Number-Theory Dark Matter

5. August 2011

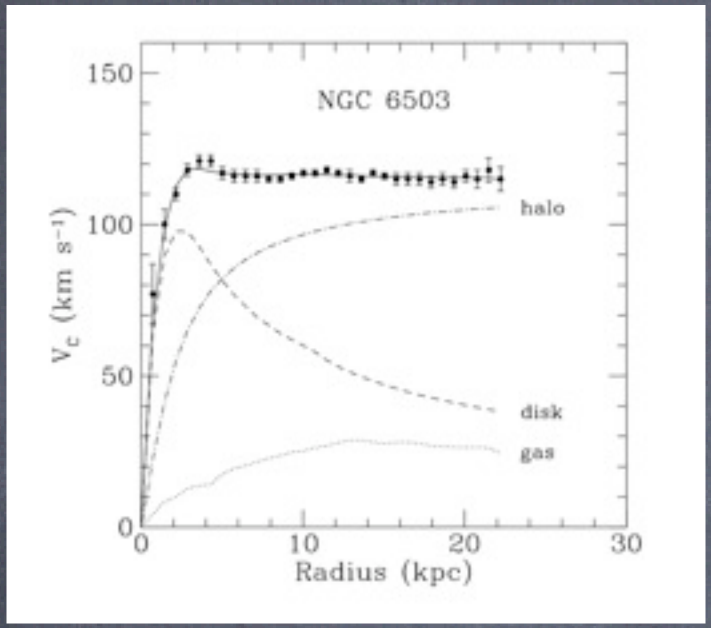
@TeVPA 2011

Fumi Takahashi
(Tohoku Univ. and IPMU)

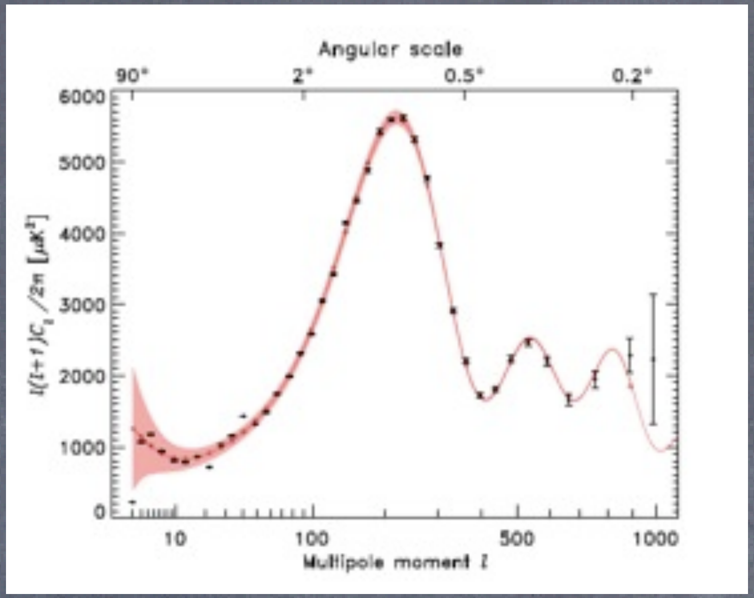
1. Introduction

Dark Matter

rotation curve



CMB

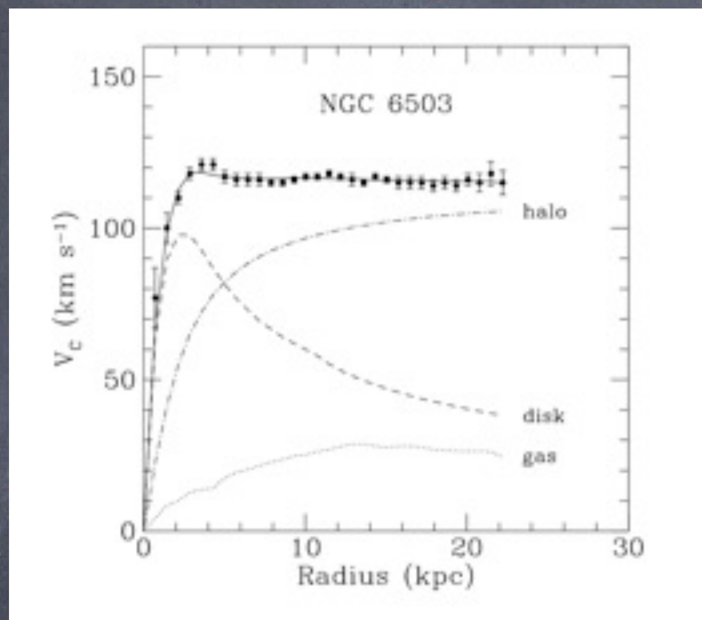


lensing

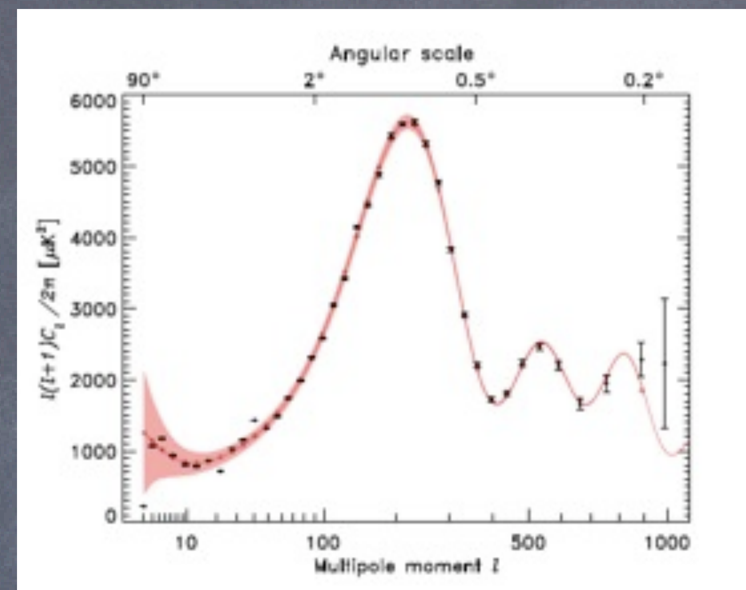
Bullet cluster

Dark Matter

rotation curve



CMB



lensing

Bullet cluster

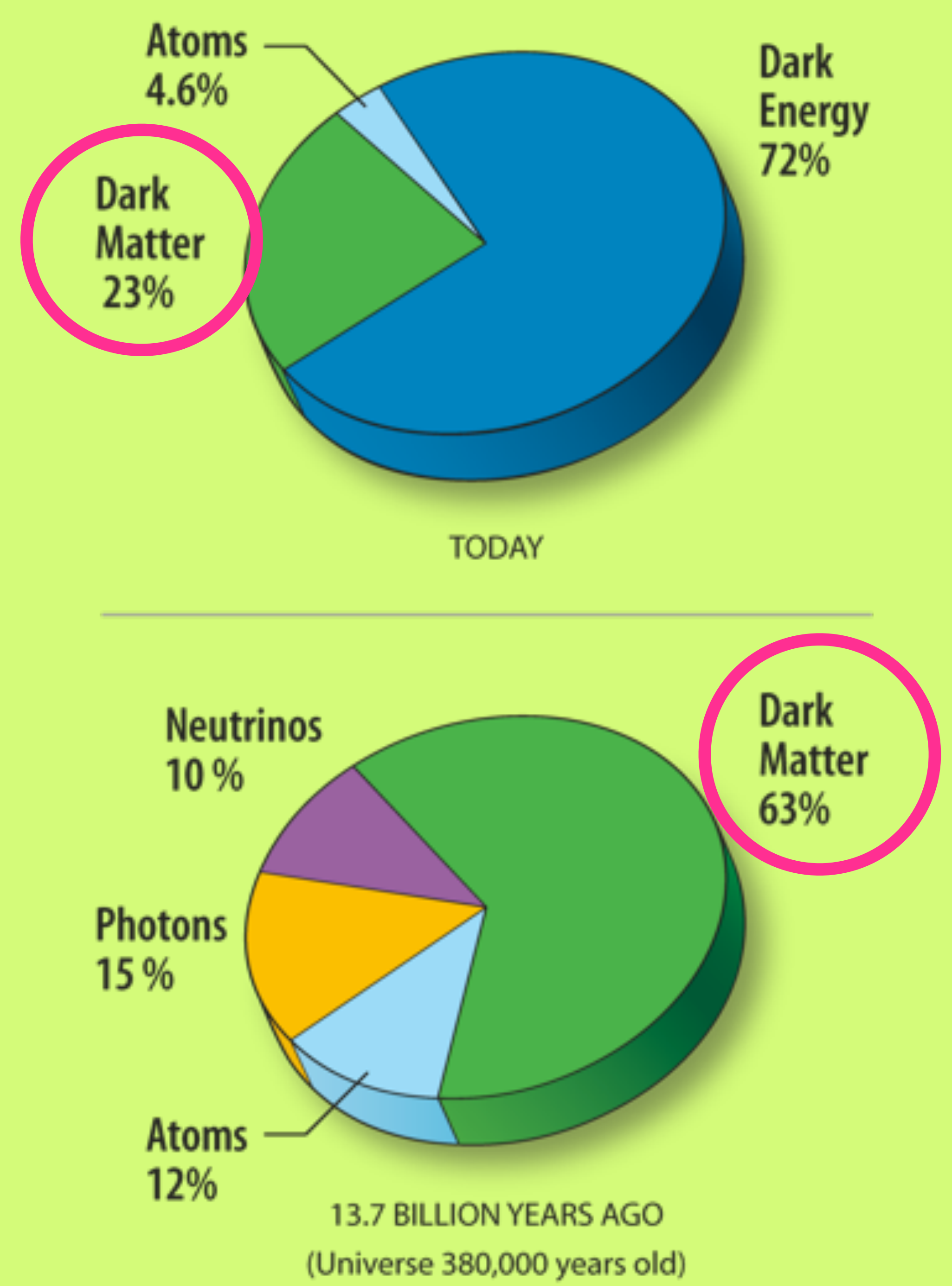
DM is now on sale

Dark Matter

The presence of DM has been firmly established.

$$\Omega_{DM} \sim 0.2$$

DM is perhaps made of an as-yet-undiscovered particle.



(taken from WMAP webpage)

What are its properties?

- Electrically neutral
- (Very)weak interactions
- Cold or warm
- Long-lived.

Why long-lived?

1) Symmetry

e.g. R-parity, KK parity: LSP, LKP.

2) Light mass

e.g. $\tau \propto 1/m^3$

3) Very weak interactions

e.g. Hidden sector, gravity sector.

These are not exclusive:
e.g. axion [2 & 3], gravitino [1 & 3 (also 2)]

Why long-lived?

1) Symmetry

Global symmetries are expected to be explicitly broken; if gauged, stable DM.

e.g. R-parity, KK parity: LSP, LKP.

2) Light mass

e.g. $\tau \propto 1/m^3$

The light mass and/or weak int. may be due to symmetry (e.g. chiral, shift sym, SUSY) or extra dim or compositeness.

3) Very weak interactions

e.g. Hidden sector, gravity sector.

These are not exclusive:

e.g. axion [2 & 3], gravitino [1 & 3 (also 2)]

2. DM model

The longevity of DM is a puzzle. It may be stabilized by an unbroken symmetry such as Z_2 symmetry.

Global symmetries are expected to be explicitly broken. So, the symmetry may be a part of new gauge symmetry.

2. DM model

The longevity of DM is a puzzle. It may be stabilized by an unbroken symmetry such as Z_2 symmetry.

Global symmetries are expected to be explicitly broken. So, the symmetry may be a part of new gauge symmetry.

→ Z_2 subgroup of $U(1)_{B-L}$

The seesaw mechanism:

T. Yanagida `79, M.Gell-Mann, P.Ramond and R.Slansky `79, (Minkowski, `77)

$$\mathcal{L} = i\bar{N}_i\gamma^\mu\partial_\mu N_i + \left(\lambda_{i\alpha}\bar{N}_i L_\alpha\,\phi - \frac{1}{2}M_{Ri}\bar{N}_i^c N_i + \mathrm{h.c.}\right),$$

$$(m_\nu)_{\alpha\beta} = \sum_i \lambda_{i\alpha}\lambda_{i\beta}\frac{\left\langle\phi^0\right\rangle^2}{M_{Ri}}.$$

Then the right-handed neutrino mass scale turns out to be close to the GUT scale $M_R \sim 10^{15}\,\mathrm{GeV}$ for $\lambda_{i\alpha} \sim 1$

Neutrino Mass = Weak scale^2 / (B-L) breaking scale

Heavy Majorana mass for right-handed neutrinos spontaneously breaks $U(1)_{B-L}$ down to Z_{2B-L}:

$$\mathcal{L} = \frac{1}{2}\kappa_i \Phi \bar{N}_i^c N_i$$

$$\langle\Phi\rangle = v_{\mathrm{B-L}} = 10^{15}\,\mathrm{GeV}$$

	Φ	N
B-L	2	-1

Since Φ has a B-L charge +2, the Z_2 subgroup of $U(1)_{B-L}$ symmetry remains unbroken, and this may make the DM stable.

- We introduce a set of chiral fermions charged under B-L, $\{\psi_i\}$, some of which are stable and become DM.
- Non-trivial constraints on the number and B-L charges of the additional fermions from the anomaly cancellation conditions of $U(1)_{B-L}$.

We show that

1) non-trivial solutions appear when at least five fermions are added.

2) One of them contains a (W)DM candidate.

Anomaly cancellation conditions:

$$\sum_{i=1}^{n} Q_i^3 = 0, \qquad \sum_{i=1}^{n} Q_i = 0,$$

where Q_i is B-L charge of ψ_i. Q_i must be a rational number.

Let us rewrite them as

$$\sum_{i=1}^{n} (Z_i)^3 = 0, \quad \sum_{i=1}^{n} Z_i = 0,$$

where $\{Z_1, \cdots Z_n\} = \{aQ_1, \cdots, aQ_n\}$ is an integer.

n=2

$$Z_1 = -Z_2$$

The following mass term is allowed by symmetry. They can be very heavy ~ M_P.

$$\mathcal{L} = \frac{1}{2} m \psi_1 \psi_2 + \text{h.c.}.$$

We exclude such vector-like solution.

n=2

$$Z_1 = -Z_2$$

The following mass term is allowed by symmetry. They can be very heavy ~ M_P.

$$\mathcal{L} = \frac{1}{2} m \psi_1 \psi_2 + \text{h.c.}.$$

We exclude such vector-like solution.

---

n=3

$$x^3 + y^3 = z^3$$

n=2

$$Z_1 = -Z_2$$

The following mass term is allowed by symmetry. They can be very heavy ~ M_P.

$$\mathcal{L} = \frac{1}{2} m \psi_1 \psi_2 + \mathrm{h.c.}.$$

We exclude such vector-like solution.

---

n=3

$$x^3 + y^3 = z^3$$

There is no integer solution; this is a special case (n=3) of the Fermat's last theorem.

n=2

$$Z_1 = -Z_2$$

The following mass term is allowed by symmetry. They can be very heavy ~ M_{P}.

$$\mathcal{L} = \frac{1}{2} m \psi_1 \psi_2 + \text{h.c.}.$$

We exclude such vector-like solution.

---

n=3

$$x^3 + y^3 = z^3$$

There is no integer solution; this is a special case (n=3) of the Fermat's last theorem.

---

n=4

$$(Z_1 + Z_2)(Z_2 + Z_3)(Z_3 + Z_1) \ = \ 0.$$

Two sets of vector-like fermions.

n=5 There are non-trivial solutions:

Table 1
Independent solutions to Eqs. (4) and (5) for $\max\{|Z_i|\} \leqslant 25$ for $n = 5$.

Z_1	Z_2	Z_3	Z_4	Z_5
−9	−5	−1	7	8
−9	−7	2	4	10
−18	−17	1	14	20
−21	−12	5	6	22
−25	−8	−7	18	22

Let us consider an integer solution, a=1.

$$(Q_1, Q_2, Q_3, Q_4, Q_5) = (-9, -5, -1, 7, 8),$$

ψ_5 is the only fermion with an even B-L charge, and so it is stable due to Z_2 (B-L) !!

n=5

There are non-trivial solutions:

Table 1
Independent solutions to Eqs. (4) and (5) for $\max\{|Z_i|\} \leqslant 25$ for $n = 5$.

Z_1	Z_2	Z_3	Z_4	Z_5
−9	−5	−1	7	8
−9	−7	2	4	10
−18	−17	1	14	20
−21	−12	5	6	22
−25	−8	−7	18	22

Let us consider an integer solution, a=1.

$$(Q_1, Q_2, Q_3, Q_4, Q_5) = (-9, -5, -1, 7, 8),$$

ψ_5 is the only fermion with an even B-L charge, and so it is stable due to Z_2 (B-L) !!

n=5

There are non-trivial solutions:

Table 1
Independent solutions to Eqs. (4) and (5) for $\max\{|Z_i|\} \leqslant 25$ for $n = 5$.

Z_1	Z_2	Z_3	Z_4	Z_5
−9	−5	−1	7	8
−9	−7	2	4	10
−18	−17	1	14	20
−21	−12	5	6	22
−25	−8	−7	18	22

Let us consider an integer solution, a=1.

$$(Q_1, Q_2, Q_3, Q_4, Q_5) = (-9, -5, -1, 7, 8),$$

ψ_5 is the only fermion with an even B-L charge, and so it is stable due to Z_2 (B-L) !!

It acquires a mass from $\frac{\Phi^{*8}}{M^7}\psi_5\psi_5,$

	Φ	ψ_5
B-L	2	8

$$m_{\psi_5} \approx 10\,\mathrm{keV}\left(\frac{v_{\mathrm{B-L}}}{3\times 10^{15}\,\mathrm{GeV}}\right)^8$$

Note that there is no mixing between ψ_5 and the other fermions.

$$Y_{\psi_5} \equiv \frac{n_{\psi_5}}{s} \sim \left.\frac{\langle\sigma v\rangle\, n_f^2/H}{\frac{2\pi^2}{45}g_* T^3}\right|_{T=T_R}$$

$$\sim 4\times 10^{-5}\left(\frac{g_*}{10^2}\right)^{-\frac{3}{2}}\left(\frac{Q_5}{8}\right)^2\left(\frac{v_{\mathrm{B-L}}}{3\times 10^{15}\,\mathrm{GeV}}\right)^{-4}\left(\frac{T_R}{4\times 10^{13}\,\mathrm{GeV}}\right)^3,$$

$$\Omega_{\psi_5}h^2 \approx 0.1\left(\frac{m_{\psi_5}}{10\,\mathrm{keV}}\right)\left(\frac{Y_{\psi_5}}{4\times 10^{-5}}\right).$$

Number-theory warm DM

Comments:

1. DM mass will be too light for other solutions with larger charges.

2. DM can be unstable if the charges of additional fermions are fractional.

Conclusions

- Longevity of dark matter may be due to some unbroken symmetry, and Z_2 subgroup of $U(1)_{B-L}$ is a prime candidate.
- At least 5 chiral fermions charged under $U(1)_{B-L}$ are needed to satisfy the anomaly cancellation condition.
- One of the chiral femrions become a DM of mass about 10keV.


Thank you very much!