A possible explanation of the atmospheric kinetic and potential energy spectra

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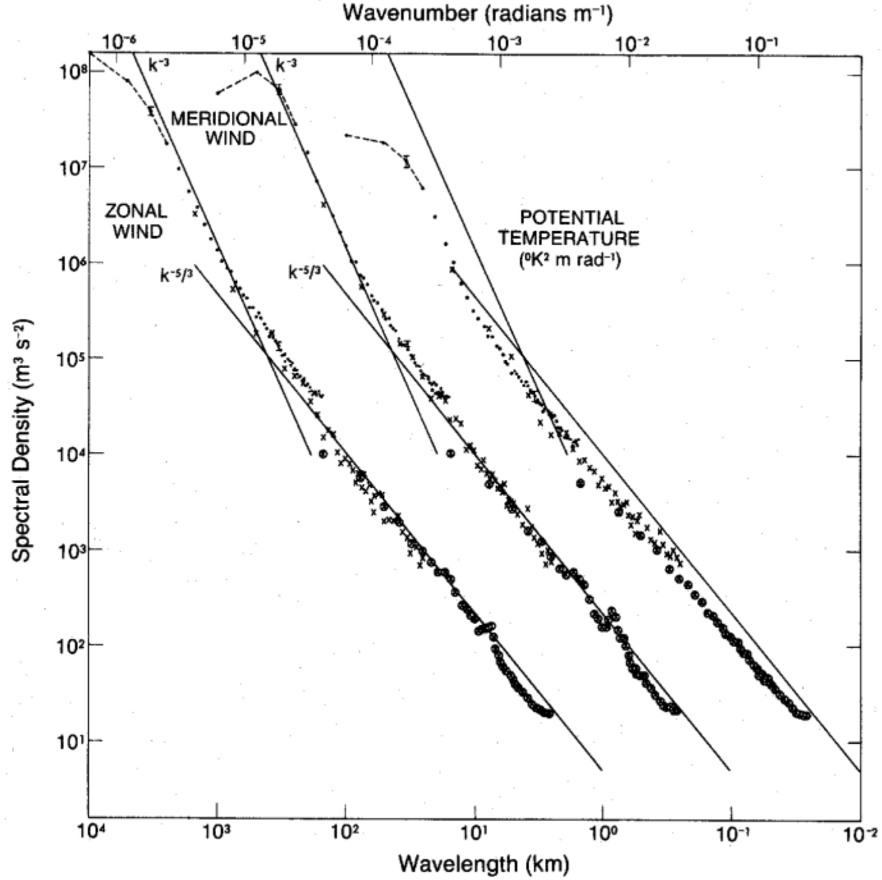
The Nastrom-Gage Spectra

Aircraft measurements of wind and potential temperature in the atmosphere display a transition between synoptic and mesoscale (~500 km)



(Nastrom *et al.*, 1984)

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Atmosphere and QG Turbulence

High Reynolds number $\Rightarrow$ Turbulence

Navier-Stokes Equations $\xrightarrow{\text{Strong rotation and stratification}}$ Quasi-Geostrophic Equations

$$\frac{\partial q}{\partial t} + u_j \frac{\partial q}{\partial x_j} = 0 \qquad \text{with } q = -\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{f}{N}\frac{\partial b}{\partial z}$$

Two quadratic invariants

$$Z = \frac{1}{2}\int_\Omega (q \cdot q)\, \mathrm{d}\Omega \qquad E = \frac{1}{2}\int_\Omega (\mathbf{u}_h \cdot \mathbf{u}_h + b \cdot b)\, \mathrm{d}\Omega$$

The long-standing problem

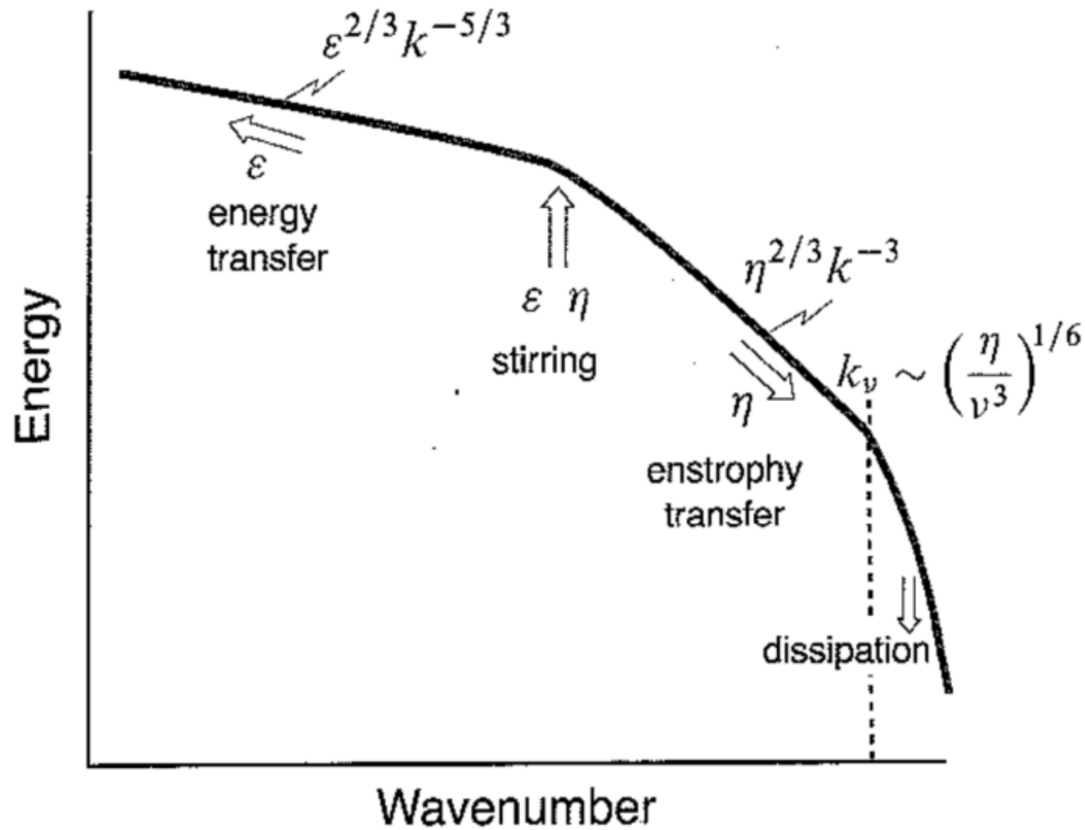
Possible explanations

Numerical code

Results

QG Turbulence

$$Z = \frac{1}{2}\int_\Omega (q \cdot q)\, \mathrm{d}\Omega \qquad E = \frac{1}{2}\int_\Omega (\mathbf{u}_h \cdot \mathbf{u}_h + b \cdot b)\, \mathrm{d}\Omega$$



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But what happens in the atmosphere?

The long-standing problem

Possible explanations

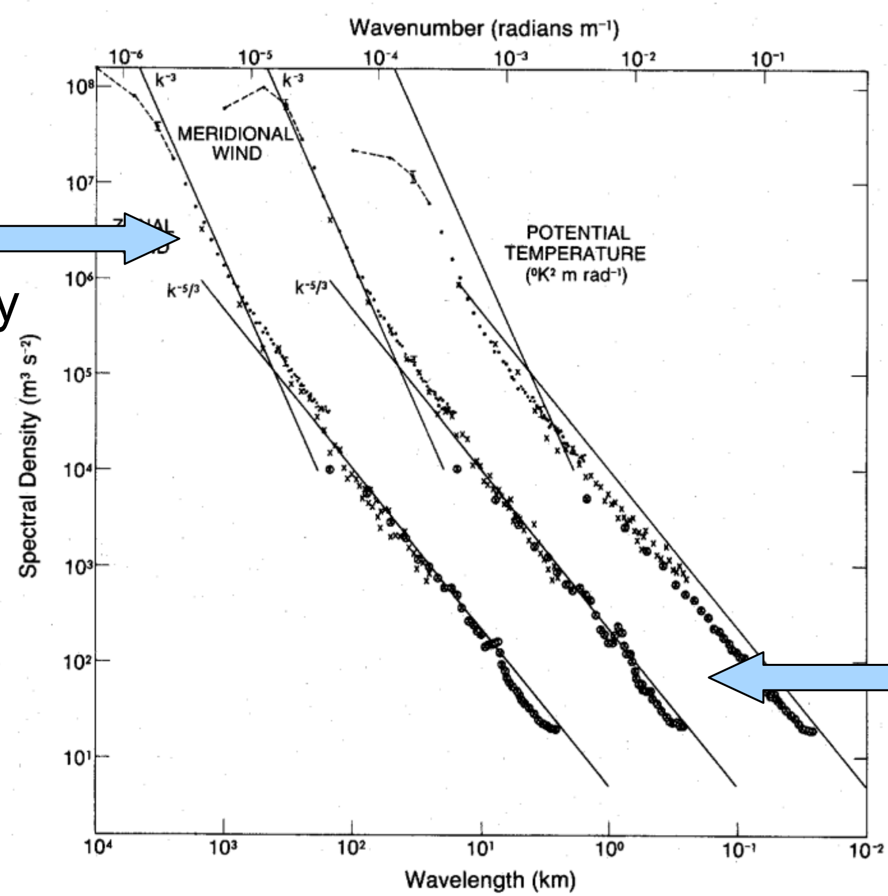
Numerical code

Results

Possible explanation 1: Lilly, 1984

Two Energy/Enstrophy sources

Direct Enstrophy Cascade



Inverse Energy Cascade

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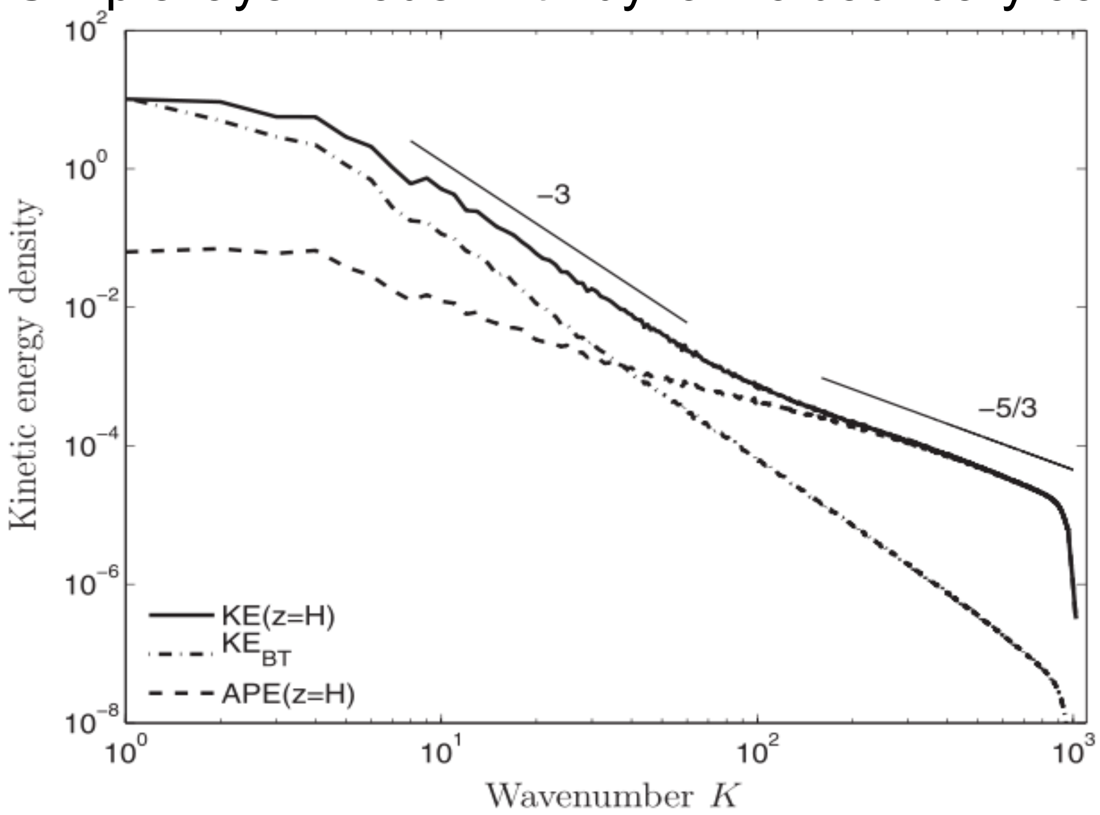
Possible explanation 2: Tulloch & Smith, 2009

Simple layer model with dynamic boundary conditions



Transition has been observed throughout the whole atmosphere and not just close to the upper boundary (Cho et al. 1999, Frehlic & Sharman 2010)

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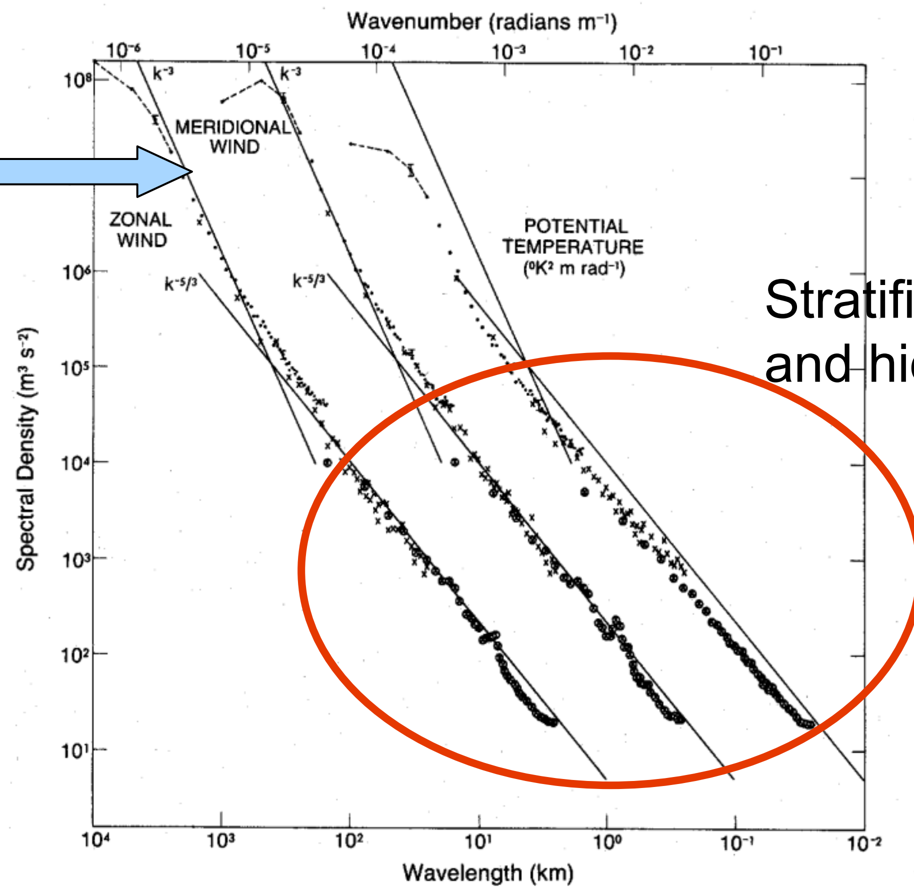
Possible explanation 3: Tung & Orlando's idea and stratified turbulence

One Energy Source



Stratified turbulence and hidden cascade

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OUR EXPLANATION

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The Primitive Equation system

$$
\begin{aligned}
\frac{D\mathbf{u}_h}{Dt} &= -\nabla_h p - f\mathbf{e}_z \times \mathbf{u}_h, \\
\cancel{\frac{Dw}{Dt}} &= -\frac{\partial p}{\partial z} + Nb, \\
\frac{Db}{Dt} &= -Nw, \\
\nabla \cdot \mathbf{u} &= 0
\end{aligned}
$$

Reformulate in terms of

$$
\begin{aligned}
q &= -\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{f}{N}\frac{\partial b}{\partial z}, \\
a_1 &= -\frac{f}{N}\frac{\partial v}{\partial z} + \frac{\partial b}{\partial x}, \\
a_2 &= \frac{f}{N}\frac{\partial u}{\partial z} + \frac{\partial b}{\partial y}.
\end{aligned}
$$

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Geostrophic scaling

$$x \sim L, \quad y \sim L, \quad z \sim f/NL, \quad t \sim L/U,$$
$$u \sim U, \quad v \sim U, \quad w \sim Ro\,U f/N, \quad b \sim U,$$
$$q \sim U/L, \quad a_1 \sim Ro\,U/L, \quad a_2 \sim Ro\,U/L,$$

where $Ro = U/fL$ is the Rossby number.

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Equations

$$\frac{\partial q}{\partial t} = \frac{\partial}{\partial y}\left(\frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + Ro\frac{\partial uw}{\partial z}\right) - \frac{\partial}{\partial x}\left(\frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} + Ro\frac{\partial vw}{\partial z}\right)$$
$$-\frac{\partial}{\partial z}\left(\frac{\partial ub}{\partial x} + \frac{\partial vb}{\partial y} + Ro\frac{\partial wb}{\partial z}\right) + \nu_S\nabla^8 q - \nu_L q,$$

$$Ro\frac{\partial a_1}{\partial t} = a_2 - \frac{\partial w}{\partial x} + \frac{\partial}{\partial z}\left(\frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} + Ro\frac{\partial vw}{\partial z}\right)$$
$$-\frac{\partial}{\partial x}\left(\frac{\partial ub}{\partial x} + \frac{\partial vb}{\partial y} + Ro\frac{\partial wb}{\partial z}\right) + Ro\nu_S\nabla^8 a_1 - Ro\nu_L a_1,$$

$$Ro\frac{\partial a_2}{\partial t} = -a_1 - \frac{\partial w}{\partial y} - \frac{\partial}{\partial z}\left(\frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + Ro\frac{\partial uw}{\partial z}\right)$$
$$-\frac{\partial}{\partial y}\left(\frac{\partial ub}{\partial x} + \frac{\partial vb}{\partial y} + Ro\frac{\partial wb}{\partial z}\right) + Ro\nu_S\nabla^8 a_2 - Ro\nu_L a_2.$$

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In the limit of Ro=0, the Charney equation is retained.

Two quadratic invariants, potential entrophy and energy.

Inverse energy cascade.

But what happens when Ro << 1 and finite?

Forward enstrophy cascade from forcing wave numbers k_f .

Enstrophy flux = Enstrophy injection rate = η

Energy injection rate = P = $k_f^{-2}\eta$

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Hypothesis

A small fraction of the injected energy will go into a downscale energy cascade. The energy flux will scale as

$$\varepsilon \sim P Ro^n$$

The rest will go into an upscale energy cascade.

The horizontal energy spectrum

$$E(k) \sim \eta^{2/3} k_h^{-3} + \varepsilon^{2/3} k_h^{-5/3}$$

Transition wave number

$$k_t \sim \sqrt{\frac{\eta}{\varepsilon}} \sim Ro^{-n/2} k_f$$

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A little about the numerics ...

Periodic boundary conditions

Pseudo-spectral code

Runge-Kutta 4th order for the time integration on NL terms

3/2 dealiasing rule

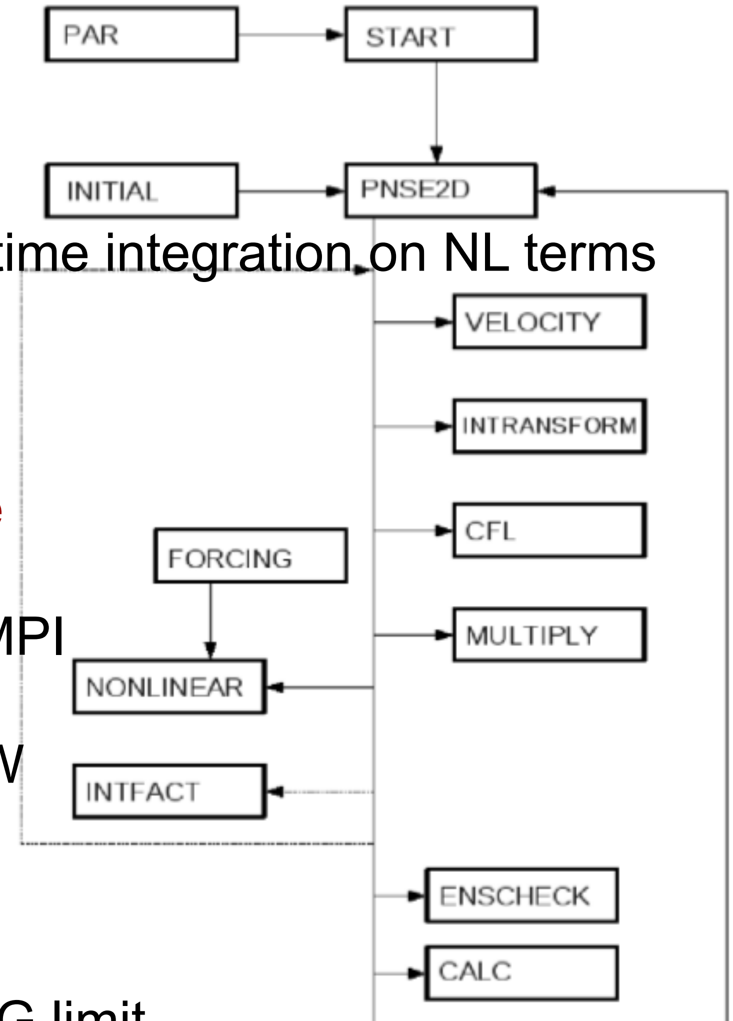
Random forcing at large scale

Heavily Parallelized through MPI

FFTs using the package FFTW

Resolution up to 1024^3

Ro ranges from 0.2 down to QG limit



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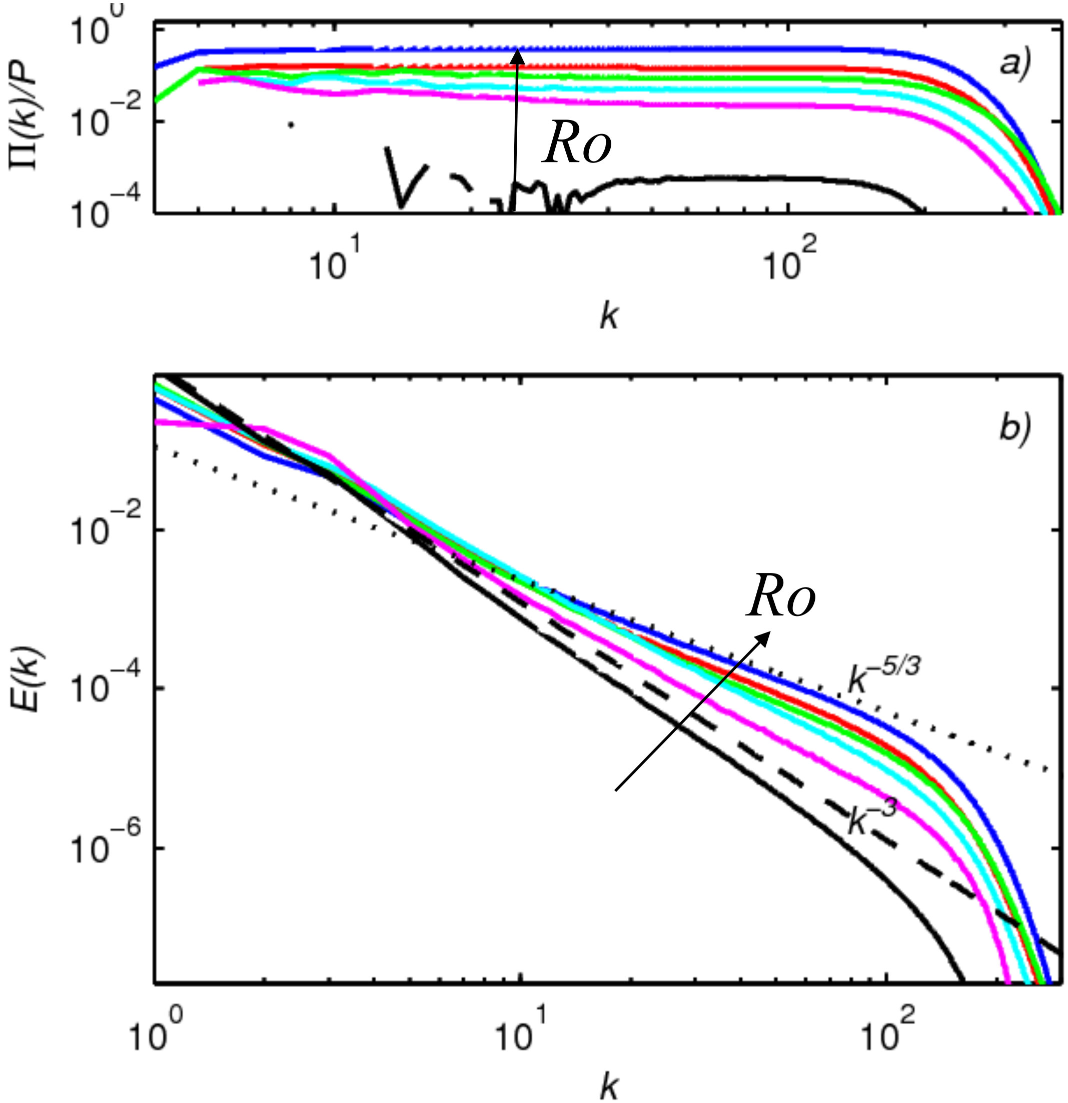
Results

Energy fluxes and spectra



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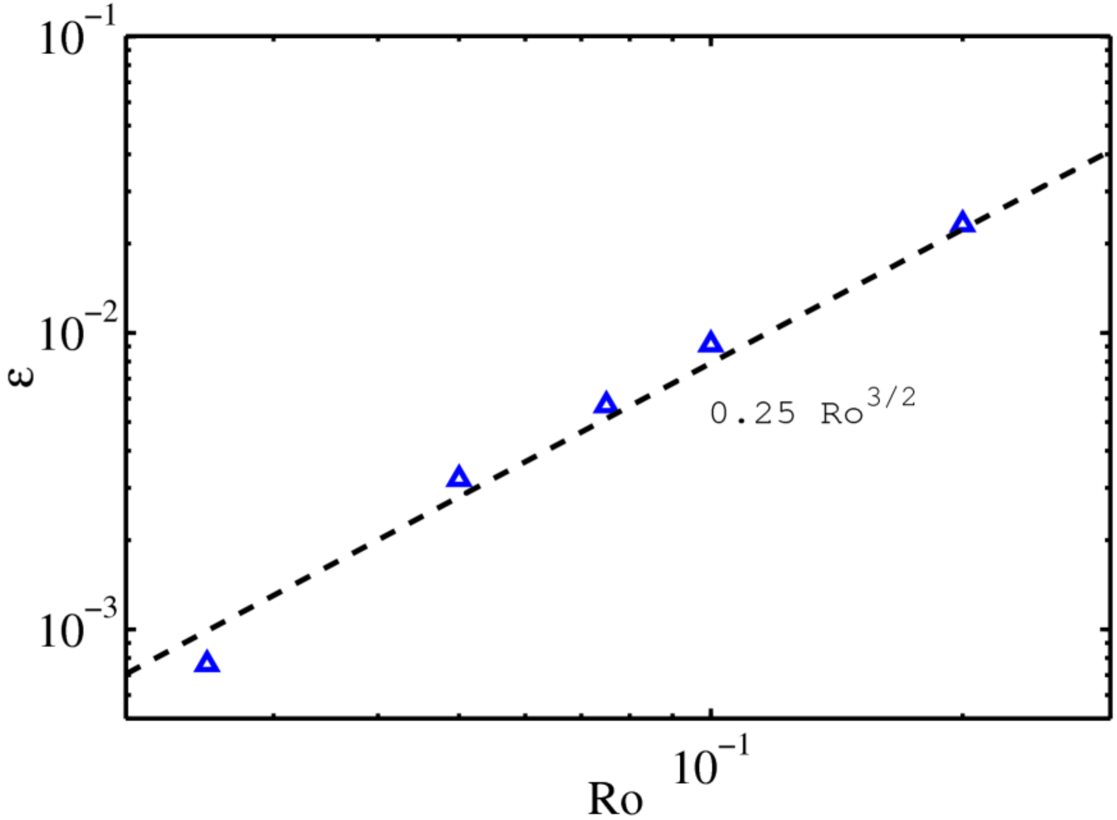
Dissipation vs Ro



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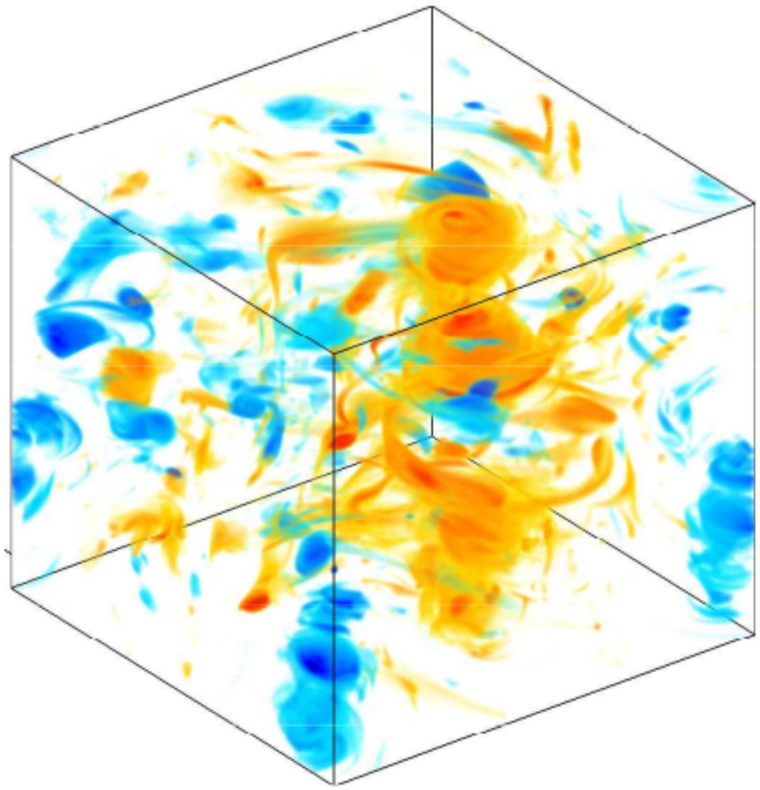
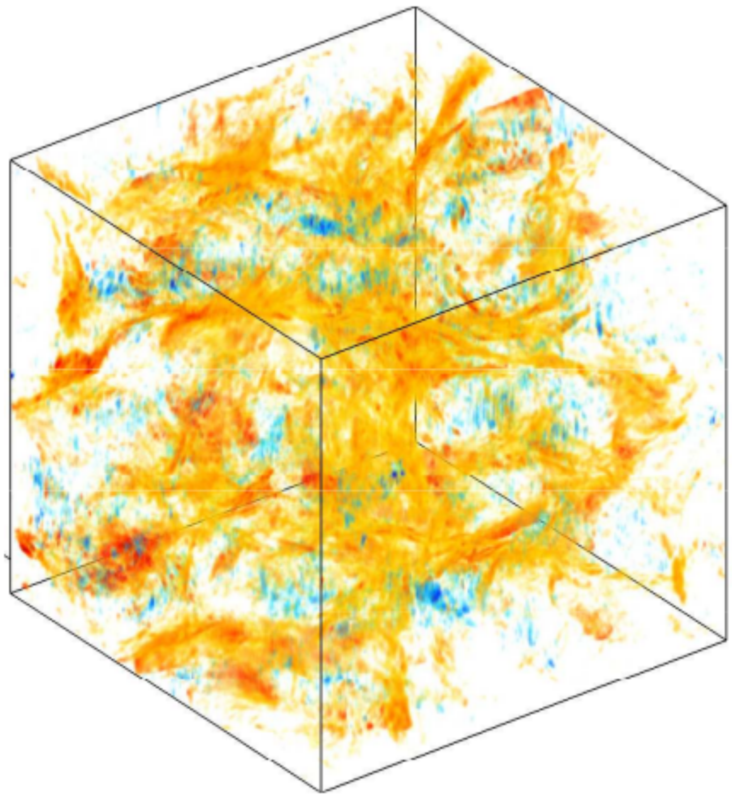
Flow fields at finite and small Rossby number



Ro = 0



Ro = 0.2

Third order structure functions

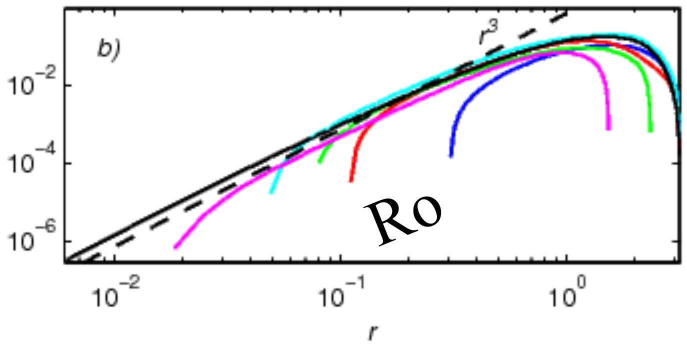
3D turbulence

$$\langle \delta u_L \delta u_L \delta u_L \rangle + 2\langle \delta u_L \delta u_T \delta u_T \rangle = -\frac{4}{3}\varepsilon r$$

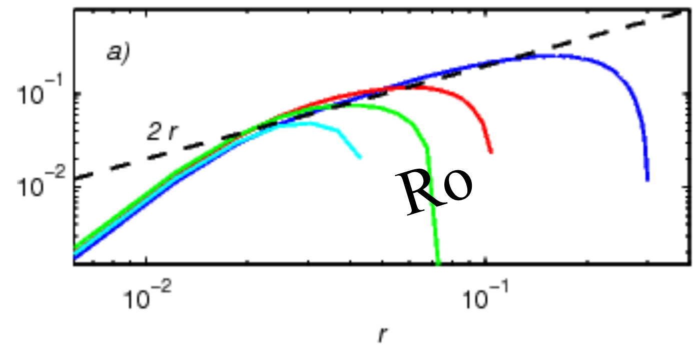
2D turbulence

$$\langle \delta u_L \delta u_L \delta u_L \rangle + \langle \delta u_L \delta u_T \delta u_T \rangle = 2\varepsilon r + \frac{1}{4}\eta r^3$$

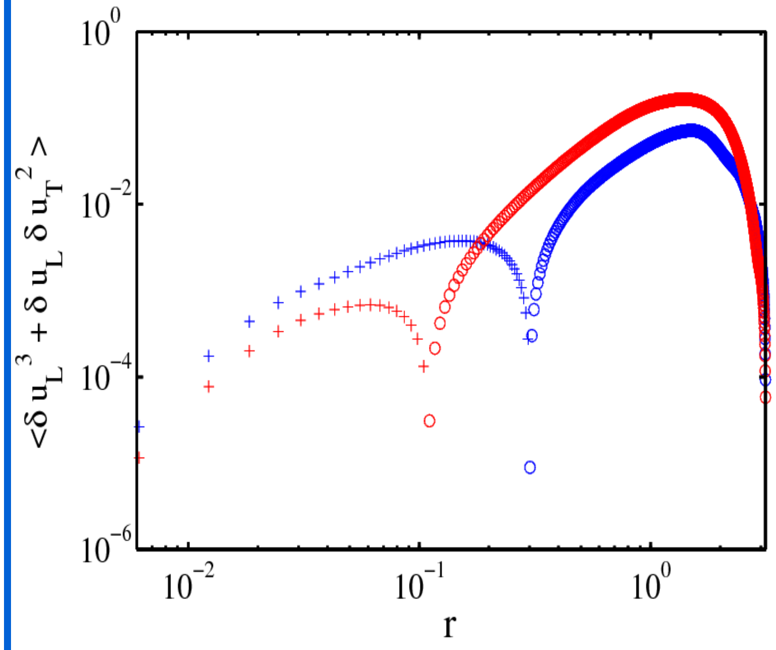
Positive

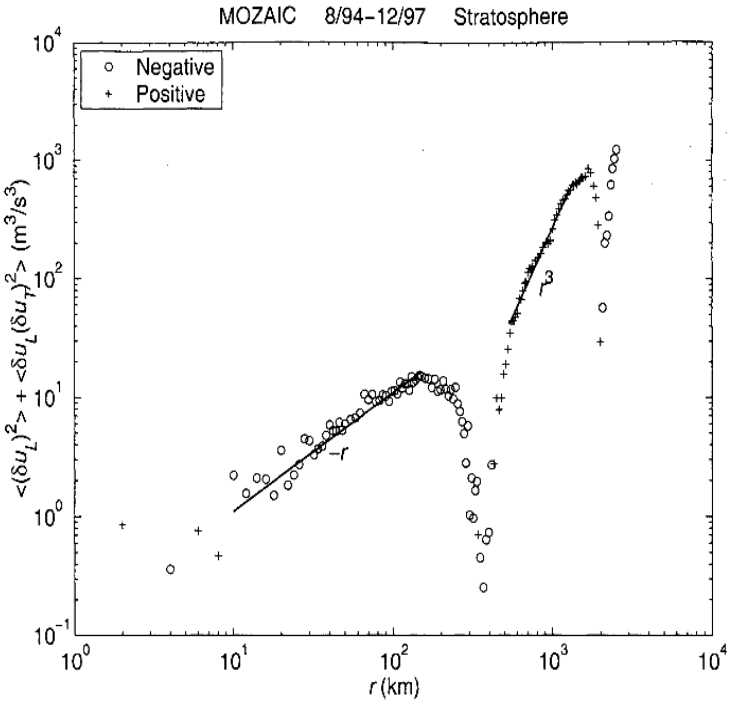


Negative



Simulations vs measurements





Conclusions

We have performed high-resolution numerical simulations of the PE equations at several Ro numbers

Energy is found to cascade more and more towards smaller scale as Ro is increased, shallowing the spectra to $k^{-5/3}$

Structure functions, in agreement with theoretical arguments are seen to scale as -r at smaller scales and r^3 at larger scales, consistent with the observations