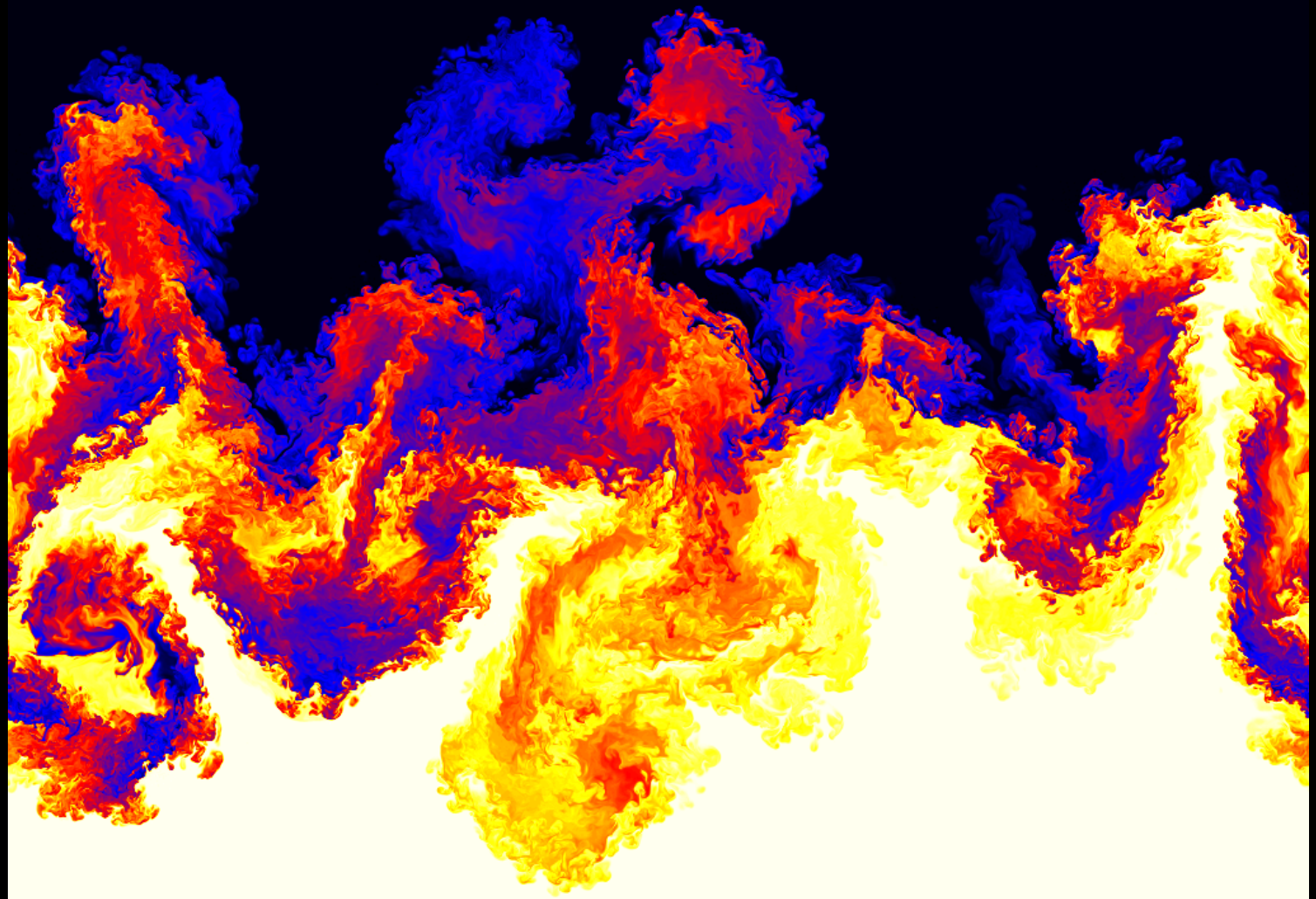
Rayleigh-Taylor turbulence in 3, 2 and 1 dimensions



Guido Boffetta (Torino)
Filippo De Lillo (Torino) Andrea Mazzino (Genova)
Stefano Musacchio (Nice) Lara Vozella (Genova)

Effects of geometrical confinement on turbulent flows

Kraichnan 1967: inversion of the energy flux in 2d

Smith, Chasonv, Waleffe 1996 &

Celani, Musacchio, Vincenzi 2010: coexistence of two cascades in thin layers

transition from 3d to 2d is a smooth function of the aspect ratio

Turbulent convection with geometrical confinement:

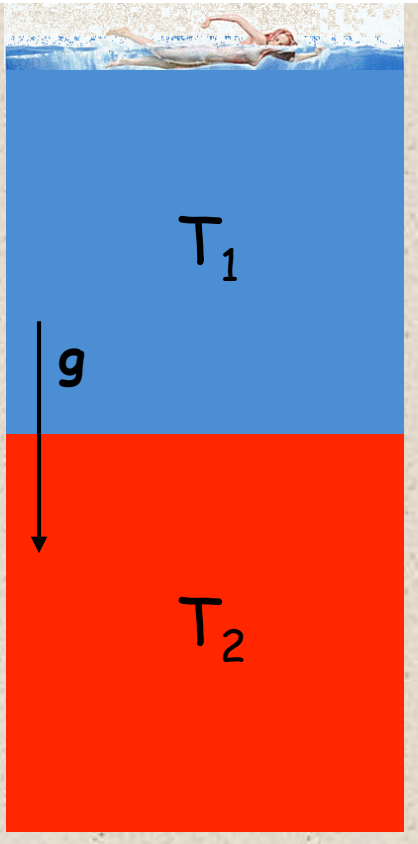
Rayleigh-Taylor turbulence

periodic boundary conditions at a given scale L (homogeneity)

Confinement of one dimension: appearance of the Bolgiano scale

Confinement of two dimensions: new phenomenology in RT mixing

Equation of motion and setup



Single fluid at two temperatures (densities)

Temperature jump: $\theta_0 = T_2 - T_1$

Atwood: $$A \equiv \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} \simeq \frac{1}{2}\beta\theta_0$$ (β: thermal expansion coef.)

For small A the Boussinesq approximation for an incompressible flow holds:

$$\begin{cases} \partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \nu\Delta\mathbf{u} - \beta \mathbf{g} T \\ \partial_t T + \mathbf{u}\cdot\nabla T = \kappa\Delta T \end{cases}$$

Time dependent turbulence with initial condition:

$$\begin{cases} \mathbf{u}(\mathbf{x},0) = 0 \\ T(\mathbf{x},0) = -(1/2)\theta_0 \, sgn(z) \end{cases}$$

Phenomenology of (3D) RT turbulence

Energy balance:
turbulent kinetic energy $E=(1/2)\langle u^2\rangle$ produced from potential energy $P=-\beta g\langle zT\rangle$

$$\varepsilon = \nu\left\langle (\nabla u)^2 \right\rangle$$

$$\frac{dE}{dt} = -\frac{dP}{dt} - \varepsilon = \beta g\left\langle wT \right\rangle - \varepsilon$$

Dimensional balance: $\frac{du^2_{rms}}{dt} \simeq \beta g \theta_0 u_{rms}$ therefore

Large scale velocity fluctuations $u_{rms}(t) \approx Agt$

Turbulent mizing layer of width h(t) $h(t) \approx Agt^2$

Kinetic energy pumped in the system at a rate $\varepsilon_I \simeq \frac{u^3}{h} \simeq (Ag)^2 t$

-> time evolving turbulence

Small scale theory of RT turbulence

M. Chertkov, PRL **91** (2003)

Ansatz: buoyancy negligible at small scales

$$\begin{cases} \partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \nu\Delta\mathbf{u} - \beta \mathbf{g} T \\ \partial_t T + \mathbf{u}\cdot\nabla T = \kappa\Delta T \end{cases}$$

$$\beta g \delta_r T \ll \frac{\delta_r u^2}{r}$$ (small Richardson number)

passive temperature in turbulent flow with time dependent flux

$$\varepsilon(t) \approx (Ag)^2 t$$

small scale fluctuations follow Kolmogorov-Obukhov scaling

$$\delta_r u(t) \simeq u_L(t)\left(\frac{r}{h(t)}\right)^{1/3} \simeq (\beta g \theta_0)^{2/3} t^{1/3} r^{1/3}$$

$$\delta_r T(t) \simeq \theta_0 \left(\frac{r}{h(t)}\right)^{1/3} \simeq \frac{\theta_0}{(\beta g \theta_0)^{1/3}} t^{-2/3} r^{1/3}$$

consistency:

$$Ri = \frac{\beta g \delta_r T(t)}{\delta_r u^2(t)/r} \simeq \left(\frac{r}{h(t)}\right)^{2/3} \to 0$$

Inconsistent in 2D where the energy flows to large scale (buoyancy dominated)

RT turbulence in 2D

M. Chertkov, PRL **91** (2003)

$$\begin{cases} \partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \nu\Delta\mathbf{u} - \beta \mathbf{g} T \\ \partial_t T + \mathbf{u}\cdot\nabla T = \kappa\Delta T \end{cases}$$

Buoyancy balances inertia at all scales

$$\beta g \delta_r T \approx \frac{\delta_r u^2}{r} \qquad \text{(Ri=O(1))}$$

direct cascade of temperature fluctuations

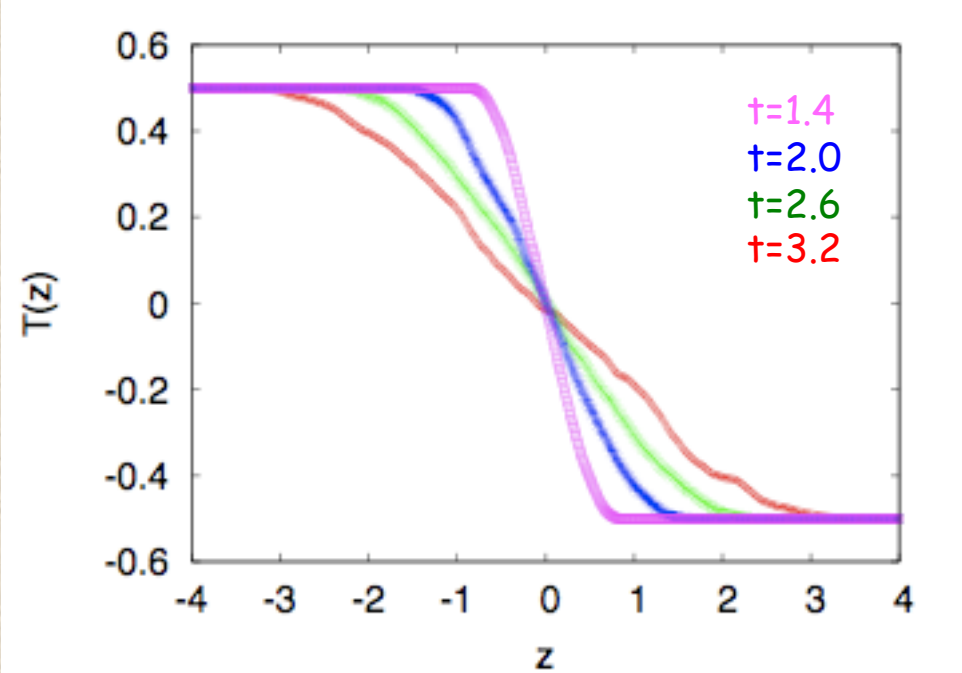
$$\varepsilon_T(t) \simeq \frac{\delta_r u \delta_r T^2}{r} \simeq \frac{\delta_r u^5}{r^3(\beta g)^2} \simeq \frac{u_L^5}{h^3(\beta g)^2}$$

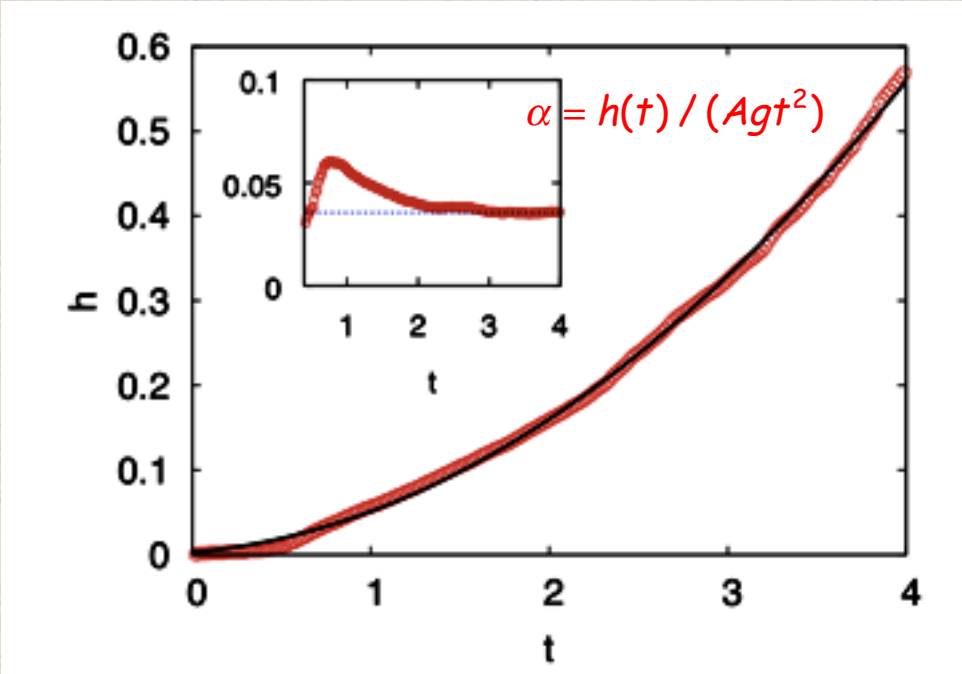
small scale fluctuations follow Bolgiano scaling

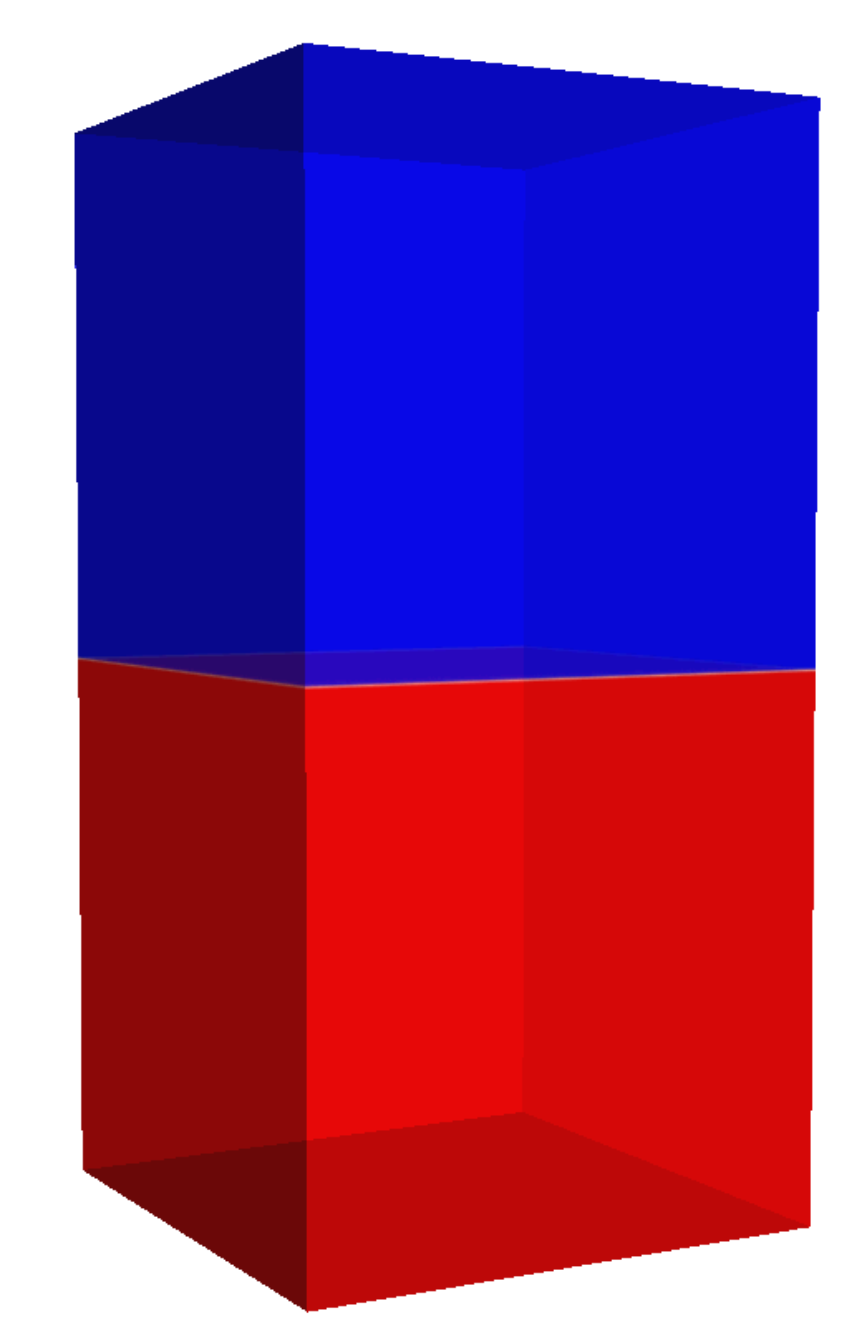
$$\delta_r u(t) \simeq u_L(t)\left(\frac{r}{h(t)}\right)^{3/5} \simeq (\beta g \theta_0)^{2/5} t^{-1/5} r^{3/5}$$

$$\delta_r T(t) \simeq \theta_0\left(\frac{r}{h(t)}\right)^{1/5} \simeq \frac{\theta_0}{(\beta g \theta_0)^{1/5}} t^{-2/5} r^{1/5}$$

3D simulations: evolution of mixing layer



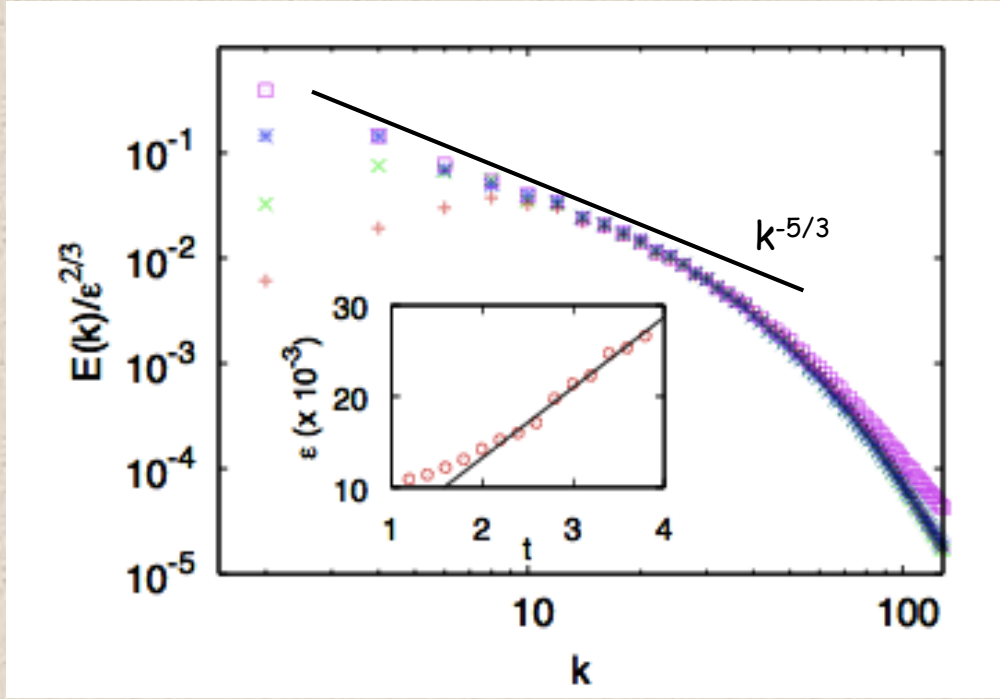


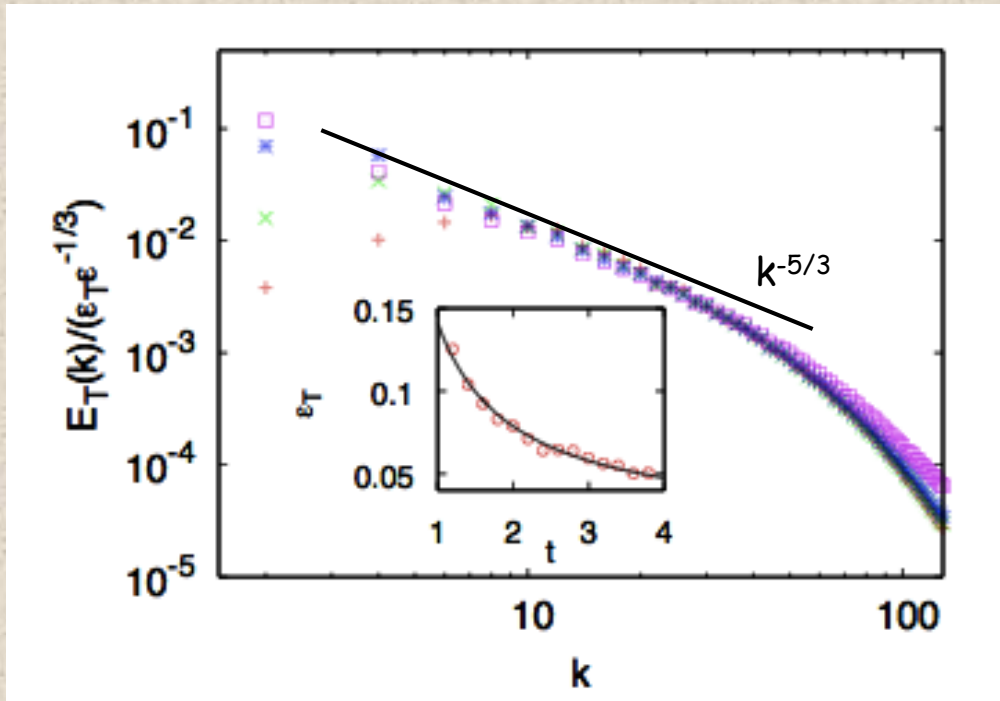


Self-similar evolution of spectra

Collapse of kinetic energy and temperature variance spectra at t/τ=1.0, 1.4, 1.8, 3.8

Insets: time evolution of kinetic energy dissipation $\varepsilon \approx t$ and temperature variance dissipation $\varepsilon_T \approx t^{-1}$





Spatial-temporal scaling in agreement with dimensional theory

$$E(k,t) \sim t^{2/3} k^{-5/3}$$
$$E_T(k,t) \sim t^{-4/3} k^{-5/3}$$

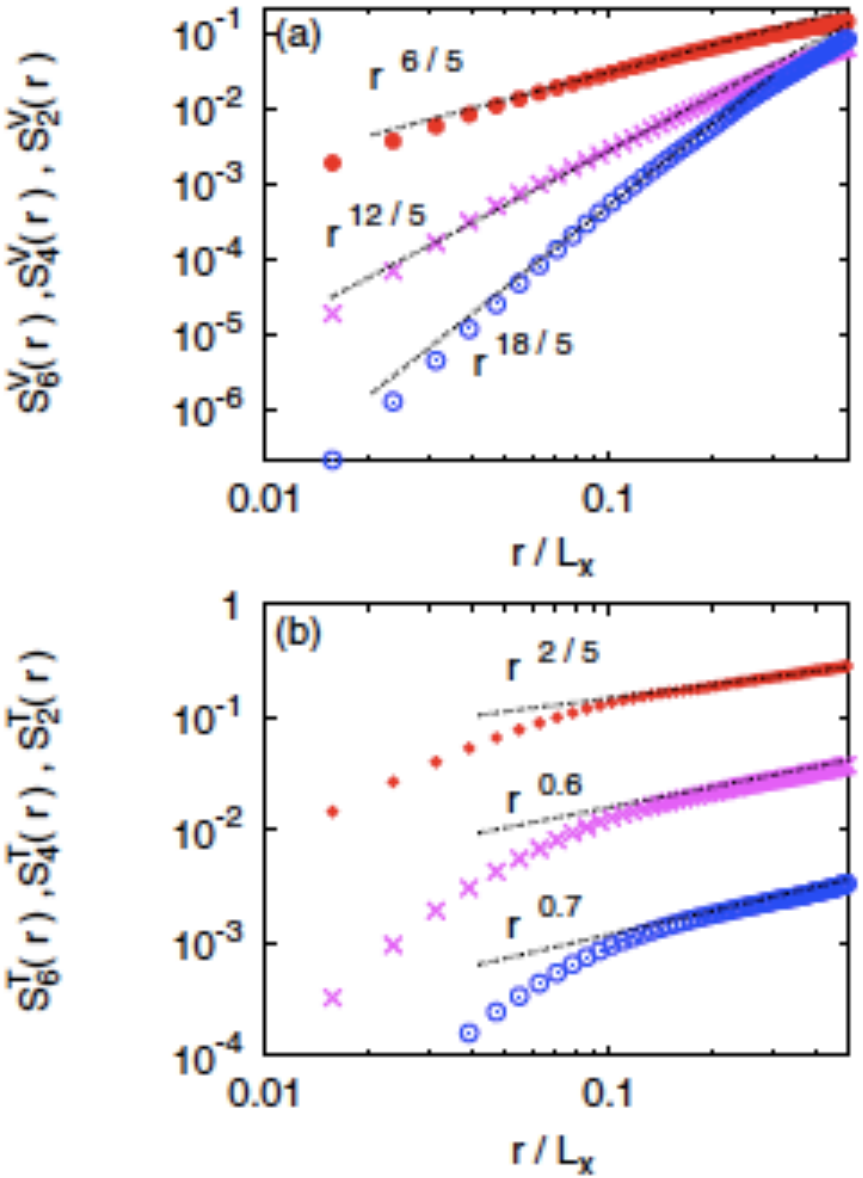
G.Boffetta, A.Mazzino, S.Musacchio, L.Vozella PRE **79**, 065301 (2009)

2D simulation of Rayleigh-Taylor turbulence: Bolgiano scaling

Bolgiano scaling observed in simulations of 2d RT turbulence

$$S_n^V(r) \sim r^{3n/5}$$

$$S_n^T(r) \sim r^{n/5-\chi_n}$$



A.Celani, A.Mazzino, L.Vozella, PRL **96** (2006)

Where is the Bolgiano scale L_B ?

$$L_B = (\beta g)^{-3/2} \varepsilon^{5/4} \varepsilon_T^{-3/4}$$

In 3D (direct cascade) $\beta g \delta_r T \ll \frac{\delta_r u^2}{r}$ $L_B \approx L$ (integral scale)

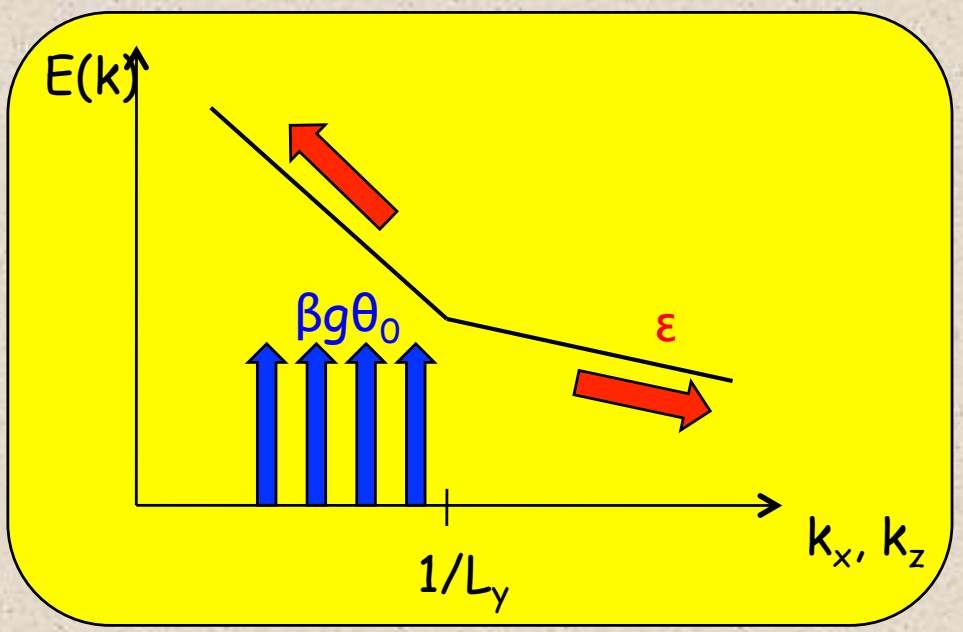
In 2D (inverse cascade) $\beta g \delta_r T \simeq \frac{\delta_r u^2}{r}$ $L_B \approx L_v$ (smallest scale)

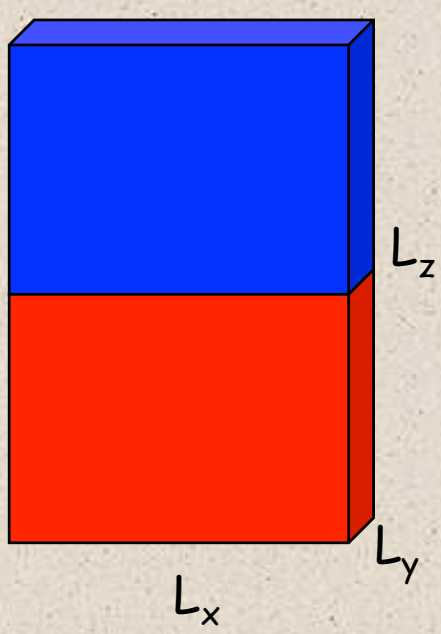
Idea: **L_B is determined by the smallest size of the box**

Setup with large aspect ratio $L_y \ll L_x , L_z$

* scales $r \ll L_y$: 3D Kolmogorov-Obukhov

* scales $r \gg L_y$: 2D Bolgiano-Obukhov





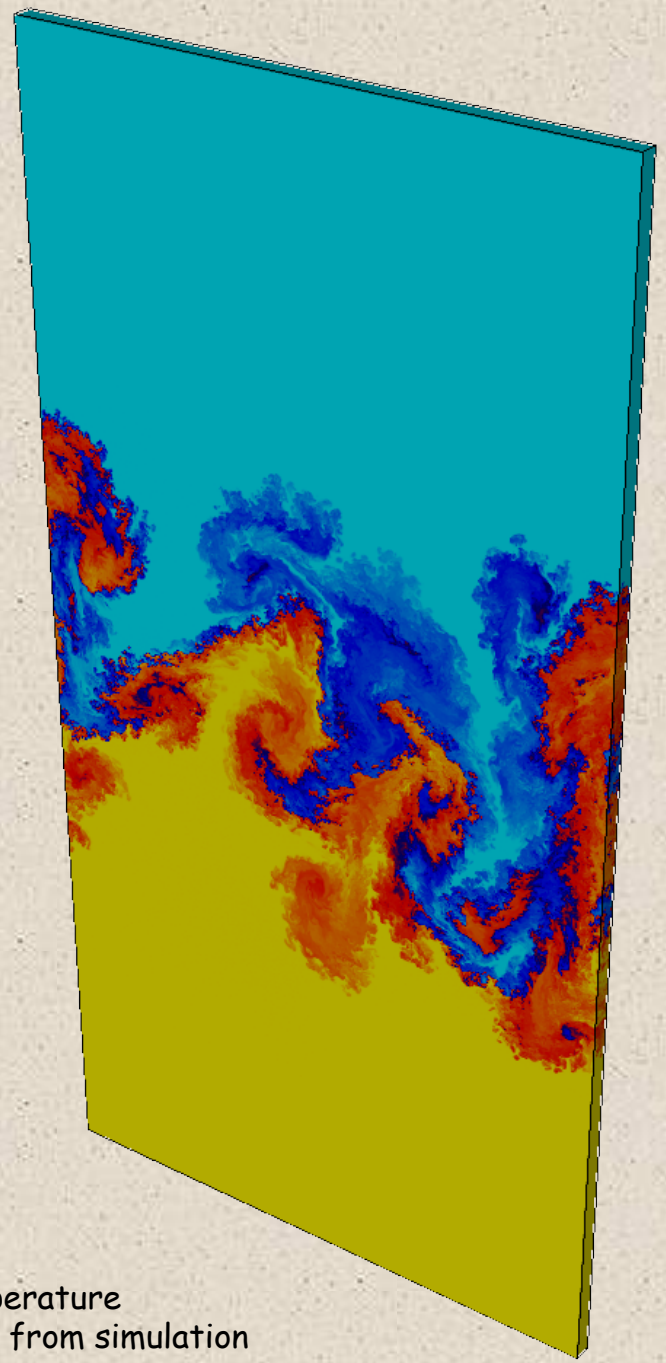
L_y becomes the Bolgiano scale

Quasi-2D Rayleigh-Taylor turbulence: the appearance of the Bolgiano scale

Rayleigh-Taylor turbulence
in a thin layer of fluid

* $h(t) < L_y$: 3D phenomenology
  - Kolmogorov scaling
  - passive temperature

* $h(t) > L_y$: 2D phenomenology
  - Bolgiano scaling
  - active temperature

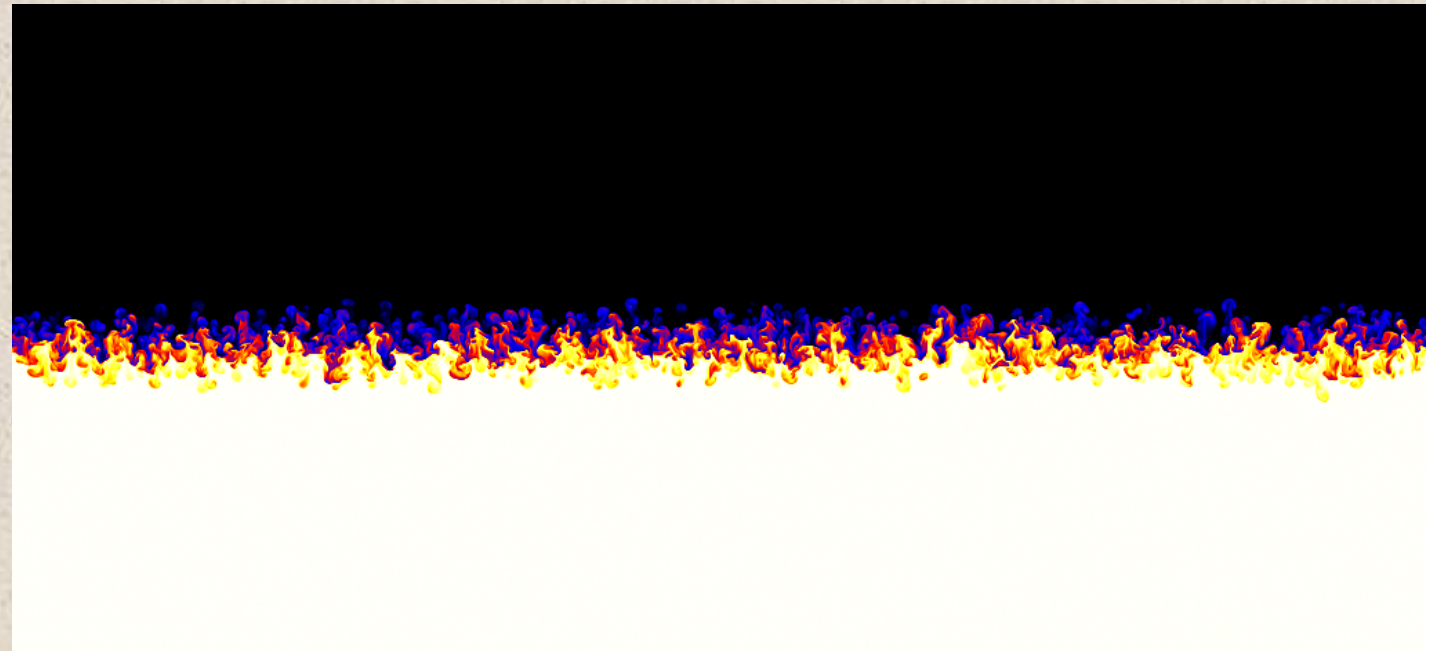


temperature
field from simulation
at 4096x128x8192 - HPC grant

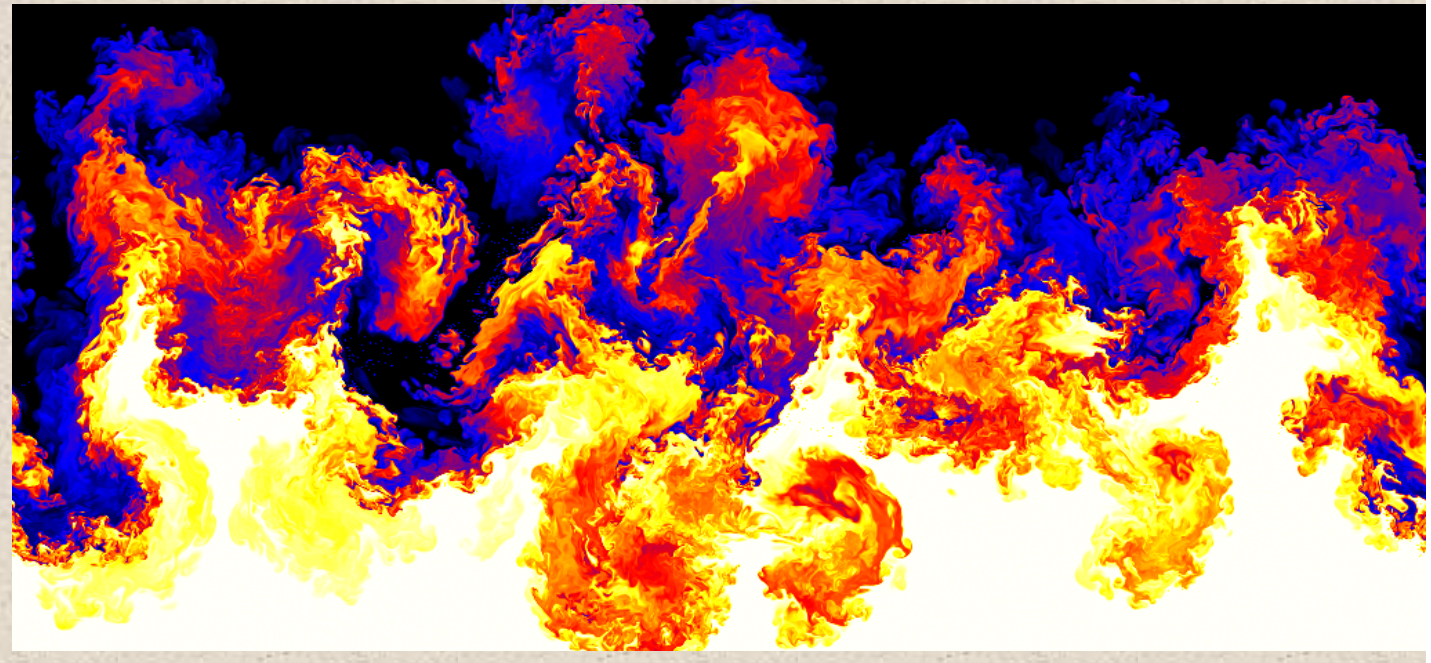
Boffetta, De Lillo, Mazzino, Musacchio, JFM (2011, in press)

A first signature of 3D – 2D transition

$h(t) < L_y$: 3D



$h(t) > L_y$: 2D



L_y

A first signature of 3D – 2D transition: energy balance

$$\frac{dE}{dt} = -\frac{dP}{dt} - \varepsilon$$

$h(t) < L_y$ 3D $\begin{cases} \dfrac{dE}{dt} \simeq t \\ \varepsilon \simeq t \end{cases}$

$h(t) > L_y$ 2D $\begin{cases} \dfrac{dE}{dt} \simeq t \\ \varepsilon \to 0 \end{cases}$

In quasi-2d a residual direct energy flux given by matching the scaling of velocity at $r=L_y$

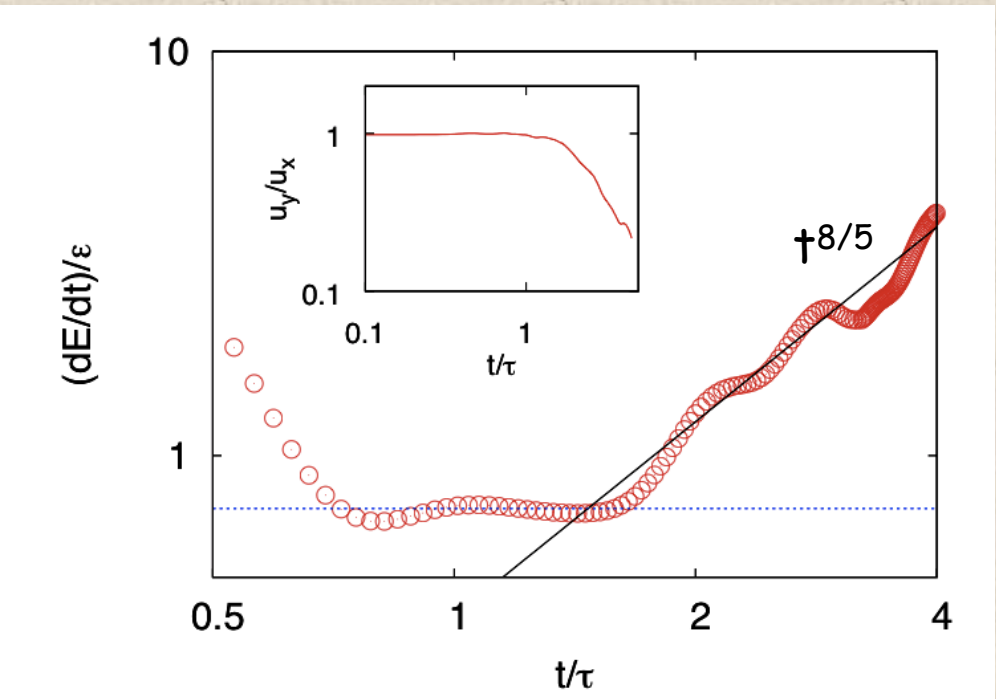
$$\delta_r u(t) \simeq \varepsilon(t)^{1/3} r^{1/3} \qquad (r \ll L_y)$$

$$\delta_r u(t) \simeq (\beta g \theta_0)^{2/5} t^{-1/5} r^{3/5} \qquad (r \gg L_y)$$

$$\varepsilon(t) \simeq (\beta g \theta_0)^{6/5} L_y^{4/5} t^{-3/5}$$

$$\left(\frac{dE}{dt}\right) / \varepsilon \sim t^{8/5}$$

when $h(t) \approx L_y$ we observe a transition from direct to inverse flux



Inversion of the flux at the Bolgiano scale

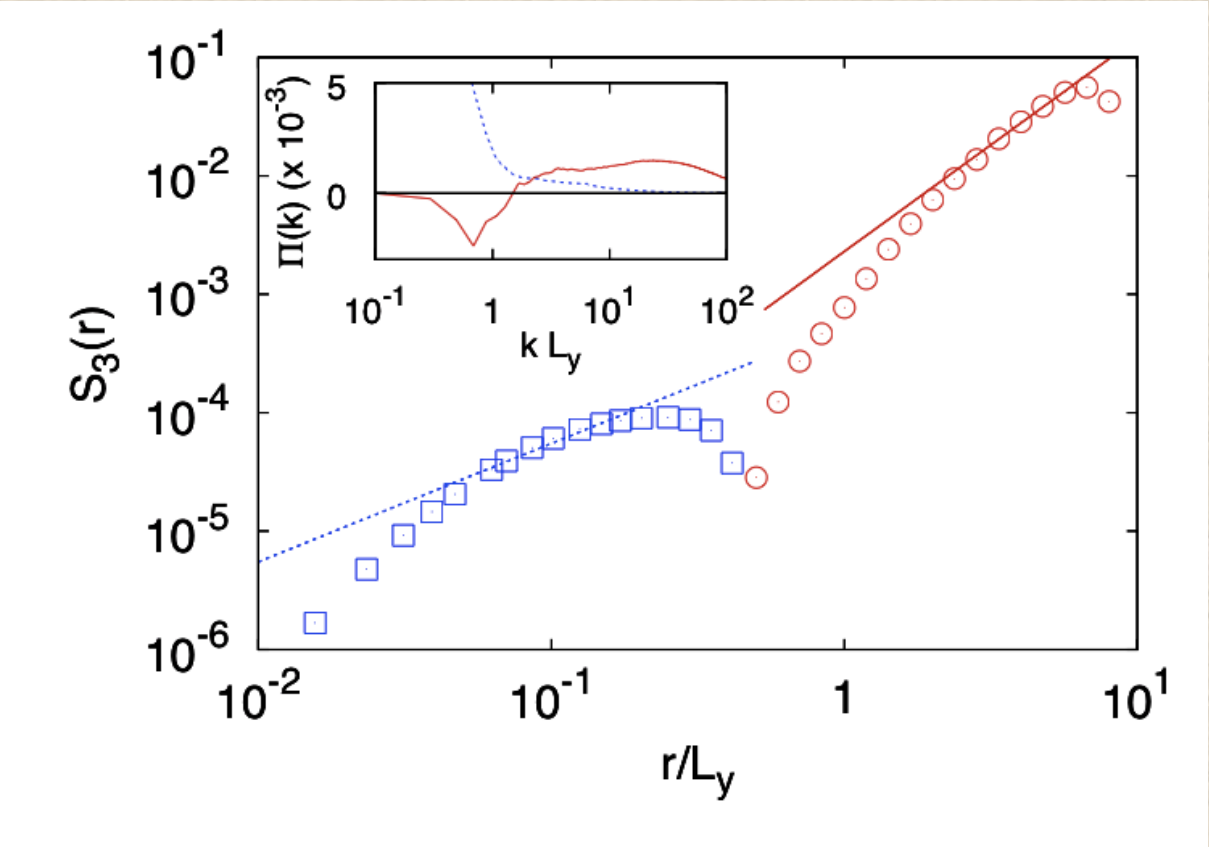
simultaneous presence of a direct and an inverse cascade

$r < L_y$

$$-S_3(r) = -\left\langle \left(\delta_r u\right)^3 \right\rangle = \frac{4}{5}\varepsilon r$$

$r > L_y$

$$S_3(r) \simeq +r^{9/5}$$



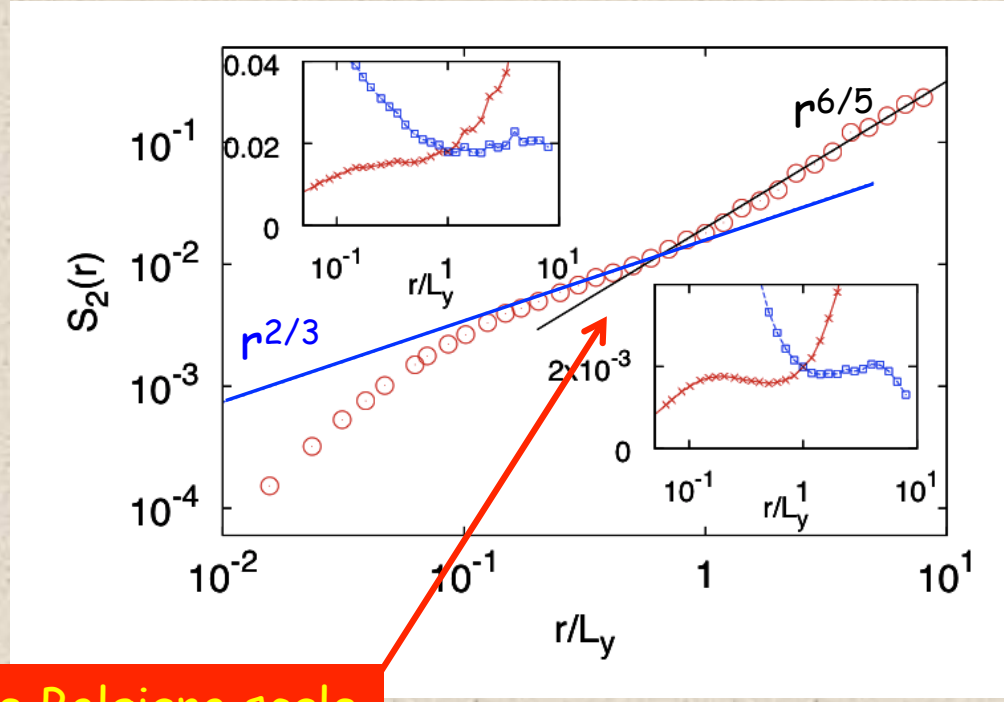
Third-order velocity SF change sign at $r=L_y$

Inset: contributions to energy flux in Fourier space by the nonlinear term and by the buoyancy term

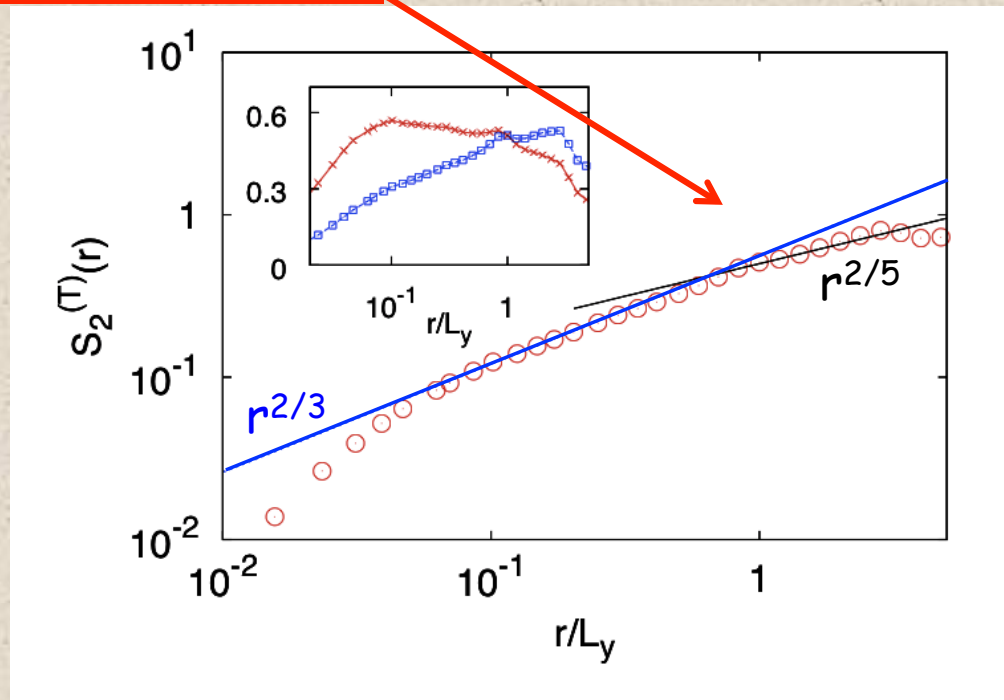
Velocity and temperature structure functions

Kolmogorov-Obukhov scaling
at small scales (passive temperature)

Bolgiano-Obukhov scaling
at large scales (active temperature)



L_y is the Bolgiano scale



First clear numerical evidence of a Bolgiano scale (i.e. two scalings) in the turbulent scales of thermal convection

From geometrical to dynamical scale

How can a geometrical scale determine the dynamical Bolgiano scale ?

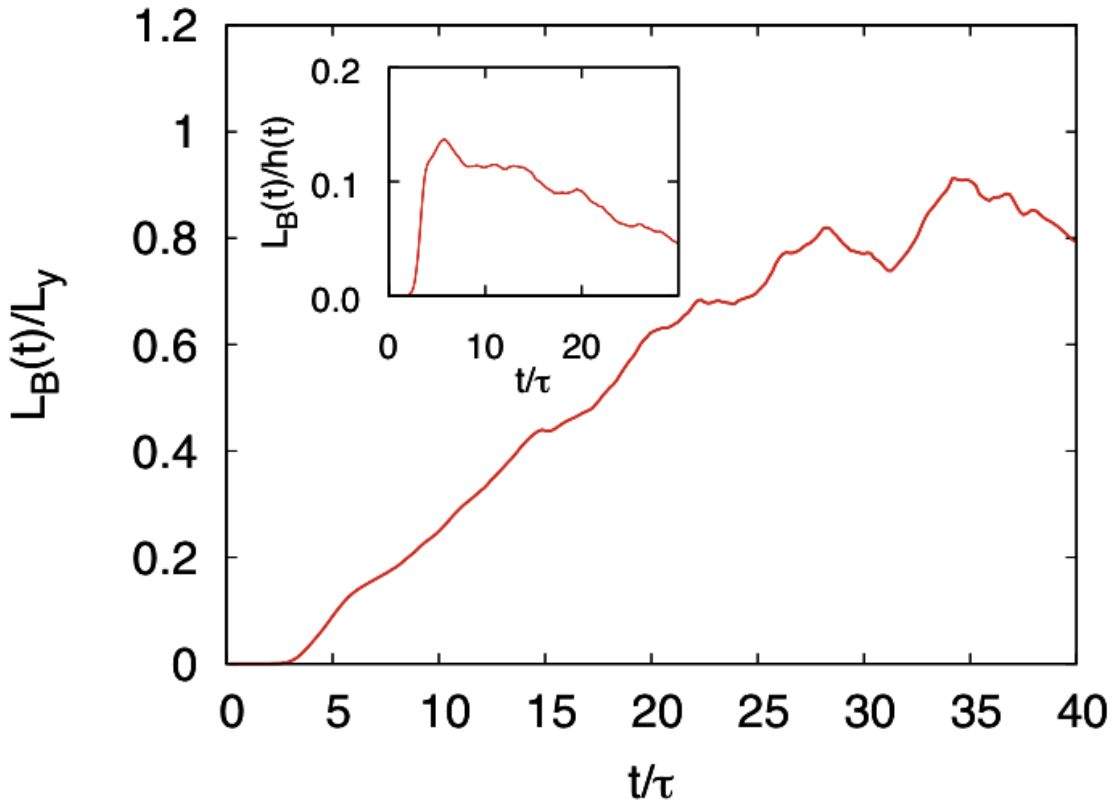
$$L_B = (\beta g)^{-3/2} \varepsilon^{5/4} \varepsilon_T^{-3/4}$$

At short times: $\begin{cases} \varepsilon \simeq (\beta g \theta_0)^2 t \\ \varepsilon_T \simeq \theta_0^2 t^{-1} \end{cases}$ and $L_B \simeq \beta g \theta_0 t^2 \propto h(t)$

At late times:

$$\begin{cases} \varepsilon \simeq (\beta g \theta_0)^{6/5} L_y^{4/5} t^{-3/5} \\ \varepsilon_T \simeq \theta_0^2 t^{-1} \end{cases}$$

and $L_B \simeq L_y$

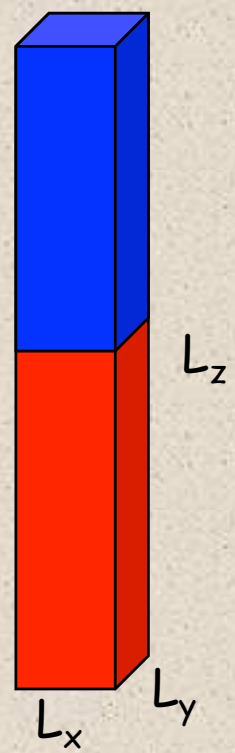


Quasi-1D Rayleigh-Taylor turbulence: anomalous growth of the mixing layer

Two-regimes:

* $h(t) < L_x$: 3D RT turbulence

* $h(t) > L_x$: ?



L_x , L_y << L_z

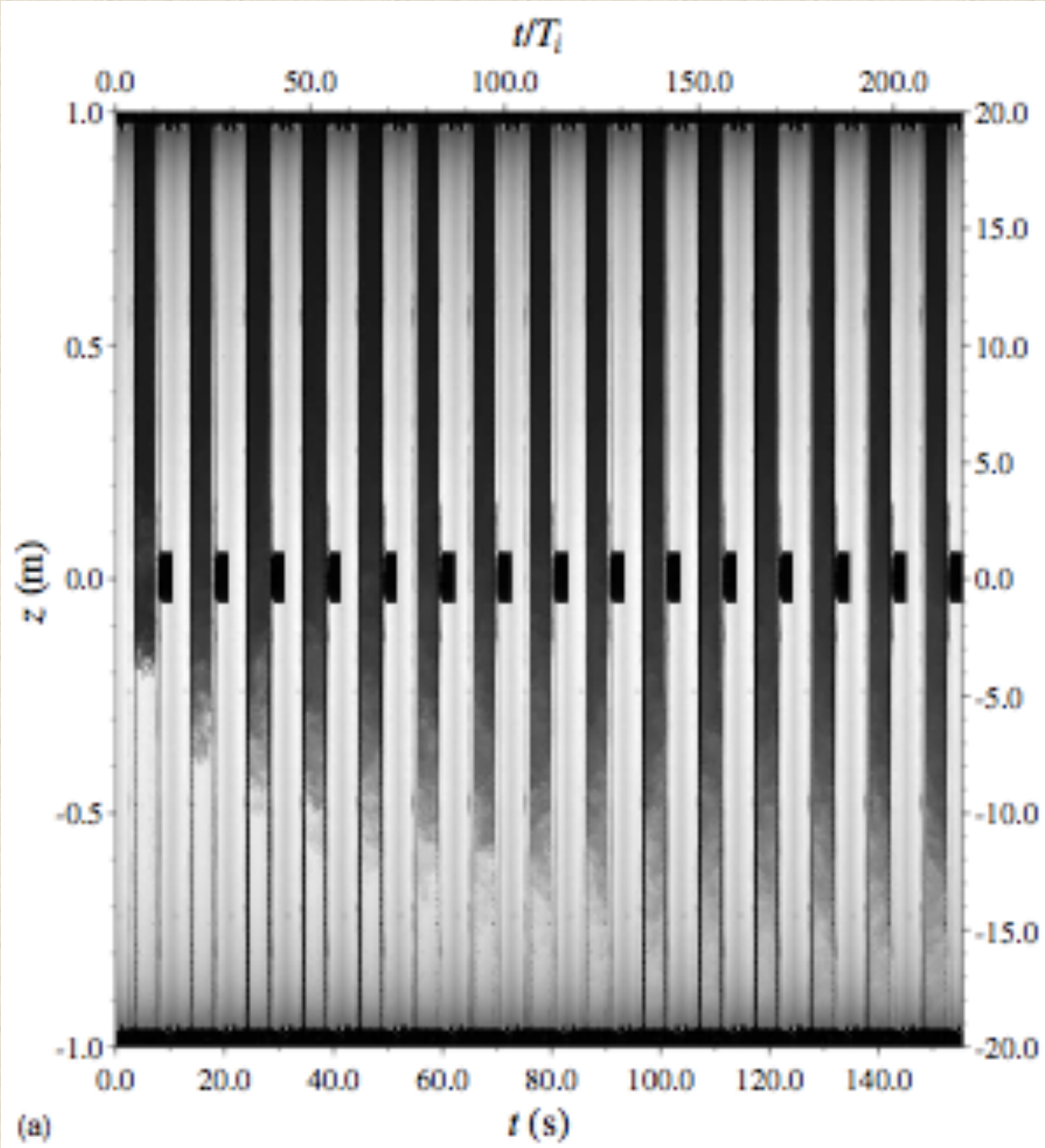
Physical motivation: mixing efficiency in stratified fluids

S.B. Dalziel, M.D. Patterson, C.P. Caulfield, I.A. Coomaraswamy, POF **20** (2008)

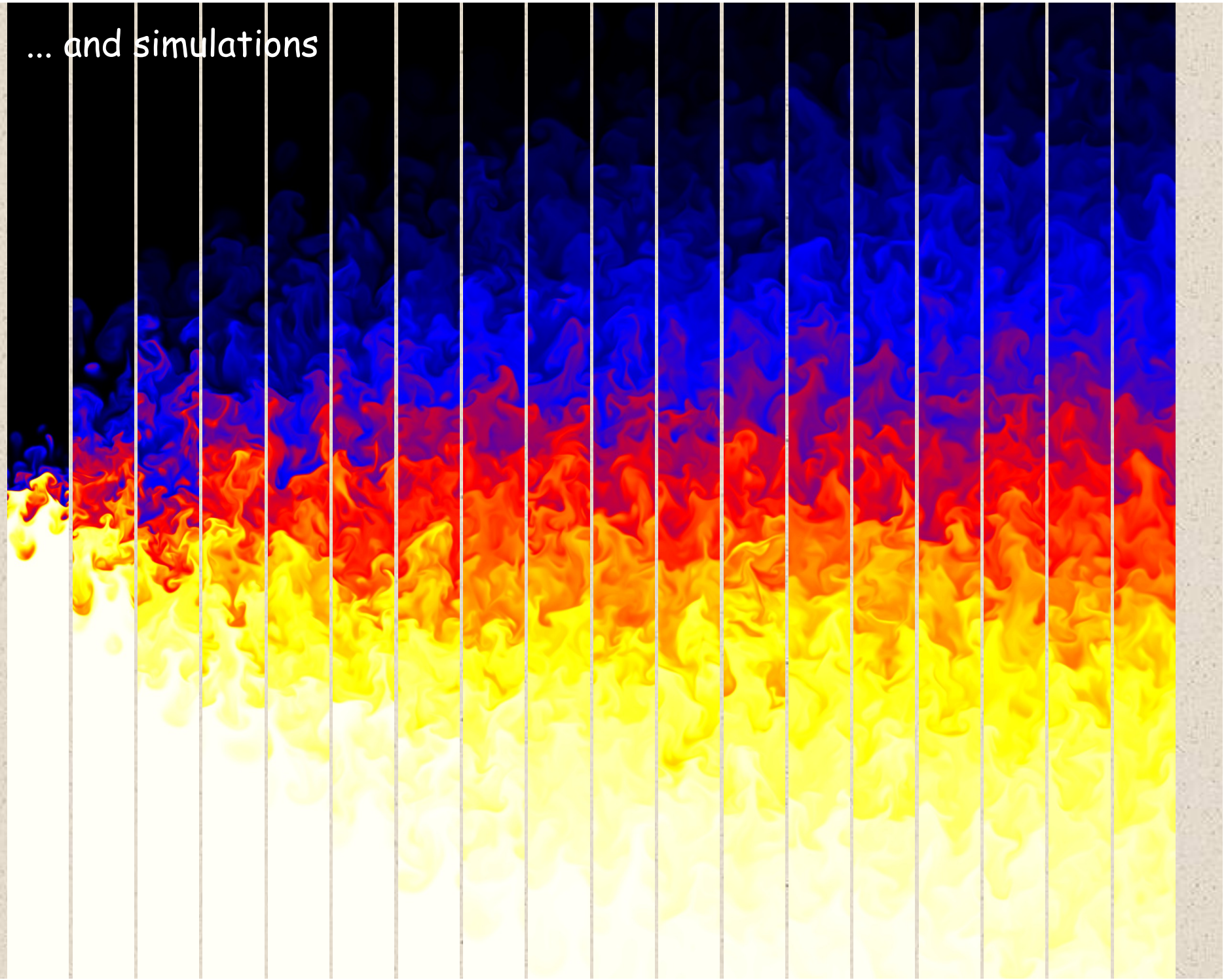
Evolution of the mixing layer: experiment

Salt water +
fresh water

A=0.01



... and simulations



Evolution of the width of mixing layer

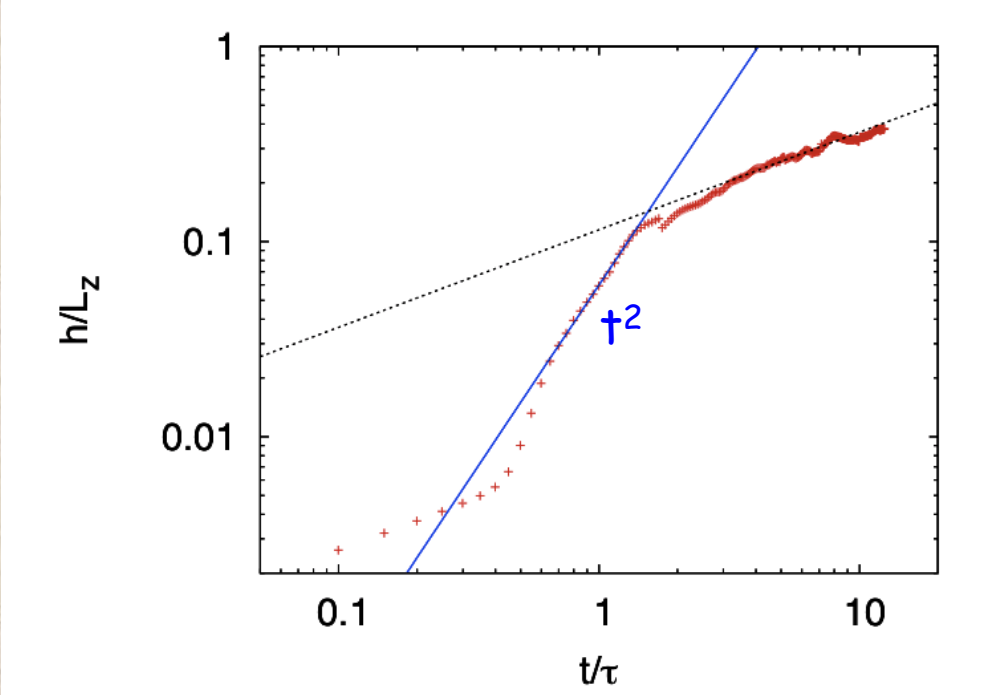
(L_z/L_x = 32)

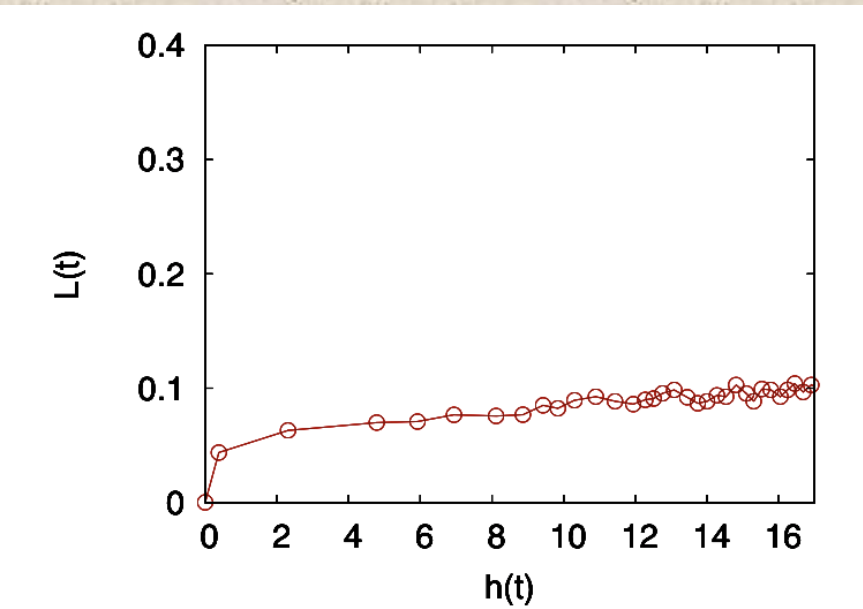
* short times

$h(t) \approx t^2$

* long times

$h(t) \approx ?$





Transition occurs when velocity correlation scale L_u saturates

velocity correlation scale
vs
mixing layer width

Late times: modeling one-dimensional mixing

Velocity fluctuations on scales $r > L_x$ are uncorrelated

Eddy diffusivity model for the mixing layer growth

$$\frac{dh^2}{dt} = K(t)$$

Modeling eddy diffusivity: $K(t) = u_{rms} L_u$

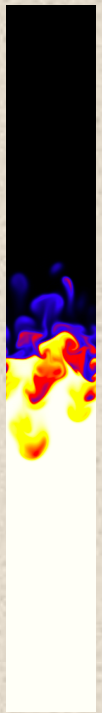
where u_{rms} is obtained dimensionally from the balance

$$\frac{u_{rms}^2}{L_u} \simeq \beta g \theta_L$$

and θ_L is the temperature jump at scale L_u

Eddy diffusivity in the two regimes

$$K(z,t) = u_{rms} L_u \quad \& \quad u_{rms} \simeq \left(\beta g \theta_L L_u\right)^{1/2}$$

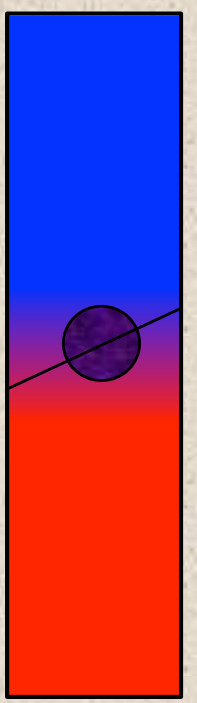


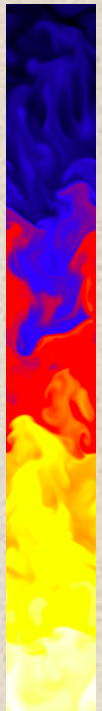
* $h(t) < L_x$

$L_u \simeq h(t)$

$\theta_L = \theta_0$

$$K \simeq (\beta g \theta_0)^{1/2} h^{3/2} \quad \text{and} \quad h(t) \simeq \beta g \theta_0 t^2$$





* $h(t) > L_x$

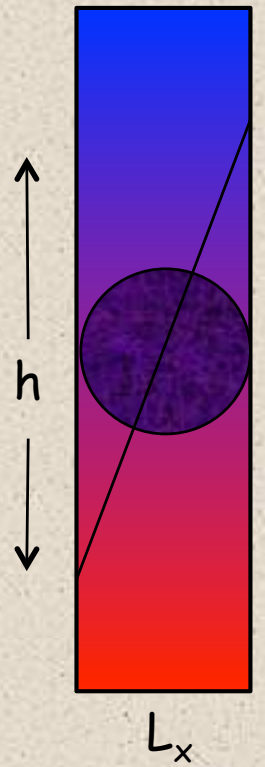
$$u_{rms} \simeq \left(\beta g \theta_0 \frac{L_x^2}{h}\right)^{1/2}$$

$L_u \simeq L_x$

$\theta_L = \theta_0 \dfrac{L_x}{h}$

$$K \simeq (\beta g \theta_0)^{1/2} \frac{L_x^2}{h^{1/2}} \quad \text{and} \quad h(t) \simeq \left(\beta g \theta_0\right)^{1/5} L_x^{4/5} t^{2/5}$$

subdiffusive growth of the mixing layer



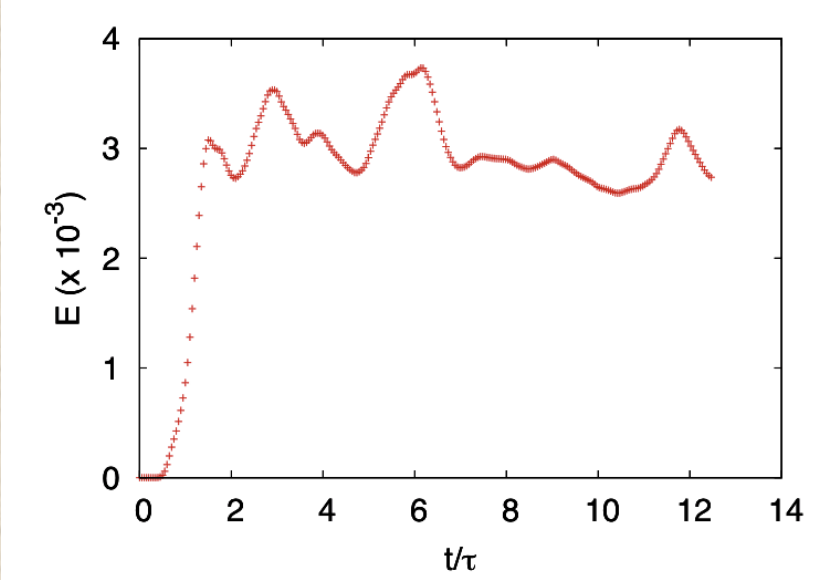
A consequence: saturation of kinetic energy

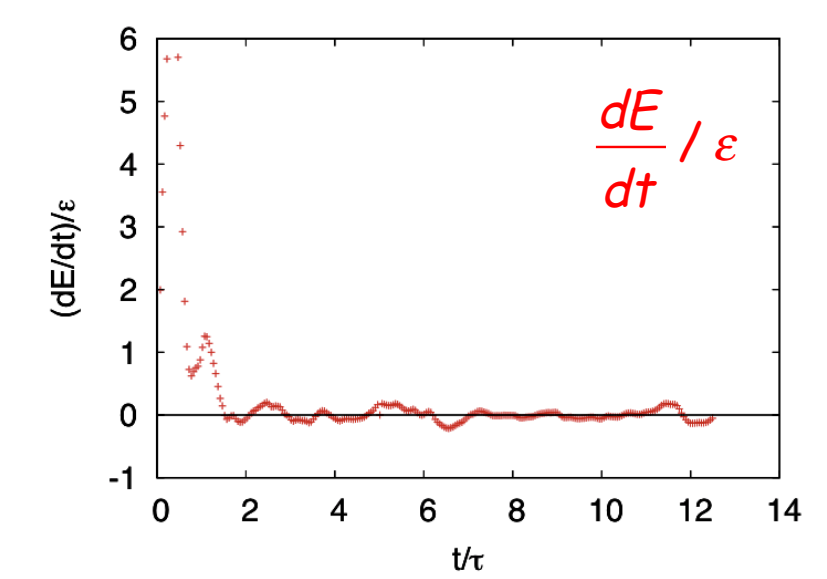
$$u_{rms}^2 \simeq \beta g \theta_0 \frac{L_x^2}{h}$$

Total kinetic energy $E = \frac{1}{2}\int d^3x\,|u|^2 \simeq \frac{3}{2} L_x^2 h(t) u_{rms}^2(t)$

becomes constant for $h(t) > L_x$:

$$E \simeq \frac{3}{2}\beta g \theta_0 L_x^4$$





Energy balance:
all potential energy is dissipated by viscosity

$$\frac{dE}{dt} = -\frac{dP}{dt} - \varepsilon$$

From global to local model

Eddy diffusivity model for mean temperature profile

$$\partial_t \overline{T}(z,t) = \partial_z \left(K(z,t) \partial_z \overline{T}(z,t) \right)$$

$$K(z,t) = u_{rms} L_u$$

In general $\partial_z \overline{T}(z,t)$ is not constant in the mixing layer

$$u_{rms} \simeq \left(\beta g \theta_L L_u \right)^{1/2}$$

a local estimation for θ_L is $\theta_L \simeq L_x \partial_z \overline{T}(z,t)$

nonlinear diffusion model

$$\partial_t \overline{T} \simeq (\beta g)^{1/2} L_x^2 \partial_z \left(\partial_z \overline{T} \right)^{3/2}$$

Self-similar solution in the form $\overline{T}(z,t) = f(z / t^{2/5})$

$$\overline{T}(z,t) = -\frac{15}{16} \vartheta_0 \left[\frac{1}{5} \left(\frac{z}{z_1} \right)^5 - \frac{2}{3} \left(\frac{z}{z_1} \right)^3 + \frac{z}{z_1} \right]$$

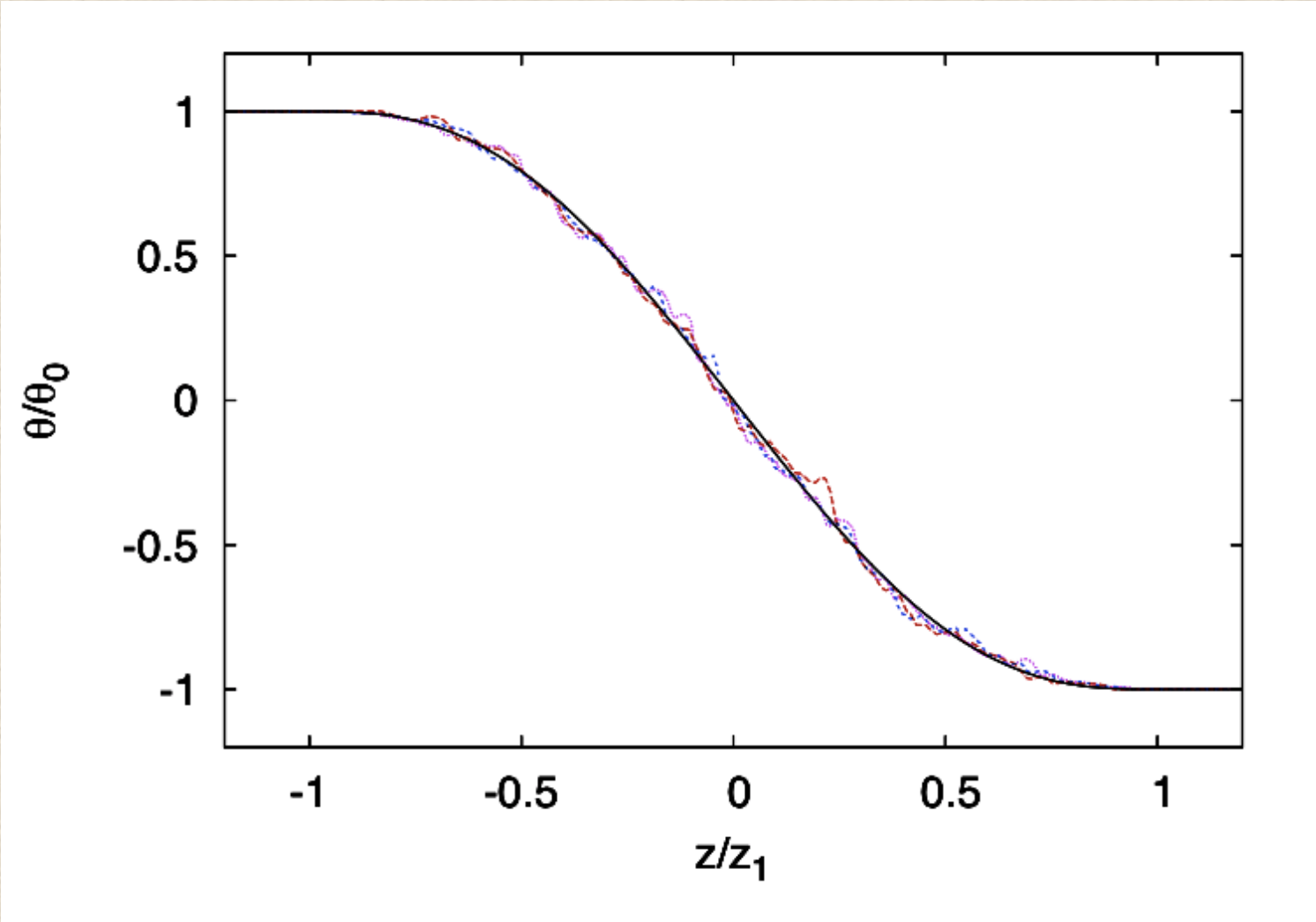
for $|z| \leq z_1$

Pattle, Q.J.Mech.Appl.Math. (1959)

$$z_1(t) = L_x^{4/5} (\beta g \vartheta_0)^{1/5} t^{2/5}$$

half width of the mixing layer

Self-similar evolution of the mixing layer

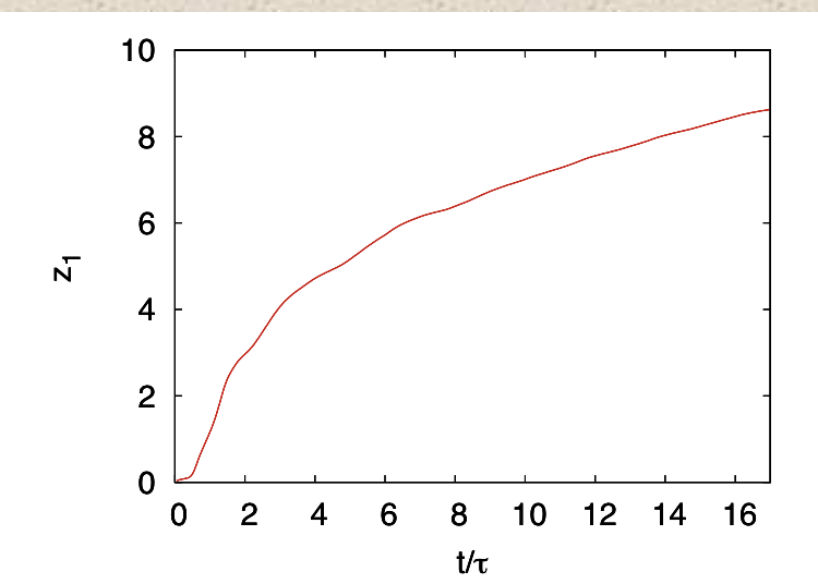


$\overline{T}(z / z_1, t)$

for different t

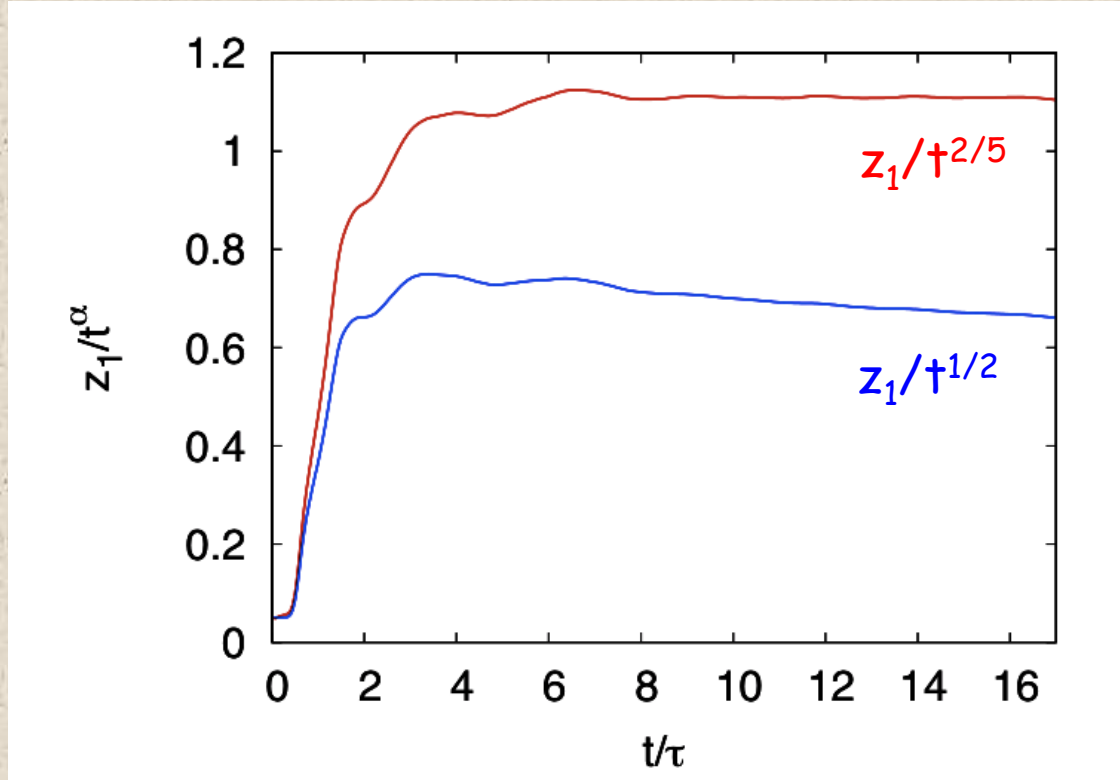
Fit with $\overline{T}(z,t) = -\frac{15}{16}\vartheta_0\left[\frac{1}{5}\left(\frac{z}{z_1}\right)^5 - \frac{2}{3}\left(\frac{z}{z_1}\right)^3 + \frac{z}{z_1}\right]$ gives $z_1(t)$

time evolution of $z_1(t)$



the nonlinear model allows for a precise determination of the temporal exponent

$$h(t) = 2z_1(t) \simeq \left(\beta g \vartheta_0\right)^{1/5} L_x^{4/5} t^{2/5}$$



Conclusions

Effects of geometrical confinement on Rayleigh-Taylor turbulence

* quasi-two dimensions

  Kolmogorov + Bolgiano scaling

  transverse scale of the box becomes the Bolgiano scale

* quasi-one dimension

  subdiffusive evolution of mixing layer

  eddy diffusivity model

G.Boffetta, A.Mazzino, S.Musacchio, L.Vozella PRE **79**, 065301 (2009)
G.Boffetta, A.Mazzino, S.Musacchio, L.Vozella JFM **643**, 127 (2010)
G.Boffetta, F.De Lillo, S.Musacchio PRL **104**, 034505 (2010)
G.Boffetta, A.Mazzino, S.Musacchio, L.Vozella *Phys. Fluids* **22** 035109 (2010)
G.Boffetta, A.Mazzino, S.Musacchio, L.Vozella PRL **104**, 184501 (2010)
G.Boffetta, F.De Lillo, A.Mazzino, S.Musacchio JFM (in press, 2011)

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