Spin-Dependent WIMP-nucleus Elastic Scattering Simplified

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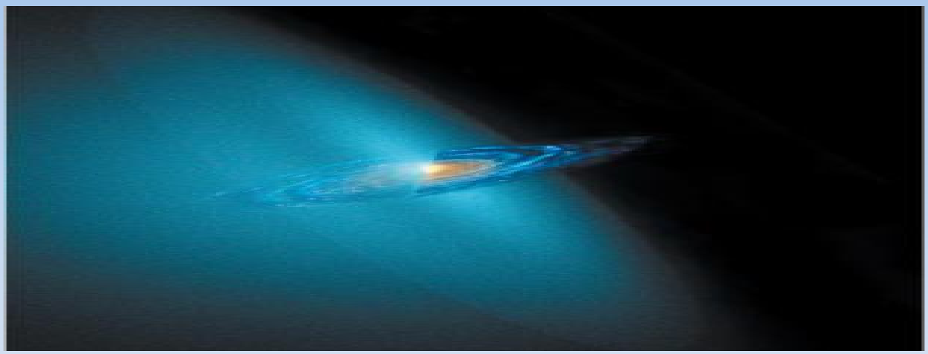
Based on:

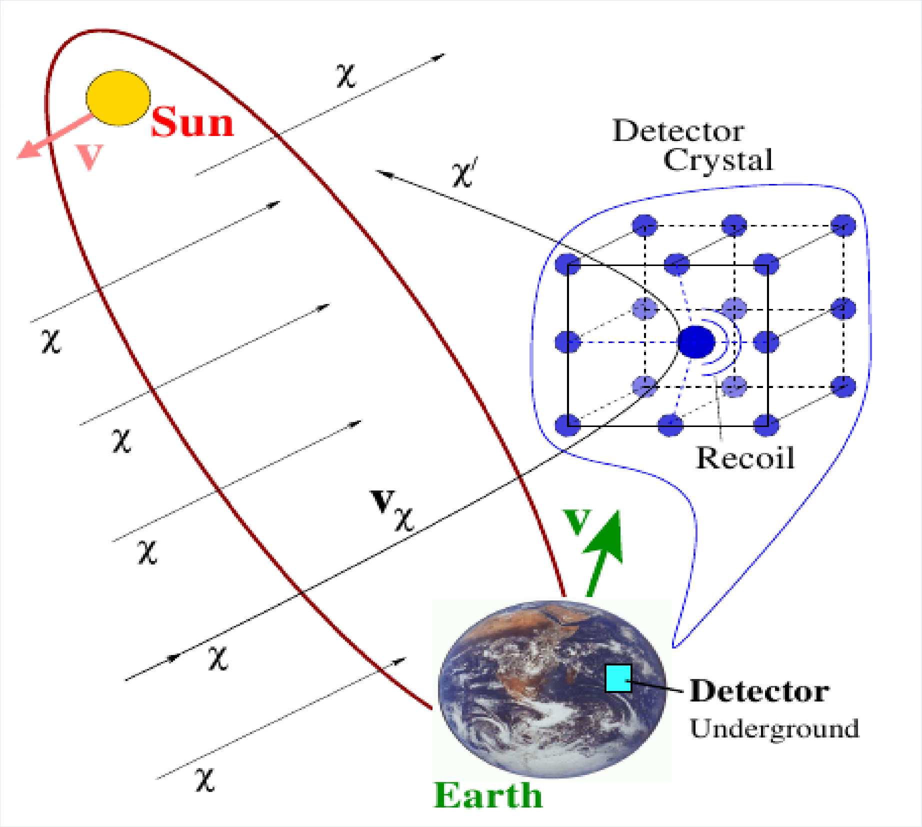
M. C., J.D. Vergados **and M. E. Gomez**
"Scheme for the extraction of WIMP-nucleon scattering cross sections from total event rates"
Phys. Rev. D 83, 075010 (2011), arXiv:1011.6108 [hep-ph]

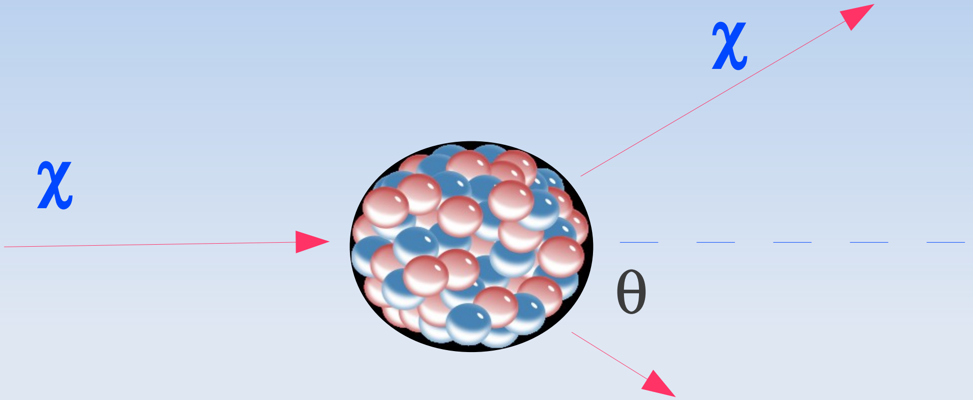
and

M.C.
"On upper limits on neutralino-nucleon spin-dependent cross sections"
In preparation

WIMP-nucleus elastic scattering





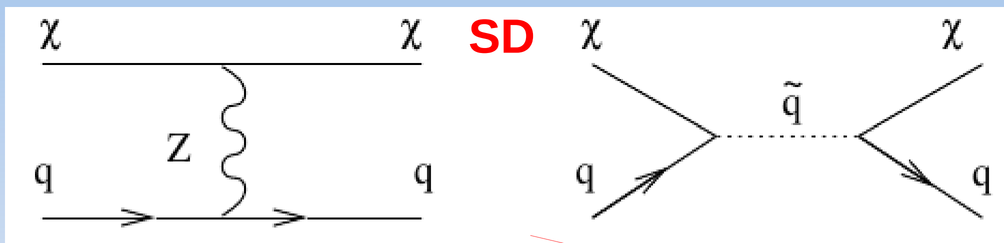


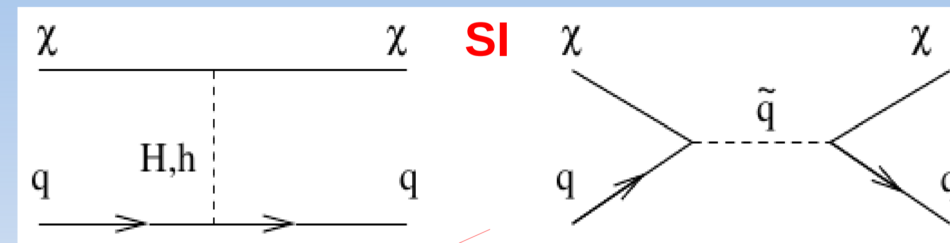
$$q = |\vec{q}| = 2\mu_A v_\chi \cos\theta$$

$$E_R = \frac{q^2}{2m_A}$$

Non relativistic WIMP scatters elastically with nuclei. The recoil of the nucleus deposits a tiny amount of energy in the detector: recoil energies are from few to 100 keV

Neutralino-nucleon cross sections





$$\mathcal{L}_{eff} = g_q(\bar{\chi}\gamma^\mu\gamma^5\chi)(\bar{q}\gamma_\mu\gamma^5 q) + h_q(\bar{\chi}\chi)(\bar{q}q)$$

Axial vector interaction:
Spin-spin interaction in the non relativistic limit

$$a_p = \sum_{q=u,d,s} g_q \Delta q^{(p)}$$

$$a_n = \sum_{q=u,d,s} g_q \Delta q^{(n)}$$

Scalar interaction,
Spin Independent

$$\lambda^{(p)} = \sum_q h_q f_q^{(p)} \simeq \lambda^{(n)} = \sum_q h_q f_q^{(n)} \equiv c_0$$

f's and Δ's factors require inputs from non perturbative QCD

$$\sigma^{SD}_{p,n} = \frac{3\mu_p^2}{\pi}|a_{p,n}|^2$$

$$\sigma^{SI} = \frac{\mu_p^2}{\pi}|c_0|^2$$

3 elementary cross sections to be extracted from experiments

WIMP-nucleus SI cross section

1) The structure of the nucleus is important if qR~1: in this case the Zero Momentum Transfer Limit (ZMTL) cross section is not a good approximation, expecially for heavy nuclei

2) The scalar SI interaction is sensible to the mass distribution inside the nucleus

$$\frac{d\sigma^{SI}_{(A)}}{dq^2} = \frac{1}{4(\mu_A v)^2}\frac{\mu_A^2}{\pi}|\lambda^p Z F^Z(q^2) + \lambda^n (A-Z) F^N(q^2)|^2$$

$$\lambda^p \simeq \lambda^n \equiv c_0$$

$$F^Z(q^2) \simeq F^N(q^2) \cong F(q^2)$$

q=0 → 1

$$\sigma^{SI}_{(A)}(0) = \frac{\mu_A^2}{\pi}|c_0|^2 A^2$$

$$\frac{d\sigma^{SI}_{(A)}}{dq^2} = \frac{1}{4(\mu_A v)^2}\sigma^{SI}_{(A)}(0) F^2(q^2)$$

The Particle Physics and the Nuclear Physics degrees of freedom are factorized

WIMP-nucleus SD cross section: standard formalism (1)

Engel PLB 264 (1991) Engel, Pittel and Vogel IJMP E 1 (1992)

$$\frac{d\sigma^{SD}_{(A)}}{dq^2} \propto |\langle J, M_Z = J| \sum_{i=1}^{A} (a_0 + a_1 \tau_i^3)\vec{\sigma}_i e^{-i\vec{q}\cdot\vec{r}_i} |J, M_Z = J\rangle|^2$$

$$\propto \frac{1}{2J+1} |\langle J|| \sum_{i=1}^{A} (a_0 + a_1 \tau_i^3)\vec{\sigma}_i e^{-i\vec{q}\cdot\vec{r}_i} ||J\rangle|^2$$

Isospin representation

$$a_0 = a_p + a_n$$

$$a_1 = a_p - a_n$$

$$\frac{d\sigma^{SD}_{(A)}}{dq^2} \propto \frac{1}{2J+1} \sum_{L_{odd}} (|\langle J||\mathcal{T}^{el5}_L(q)||J\rangle|^2 + |\langle J||\mathcal{L}^5_L(q)||J\rangle|^2)$$

$$S(q) = a_0^2 S_{00}(q) + a_0 a_1 S_{01}(q) + a_1^2 S_{11}(q)$$

Donnelly and Walecka NPA 274 (1976)
Donnelly and Peccei, Phys. Rep. 50 (1979)
Donnelly and Haxton At.Data Nucl. Data Tables 23 (1979)

WIMP-nucleus SD cross section: standard formalism (2)

$$S(q) = a_0^2 S_{00}(q) + a_0 a_1 S_{01}(q) + a_1^2 S_{11}(q)$$

q=0

$$S(0) = \frac{2J+1}{\pi} J(J+1) \left[\frac{a_0(\langle \vec{S}_p \rangle + \langle \vec{S}_n \rangle) + a_1(\langle \vec{S}_p \rangle - \langle \vec{S}_n \rangle)}{2J} \right]^2$$

$$\sigma_{(A)}^{SD}(0) = \frac{4\mu_A^2}{2J+1} S(0)$$

$$\frac{d\sigma_{(A)}^{SD}}{dq^2} = \frac{1}{4(\mu_A v)^2} \sigma_{(A)}^{SD}(0) \frac{S(q)}{S(0)}$$

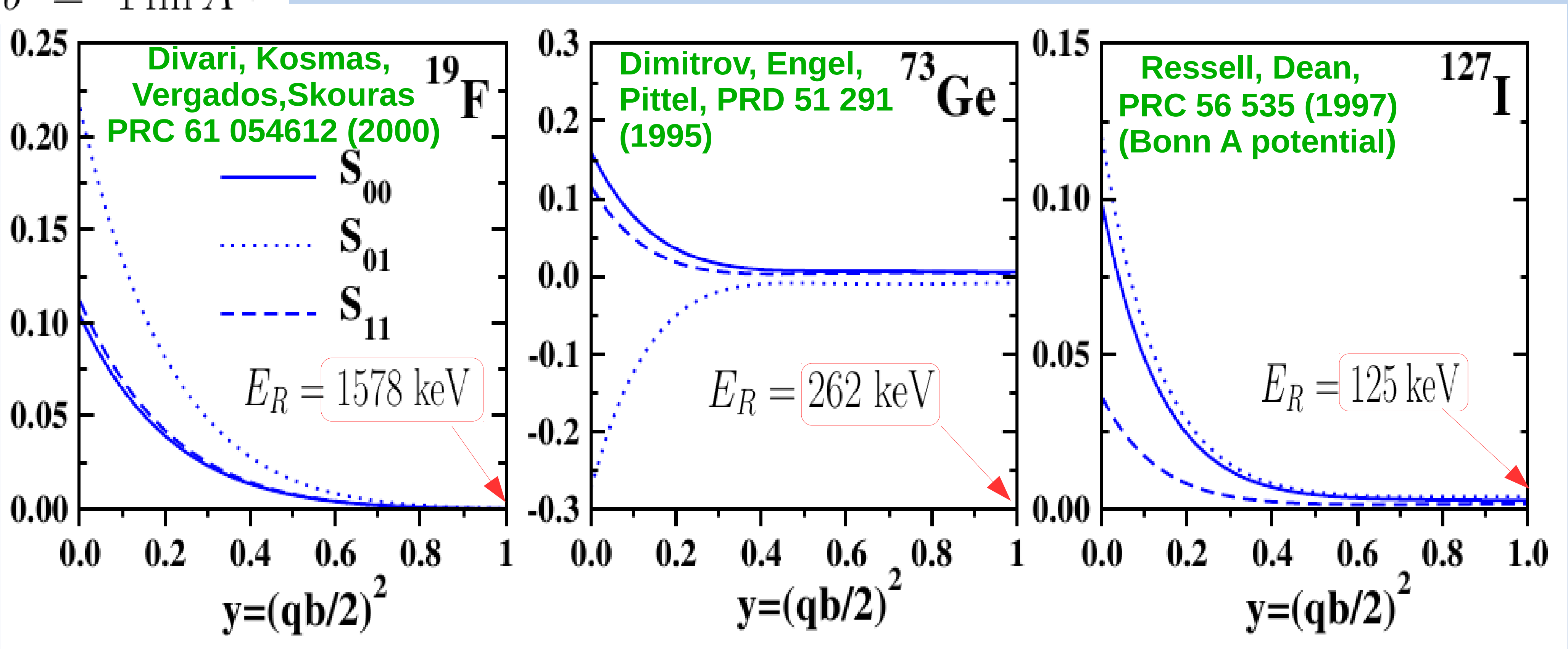
1) To have a spin-dependent “form factor” normalized to one it is necessary to divide by S(0)

2) The nuclear physics degrees of freedom are not decoupled from the particle physics ones as in the SI case with the form factor

WIMP-nucleus SD cross section: standard formalism (2)

The structure functions are furnished as polynomial fits to the results of the shell model calculations in terms of the variable $y = (qb/2)^2$

$b = 1\,\mathrm{fm}\,A^{1/6}$



1) The three momentum-dependent structure functions S_{ij} look different
2) The interference term can be negative

WIMP-nucleus cross section: Vergados formalism (1)

$$\frac{d\sigma^{SD}_{(A)}}{dq^2} \propto |\langle J, M_Z = J| \sum_{i=1}^{A} (a_0 + a_1 \tau_i^3) \vec{\sigma}_i e^{-i\vec{q}\cdot\vec{r}_i} |J, M_Z = J\rangle|^2$$

$$\propto \frac{1}{2J+1} |\langle J|| \sum_{i=1}^{A} (a_0 + a_1 \tau_i^3) \vec{\sigma}_i e^{-i\vec{q}\cdot\vec{r}_i} ||J\rangle|^2$$

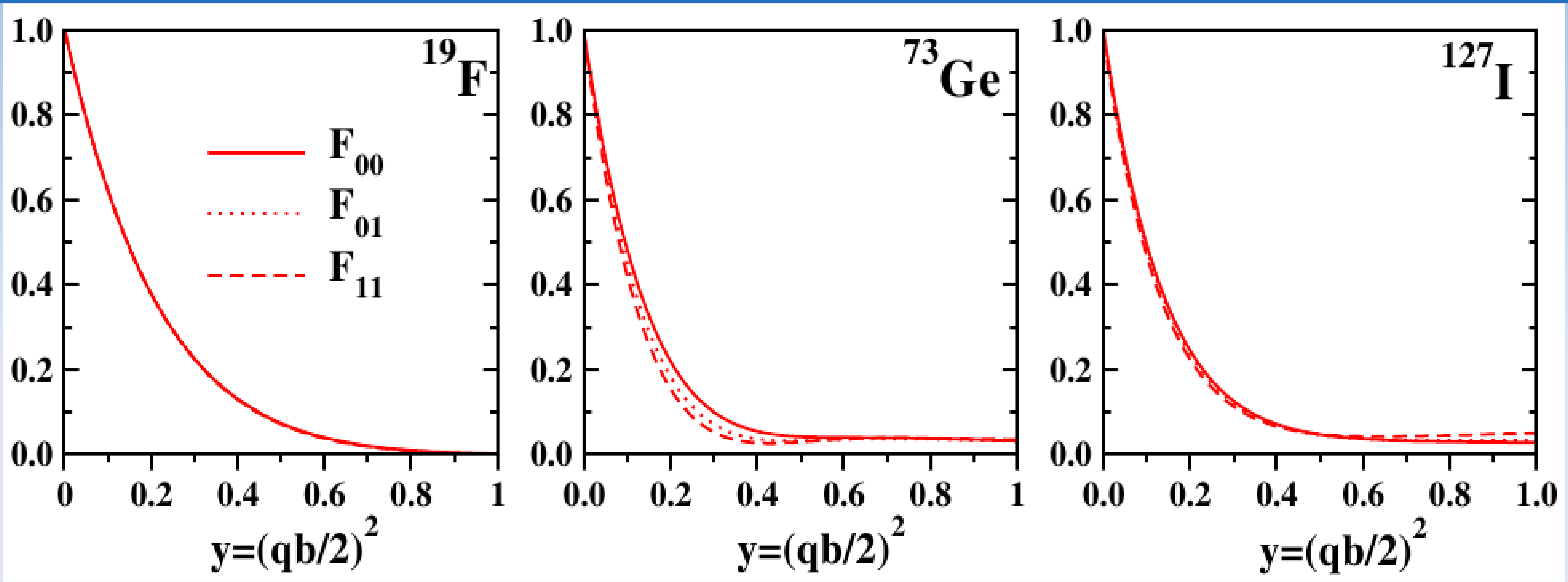
$$\frac{d\sigma^{SD}_{(A)}}{dq^2} \propto (a_0 \Omega_0(q) + a_1 \Omega_1(q))^2$$

Kosmas and Vergados PRD 55 (1997)
Divari, Kosmas, Vergados, Skouras PRC 61 (2000)

$$a_0^2 \Omega_0^2(0) \left(\frac{\Omega_0(q)}{\Omega_0(0)}\right)^2 + 2a_0 a_1 \Omega_0(0)\Omega_1(0) \left(\frac{\Omega_0(q)\Omega_1(q)}{\Omega_0(0)\Omega_1(0)}\right) + a_1^2 \Omega_1^2(0) \left(\frac{\Omega_1(q)}{\Omega_1(0)}\right)^2$$

$F_{00}(q)$ $F_{01}(q)$ $F_{11}(q)$

WIMP-nucleus SD cross section: connection between the two formalisms



$$S_{00}(q) = \frac{2J+1}{16\pi}\Omega_0^2 \; F_{00}(u)$$

$$S_{01}(q) = \frac{2J+1}{8\pi}\Omega_0\Omega_1 \; F_{01}(u)$$

$$S_{11}(q) = \frac{2J+1}{16\pi}\Omega_1^2 \; F_{11}(u)$$

$$\Longrightarrow$$

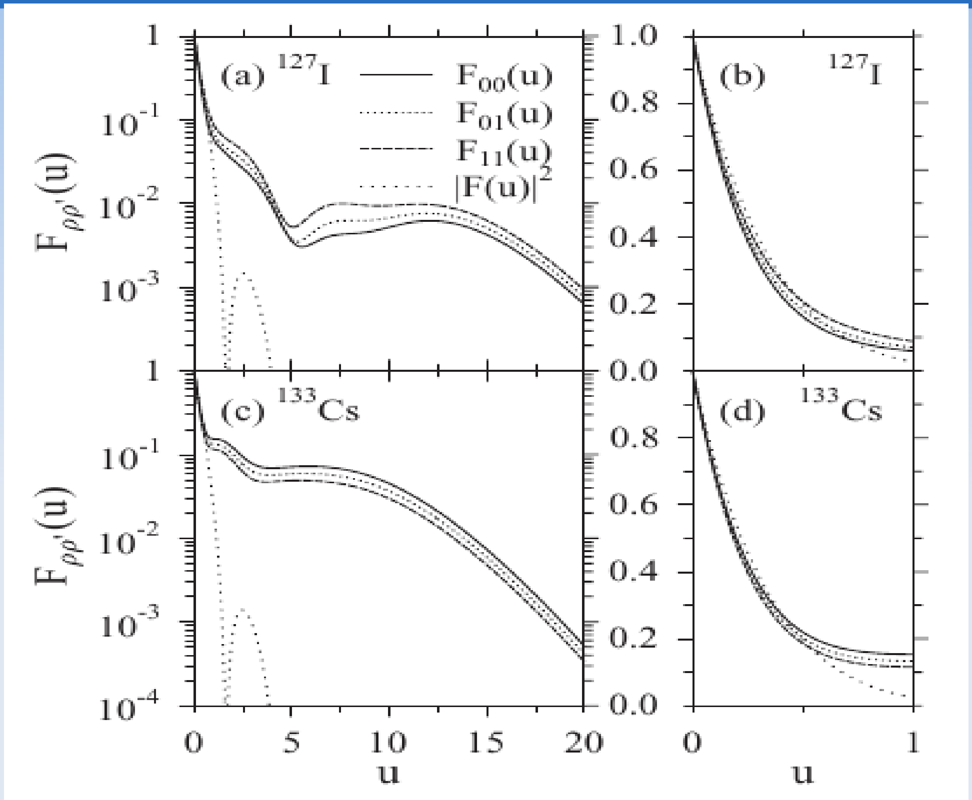
$$\frac{S_{00}(q)}{S_{00}(0)} = F_{00}(u)$$

$$\frac{S_{01}(q)}{S_{01}(0)} = F_{01}(u)$$

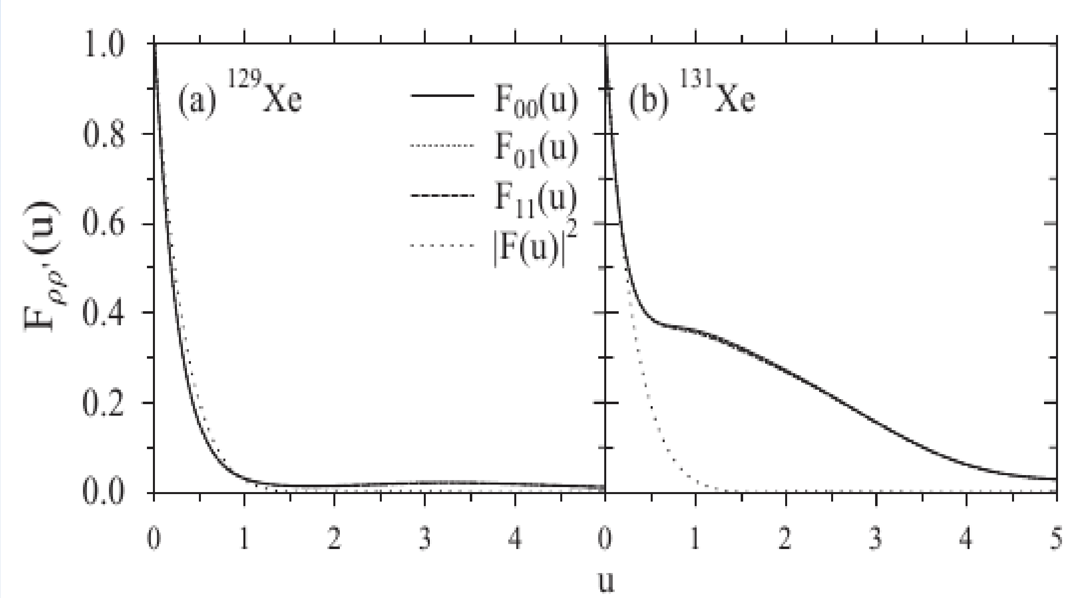
$$\frac{S_{11}(q)}{S_{11}(0)} = F_{11}(u)$$

$$u = q^2b^2/2$$

A recent calculation for heavy nuclei



P. Toivanen, Kortelainen, Suohnen, J. Toivanen PRC 79 (2009)



1) **At high momentum transfer there can be larger differences depending on the nucleus**
2) **There are uncertainties due to the nucleon-nucleon potential used in the shell model calculation**
3) **These regions are less interesting for experiments because cross sections are suppressed and with bigger background**

Advantages of the normalized functions

$$F_{00}(u) \simeq F_{01}(u) \simeq F_{11}(u)$$

One only needs one structure function, that by definition is normalized to one at zero momentum transfer

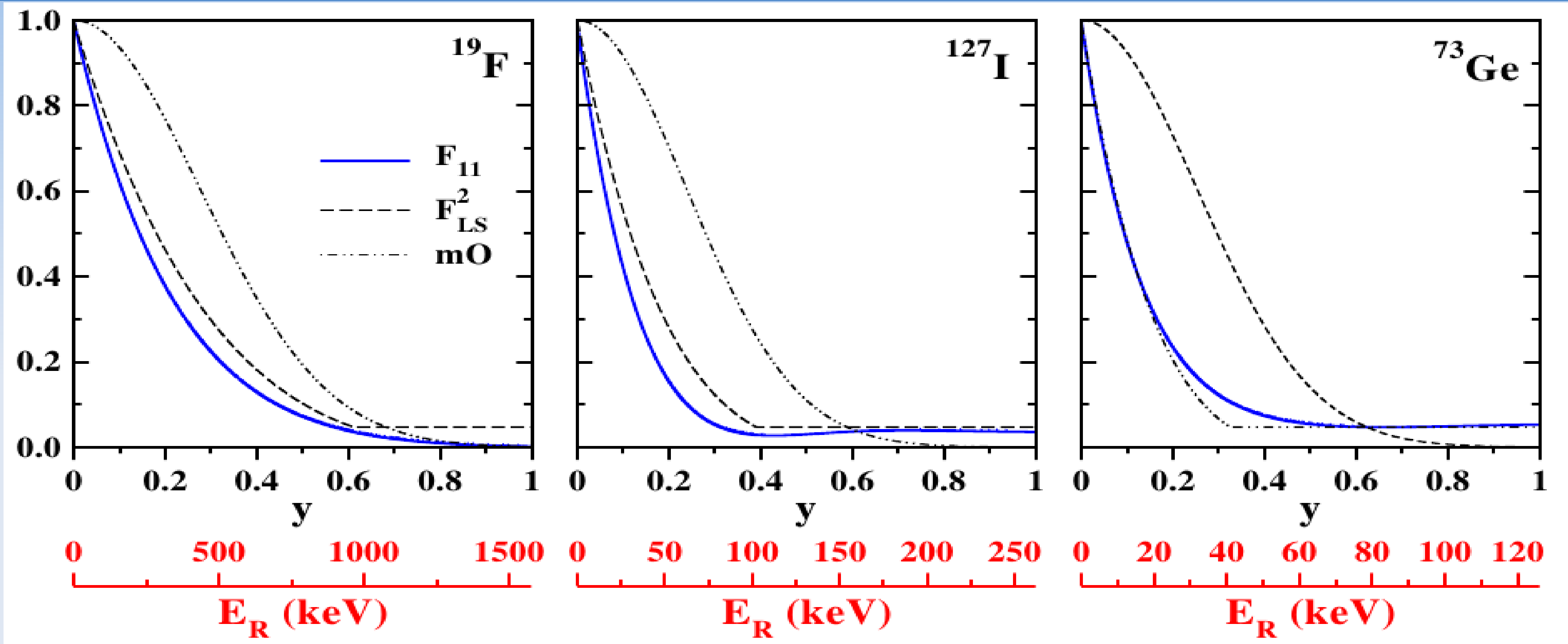
$$\frac{d\sigma^{\mathrm{SD}}_{(A)}}{dE_R} = \frac{1}{2\mu_A^2 v^2}\sigma^{\mathrm{SD}}_{(A)}(0) F_{11}(E_R)$$

Particle physics and nuclear physics are factorized. The cross section has the same form as in the SI case

$$F_{11}(E_R)=S_{11}(E_R)/S_{11}(0)$$

is all that you need for the "spin-dependent form factor"

No need of other parametrizations...



$$F^2_{LS}(qr_n) = \begin{cases} \left(\frac{\sin(qr_n)}{qr_n}\right)^2 & qr_n < 2.55,\ qr_n > 4.5 \\ 0.047 & 2.55 \leq qr_n \leq 4.5 \end{cases}$$

J. D. Lewin, P. F. Smith, Astropart. Phys. 6 (1996)

$$\frac{S_{ij}(q)}{S_{ij}(0)} = \exp\left(-\frac{q^2 R_A^2}{4}\right)$$

MicrOMEGAs, G.~Belanger et al. Comput. Phys. Commun. 180 (2009)

Notation for the total rate

Nuclear cross sections at zero momentum transfer

$$\sigma^{SI}_{(A)}(0) = \left(\frac{\mu_A}{\mu_p}\right)^2 A^2 \sigma^{SI}$$

$$\sigma^{SD}_{(A)}(0) = \left(\frac{\mu_A}{\mu_p}\right)^2 \frac{1}{3}\left(\Omega^A_p \sqrt{\sigma^{SD}_p} + \varrho \Omega^A_n \sqrt{\sigma^{SD}_n}\right)^2$$

Truncated Maxwellian velocity distribution function

$$f(v) = \frac{1}{\kappa v_E}\left(e^{-\frac{(v-v_E)^2}{v_0^2}} - e^{-\frac{(v+v_E)^2}{v_0^2}}\right)$$

Tipycal recoil energy

$$\epsilon_0 = 2\mu_A v_0^2 \left(\frac{\mu_A}{m_A}\right)$$

The integral are dimensionless

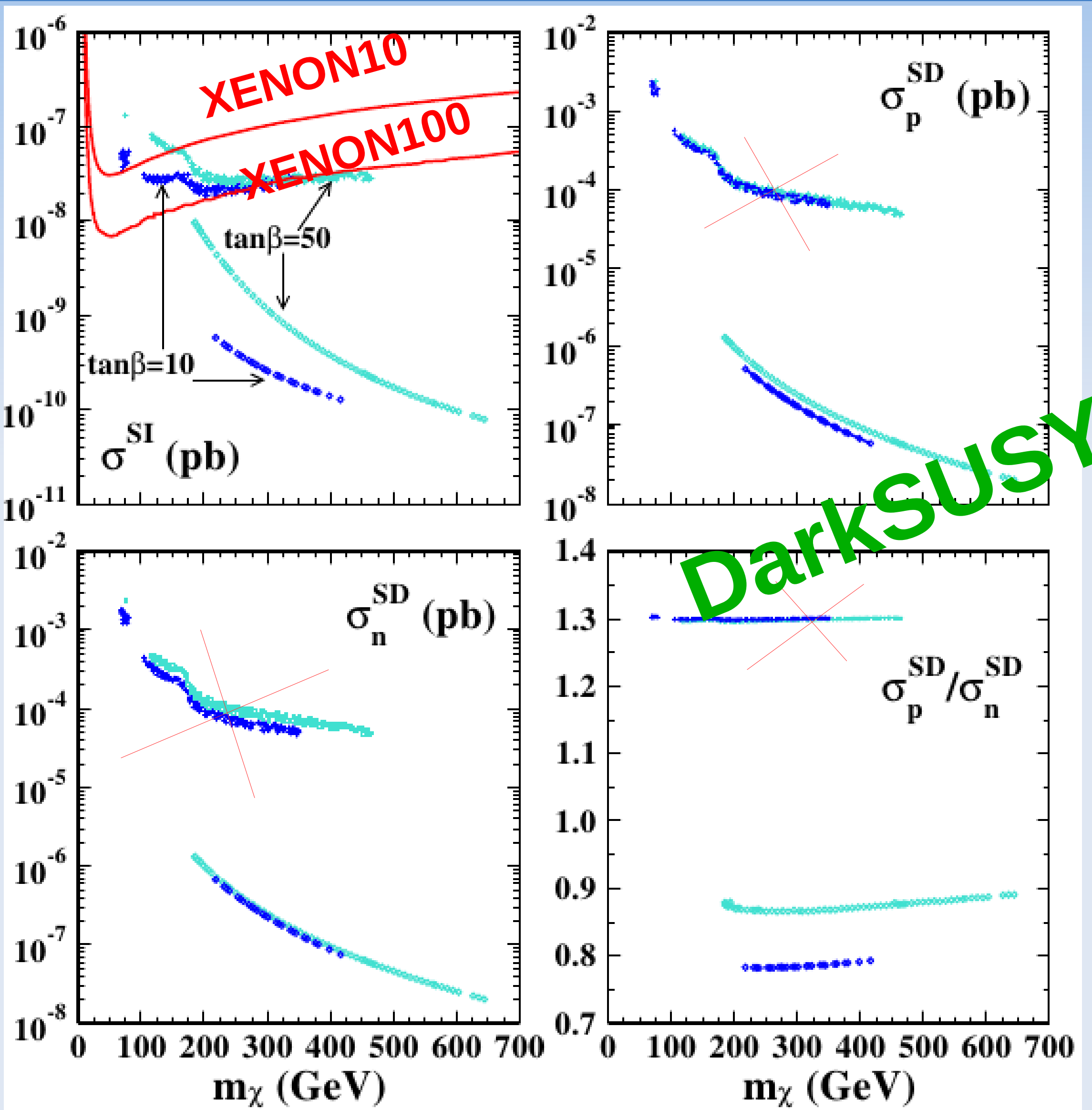
$$t^{SD}_A = \int_{E_1}^{E_2} \frac{dE_R}{\epsilon_0} F_{11}(E_R) \int_{v_{min}(E_R)}^{v_{max}} dv f(v)$$

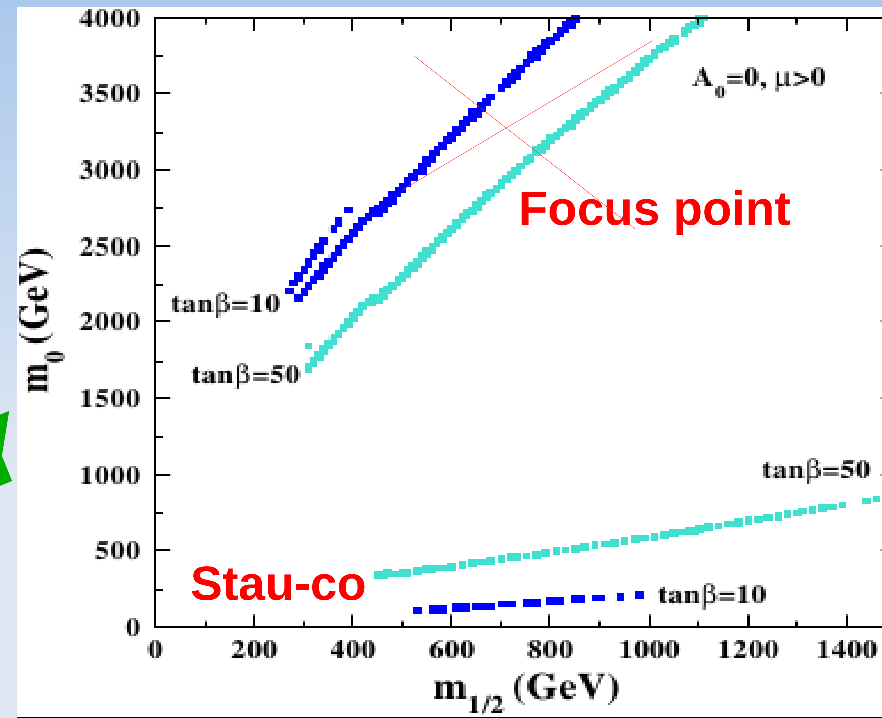
$$t^{SI}_A = \int_{E_1}^{E_2} \frac{dE_R}{\epsilon_0} F^2(E_R) \int_{v_{min}(E_R)}^{v_{max}} dv f(v)$$

Total event rate

$$R = \frac{\rho_0 v_0}{m_\chi m_A}\left(\sigma^{SI}_{(A)}(0) t^{SI}_A + \sigma^{SD}_{(A)}(0) t^{SD}_A\right)$$

Constrained MSSM: the stau co-annihilation region





$$\mathrm{WMAP}(3\sigma):\ 0.094 < \Omega h^2 < 0.128$$

$$a_n > 0$$

$$a_p < 0$$

$$\varrho = \mathrm{sgn}(a_p/a_n) = \pm 1$$

Can the relative sign between the SD amplitudes and the three cross sections be determined experimentally?

Spin matrix elements

$$\Omega_{p,n} = 2\sqrt{\frac{J+1}{J}}\langle \vec{S}_{p,n} \rangle$$

DAMA, KIMS
COUPP, ANAIS

$$^{127}\mathrm{I} \Longleftrightarrow A_1:\ \Omega_p^{127} = 0.731\ \ \Omega_n^{127} = 0.177$$

Ressell, Dean, PRC 56 535 (1997) (Bonn A potential)

The only nucleus with large protons and neutrons matrix elements

CDMS, COGENT
EDELWEISS

$$^{73}\mathrm{Ge} \Longleftrightarrow A_3:\ \Omega_p^{73} = 0.066\ \ \Omega_n^{73} = 0.836$$

Dimitrov, Engel, Pittel, PRD 51 291 (1995)

Protons contribution is negligible

COUPP, PICASSO
SIMPLE,DRIFT

$$^{19}\mathrm{F} \Longleftrightarrow A_2:\ \Omega_p^{19} = 1.646\ \ \Omega_n^{19} = -0.030$$

Divari, Kosmas, Vergados,Skouras PRC 61 054612 (2000)

Neutrons contribution negligible

WARNING!

^{19}F ($L_J = S_{1/2}$)	$\langle \mathbf{S}_p \rangle$	$\langle \mathbf{S}_n \rangle$	μ (in μ_N)
ISPSM, Ellis–Flores [28, 39]	1/2	0	2.793
OGM, Engel–Vogel [29]	0.46	0	$(2.629)_{\rm exp}$
EOGM ($g_A/g_V = 1$), Engel–Vogel [29]	0.415	−0.047	$(2.629)_{\rm exp}$
EOGM ($g_A/g_V = 1.25$), Engel–Vogel [29]	0.368	−0.001	$(2.629)_{\rm exp}$
SM, Pacheco-Strottman [14]	0.441	−0.109	
SM, Divari et al. [5]	0.4751	−0.0087	2.91

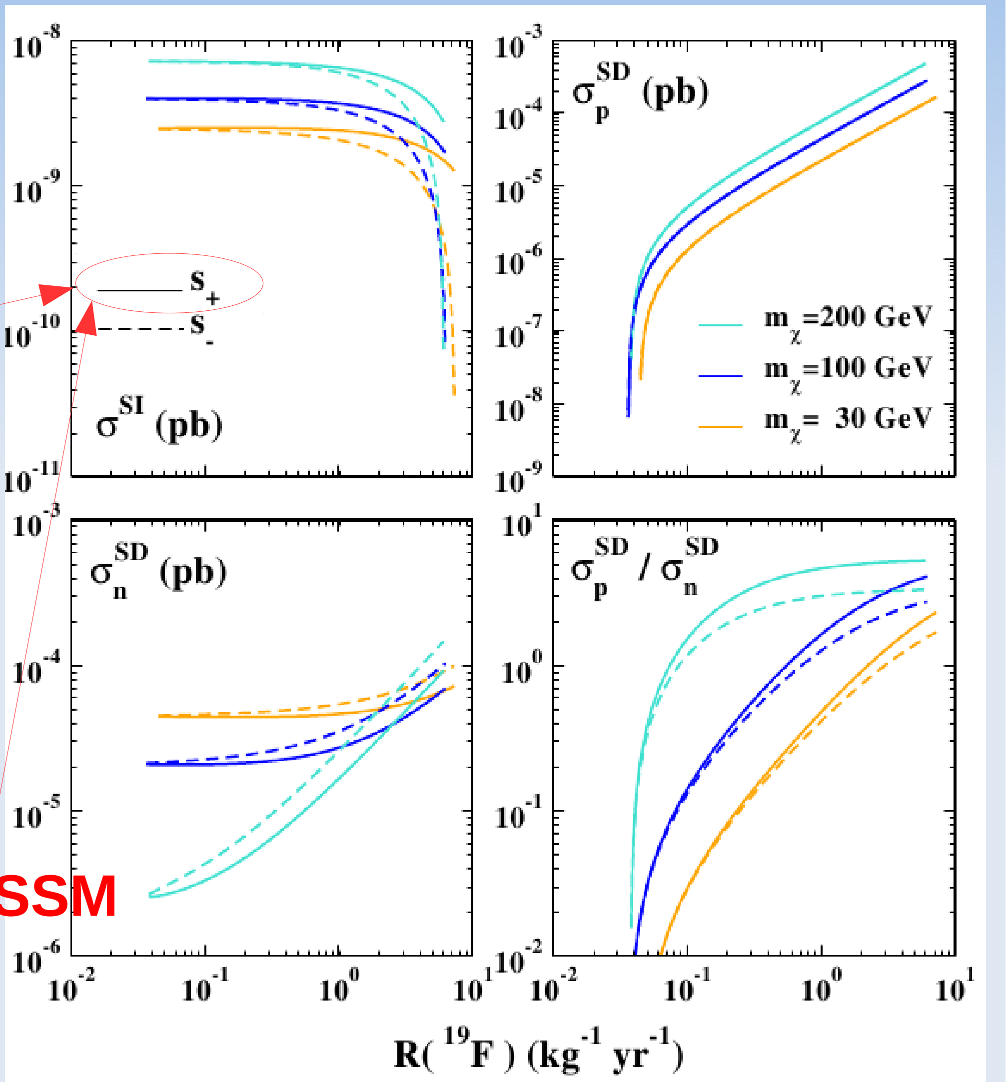
In some papers it is stated the opposite using the Pacheco-Strottman matrix elements.

The calculations of Divari, Kosmas, Vergados, Skouras are more accurate and do not support this conclusion.

Application (1): Cross sections from rates

$$\begin{cases} \text{I} \quad \sigma^{SI} + \mathcal{R}_{A_1}\left(\Omega_p^{A_1}\sqrt{\sigma_p^{SD}} + \varrho\,\Omega_n^{A_1}\sqrt{\sigma_n^{SD}}\right)^2 - \mathcal{S}_{A_1} = 0 \\ \text{F} \quad \sigma^{SI} + \mathcal{R}_{A_2}(\Omega_p^{A_2})^2\sigma_p^{SD} - \mathcal{S}_{A_2} = 0 \\ \text{Ge} \quad \sigma^{SI} + \mathcal{R}_{A_3}(\Omega_n^{A_3})^2\sigma_n^{SD} - \mathcal{S}_{A_3} = 0 \end{cases}$$

$$s_+ : \begin{cases} \sigma_+^{SI} = \dfrac{\mathcal{B}+\sqrt{\mathcal{B}^2-\mathcal{A}\mathcal{C}}}{\mathcal{A}} \\ \sigma_{p,+}^{SD} = \dfrac{\mathcal{S}_{A_2}-\sigma_+^{SI}}{\mathcal{R}_{A_2}(\Omega_p^{A_2})^2} \\ \sigma_{n,+}^{SD} = \dfrac{\mathcal{S}_{A_3}-\sigma_+^{SI}}{\mathcal{R}_{A_3}(\Omega_n^{A_3})^2} \end{cases} \qquad s_- : \begin{cases} \sigma_-^{SI} = \dfrac{\mathcal{B}-\sqrt{\mathcal{B}^2-\mathcal{A}\mathcal{C}}}{\mathcal{A}} \\ \sigma_{p,-}^{SD} = \dfrac{\mathcal{S}_{A_2}-\sigma_-^{SI}}{\mathcal{R}_{A_2}(\Omega_p^{A_2})^2} \\ \sigma_{n,-}^{SD} = \dfrac{\mathcal{S}_{A_3}-\sigma_-^{SI}}{\mathcal{R}_{A_3}(\Omega_n^{A_3})^2} \end{cases}$$



In the case of I, F, Ge, the solutions are connected with the relative sign between the proton and neutron amplitudes appearing in the equation for I

CMSSM

$\varrho = +1 \Leftrightarrow s_-$ $\qquad$ $\varrho = -1 \Leftrightarrow s_+$

$R(^{127}\text{I}) = R(^{73}\text{Ge}) = 1\ \text{kg}^{-1}\ \text{y}^{-1}$

Application (2):Model independent UL are...naturally model independent!

Suppose we can neglect the SI rate

$$R^{\rm SD} = \phi_A \left(\mathcal{C}_A^p \sqrt{\sigma_p^{\rm SD}} \pm \mathcal{C}_A^n \sqrt{\sigma_n^{\rm SD}} \right)^2 t_A^{\rm SD}$$

$$\phi_A = \frac{\rho_0 v_0}{m_\chi m_A}$$

$$\mathcal{C}_A^{p,n} = \frac{\mu_A}{\mu_p} \frac{\Omega_{p,n}^A}{\sqrt{3}}$$

If an experiment with exposure ε sets an upper limit on the number of events then:

$$R^{SD} \times \mathcal{E}_A < N^{UL}$$

$$\left(\mathcal{C}_A^p \sqrt{\sigma_p^{\rm SD}} \pm \mathcal{C}_A^n \sqrt{\sigma_n^{\rm SD}} \right)^2 < \frac{N^{UL}}{\phi_A t_A^{\rm SD} \mathcal{E}_A} = \sigma_A^{\rm lim} \equiv \frac{N^{UL}}{\phi_A t_A^{\rm SD} \mathcal{E}_A},$$

Define also

$$\sigma_{p,n}^{\rm lim(A)} \equiv \frac{\sigma_A^{\rm lim}}{(\mathcal{C}_A^{p,n})^2}$$

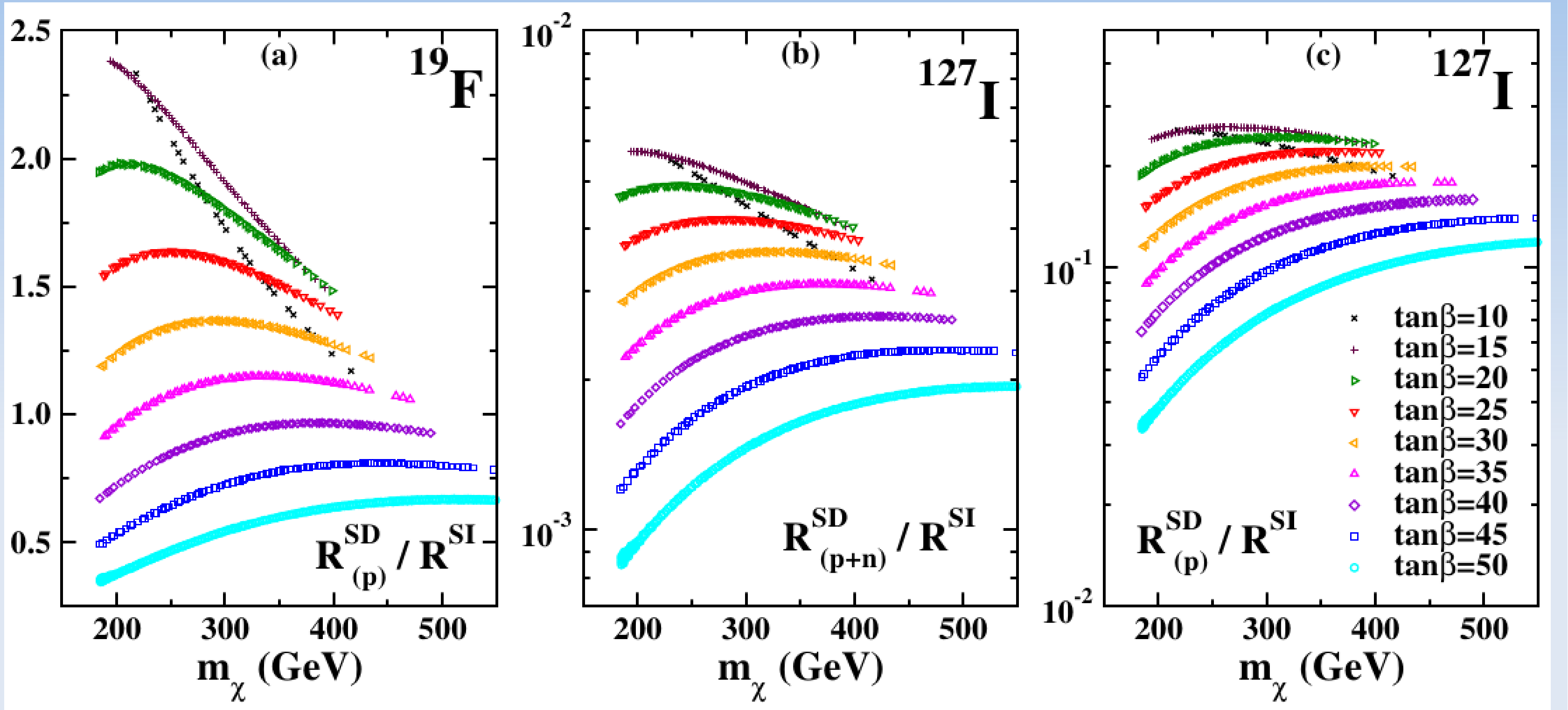
$$\left(\frac{\sqrt{\sigma_p^{\rm SD}}}{\sqrt{\sigma_p^{\rm lim(A)}}} \pm \frac{\sqrt{\sigma_n^{\rm SD}}}{\sqrt{\sigma_n^{\rm lim(A)}}} \right)^2 < 1$$

1) The formula of Tovey et al. PLB 488 (2000) follows naturally from the factorization.
   The model independent method... is not a method, after all.
2) Not other assumptions are needed to derive it. The correct nuclear physics is taken into account

Fluorine, Iodine and the stau co-annihilation region

$$\sigma_{p,n}^{\lim(A)} \equiv \frac{\sigma_A^{\lim}}{(C_A^{p,n})^2}$$

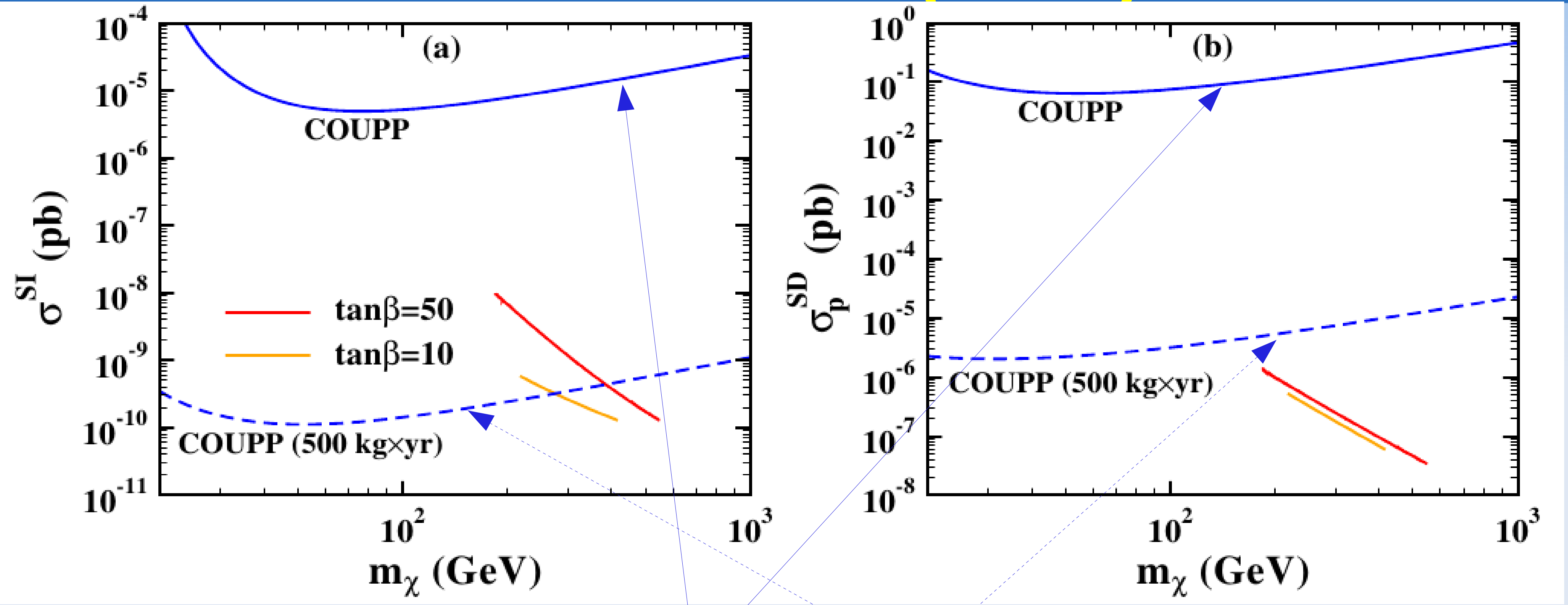
Are these "universal" upper limits?



1) In Fluorine the SI rate is never negligible: bigger than the SD for tanβ>35 and smaller for tanβ<35. Limits on the proton SD cross section are obtained assuming no SI rate....

2) In Iodine there is a cancellation in the SD rate that is two orders of magnitude smaller than the SI: the limits on the single nucleon SD cross sections derived by Iodine are meaningless for this model

The stau co-annihilation ragion and COUPP (CF_3I)



COUPP Collaboration PRL 106 (2011)

$\mathcal{E} = 28.1$ kg×days $\quad N^{UL}_{90\%\text{C.L.}} = 6.7$

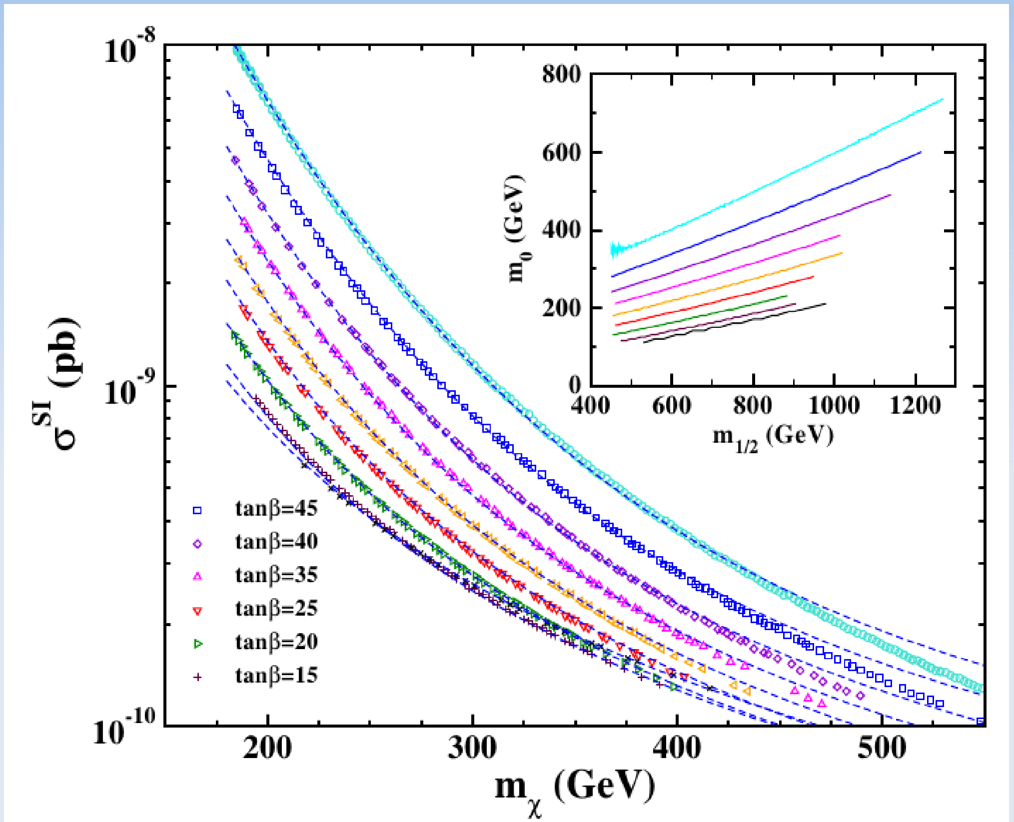
50% efficiency $\quad E_{th} = 21$ keV

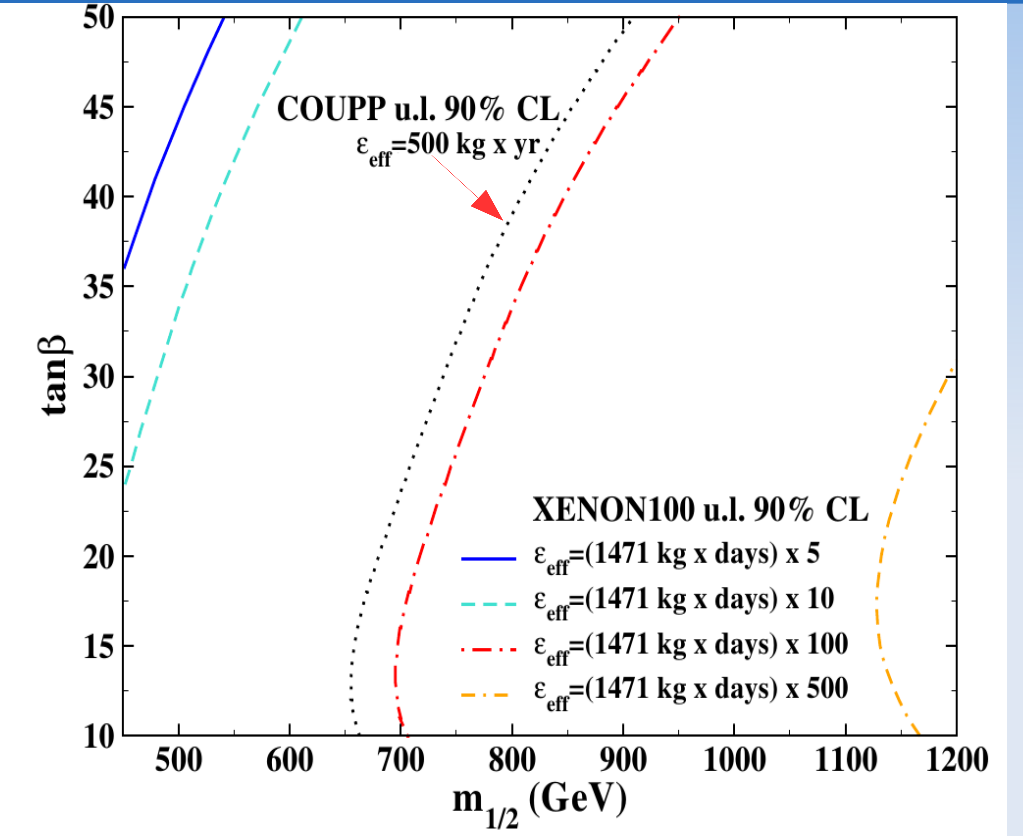
$\mathcal{E} = 500$ kg×year $\quad N^{UL}_{90\%\text{C.L.}} = 6.7$

100% efficiency $\quad E_{th} = 7$ keV

COUPP can probe the model only through SI interaction with Iodine but not through the SD interaction with Fluorine

Constraining directly the ($m_{1/2}$,tanβ) plane





Fit as a function of tanβ and $m_{1/2}$

$$\sigma^{SI}(\tan\beta, m_\chi) = \sum_{k=2}^{4}\left[\sum_{i=0}^{3}\sigma_{ki}\,(\tan\beta)^i\left(\frac{100\ \mathrm{GeV}}{m_\chi}\right)^k\right]$$

$$m_\chi \simeq 0.44\, m_{1/2} - 15\ \mathrm{GeV}$$

Translate an upper limit N_{UL} directly into a constrain on the ($m_{1/2}$ – tanβ) plane