

~~Thermodynamic limit~~

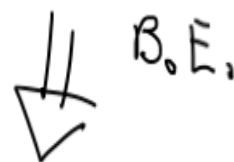
$$N \rightarrow \infty$$

Use analytics:

$$\{I\} \quad \dots \circ \dots$$

Intuitive picture:

$$\{I\} \quad \dots \dots \dots$$



Rewrite BE: as $N \rightarrow \infty$, treat $\frac{I_\alpha}{N} \rightarrow \lambda$

$$\lambda \in \mathbb{R}$$

Define a density $\rho(x) \equiv \frac{1}{N} \sum_{a=1}^M \delta\left(x - \frac{I_a}{N}\right)$

Rewrite BE using ρ :

$$\left[\Theta_1(\lambda(x)) - \int_{-x}^{x_+} dy \Theta_2(\lambda(x) - \lambda(y)) \rho(y) - 2\pi x \right]_{x = \frac{I_a}{N}} = 0$$

$$\lambda\left(x = \frac{I_a}{N}\right) \equiv \lambda_a$$

Let $\lambda(x)$ be defined on whole x axis:

$$\Theta_1(\lambda(x)) - \int_{-x}^{x_+} dy \Theta_2(\lambda(x) - \lambda(y)) \rho(y) = 2\pi x$$

$$\text{or } \Theta_1(\lambda) - \text{''} \text{''} \text{''} = 2\pi x(-1)$$

To go further: define hole & total densities:

given a set $\{I_a\}$,

$$\rho(x) = \frac{1}{N} \sum_{n \in \{I\}} \delta\left(x - \frac{n}{N}\right)$$

$$\rho_h(x) = \frac{1}{N} \sum_{n \notin \{I\}} \delta\left(x - \frac{n}{N}\right)$$

$$\rho_T(x) = \rho(x) + \rho_h(x) \xrightarrow{N \rightarrow \infty} 1$$

Rewrite everything in rapidity space:

$$\rho(\lambda) = \rho(x(\lambda)) \frac{dx(\lambda)}{d\lambda} \quad \& \quad \rho_h(\lambda) \quad \rho_T(\lambda) = \frac{dx(\lambda)}{d\lambda}$$

$$\stackrel{BE}{\Rightarrow} \Theta_1(\lambda) - \int_{-\lambda}^{\lambda} d\lambda' \Theta_2(\lambda - \lambda') \rho(\lambda') = 2\pi x(\lambda)$$

Take $\frac{d}{dx}$ of this:

$$a_1(x) = \int_{-x}^{x} dx' a_2(x-x') \rho(x') = \rho(x) + \rho(-x)$$

$$a_n(x) \equiv \frac{1}{2\pi} \frac{d}{dx} \Theta_n(x) = \frac{1}{2\pi} \frac{n}{x^2 + n^2/4} \quad n=1, 2, \dots$$

Energy of a state: $\frac{E}{N} = -J\pi \int_{-x}^{x} dx a_1(x)$ (1)

See notes for further calculations:
- GSP
- simple excitations

Naive defⁿ of QI: \exists set $\{\hat{I}\}$ of conserved charges

$$[H, \hat{I}] = 0 \text{ \& } [\hat{I}_m, \hat{I}_n] = 0 \quad \forall m, n$$

Define a function $\hat{Z}(\lambda)$ taking operator values
for $\lambda \in \mathbb{C}$, i.e.: ∇ transfer matrix

$$\hat{Z}(\lambda) = \exp \sum_{n=0}^{\infty} \frac{i_n}{n!} \hat{I}_n (\lambda - \xi)^n$$

i_n : parameters

conserved charges

ξ : some extra parameter

$$\hat{I}_n = \frac{d^n}{d\lambda^n} \ln \hat{Z}(\lambda) \Big|_{\lambda=\xi}$$

Since $[I, I] = 0$, we have

$$[\hat{z}(\lambda), \hat{z}(\mu)] = 0 \quad \forall \lambda, \mu.$$

monodromy matrix

Introduce auxiliary space \mathcal{H} s.t. $\hat{z}(\lambda) = \text{Tr}_A T(\lambda)$

$$[(\text{Tr}_A T(\lambda)), (\text{Tr}_A T(\mu))] = 0$$

Hilbert space

View this as a relation in $A_1 \otimes A_2 \otimes \mathcal{H}$

$$\text{Define } T_1(\lambda) = T(\lambda) \otimes \mathbb{1}_2$$

$$T_2(\mu) = \mathbb{1}_1 \otimes T(\mu)$$

$$\Rightarrow \text{Tr}_{A_1 \otimes A_2} [T_1(\lambda), T_2(\mu)] = 0$$

$$\text{or } T_{A_1 \otimes A_2} T_1(\lambda) T_2(\mu) = T_{A_1 \otimes A_2} T_2(\mu) T_1(\lambda)$$

General solⁿ: \exists similarity $R_{12}(\lambda, \mu)$ in $A_1 \otimes A_2$

$$\text{s.t. } R_{12}(\lambda, \mu) T_1(\lambda) T_2(\mu) R_{12}^{-1}(\lambda, \mu) = T_2(\mu) T_1(\lambda)$$

$$\text{or } R_{12}(\lambda, \mu) T_1(\lambda) T_2(\mu) = T_2(\mu) T_1(\lambda) R_{12}(\lambda, \mu)$$

Interesting relation or Yang-Baxter relⁿ

