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Role of polymers in the mixing of Rayleigh-Taylor turbulence

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Outline of the talk

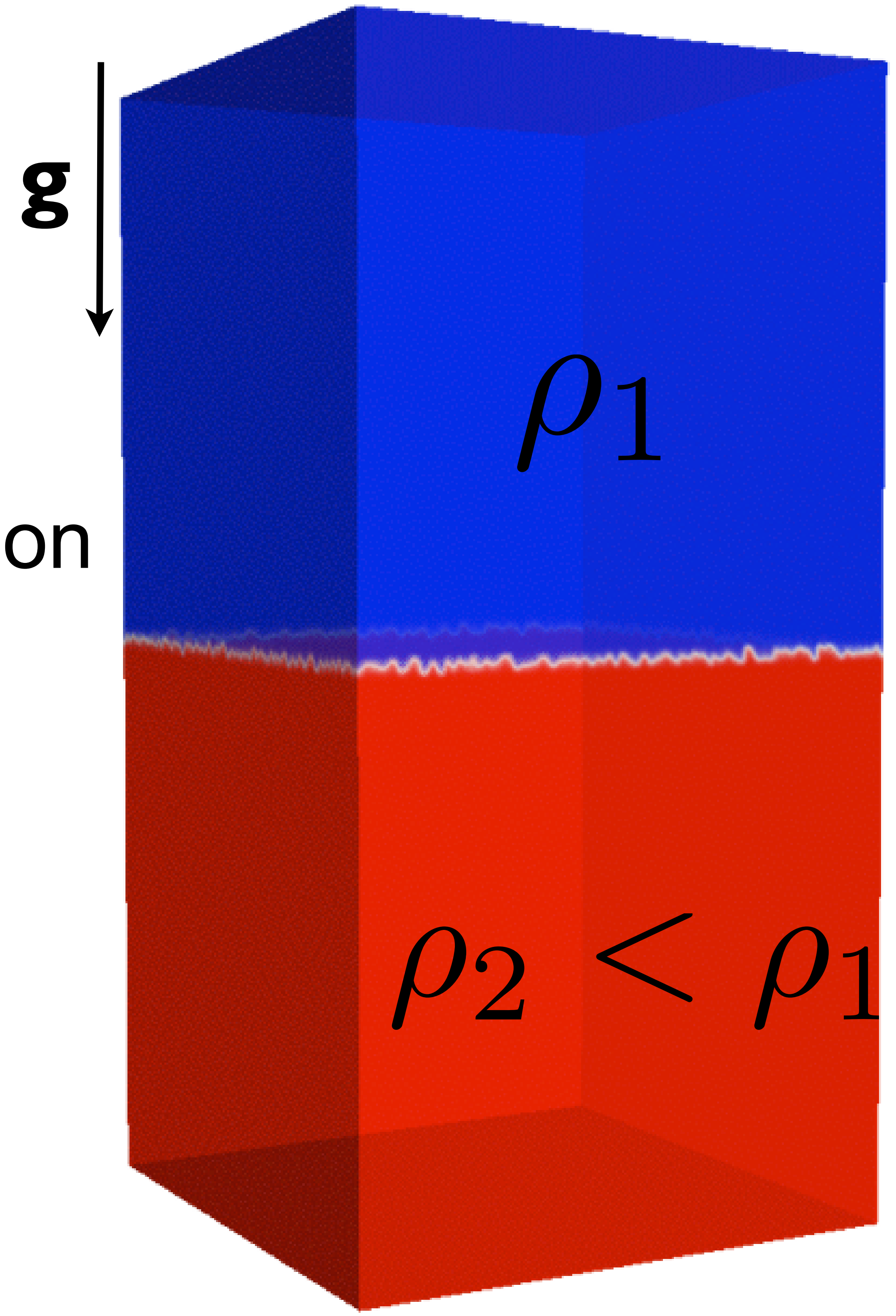
1) Rayleigh-Taylor turbulence: quick view to the Newtonian case

2) Viscoelastic Rayleigh-Taylor turbulence

- *Effect of Polymers on mixing*
- *Heat transfer enhancement*

Rayleigh-Taylor instability

Instability on the interface of two fluids of different densities with relative acceleration.



Rayleigh (1883): unstable stratification in gravitational field
Taylor (1950): generalization to all acceleration mechanisms

Atwood number $\mathrm{A} \equiv \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$

Linear analysis: exponential growth with rate $\sqrt{Agk}$

Rayleigh-Taylor turbulence (2D case)

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \partial \mathbf{u} = -\partial p + \nu \partial^2 \mathbf{u} - \beta \mathbf{g} T$$

$$\partial_t T + \mathbf{u} \cdot \partial T = \kappa \partial^2 T$$

Buoyancy balances inertia at all scales

- *Temperature cascades toward small scales*

M. Chertkov, PRL 91 (2003)

- *Velocity: inverse energy cascade (flux not constant)*

small scale fluctuations follow Bolgiano scaling

$$\delta_r u(t) \sim (Ag)^{2/5} t^{-1/5} r^{3/5}$$

$$\delta_r T(t) \sim \theta_0 (Ag)^{-1/5} t^{-2/5} r^{1/5}$$

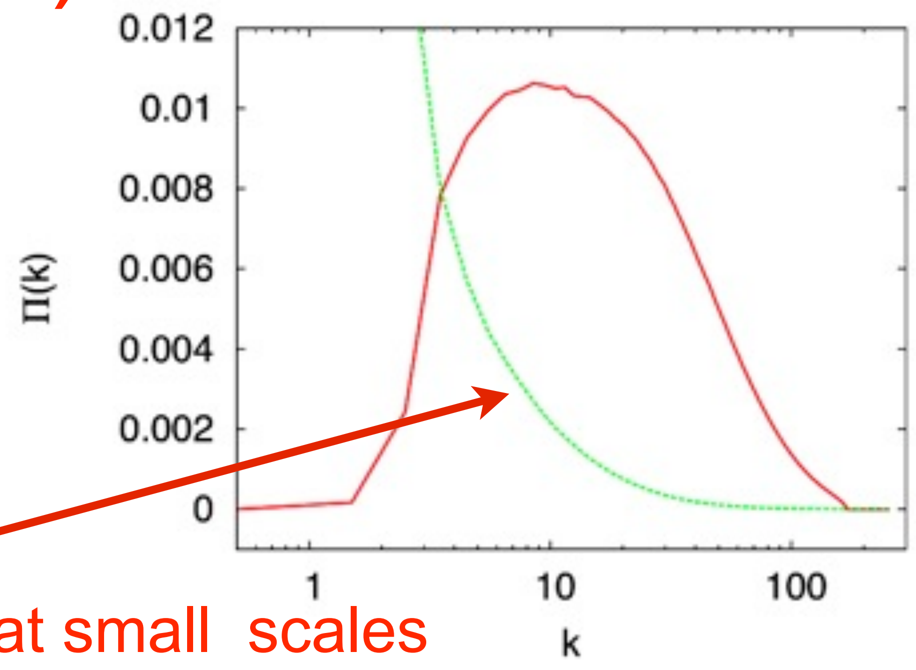
A. Celani, A. Mazzino, L. Vozella, PRL 96 (2006)

Biferale et al, PF 22 (2011)

Rayleigh-Taylor turbulence (3D case)

$$\partial_t \mathbf{u} + \mathbf{u}\cdot\partial\mathbf{u} = -\partial p + \nu\partial^2\mathbf{u} - \beta\mathbf{g}T$$

$$\partial_t T + \mathbf{u}\cdot\partial T = \kappa\partial^2 T$$



Buoyancy is negligible at small scales

- *Temperature cascades toward small scales*

M. Chertkov, PRL 91 (2003)

- *Velocity: direct energy cascade (constant flux)*

Rayleigh-Taylor turbulence (3D case)

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \partial \mathbf{u} = -\partial p + \nu \partial^2 \mathbf{u} - \beta \mathbf{g} T$$

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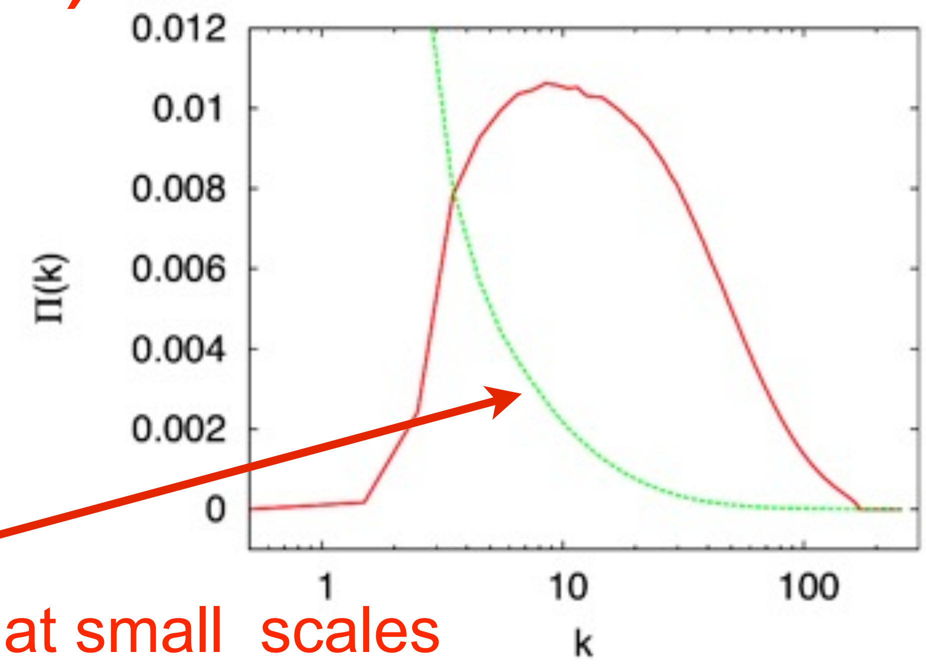


Buoyancy is negligible at small scales

- *Temperature cascades toward small scales*

M. Chertkov, PRL 91 (2003)

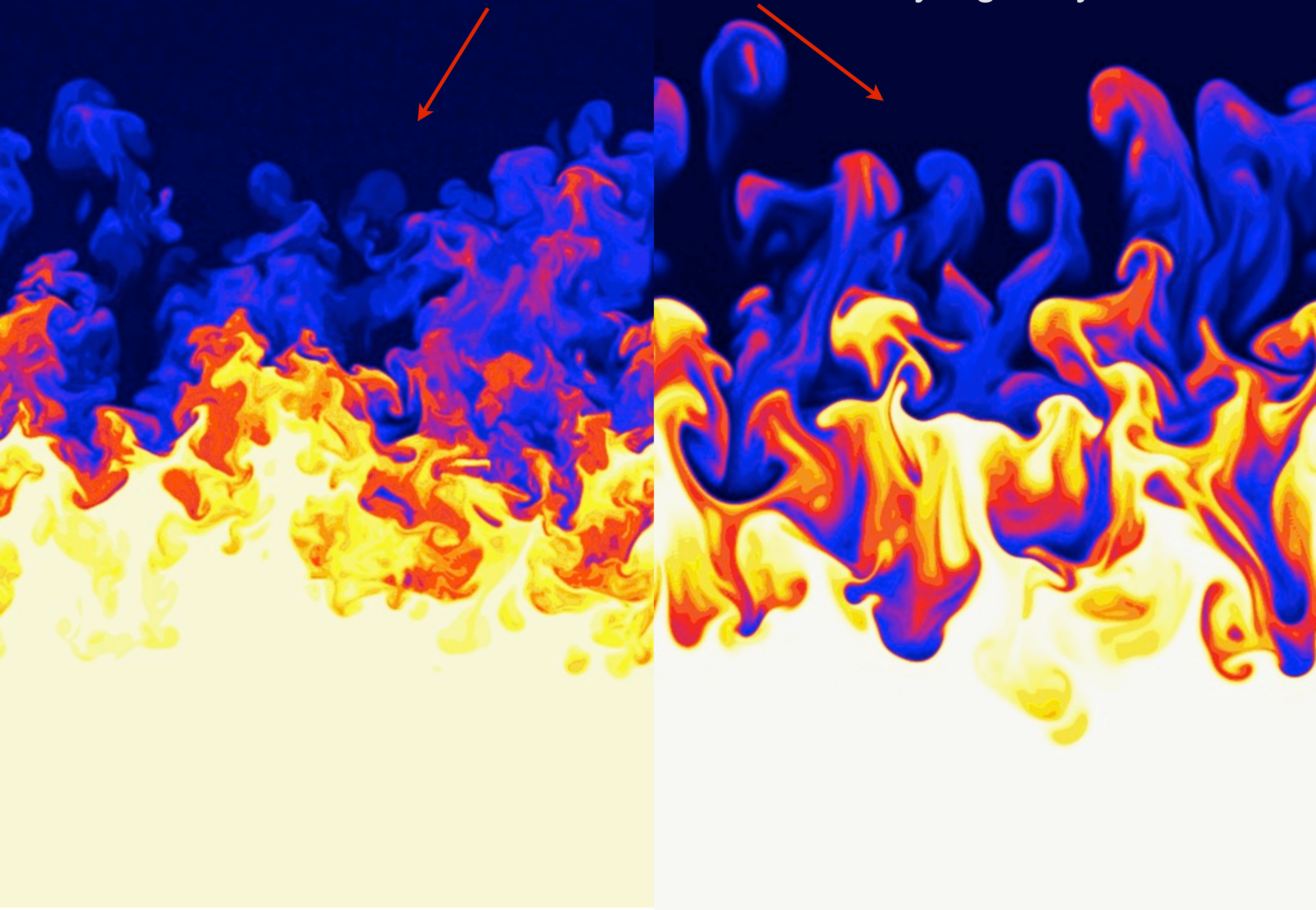
- *Velocity: direct energy cascade (constant flux)*

small scale fluctuations follow Kolmogorov-Obukhov scaling

$$\delta_r u(t) \sim (Ag)^{2/3} t^{1/3} r^{1/3}$$
$$\delta_r T(t) \sim \theta_0 (Ag)^{-1/3} t^{-2/3} r^{1/3}$$

G. Boffetta, A. Mazzino, S. Musacchio, L. Vozella, PRE 79 (2009)

From Newtonian to viscoelastic Rayleigh Taylor



Turbulence in dilute polymer solutions

Drag reduction (Toms 1949)
polymers can reduce the friction drag in a pipe flow up to 80%

Dilute polymer solutions
polymers are stretched by velocity gradients
they can store and dissipate elastic energy

Viscoelastic models: Oldroyd-B
viscous + elastic stress tensor

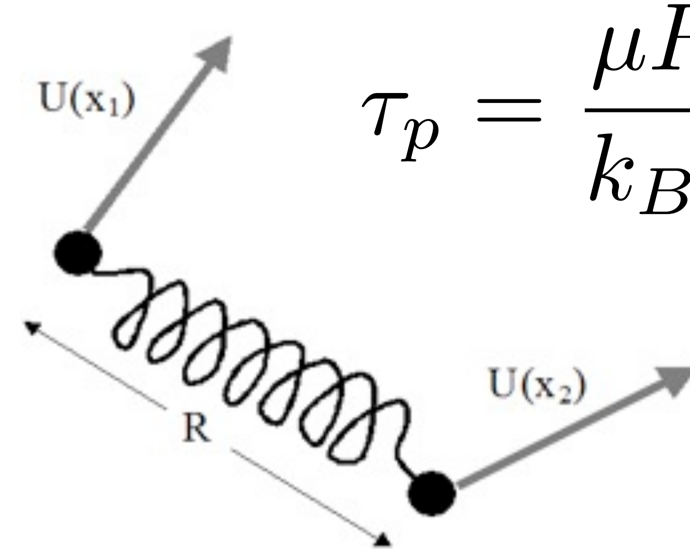
$$\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \nu\Delta\mathbf{u} - \beta\mathbf{g}T + \frac{2\nu\eta}{\tau_p}\nabla\cdot\sigma$$

$$\partial_t T + \mathbf{u}\cdot\nabla T = \kappa\Delta T$$

$$\partial_t \sigma + \mathbf{u}\cdot\nabla\sigma = (\nabla\mathbf{u})^T\cdot\sigma + \sigma\cdot(\nabla\mathbf{u}) - \frac{2}{\tau_p}(\sigma - \mathbb{I})$$

Zimm relaxation time

$$\tau_p = \frac{\mu R_0^3}{k_B T}$$



Conformation tensor

$$\sigma_{\alpha\beta} \equiv \langle R_\alpha R_\beta \rangle$$

Viscoelastic Rayleigh-Taylor: linear stability

Linear stability analysis on the Oldroyd-B $\longrightarrow$ perturbation growth-rate α

$$(\alpha\tau_p)^3 + 2(\alpha\tau_p)^2(1+\nu k^2\tau_p) + \alpha\tau[4\nu(1+\eta)k^2\tau_p - \omega^2\tau_p^2] - 2\omega^2\tau_p^2 = 0$$

zero-shear polymer contribution to total viscosity

Zimm relaxation time

$$\omega = \sqrt{Agk}$$

Relevant asymptotics:

1) $\tau_p \longrightarrow 0$ (Newtonian case) $\alpha_0 = -\nu(1+\eta)k^2 + \sqrt{\omega^2 + [\nu(1+\eta)k^2]^2}$

2) $\tau_p \longrightarrow \infty$ (pure solvent Newt. case) $\alpha_\infty = -\nu k^2 + \sqrt{\omega^2 + \nu^2 k^4} \geq \alpha_0$

Viscoelastic Rayleigh-Taylor: linear stability

From implicit differentiation $\longrightarrow$ $\alpha(\tau_p)$ is monotonic

$$\alpha_\infty \geq \alpha_0$$

Viscoelastic Rayleigh-Taylor: linear stability

From implicit differentiation $\longrightarrow$ $\alpha(\tau_p)$ is monotonic

Because $\alpha_\infty \geq \alpha_0$

$\downarrow$

G. Boffetta, A. Mazzino, S. Musacchio, L. Vozella, JFM 643 (2010)

The instability growth-rate increases with the elasticity

Viscoelastic Rayleigh-Taylor: linear stability

From implicit differentiation → $\alpha(\tau_p)$ is monotonic

Because $\alpha_\infty \geq \alpha_0$

↓

G. Boffetta, A. Mazzino, S. Musacchio, L. Vozella, JFM 643 (2010)

The instability growth-rate increases with the elasticity

?? Similar speed-up in turbulence ??

Heuristics in the “passive case” (2D)

From mean field arguments:

2D RT obeys Bolgiano-Obukhov phenomenology: $\delta_r u(t) \sim (Ag)^{2/5} t^{-1/5} r^{3/5}$

Energy flux flows toward large scales at non-constant flux $\epsilon \sim (Ag)^{6/5} t^{-3/5} r^{4/5}$

Viscous Kolmogorov scale: $\eta \sim \frac{\nu}{\delta_\eta u} = \nu^{5/8} t^{1/8} (Ag)^{-1/4}$

Kolmogorov time scale: $\tau_\eta \sim \frac{\eta}{\delta_\eta u} = \nu^{1/4} t^{1/4} (Ag)^{-1/2}$

Heuristics in the “passive case” (2D)

Weissemberg number: $\boxed{Wi \equiv \frac{\tau_p}{\tau_\eta} \sim t^{-1/4}}$

No coil-stretch transition is expected in 2D

Lumley scale: $\frac{l_L}{\delta_{l_L} u} \sim \tau_p$ $\boxed{l_L \sim \tau_p^{5/2}(Ag)t^{-1/2}}$

At late times $l_L \ll \eta$

polymers: only contribute to viscosity renormalization

Heuristics in the "passive case" (3D)

From mean field arguments:

3D RT obeys K41 phenomenology: $\delta_r u(t) \sim \epsilon^{1/3} r^{1/3}$

Energy flux flows toward small scales at constant flux $\epsilon \sim (Ag)^2 t$

Viscous Kolmogorov scale: $\eta \sim \dfrac{\nu}{\delta_\eta u} \sim \nu^{3/4}\epsilon^{-1/4} \sim \nu^{3/4}(Ag)^{-1/2} t^{-1/4}$

Kolmogorov time scale: $\tau_\eta \sim \dfrac{\eta}{\delta_\eta u} = \nu^{1/2} t^{-1/2} (Ag)^{-1}$

Heuristics in the "passive case" (3D)

Weissemberg number: $\boxed{Wi \equiv \frac{\tau_p}{\tau_\eta} \sim t^{1/2}}$

Coil-stretch transition is expected in 3D

Lumley scale: $\frac{l_L}{\delta_{l_L} u} \sim \tau_p$ $\boxed{l_L \sim \tau_p^{3/2} (Ag) t^{1/2}}$

$$\eta \ll l_L \ll L$$

well within the inertial range of scales

DNS of 3D viscoelastic RT turbulence

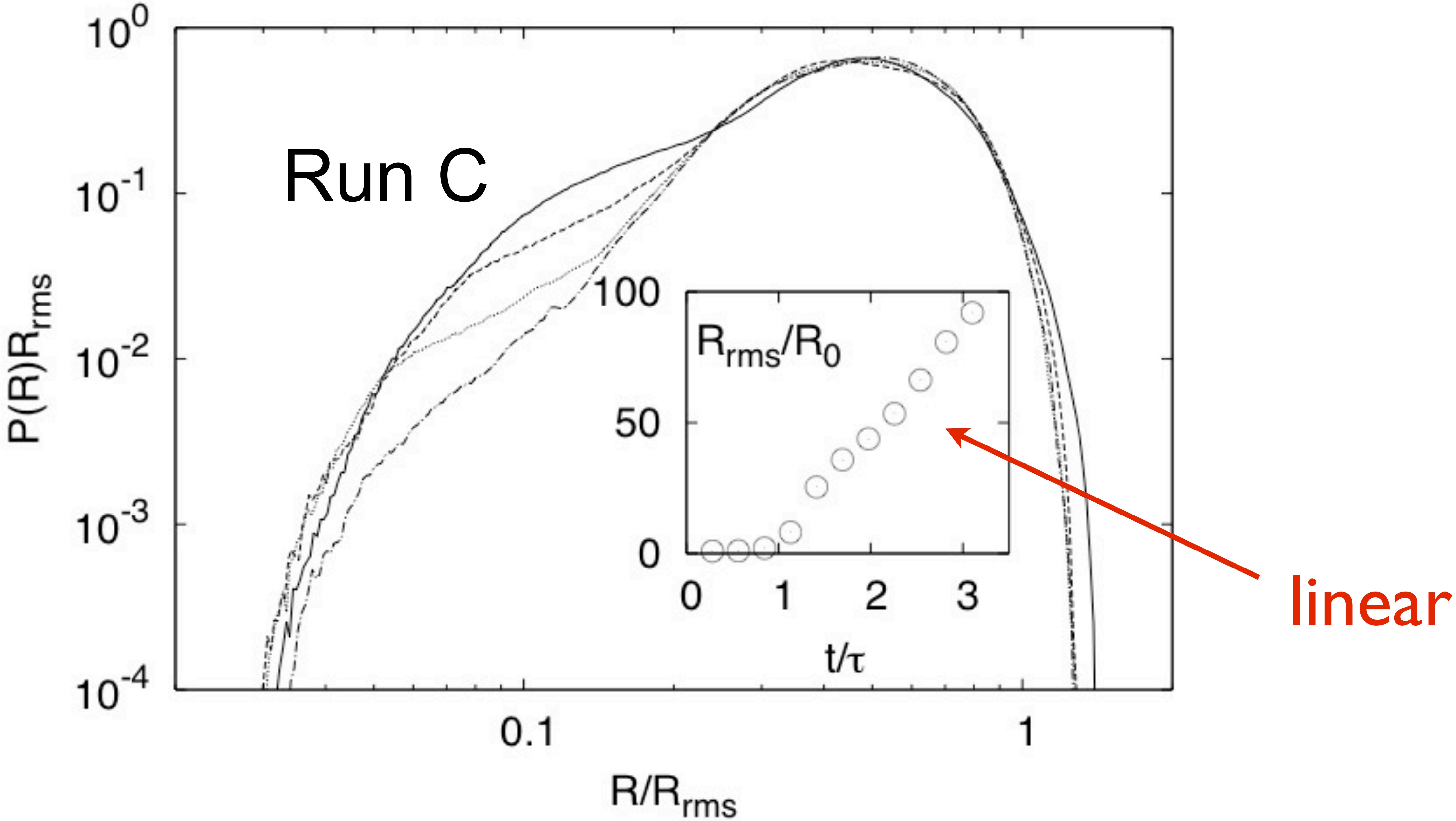
G. Boffetta, A. Mazzino, S. Musacchio, PRE (2011)

- *Parallel pseudo-spectral code*
- *Second-order Runge-Kutta temporal scheme*
- *Interface temperature initial perturbation: 10% white noise*
- *Simulations halted at L(t) ~ 80% Ly*

Main parameters:

Run	$N_{x,y}$	N_z	$L_{x,y}$	L_z	θ_0	βg	$\nu = \kappa$	κ_p	η	τ_p
N	512	1024	2π	4π	1	0.5	$3 \cdot 10^{-4}$	–	–	–
A	512	1024	2π	4π	1	0.5	$3 \cdot 10^{-4}$	10^{-3}	0.2	1
B	512	1024	2π	4π	1	0.5	$3 \cdot 10^{-4}$	10^{-3}	0.2	2
B2	512	1024	2π	4π	1	0.5	$3 \cdot 10^{-4}$	$3 \cdot 10^{-3}$	0.2	2
C	512	1024	2π	4π	1	0.5	$3 \cdot 10^{-4}$	10^{-3}	0.2	10

PDF of polymers elongation



Coil-stretch transition at $t \sim \tau$

An intrinsic exp cutoff emerges for polymers elongation

Energy balance in viscoelastic RT

Kinetic end elastic energy $$E = K + \Sigma = 1/2\langle u^2 \rangle + (\nu\eta/\tau_p)\langle tr\sigma \rangle$$

produced from potential energy $$P = -\beta g \langle zT \rangle$$

Effects of polymers:

•Speed-up of potential energy consumption

•Increase of kinetic energy

•Reduction of viscous dissipation (not shown)

"Drag reduction"

Benzi, De Angelis, Govindarajan and Procaccia, PRE (2003);
De Angelis, Casciola, Benzi and Piva, JFM (2005)

Energy balance in viscoelastic RT

Kinetic end elastic energy $E = K + \Sigma = 1/2\langle u^2\rangle + (\nu\eta/\tau_p)\langle tr\sigma\rangle$

produced from potential energy $P = -\beta g\langle zT\rangle$

$$-\frac{dP}{dt} = \beta g\langle wT\rangle = \frac{dE}{dt} + \epsilon_\nu + \frac{2\nu\eta}{\tau_p^2}[\langle tr\sigma\rangle - 3]$$

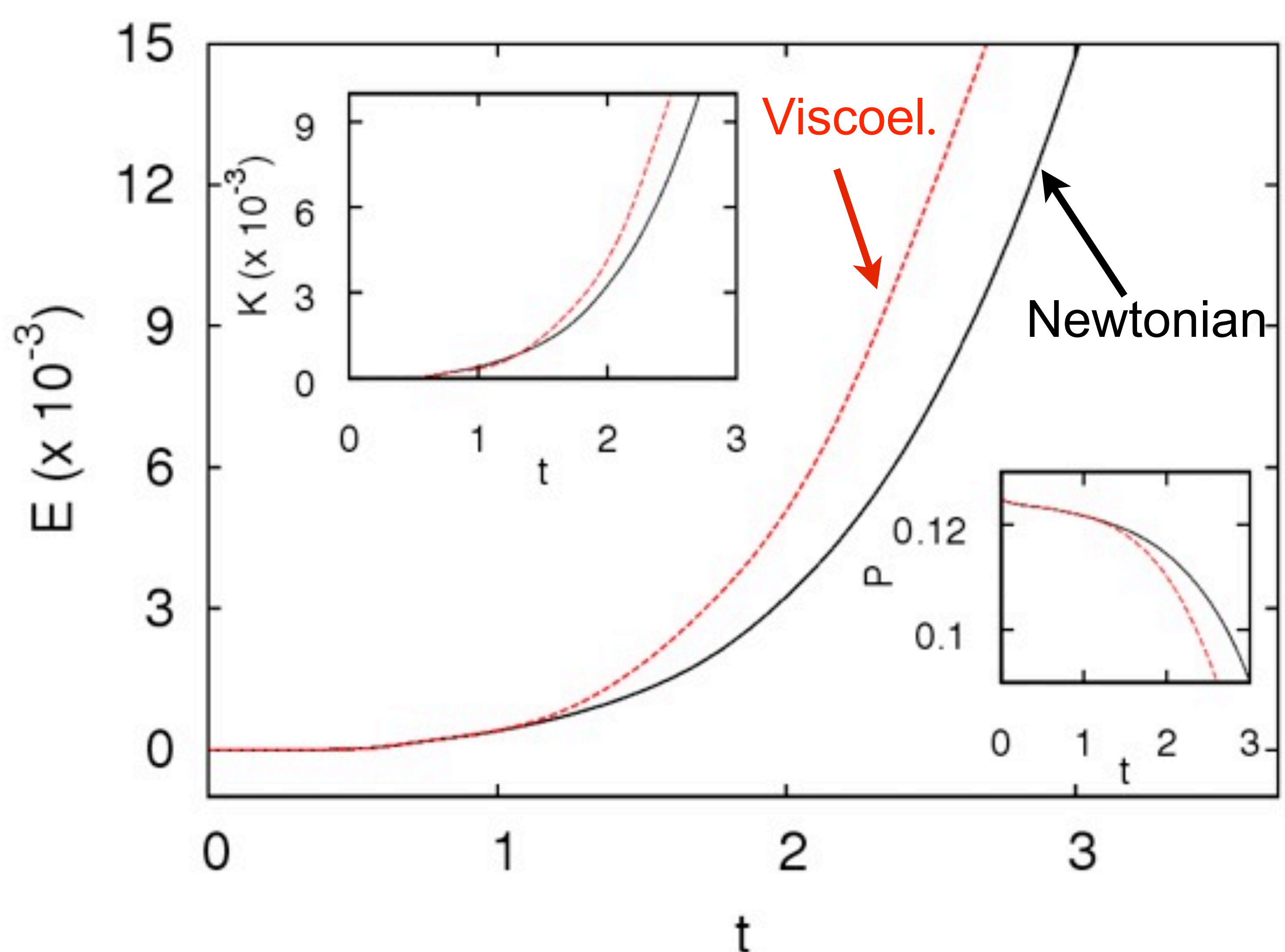
Effects of polymers:

•Speed-up of potential energy consumption

•Increase of kinetic energy

•Reduction of viscous dissipation (not shown)

“Drag reduction”



Benzi, De Angelis, Govindarajan and Procaccia, PRE (2003);
De Angelis, Casciola, Benzi and Piva, JFM (2005)

Quantification of Drag Reduction (DR)

DR = loss of potential energy/plumes kinetic energy

Potential energy: $P = -\beta g \langle zT \rangle$

Kinetic energy: $K \sim 1/2[\dot{h}(t)]^2$ (Fermi and von Neumann, 1955)

Assuming linear vertical profile for T: $P(t) \sim -1/6Agh(t)$

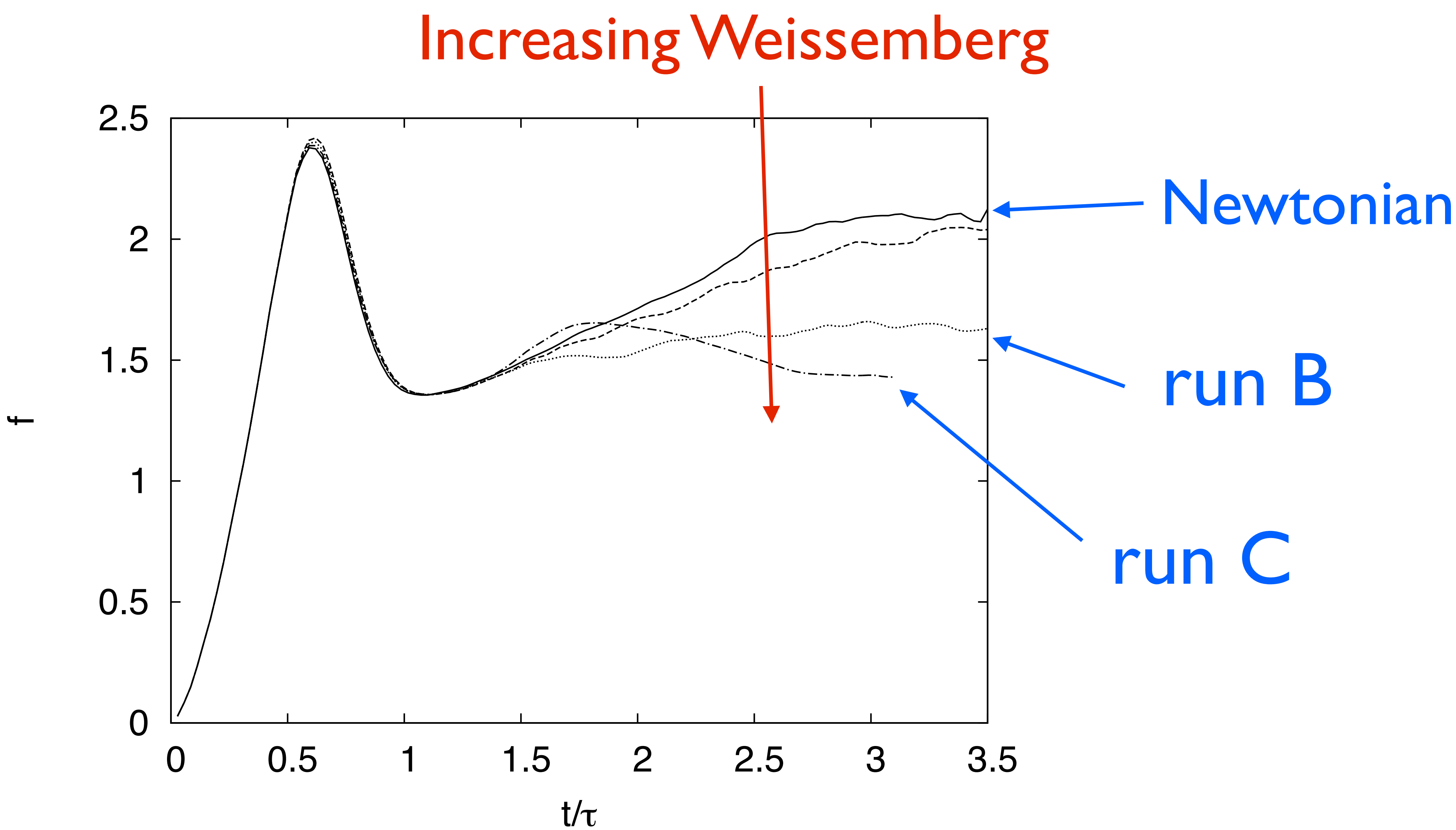
$$f \equiv \frac{\Delta P}{K} = 1/3Ag\frac{h}{\dot{h}^2} = \frac{1}{12\alpha} \qquad (h(t) = \alpha A g t^2)$$

α is thus related to f

Quantification of Drag Reduction

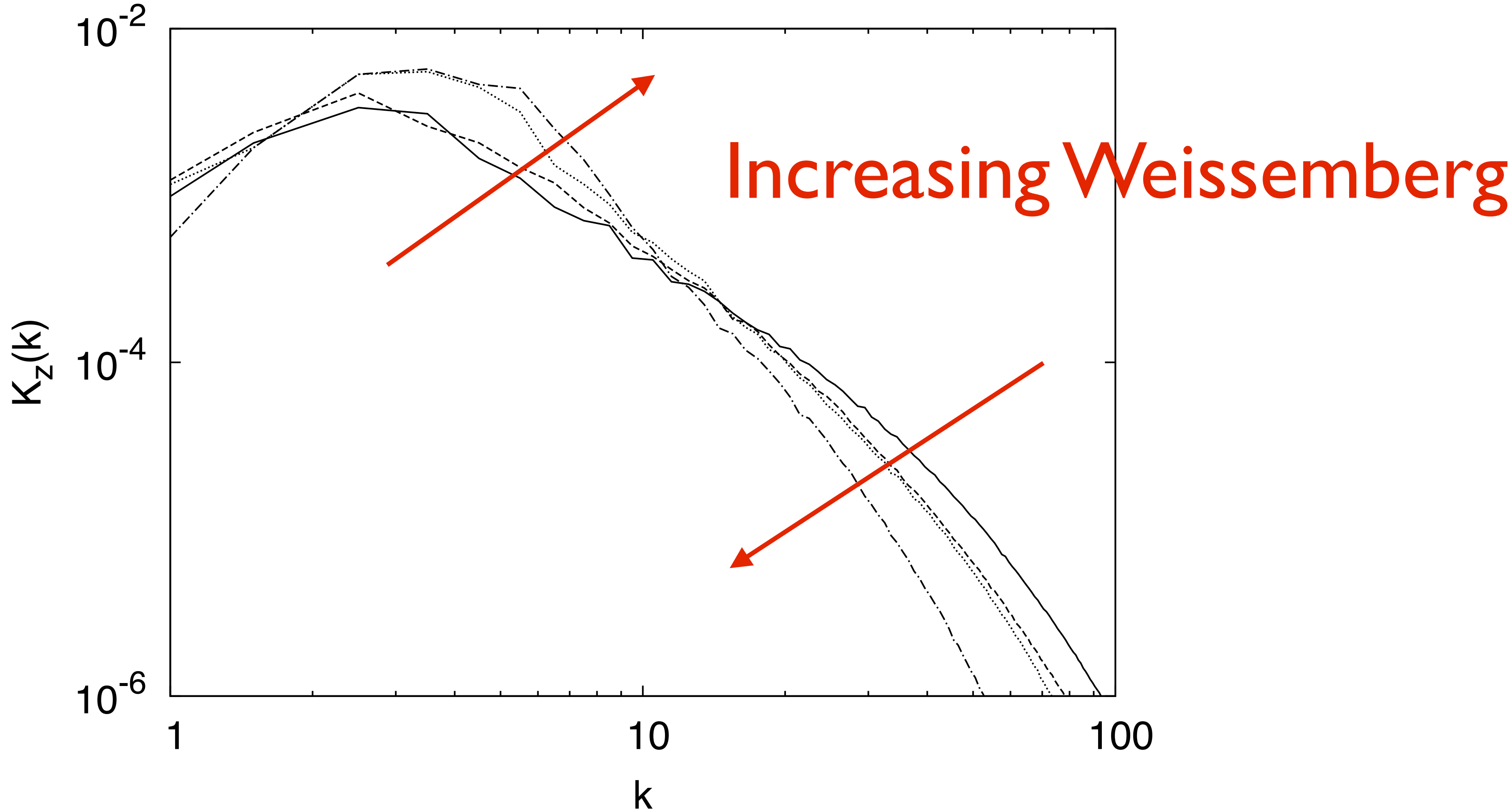
22 % of DR for run B

30 % of DR for run C



Quantification of Drag Reduction

Energy spectra



Polymers:

- *More efficient conversion from potential to kinetic energy*
- *Reduced energy transfer to small scales (reduced dissipation)*

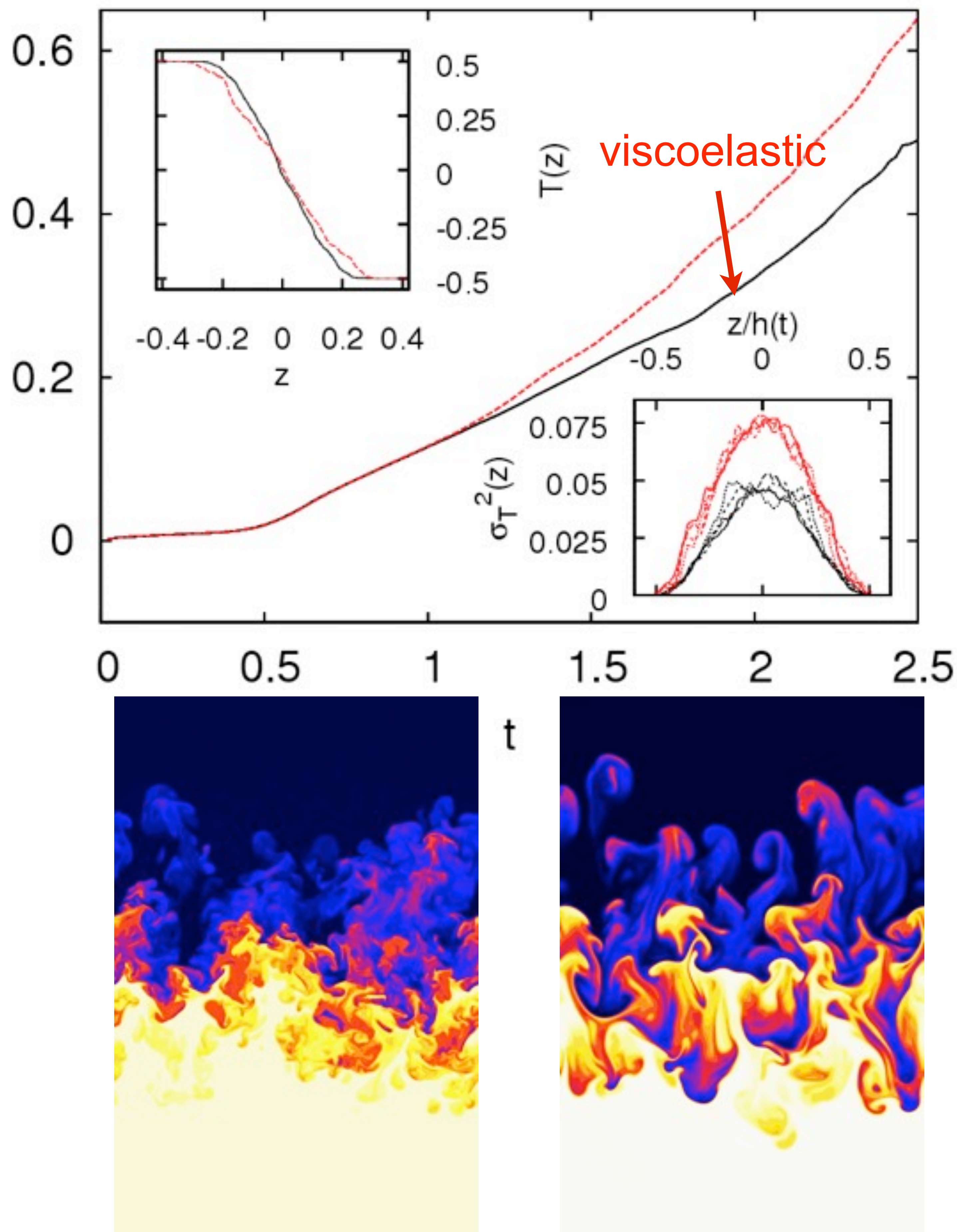
Similarities with isotropic homogeneous turbo: Benzi, De Angelis, Govindarajan and Procaccia, PRE (2003); De Angelis, Casciola, Benzi and Piva, JFM (2005)

Faster growth of mixing layer

Faster growth of mixing layer h(t)

More efficient mass transfer

a)



Faster growth of mixing layer

a)

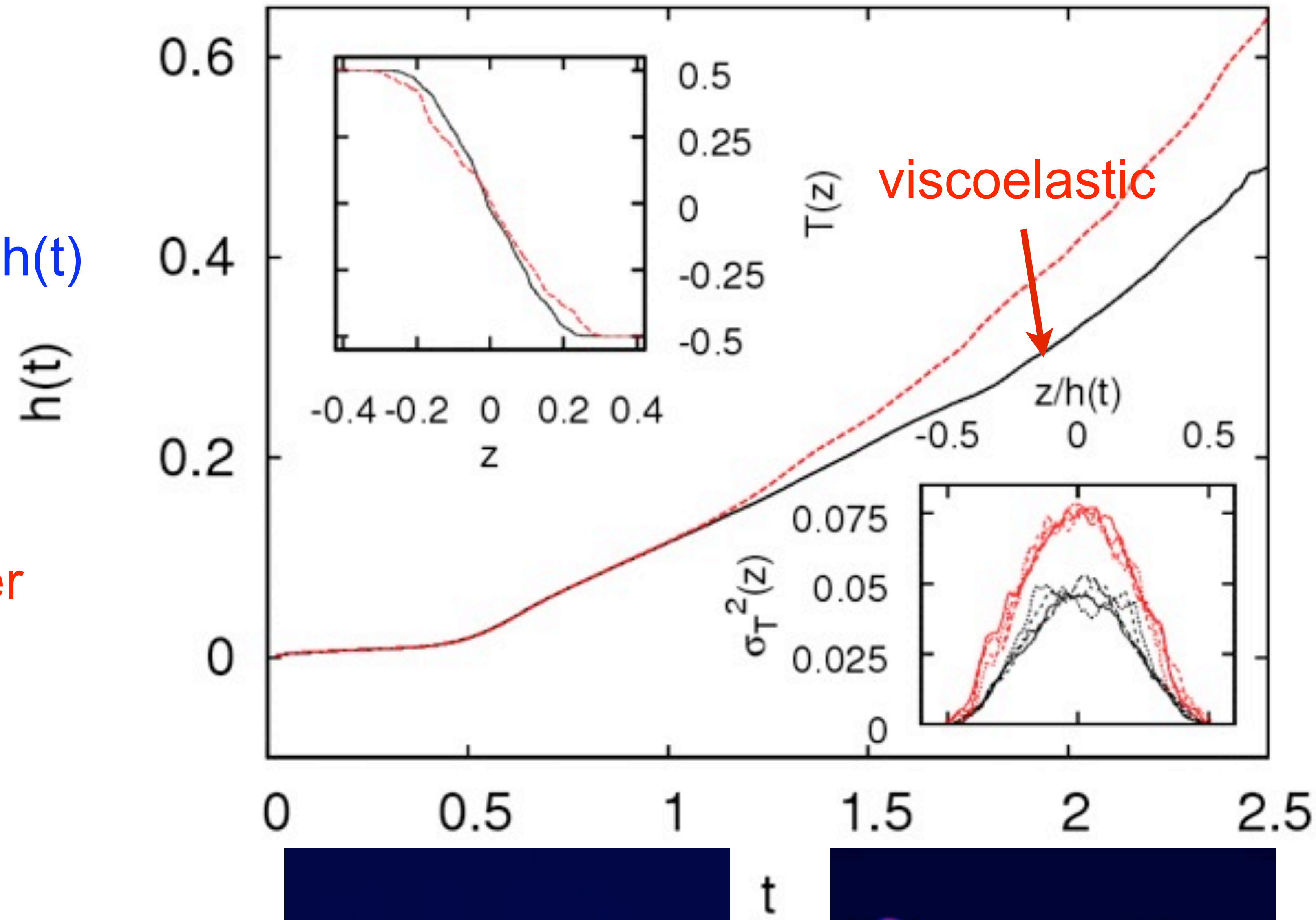
Faster growth of mixing layer h(t)

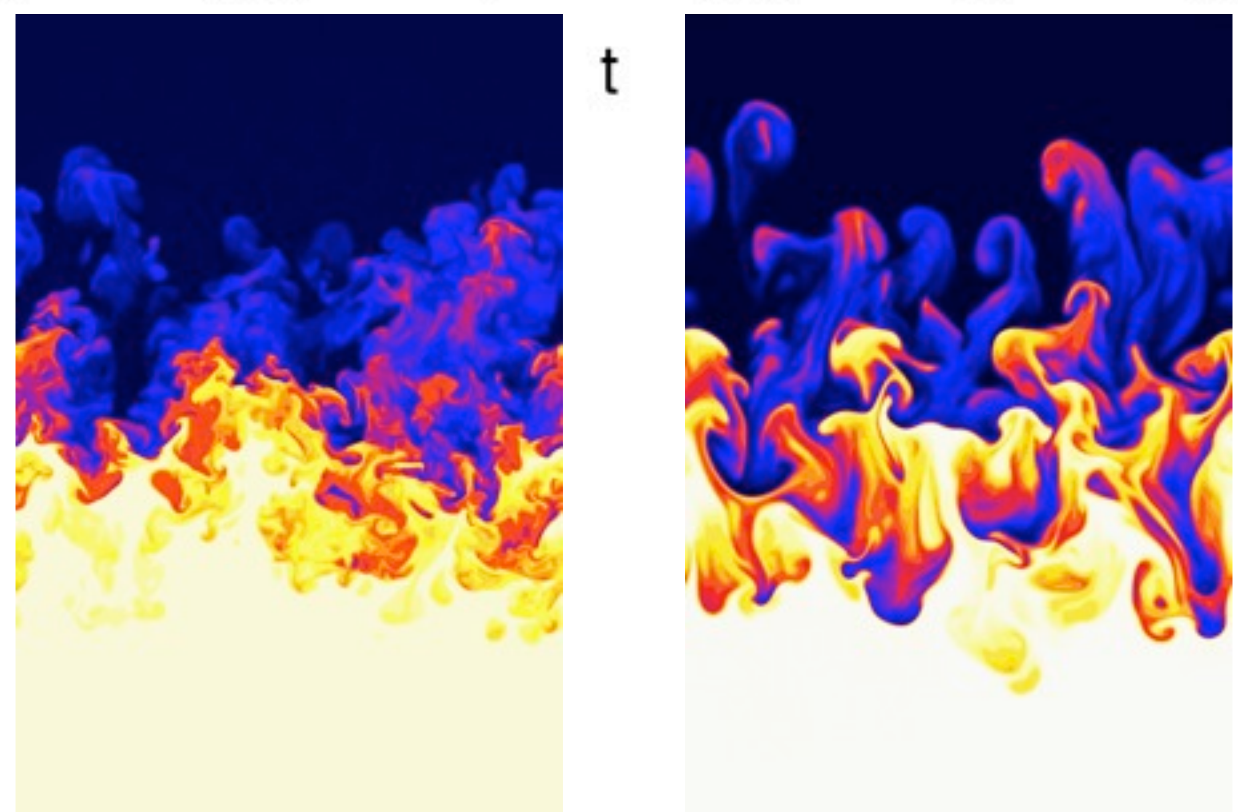
More efficient mass transfer

b)

Larger temperature variance σ_T

Reduced mixing efficiency at small scale



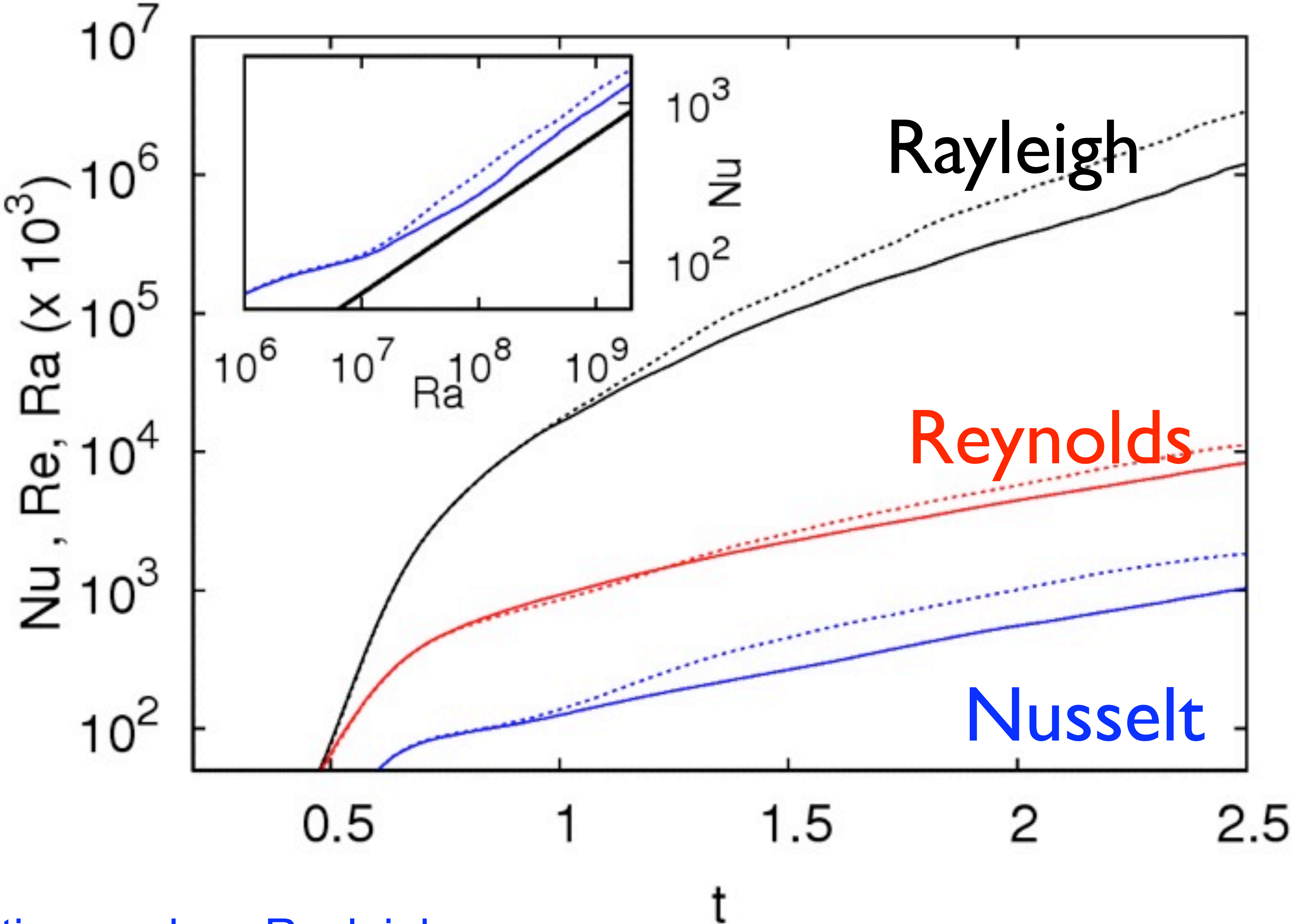


Heat transfer enhancement

$$\text{Ra} \equiv \frac{\beta g \theta_0 h^3}{\kappa \nu}$$

$$\text{Re} \equiv \frac{u_{rms} h}{\nu}$$

$$\text{Nu} \equiv \frac{Corr(wT) h w_{rms}}{\kappa}$$



Nusselt increases both vs time and vs Rayleigh

because: $\text{Corr}(\text{w T})$, w_{rms}, h increase

w.r.t. RB convection:
agreement (Benzi et al, PRL 2010); disagreement (Ahlers and Nikolaenko, PRL 2010)

Main Conclusions

- Speed-up of Rayleigh-Taylor instability due to Polymers (linear analysis)
- Polymers increase the rate of large-scale mixing and reduce small-scale mixing in the fully developed turbulence stage
- Many analogies with DR in homogeneous isotropic turbulence
- Heat transport enhancement

Perspectives

- Extension to elastic fibers
- Extension to immiscible Newtonian fluids

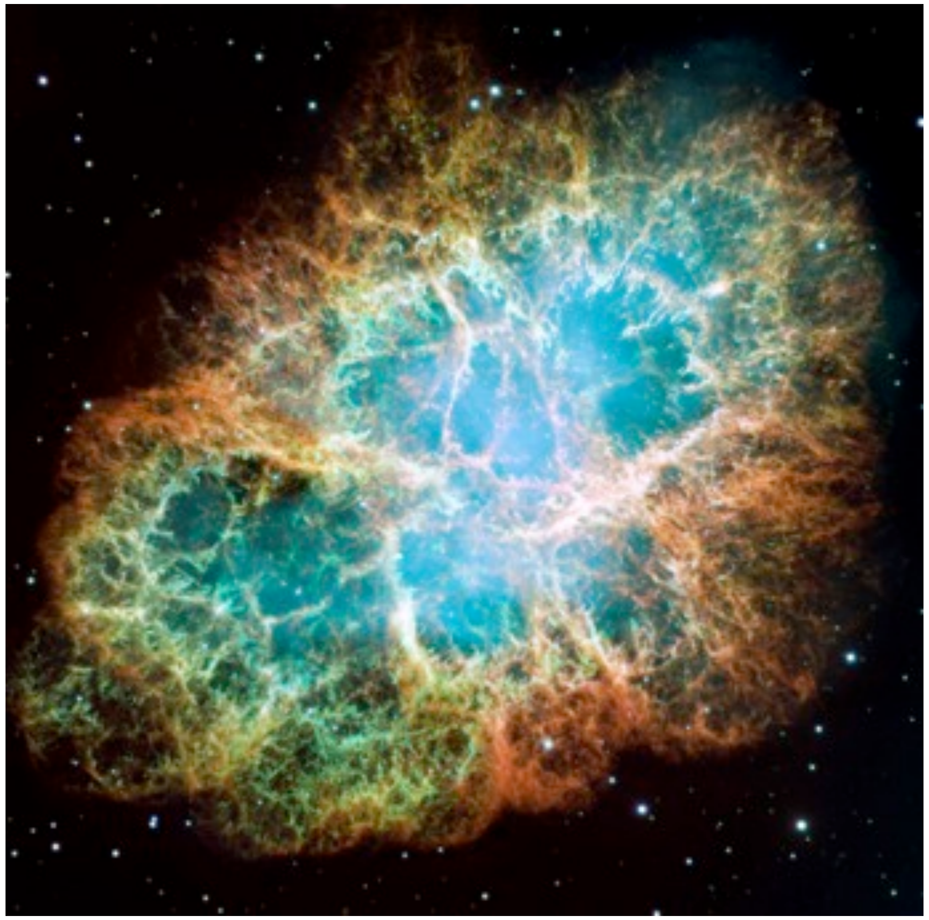
Details on the viscoelastic RT:

* *Rayleigh--Taylor instability in a viscoelastic binary fluid*
  G. Boffetta, A. Mazzino, S. Musacchio and L. Vozella
  J. Fluid Mech. 643, 127-136 (2010)

* *Polymer Heat Transport Enhancement in Thermal Convection: The Case of Rayleigh-Taylor Turbulence*
  G. Boffetta, A. Mazzino, S. Musacchio and L. Vozella
  Phys. Rev. Lett. 104, 184501 (2010)

* Effects of polymer additives on Rayleigh-Taylor turbulence
  G. Boffetta, A. Mazzino and S. Musacchio
  Phys. Rev. E 83, 056318 (2011)

Relevance of Rayleigh-Taylor instability

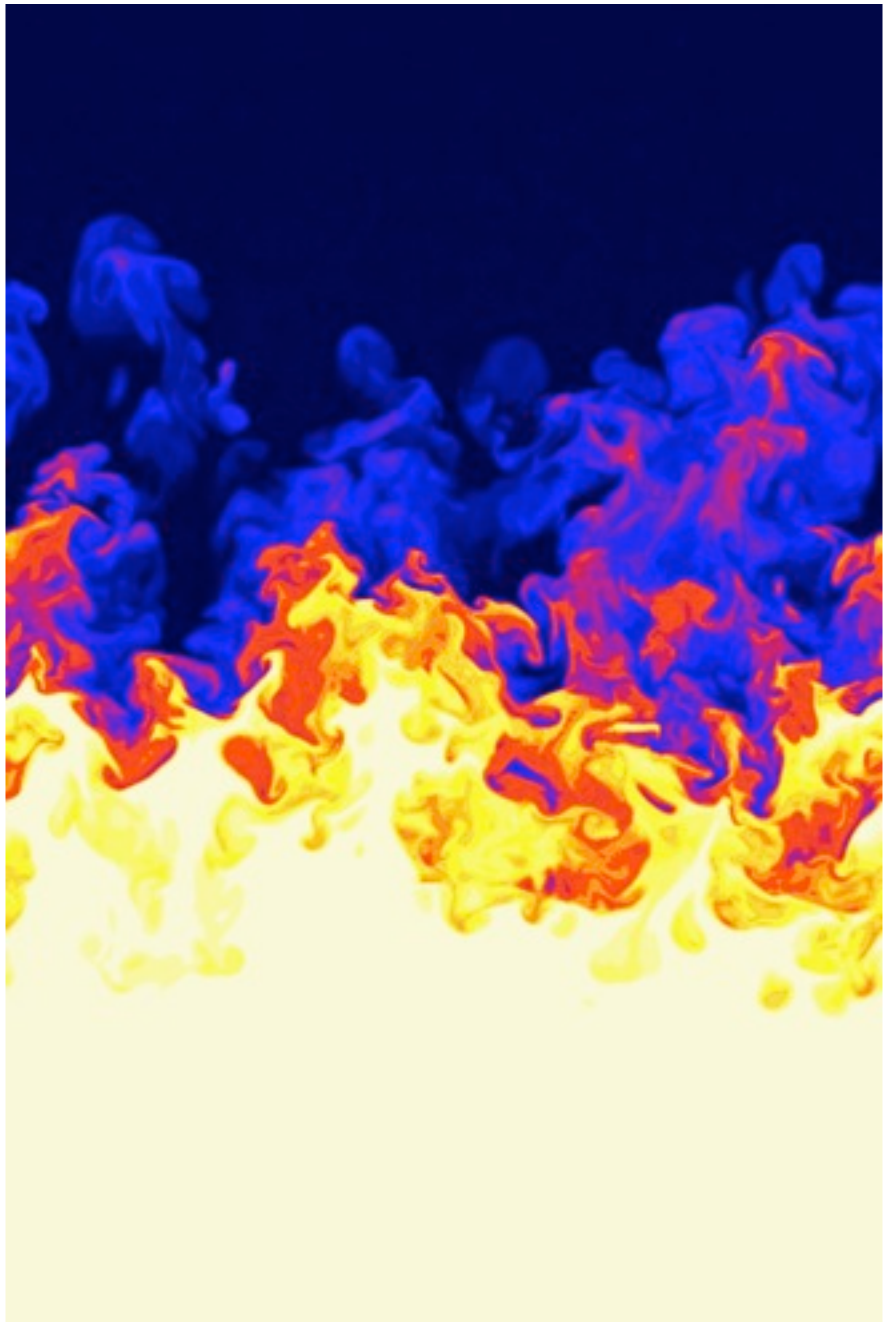


Many applications in natural phenomena and technological problems:

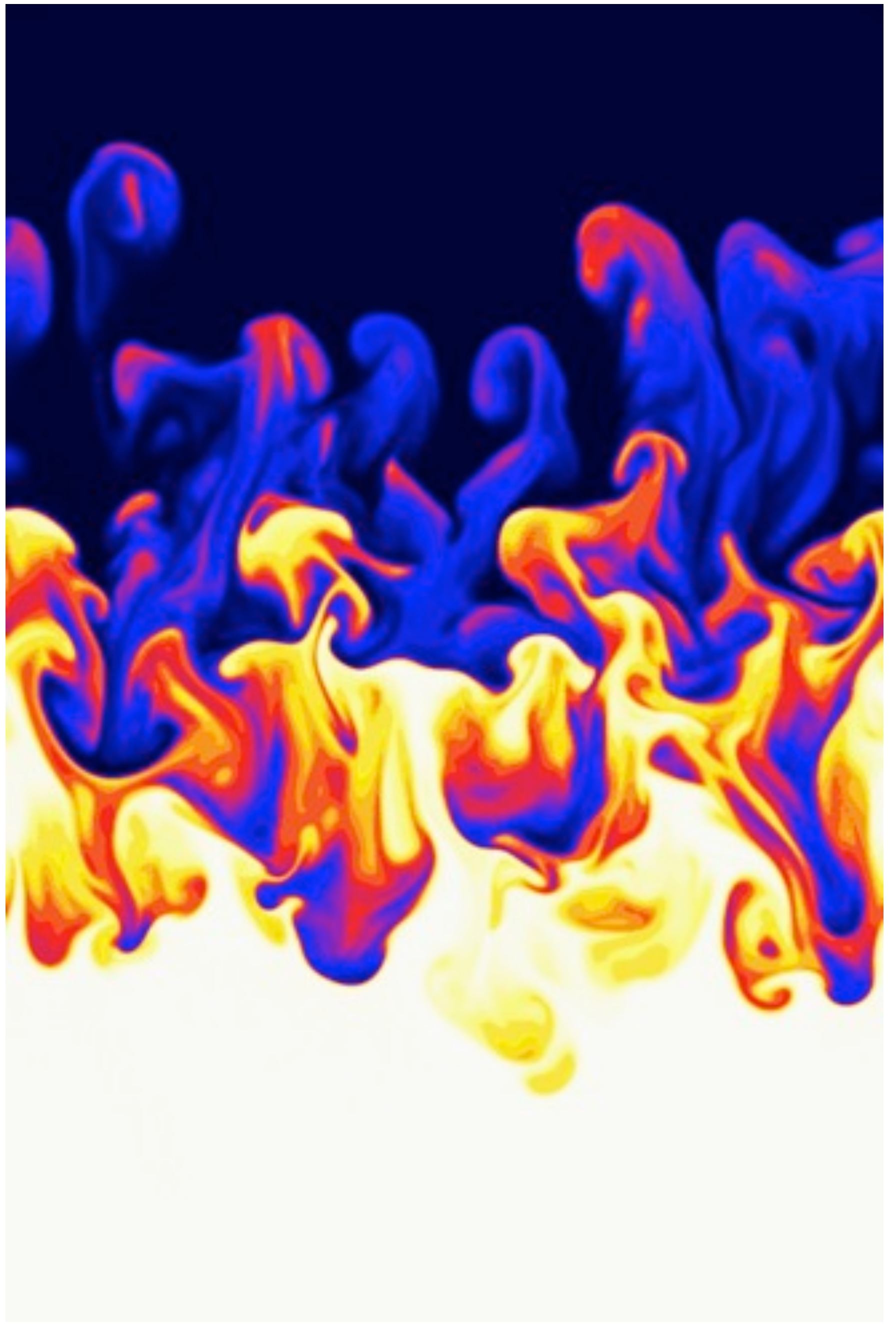
- supernova explosion
- acceleration mechanism for thermonuclear flame front
- atmospheric physics (mammatus clouds)
- solar corona heating
- intertial confinement fusion
- etc.

Viscoelastic Rayleigh-Taylor turbulence

Newtonian

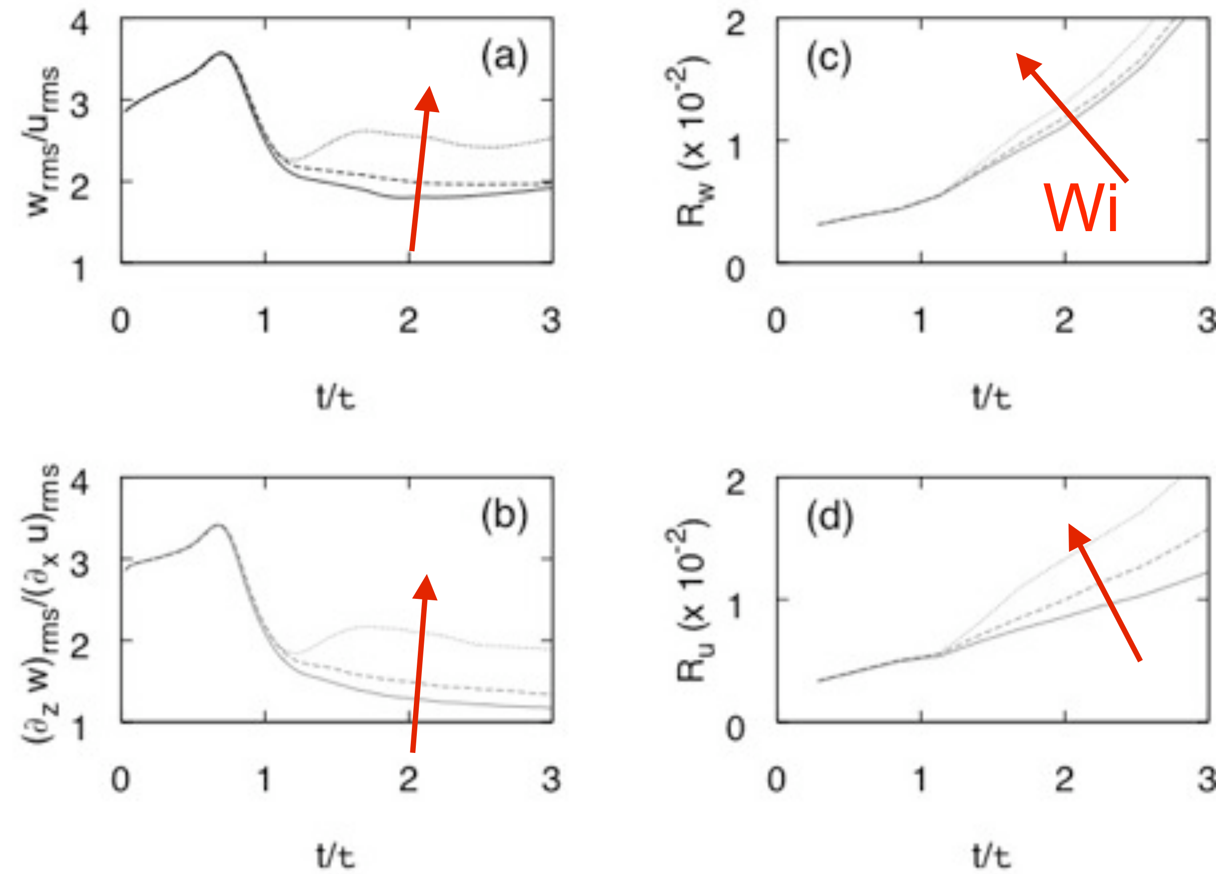


Viscoelastic



Faster and more coherent plumes

Horizontal velocities u_{rms} are depleted
Vertical velocities w_{rms} are enhanced



R_u , R_w half width vel. correlat.

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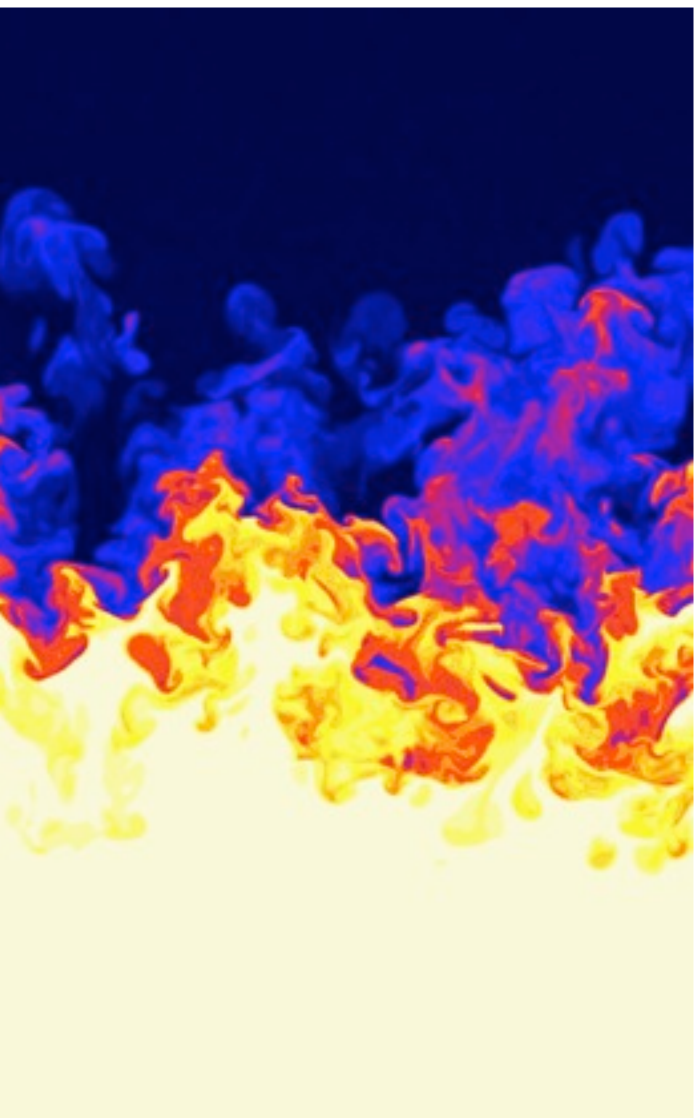
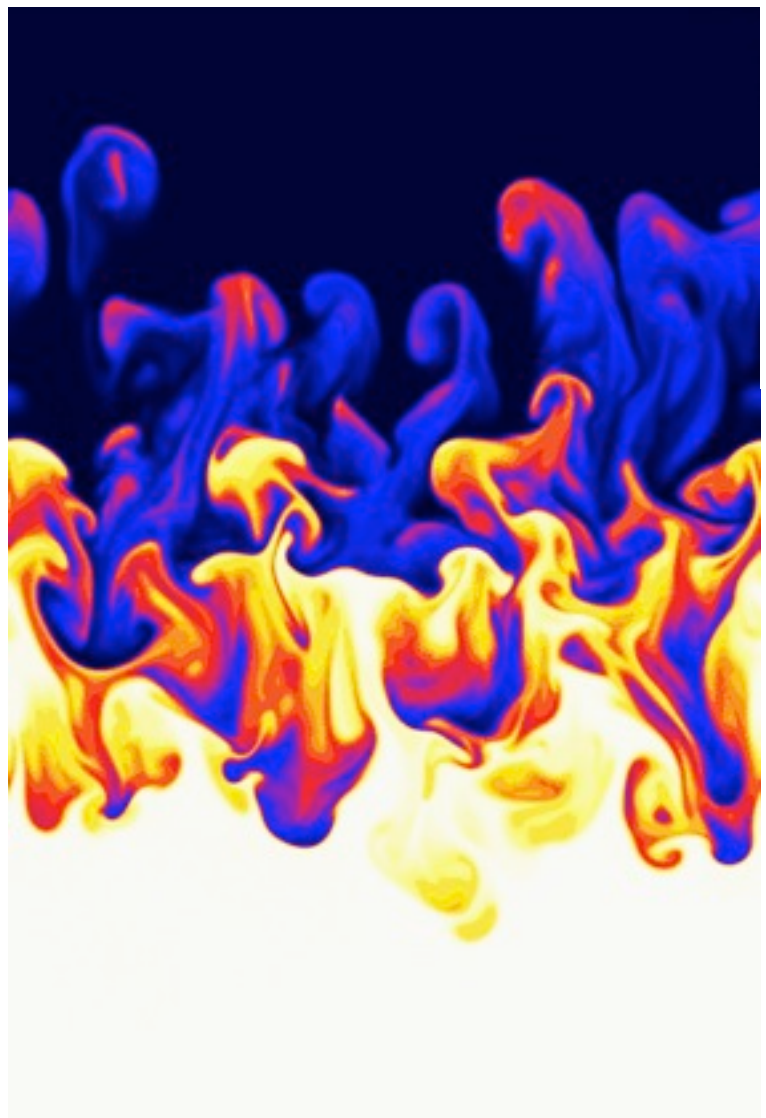
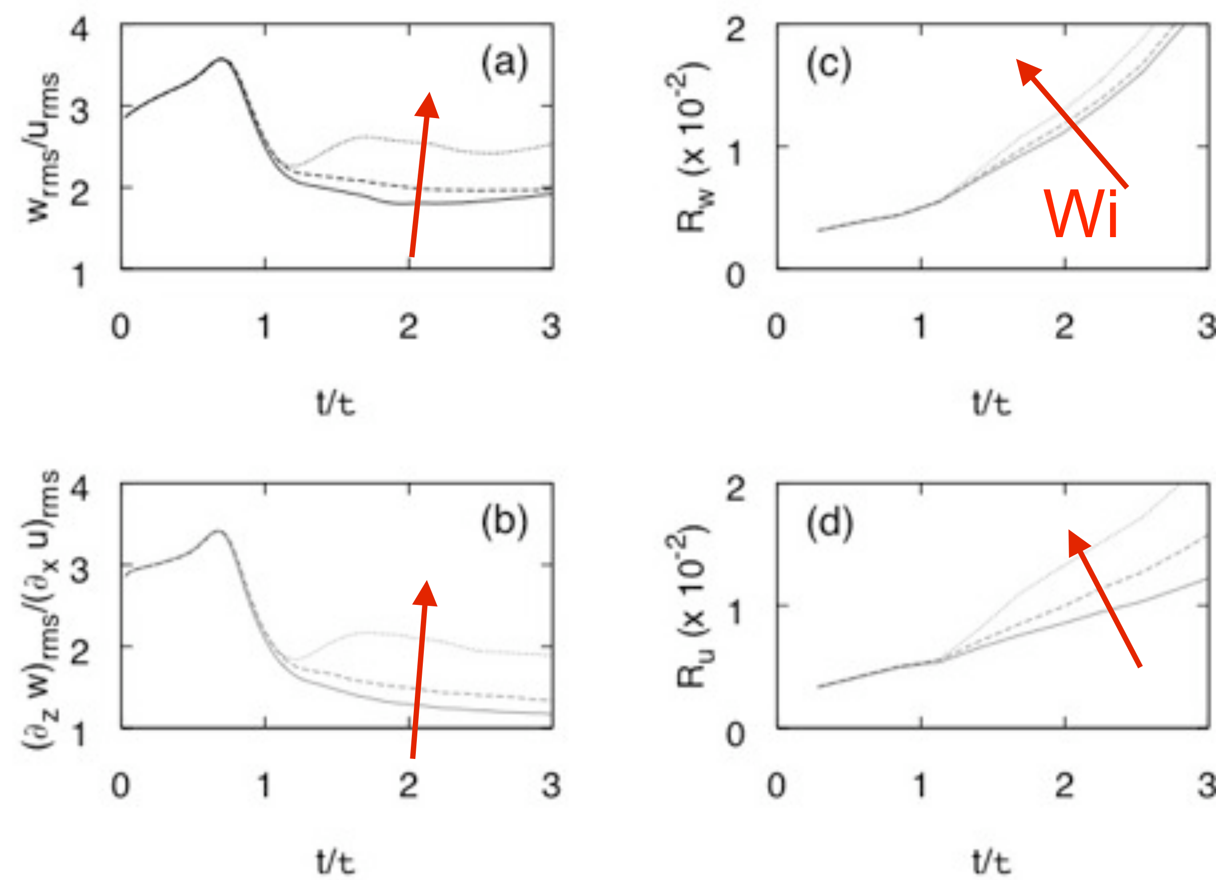
Faster and more coherent







R_u , R_w half width vel. correlat.