Rotating turbulence and the return to isotropy

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Stockholm, Uriel's Fest, October 17-21, 2011

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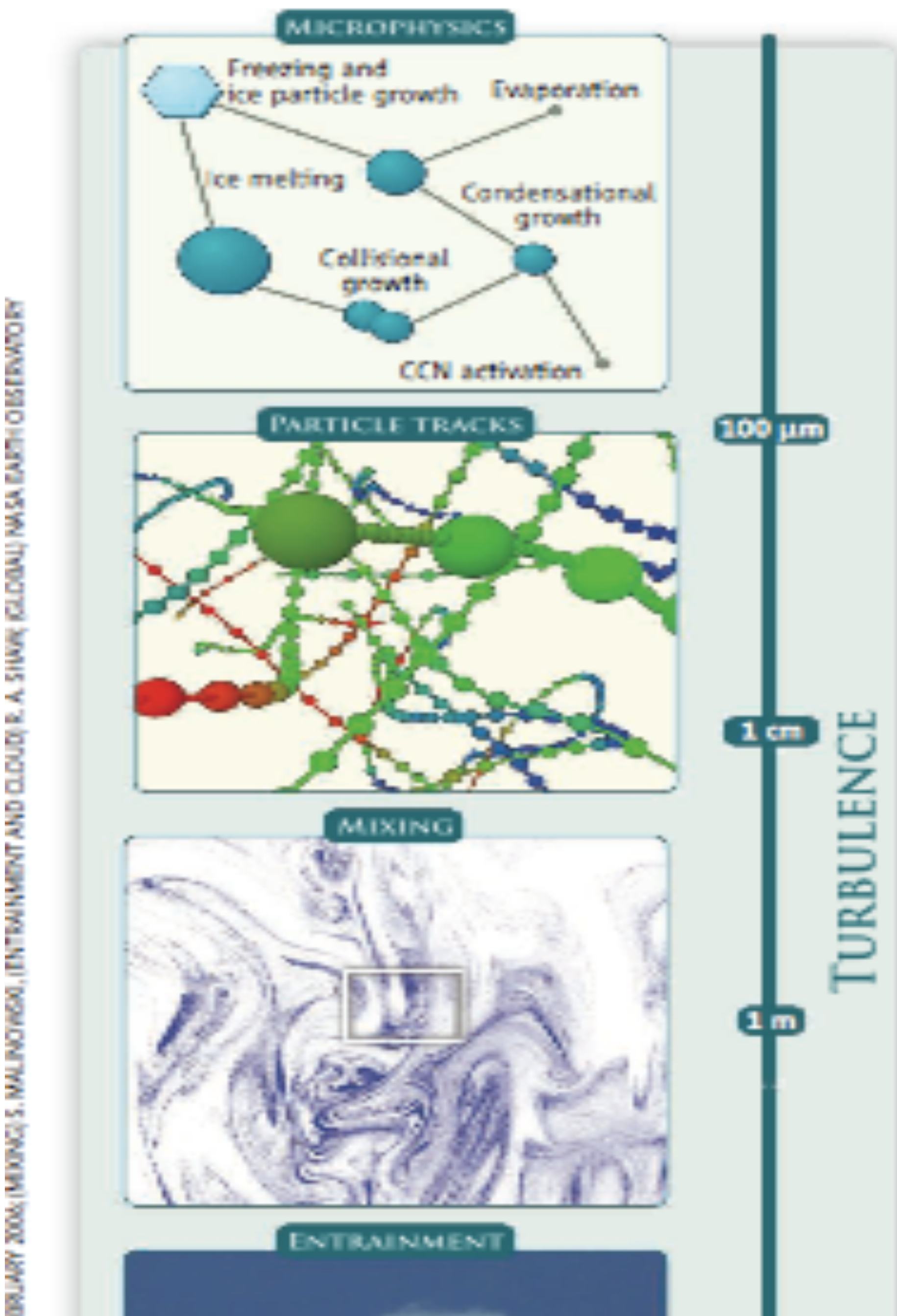
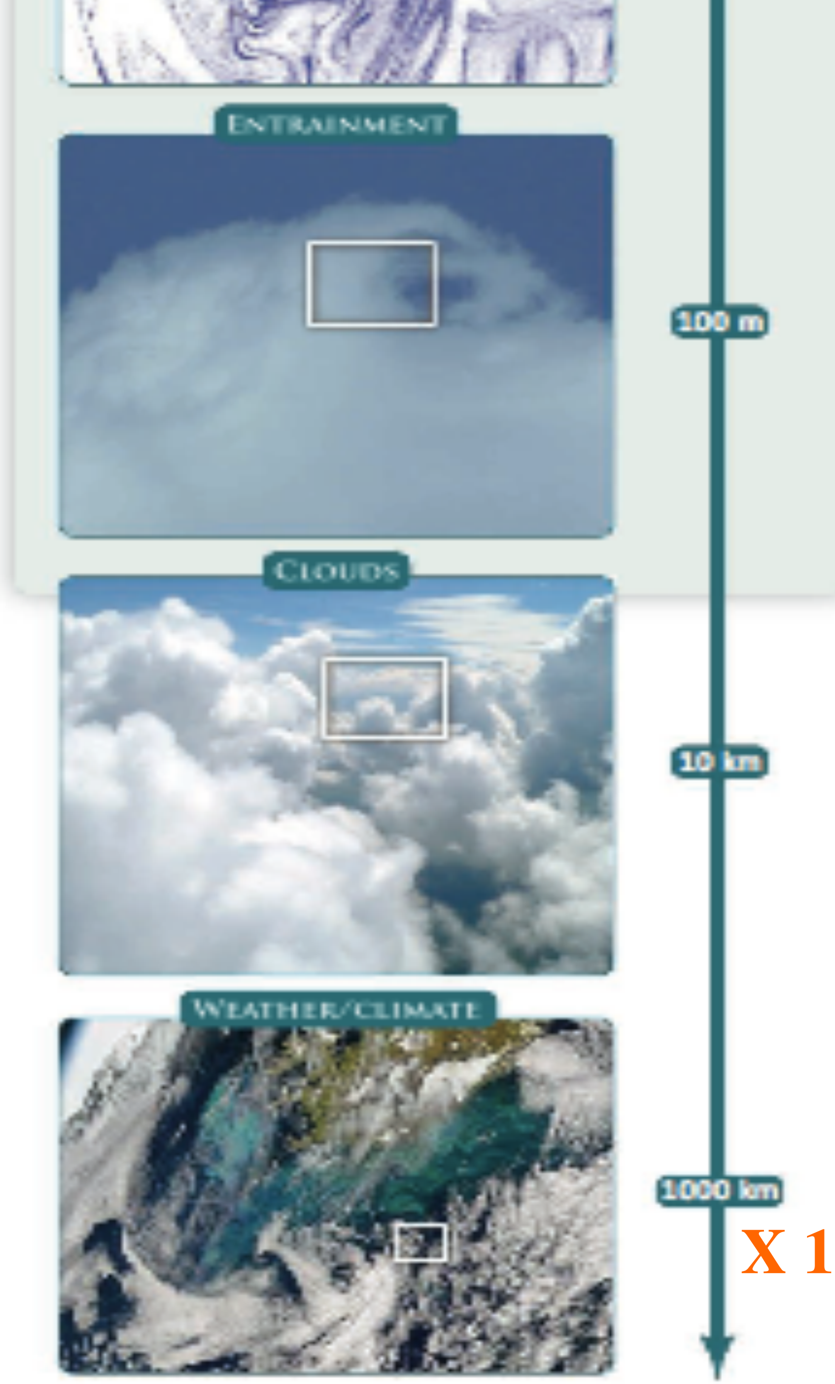
Bodenschatz et al., Science, 2010





X 10^{10}

"Can we understand clouds w/o turbulence?"

OUTLINE

I – Turbulence and inertial waves, *with or without helicity*

II – Is there a return to small-scale isotropy when the wave turbulence regime breaks down?

III – Remarks and questions

Invariants of the **Euler** equation

$$D_t \, v = 0$$

Invariants in the absence of dissipation & forcing (ν=0=F):

* Kinetic energy $E^V = \langle v^2 \rangle / 2$, together with:

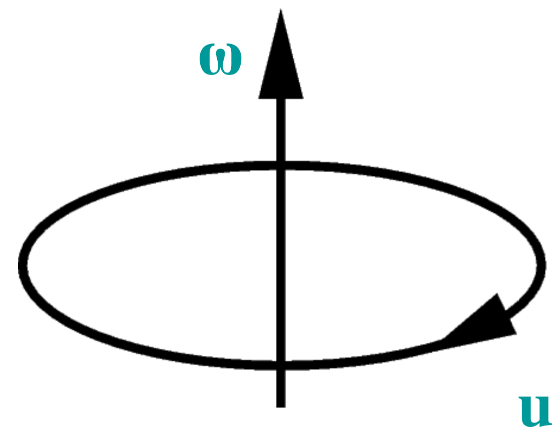
• In three dimensions: **kinetic helicity** $H^V = \langle \mathbf{v} . \boldsymbol{\omega} \rangle$

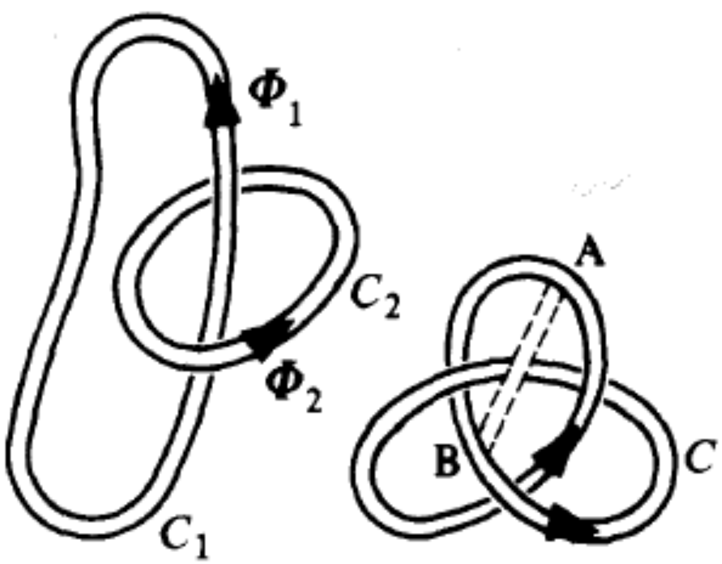
(mid 60s, Moreau; Moffatt; after Woltjer for MHD, mid 50s)

Helicity dynamics

H is a pseudo (axial) scalar

$$H = \int \boldsymbol{\omega} \cdot \mathbf{u} \, dV$$





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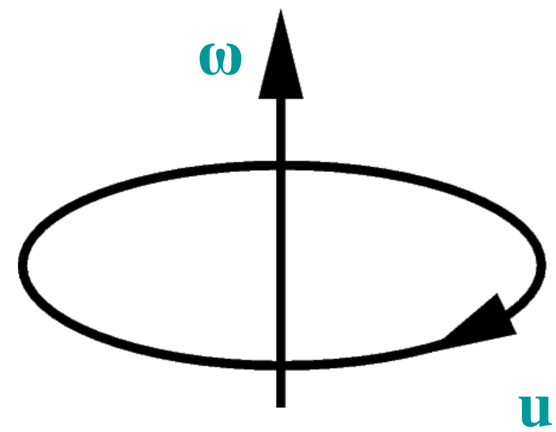
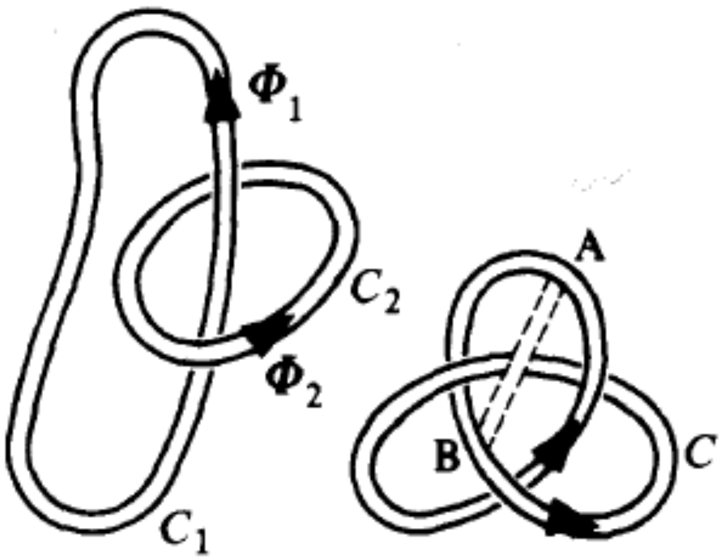


$$H = \int \boldsymbol{\omega} \cdot \mathbf{u} \, dV$$

$\langle u_i(\mathbf{k}) u_j^*(-\mathbf{k}) \rangle = U_E(|k|)\, P_{ij}(|k|)$



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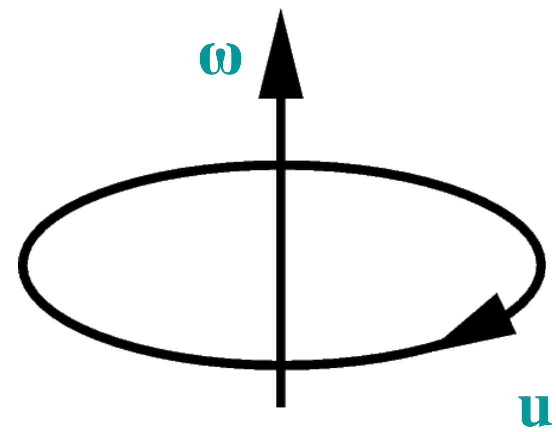
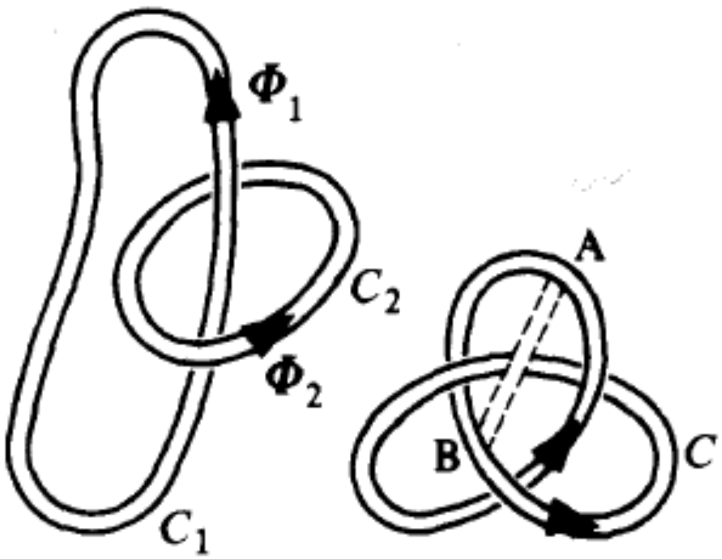


$$H = \int \boldsymbol{\omega} \cdot \mathbf{u} \, dV$$

$<u_i(\mathbf{k})u_j^*(-\mathbf{k})> = U_E(|k|)\, P_{ij}(|k|) + \varepsilon_{ijl} k_l\, U_H(|k|)$



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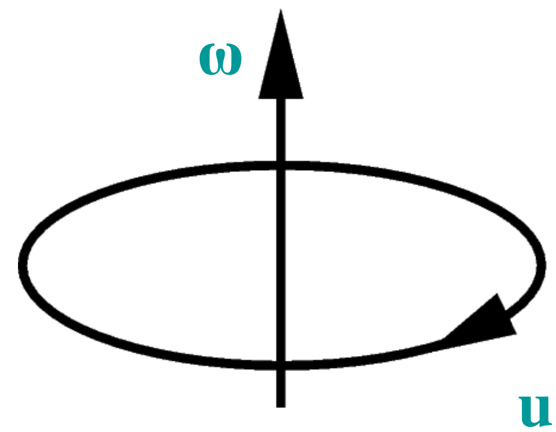

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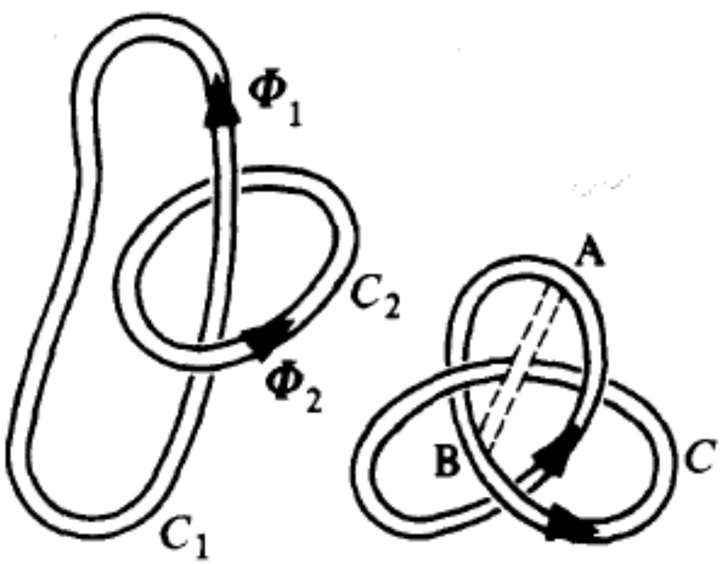


Two defining functions: $U_E(k)$ & $U_H(k)$,
or E(k) and H(k) after integration

→ *A priori* two different scaling laws …

Helicity in tropical cyclones versus *small or large* shear

Molinari & Vollaro, 2010

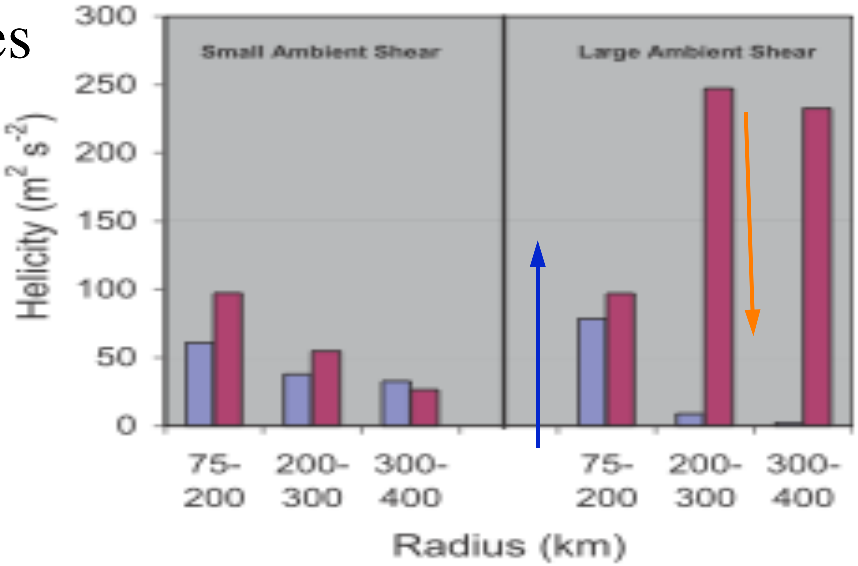
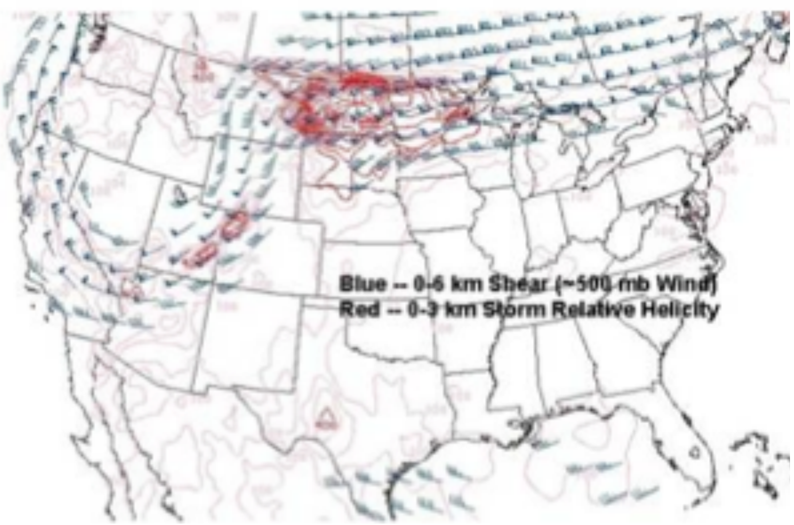
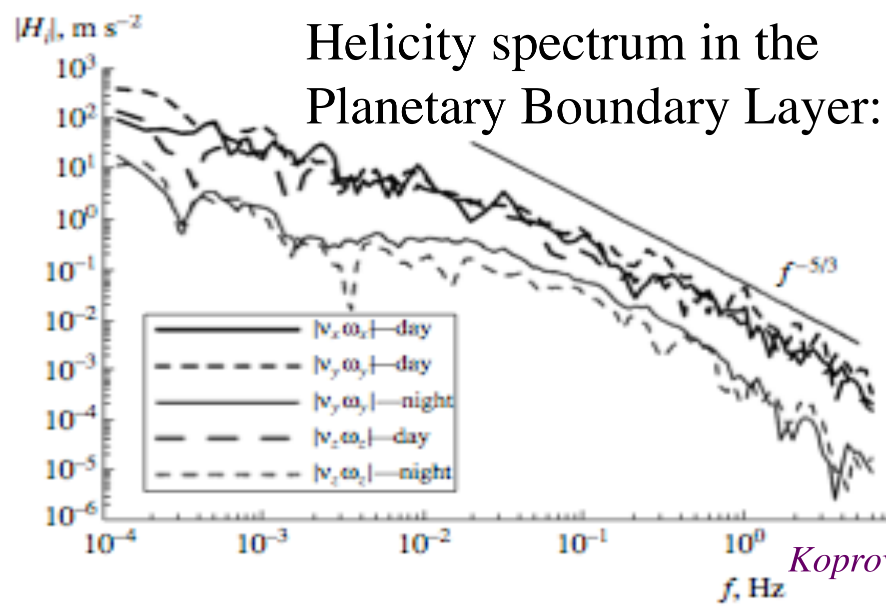


FIG. 4. Radial variation of 0–3-km helicity ($m^2 s^{-2}$) averaged over 75–200, 200–300, and 300–400 km for TCs experiencing (left) small and (right) large ambient shear. Upshear means are in blue and downshear in red.

Shear & helicity in the atmosphere



Helicity spectrum in the Planetary Boundary Layer: *K41*



Koprov, 2005

Fig. 4. Spectra of helicity components.

Helicity dynamics & alignment

- Evolution equation for the local helicity <u>density</u>
  (Matthaeus et al., PRL 2008)

$$\partial_t(\mathbf{v}.\,\omega) + \mathbf{v}.\,\mathbf{grad}(\mathbf{v}.\,\omega)$$
$$= \omega.\mathbf{grad}(\mathbf{v}^2/2 - P) + \nu\Delta\,(\mathbf{v}.\,\omega)$$
$$+ \text{forcing}$$

→ ***v. ω (x)*** *can grow* ***locally***
on a fast (nonlinear) time-scale

$h_r = cos(\mathbf{v}, \omega)$, non-helical TG flow

Blue, $h_r > 0.95$; Red, $h_r < -.95$

In search of a small parameter

The theoretically solvable case of weak/wave turbulence

But is it useful?

Isotropic phenomenology of turbulence with waves

- Assumption: $\varepsilon = \tau_W / \tau_{NL} << 1$; transfer time T_{tr} evaluated as

$$\boxed{T_{tr} = T_{NL} / \varepsilon = T_{NL} * (T_{NL}/T_W)}$$

with $T_{NL} = l/u_l$ *and* $T_W = 1/\Omega$

Inertial waves, rotation Ω

- Constant energy flux: $\varepsilon_* = DE/Dt \sim k*E(k) / T_{tr}$

$\longrightarrow$ $E(k) \sim [\varepsilon_* \Omega]^{1/2} k^{-2}$

(Dubrulle & Valdetarro, 1992; Zhou, 1995)

Structure functions: $<\delta u(l)^p> \sim l^{\zeta p}$, $\boxed{\zeta_p = p/2}$

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Kolmogorov: $\zeta_p = p/3$, $E(k) \sim \varepsilon_*^{2/3} k^{-5/3}$

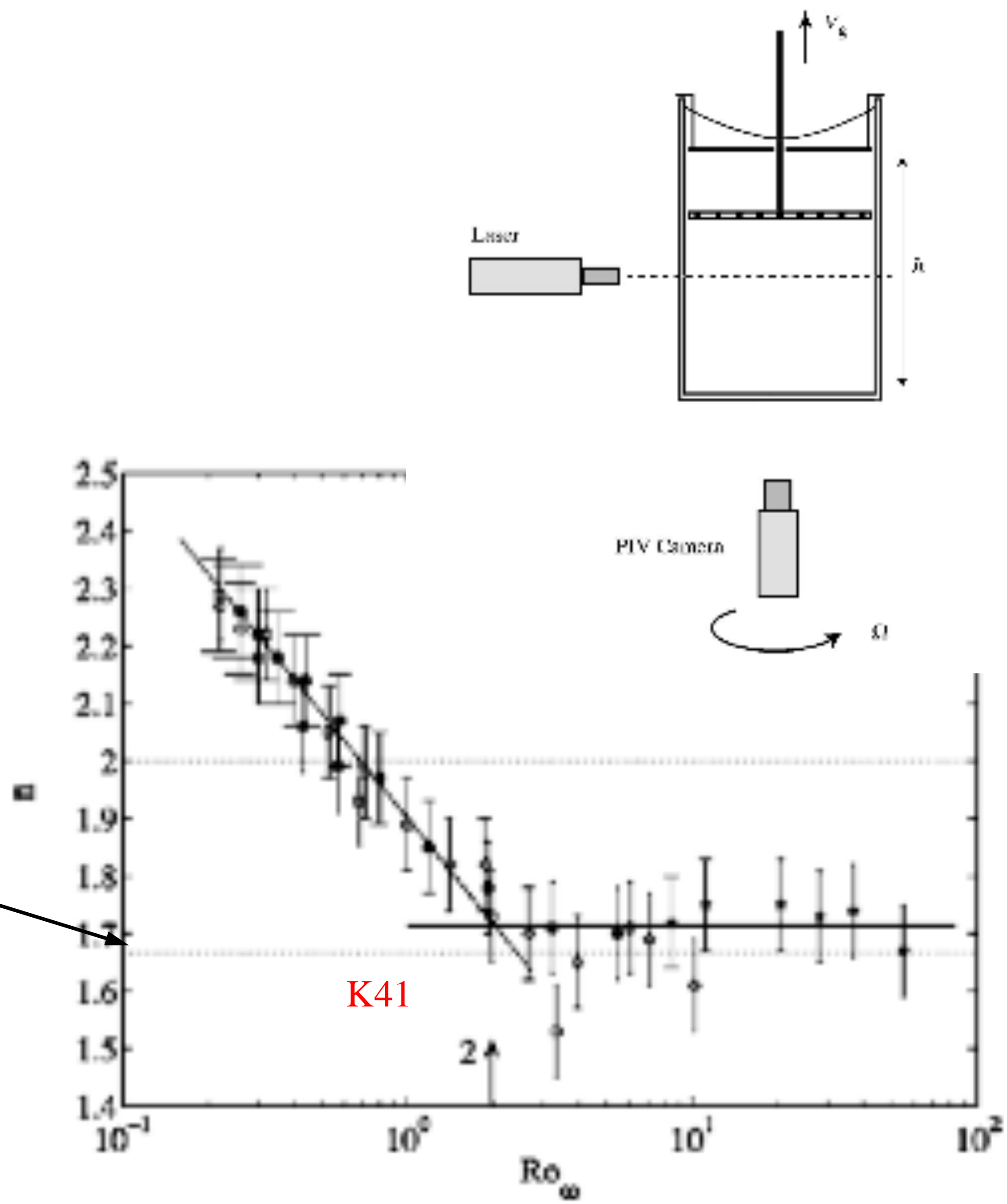
So, what about data?

Both experimental and numerical

Scaling of the energy spectrum at high enough rotation rate

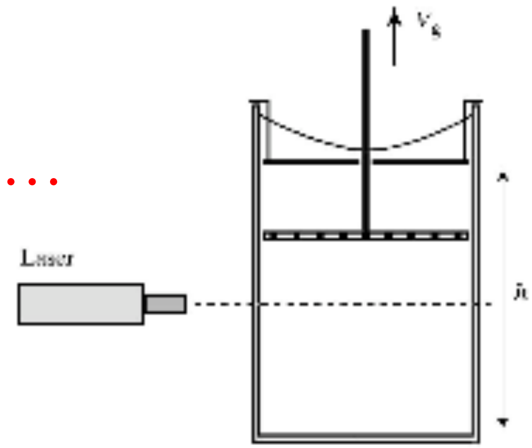
can differ from the classical Kolmogorov spectrum,

i.e. $E(k) \neq k^{-5/3}$



(Morize et al., 2005)

But it does not stop at k^{-2} …



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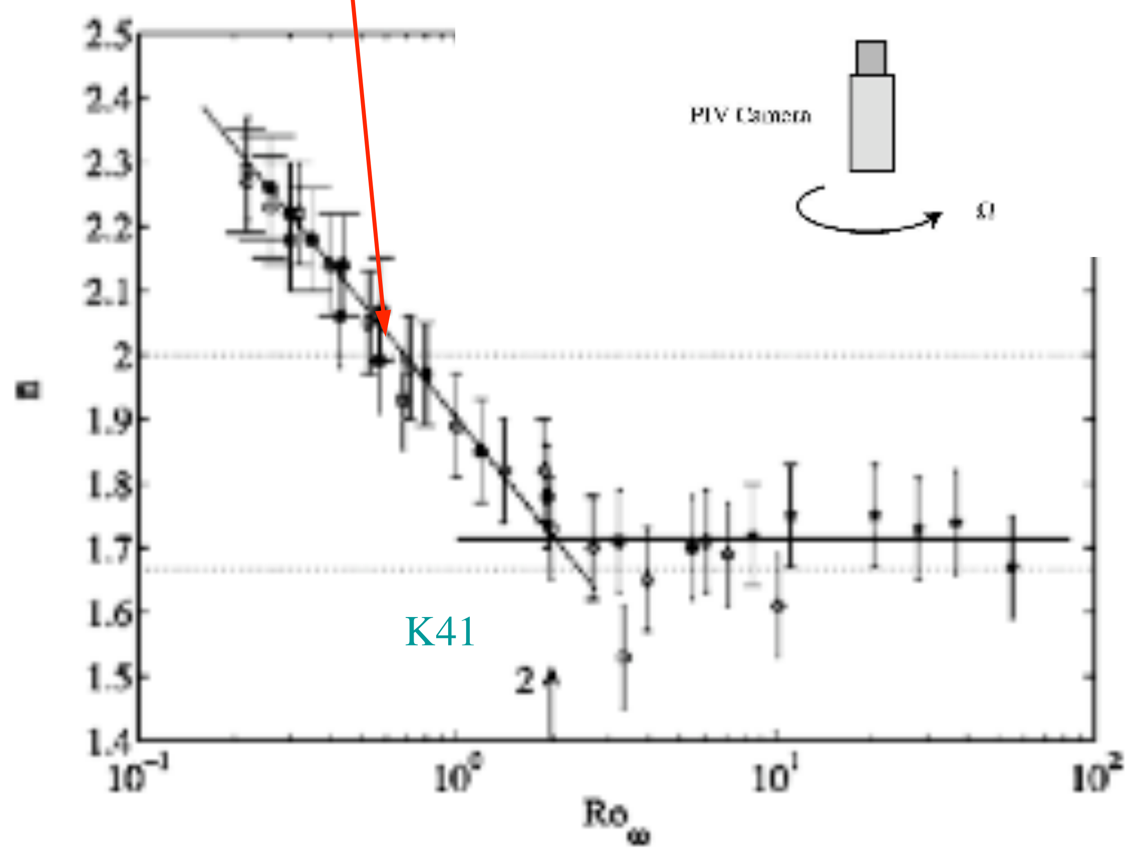
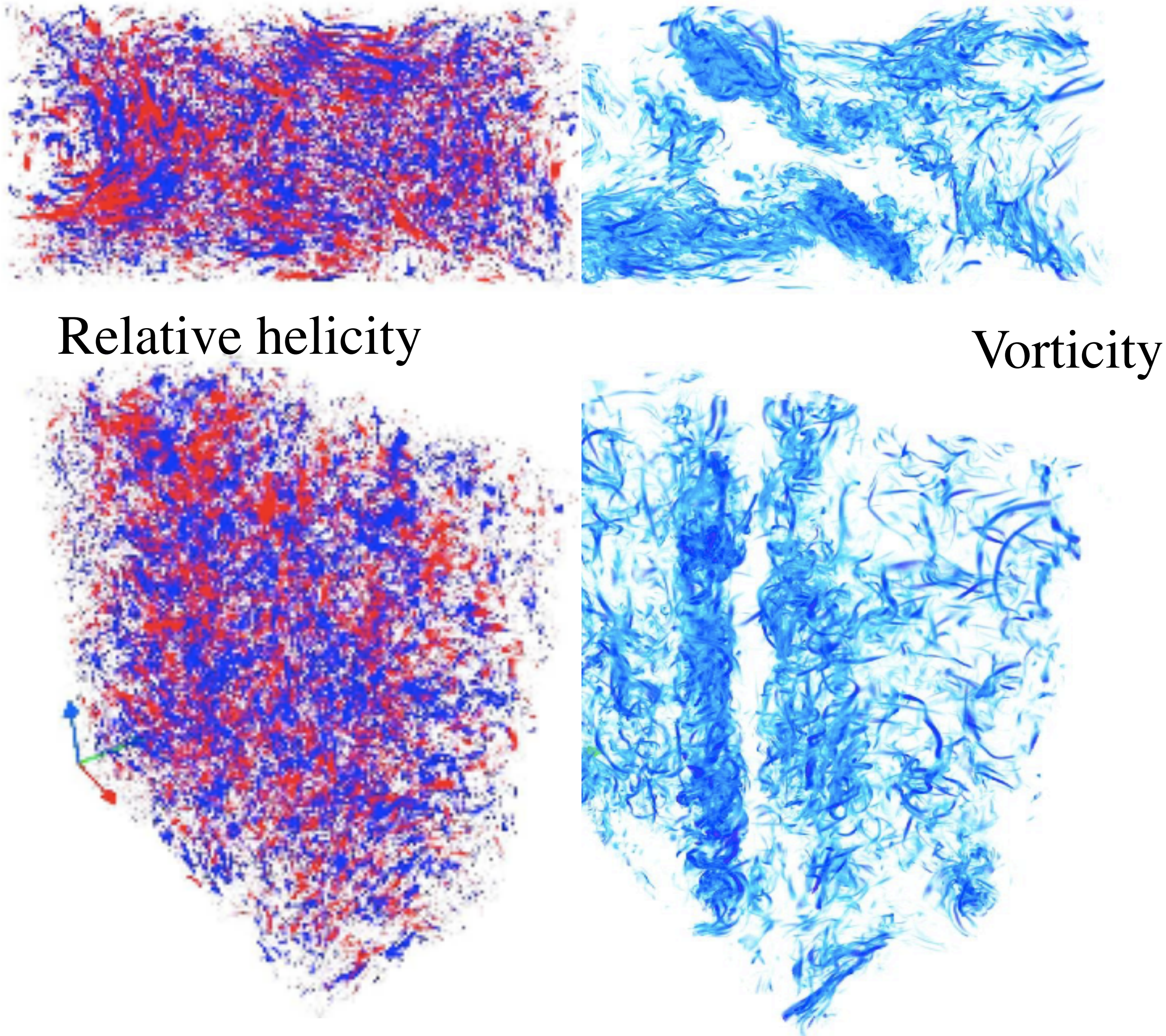


(Morize et al., 2005)

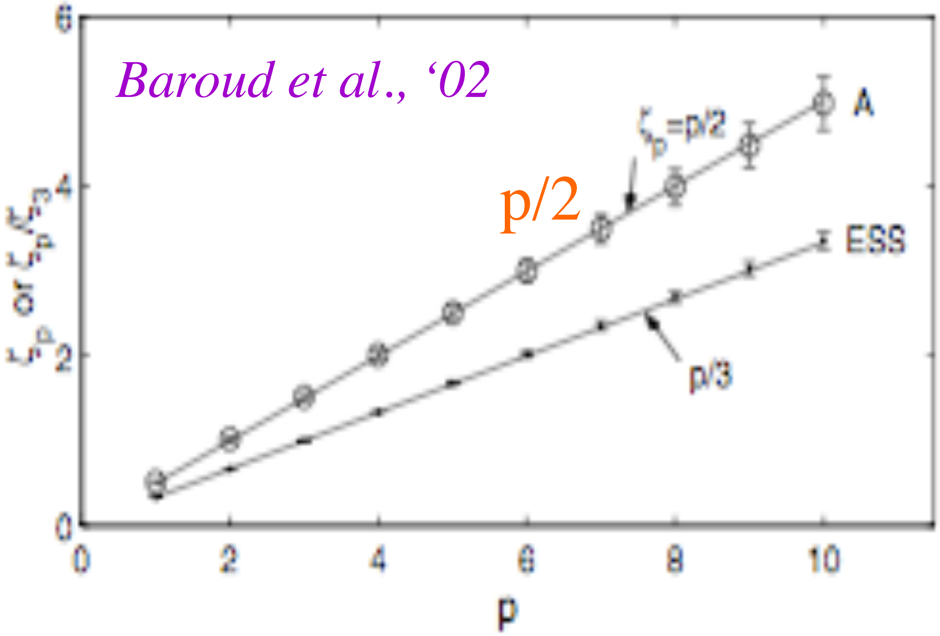
Top view

Side view

Parallel or opposite alignment of $\boldsymbol{v}$ *&* $\boldsymbol{\omega}$



Taylor-Green non-helical forcing, $k_0=4$*,* 512^3 *grid,* $Ro=0.35$

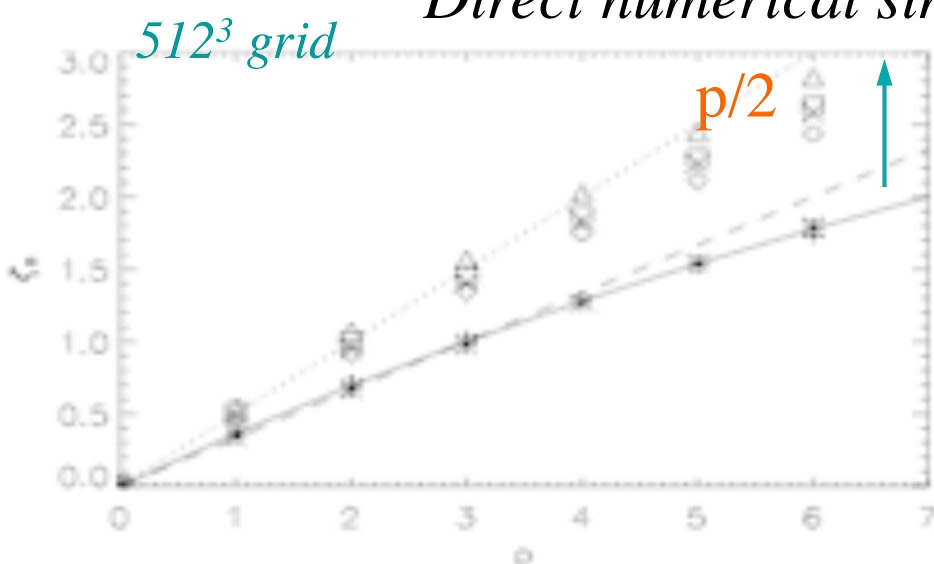


Scaling of structure functions

in rotating turbulence

←*Experiment*

Direct numerical simulation



As time evolves

$<\delta f^p=[f(x+r)-f(x)]^p> \sim r^{\zeta p}$

for the velocity, f=v

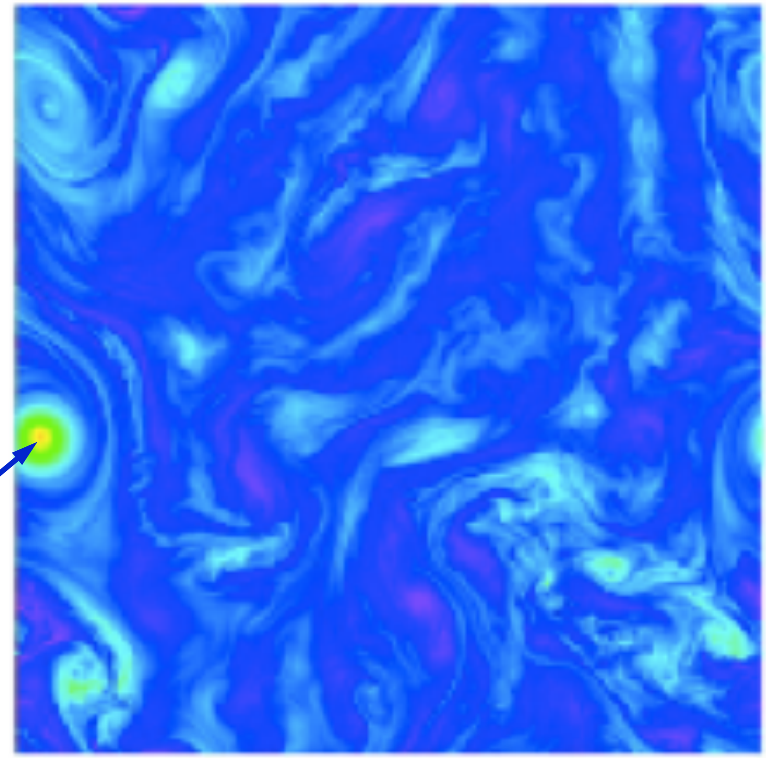
*Mininni+AP, PRE **79** '09*

From Taylor-Green forcing
(globally non helical)

to ABC forcing
(Beltrami flow, fully helical)

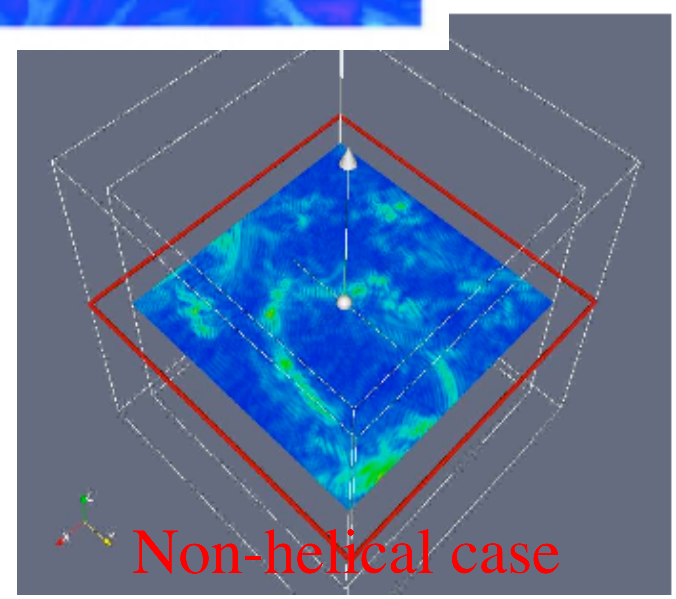
$$u_x = B_0 \, cos \, k_0 \, y + C_0 \, sin \, k_0 \, z$$
$$u_y = C_0 \, cos \, k_0 \, z + A_0 \, sin \, k_0 \, x$$
$$u_z = A_0 \, cos \, k_0 \, x + B_0 \, sin \, k_0 \, y$$

With helicity, strong coherent structures form that are organized:



V_z

Beltrami Core Vortices



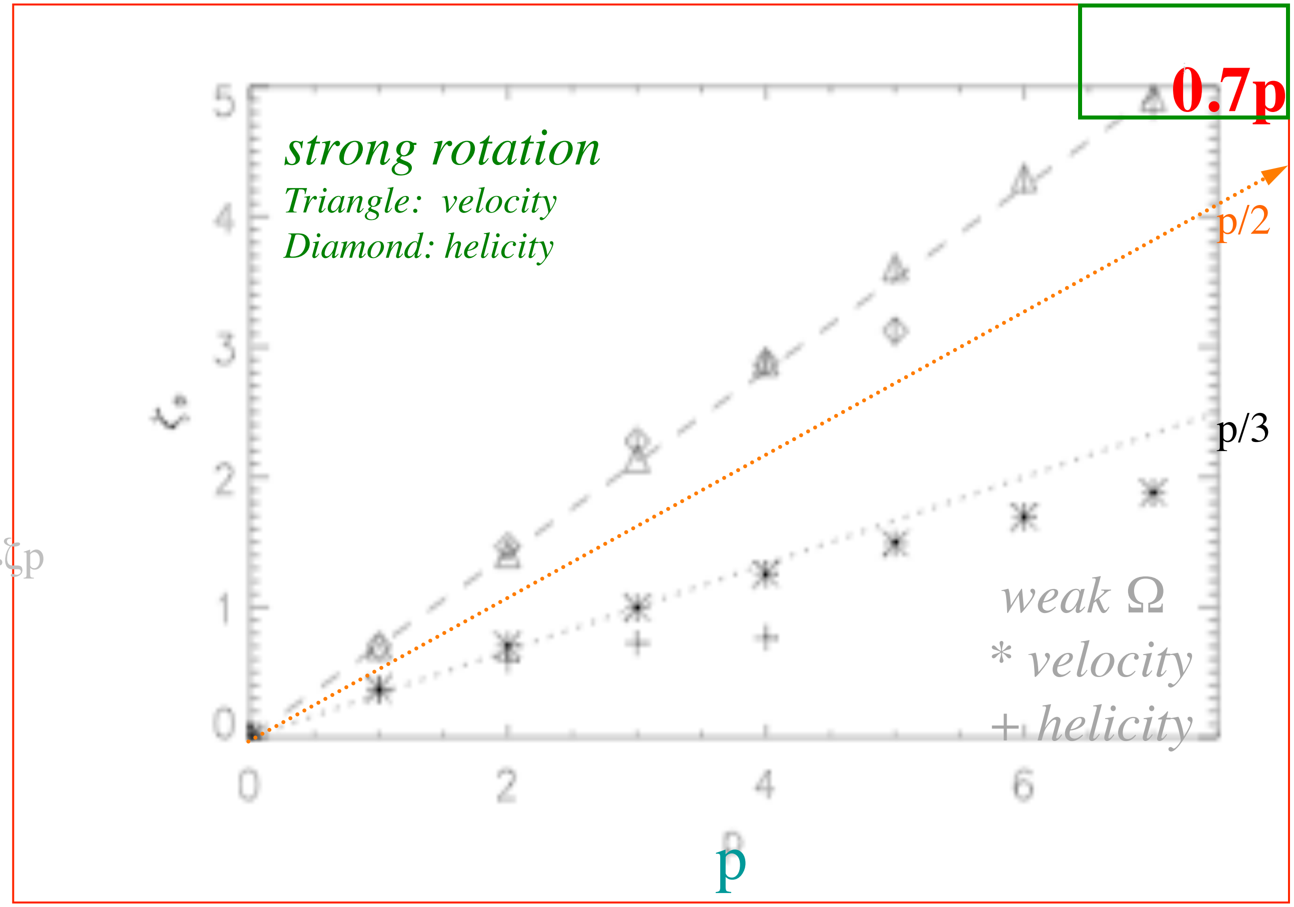
Scaling exponents
of structure functions,

1536^3 grid

$<\delta f=f(x+r)-f(x)]^p> \sim r^{\zeta p}$

of velocity

and helicity
$<\delta u^2(l)> \sim l^{1.4}$

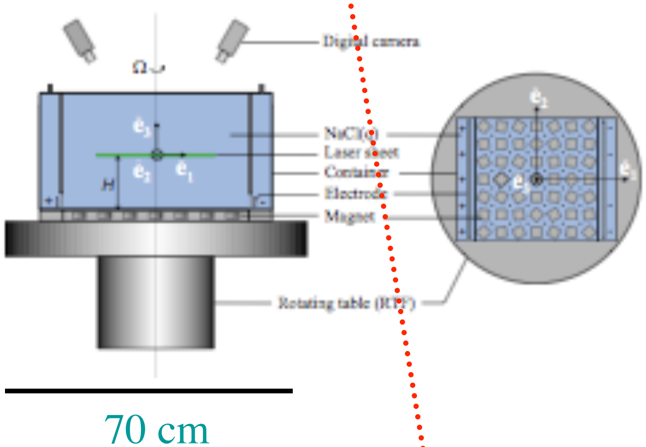


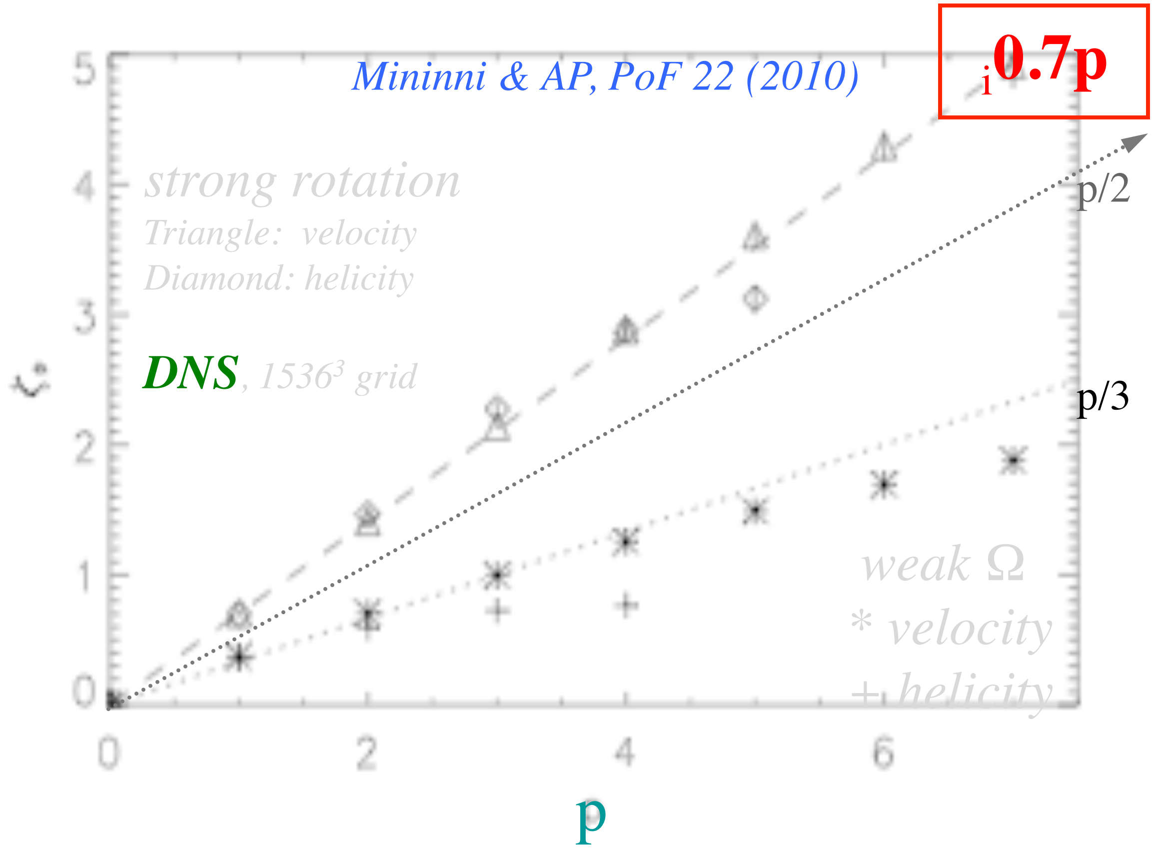
The energy in the <u>direct cascade is again **self-similar**</u> for strong rotation,

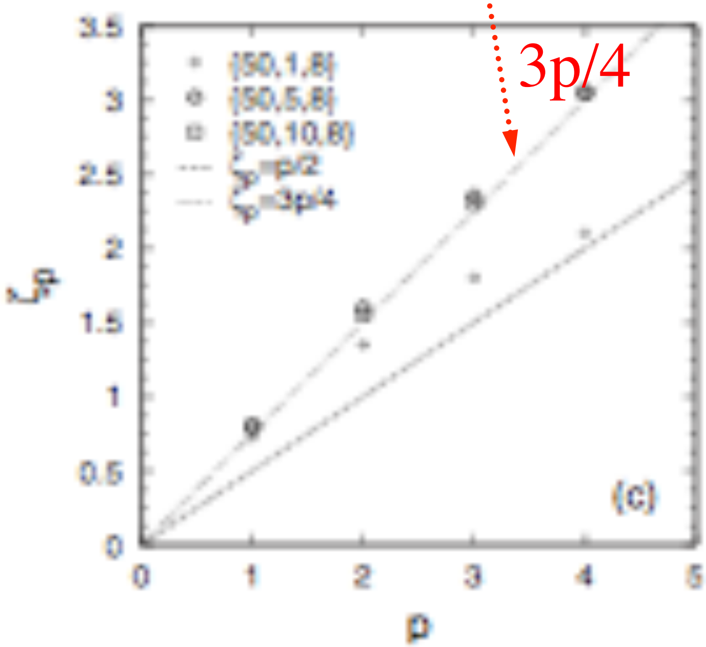
with a scaling which is ***different*** from the *non-helical case* *($\zeta_p = p/2$)*

Mininni & AP, PoF 22 (2010)

Scaling exponents
of structure functions,
experiment







$\zeta_p \sim 3p/4$ at high Ω

van Bhokhoven et al. 2009

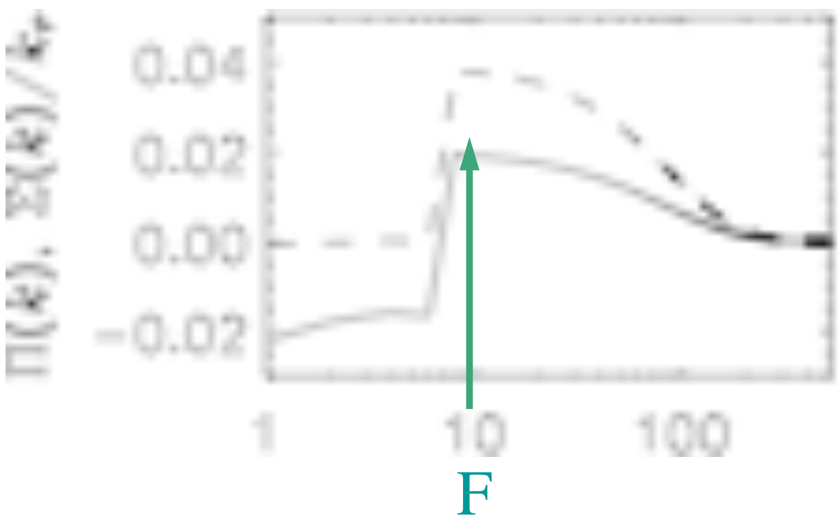
So, what's happening?

→ New spectral law for energy and helicity at high rotation

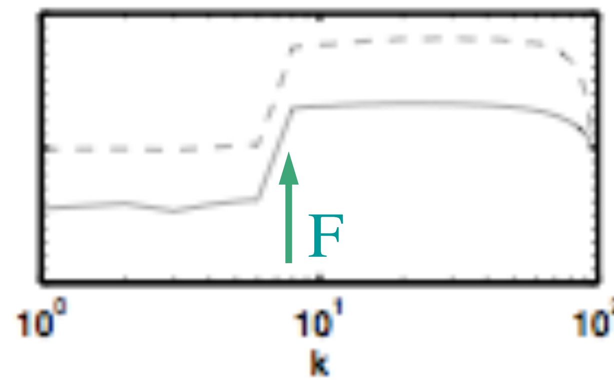
1536^3 DNS run

- k_F=7
- Re=5100
- Ro=0.06

Fluxes of normalized helicity Π_H/k_F (dash) and of energy Π_E (solid)



Mininni & AP,
Phys. Fluids 2010



Spectral **model**,
higher effective
Reynolds number,
Baerenzung et al. JAS 2011

1536^3 DNS run

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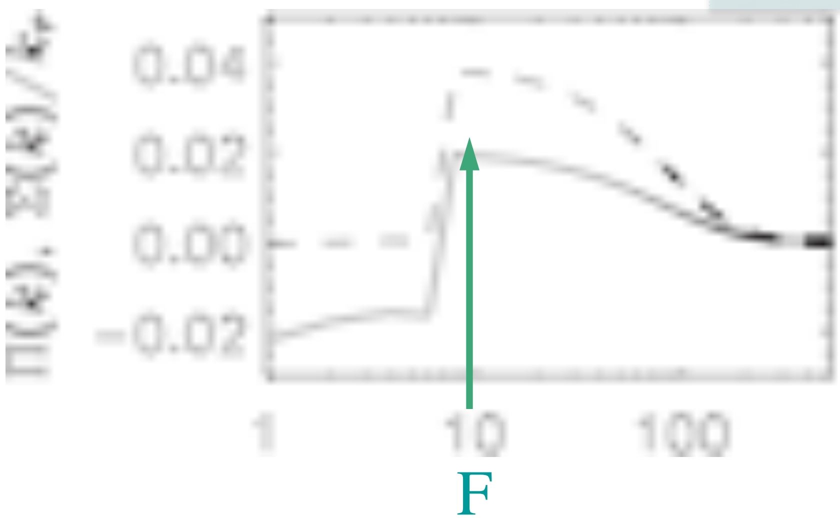
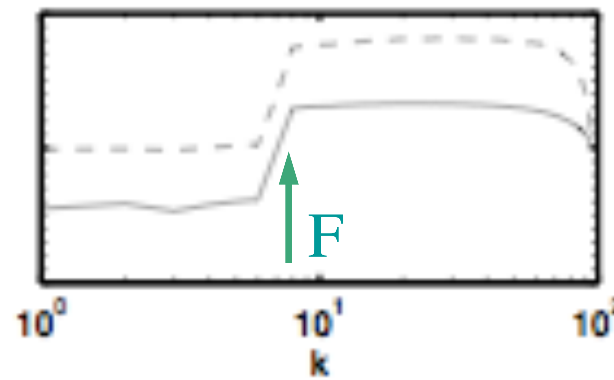
Fluxes of normalized helicity Π_H/k_F (dash) and of energy Π_E (solid)

$$\Pi_E < \Pi_H/k_F$$



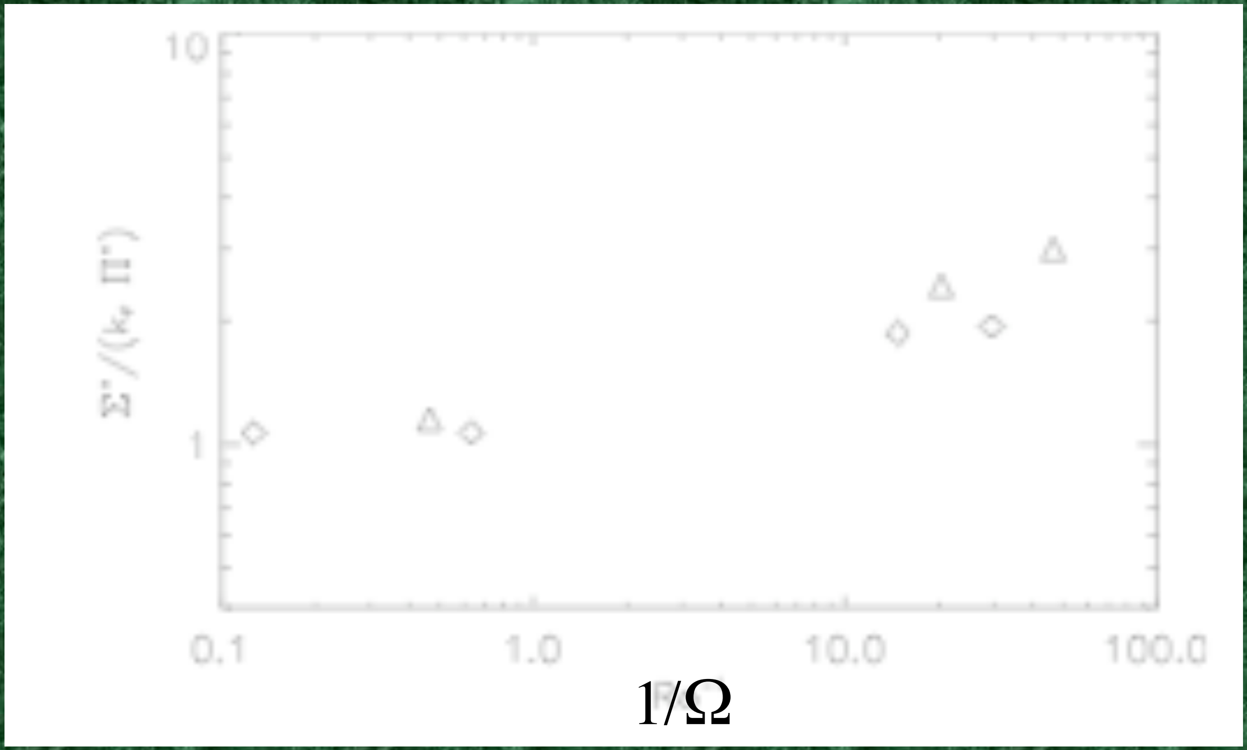


Mininni & AP, Phys. Fluids 2010

Spectral **model**, higher effective Reynolds number, *Baerenzung et al. JAS 2011*

NORMALIZED RATIO OF HELICITY TO ENERGY TO SMALL SCALES

as a function of inverse rotation



Mininni & AP, Phys. Fluids 2010

A **helical twist** of wave turbulence **phenomenology**

- Small parameter: $\varepsilon = \tau_W / \tau_{NL}$; transfer time T_{tr} evaluated as:

$$T_{tr} = T_{NL} / \varepsilon = T_{NL} * (T_{NL}/T_W) \qquad \textit{with } T_{NL}=l/u_l \textit{ and } T_W = 1/\Omega$$

- **Constant helicity flux:** $\varepsilon^H = DH/Dt \sim k^* H(k) / T_{tr}$
- Assume $E(k) \sim k^{-e}$, $H(k) \sim k^{-h}$

→ **$e + h = 4$** in the helical case with rotation

Assuming now maximal helicity [$H(k)=kE(k)$] leads to $e=5/2$
and structure functions: $\langle \delta u(l)^p \rangle \sim l^{\zeta_p}$, $\zeta_p = 3p/4$ *(Mininni & AP, 2009)*

But is maximal helicity a reachable solution?

The end of wave turbulence

The weak turbulence (WT) regime: $\varepsilon = \tau_W/\tau_{NL} \ll 1$

with $\tau_W \sim 1/\Omega$ and $\tau_{NL} \sim \lambda/u_\lambda$

WT breaks down, since $\tau_W \sim \lambda^m$ and $\tau_{NL} \sim \lambda^n$, $m \neq n$:

non-uniformity in scale of the theory

$\tau_W \sim \tau_{NL}$ at scale λ_w, called Zeman scale for rotating flows,

& Ozmidov scale for stratified flows

Recovery of isotropy at small scale

- The Zeman scale l_Ω at which $\tau_W=\tau_{NL}$ → $\boxed{l_\Omega = [\varepsilon/\Omega^3]^{1/2}}$

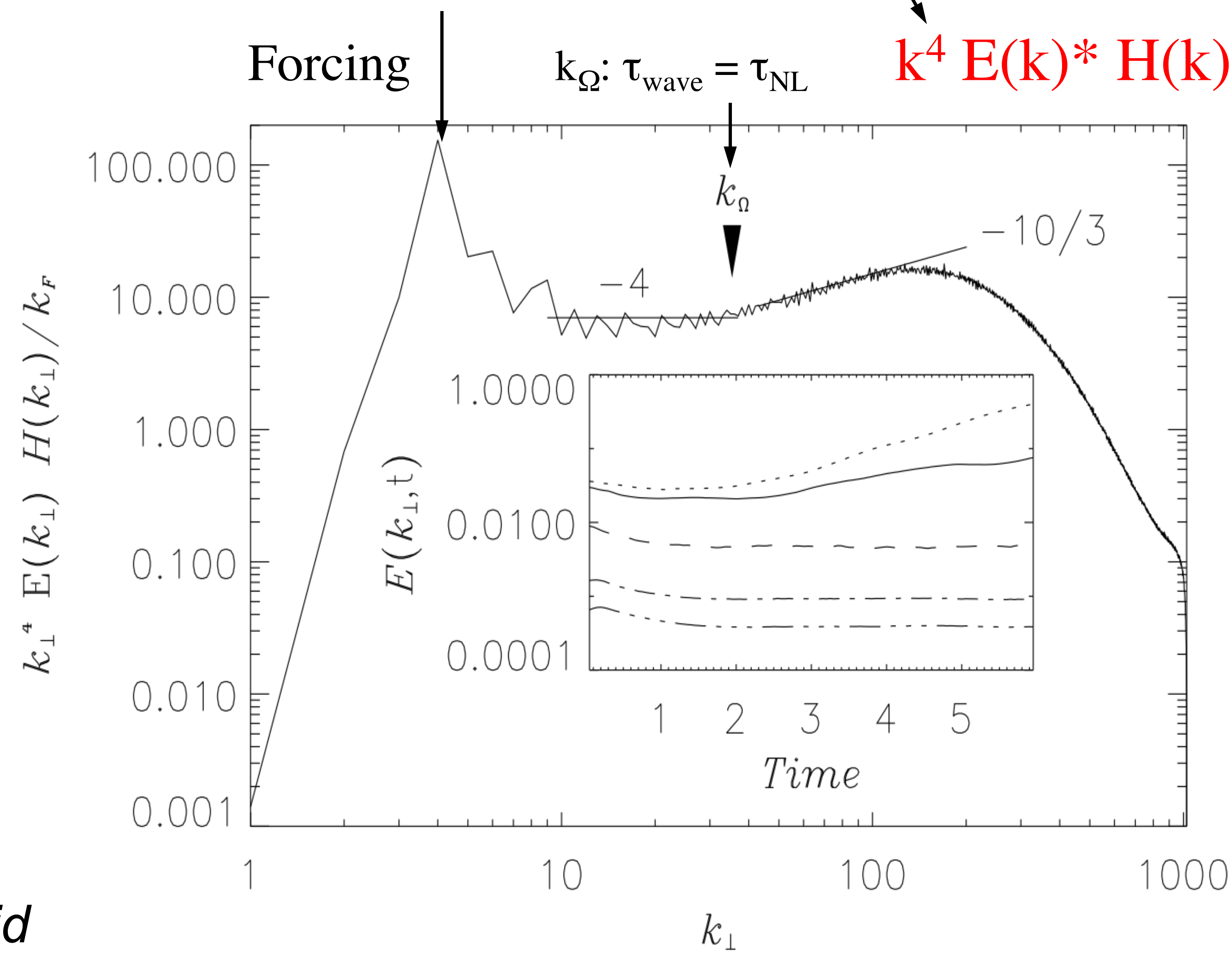
- Large run to resolve, *each moderately*:
  - (i) the inverse cascade range,
  - (ii) the wave-modulated anisotropic inertial range,
  - (iii) the presumably isotropic inertial range, &
  - (iv) the dissipation range

- 3072^3 grid points, Tera-grid allocation of 21 million hours, 30,000 processors (700 hours of clock time, ~ 5 weeks)

*Helical twist in phenomenology: E(k)*H(k) ~ k^{-4}*

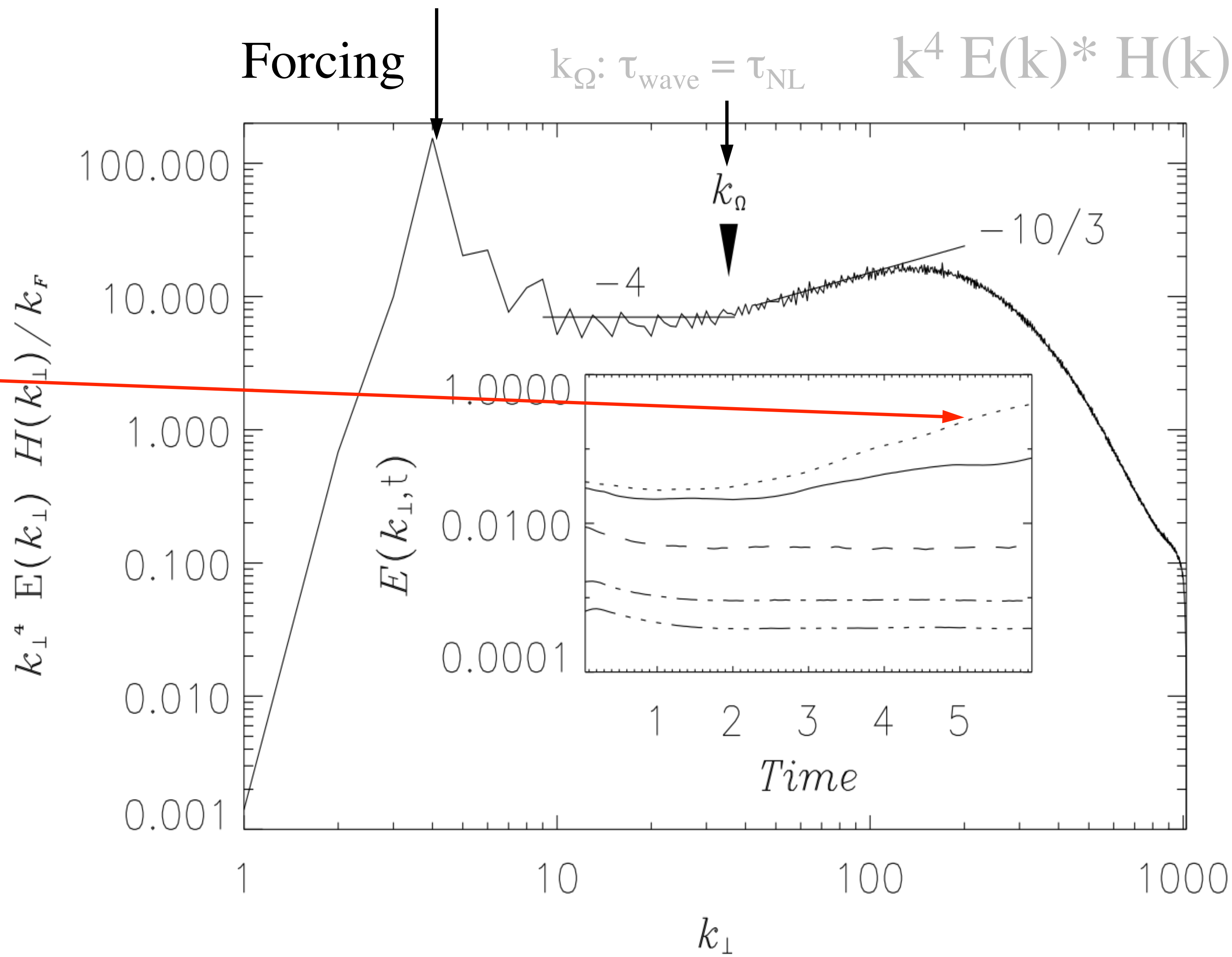
Return to isotropy & K41 in the small scales,

3072^3 grid,
Ro ~ 0.07,
Re ~ 24000,
NSF Tera-grid



*Helical twist in phenomenology: E(k)*H(k) ~ k^-4*

Growth of large-scale modes



3072^3 grid,

Ro ~ 0.07,

Re ~ 24000,

NSF Tera-grid

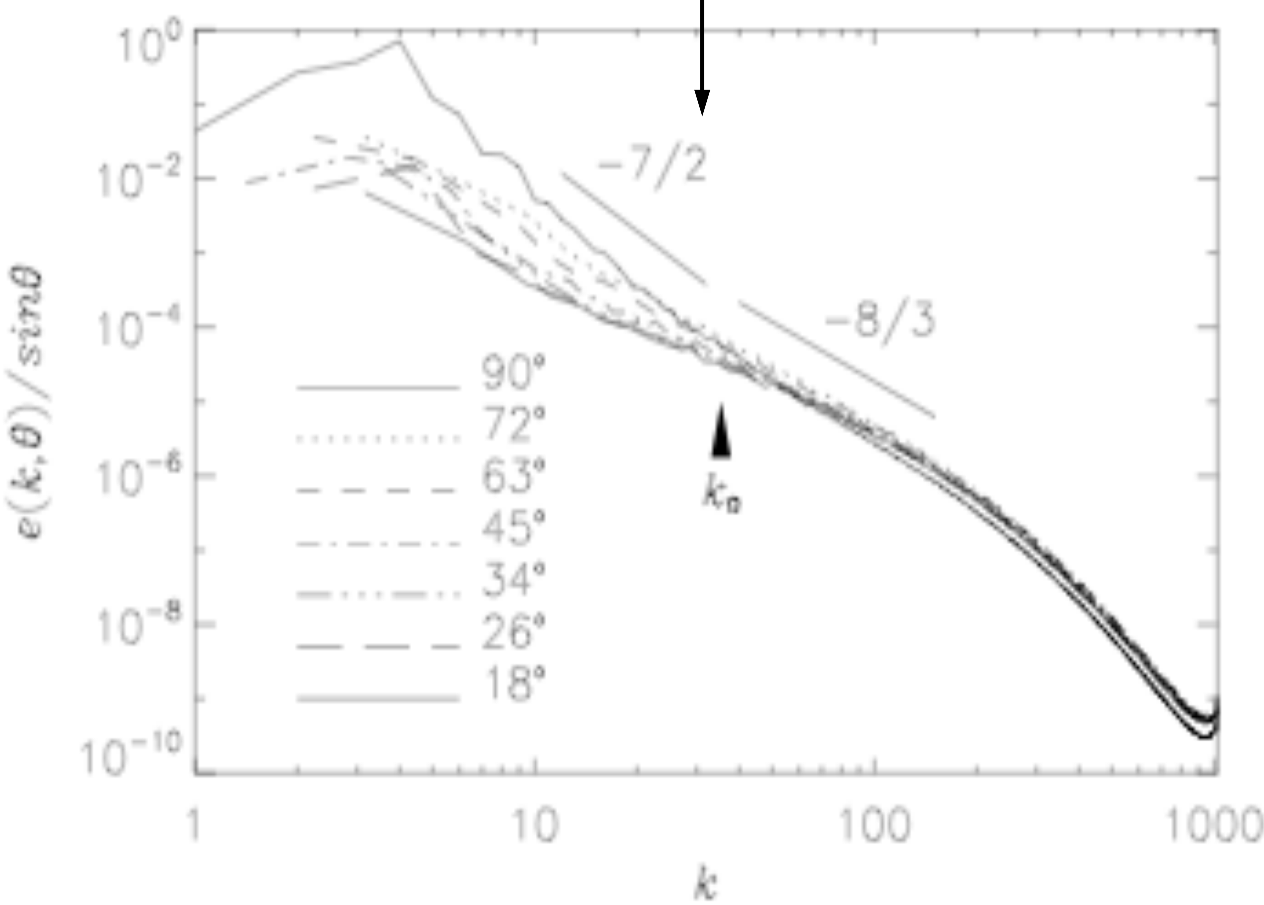
Isotropy & K41 in the small scales:

angular variation, with

$\Theta = (\Omega, k)$

Forcing

Zeman wvnb k_Ω



3072^3 grid

Ro ~ 0.07

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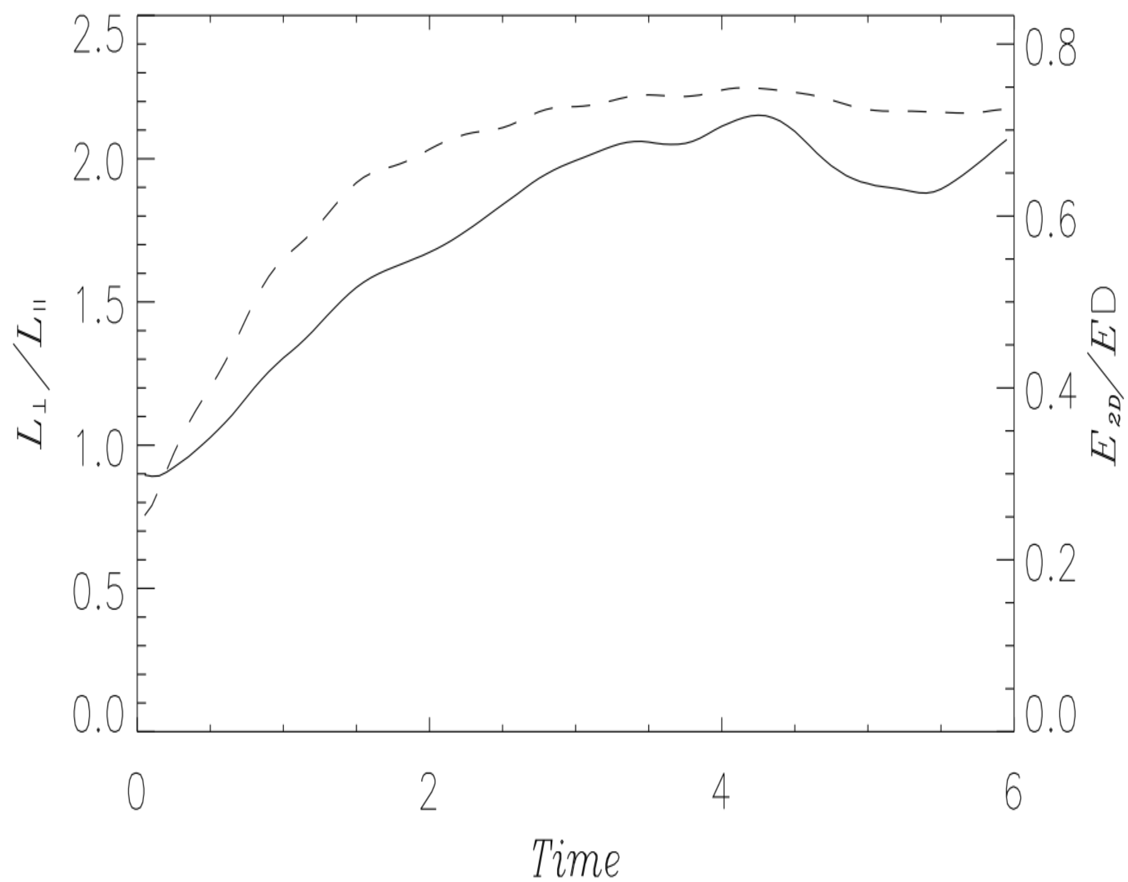
NSF Tera-grid

Isotropy & K41 in the small scales:
ratios of

Perp. to parallel length scales, & 2D to 3D *(dash)* energy ratio



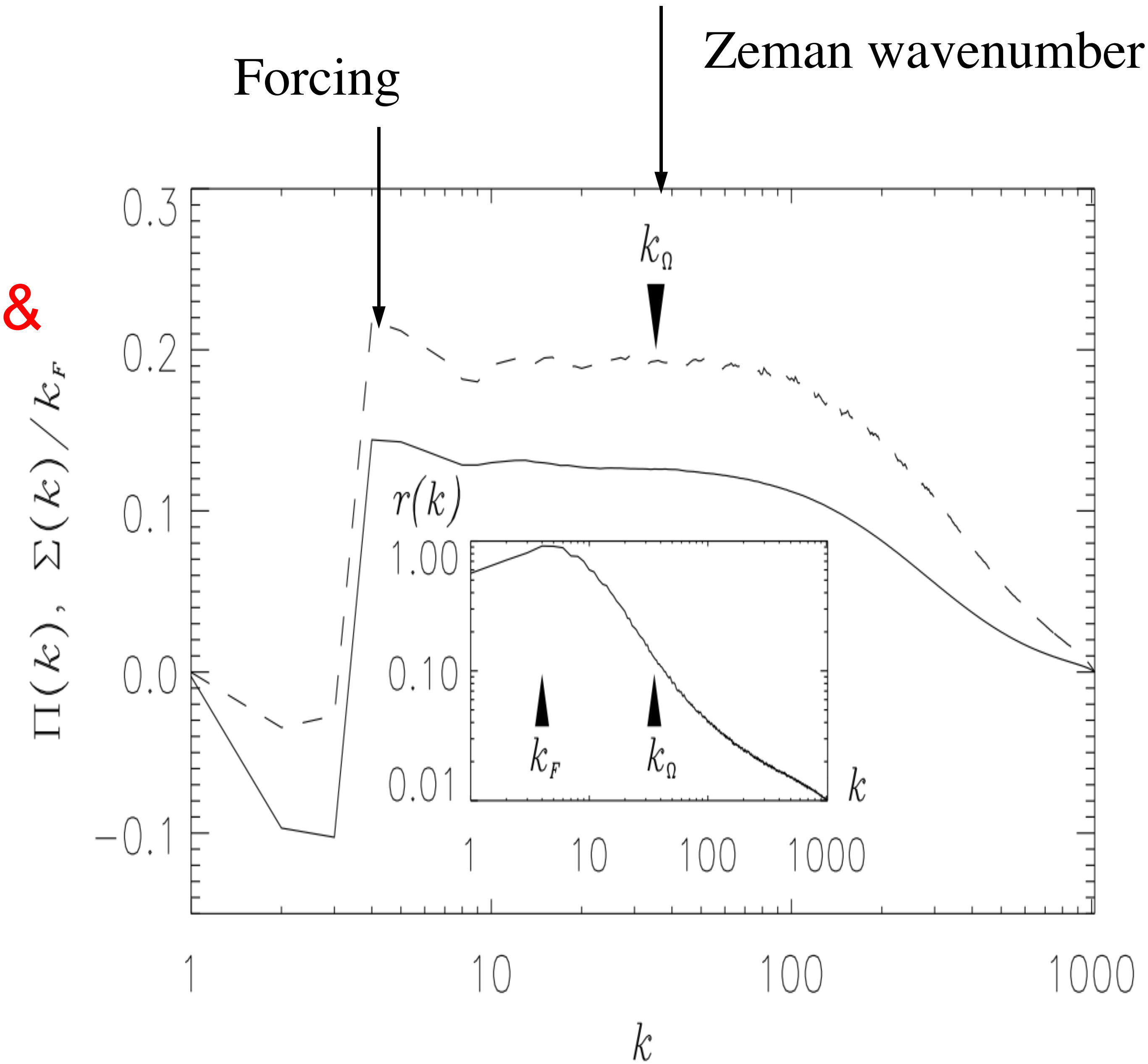
3072^3 grid
Ro ~ 0.07
Re ~ 24000
NSF Tera-grid

Isotropy in the small scales

Helicity *(dash)* & energy *(solid)* fluxes,

and

relative helicity
r(k)=H(k) / [kE(k)]



3072^3 grid
Ro ~ 0.07
Re ~ 24000
NSF Tera-grid

Isotropy & K41 in the small scales:

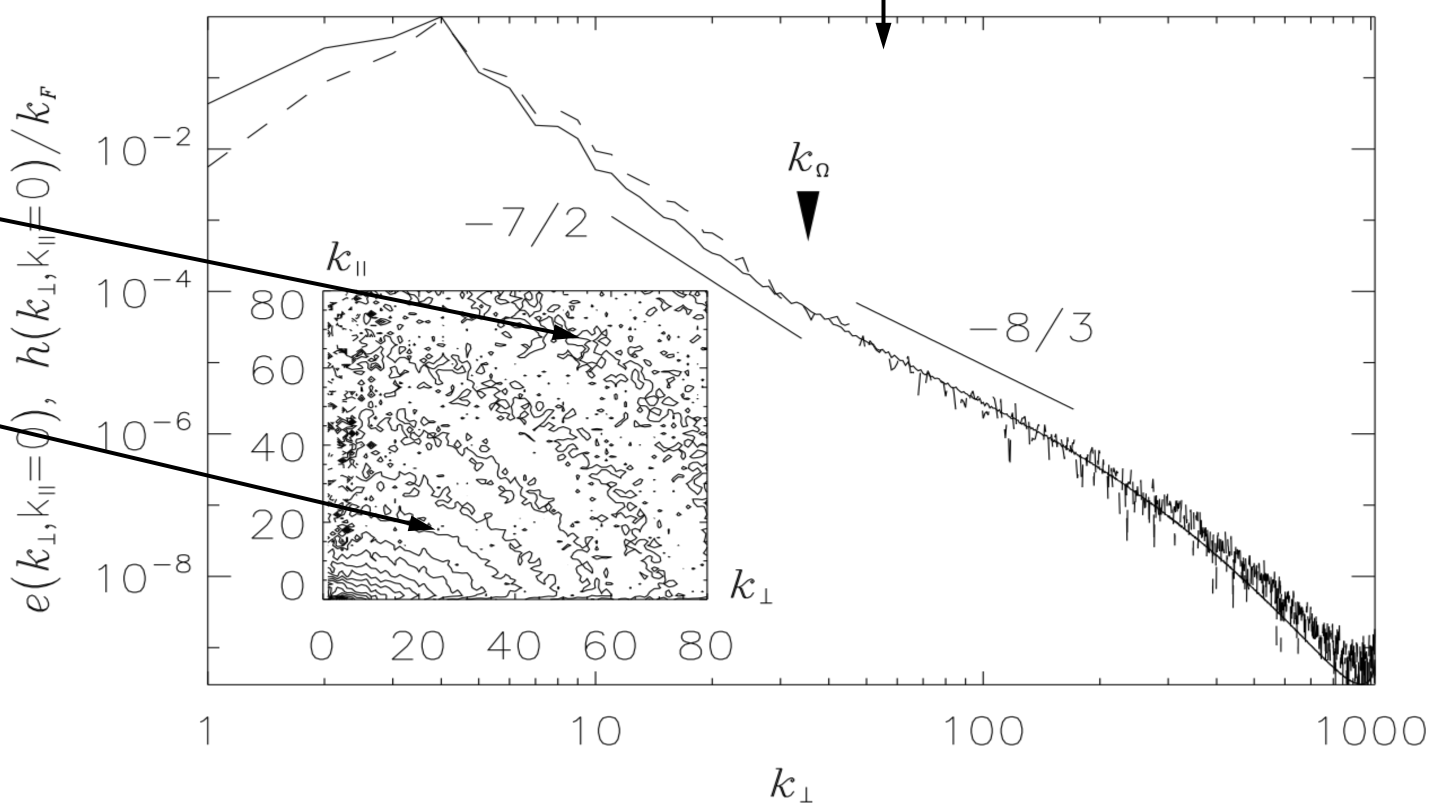
Angular spectra

Forcing

Zeman wvnb k_Ω

Noisy

ellipses



3072^3 grid

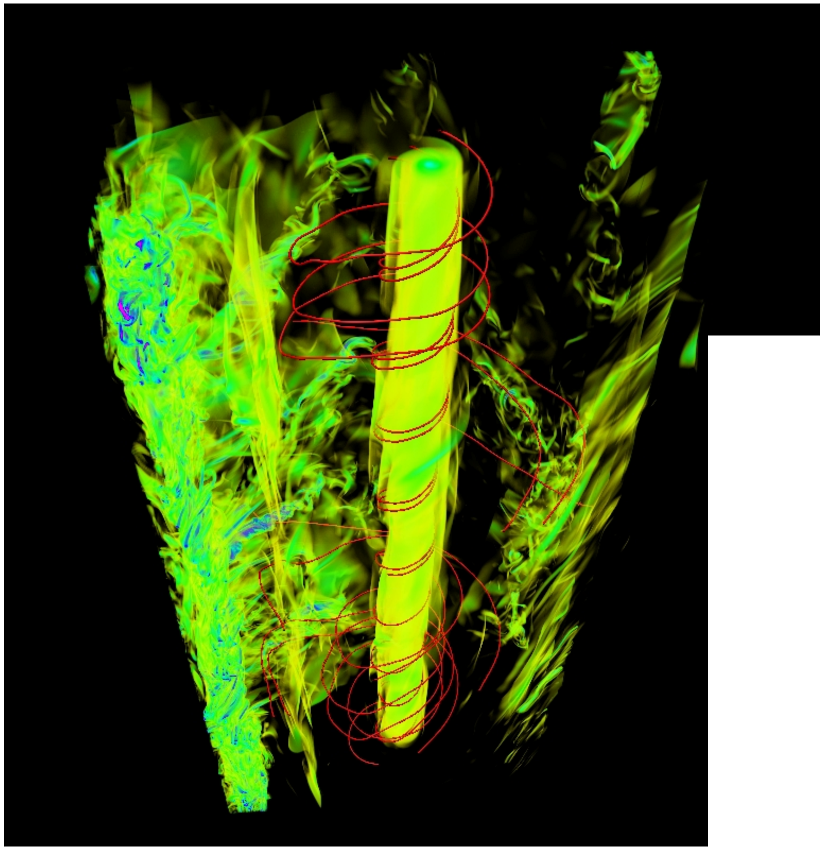
Ro ~ 0.07

Re ~ 24000

NSF Tera-grid

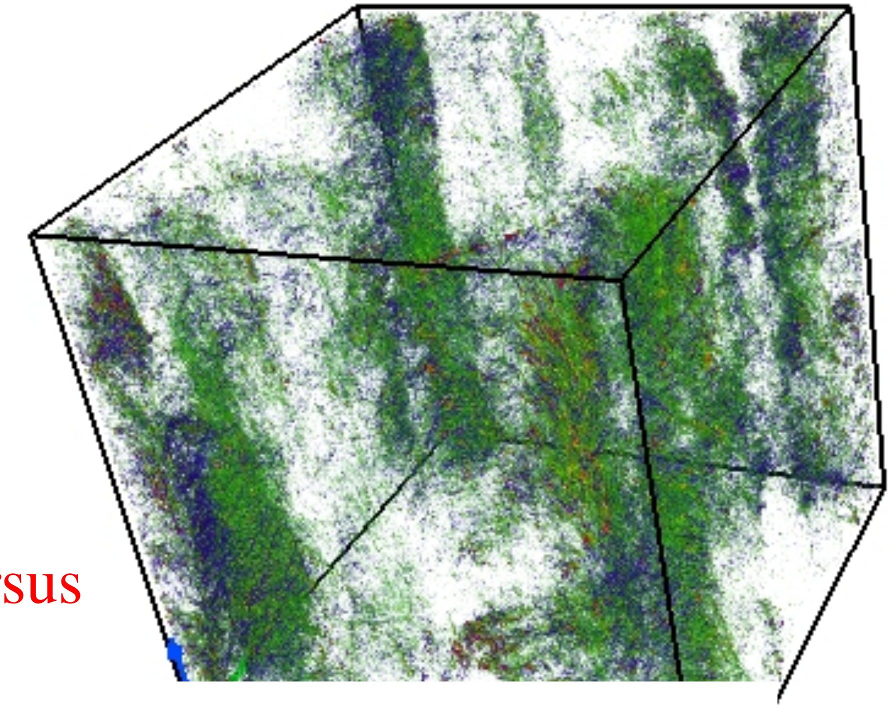
Mininni & AP,
Phys. Fluids 22 (2010)

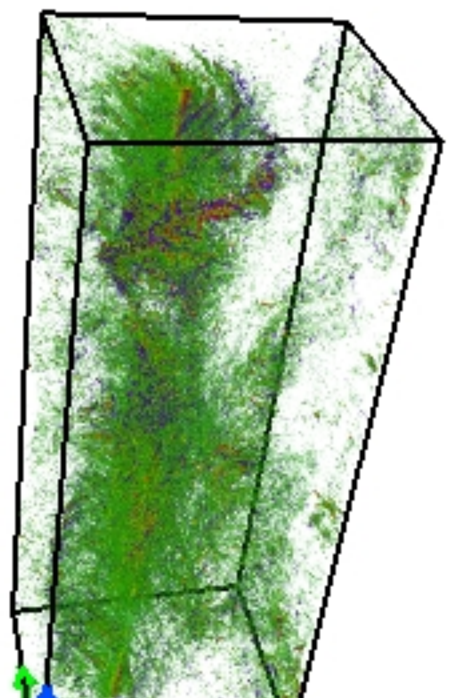
1536^3 grid, k_F=7,
Re=5100,
Ro=0.06,



$L_\Omega \sim l_{min}$

versus





$L_\Omega < l_{min}$

3072^3 grid,
Re~ 24000, Ro~0.07
Mininni et al., 2011

Summary of results

- In the presence of **helicity and rotation**, the direct transfer to small scales is dominated by the helicity cascade and the energy cascade to small scales is quenched since it undergoes an inverse cascade to large scales
- This provides a *``small'' parameter* for the problem (the normalized ratio of energy to helicity fluxes), besides the small Rossby number

Summary of results

- In the presence of **helicity and rotation**, the direct transfer to small scales is dominated by the helicity cascade and the energy cascade to small scales is quenched since it undergoes an inverse cascade to large scales
- This provides a ``*small" parameter* for the problem (the normalized ratio of energy to helicity fluxes), besides the small Rossby number

- The direct energy cascade is non-intermittent and conformal invariant *(when properly analyzed using* $<\omega_z>_z$*).*

- The intermediate (larger) small scales follow a law predicted by a wave-induced helical model, with a possible breaking of universality and with a *possible* $e \leq 7/3$, $h \geq 5/3$ *limit*

- The flow produces strong organized long-lived columnar helical structures, Beltrami Core Vortices, at scales slightly smaller than the injection scale, *with also a growth of structures at large scales*

- Isotropy & K41 recover at scales smaller than the Zeman scale, if resolved

Some questions

- Can helicity help in interpreting results from laboratory experiments and atmospheric data?

- What is the **large Reynolds number limit** at fixed rotation?

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- Can helicity help in interpreting results from laboratory experiments and atmospheric data?
- What is the **large Reynolds number limit** at fixed rotation?
- Is there a change of dynamics in terms of the **relative alignment between velocity and vorticity**? *(role of polarization anisotropy)*
- Is the direct energy cascade different in
  - the non-helical case,
  - in the moderately helical case, and
  - the (presumably) self-similar energy inverse cascade to large scales?

Some more questions

* Does the kind of imposed forcing at large scale play a role?

  Helical or not: yes. *Random vs. deterministic?* *2D vs 3D?*

- What happens locally in space? *What structures transfer to small vs. large scales? What are the Beltrami Core Vortex structures made of? How do they evolve and interact to lead to both a direct and an inverse cascade?*

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Helical or not: yes. *Random vs. deterministic?* *2D vs 3D?*

- What happens locally in space? *What structures transfer to small vs. large scales? What are the Beltrami Core Vortex structures made of? How do they evolve and interact to lead to both a direct and an inverse cascade?*

- Universality? *p/2 vs. 3p/4 vs. 2p/3 vs. ???*

- Modeling:
  - isotropic vs. anisotropic *(perhaps not)*?
  - Need / expression of helical contribution to transport coefficients in models?

- What happens when helicity is neither zero nor maximal?

GHOST

- **G**eophysical **H**igh **O**rder **S**uite for **T**urbulence

- *Community code*
- Pseudo spectral, incompressible Navier-Stokes (including rotation, passive scalar & Boussinesq); magnetic fields (MHD with Hall term). It also includes some LES (``alpha'' filtering & variants; helical spectral model)

- Linearly parallelization up to 30,000 processors using hybrid Open-MP / MPI *(Mininni et al. 2011, Parallel Computing 37, 2011)*

- Community Data: *2048^3 forced Navier-Stokes turbulence with and without helicity; 1536^3 and 3072^3 helically forced rotating turbulence; 1536^3 decaying turbulence with a magnetic field, 2048^3 MHD with TG symmetries. 3D visualization with VAPOR NCAR freeware.*

Thank you for your attention!