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Relaxation of Galerkin-Truncated Gyrokinetic, and back to Fluids

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dedicated to **Frisch, Uriel**

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Outline

- 2D Gyrokinetic Absolute Equilibrium (GKAE): Theory
  - GKAE in **k**-v and k-v_i:
  - GKAE in **k**-b and k-b_i: Fourier-Hankel space
  - GKAE in **k**-z and k-z_(k): Fourier-Bessel space

- Simulations of Galerkin-truncated gyrokinetics
  - Statistical dynamics with moderate large number of modes
  - Statistical dynamics with large number of modes: long time v.s. infinite time
  - Validness and choice of Gibbs statistics

- Two-fluid Magnetohydrodynamics (TFMHD) absolute equilibrium
  - "Trivial" calculations but nontrivial physics: helicities
  - Further considerations: Bessel space for TFMHD in cylindrical geometry? starting point for two-specie GKAE? "(self-)helicity" in GK?..

- Further Discussions
  - Relevance to Turbulence
  - Helicities of Plasmas…

work/thoughts underway

Due to (working/presenting-)time limit…

Why GKAE

- Magnetized plasma turbulence is highly complicated with nontrivial mathematical structures (GK) and rich physics, very challenging for theoretical and numerical approaches in which baselines/benchmarks are wanted

- Successful experiences with absolute equilibrium in understanding turbulence transfers and large- & small-scale properties

- GK is quite a useful but less explored model for magnetized plasmas with sufficient complexity and tractability

hard to solve directly so to find some relevant things

Turbulence … , not to mention that AE itself is interesting and useful

A mixture of equilibrium and “inertial” fluctuations:
Not “black or white”

Skandinaviska halvön

Equilibrium and non-equilibrium fluctuations are self-organized in a particular way

persistence

What T.-D. Lee did for Euler case

- Showed Liouville theorem
  - Appropriately tracked the dynamics of the phase space spanned by the real and imaginary parts of the Fourier modes
- Given ("rugged") conserved quantities
  - Knew only energy (for 3D)…
- Concluded energy equipartition
  - calculations

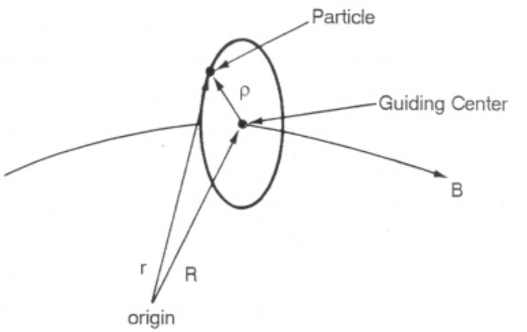
One-line introduction to GK

GK games:

$$\frac{\partial f_s}{\partial t} + \mathbf{v}\cdot\frac{\partial f_s}{\partial \mathbf{r}} + \frac{q_s}{m_s}(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c})\cdot\frac{\partial f_s}{\partial \mathbf{v}} = C[f_s]$$

A physical space averaging method that "solved" the subgrid closure problem:

$$\langle\Psi\rangle_{\mathbf{R}} = \frac{\int \Psi(\mathbf{r})\delta(\mathbf{r}_{\|}-\mathbf{R}_{\|})\delta[|\mathbf{r}_{\perp}-\mathbf{R}_{\perp}|-\rho(\mathbf{R})]d^3\mathbf{r}}{\int \delta(\mathbf{r}_{\|}-\mathbf{R}_{\|})\delta[|\mathbf{r}_{\perp}-\mathbf{R}_{\perp}|-\rho(\mathbf{R})]d^3\mathbf{r}=2\pi\rho(\mathbf{R})}$$



Btw: all integrals are definite integrals

$$f = F_0 \exp\{-q\varphi/T\} + h + h.o.t.$$
$$g = h - F_0 q\langle\varphi\rangle_R/T$$

Electrostatic GK (GK Vlasov-Poisson with uniform background magnetic field) in **R**-**v**-t:

$$\frac{\partial g}{\partial t} + v_{\|}\frac{\partial g}{\partial z} + \left(\hat{\mathbf{z}}\times\frac{\partial\langle\phi\rangle_{\mathbf{R}}}{\partial\mathbf{R}}\right)\cdot\frac{\partial g}{\partial\mathbf{R}} = -v_{\|}\frac{\partial\langle\phi\rangle}{\partial z}F_0$$

Then we will practice the Lee paradigm for gyrokinetics

2D GK model in **X**-v-t

Guiding center dynamics due to gyro-averaged **E** X **B** drift across the magnetic field:

$$\frac{\partial g}{\partial t} + \mathbf{z} \times \nabla \langle \varphi \rangle_{\mathbf{R}} \cdot \nabla g = 0,$$

with the quasi-neutrality condition

$$2\pi \int v dv \langle g \rangle_{\mathbf{r}} = \alpha\varphi - \Gamma\varphi.$$

Invariants: $\mathrm{F}(v) \;=\; \int d^2\mathbf{R} F[g(v)]$

(, especially $\mathrm{G}(v) \;=\; \int \frac{d^2\mathbf{R}}{2V} g^2$,)

$$\mathrm{E} \;=\; \int d^2\mathbf{r}[(1\,+\,\tau)\varphi^2 \;-\; \varphi\Gamma\varphi]$$

Exact GKAE in k-v

2D GK model in **k**-v-t:

$$\partial_t \hat{g}(\mathbf{k}, v) = \hat{\mathbf{z}} \times \sum_{\mathbf{p}+\mathbf{q}=\mathbf{k}} \mathbf{p} J_0(pv) \hat{\varphi}(\mathbf{p}) \cdot \mathbf{q}\, \hat{g}(\mathbf{q}, v)$$

$$\hat{\varphi}(\mathbf{k}) = \beta(\mathbf{k}) \int v dv J_0(kv) \hat{g}(\mathbf{k}, v),$$

$$\beta(k) = \frac{2\pi}{\tau + 1 - \hat{\Gamma}(k)}$$

✓Liouville theorem in the space spanned by real and imaginary parts…
✓Quadratic invariants are rugged:

$$\mathrm{G}(v) = \int \frac{d^2\mathbf{R}}{2V} g^2 = \tilde{\sum} \frac{1}{2} |\hat{g}(\mathbf{k}, v)|^2$$

$$\mathrm{E} = \int \frac{d^2\mathbf{r}}{2V} [(1+\tau)\varphi^2 - \varphi\Gamma\varphi] = \tilde{\sum} \frac{\pi}{2\beta(\mathbf{k})} |\hat{\varphi}(\mathbf{k})|^2$$

where $\tilde{\bullet}$ means operating only on a subset, say, 0<|**k**|<K

Canonical equilibrium distribution:

$$Z^{-1}\exp\{-\mathcal{S}\},\ Z = \int D\sigma \exp\{-\mathcal{S}\}, \text{ with } \mathcal{S} = \int \alpha(v)\mathrm{G}(v)dv + \alpha_0 \mathrm{E}$$

and that

$$\mathcal{S} = \frac{1}{2}\tilde{\sum}_{\mathbf{k}}\left\{ \int \alpha(v)|\hat{g}(\mathbf{k},v)|^2 dv + \alpha_0 2\pi\beta(k)\int v dv J_0(kv)\hat{g}(\mathbf{k},v)\int v dv J_0(kv)\hat{g}^*(\mathbf{k},v)\right\}$$

$$= \tilde{\sum}_{\mathbf{p}}\tilde{\sum}_{\mathbf{k}}\frac{1}{2}\iint \hat{g}(\mathbf{p},u)\mathbf{M}(\mathbf{p},\mathbf{k},u,v)\hat{g}^*(\mathbf{k},v)dudv,$$

with "*" denoting the complex conjugate. Clearly, $\mathbf{M}(\mathbf{p},\mathbf{k},u,v) = \delta_{\mathbf{p},\mathbf{k}}C^{\mathcal{I}}(\mathbf{k},u,v)$, with

$$C^{\mathcal{I}}(\mathbf{k},u,v) = \alpha(v)\delta(u-v) + \alpha_0 2\pi\beta(k)vuJ_0(kv)J_0(ku).$$

functional inversion:

$$C(\mathbf{k},u,v) = \frac{\delta(u-v)}{\alpha(v)} - \frac{2\pi\alpha_0\beta(k)uJ_0(uk)vJ_0(vk)}{\alpha(u)\alpha(v)\left[1 + 2\pi\alpha_0\beta(k)\int \frac{x^2 J_0^2(kx)}{\alpha(x)}dx\right]}$$

Remarkable results and subtleties:

the distribution over v of the spectra density (over $\mathbf{k}$) of $g^2(\mathbf{R}, v)$ is

$$\mathrm{G}(\mathbf{k}, v) = 1/\alpha(v)$$

BECAUSE (not yelling at all):

infinitesimal

$$\lim_{u \to v} \langle \hat{g}(\mathbf{k}, u) du \hat{g}^*(\mathbf{k}, v) dv \rangle = dv/\alpha(v) + O[(dv)^2]$$; or,

$$\int \int_{\lim u \to v} C(\mathbf{k}, u, v) T(u, v) du dv = \lim_{\Delta u \to 0} \int dv \int_{v - \Delta u/2}^{v + \Delta u/2} [\frac{\delta(u-v)}{\alpha(v)} + \bar{C}(\mathbf{k}, u, v)] T(u, v) du$$

$$= \int \frac{1}{\alpha(v)} T(v, v) dv + \lim_{\Delta u \to 0} \int \{\bar{C}(\mathbf{k}, v, v) T(v, v) \Delta u + O[(\Delta u)^2]\} dv.$$

Subtleties concerning the 1D v.s. 2D descriptions and the covariance function:

- G(v) was introduced as a 1D density in S: The prescription of discretization, in (numerical) calculations, should be $\delta_{i,j}/m_i \leftrightarrows \delta(u-v)$; and , one should NOT use G(v_i)=(g_i)^2;

- So, don't be scared by the Dirac delta function in C(**k**,u,v) which is a 2D density for convenience of calculations;

Approximation in the sense of discretization/quantization of v

- GK system can be discretized from the beginning (as the present continuum GK codes do) and perform the corresponding calculations: Zhu&Hammett "Gyrokinetic statistical absolute equilibrium and turbulence" *Phys. Plasmas* **17**, 122307--122319 ;

- We can also discretize the final results from the calculations with continuous v

$$c_{i,j}(\mathbf{k}) = \langle \overbrace{\tilde{\hat{g}}(\mathbf{k}, v_i)}^{\boxed{\text{or } \tilde{\hat{g}}_{\mathbf{k},i}}} \tilde{\hat{g}}^*(\mathbf{k}, v_j) \rangle$$

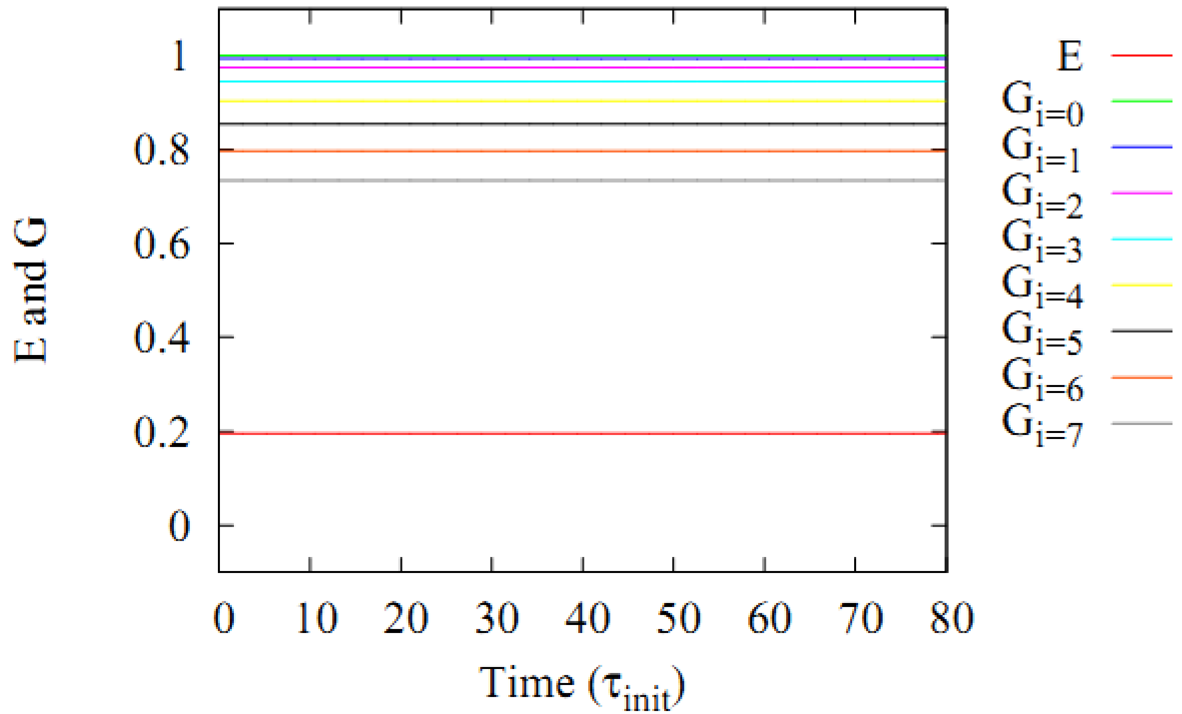
$$= \frac{\delta_{i,j}}{\alpha_i} \boxed{- \frac{\tilde{\alpha}_0 2\pi\beta(k) w_i \alpha_i^{-1} w_j \alpha_j^{-1}}{1 + \tilde{\alpha}_0 2\pi\beta(k) \sum_l w_l^2 \alpha_l^{-1}}}$$

Contributes an extra term for the discretized 1D distribution [c_{i,i}\Delta v for m_i=\Delta v] compared to the 1D continuous distribution G(k,v) and may have significant physics.

The two approaches lead to the same results: Zhu 2010' @ arXiv:1008.0330

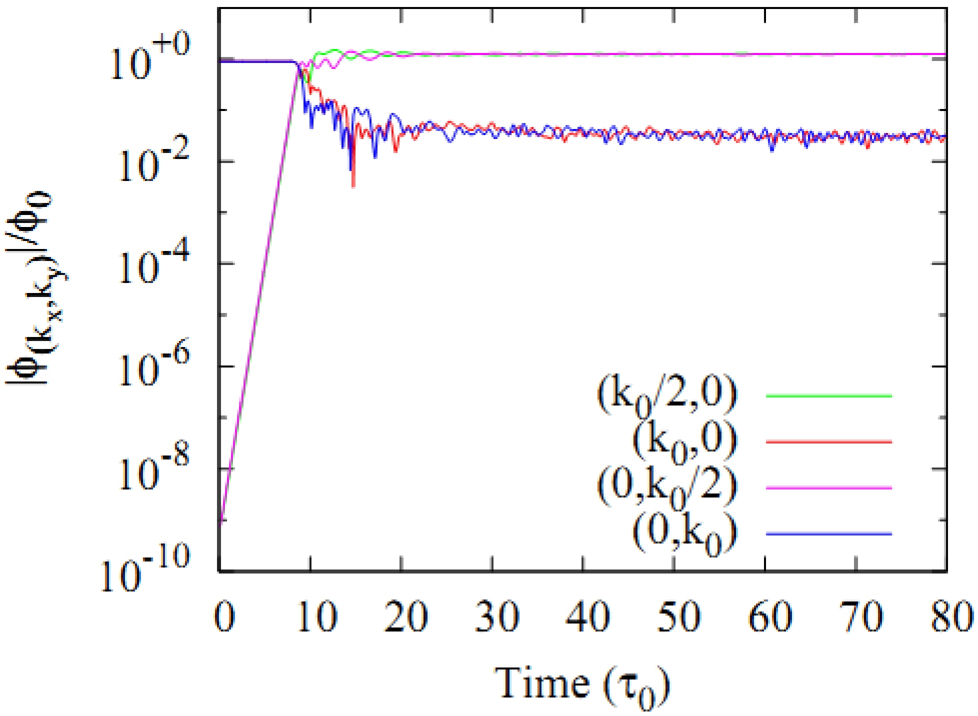
There are lot of turbulence relevance, including zonal flows et al., but let’s look at the

numerical results



Conservation check

Physical and numerical parameters used in the present simulations are $\tau = 0$, $k_0 = \sqrt{2}\rho^{-1}$, the minimum wavenumber, $k_{\min} = k_0/2 = (1/\sqrt{2})\rho^{-1}$, the time step size, $\Delta t = 2.88 \times 10^{-4}\tau_0$, the maximum perpendicular velocity, $v_{\perp,\max} = 5v_t$, the velocity grid numbers, $N_v = 64$, the number of Fourier modes, $(N_x, N_y) = (42, \pm 42)$, in the x and y directions, respectively.



Time histories

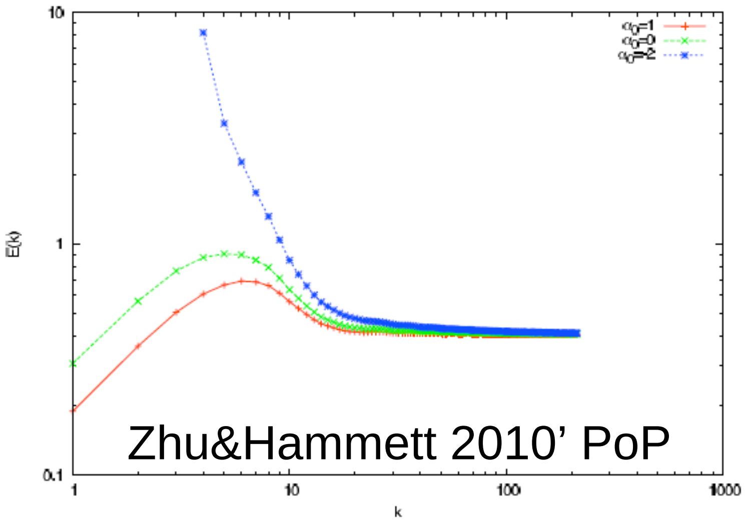
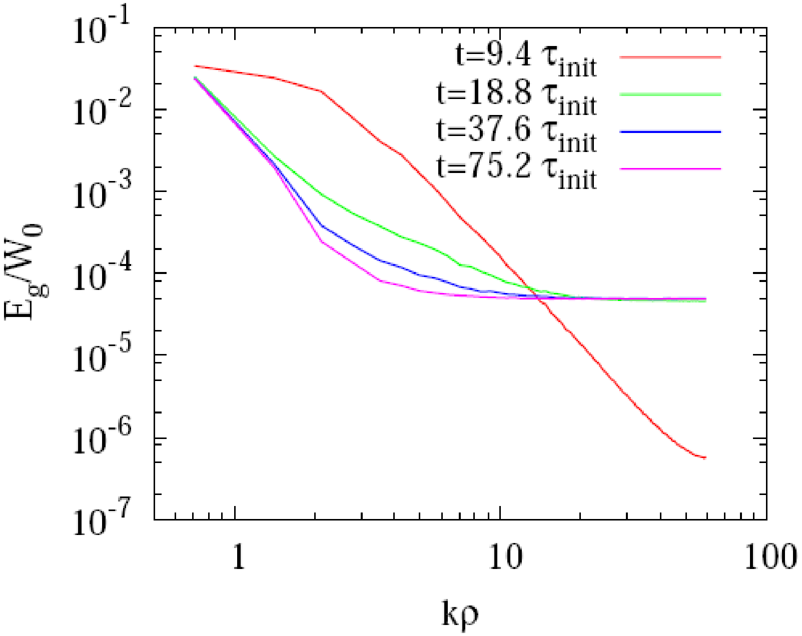
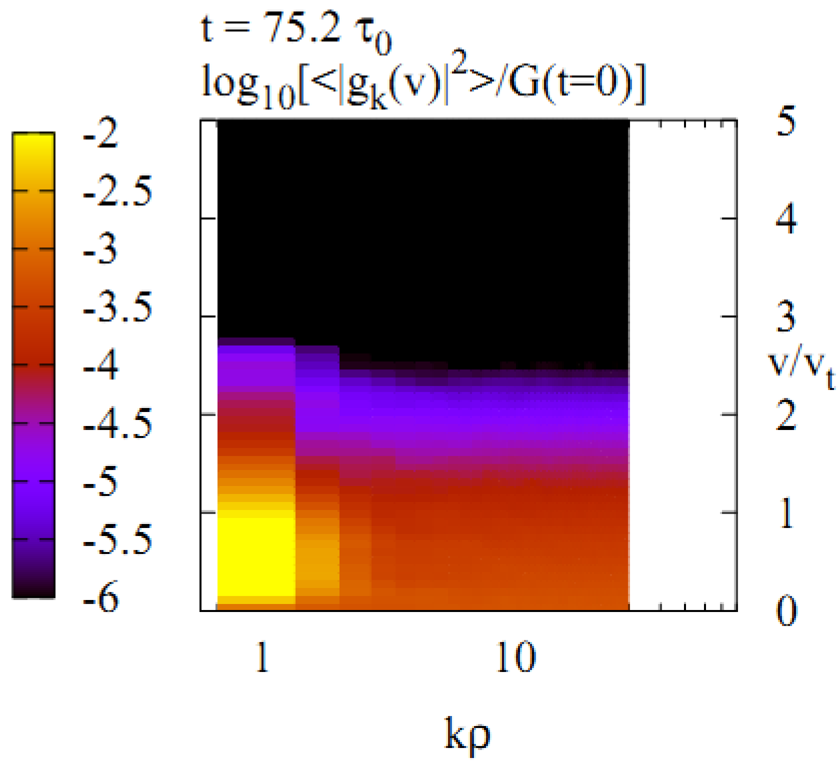


FIG. 1. (Color online) Example spectra for various values of α_0, with $\alpha_i = 10^3 \exp\{v_i^2/2\}$ for $i > 0$, and $\rho_0 = 10^{-1}$, $\tau = 1$. Negative α_0 state can occur that correspond to condensation of most of the energy into the longest wavelength modes.



Evolution of “energy” spectrum



- We can also discretize the final results from the calculations with continuous v

or $\tilde{\hat{g}}_{\mathbf{k},i}$

$$c_{i,j}(\mathbf{k}) = \langle \boxed{\tilde{\hat{g}}(\mathbf{k}, v_i)} \tilde{\hat{g}}^*(\mathbf{k}, v_j) \rangle$$

$$= \frac{\delta_{i,j}}{\alpha_i} \boxed{- \frac{\tilde{\alpha}_0 2\pi\beta(k) w_i \alpha_i^{-1} w_j \alpha_j^{-1}}{1 + \tilde{\alpha}_0 2\pi\beta(k) \sum_l w_l^2 \alpha_l^{-1}}}$$

Contributes an extra term for the discretized 1D distribution [c_{i,i}\Delta v for m_i=\Delta v] compared to the 1D continuous distribution G(k,v) and may have significant physics.

Exact GKAE in k-b

Hankel transform: $\breve{\hat{g}}(\mathbf{k}, b) = \int v dv J_0(bv) \hat{g}(\mathbf{k}, v)$

$$
\begin{aligned}
E &= \sum_{\mathbf{k}} \beta(\mathbf{k}) \int \delta(b - k) |\breve{\hat{g}}(\mathbf{k}, b)|^2 db \\
W &= \sum_{\mathbf{k}} \int |\breve{\hat{g}}(\mathbf{k}, b)|^2 b db
\end{aligned}
$$

The corresponding canonical distribution for the absolute equilibrium:

$$\sim \exp\{-\gamma_E \tilde{\sum}_{\mathbf{k}} \beta(\mathbf{k}) \tilde{\int} \delta(b - k) |\breve{\hat{g}}(\mathbf{k}, b)|^2 db - \gamma_W \tilde{\sum}_{\mathbf{k}} \tilde{\int} |\breve{\hat{g}}(\mathbf{k}, b)|^2 b db\}$$

which gives (clean result for telling stories about negative temperature et al.)

$$\langle |\breve{\hat{g}}(\mathbf{k}, b)|^2 \rangle = \frac{1}{\gamma_W b + \gamma_E \beta(k) \delta_{k,b}}$$

where $\delta_{k,b}$ acquires 1 for $k = b$, and 0 otherwise.

“Useless” discretization of b

b can be discretized into b_i with a lattice parameterized by \Delta b

$$\langle|\check{\hat{g}}(\mathbf{k}, b_i)|^2\rangle = \frac{1}{\Gamma_W b_i \Delta b + \Gamma_E \beta(k) \delta_{i_k, i}}$$

$(k$ falls into the lattice of b indexed by $i_k)$

$\Gamma_W \Delta b \to \gamma_W$ and $\Gamma_E \to \gamma_E$, and, $\delta_{i_k,i} \to \delta_{k,b}$: $\delta_{i_k,i} \to \delta_{k,b}$ can be understood as $\delta_{i,i_k}/\Delta b \to \delta(k-b)$ and $\delta(k-b)\Delta b \to \delta_{k,b}$, in the continuous limit $\Delta b \to 0$.

$$\langle|\check{\hat{g}}(\mathbf{k}, b)|^2\rangle = \frac{1}{\gamma_W b + \gamma_E \beta(k) \delta_{k,b}}$$

V Bounded case: use Bessel series and do truncation

The idea: velocity is bounded somehow … Fourier-Bessel series…

$$\hat{g}(\mathbf{k}, v) \;\; = \sum_z 2V^{-2}[J_1(z)]^{-2}\breve{\hat{g}}(\mathbf{k}, z)J_0(zv/V)$$

Using **very special k dependent upper bound V(k)** as in G. Plunk and T. Tatsuno, Phys. Rev. Lett. **106**, 165003 (2011) ("LPT"):

for each k there is some zero z_k of J_0 such that $k = \frac{z_k}{V(\mathbf{k})}$.

$\tilde{E} = \tilde{\sum}_{\mathbf{k}}\tilde{\sum}_z \pi\beta(\mathbf{k})\delta_{z,z_k}|\breve{\hat{g}}(\mathbf{k}, z)|^2$ (with $z_k = kV(\mathbf{k})$, and, δ_{z,z_k} acquires 1 for $z = z_k$ and 0 otherwise,) $\tilde{W} = \tilde{\sum}_{\mathbf{k}}\tilde{\sum}_z 2\pi^2 J_1^{-2}(z)|g(\mathbf{k}, z)|^2$, where $\tilde{\bullet}$ means operating only $\cdots\cdots$

If V(k) are **not that specially chosen**, say, V(k)=V is taken uniformly for all k, then E is not rugged and the absolute equilibrium is a special case of the result from the above very special treatment.

Absolute equilibria in k-z space

The corresponding canonical distribution for the absolute equilibrium:

$$\sim \ \exp\{-\alpha_E \tilde{E} - \alpha_W \tilde{W}\}$$

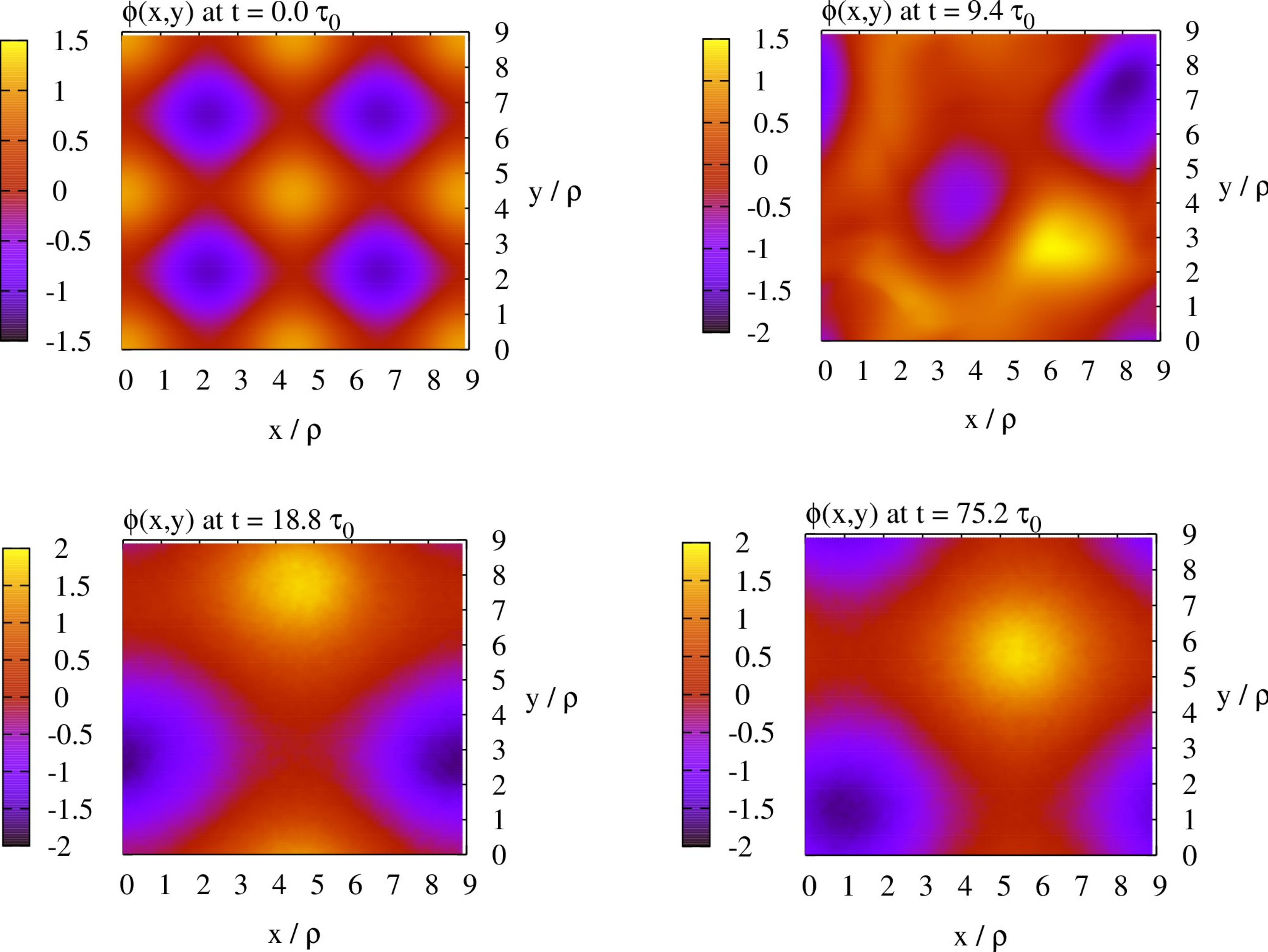
gives the spectral density of $\tilde{E}$ and $\tilde{W}$:

$$E(\mathbf{k}) \triangleq \langle \pi\beta(\mathbf{k}) |\breve{\tilde{g}}(\mathbf{k}, z_k)|^2 \rangle = \frac{\beta(\mathbf{k})}{2\alpha_E \beta(\mathbf{k}) + 4\pi\alpha_W J_1^{-2}[kV(\mathbf{k})]}$$

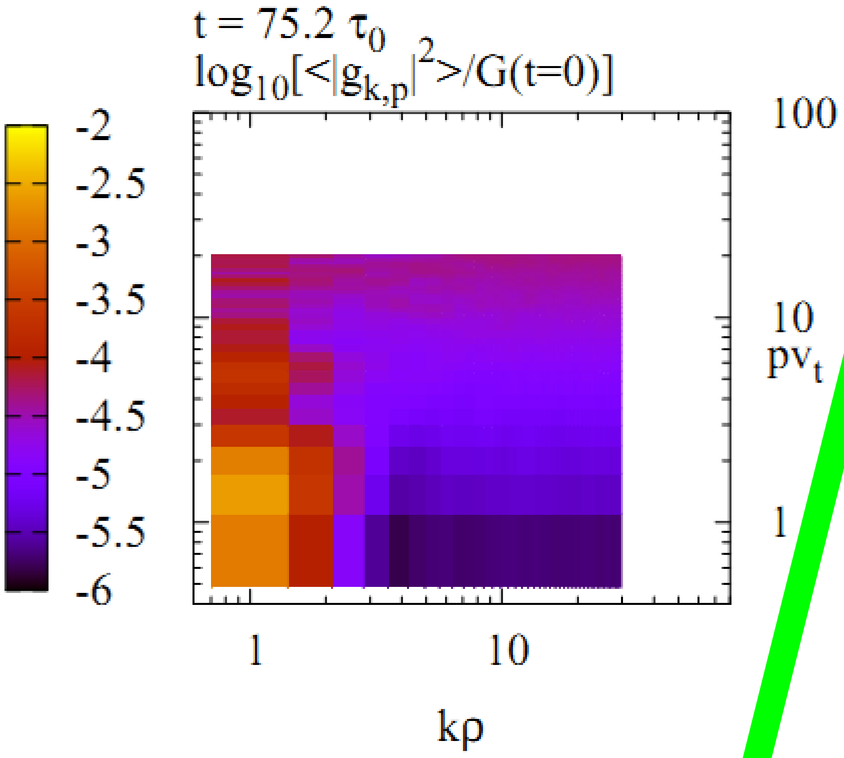
with $z_k = kV(\mathbf{k})$ and

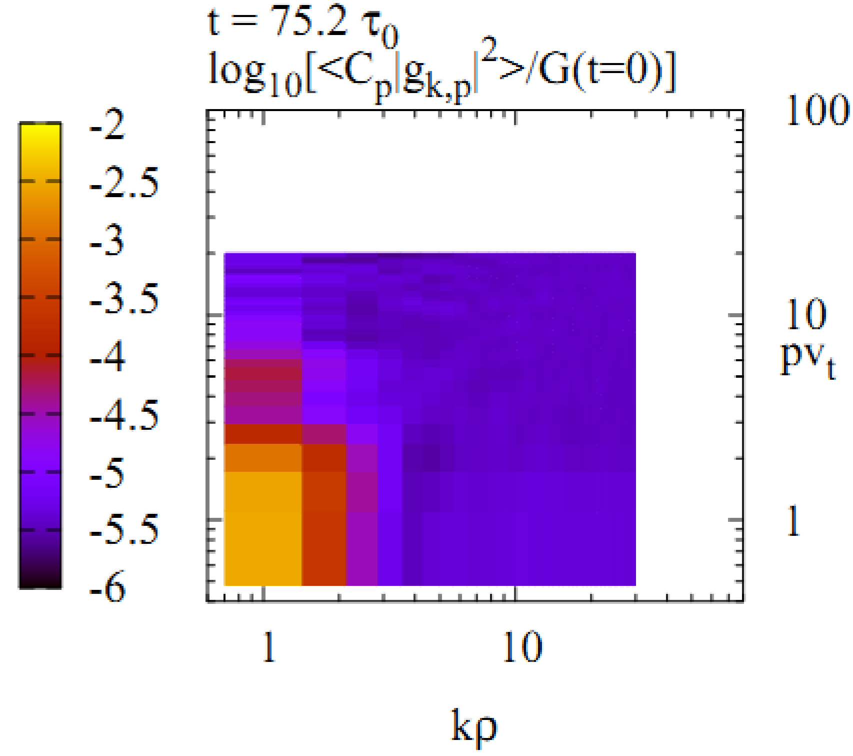
$$W(\mathbf{k}, z) \triangleq \langle 2\pi^2 J_1^{-2}(z) |\breve{\tilde{g}}(\mathbf{k}, z)|^2 \rangle = \frac{2\pi}{2\alpha_E \beta(\mathbf{k}) J_1^2(z) \delta_{z, z_k} + 4\pi\alpha_W}.$$

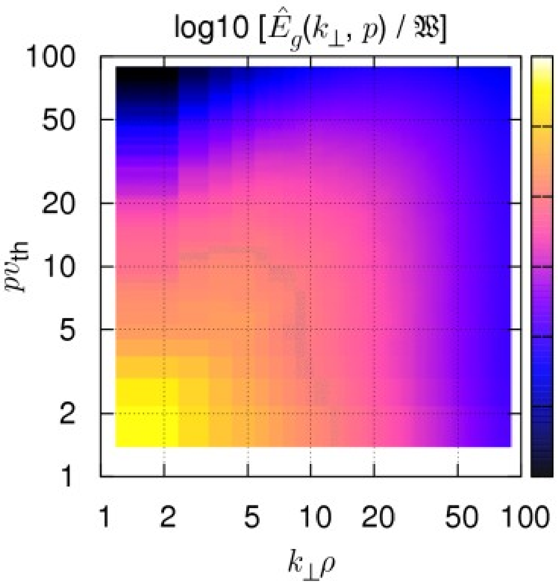
\alpha_E=0 corresponds to the case with E not rugged



φ(x,y) at t = 0.0 τ0
φ(x,y) at t = 9.4 τ0
φ(x,y) at t = 18.8 τ0
φ(x,y) at t = 75.2 τ0
x / ρ
y / ρ







arXiv:1003.3933v2 [physics.plasm-ph]

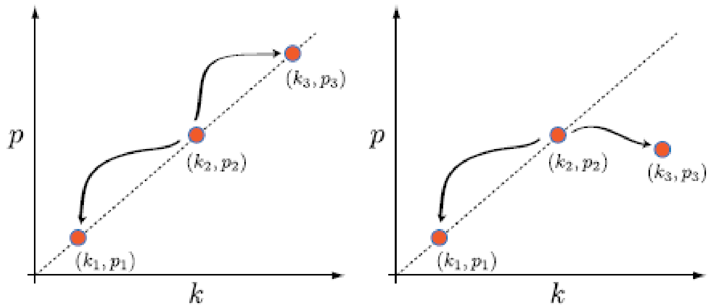
$$W(\mathbf{k},z) \triangleq \langle 2\pi^2 J_1^{-2}(z)|\breve{\hat{g}}(\mathbf{k},z)|^2\rangle = \frac{2\pi}{2\alpha_E\beta(\mathbf{k})J_1^2(z)\delta_{z,z_k}+4\pi\alpha_W}.$$

$$\langle|\breve{\hat{g}}(\mathbf{k},b)|^2\rangle = \frac{1}{\gamma_W b+\gamma_E\beta(k)\delta_{k,b}}$$

where $\delta_{k,b}$ acquires 1 for $k=b$, and 0 otherwise.

Symmetry v.s. asymmetry

The calculated absolute equilibria seem to be ready to explain many aspects of the numerical results of LPT (“p” here is our “b”):



(a) I: Fjørtoft-type (b) II: Kinetic-type

FIG. 1 (color online). Constrained energetic transitions involving three scales. The dotted diagonals indicate $k = p$.

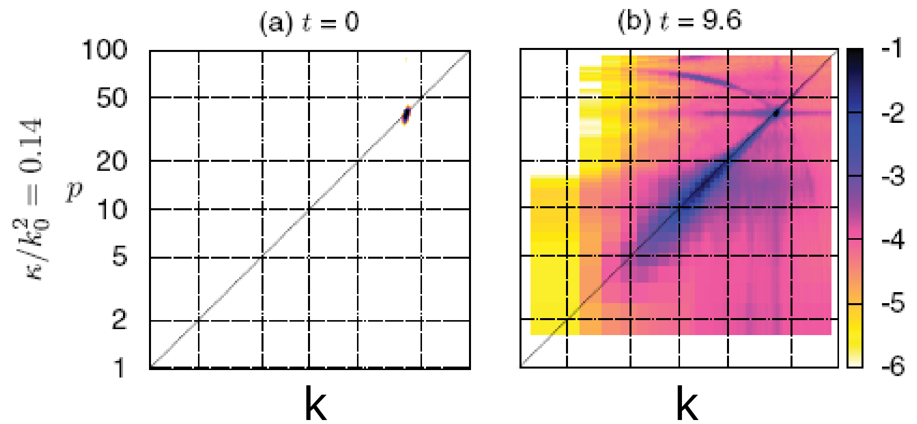
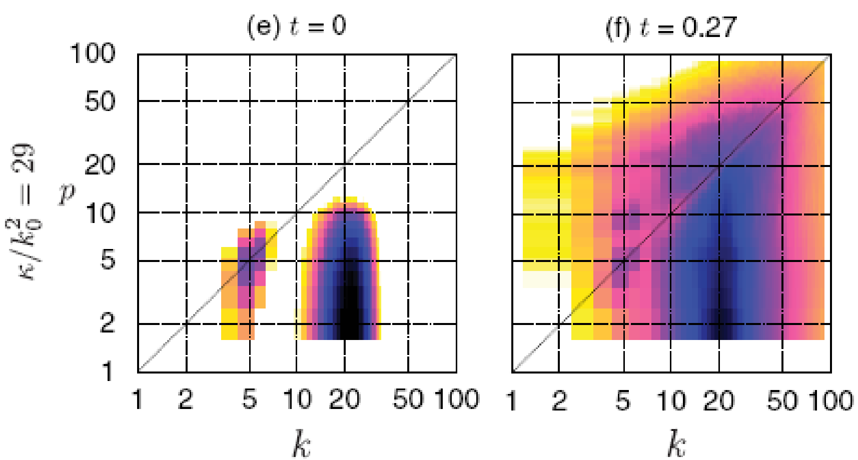
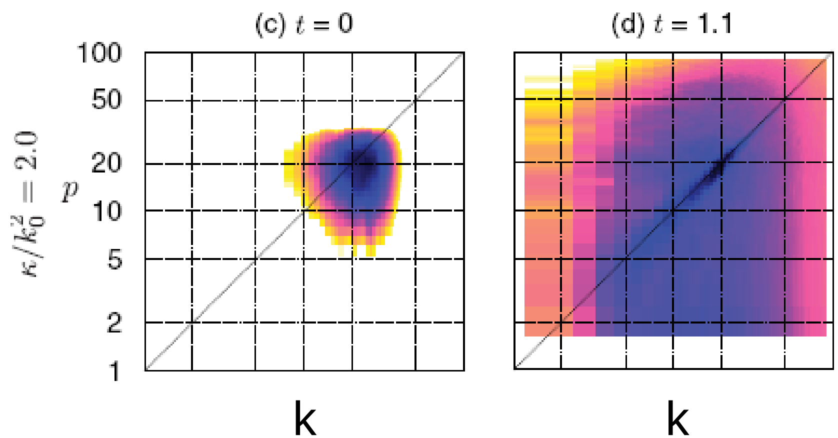


FIG. 2 (color online). Example spectral distributions $\log_{10}[W(k, p)/W]$ and initial evolution. Diagonals marked by dotted lines.





Courtesy http://arxiv.org/abs/1007.4787v3

Sub-summary

- We have four versions of GKAE: 1) k-v, 2) k-v_i, 3) k-b, 4) k-z; all are good and give some similar results;
- K-v and k-b are exact and mathematically elegant; k-v_i is best for numerical validation of the current continuum codes; k-b is most convenient for physical discussion; k-z is most appropriate for commenting the LPT and for validating the complete (pseudo-)spectra (both X -> k and v -> b) codes (have not appeared yet);
- Comparisons among themselves and to other statistical mechanics may be helpful for…benchmarks for gyrofluid?

We need the corresponding precise statistical solutions to different treatments of the same system.

Helicities in Two-fluid MHD

Why Two-fluid MHD (TFMHD)

eulerS+Maxwell's

- Our object is not only to understand some statistical properties of some particular model but the statistical dynamics of plasmas whose description is accomplished by various models;

- Two-fluid MHD is needed for good understanding of reconnection, small-scale plasma waves and turbulence;

- Some issues, such as the effects and roles of various helicities, in MHD, Hall MHD et al. can be further clarified in such a more general model. ……

Starting from the Boltzmann equations

$$(\partial_t + \mathbf{u}_i \cdot \nabla)\mathbf{u}_i = +\frac{e}{m_i}\left(\mathbf{E} + \frac{1}{c}\mathbf{u}_i \times \mathbf{B}\right) - \nabla p_i + \nu_i \Delta \mathbf{u}_i - \frac{1}{\tau_i}(\mathbf{u}_i - \mathbf{u}_e), \quad \nabla \cdot \mathbf{u}_i = 0,$$

$$(\partial_t + \mathbf{u}_e \cdot \nabla)\mathbf{u}_e = -\frac{e}{m_e}\left(\mathbf{E} + \frac{1}{c}\mathbf{u}_e \times \mathbf{B}\right) - \nabla p_e + \nu_e \Delta \mathbf{u}_e - \frac{1}{\tau_e}(\mathbf{u}_e - \mathbf{u}_i), \quad \nabla \cdot \mathbf{u}_e = 0,$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0 \qquad \nabla \cdot \mathbf{E} = \frac{\varrho_c}{\varepsilon_0}$$

$$\frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} - \nabla \times \mathbf{B} = -\mu_0 \sum_{\mathbf{s}} \frac{\mathbf{q_s}}{\mathbf{m_s}} \rho_{\mathbf{s}} \mathbf{u_s} \qquad \nabla \cdot \mathbf{B} = 0.$$

Some fundamentals and techniques in two-fluid MHD

- Manuel Nunez 2008': proves "a theorem of existence and uniqueness of solutions for a finite time by means of a fixed point argument in an appropriate functional setting;"
- Greg Eyink 2009': extends Contantin-Iyer formulation for TFMHD – "Stochastic line motion and stochastic flux conservation for nonideal hydromagnetic models;"
- numerical methods to solve them in various situations;
- K. Avinash&J.B. Taylor, 1991'; L. C. Steinhauer&A. Ishida, 1997' on selective decay argument for relaxed states with plasma flow:

$$E = \frac{1}{2}\sum_{\mathbf{k}}[m_a n_a \vec{u}_a(\vec{k})\cdot\vec{u}_a(\vec{k})^* + \vec{B}(\vec{k})\cdot\vec{B}(\vec{k})^* + \vec{E}(\vec{k})\cdot\vec{E}(\vec{k})^*], \quad (a = i, e)$$

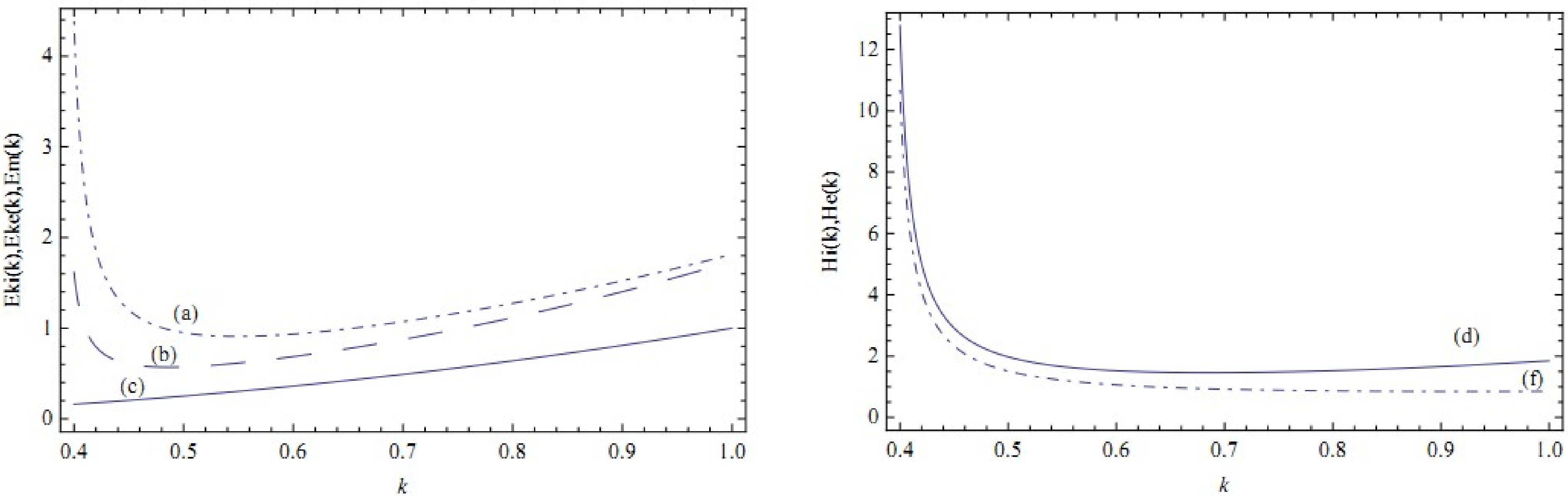
m_\alpha \omega_{g\alpha}

$$H_a = \frac{1}{2}\int d^3x(m_a\vec{u}_a + q_a\vec{A})(m_a\vec{w}_a + \boxed{q_a\vec{B}}).$$

"spin"? "gyrofluid"?

$$[\ H_i - H_e = (m_i + m_e)e\int \vec{u}\cdot\vec{B}dV + \int (m_i{}^2\vec{u}_i\cdot\vec{w}_i - m_e{}^2\vec{u}_e\cdot\vec{w}_e)dV\ ]$$

Inverse energy and/or helicity cascade in tfmhd: (re)covering MHD, Hall MHD and EMHD



effect of $H_i - H_e$

with remarks on Uriel and Annick et al.’s work 70/2 years ago

Discussions of two-fluid effects; “?””!”two-species gyrokinetics…

Won’t advertize anything more, unless asked, though we have a thirty-page working manuscript at hand.

Further discussions

- Orally, unformatted

Summary

- Tedious?

- Gorgeous: The paradigm works also for kinetic model(s) and is able to explore physical effects precisely in some particular situations !

- Cautious: AE is not exactly real turbulence !?!?...

- Courageous.

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