Lagrangian/Eulerian estimates of turbulent breakup of small particles

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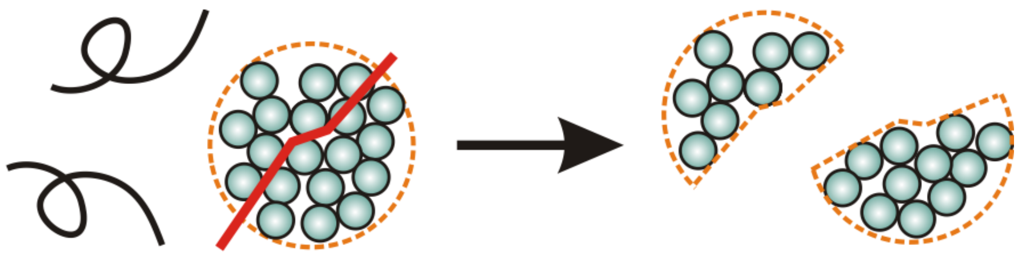
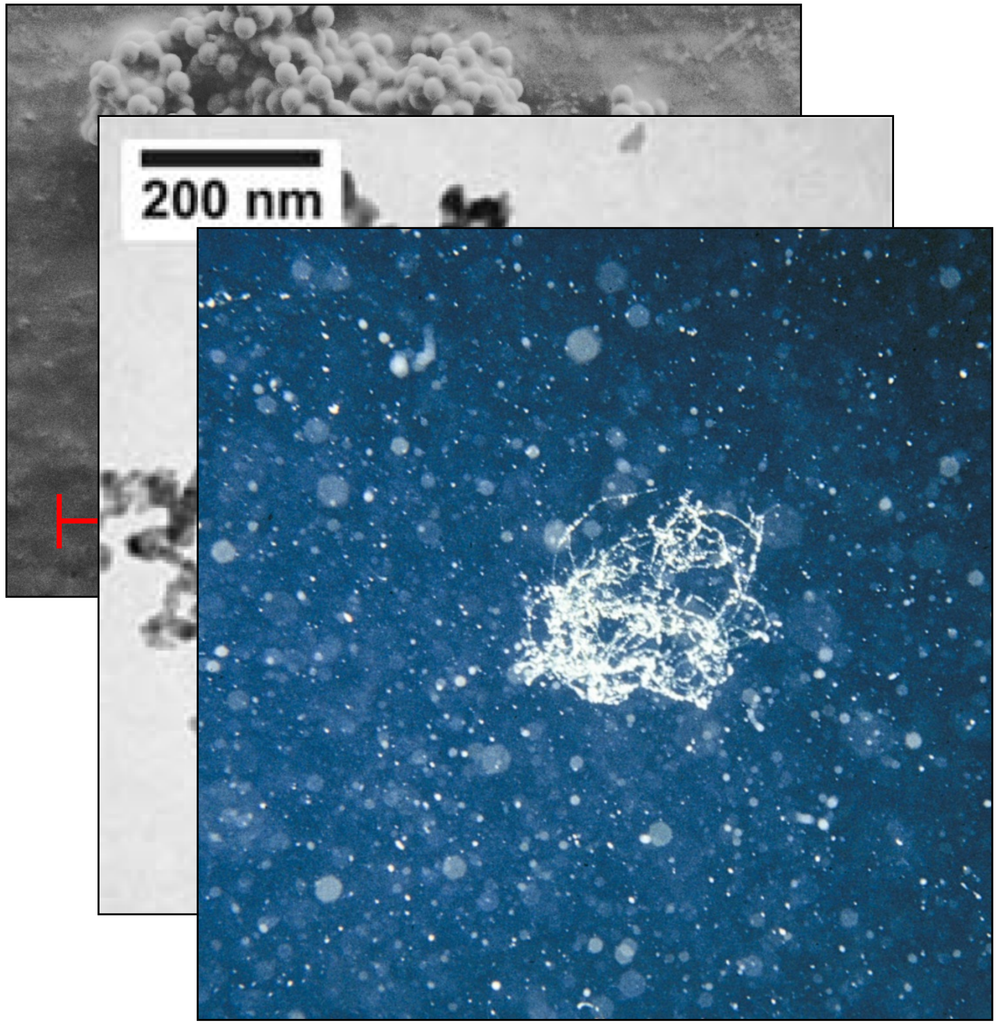
Breakup of aggregates in turbulent flows



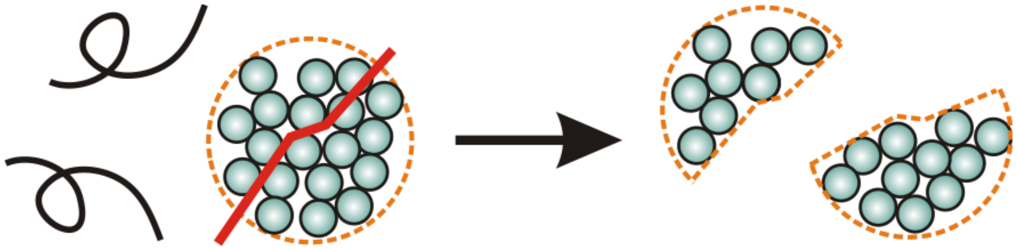
- Processing of industrial colloids, e.g., polymer colloids
- Nano-particle and powder manufacturing, dispersion of dust and aerosols
- Suspended particulate matter in environmental systems, e.g., marine snow
- Mixing/Un-mixing devices

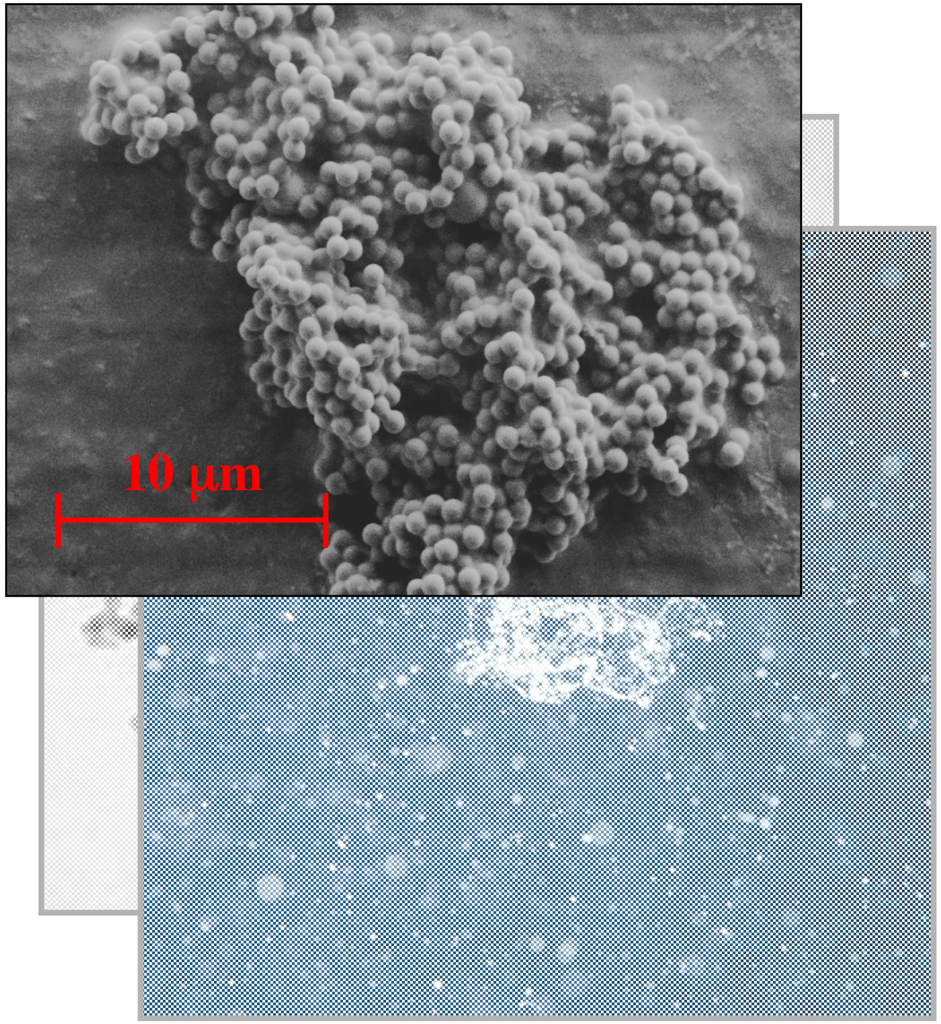


J. März, Dissertation Univ. Oldenburg (2010)

Picture: H. P. Grossart, IGB, Leibniz-Institute of Freshwater Ecology and Inland Fisheries

Breakup of aggregates in turbulent flows





- Neutrally buoyant aggregate $\rho_p \sim \rho_f$
- Aggregate size d $\ll \eta$

--> *tracer particles*

- Break-up due to local shear stress

$$\sigma = \mu \langle s_{ij} s_{ji} \rangle^{1/2} \rightarrow \mu \sqrt{\frac{2\epsilon}{15\nu}}$$ *Homog & Iso Turb*

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Population balance equation

$$n(\xi, \mathbf{x}, \mathbf{u}, t) d\xi \, d\mathbf{x} \, d\mathbf{u}$$

PDF at time t of number of particles of mass ξ, located at [**x**, **x** + d**x**], with velocity [**u**, **u** + d**u**]

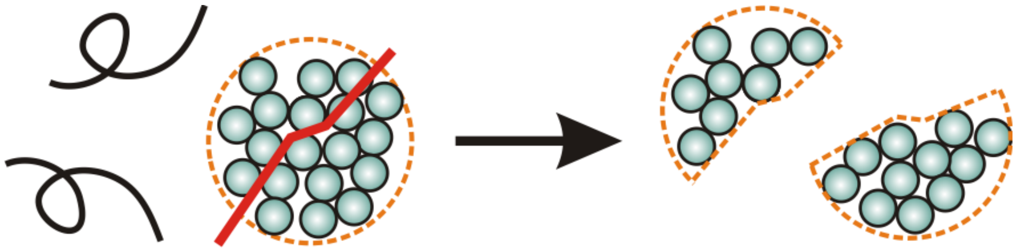
$$\frac{\partial n}{\partial t} + \nabla_{\mathbf{x}} \cdot (\mathbf{u}\, n) + \nabla_{\mathbf{u}} \cdot (\mathbf{F}\, n) = \dot{Q}_k + \dot{Q}_g + \Gamma$$

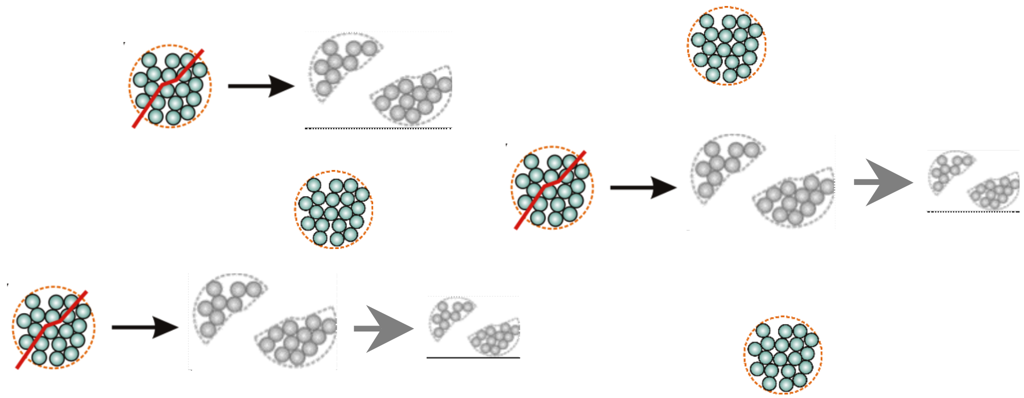
$\dot{Q}_k$: *coalescence*

$\dot{Q}_g$: *break-up*

Γ: *collisions* without coalescence

Turbulent break-up only: working ansatz





- Neutrally buoyant $\rho_p \sim \rho_f$
- Aggregate size **d** $\ll \eta$
- Dilute suspension
- Break-up due to local shear stress

$$\sigma_{crit} = \mu \sqrt{\frac{2\,\epsilon_{crit}}{15\,\nu}}; \qquad \epsilon_{crit}(\xi) \sim \xi^{-m}$$

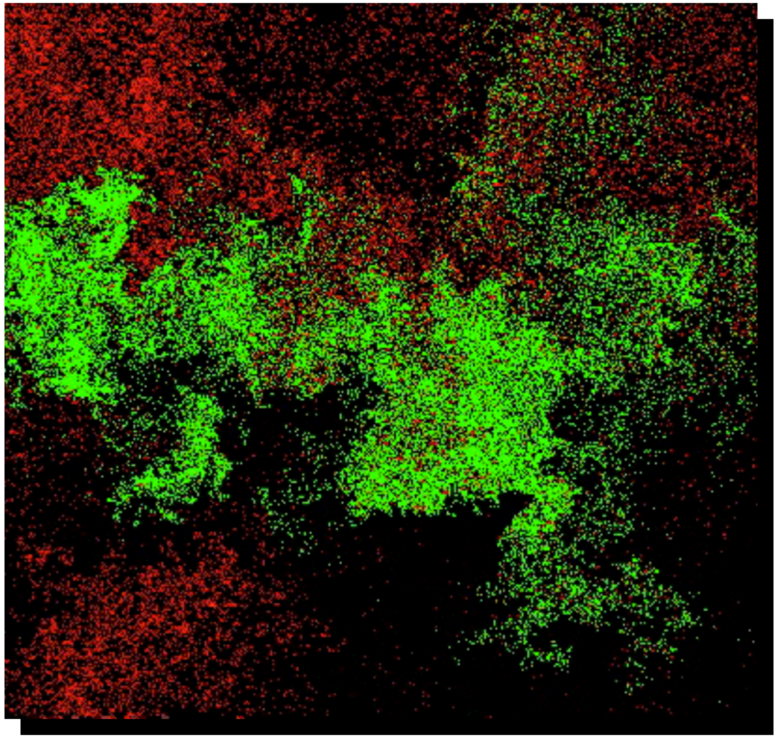
- Evaluate decay of the number of aggregates of a given mass

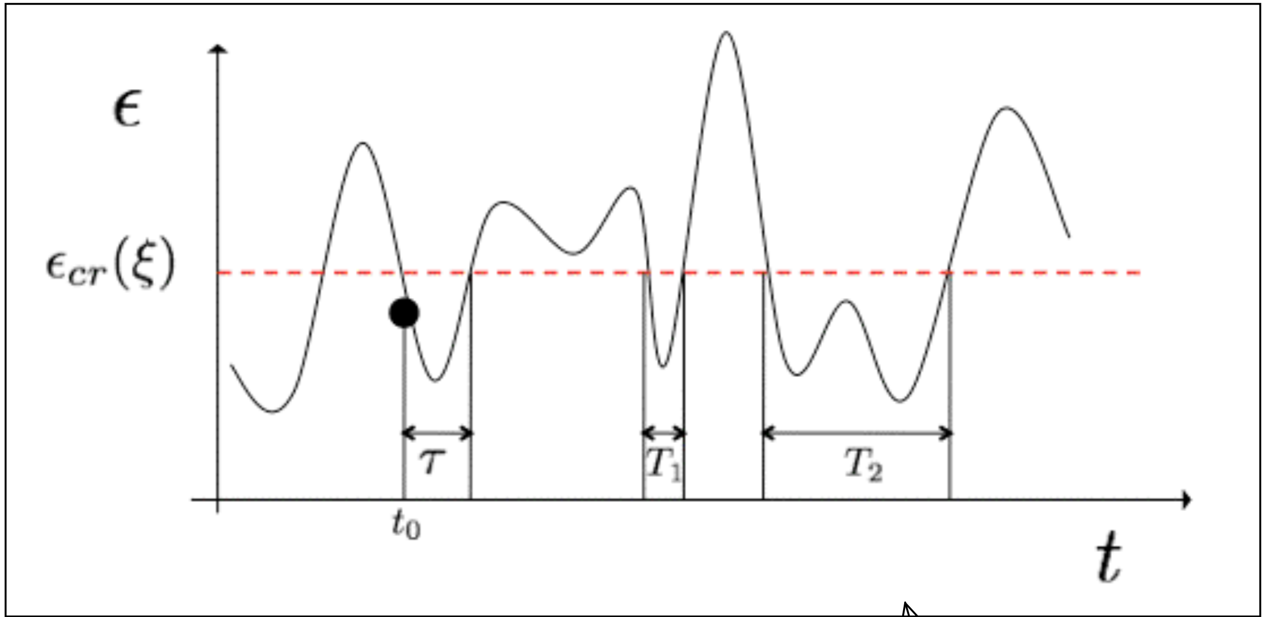
$$\frac{dn_\xi}{dt} = -f_\xi n_\xi$$

break-up rate

- Re-aggregation is neglected
- Previous break-up is neglected
- Break-up is instantaneous

An operational method to estimate the break-up rate





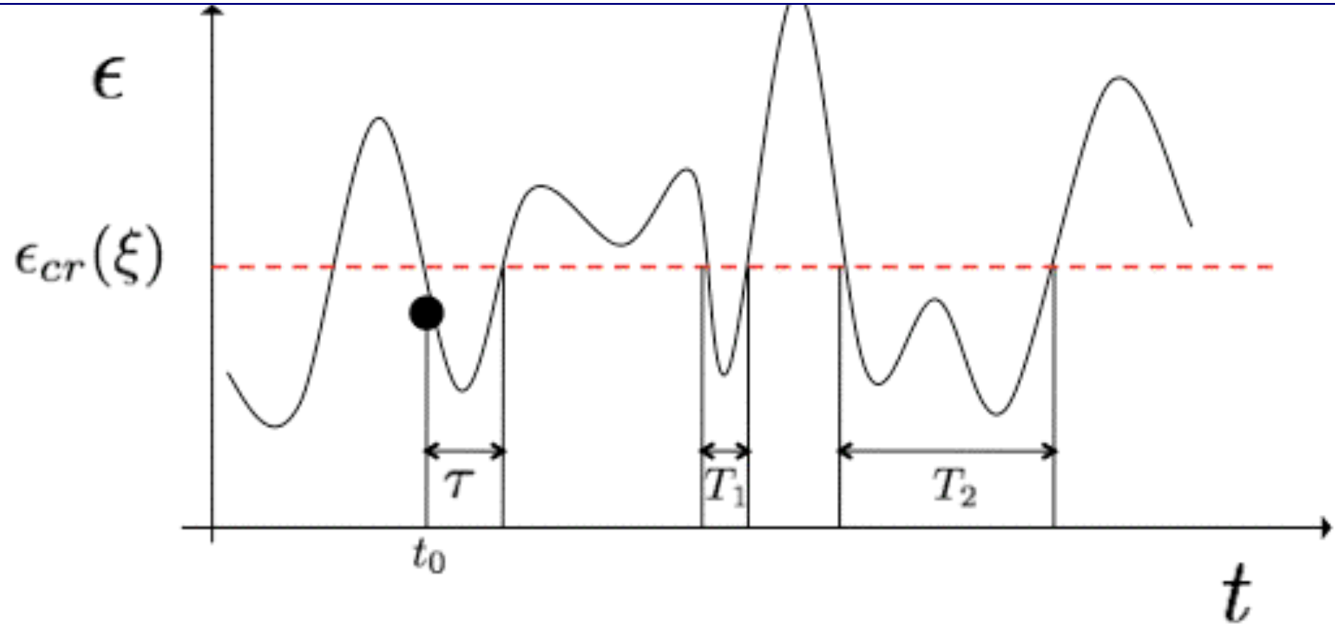
- Instantaneous break-up
  (e.g. brittle materials, no prior deformation)

T DIVING TIME

τ FIRST EXIT-TIME

Lagrangian measure: from MEAN FIRST EXIT-TIME

$$f(\varepsilon_{\rm cr}) = \left[\int_0^\infty d\tau\ \tau \mathcal{P}_{\varepsilon_{\rm cr}}(\tau)\right]^{-1} = \frac{1}{\langle \tau(\varepsilon_{\rm cr})\rangle_e}$$



Eulerian estimate: from MEAN DIVING TIME

$$f_{Eul}(\epsilon_{cr}) = \frac{1}{\langle T(\epsilon_{cr})\rangle} = \frac{\int_0^\infty d\dot\epsilon\, \dot\epsilon\, p_2(\epsilon_{cr}, \dot\epsilon)}{\int_0^{\epsilon_{cr}} d\epsilon\, p(\epsilon)}$$

V.I. Loginov (1985) + S.O. Rice formula for upcrossings number estimate (1945)

Closure Models for Loginov/Rice Eulerian estimate

$$f_{Eul}(\epsilon_{cr}) = \frac{1}{\langle T(\epsilon_{cr})\rangle} = \frac{\int_0^\infty d\dot{\epsilon}\,\dot{\epsilon}\,p_2(\epsilon_{cr},\dot{\epsilon})}{\int_0^{\epsilon_{cr}} d\epsilon\,p(\epsilon)}$$

Modelling joint distribution of critical energy dissipation, and its derivative

I. Dimensional estimate

$$p_2(\epsilon,\dot{\epsilon}) \simeq \frac{1}{2}p(\epsilon)\delta(|\dot{\epsilon}| - \epsilon/\tau_\eta(\epsilon)) \longrightarrow f^{I}_{Eul}(\epsilon_{cr}) \simeq \frac{\epsilon_{cr}\,p(\epsilon_{cr})/\tau_\eta(\epsilon_{cr})}{2\int_0^{\epsilon_{cr}} d\epsilon\,p(\epsilon)}$$

Value of critical energy dissipation dominates

II. Engulfment model

Any fluid region where $\varepsilon > \varepsilon_{cr}$ is active matters to engulf aggregates at a rate ~ $1/\tau_\eta(\varepsilon)$

$$\longrightarrow f^{II}_{Eul}(\epsilon_{cr}) \simeq \frac{\int_{\epsilon_{cr}}^\infty d\epsilon\,p(\epsilon)/\tau_\eta(\epsilon)}{\int_0^{\epsilon_{cr}} d\epsilon\,p(\epsilon)}$$

(Bäbler et al., JFM 2008)

Numerical Experiment: 3D incompressible HIT

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} P + \nu \nabla^2 \boldsymbol{u} + \boldsymbol{F} \qquad \nabla \cdot \mathbf{u} = 0$$

$$\dot{\mathbf{X}} = \mathbf{u}(\mathbf{X}, t)$$

tracers

$$\begin{cases} \dot{\mathbf{X}} = \mathbf{V}(\mathbf{X}, t) \\ \dot{\mathbf{V}} = \dfrac{\mathbf{u}(\mathbf{X}, t) - \mathbf{V}(\mathbf{X}, t)}{\tau_a} \end{cases} \qquad \tau_a = \frac{2}{9} \frac{a^2 \rho_a}{\rho_f \nu}$$

Heavy spherical particles:
radius a << η
dilute solution: no collisions
small particle Reynolds number Re_a
very heavy $\rho_a >> \rho_f$

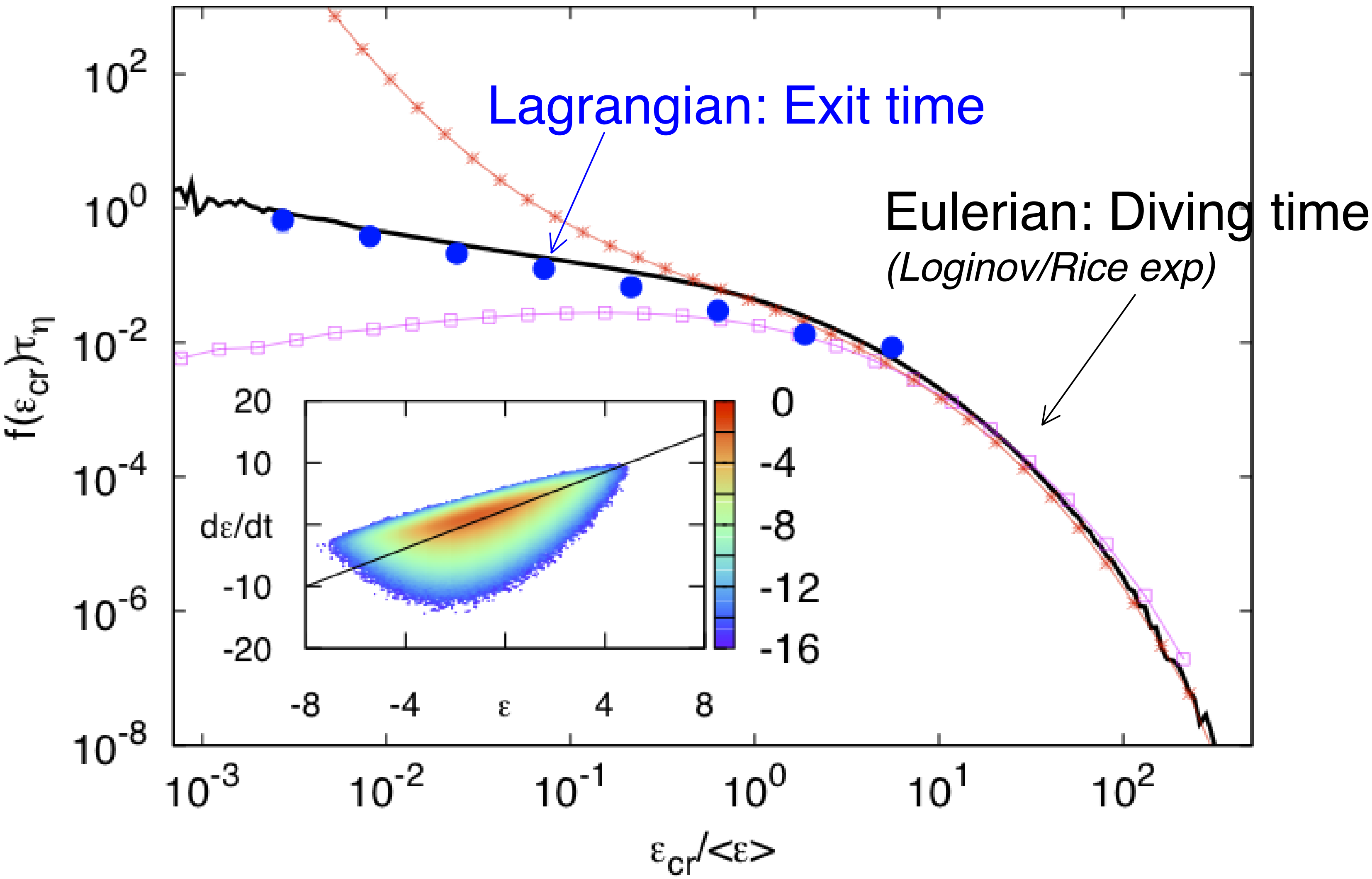
CONTROL PARAMETERS: Re_λ & $St = \dfrac{\tau_a}{\tau_\eta}$

	N^3 [Re_λ]	St	# particles per St
RUN I	**512^3 Re_λ =185**	**0, 0.16 - 3.3**	**7.5 x 10^6**
RUN II	**2048^3 Re_λ =400**	**0, 0.16 - 70**	**10^8**

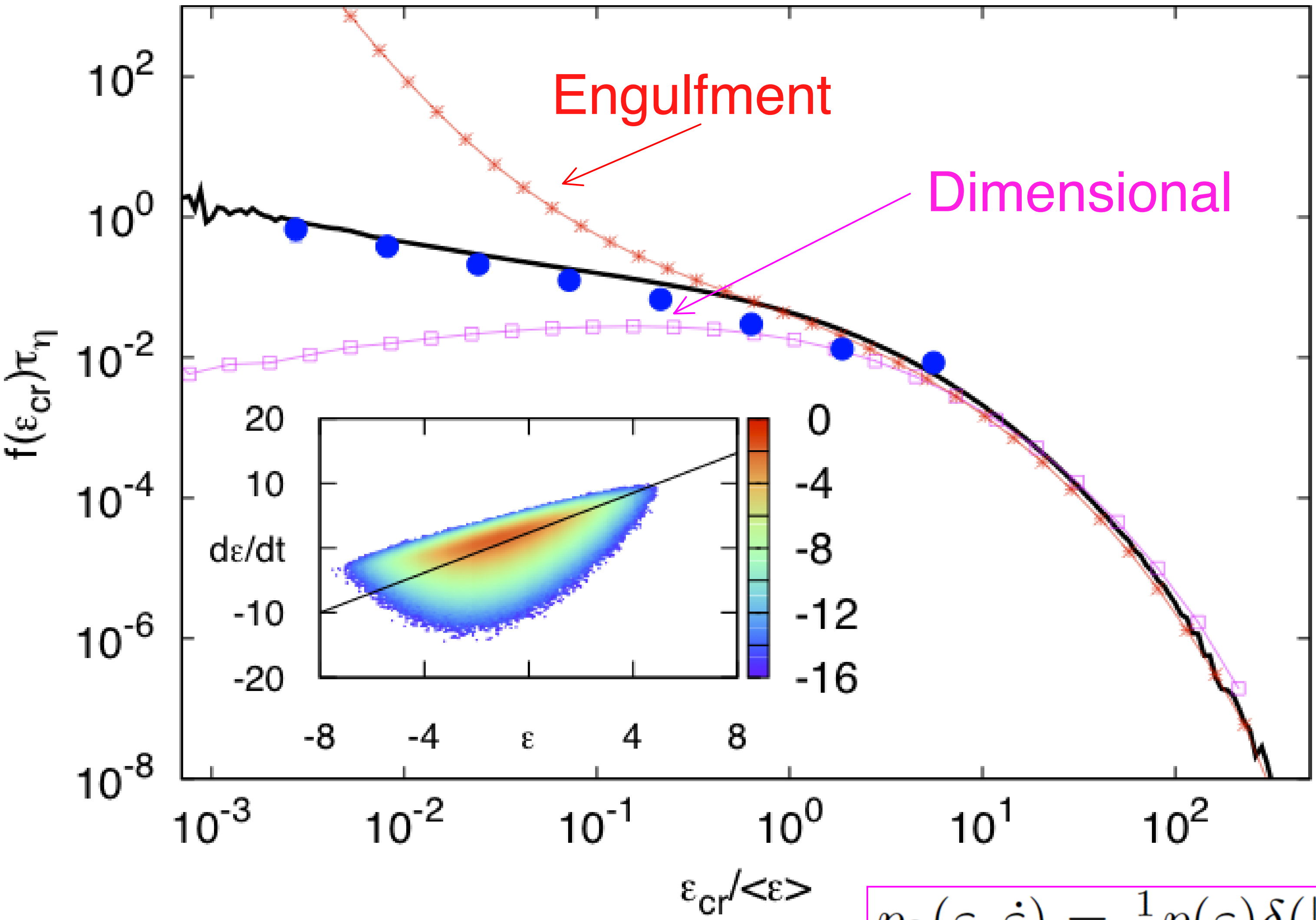
Velocity gradient matrix measured along Lagrangian trajectories

Fragmentation rate:
Lagrangian measure vs Eulerian estimate

Fix thresholds for critical energy dissipation and measure frag. rates.

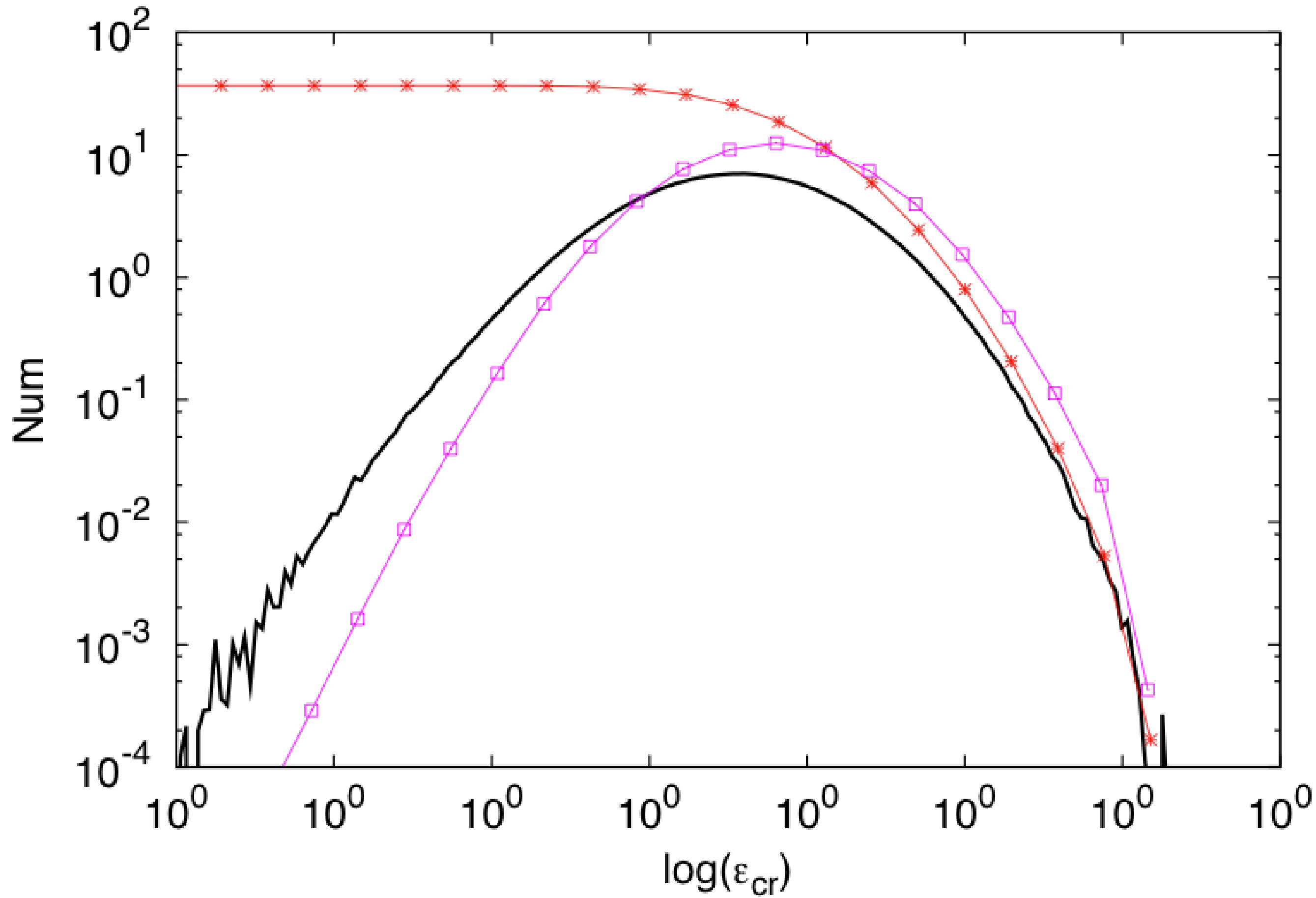


Fragmentation rate:
testing dimensional & engulfment closures



$$p_2(\varepsilon, \dot{\varepsilon}) = \frac{1}{2} p(\varepsilon)\delta(|\dot{\varepsilon}| - \varepsilon/\tau_\eta(\varepsilon))$$

Eulerian break-up rates : origin of differences



$$\int_0^\infty d\dot{\epsilon}\,\dot{\epsilon}\,p_2(\epsilon_{cr},\dot{\epsilon})$$

Engulfment

$$\int_{\epsilon_{cr}}^\infty d\epsilon\; p(\epsilon)/\tau_\eta(\epsilon)$$

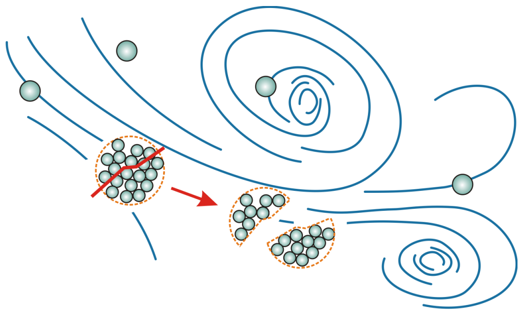
Dimensional

$$\epsilon_{cr}\,p(\epsilon_{cr})/\tau_\eta(\epsilon_{cr})$$

Coalescence/break-up dynamics

--> averaging over space/velocity ensembles

Smoluchowski Equation for adimensional mass distribution *n(ξ,t)*



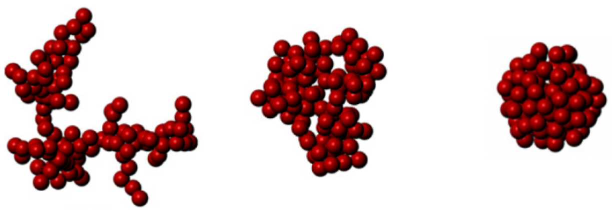
$$\frac{\partial n(\xi,t)}{\partial t} = \frac{1}{2}\int_0^{\xi} k(\vartheta,\xi-\vartheta)n(\vartheta,t)n(\xi-\vartheta,t)d\vartheta \;-\; n(\xi,t)\int_0^{\infty} k(\xi,y)n(\vartheta,t)d\vartheta$$

$$-\, f(\xi)n(\xi,t) \;+\; \int_{\xi}^{\infty} g(\xi,\vartheta)f(\vartheta)n(\vartheta,t)d\vartheta$$

Aggregation rate:
Saffman-Turner type

$$k(\xi,\vartheta) = k_0/\tau_\eta\Big(R(\xi)+R(\vartheta)\Big)^3$$

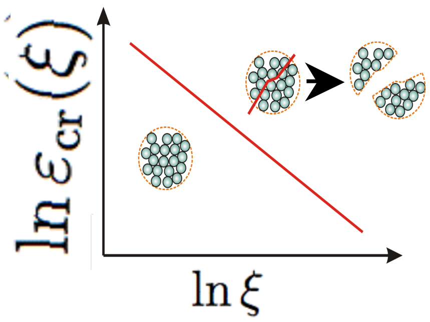
$$R(\xi) \sim \xi^{1/d_f}$$



d_f=1.8 d_f=2.5 d_f=3.0

Aggregate strength:
power law relation

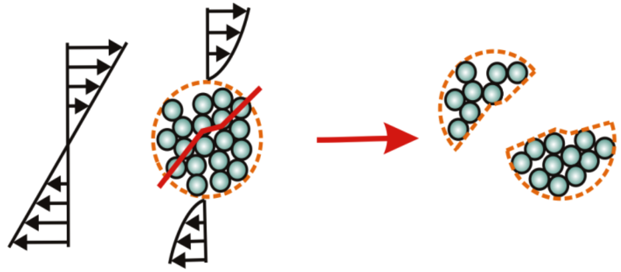
$$\varepsilon_{cr}(\xi) \sim \xi^{-m}$$ *solid*



$$\varepsilon_{cr}(\xi) \sim \xi^{-2/3}$$ *liquid*

Fragmentation:
binary process

$$g(\xi,\vartheta) = 2\delta\Big(\xi - \vartheta/2\Big)$$

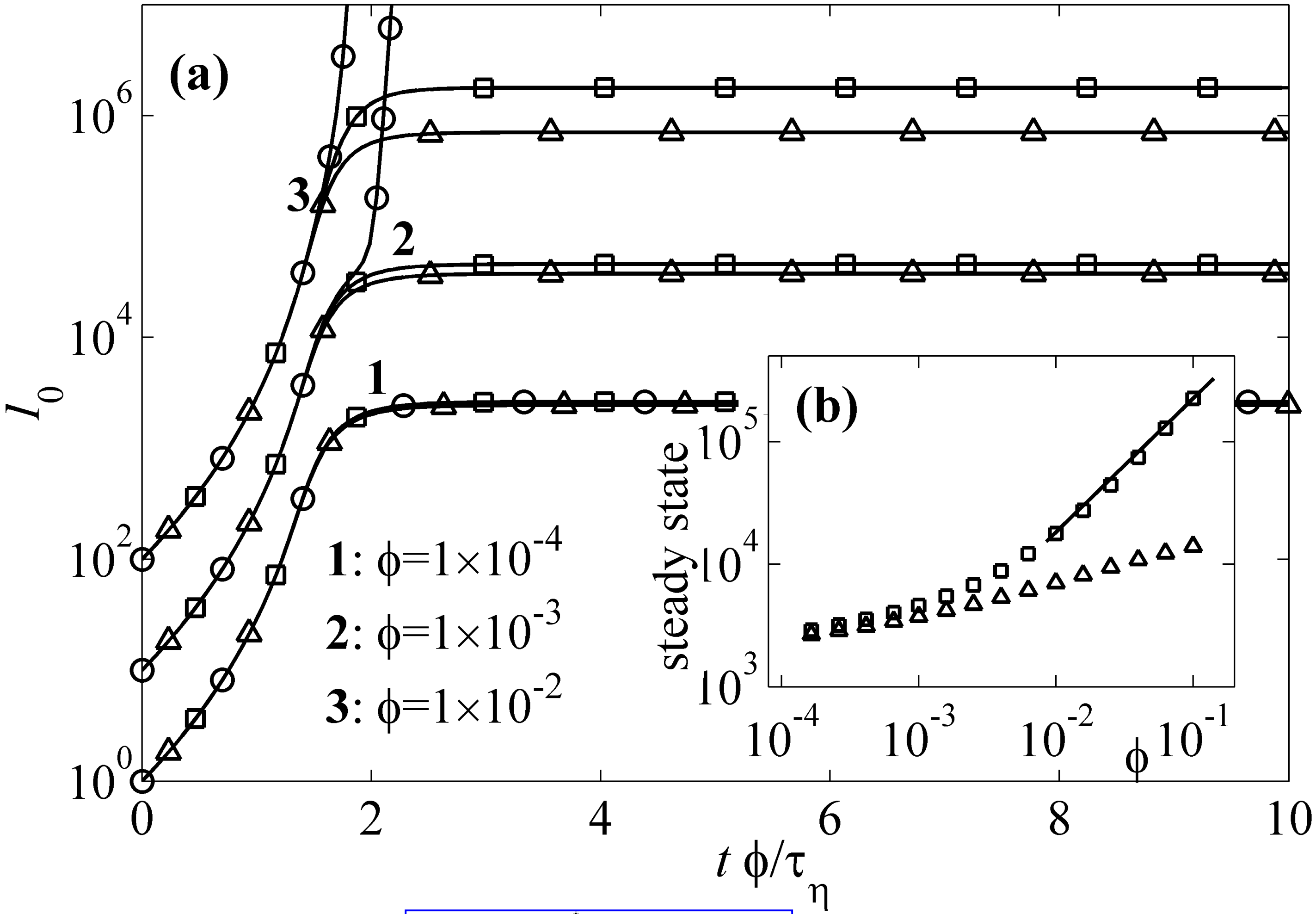


Evolution of aggregate mass distribution

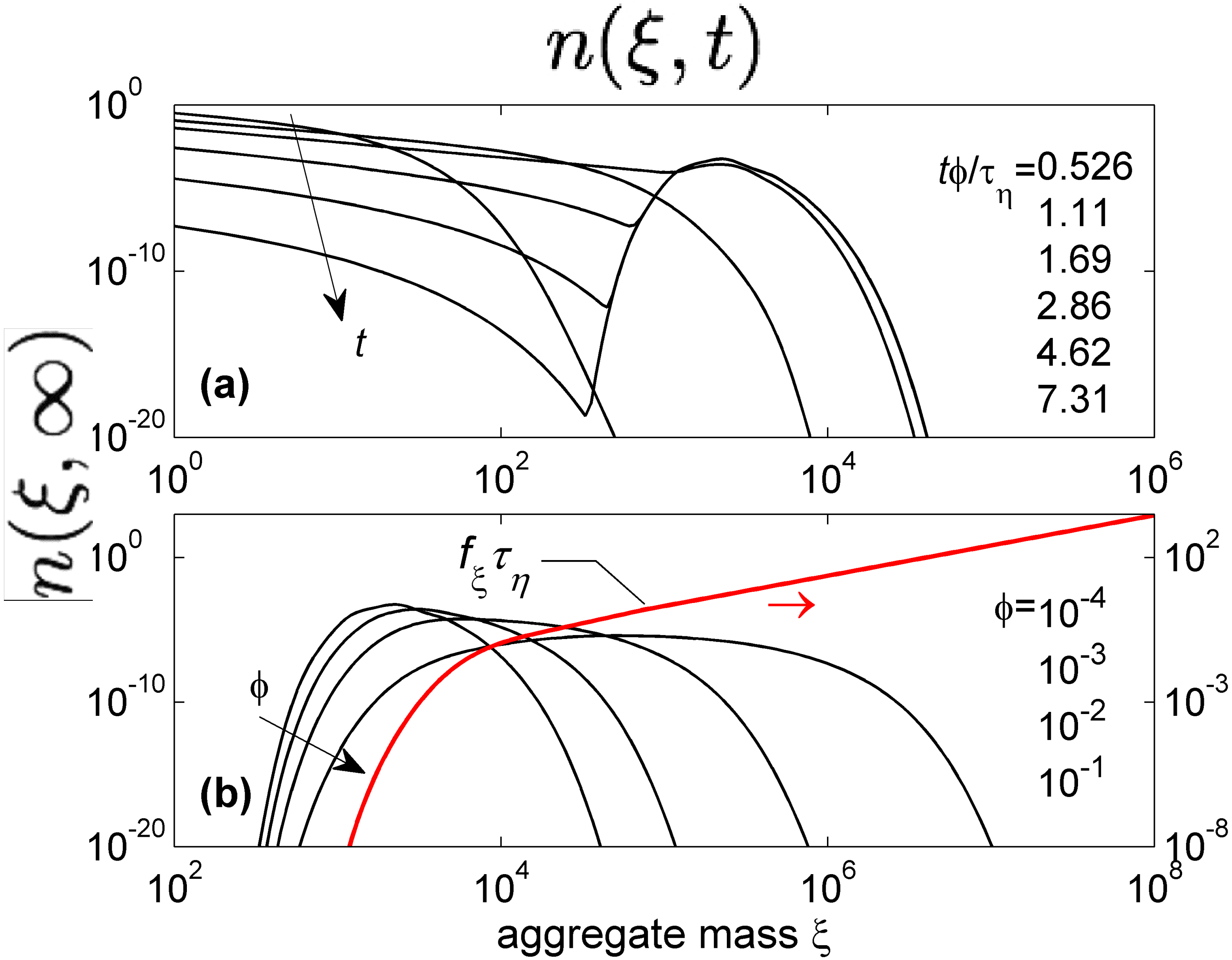
(Volume weighted) Mean aggregate mass from zero-angle light scattering

$$I(0) = \int_0^\infty d\xi \; \xi^2 n_\xi(t$$

- △ Eul. Loginov/Rice
- ○ Dimensional
- □ Engulfment



Control parameter: solid volume fraction $\phi = \frac{4}{3}\pi d^3 N_0$



$n(\xi, t)$
$n(\xi, \infty)$
$t\phi/\tau_\eta$=0.526
1.11
1.69
2.86
4.62
7.31
t
(a)
(b)
$f_\xi \tau_\eta$
ϕ=10^{-4}
10^{-3}
10^{-2}
10^{-1}
ϕ
aggregate mass ξ

Summary

- We have introduced a operational definition of break-up rates for small particles diluted in a turbulent flow.
  Lagrangian first-exit time definition is a natural one, but requires high sampling of the aggregate dynamics along trajectories.

  Eulerian proxy is useful.

- A study of the aggregate mass distribution, averaged over space & velocity fluctuations, reveal a non-trivial influence on fragmentation/caoalescence dynamics of
  breakup events driven by small values of the dissipation

- Ongoing work: i) effects of inertia (local shear and local drag stress)
  ii) beyond Smoluchowski approach ?

M.U Bäbler, L.Biferale, A.S. Lanotte
Breakup of small aggregates driven by hydrodynamical stress submitted to PRL (2011)

For Uriel jubilee conference

"O frati", dissi "che per cento milia
perigli siete giunti a l'occidente,
a questa tanto picciola vigilia

d'i nostri sensi ch'è del rimanente,
non vogliate negar l'esperienza,
di retro al sol, del mondo sanza gente.

Considerate la vostra semenza:
fatti non foste a viver come bruti,
ma per seguir virtute e canoscenza."

(Dante Alighieri, Inferno, Canto XXVI)

"O brothers, who amid a hundred thousand
Perils," I said, "have come unto the West,
To this so inconsiderable vigil

Which is remaining of your senses still
Be unwilling to deny the knowledge,
Following the sun, of the unpeopled world.

Consider the seed from which you sprang;
You were not made to live like unto brutes,
But for pursuit of virtue and of knowledge."