"A GLIMPSE INTO TOPOLOGICAL QUANTUM COMPUTATION"

LECTURES AT NORDITA, JAN 2011, J.K. SLINGERLAND

MOTIVATION

QUANTUM COMPUTATION PROVIDES DRAMATIC SPEEDUP OF SOME TYPES OF COMPUTATION

- FACTORING ($ → DECRYPT THE WORLD'S SECRETS)
- SEARCH (NOT SO DRAMATIC, BUT GOOD FOR ILLUSTRATION SEARCH THROUGH N UNORDERED OBJECTS IN $O(\sqrt{N})$ TIME)
- SIMULATION OF QUANTUM SYSTEMS ! (THE REALLY BIG ONE)

IDEA (QUANTUM CIRCUIT MODEL) (STANDARD "PARADIGM")

- FIND SOME NICE 2-LEVEL SYSTEM (SPIN, ATOMIC GS/EXCITED STATE) etc. etc.
  ↳ QUBIT $|0\rangle, |1\rangle$, or of course $\alpha|0\rangle + \beta|1\rangle$, ∈S V_2
- MULTIPLE QUBITS → TENSOR PRODUCT $V_2^{\otimes n}$
  now have all binary numbers of length n ($|0\rangle|1\rangle|0\rangle|0\rangle$ etc.)
  and all their superpositions ⟶ QUANTUM PARALLELISM
- ACT ON THESE STATES WITH UNITARY TIME EVOLUTION
  (ONE QUBIT GATES ACT ON ONLY ONE TENSOR FACTOR
  e.g. $U \otimes \mathbb{1} \otimes \mathbb{1} \ldots \otimes \mathbb{1}$, $\mathbb{1} \otimes \mathbb{1} \otimes U' \otimes \mathbb{1} \otimes \ldots \mathbb{1}$
  TWO QUBIT GATES ACT ON TWO TENSOR FACTORS etc.
- WANT UNIVERSAL QUANTUM COMPUTATION: A SET OF GATES WHICH CAN BE PHYSICALLY IMPLEMENTED + GENERATE A DENSE SUBSET OF $U(2^n)$ ⟶ CAN APPROX. ANY UNITARY TO ARB. ACC.
  UNIVERSAL GATE SETS WITH FINITELY MANY GATES ARE KNOWN
  (IN GENERAL ALL 1-QUBIT GATES + 1 NONTRIV 2-QUBIT GATE IS ENOUGH)

NADIYA TQC)

PROBLEM: VERY SENSITIVE TO ERROR

- ERRORS ARE NOT DISCRETE $\alpha|0\rangle + \beta|1\rangle \to (\alpha+\varepsilon)|0\rangle + (\beta+\varepsilon)|1\rangle$

instead of $0 \to 1$

- QUANTUM COMPONENTS ARE SENSITIVE TO NOISE (EVEN IF WE DON'T DO ANYTHING)
- DIFFICULT TO APPLY GATES ACCURATELY (EG LASER PULSE FOR PRECISE AMOUNT OF TIME)

RESULTS: → NO WORKING QC FOR $\gtrsim 5$ QUBITS

SOLUTIONS: 1. QUANTUM ERROR CORRECTION (TOMORROW)

AMAZINGLY, CAN CORRECT QUANTUM ERRORS, EVEN WHILE CALCULATING

NEED FIDELITY THRESHOLD, THEN CAN DO THIS INDEFINITELY.

(THRESHOLD THMS)

OBVIOUSLY, IT IS HARD TO GET TO THE THRESHOLD WITH CURRENT IMPLEMENTATIONS (OR WOULD HAVE >5 QUBITS)

2. MAKE SYSTEM INHERENTLY INSENSITIVE TO NOISE → TQC

- BUILT-IN ERROR CORRECTION (FAULT TOLERANCE (AGAIN, TOMORROW)

HOW? STORE Q-INF IN TOPOLOGICAL (NON-LOCAL) D.O.F.

THESE ARE UNAFFECTED BY LOCAL NOISE (= MOST NOISE)

PROBLEM IS NOW TO MANIPULATE THE INFORMATION

- CANNOT DO THIS BY LOCAL PROBES. (+ READ OUT)

THIS IDEA LEADS TO LOTS OF INTERESTING CM PHYSICS. (MY MOTIVATION)

- WHAT CM SYSTEMS HAVE SUITABLE TOPOLOGICAL QUANTUM NUMBERS?
- HOW TO MEASURE + MANIPULATE THESE? - IN THEORY, TO GET QC
  - IN PRACTICE. (MODELS)

TODAY. THEORY, KIND OF "TQC PARADIGM" BASED ON "ANYONS"

SOME REFS: NAYAK, STERN, FREEDMAN, DAS SARMA, REV MOD PHYS 80, 1083 (2008)

PRESKILL, LECTURE NOTES FOR PHYSICS 219 (CALTECH)

Zhenghan WANG, BOOK: "TOPOLOGICAL QUANTUM COMPUTATION" (CBMS CONF SERIES) (MATH)

2)

NORDITA TQC) — LEINAAS / MYRHEIM, NUOV. CIM 1977
WILCZEK (NAME)

ANYONS PARTICLE LIKE OBJECTS (LOCALISED NR PT.) in 2D
WITH PROPERTY THAT THE WAVE FUNCTION
CHANGES BY NONTRIV PHASE (UNITARY MATRIX) ON EXCHANGING (THEM)

(ABELIAN) ANYONS — NONABELIAN ANYONS (ORDER OF EXCHANGES MATTERS)

PATH MATTERS (NOT NEC. EXC. ANY CLOSED PATH)

EXAMPLE: REALISED IN FQHE.

MAGNETIC VORTEX (FLUX TUBE) BOUND TO EL. CHARGE

Φ — charge depletion $e^* = \nu e$, e.g. $e^* = \frac{e}{3}$

2D e^--gas

TAKE ONE AROUND ANOTHER → AHARONOV-BOHM EFFECT

CAUSES PHASE $e^{\frac{ie^*\Phi}{\hbar c}}$

HERE $\boxed{e^{2\pi i/3}}$

$\Phi = \frac{hc}{e}$ (1 flux quantum)

$\frac{h}{\hbar} = 2\pi$

OBVIOUSLY IN OTHER DIRECTION, GET $\underline{e^{-2\pi i/3}}$

IF FLUX IS LOCALISED ON PTCLS (+ CHARGE)
THE PHASES ARE "PATH INDEPENDENT", THAT IS, INDEPENDENT
OF (~~SMALL~~) DEFORMATIONS OF THE PATH (SAME FLUX ENCLOSED)

IN GENERAL (NOT NEC. FQH)
CONSIDER N "ptcls" positions $x_1 \ldots x_N$, dimension d
ASSUME THEY ARE "INDISTINGUISHABLE"
So $\Psi(x_1 \ldots x_N)$ is the same state as $\Psi(\text{permutation}(x_1 \ldots x_N))$
DO NOT ALLOW THEM TO BE IN THE SAME POSITION
→ CONFIGURATION SPACE OF THE "PTCLS"
IS $M_N = \left((\mathbb{R}^d)^N \setminus D\right) / S_N$

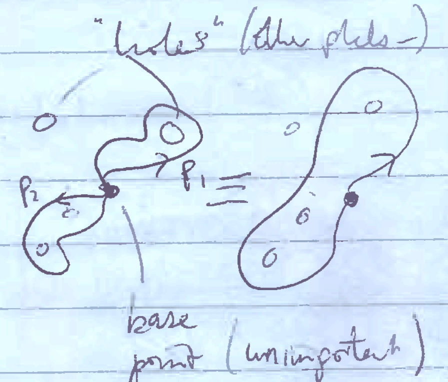
"diagonal"

3)

MAJORANA TQC

AN EXCHANGE IS A PATH THROUGH M_N WHICH RETURNS TO STARTING POINT → LOOP, WE ARE INTERESTED IN LOOPS UP TO DEFORMATION.

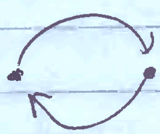
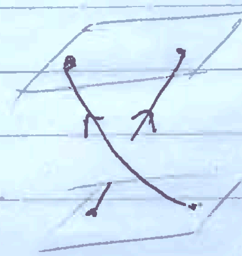
THESE FORM A GROUP $\pi_1(M_N)$ (FUNDAMENTAL GROUP)



GROUP STRUCTURE:

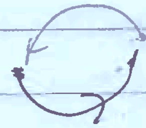
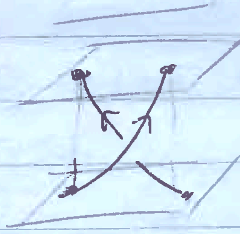
- MULTIPLICATION = COMPOSITION OF PATHS
- INVERSE = REVERSE PATH
- TRIV LOOP (NO MOTION) = e (UNIT ELT)

EXCHANGE GROUPS LIKE $\pi_1(M_N)$
GENERATED BY SIMPLE EXCHANGES

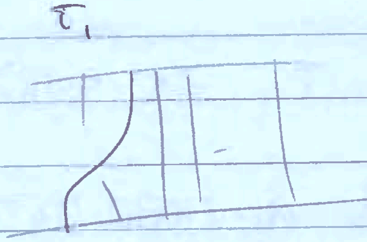
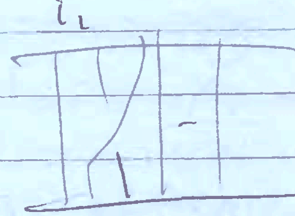
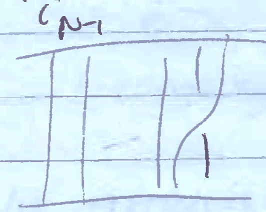
 → WORLDLINES 

FOR 2D SYSTEM THIS IS THE PATH → BRAID GROUP (higher d, must remember extra dim)

inverse

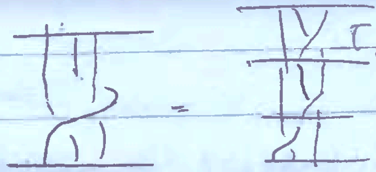
 → 

SO ARRANGING PTCLS LEFT → RIGHT
GET GENERATORS

τ_1  τ_2  ... τ_{N-1} 

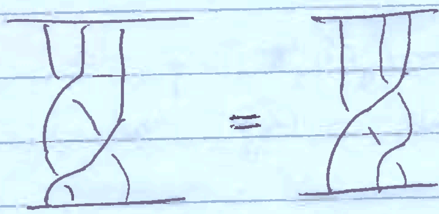
NOTATION TQC

ANY EXCHANGE PROCESS PRODUCT OF GENS

EG  $\tau_1\tau_2$ / τ_2 / τ_1 $\quad \tau_1\tau_2^2$ (MAY NEED INV.)

RELATIONS

ALWAYS HAVE:



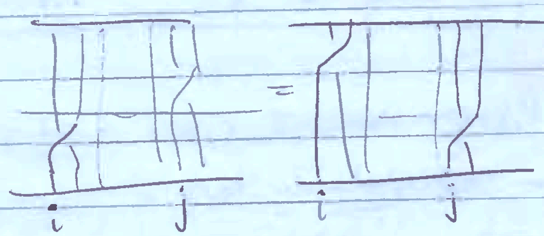
$\tau_1\tau_2\tau_1 \qquad \tau_2\tau_1\tau_2$

(deform middle strand left → right)

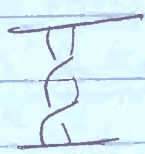
MORE GEN

$\tau_i\tau_{i+1}\tau_i = \tau_{i+1}\tau_i\tau_{i+1}$ (YBE)

TOGETHER: BRAID GROUP (∞) B_N

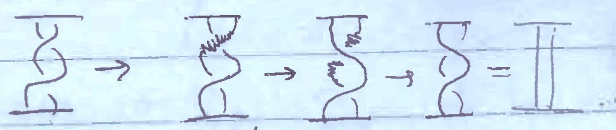
ALSO  $\tau_i\tau_j = \tau_j\tau_i$ for $|i-j| \geq 2$

IN $d>2$, ALSO HAVE



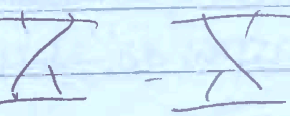
τ_i^2

$\tau_i^2 = e$ (triv elt) → DO COLOUR FOR DIMENSION THING

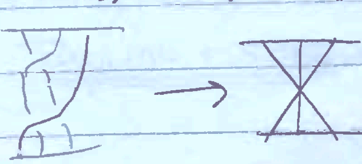


particle moved to different plane at …

(ALSO "CANNOT LASSO A POINT")

SAME AS $\tau_i = \tau_i^{-1}$  obvious by same arg

THIS MEANS IN $d>2$ BRAID ARE JUST PERMUTATIONS

Eg)  (which loses us all info)

PERMUTATION GROUP S_N (quotient of B_N)

NORDITA TQC

ACTION OF EXCHANGE PROCESSES ON SYSTEM STATE ψ
→ BY REPRESENTATION ρ OF $\pi_1(M_N)$ (π_1 (CONF SPACE) & CAN HAVE π_1's ON TORUS → MORE NONTRIV LOOPS)

ie $\rho : \pi_1(M_N) \to U(n)$ (n×n unitaries)
Simplest: just a phase $U(1)$, but if we have n states for N parts in fixed positions $U(n)$

REPRESENTATION: $\rho(a \cdot b) = \rho(a)\rho(b)$ $\rho(e) = \mathbb{1}$

IF $n = 1$ ρ a phase, get

$\tau_i \tau_{i+1} \tau_i = \tau_{i+1} \tau_i \tau_{i+1} \Rightarrow \rho(\tau_i)\rho(\tau_{i+1})\rho(\tau_i) = \rho(\tau_{i+1})\rho(\tau_i)\rho(\tau_{i+1})$
here $\rho(\tau_i)$ are just numbers so $\Rightarrow \rho(\tau_{i+1}) = \rho(\tau_i)$
$\Rightarrow \rho(\tau_i) = e^{i\theta}$ (some fixed phase θ) for all i
exchange phase same for all anyons.
here $\tau_i \tau_j = \tau_j \tau_i$ for $|i-j| \geq 2$ automatically ok
in $d>2$ $\tau_i^2 = 1 \Rightarrow \rho(\tau_i)^2 = 1 \Rightarrow e^{2i\theta} = 1$
$\Rightarrow \begin{cases} \theta = 0 & \text{bosons} \\ \text{or } \theta = \pi & \text{fermions} \end{cases}$

FOR $n > 1$ GET NONTRIVIAL REPS OF B_N, S_N (also S_N!)
FOR B_N CAN BE UNIVERSAL, $\rho(B_N) = U(n)$ (or $U(n')$ ($n' < n$))
FOR S_N CANNOT BE UNIVERSAL (FINITE IMAGE)
(BUT COULD STILL BE INTERESTING.

FIRST, WANT SOME IDEA WHERE $N>1$ COMES FROM:
FUSION + TOPOLOGICAL CHARGE

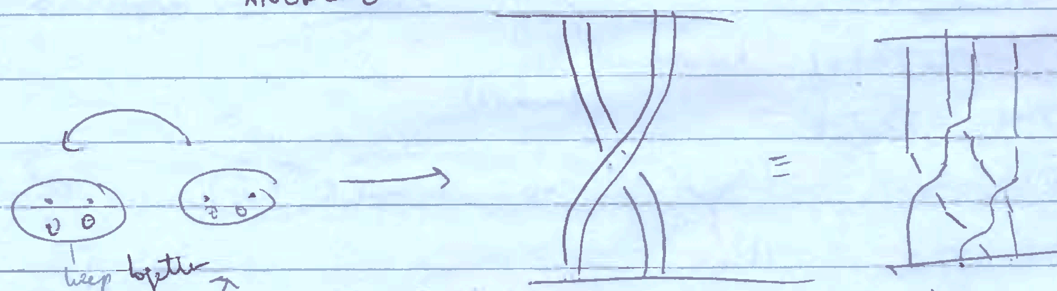
NONABELIAN TQC)

FUSION + TOPOLOGICAL CHARGE

USUALLY, HAVE MULTIPLE TYPES OF ANYONS / ANYONIC SECTORS
→ DIFFERENT "TOPOLOGICAL CHARGES"

CAN RELATE THESE BY FUSION : CONSIDERING SEVERAL ANYONS TOGETHER AS ONE ANYON.

SIMPLEST : ABELIAN ANYONS, (VERY SIMPLEST FERMIONS → "BOSONS")
ANGLE θ



keep together

4x left over right exchange:
has statistical angle 4θ

Similarly → 9θ , → $n^2\theta$

So typically have multiple "anyonic sectors" (types of anyons with statistical angles $\theta, 4\theta, 9\theta, \ldots n^2\theta$

If we want finite # of sectors, must have $n^2\theta = 2k\pi$, some n, k

Simplest, only 2 sectors (no anyon vs 1 anyon like boson/fermion)

Then have θ, $4\theta = 2k\pi$ (some k) $\rightarrow \theta = \frac{k}{2}\pi$

Can take $-\pi < \theta \leq \pi$ gives $\theta = 0$, $\theta = \pm\frac{\pi}{2}$, $\theta = \pi$ ($= -\pi$)

$\theta = \pi \rightarrow$ boson/fermion, $\theta = 0 \rightarrow$ 2 types of boson ($\mathbb{Z}_2$ symmetry?)

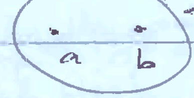
$\theta = \pm\frac{\pi}{2} \rightarrow$ Semions (half fermions) gives —

Can introduce top charges as operators

$\hat{a}$ operator determines type of a (eigenvalues = types of a)

NORDITA TQC)

NONABELIAN ANYONS + FUSION



– charge operator

Can now find different charges c

N_{ab}^{c} $\propto$ dim size of eigenspace of op. for charge c (idealized system)

(FINITE EXPECT OPERATORS ON NON OVERLAPPING SUPPORTS TO COMMUTE)

(number of states of quantum medium with anyons a, b st they fuse to c)

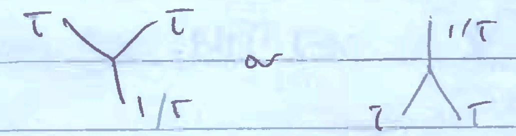
Write schematically $a \times b = \sum_c N_{ab}^{c}\, c$ FUSION RULES

Examples: 2 types $1, \tau$ FIBONACCI THEORY.

trivial (vacuum), nontrivial

$$\begin{cases} 1 \times \tau = \tau \times 1 = \tau & \text{(obv)} \\ \tau \times \tau = 1 + \tau \end{cases}$$

So $(\tau\ \tau)_{1/\tau}$ or



3 types $1, \sigma, \psi$ ISING FUSION RULES.

$$\begin{cases} 1 \times \text{anything} = \text{anything} \times 1 = a\,(\text{anything}) & (a = \sigma, \psi) \\ \psi \times \psi = 1 \\ \psi \times \sigma = \sigma = \sigma \times \psi & (\text{"}\sigma\text{ absorbs a }\psi\text{"}) \\ \sigma \times \sigma = 1 + \psi & \end{cases}$$

$\rightarrow$ similar to spin $\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$

but with "$1 \times 1 = 0$"

QDIM MIGHT BE TEMPTED TO INTRODUCE LOCAL "INTERNAL" HILBERT SPACES FOR ANYONS + THINK OF FUSION AS "TENSOR PRODUCT DECOMPOSITION" (LIKE CLEBSCH-GORDAN)

OFTEN GOOD ANALOGY BUT MUST BE CAREFUL

EG. $\psi \times \psi = 1 \Rightarrow d_\psi = 1$ $\quad (1 \times 1 = 1 \rightarrow d_1 \times d_1 = d_1 \Rightarrow d_1 = 1$

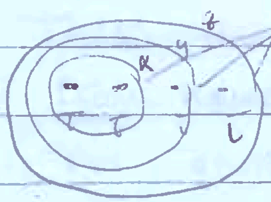
$\sigma \times \sigma = 1 + \psi \Rightarrow d_\sigma d_\sigma = 1 + 1 = 2$ $\quad \psi \times \psi = 1 \Rightarrow d_\psi d_\psi = d_1 = 1$

$\rightarrow d_\sigma = \sqrt{2}$? So CALLED QUANTUM DIMENSION

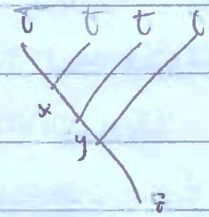
SIMILARLY $d_\tau \times d_\tau = 1 + d_\tau \Rightarrow d_\tau = \frac{1 \pm \sqrt{5}}{2}$ $\quad \frac{1+\sqrt{5}}{2}$ golden ratio

NORDITA TQC

MULTI ANYON STATES

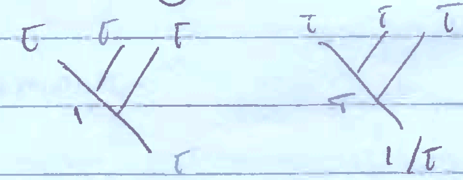


maximal set of commuting top charges determines anyonic state



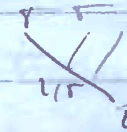
NON-OVERLAPPING SUPPORTS COMMUTE → ALSO GIVES $a \times b = b \times a$

eg 3 τ's



→ 3 states.

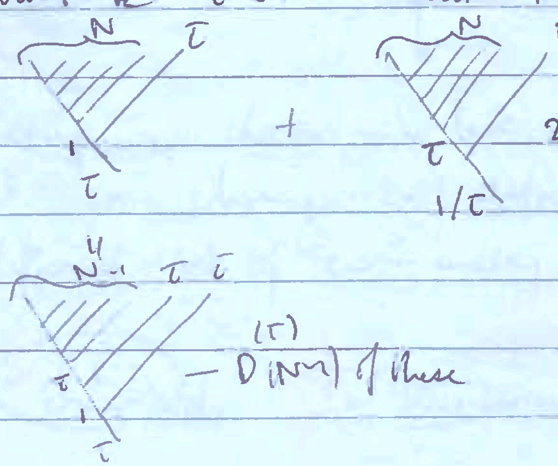
FIX OVERALL CHARGE TO τ THEN



QUBIT

MULTIPLE τ-s → COUNT DIMENSION FOR $(N+1)$ τ-s $D^{(\tau)}_{N+1}$, $D^{(1)}_{N+1}$ OVERALL CHARGE

States:



$2 \times D^{(1)}_N$ of these giving

$$D^{(1)}_{N+1} = D^{(\tau)}_N$$

$$D^{(\tau)}_{N+1} = D^{(\tau)}_N + D^{(\tau)}_{N-1}$$

FIBONACCI RECURSION

— $D^{(\tau)}_{(N-1)}$ of these

$D^{(\tau)}_N$: $D^{(\tau)}_1 = 1$, $D^{(\tau)}_2 = 1$, $D^{(\tau)}_3 = 1+1 = 2$, then $2+1=3$, $3+2=5$, $5+3=8$

$1 = D^{(1)}_2$, $1 = D^{(1)}_3$

N: 2 3 4 5 6 7 8
N_τ: 1 2 3 4 5 6 7

So FIB NUMBERS 1, 1, 2, 3, 5, 8, 13

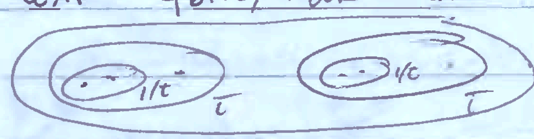
THESE GROW $\sim \left(\frac{1+\sqrt{5}}{2}\right)^N = (d_\tau)^N$

MEANING OF QDIM

NOTICE: NOT A TENSOR PRODUCT SPACE

→ IF WE WANT QBITS, MUST CHOOSE A COMPUTATIONAL SUBSPACE

EG 2 QUBITS



$1/\tau$ → (choose) (OVERALL CHARGE OF MEDIUM)

$2 \times 2 = 4D$ SUBSPACE OF 5D — overall 1, or 8D — overall τ SPACE → MUST WORRY ABOUT LEAKAGE

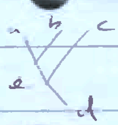
NORDITA TQC

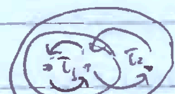
WANT TO DO QC BY BRAIDING

TURNS OUT FUSION, BRAIDING TIGHTLY CONNECTED — UBTC (UNITARY BRAIDED TENSOR CATEGORY)

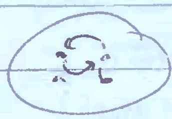
RIGID STRUCTURE. ONLY FINITELY MANY POSSIBILITIES FOR BRAIDING, GIVEN FUSION (OFTEN NONE, e.g. $\tilde{\tau}\times\tilde{\tau} = 1 + 2\tilde{\tau}$ is inconsistent)

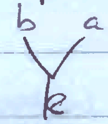
E.G. 3 PTCL BRAIDING





τ_1 does not interfere with any topological charges → gives an overall phase R^{ab}_c - R-SYMBOLS

 e

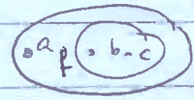
 $= R^{ab}_e$ 

FIB $R^{\tau\tau}_1 = e^{-4\pi i/5}$

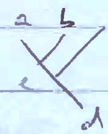
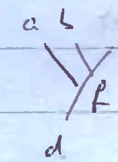
$R^{\tau\tau}_\tau = e^{3\pi i/5}$

(are complex conjugates)
(turn over plane)
$(R^{\tau 1}_\tau = R^{1\tau}_\tau = R^{11}_1 = 1)$

exchange τ_2 does interfere with charge e.
but can change to different basis
(different set of commuting operators)



In this basis can use R-Symbols again (R^{bc}_f)

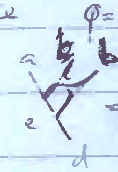
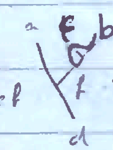
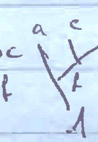
BUT NEED BASIS CHANGE MATRIX  $= \sum [F^{abc}_d]_{ef}$ 

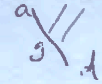
ELEMENTS $[F^{abc}_d]_{ef}$: F-SYMBOLS

FOR FIB, $[F^{\tau\tau\tau}_\tau] = \frac{1}{\varphi}\begin{array}{c|cc} & 1 & \tau \\ \hline 1 & 1 & \sqrt{\varphi} \\ \tau & \sqrt{\varphi} & -1 \end{array}$ with $\varphi = \frac{1+\sqrt{5}}{2} = d_\tau$.

NOTE $\left(\frac{1}{\varphi}\right)^2 + \left(\frac{\sqrt{\varphi}}{\varphi}\right)^2 = \frac{1}{\varphi^2} + \frac{1}{\varphi} = \frac{1}{\varphi^2}(1+\varphi) = 1$

(Since $\varphi = d_\tau$, $d_\tau \times d_\tau = 1 + d_\tau$) So UNITARY MATRIX

NOW CAN DO  $= \sum_f [F^{abc}_d]_{ef}$  $= \sum_f [F^{abc}_d]_{ef} R^{bc}_f$ 

$= \sum_{f,g} [F^{abc}_d]_{ef} R^{bc}_f [F^{-1}{}^{acb}_d]_{fg}$ 

NON-ABELIAN TQC

THIS GIVES τ_2, SO NOW HAVE FULL 3-PARTICLE BRAIDING

→ TURNS OUT IT IS UNIVERSAL FOR (1-qubit) QC (WON'T SHOW HERE)

THOUGH IT'S NOT EASY TO MAKE "SIMPLE" GATES LIKE QNOT $\begin{pmatrix}0&1\\1&0\end{pmatrix}$

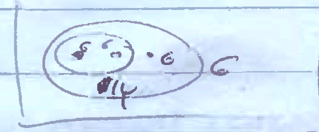
(APPROX BY LONG BRAIDS + MANY τ_1, τ_2's)

NOTE: ISING NOT UNIVERSAL (FINITE IMAGE)

$R^{\sigma\sigma}_1 = e^{-i\pi/8}$ $\quad (F^{\sigma\sigma\sigma}_\sigma) = \frac{1}{\sqrt{2}}\begin{array}{cc|c} 1 & \psi & \\ \hline 1 & 1 & 1 \\ 1 & -1 & \psi \end{array}$

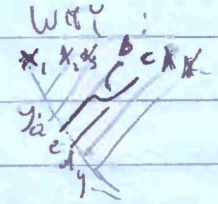
$R^{\sigma\sigma}_\psi = e^{3\pi i/8}$

IF YOU WANT TO CHECK



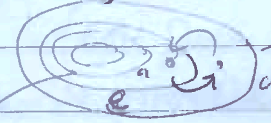
ACTUALLY F, R SYMBOLS DETERMINE ALL BRAIDINGS (AND EVEN AMPLITUDES FOR PROCESSES WITH FUSION) (FULL UMTC STRUCTURE).

WHY: LOCALLY ALWAYS LOOKS LIKE 3 PTCLS



all x's, y's unaffected by this braiding.

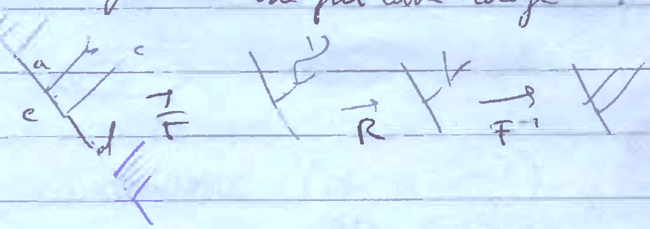
Only e changes



— total charge d (further out doesn't matter)

just like one ptcl with charge a.

Can change basis



etc.

WHY SUCH A RIGID STRUCTURE?

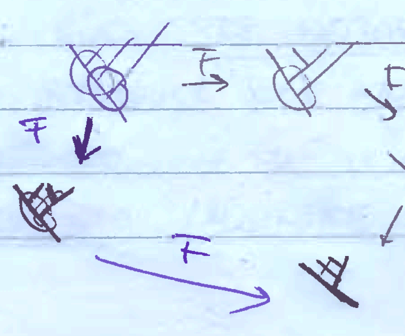
→ MANY CONSTRAINTS ON F, R SYMBOLS

EG MUST GET REPRESENTATION OF B_N ABOVE (ANY N).

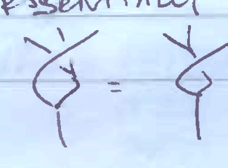
COMPLETE SET OF INDEP CONSTRAINTS: PENTAGON / HEXAGON EQNS (G. MOORE / N. SEIBERG)

EG PENTAGON:

2 WAYS OF DOING SAME BASIS TRANSF



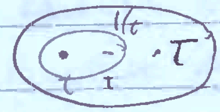
HEXAGONS ESSENTIALLY

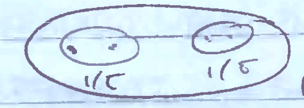


So, "STANDARD PARADIGM" OF TQC

- CHOOSE ANYON MODEL (= FUSION RULES, F, R-SYMBOLS)

- MAKE QUBITS FROM SMALL GROUPS OF ANYONS

e.g. 

or 

[DO SINGLE QUBIT OPS IN QUBIT IF POSS → PREVENT LEAKAGE)

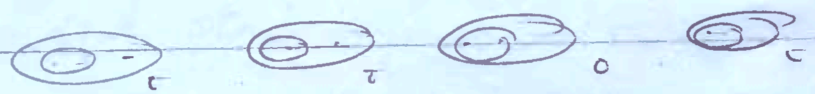
Q: HOW MANY ANYONS / QUBIT? 2 IS MIN. BUT ONLY PHASES R^{ab}_c FROM BRAIDING

3 is better (CAN HAVE UNIVERSALITY)

4 ALLOWS OVERALL CHARGE 1 (CAN MOVE QUBITS AROUND)

Though can use QDITS (dim d bits) ← 5 is IMPOSSIBLE! (ANY ANYON MODEL)

- ACT ON COMPUTATIONAL SUBSPACE BY BRAIDINGS (OR MEASUREMENT)



etc.

LEAKAGE USUALLY UNAVOIDABLE (OR IS IT?)
OPEN QUESTION

CAN SHOW
COMPUTATIONAL UNIVERSALITY FOR MANY ANYON MODELS
(BUT SOME IMPORTANT ONES NOT) (e.g. ISING)
→ ALSO ALLOWS LEAKAGE AVOIDANCE
NON-UNIVERSAL STILL GIVES TOP. MEMORY.
VARIOUS SCHEMES TO INTRODUCE SOME NON-PROTECTED OPS
(STILL RAISES THRESHOLD (UP TO 10% FIDELITY))
CAN NOW DO QUANTUM INFO (ALGORITHMS ETC.) NEXT: PHYSICAL IMPLEMENTATION

MODITA TQC

SOMETHING ABOUT EXPERIMENTS - WHERE ARE WE?

- $\exists$ SYSTEMS WHICH ARE BELIEVED TO HOST NON-ABELIAN ANYONS
  FQHE, plateau at $\nu = \frac{5}{2}$ (Ising-like) $\nu = \frac{12}{5}$ (could be Ising or Fib like)
  (potentially more)

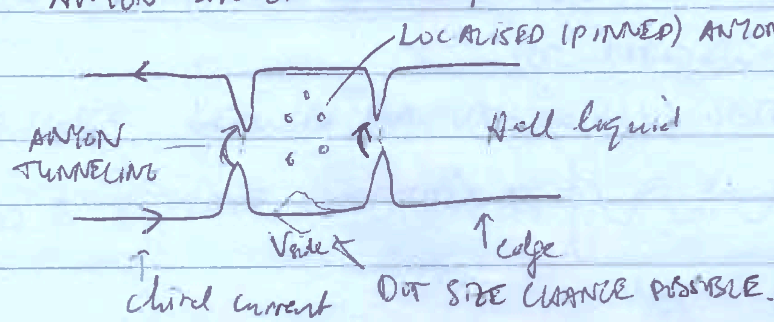
  OTHER EXISTING SYSTEMS (THEORETICALLY WORSE OR EXPERIMENTALLY LESS MATURED)

ALL SPIN LIKE
- TOPOLOGICAL INSULATORS (IN PROXIMITY TO S.C.)
- SPIN-ORBIT COUPLED SYSTEMS WITH PROXIMITY INDUCED GAP (1D, 2D)
- p-wave SCs (STRONTIUM-RUTHENATE)
- ROTATING BECs (LIKE FQHE FOR BOSONS) AT VERY HIGH ANGULAR FREQUENCY.
- He^3 SURFACE.
- GRAPHENE?

"ENGINEERING" APPROACHES
- JOSEPHSON JUNCTIONS (HAVE EXPERIMENTS)
- ATOMIC / MOLECULAR LATTICES (LOTS OF THEORY, NO EXP)
- YOUR IDEA!

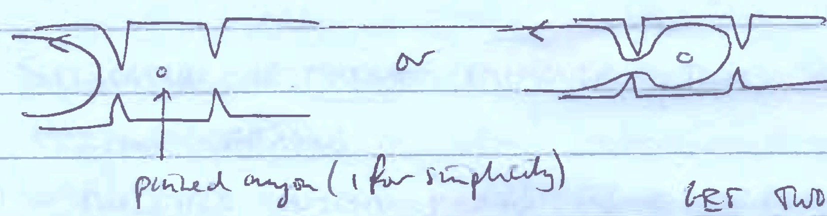
WHAT KINDS OF EXPERIMENTS? (FQA)
- MEASUREMENT OF CHARGE OF ANYONS, HEAT TRANSPORT, ETC.
  QUANTITIES WHICH ARE NOT DIRECT EVIDENCE FOR NON-AB ANYONS
  (BUT ARE FOR THE MODELS WHICH INCLUDE THEM

- ANYON INTERFEROMETRY.

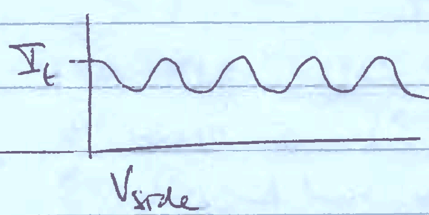


NORDITA TQC

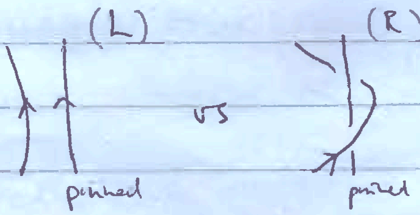
FOR TUNNELING CURRENT, CALCULATE $|\text{AMPLITUDE}|^2$
FOR INCOMING 'ANYON' TO REACH THE OTHER SIDE (ANY PATH)
TWO PATHS POSSIBLE



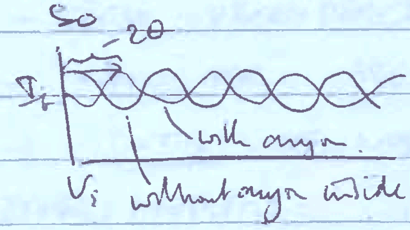
GET TWO DIRECT TERMS $(\text{SINGLE PATH AMP})^2$
+ INTERFERENCE TERM → OVERLAP BETWEEN DIFFERENT PATHS
EVEN WITHOUT (ENCLOSED) PINNED ANYON, WILL HAVE PHASE DIFF
WHICH DEPENDS ON GEOMETRY (PATH LENGTH, ENCLOSED FLUX ...)



WITH ANYON INSIDE, COMPARE



FOR ABELIAN ANYONS, THIS GIVES
AN EXTRA PHASE $e^{2i\theta}$ ← STATISTICS PARAM OF THE ANYONS
SO

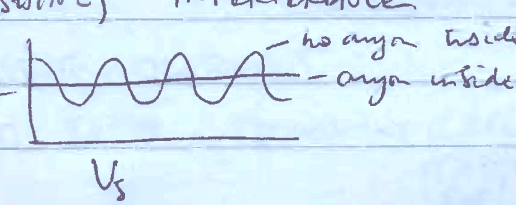


DIFFICULT TO MEASURE (OTHER THINGS CAUSE PHASE SHIFT TOO)

FOR NON-ABELIAN ANYONS, THE AMPLITUDES (L) (R) CAN BE ORTHOGONAL
(OR PARTIALLY ORTHOGONAL)

→ GET REDUCED (OR VANISHING) INTERFERENCE

SO IN EXTREME CASE (ISING)



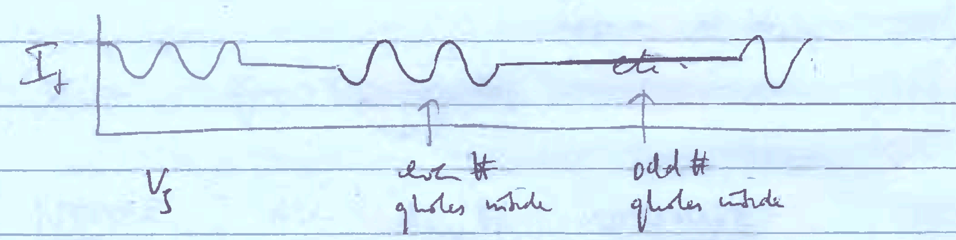
NOTICE : IF WE CAN DETERMINE THE TYPE OF ANYON INSIDE CAN "MEASURE TOP CHARGE"
∃ MEASUREMENT-ONLY TQC SCHEME (BONDERSON –)

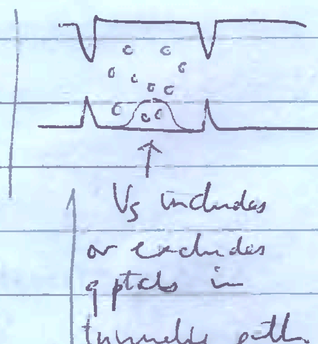
NUKOTTA TQC

ACTUAL EXPERIMENTS

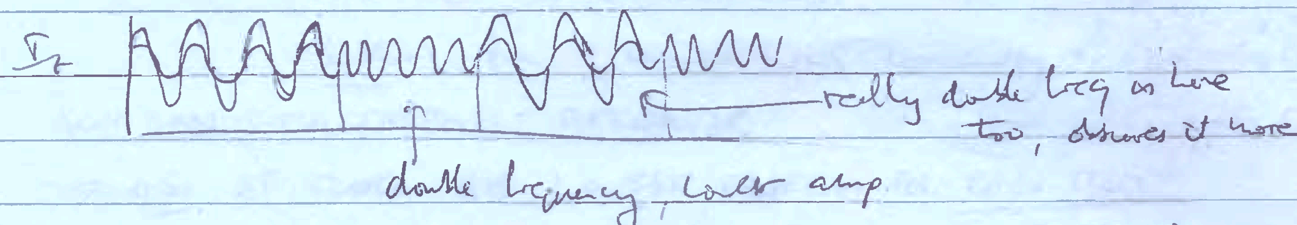
WILLETT, PFEIFFER, WEST, PNAS 0812599106, Arxiv: 0807.0221
+ PRB 82, 205301, 2010
(THEORY ; BISHARA, BONDERSON, NAYAK, SHTENGEL, JKS, PRB 80, 155303, 2009.)

SITUATION IS MUCH MURKIER, BUT STILL PROMISING
VARIOUS REASONS
– MULTIPLE ANYONS PINNED INSIDE → EVEN/ODD EFFECT.





– MULTIPLE TYPES OF ANYONS TUNNEL
AT LEAST 2 $\left(\frac{e}{4}, \frac{e}{2}\right)$
$\frac{e}{2}$ does not have non-AB braiding → OSCILLATION ALWAYS THERE.



– TEMP DEPENDENCE (AT HIGH TEMP, $\frac{e}{2}$ dominates $\frac{e}{4}$)
– CAN "SEE" ABOVE WITH MUCH GOOD WILL, BUT
LOTS OF NOISE (PICTURE IN UPDATED NOTES)

OTHER PROBLEMS :
– HIGH DENSITY OF ANYONS IN INTERF. (NOT ISOLATED ANYONS
+ COUPLE TO EDGE
– "HIGH" T (~ 10 mK) → SHOULD SPOIL SIGNAL)
– VERY SAMPLE / MEASUREMENT TECHNIQUE DEPENDENT
(ONLY ONE GROUP OBSERVES THIS SO FAR, THOUGH → KANG – (DIFF EXP) RESULTS –

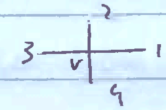
NORDITA TQC (IF TIME LEFT)

ACTUAL MODEL WITH TOPOLOGICAL PHASE (+ANYON-LIKE PARTICLES)

KITAEV'S TORIC CODE.

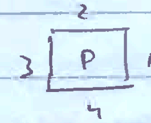
SPINS ON LINKS OF SQUARE LATTICE

INTERACTIONS (4-SPIN) AROUND VERTEX



$$A_v = \prod_{i=1}^{4} \sigma_z^{(i)}$$

AROUND PLAQUETTE



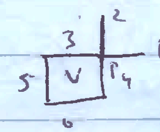
$$B_p = \prod_{i=1}^{4} \sigma_x^{(i)}$$

$$H = -\sum_v A_v - \sum_p B_p$$

NOTICE ALL A_v, B_p COMMUTE

ONLY NONTRIV FOR ADJACENT PLAQUETTE + VERTEX

v, p always share two links



So $A_v B_p = B_p A_v$

(from 2 anticommutations)

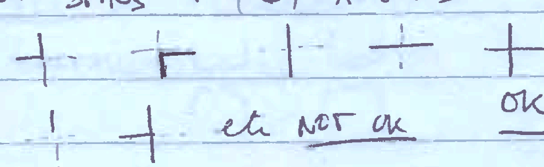
better, from 2 minus signs commuting $\sigma_x^3 \sigma_z^3 = -\sigma_z^3 \sigma_x^3$ and $\sigma_x^4 \sigma_z^4 = -\sigma_z^4 \sigma_x^4$

SO CAN SIMULTANEOUSLY DIAGONALIZE

→ GS OF MODEL HAS LOWEST ENERGY FOR EACH TERM

IN GS $A_v = 1$ REQUIRES EVEN # OF SPINS ↑ (↓) AROUND A VERTEX

(— = $|\uparrow\rangle$ --- = $|\downarrow\rangle$)



etc NOT OK   OK.

MEANS OVERALL REQUIRE LOOPS OF $|\uparrow\rangle$ (or ↓)

(CAN SELF CROSS → COULD GO TO TRIVALENT LATTICE ...)

$B_v = 1$ REQUIRES EQUAL SUPERPOSITION OF "PLAQUETTE FLIPPED" STATES, ie

etc

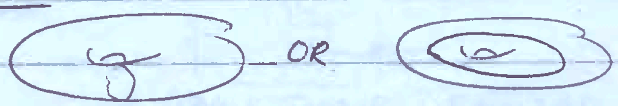
⇒ MEANS WE CAN DEFORM LOOPS  (ALL DEFORMED VERSIONS IN EQUAL SUPERPOSITION)

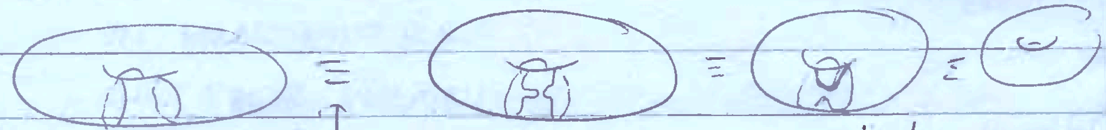
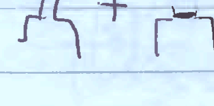
+ NUCLEATE THEM  → ALL NUMBERS OF LOOPS IN SUP.

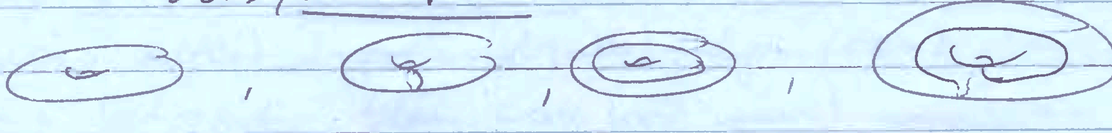
NORDITA TQC

So: TC ground states have
equal superposition of ALL loop configurations
related by
- deformation
- nucleation of small loops

On plane, this gives superposition of ALL loop configs
so unique GS

On torus (also higher genus) get degenerate GS
Eg the GS with amplitude for empty torus  no loops
+ all loops got from nucleating
at plaquettes + deforming → Eg 
does not contain amplitude for

(nontriv windings not related to  by cont def.)

In principle can start from a loop
with any winding (or multiple loops with winding)
but many are in the same GS because B terms
also reconnect (surgery)
Eg 
↑ in same GS as
Local detail 

So in the end, have 4 GS corr to


This degeneracy is topological ('homology'), not due to local sym (clearly no symop)
+ totally stable against perturbation (in pert theory to order ~ L_{torus})
local

17

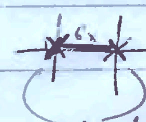
NORDITA TQC | STABILITY BECAUSE OBLIVIOUS TO LOCAL OPS WHICH PRESERVE ENERGY → CLOSED LOOP OPS (NOT AROUND TORUS)

ALSO NOTE: SYSTEM IS GAPPED → EXCITED STATES HAVE OTHER EIGENVALUES OF A_v, B_p
→ $\Delta E \geq 2$ (or $2J_v, 2J_p$)

So AT $k_B T \ll \Delta$ (gap) HAVE PERFECT TOPOLOGICAL MEMORY (2 QUBITS, MORE AT HIGHER GENUS...)

- EXCITATIONS: LOCAL "VIOLATIONS" OF SOME A_v, B_p

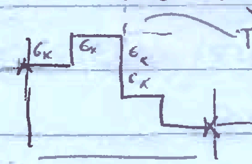
- vertices (A_v): CAN CREATE BY FLIPPING A LINK



vertices have odd #(↑)(↓) states

MAKES "LOOSE ENDS"

CAN MOVE BY FLIPPING ADJACENT LINKS

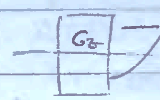


THIS VERTEX HAS EVEN # OF ↑,↓

ONLY THESE VERTICES VIOLATED

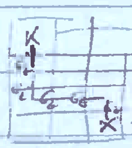
(NOTE σ_x DOES NOT AFFECT B_p EV. → COMMUTES)

- plaquettes (B_p) APPLY σ_z TO LINK

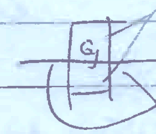


these plaquettes violated

CAN MOVE ALONG PATH ON DUAL LATTICE



ONLY THESE VIOLATED



plaquette violated

vertices violated

CAN ALSO MAKE COMBINED VIOLATIONS

GIVES 4 TYPES OF PARTICLES

Vac, "electric" (vertex), "magnetic" (plaquette), dyon ($e \times m$)

FUSION $e \times e = m \times m = 1$ (can create from vacuum)

$e \times m = em$ ($e \times em = (e \times e) \times m = 1 \times m = m$ etc.)

JUST 1D FUSION SPACES SO AT MOST ABELIAN ANYONS

KITAEV TQC

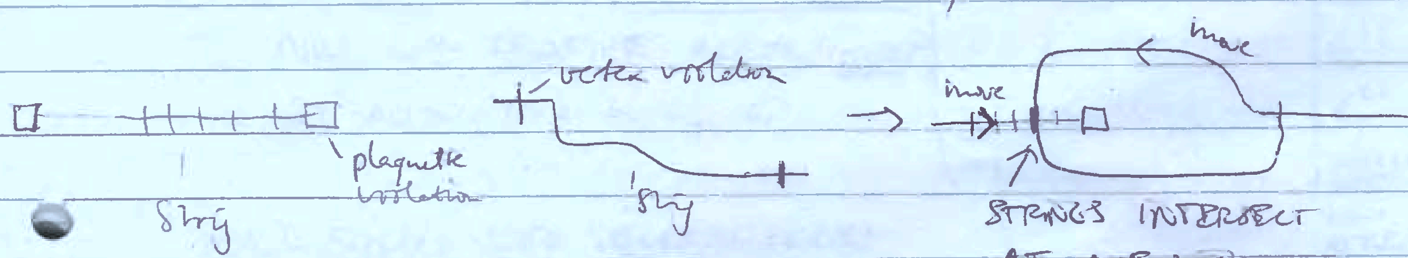
2 INTERESTING PROCESSES

- CAN ACT ON GS-s BY CREATING PARTICLES AND MOVING THEM AROUND NONTRIV LOOPS, THEN ANNIHILATING.

e.g. vertex excitations (or plaquette –)

(DOES NOT GIVE UNIVERSAL COMPUTATION)

- CAN CHECK STATISTICS (TURNS OUT e, m "boson", "em fermion" BUT ;)



STRINGS INTERSECT AT ONE LINK THERE DIFFERENT ORDER OF MOTION (PLAQUETTE ENCLOSED OR NOT) GIVES

$|\sigma_x \sigma_z$ (□ MOVES AFTER + NOT ENCLOSED)

$|\sigma_z \sigma_x$ (□ MOVES BEFORE + NOW ENCLOSED)

SINCE $\sigma_x \sigma_z = -\sigma_z \sigma_x$ GET MINUS SIGN WHEN ENCLOSED

SO MUTUAL STATISTICAL PHASE GIVES −1

→ e, m ARE MUTUALLY "FERMIONIC"

NORDITA TQC

"CODE" PERSPECTIVE

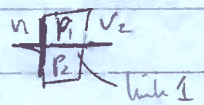
CAN THINK OF ENTIRE TC-SYSTEM AS WAY TO ENCODE THE 4 GS. (REST IS REDUNDANCY TO IMPROVE STABILITY AGAINST ERROR + ALLOW ERR CORRECTION.)

NOTE: IF $\exists$ ERROR AT SOME QUBIT / SPIN)

SO $|S\rangle \to (c_1 \mathbb{1} + c_x \sigma_x + c_y \sigma_y + c_z \sigma_z)|S\rangle$

THEN DETECT THAT EASILY BY MEASURING A_p, B_v

$\to$ WILL SEE VIOLATION OF A_p OR B_v FOR p, v NR THE AFFECTED SPIN.



Eg MEASURE

ALSO PROJECT ON DISCR ERR

NOW CAN RESTORE ERROR (CORRECT)

BY ACTING WITH $\sigma_x^{(1)}, \sigma_y^{(1)}, \sigma_z^{(1)}$

A_{v_1}	A_{v_2}	B_{p_1}	B_{p_2}	
1	1	1	1	OK
−1	−1	1	1	$\sigma_x^{(1)}$ acted
1	1	−1	−1	$\sigma_z^{(1)}$ "
−1	−1	−1	−1	$\sigma_y^{(1)}$ "
1	−1	1	1	ERROR MUST LOOK AT REST OF A_v

STILL WORKS WITH SEVERAL ERRORS.

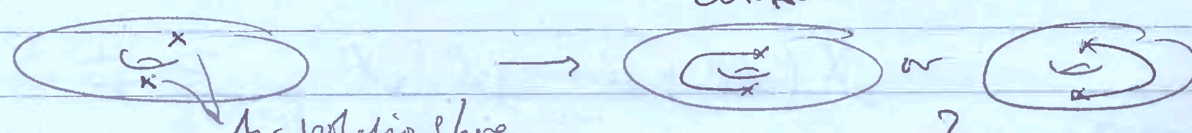
GOES WRONG FOR

ERRORS ALL THE WAY ROUND

$\to$ NOT DETECTED BY A_v, A_p

Err = σ_z here (or σ_x)

OR MOST OF HALF WAY ROUND



A_v violations here

SEE ERROR

? CORRECTION NOT POSSIBLE.

MKS