

# Constraining the 2HDM parameter space

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work in progress with A. El Kaffas, O. M. Øgreid

Bottom line:

Parameter space very constrained

# Define model: 2HDM (II)

Potential:

$$\begin{aligned} V = & \frac{\lambda_1}{2}(\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\ & + \frac{1}{2} \left[ \lambda_5(\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right] + \left\{ \left[ \lambda_6(\Phi_1^\dagger \Phi_1) + \lambda_7(\Phi_2^\dagger \Phi_2) \right] (\Phi_1^\dagger \Phi_2) + \text{h.c.} \right\} \\ & - \frac{1}{2} \left\{ m_{11}^2(\Phi_1^\dagger \Phi_1) + \left[ m_{12}^2(\Phi_1^\dagger \Phi_2) + \text{h.c.} \right] + m_{22}^2(\Phi_2^\dagger \Phi_2) \right\} \end{aligned}$$

Allow CP violation:

$\lambda_5, \lambda_6, \lambda_7, m_{12}^2$  may be **complex**

Neutral sector:  $3 \times 3$  mixing matrix  $\alpha = (\alpha_1, \alpha_2, \alpha_3)$

$$-\frac{\pi}{2} < \alpha_i \leq \frac{\pi}{2}, \quad i = 1, 2, 3$$

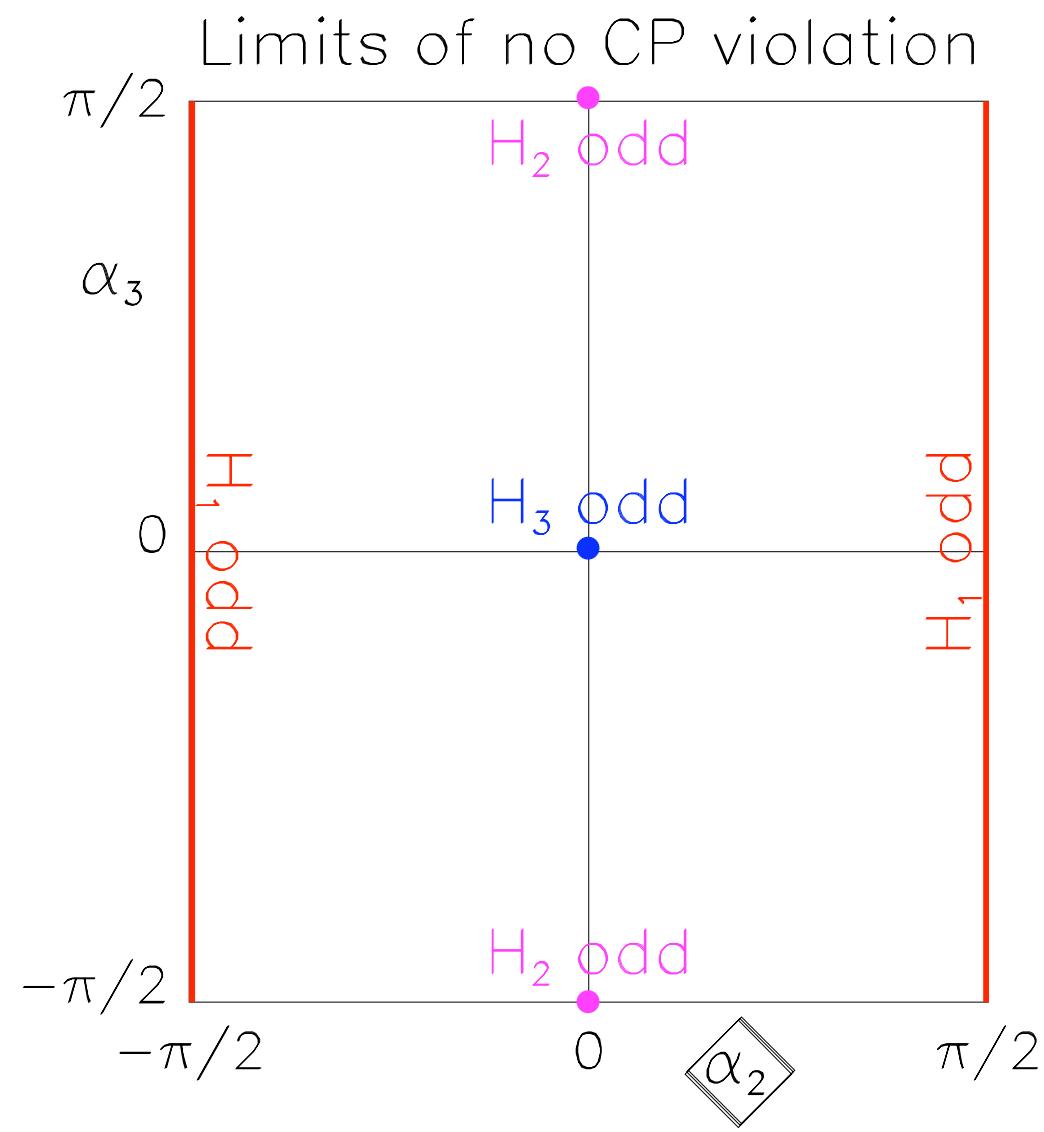
Today:  $\lambda_6 = \lambda_7 = 0$

# Rotation matrix

$$R\mathcal{M}^2R^T = \mathcal{M}_{\text{diag}}^2 = \text{diag}(M_1^2, M_2^2, M_3^2)$$

$$R = \begin{pmatrix} c_1 c_2 & s_1 c_2 & s_2 \\ -(c_1 s_2 s_3 + s_1 c_3) & c_1 c_3 - s_1 s_2 s_3 & c_2 s_3 \\ -c_1 s_2 c_3 + s_1 s_3 & -(c_1 s_3 + s_1 s_2 c_3) & c_2 c_3 \end{pmatrix}$$

$$c_i = \cos \alpha_i, \quad s_i = \sin \alpha_i$$



**Parameters:**

$$\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \mu^2, \tan \beta$$

↑  
complex

or:

$$M_1 \leq M_2 \leq M_3, M_{H^\pm}, \tan \beta, \mu^2, \alpha_1, \alpha_2, \alpha_3$$

**Input parameters:**

$$M_1 \leq M_2, M_{H^\pm}, \tan \beta, \mu^2, (\alpha_1, \alpha_2, \alpha_3)$$

**Calculate:**

$$M_3, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$$

**Conditions:**

$$M_2 \leq M_3$$

$$V(\Phi_1, \Phi_2) > 0$$

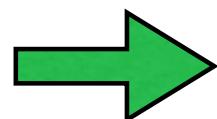
# Special property

Neutral sector mass matrix (squared)  
given by second derivatives of potential

For  $\text{Im } \lambda_5 \neq 0$

have

$$\mathcal{M}_{13}^2 = \tan \beta \mathcal{M}_{23}^2$$



$$\sum_k M_k^2 R_{k3} (R_{k1} - R_{k2} \tan \beta) = 0$$

Determine  $M_3$  from  $M_1 \leq M_2, \tan \beta, (\alpha_1, \alpha_2, \alpha_3)$

## Yukawa couplings (Model II)

$$H_j b \bar{b} : \quad \frac{1}{\cos \beta} [R_{j1} - i \gamma_5 \sin \beta R_{j3}]$$

$$H_j t \bar{t} : \quad \frac{1}{\sin \beta} [R_{j2} - i \gamma_5 \cos \beta R_{j3}]$$

$$H^+ b \bar{t} : \quad \frac{ig}{2\sqrt{2}m_W} [m_b(1 + \gamma_5) \tan \beta + \textcolor{red}{m_t}(1 - \gamma_5) \cot \beta]$$

$$H^- t \bar{b} : \quad \frac{ig}{2\sqrt{2}m_W} [m_b(1 - \gamma_5) \tan \beta + \textcolor{red}{m_t}(1 + \gamma_5) \cot \beta]$$

Important at low  $\tan \beta$

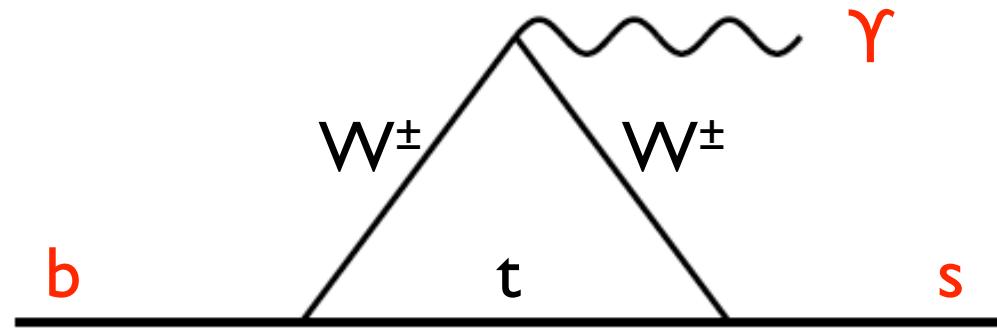
# Constraints (three killers):

- Positivity
- Perturbative unitarity
- Experimental constraints

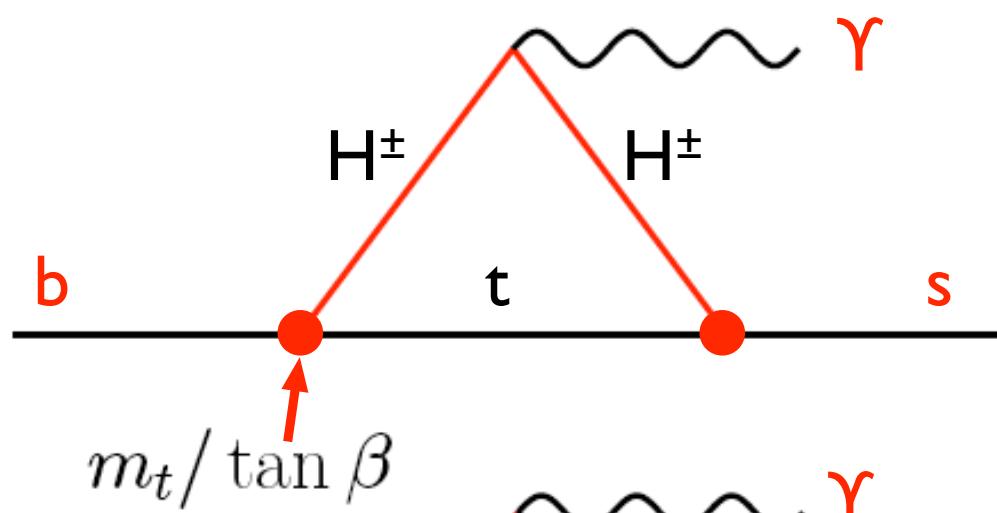
# Experimental constraints:

Independent of neutral sector	● $B \rightarrow X_s \gamma$	excludes low $M_{H^\pm}$
	● $B \rightarrow \bar{B}$ oscillations	excludes low $\tan\beta$
	● $B \rightarrow \tau V$	excludes high $\tan\beta$ , low $M_{H^\pm}$
Depend on neutral sector	● $\Gamma_Z \rightarrow b\bar{b}$	excludes low $\tan\beta$
	● LEP2 non-discovery	light H decouples
	● $\Delta\rho$	spectrum compact
	● $(g-2)_\mu$	rel. only at very large $\tan\beta$

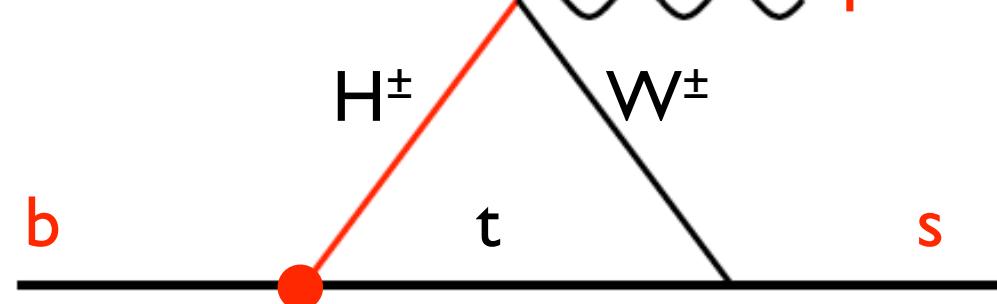
$B \rightarrow X_s \gamma$



Sensitive to  
BSM physics



$m_t / \tan \beta$



$B \rightarrow X_s \gamma$

Misiak et al, 2006 (NNLO):

$$\mathcal{B}(\bar{B} \rightarrow X_s \gamma) = \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \frac{6\alpha_{\text{e.m.}}}{\pi C} \left\{ P(E_0) + N(E_0) \right\}$$

$$\times \mathcal{B}(\bar{B} \rightarrow X_c e \bar{\nu}_e)_{\text{exp}}$$

$|C_7^{(0)\text{eff}}(\mu_b)|^2$  at LO

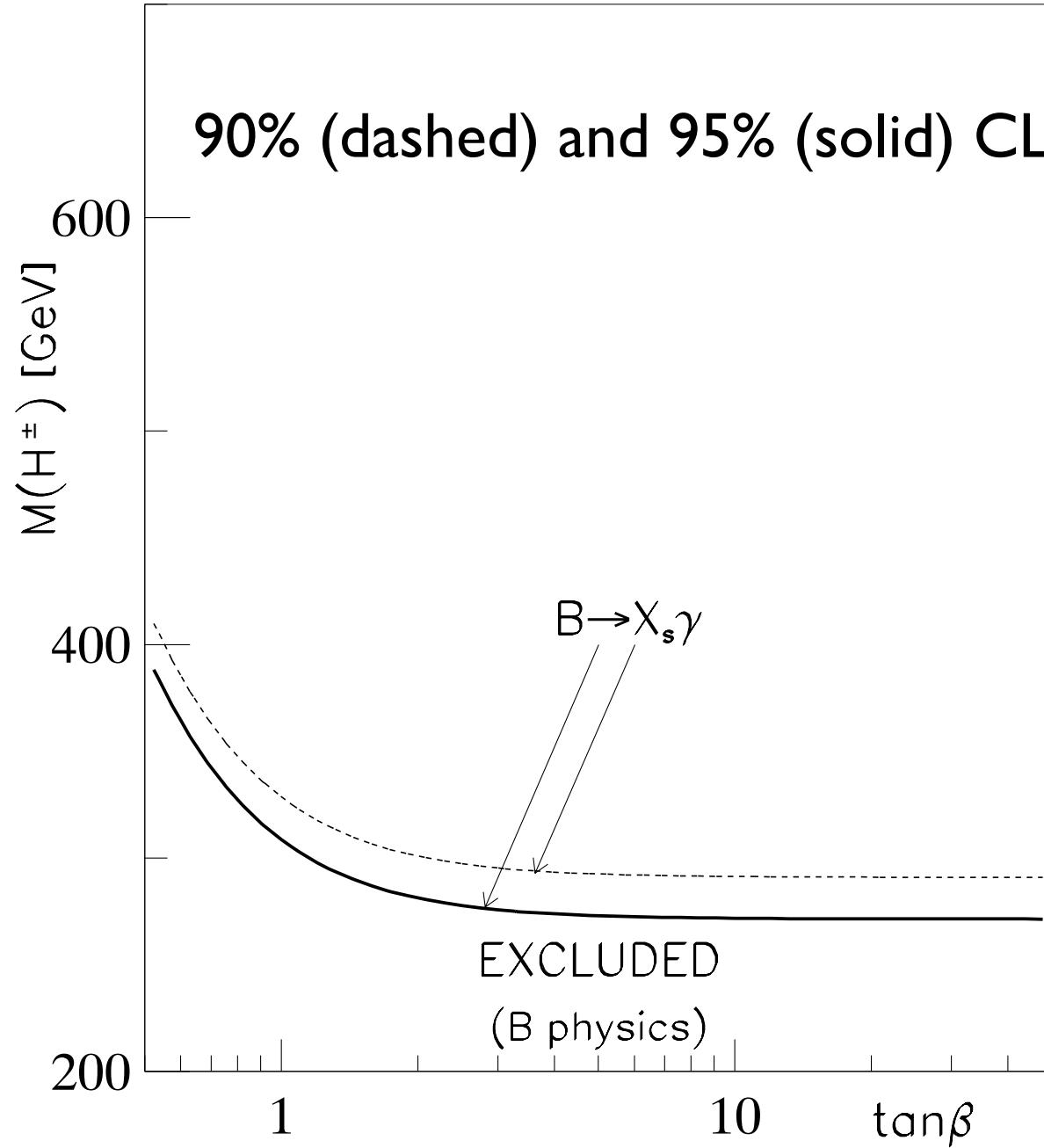
Misiak et al:  $\mathcal{B}(\bar{B} \rightarrow X_s \gamma) = (3.15 \pm 0.23) \times 10^{-4}$

HFAG (exp):  $\mathcal{B}(\bar{B} \rightarrow X_s \gamma) = (3.55 \pm 0.24 \pm \dots) \times 10^{-4}$

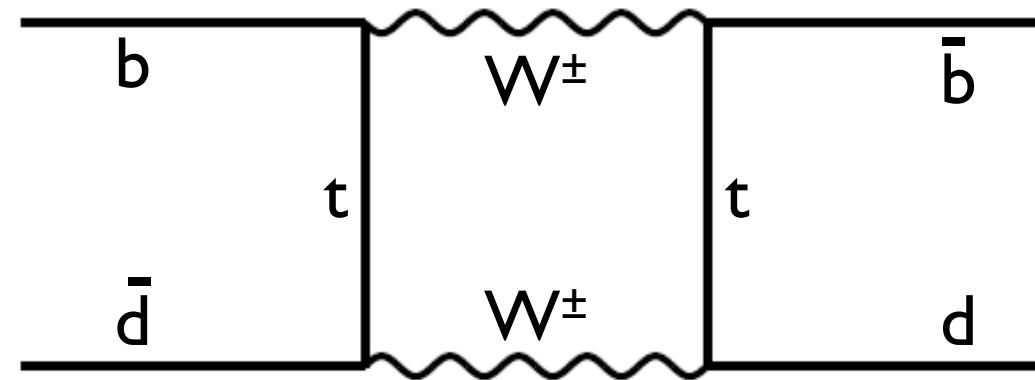
Measure of discrepancy:

$$\chi^2_{b \rightarrow s \gamma} = \frac{[\mathcal{B}(\bar{B} \rightarrow X_s \gamma)_{\text{2HDM}} - \mathcal{B}(\bar{B} \rightarrow X_s \gamma)_{\text{ref}}]^2}{\{\sigma[\mathcal{B}(\bar{B} \rightarrow X_s \gamma)]\}^2}$$

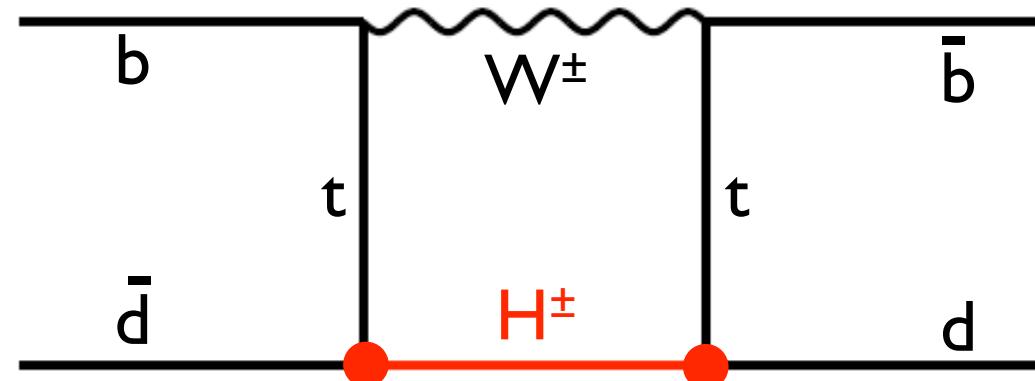
$$\sigma[\mathcal{B}(\bar{B} \rightarrow X_s \gamma)] = 0.25 \times 10^{-4}$$



$B \rightarrow \bar{B}$



Sensitive to  
BSM physics



$B \rightarrow \bar{B}$  Urban, Krauss, Jentschura, Soff 1998 (NLO):

$$x_d \equiv \frac{\Delta m_{B_d}}{\Gamma_B} = \frac{G_F^2}{6\pi^2} |V_{td}^*|^2 |V_{tb}|^2 f_B^2 B_B m_B \eta \tau_B M_W^2 S_{2HDM}$$



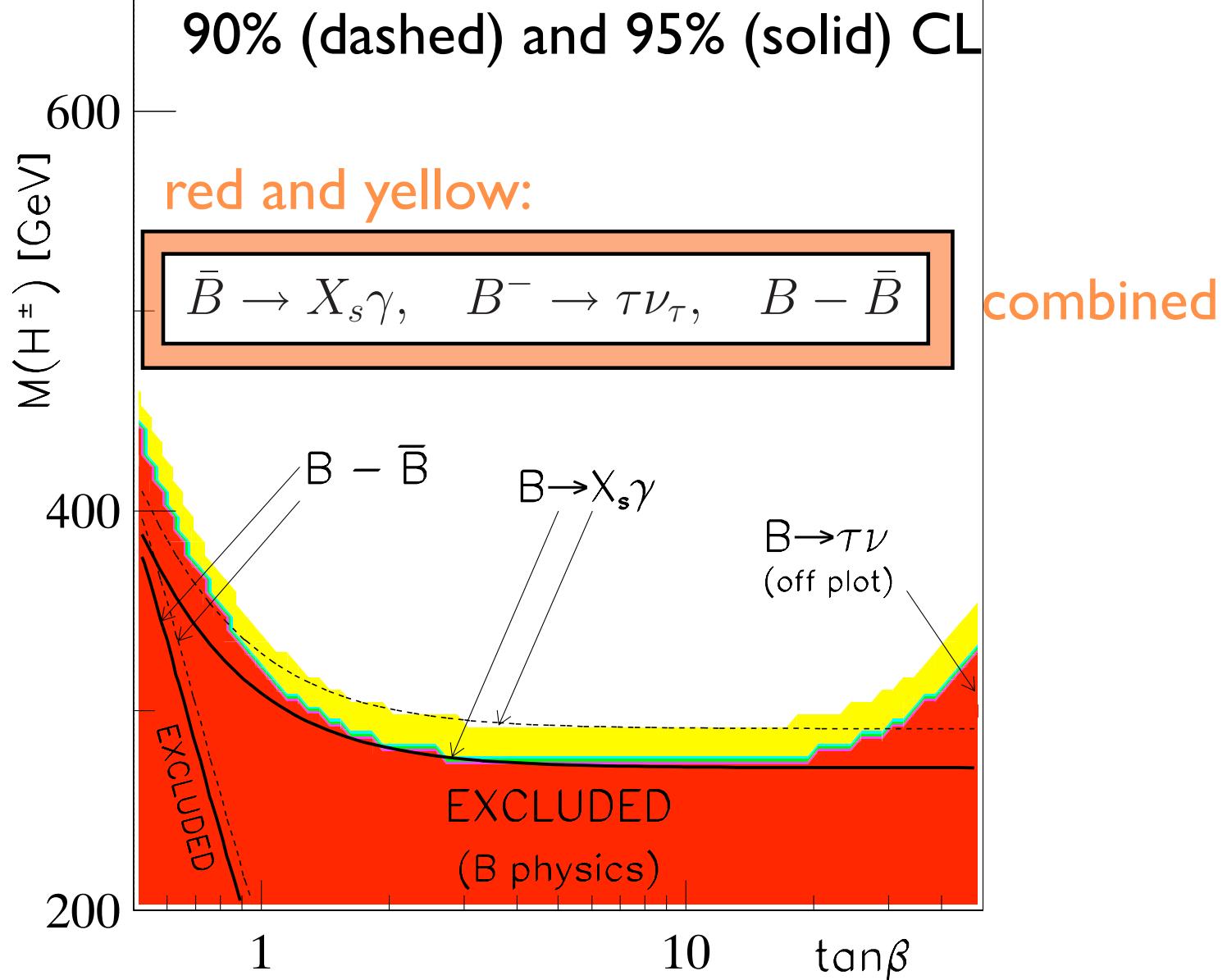
QCD corrections

Inami-Lim functions:

$$S_{2HDM} = S_{WW} + 2S_{WH} + S_{HH}$$
$$\uparrow \quad \uparrow$$
$$1/\tan^2 \beta \quad 1/\tan^4 \beta$$

from Higgs Yukawa couplings

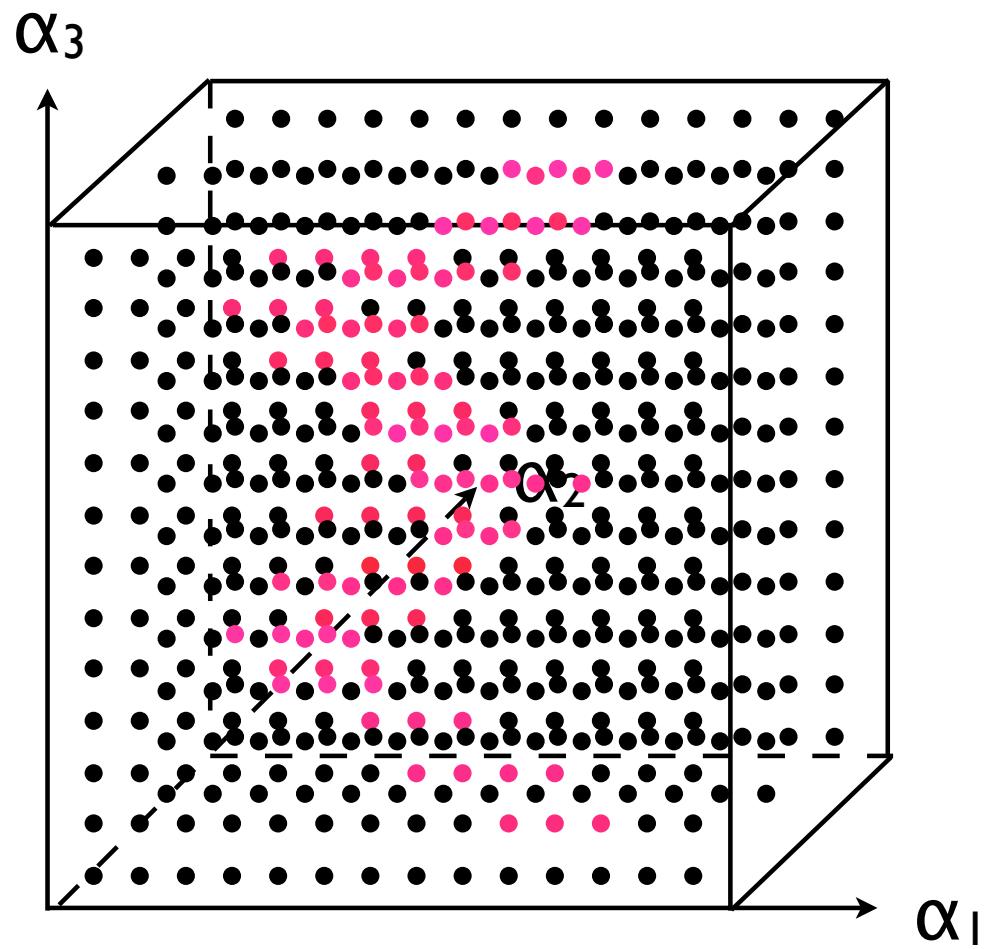
$\eta \times S_{2HDM}$  has weaker dependence on  
 $\tan \beta$  and  $M_{H^\pm}$  than  $S_{2HDM}$  2.7 vs. 4.5



# Consistency of Neutral Sector

## Positivity

- Choose  $M_1 \leq M_2, \mu^2$
- Loop over  $\tan \beta, M_{H^\pm}$
- For each  $\tan \beta, M_{H^\pm}$  scan over  $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$



# Consistency of Neutral Sector

## Positivity

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- For each  $\tan \beta, M_{H^\pm}$  scan over  $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$
- Count fraction of points where positivity is satisfied.
- Result: about 20%, denote these points  $\boldsymbol{\alpha}_+$

# Reference (mass) points

Name	$M_1$ [GeV]	$M_2$ [GeV]	$\mu^2$ [GeV] <sup>2</sup>
“100-300”	100	300	0 [ $\pm(200)^2$ ]
“150-300”	150	300	0 [ $\pm(200)^2$ ]
“100-500”	100	500	0 [ $\pm(200)^2$ ]
“150-500”	150	500	0 [ $\pm(200)^2$ ]

Table 1: Reference masses.

Recall:  $M_1 \leq M_2 \leq M_3$



calculated from  $(\alpha_1, \alpha_2, \alpha_3)$

# Unitarity

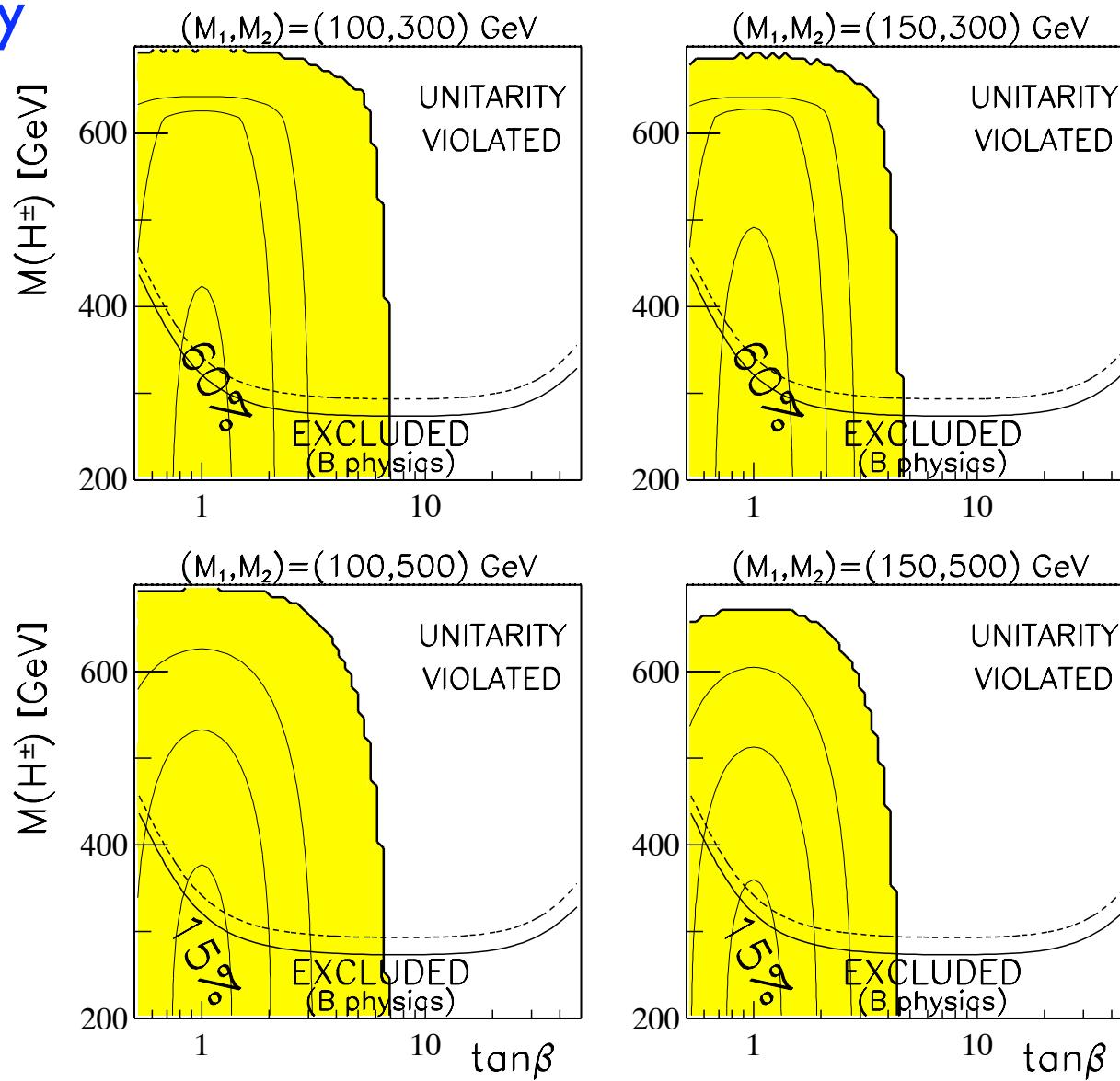
Higgs-Higgs scattering

Kanemura, Kubota, Takasugi (1993);  
Akeroyd, Arhrib, Naimi (2000);  
Ginzburg, Ivanov (2003, 2005)

Now focus on region not excluded by charged-Higgs constraints

- Choose  $M_1 \leq M_2, \mu^2$
- Loop over  $\tan \beta, M_{H^\pm}$
- Scan over  $\alpha_+$
- Count fraction of points where unitarity is satisfied.
- Result: up to 60%  $\hat{\alpha} \in \alpha_+ \in \alpha$
- Peaked at low  $\tan \beta$
- Cut off at high  $\tan \beta$  and high  $M_{H^\pm}$

## Unitarity



Isospin 0,  
hypercharge 0  
channel most  
restrictive at  
 $\tan\beta$

$$\mu^2 = 0$$

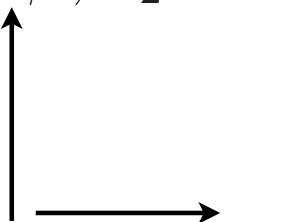
Note cut-off at 'large'  $\tan\beta$

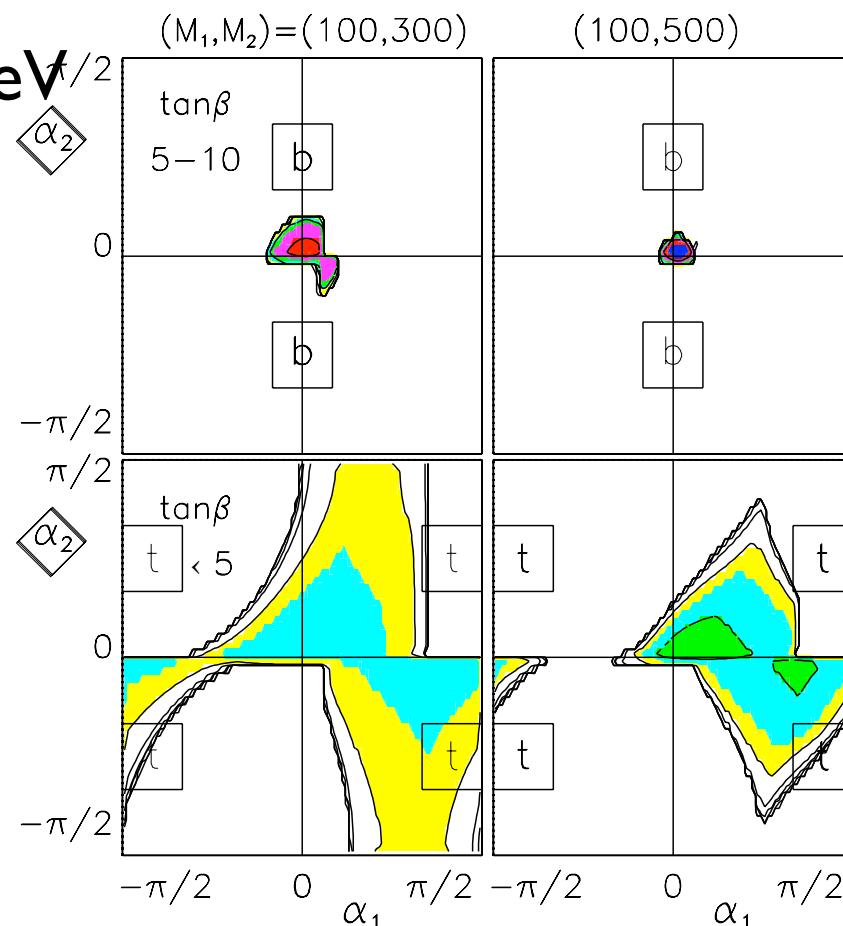
Regions in  $(\alpha_1, \alpha_2)$   
populated by allowed  
(by unitarity) solutions

$M_1, M_2 = 100, 300 \text{ & } 500 \text{ GeV}^{1/2}$

$\mu=0$

2 slices of  $\tan\beta$

$\tan\beta, \alpha_2$   


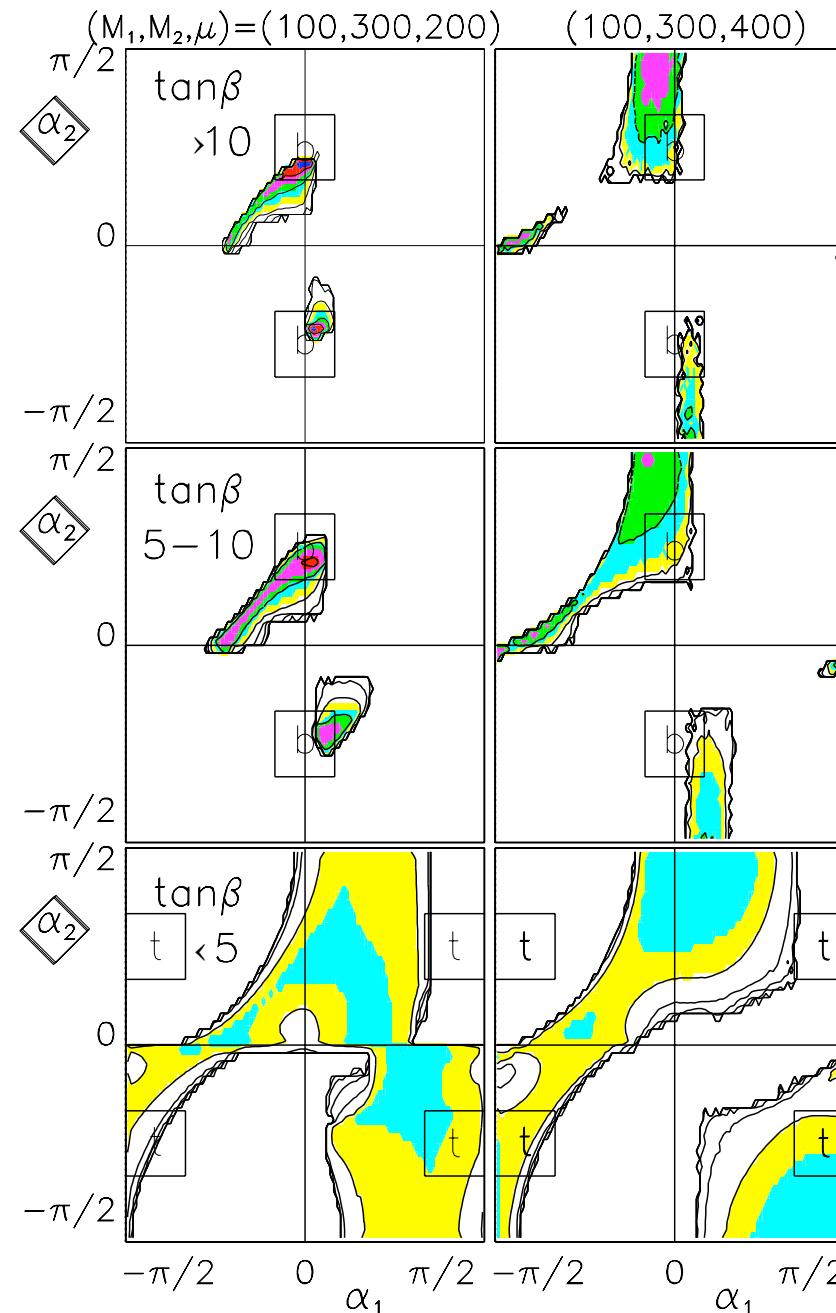
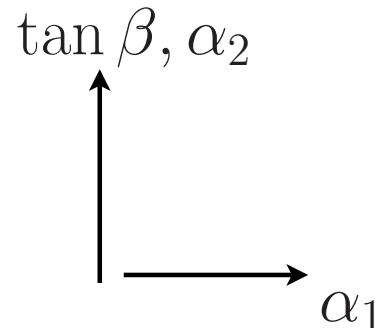


Regions in  $(\alpha_1, \alpha_2)$   
populated by allowed  
(by unitarity) solutions

$M_1, M_2 = 100, 300 \text{ GeV}$

$\mu = 200, 400 \text{ GeV}$

3 slices of  $\tan\beta$



# Experimental Bounds depending on Neutral Sector

- Choose  $M_1 \leq M_2, \mu^2$
- Loop over  $\tan \beta, M_{H^\pm}$
- Scan over  $\hat{\alpha} \in \alpha_+ \in \alpha$
- Form  $\chi^2$
- Select point in  $\hat{\alpha}$  with lowest  $\chi^2$   
*(try to be as generous as possible)*

$M_1, M_2, M_3 =$   
 $100, 300, 500 \text{ GeV}$

White: positivity violated

Yellow: unitarity violated

Red: LEP2 search violated

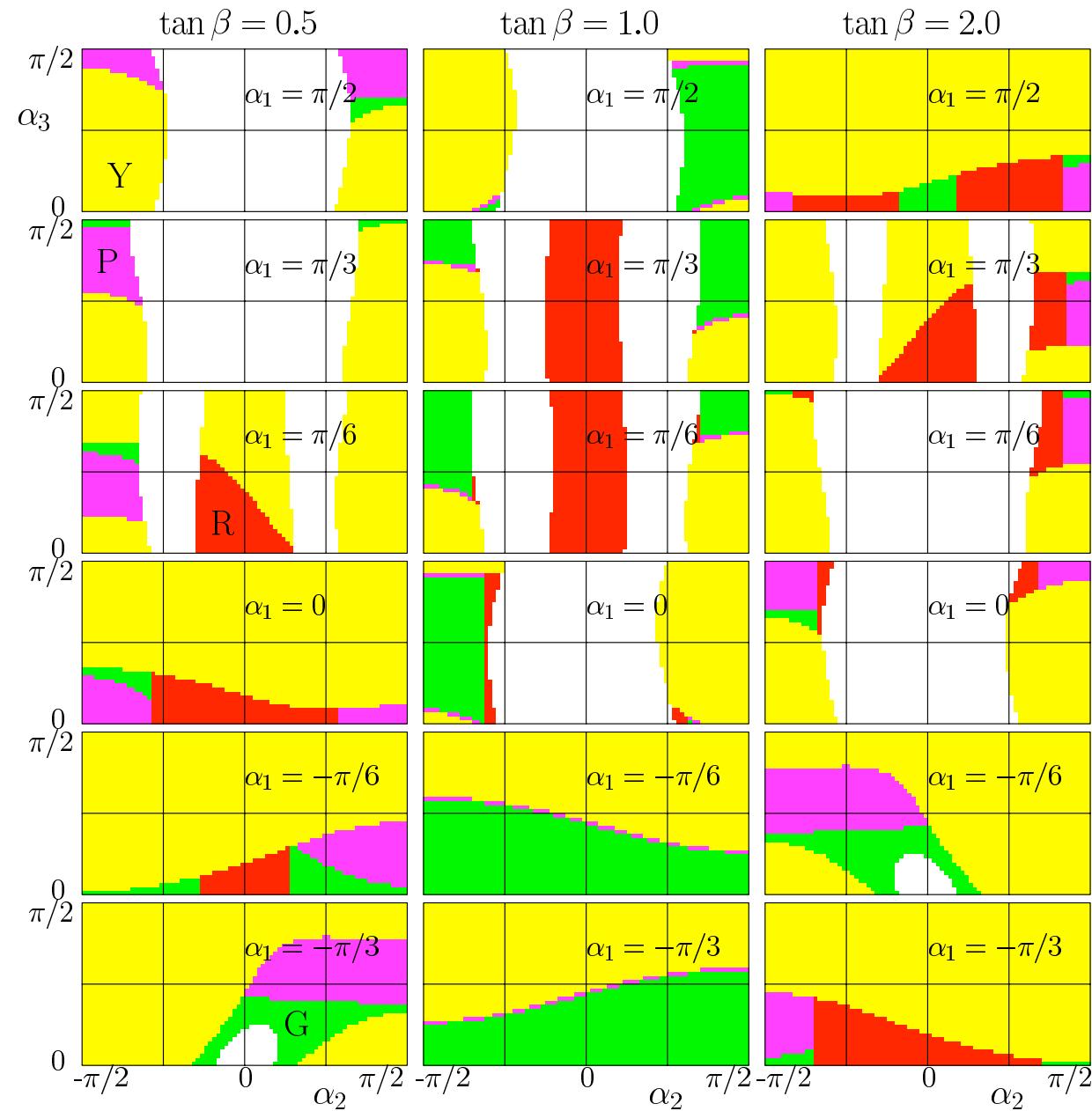
Purple:  $\Delta\rho$  violated

Green: OK

$\alpha_1, \alpha_3$



$\tan\beta, \alpha_2$

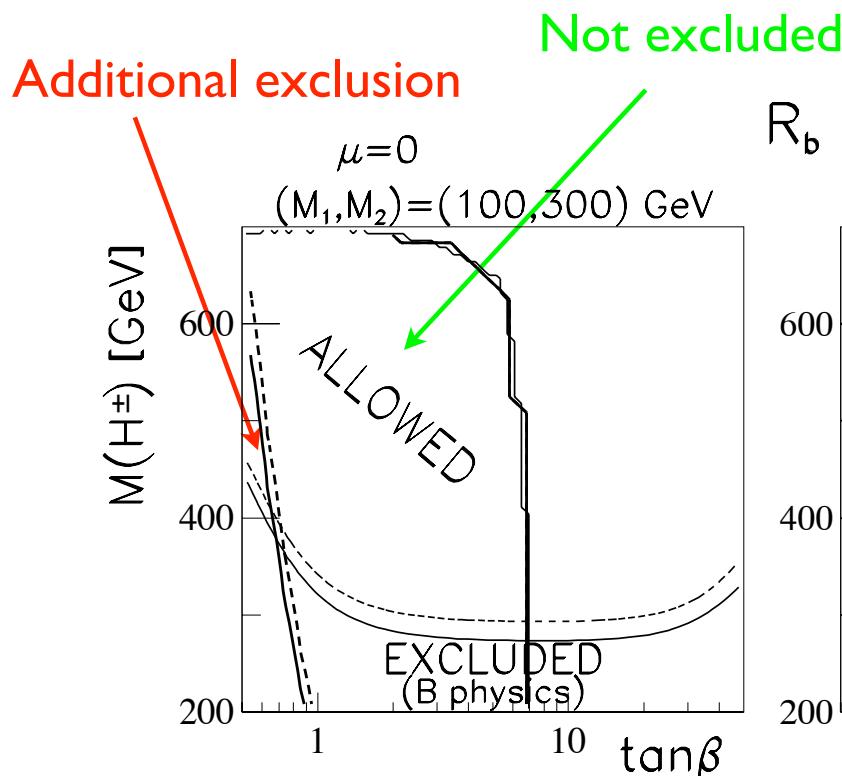


- For fixed  $\tan \beta$  and  $M_{H^\pm}$  take

$$\hat{\chi}_i^2 = \min_{\hat{\alpha} \in \alpha_+} \chi_i^2$$

where  $\chi_i^2$  is minimized over the part  $\hat{\alpha}$  of the  $\alpha_+$  space for which positivity and also unitarity are satisfied.

$\Gamma_{Z \rightarrow b\bar{b}} \text{ or } R_b$

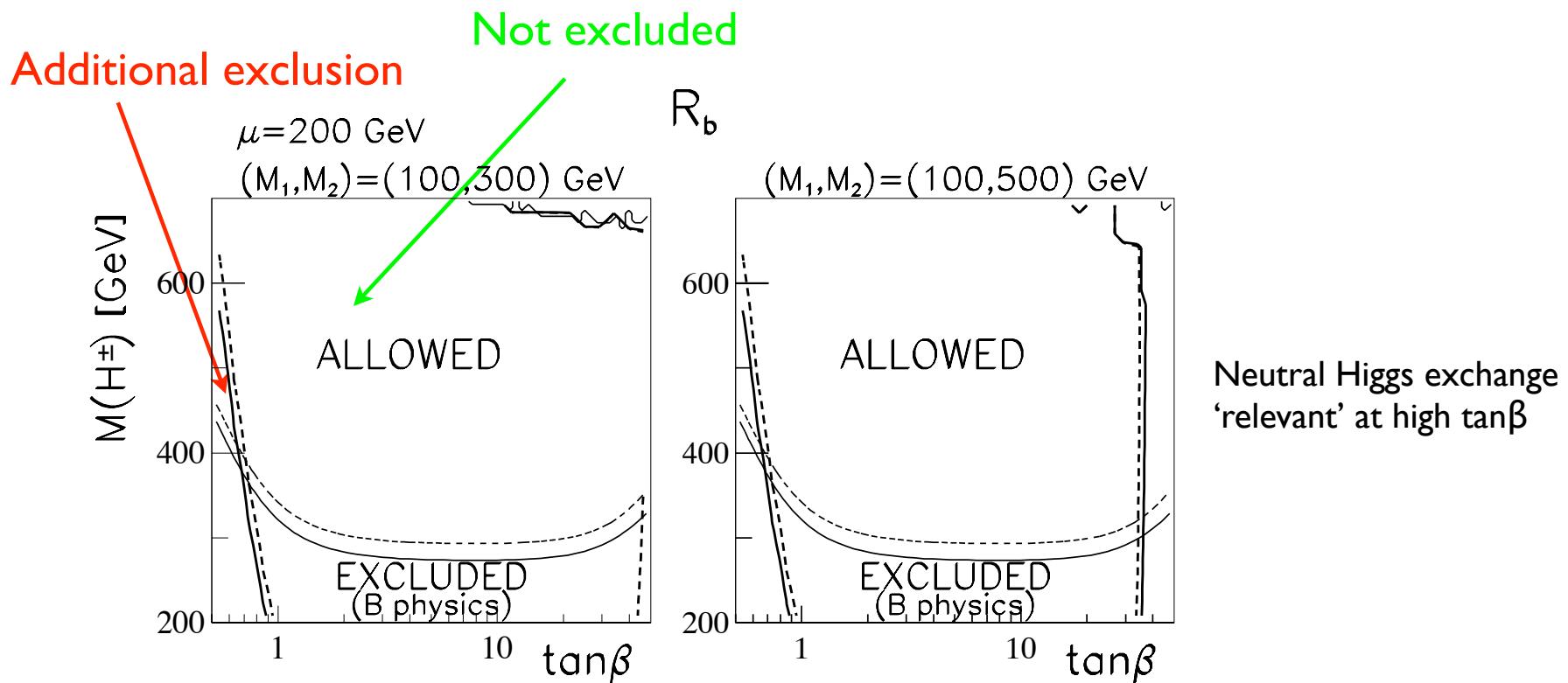


$$\mu^2 = 0$$

Denner, Guth, Hollik,  
Kuhn (1991) extended  
to CP-violating case in  
El Kaffas et al, hep-ph/  
0605142  
(Neutral Higgs exchange  
only ‘relevant’ at high  
 $\tan\beta$ )

Consider as not excluded: region where  
some  $(\alpha_1, \alpha_2, \alpha_3) \in \hat{\alpha}$  satisfies  $\chi^2 < \chi^2(90\%, 95\% \text{CL})$

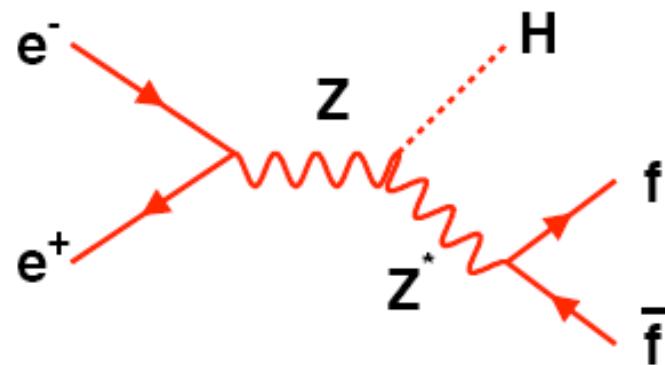
$\Gamma_{Z \rightarrow b\bar{b}} \text{ or } R_b$



$$\mu^2 = (200 \text{ GeV})^2$$

## LEP2

### Production mechanism at LEP



No SM Higgs particle found up to  $M=115$  GeV

Less exclusion in 2HDM

2HDM:

$$\sigma_{Z(h \rightarrow X)} = \sigma_{Zh}^{\text{ew}} \times C_{Z(h \rightarrow X)}^2$$

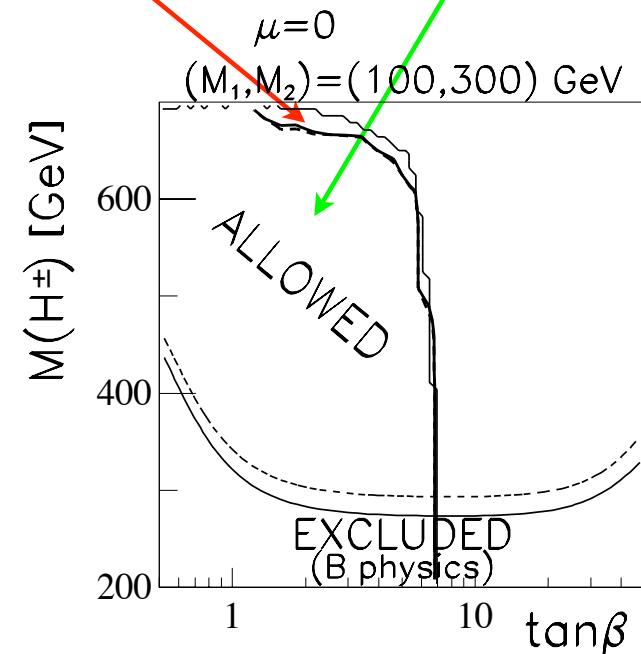
suppressed production:  $H_j ZZ : [ \cos \beta R_{j1} + \sin \beta R_{j2} ], \quad \text{for } j = 1$

modified  $b\bar{b}$  decay rate:  $\frac{1}{\cos^2 \beta} [R_{11}^2 + \sin^2 \beta R_{13}^2]$

## LEP2 non-discovery

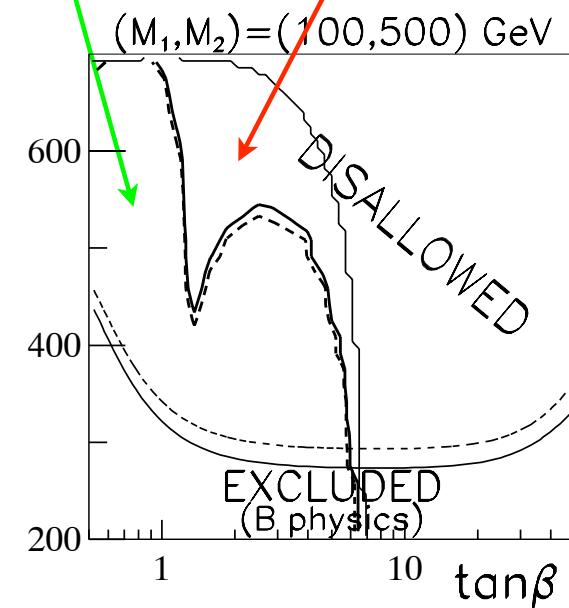
Additional exclusion

Not excluded



LEP2

Additional exclusion

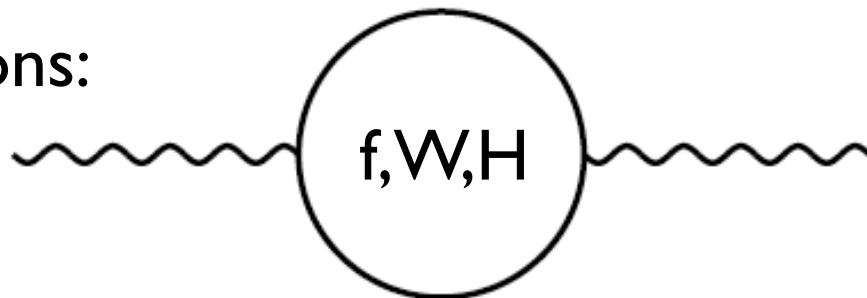


$$\mu^2 = 0$$

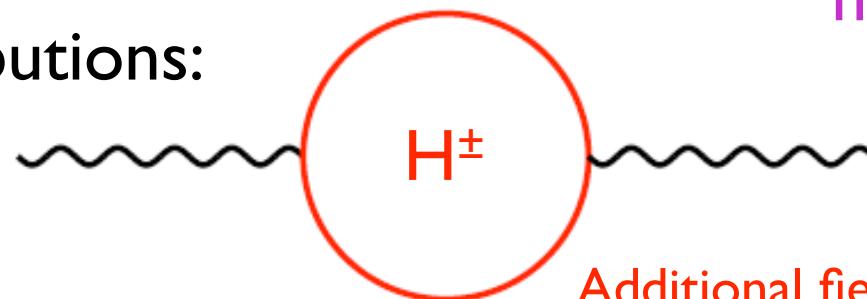
$$\Delta\rho$$

$\rho$  parameter measures difference of self energies of W and Z

SM contributions:



2HDM contributions:

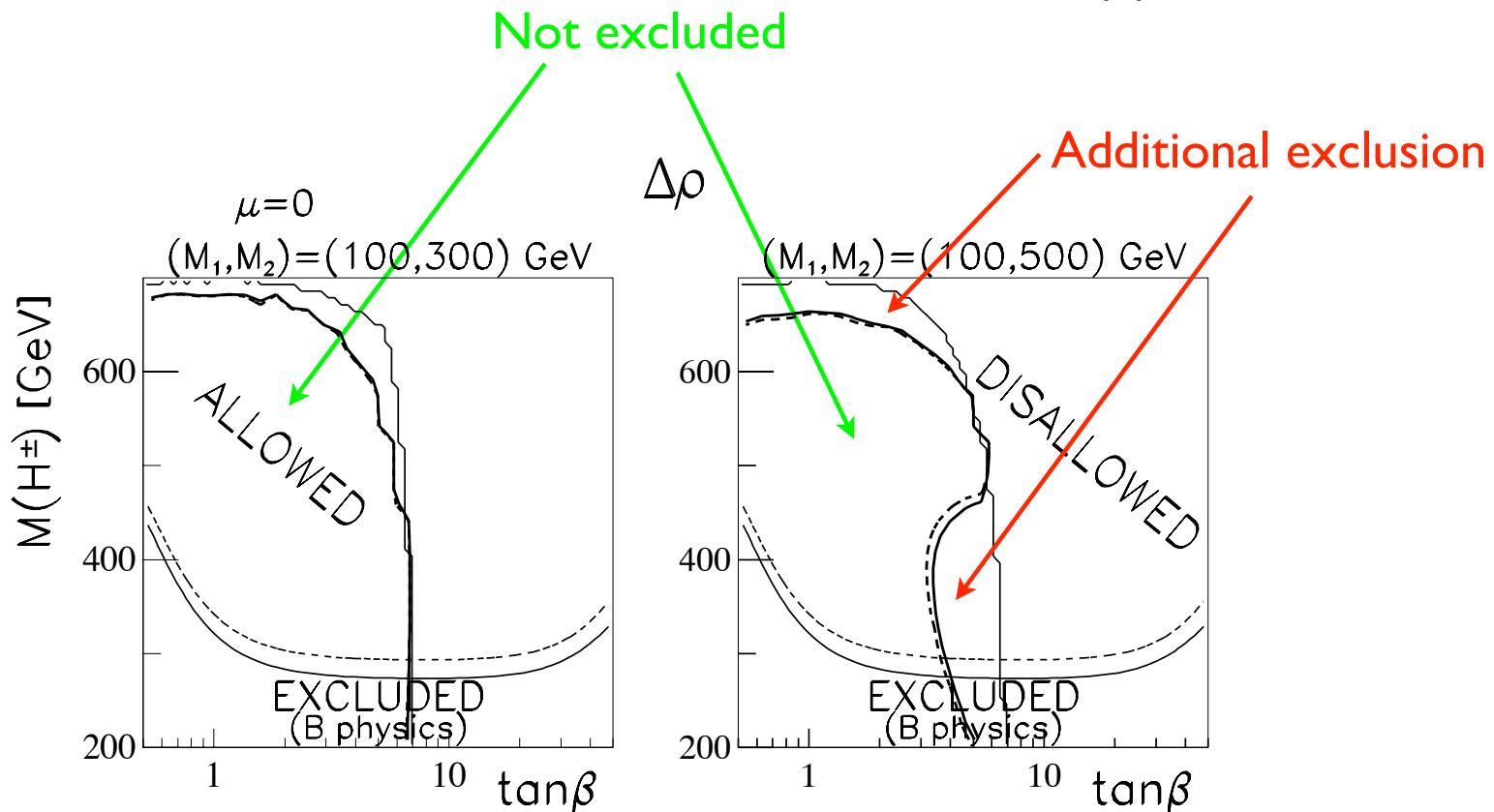


many diagrams

Additional fields must have masses  
“close” to  $M_W, M_Z$  and  
masses not too widely spaced

$\Delta\rho$

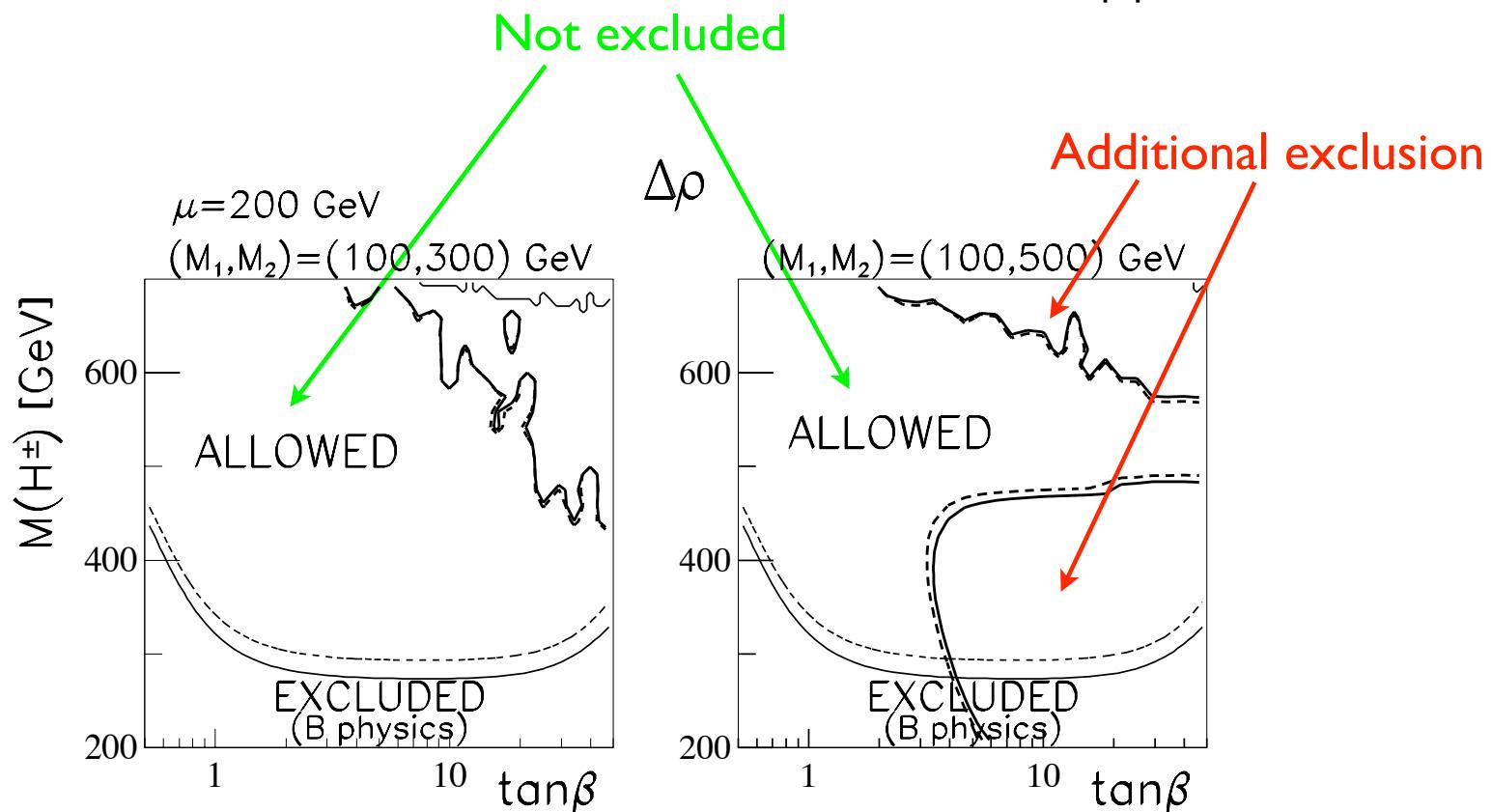
Bertolini (1986)  
extended to CP-violating  
case in El Kaffas et al,  
hep-ph/0605142



$$\mu^2 = 0$$

$\Delta\rho$

Bertolini (1986)  
extended to CP-violating  
case in El Kaffas et al,  
hep-ph/0605142



$$\mu^2 = (200 \text{ GeV})^2$$

$\Delta\rho$  contributions       $\tan\beta \gg 1$

$$\begin{aligned}
 & A_{WW}^{HH}(0) - \cos^2 \theta_W A_{ZZ}^{HH}(0) \\
 & \rightarrow \frac{g^2}{64\pi^2} \sum_j \left[ (R_{j1}^2 + R_{j3}^2) F_{\Delta\rho}(M_{H^\pm}^2, M_j^2) \right. \\
 & \quad \left. - \sum_{k>j} (R_{j1}R_{k3} - R_{k1}R_{j3})^2 F_{\Delta\rho}(M_j^2, M_k^2) \right]
 \end{aligned}$$

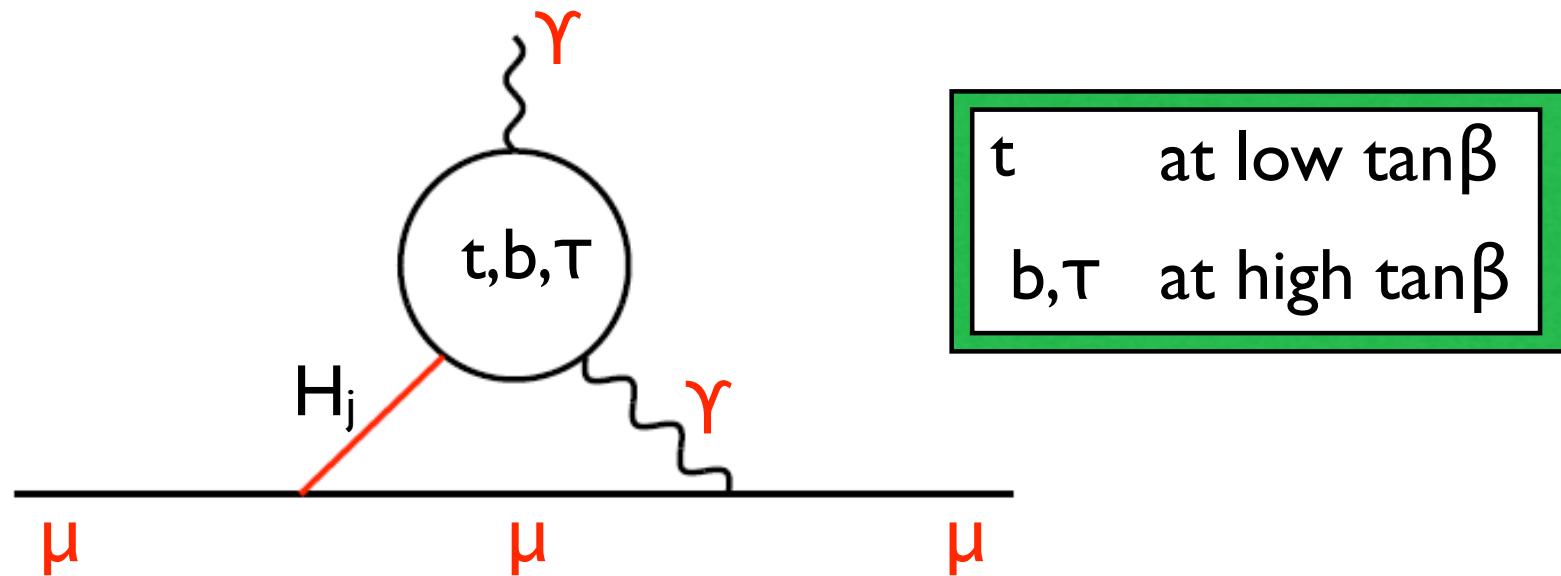
$$\begin{aligned}
 & A_{WW}^{HG}(0) - \cos^2 \theta_W A_{ZZ}^{HG}(0) \\
 & \rightarrow \frac{g^2}{64\pi^2} \left[ \sum_j R_{j2}^2 \left( 3F_{\Delta\rho}(M_Z^2, M_j^2) - 3F_{\Delta\rho}(M_W^2, M_j^2) \right) \right. \\
 & \quad \left. + 3F_{\Delta\rho}(M_W^2, M_0^2) - 3F_{\Delta\rho}(M_Z^2, M_0^2) \right]
 \end{aligned}$$

$$F_{\Delta\rho}(m_1^2, m_2^2) = \frac{1}{2}(m_1^2 + m_2^2) - \frac{m_1^2 m_2^2}{m_1^2 - m_2^2} \log \frac{m_1^2}{m_2^2}$$

Unimportant  
M<sub>W</sub> ≈ M<sub>Z</sub>

$(g-2)_\mu$

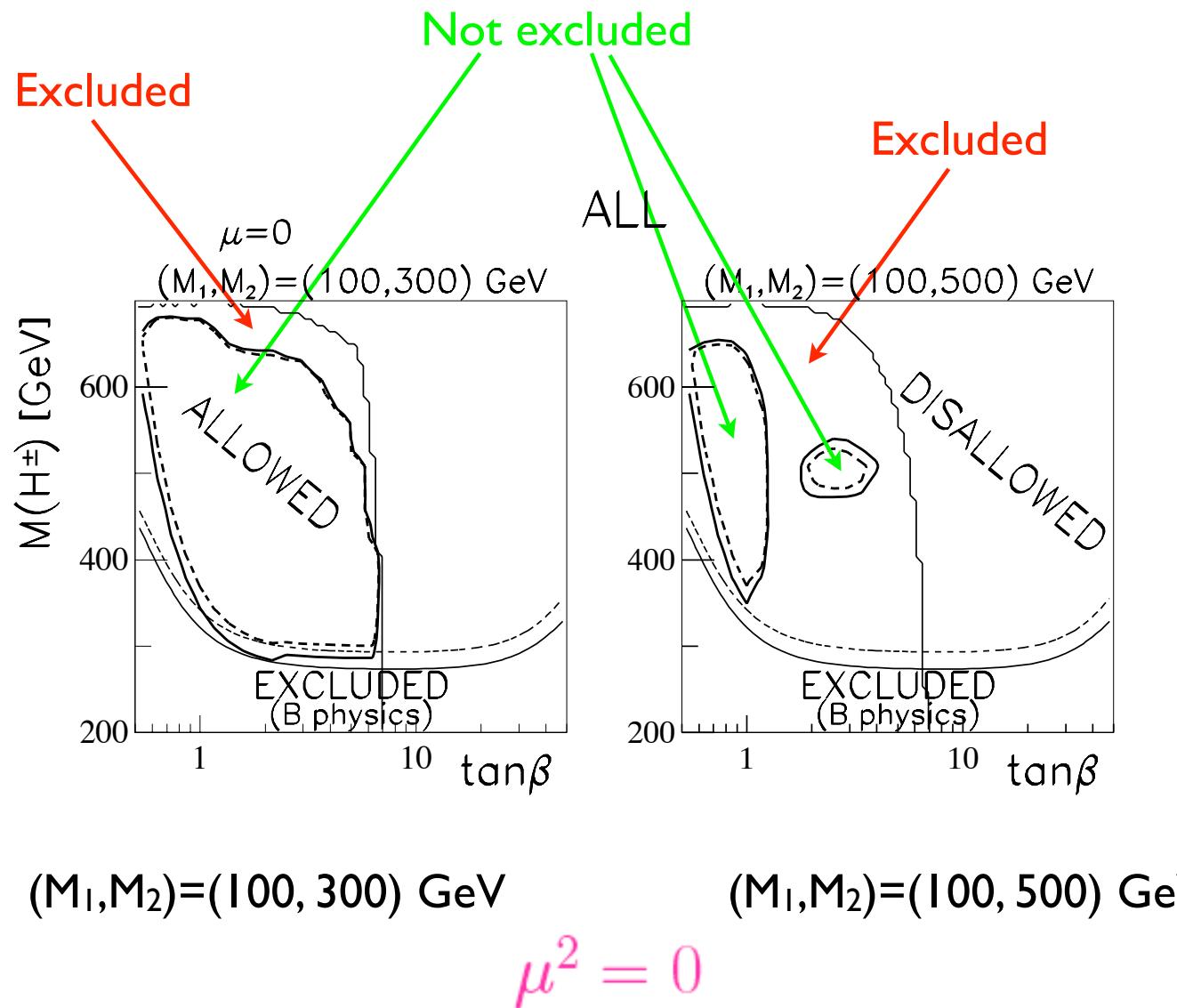
Two-loop Barr-Zee effect



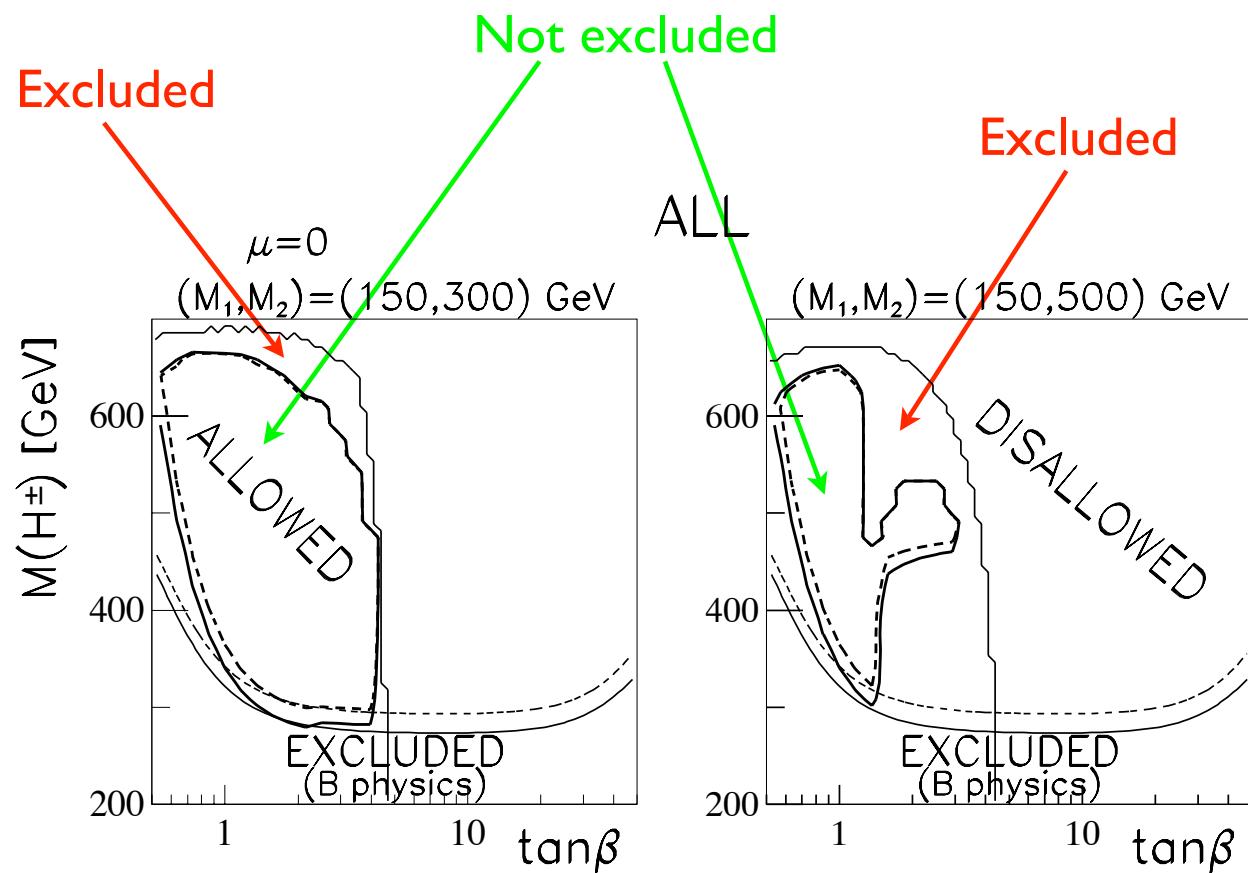
may produce effect comparable with (very high) precision

Only relevant at very high  $\tan\beta$  and/or low  $M_{H^\pm}$

# Combine all constraints



# Combine all constraints

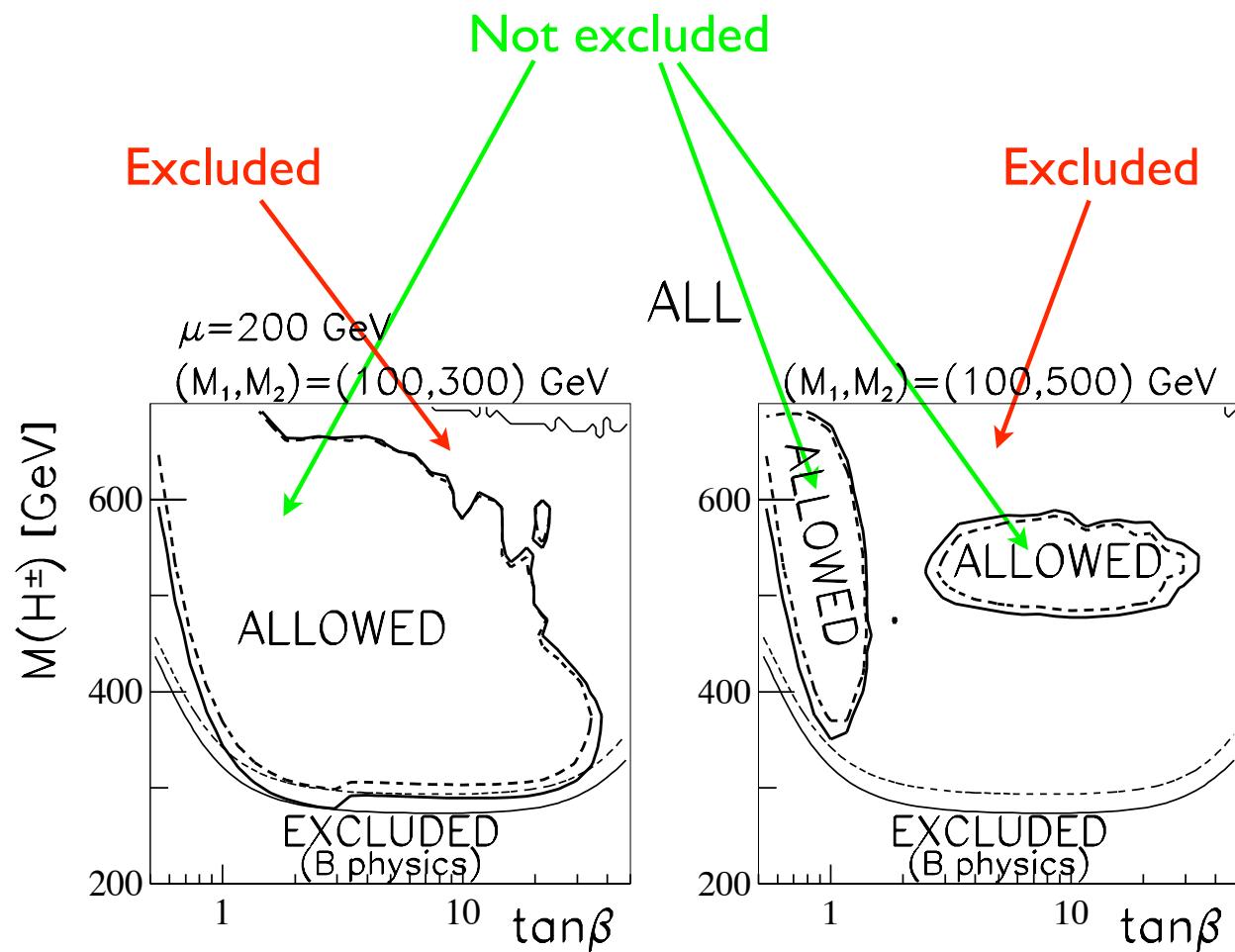


$(M_1, M_2) = (150, 300)$  GeV

$$\mu^2 = 0$$

$(M_1, M_2) = (150, 500)$  GeV

# Combine all constraints

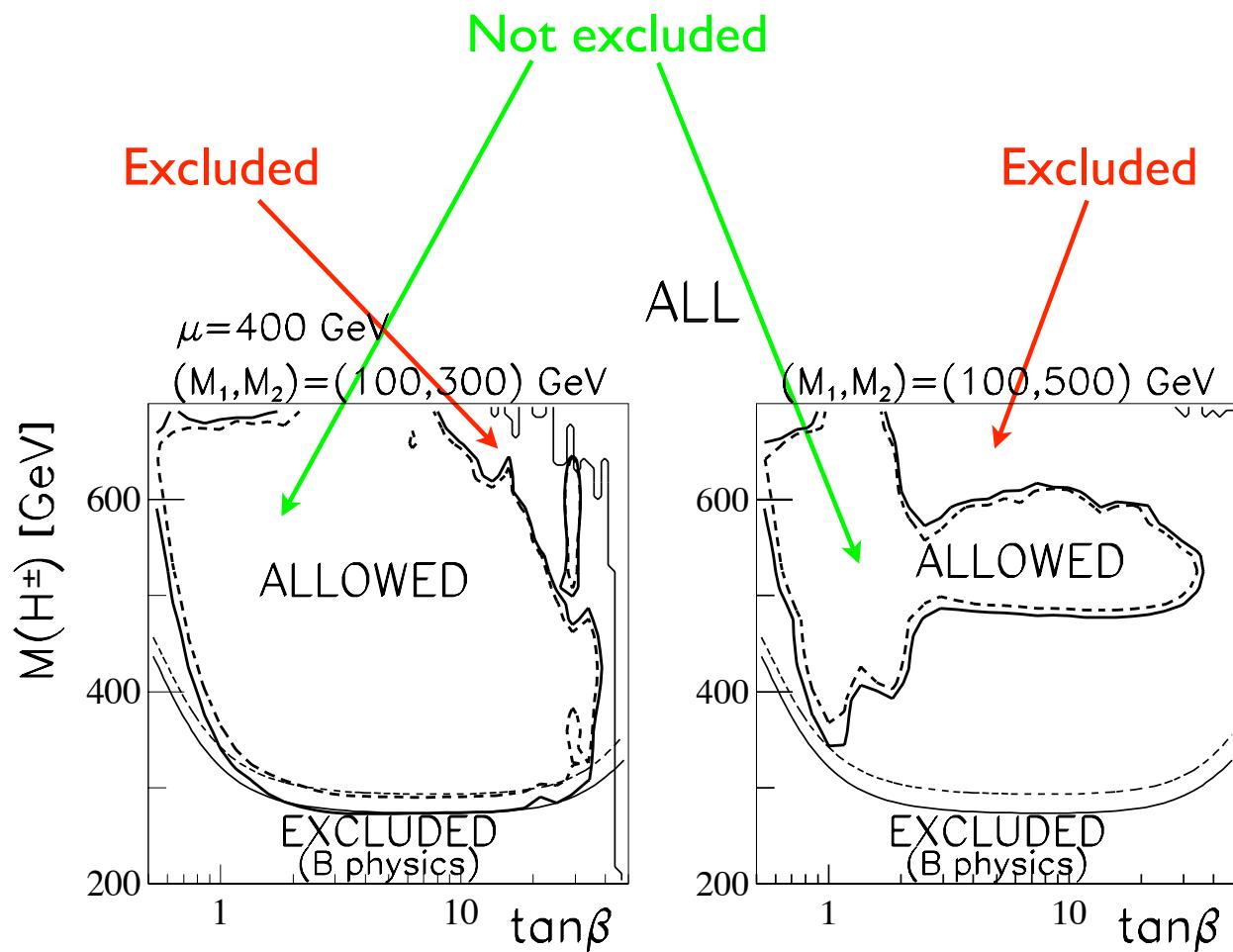


$(M_1, M_2) = (100, 300)$  GeV

$$\mu^2 = (200 \text{ GeV})^2$$

$(M_1, M_2) = (100, 500)$  GeV

# Combine all constraints

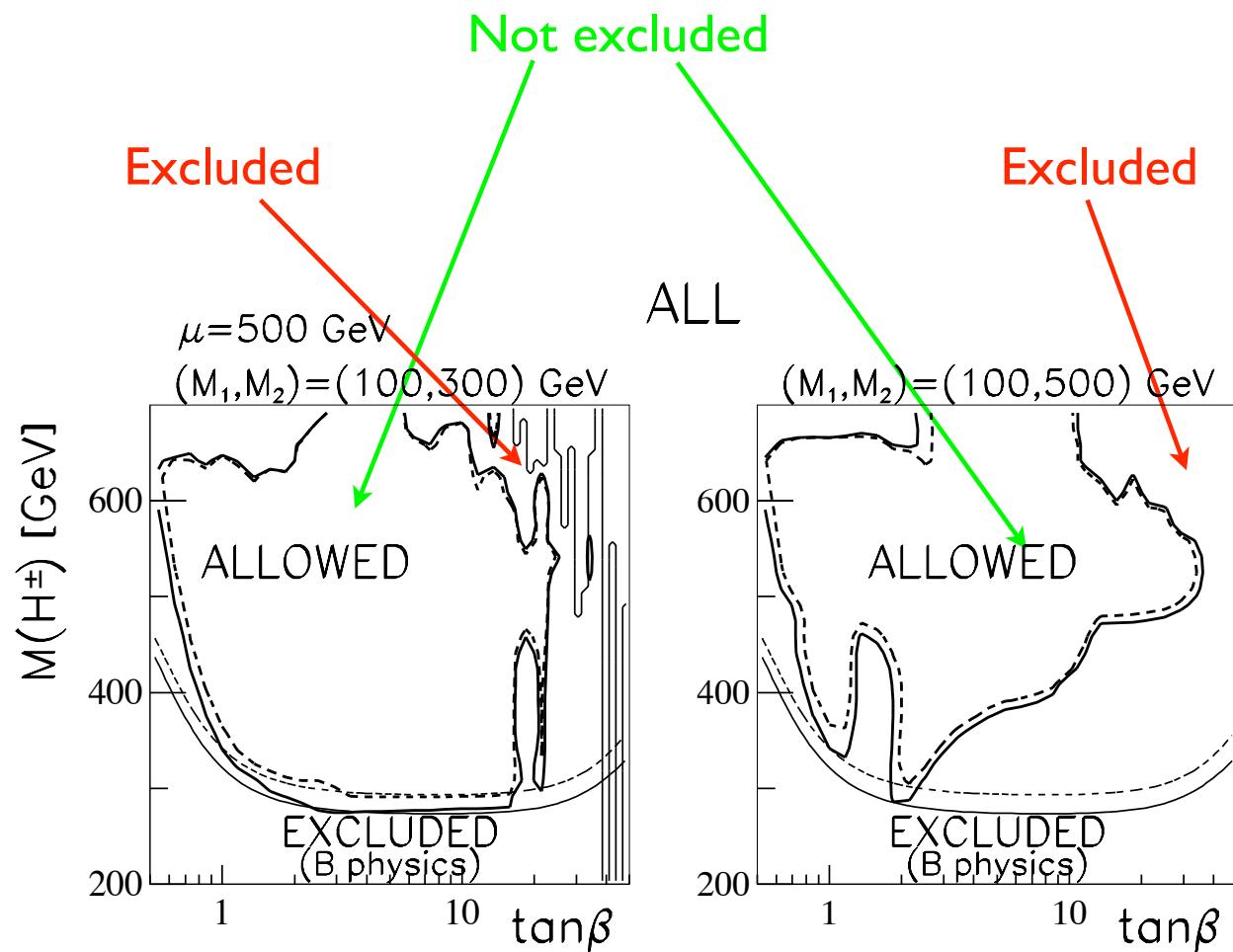


$$(M_1, M_2) = (100, 300) \text{ GeV}$$

$$\mu^2 = (400 \text{ GeV})^2$$

$$(M_1, M_2) = (100, 500) \text{ GeV}$$

# Combine all constraints

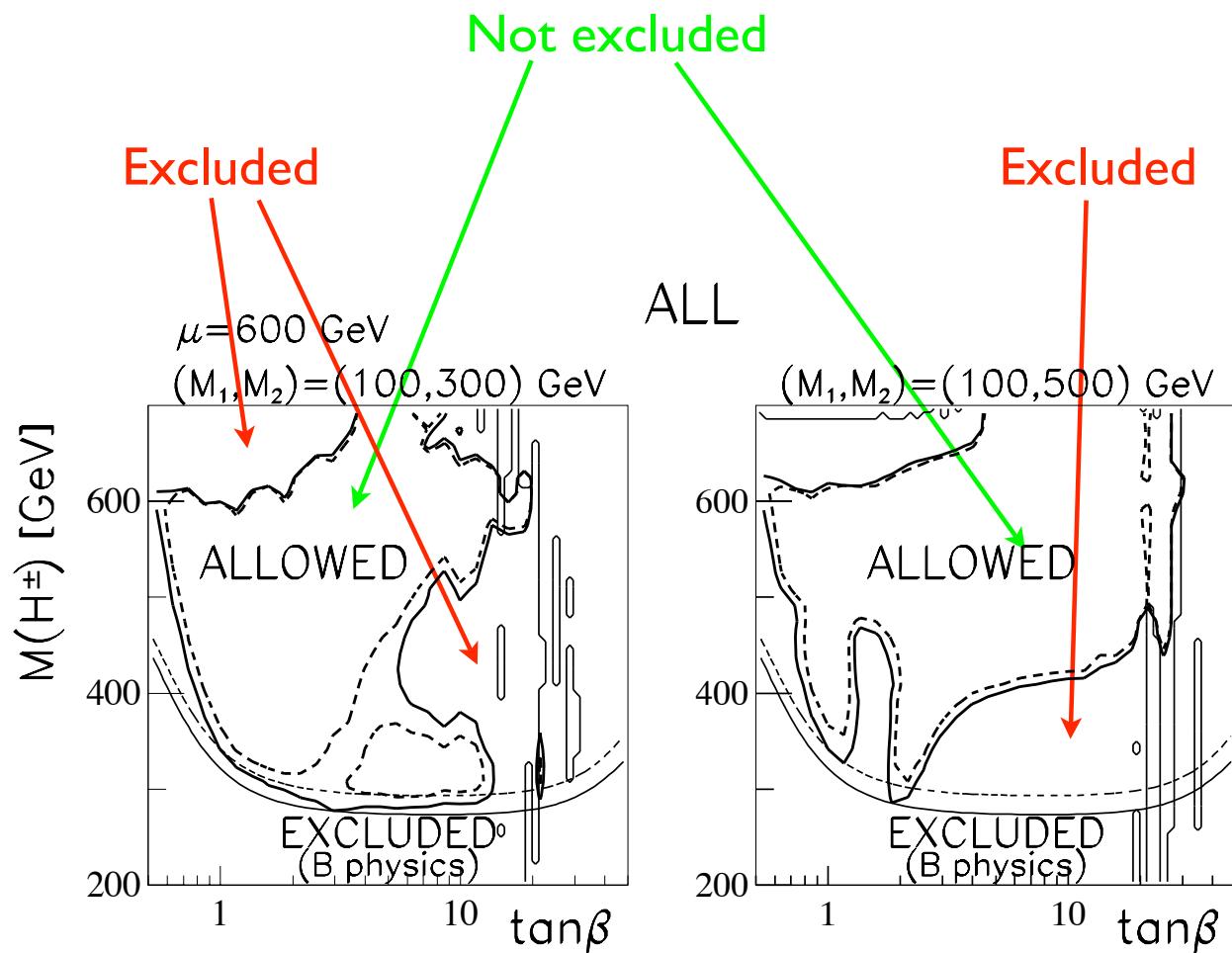


$$(M_1, M_2) = (100, 300) \text{ GeV}$$

$$\mu^2 = (500 \text{ GeV})^2$$

$$(M_1, M_2) = (100, 500) \text{ GeV}$$

# Combine all constraints

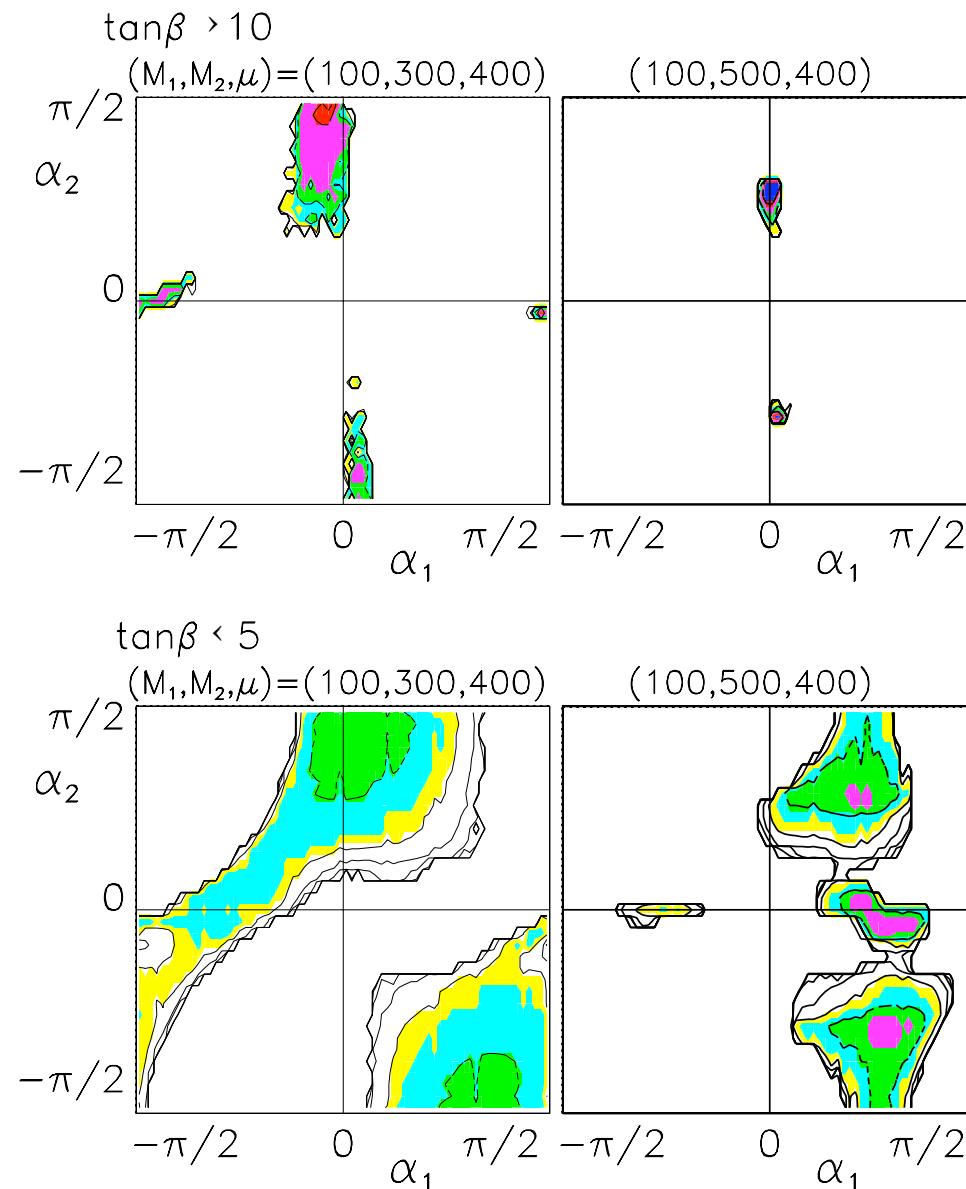


$$(M_1, M_2) = (100, 300) \text{ GeV}$$

$$\mu^2 = (600 \text{ GeV})^2$$

$$(M_1, M_2) = (100, 500) \text{ GeV}$$

# Profile of surviving parameter space



# Summary

- B physics data exclude low  $M_H^\pm$  and low  $\tan\beta$
- Unitarity excludes high  $\tan\beta$  and high  $M_H^\pm$
- Neutral sector constraints allow only  $\alpha_i \in \hat{\alpha}$   
 $i = \{\Delta\Gamma_b, \text{LEP2}, \Delta\rho, (g-2)_\mu\}$

Do the not-excluded  $\alpha_i$  have any overlap?

“Yes”

$(g-2)_\mu$  irrelevant

$\Delta\rho$  very constraining

LHC may provide total exclusion (or discovery)