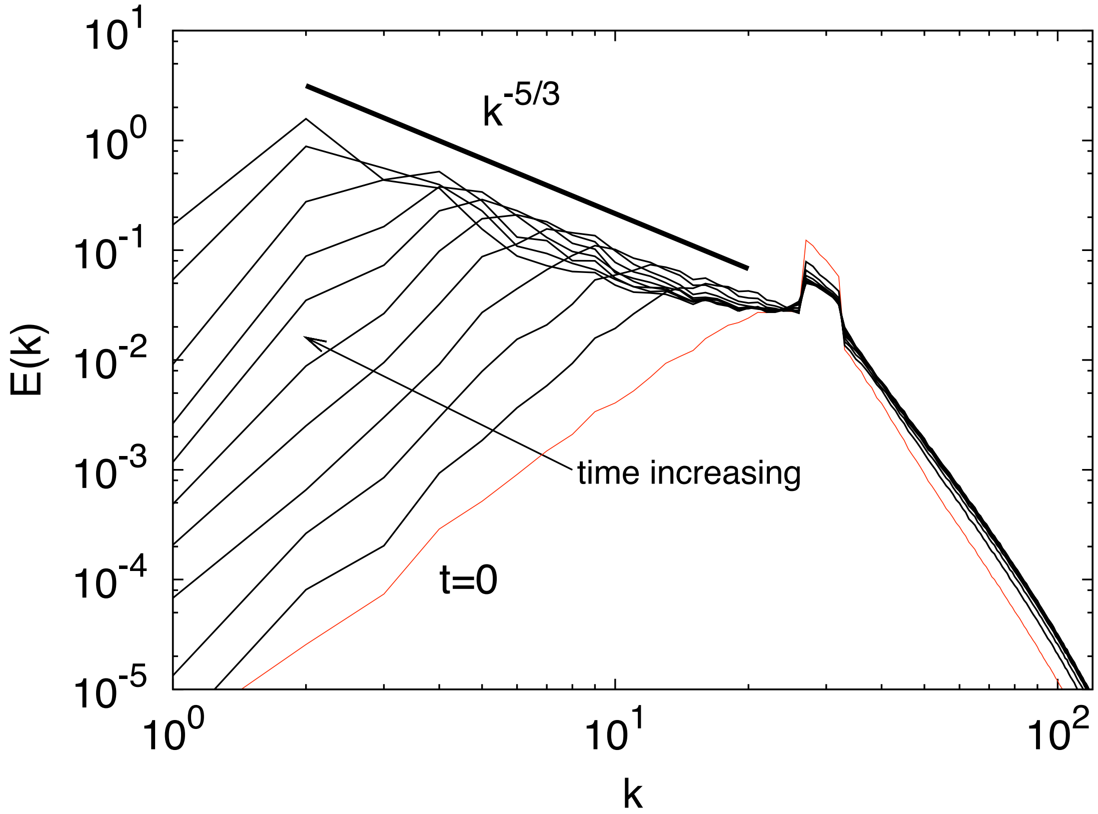
LOCAL BELTRAMIZATION (IN FOURIER)

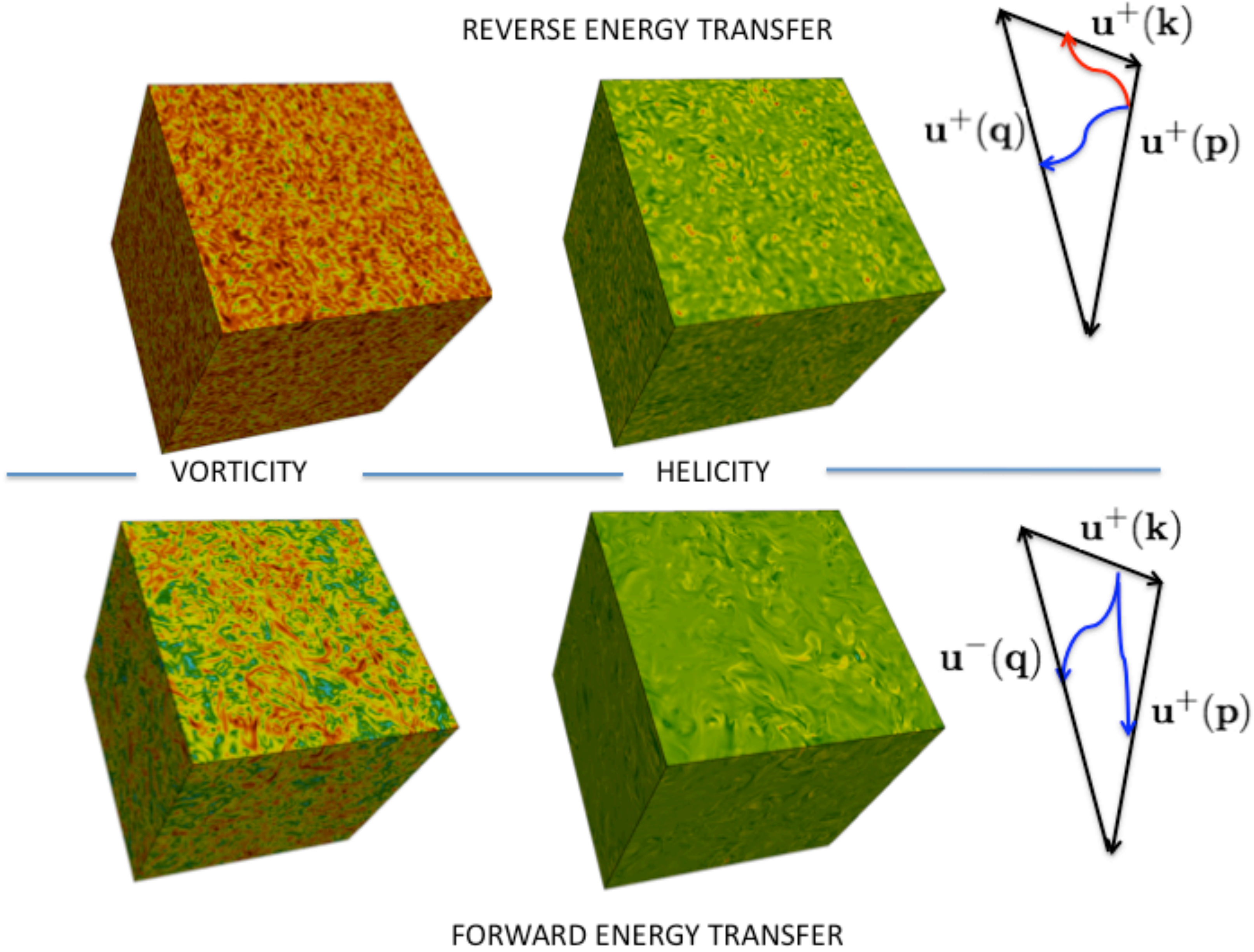
$$\partial_t \boldsymbol{v}^{+} = (-\boldsymbol{v}^{+} \cdot \boldsymbol{\nabla} \boldsymbol{v}^{+} - \boldsymbol{\nabla} p)^{+} + \nu \Delta \boldsymbol{v}^{+} + \boldsymbol{f}^{+}$$

$$\frac{d}{dt} u^{s_k}(\mathbf{k}) + \nu k^2 u^{s_k}(\mathbf{k}) = \sum_{\mathbf{k}+\mathbf{p}+\mathbf{q}=0} \sum_{s_p, s_q} g_{\mathbf{k},\mathbf{p},\mathbf{q}}(s_p p - s_q q) \times [u^{s_p}(\mathbf{p}) u^{s_q}(\mathbf{q})]^{*}. \qquad (15)$$

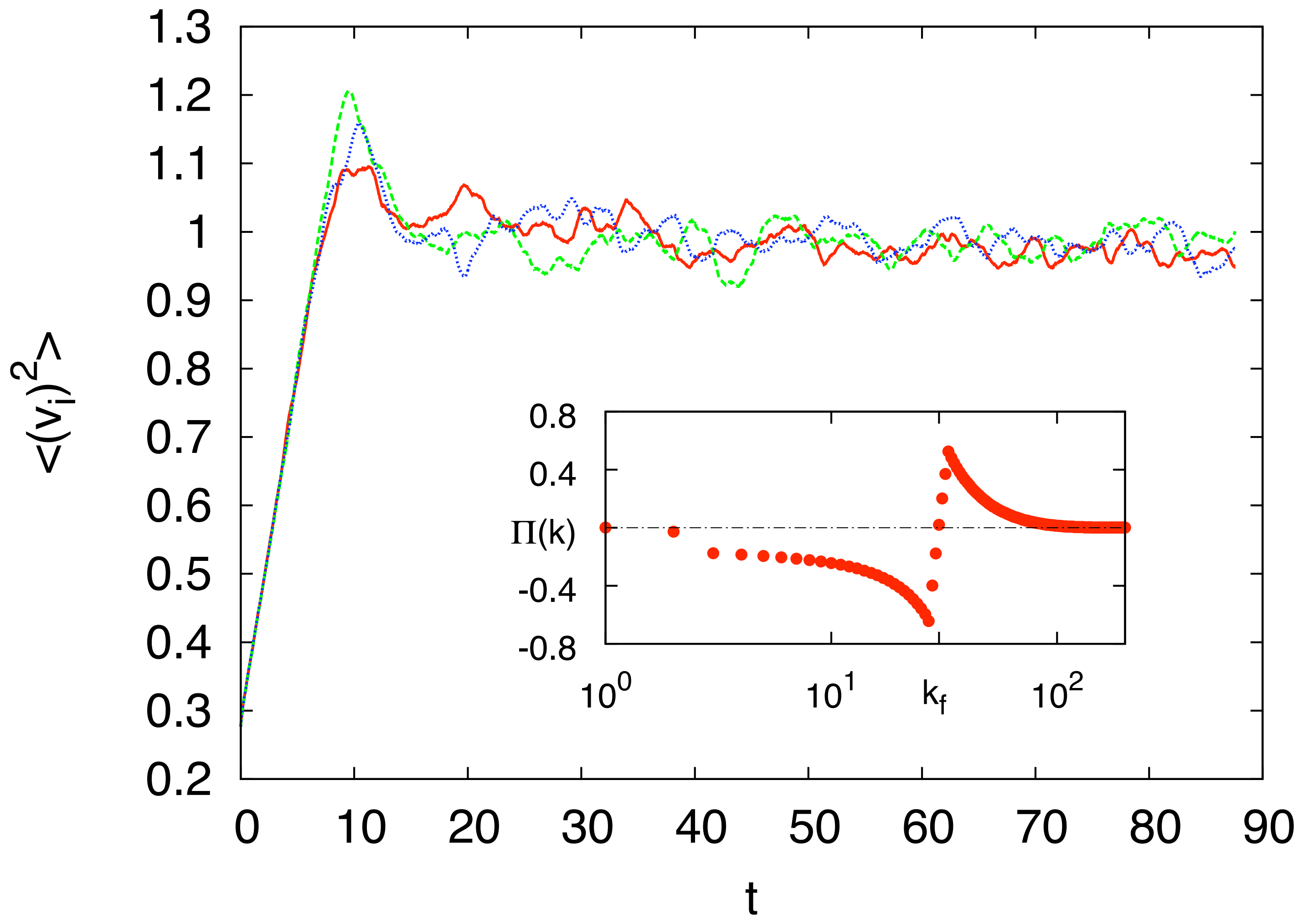
$$\boxed{s_p = s_q = s_k = +}$$

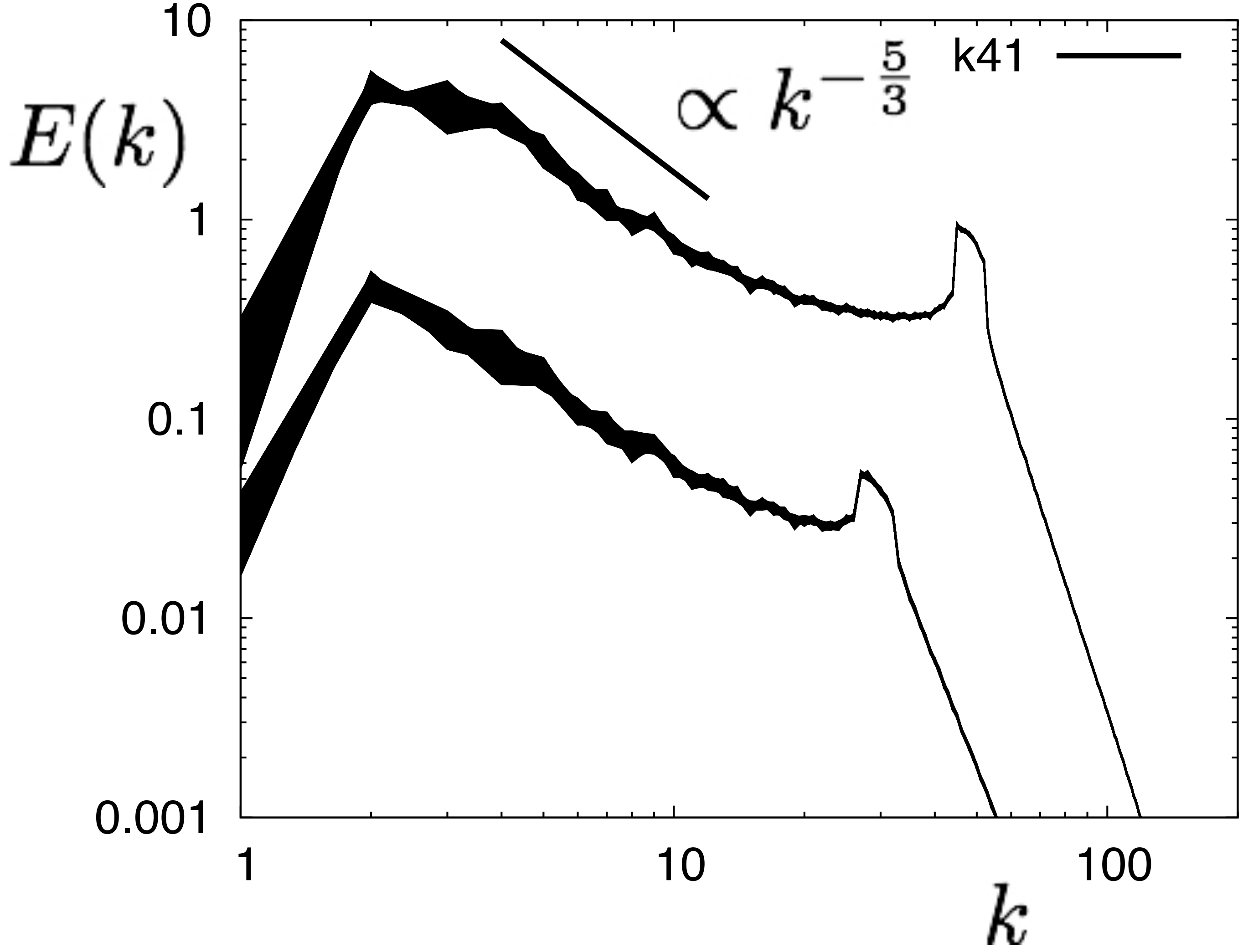
$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + \cancel{|u^-(\boldsymbol{k})|^2}; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - \cancel{|u^-(\boldsymbol{k})|^2}). \end{cases}$$



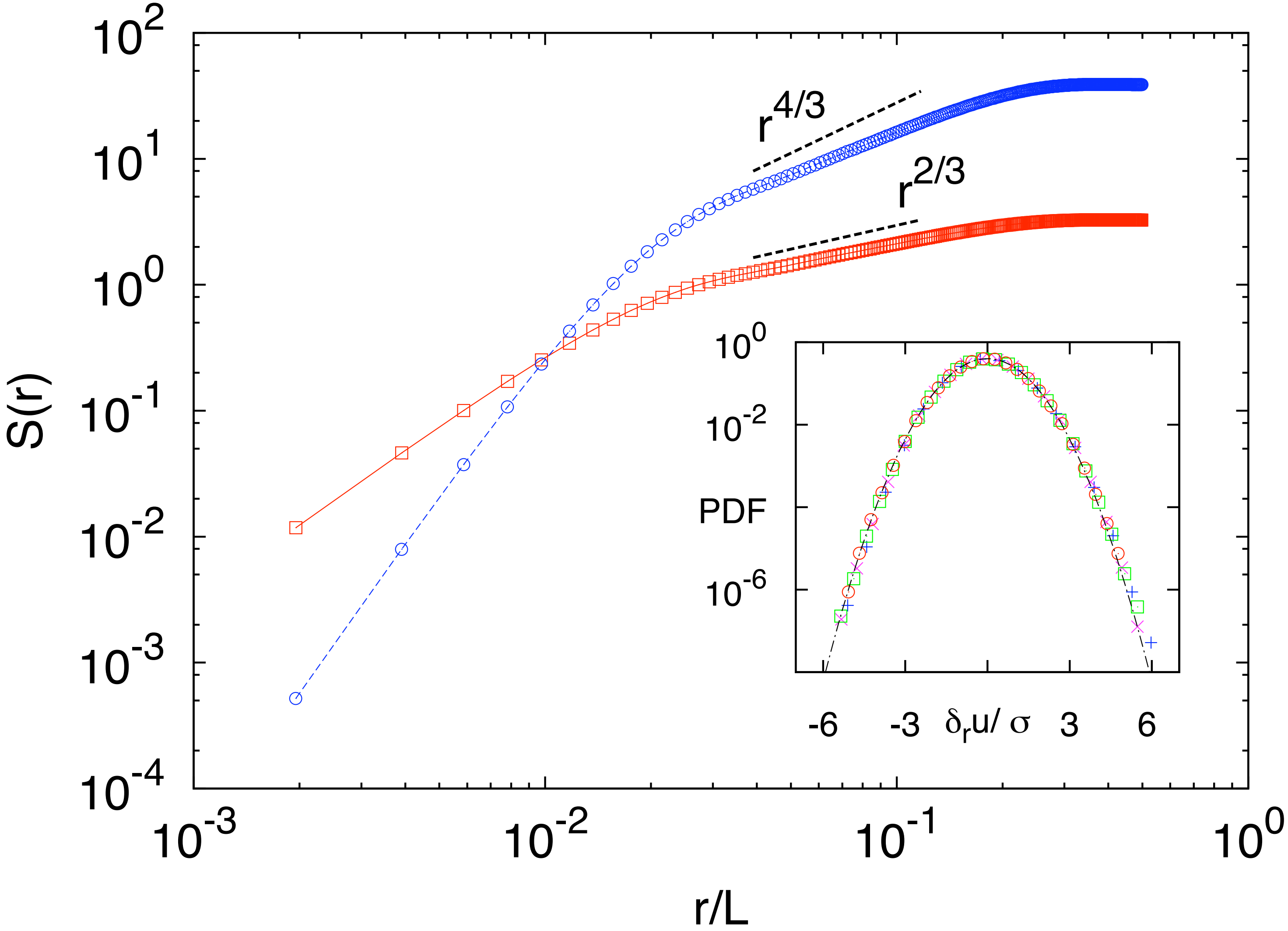

REVERSE ENERGY TRANSFER
$\mathbf{u}^+(\mathbf{k})$
$\mathbf{u}^+(\mathbf{q})$
$\mathbf{u}^+(\mathbf{p})$
VORTICITY
HELICITY
$\mathbf{u}^+(\mathbf{k})$
$\mathbf{u}^-(\mathbf{q})$
$\mathbf{u}^+(\mathbf{p})$
FORWARD ENERGY TRANSFER

PERFECTLY 3D AND PERFECTLY ISOTROPIC





$E(k)$
$\propto k^{-\frac{5}{3}}$
k41
10
1
0.1
0.01
0.001
1
10
100
k



S(r)
r/L
r^{4/3}
r^{2/3}
PDF
δ_r u/ σ
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-3
10^-2
10^-1
10^0
10^0
10^-2
10^-6
-6
-3
3
6

$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + \cancel{|u^-(\boldsymbol{k})|^2}; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - \cancel{|u^-(\boldsymbol{k})|^2}). \end{cases}$$

BELTRAMIZATION: $u^-(\mathbf{k}) = 0$

BIDIMENZIONALIZATION: $u^-(\mathbf{k}) = u^+(\mathbf{k})$

BOTH CASES YOU PUT A CONSTRAINTS BETWEEN THE TWO INVARIANTS AND ONE CANNOT SUSTAIN ANYMORE A TWO FORWARD-CASCADE REGIME.

-FULLY 3D SYSTEM WITH INVERSE ENERGY CASCADE : ROLE OF HELICITY
-INVERSE ENERGY CASCADE CAM BE MADE STATIONARY BY A HYPOVISCOSITY
-NON INTERMITTENT

QUESTIONS:

WHAT ABOUT THE OTHER SUB-NS SYSTEMS?

$$\partial_t \boldsymbol{v} = \Phi(\boldsymbol{v}, \boldsymbol{v}) - \Phi^+(\boldsymbol{v}^+, \boldsymbol{v}^+) - \Phi^-(\boldsymbol{v}^-, \boldsymbol{v}^-) + \nu \Delta \boldsymbol{v} + \boldsymbol{f}$$

$$\Phi(\boldsymbol{v}, \boldsymbol{v}) = -\boldsymbol{v} \cdot \boldsymbol{\nabla} \boldsymbol{v} - \boldsymbol{\nabla} p$$

WHAT ABOUT A 2D-LIKE CONSTRAINT IN 3D? $u^-(\mathbf{k}) = u^+(\mathbf{k})$

DOES INTERMITTENCY DEPEND ON THE TRIAD FAMILY USED TO TRANSFER ENERGY?